Quantitative Macroeconomics – Homework 3

Transitions in a Representative Agent Economy

The model:

The infinitely lived large number of households want to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\}$$,where $u(c_t) = \ln c_t$

Subject to:

$$c_t + i_t = y_t$$

$$y_t = k_t^{1-\theta} \ (z\ h_t)^{\theta}$$

$$i_t = k_{t+1} - (1-\delta)\ k_t$$

Part 1.a

In order to obtain the given capital-output ratio, $\frac{k_t}{y_t} = 4$, and investment-output ratio, $\frac{i_t}{y_t}$=0.25, we are going to normalize output to one. Therefore at the first steady state capital (k) will be equal to 4, and investment (i) will equalize 0.25. From the value of investment and capital at steady state we obtain δ=1/16.

From the production function and for an initial guess we obtain the value of the productivity parameter, z, which matches the given ratios. Once we obtain the value of z, we compute the steady state levels of output and consumption.

Finally, making use of the Euler equation at the steady state, we obtain the discount factor β.

The initial steady state equilibrium values are the following:

Steady State Equilibrium	
z	1,6297
k	4
y	1
i	0,25
c	0,75
β	0,9803

Part 1.b

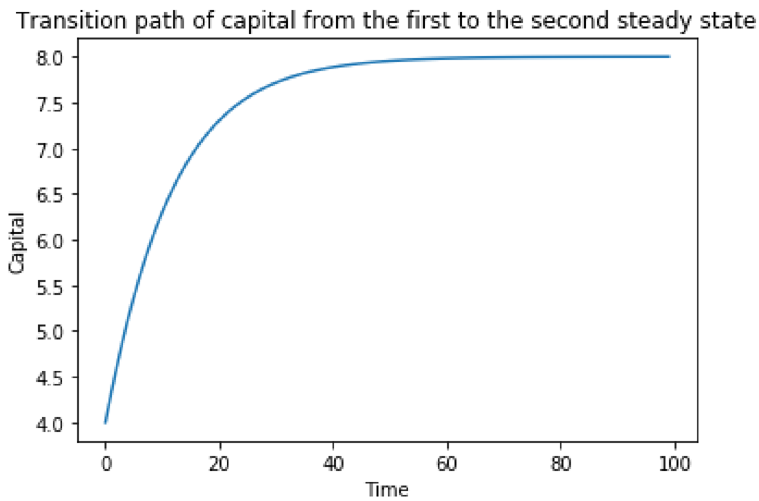
Doubling the productivity level lead to a new steady state equilibrium in which levels of capital, investment, output and consumption are also the double of the level in the first steady state. Therefore, the new steady state equilibrium values are the following:

New Steady State Equilibrium	
z	3,259
k	8
y	2
i	0,5
c	1,5
β	0,9803

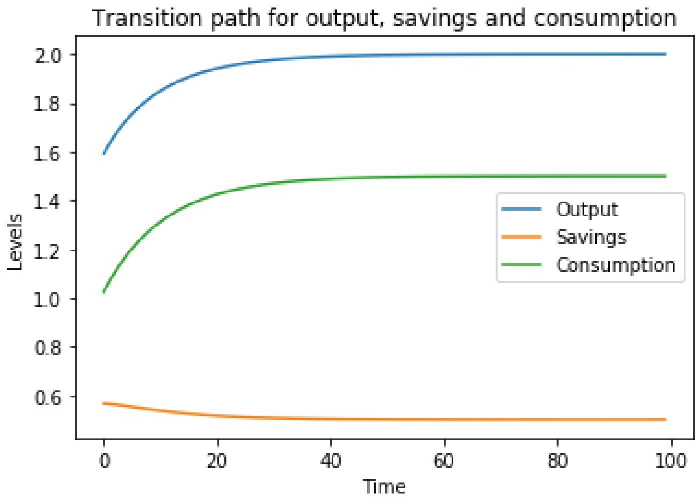
Part 1.c

In order to compute the transition of capital from the first to the second steady state, we solve a system of Euler equations for a chosen horizon of 100 periods. The initial value of capital is set equal to 4 and the last one equal to 8.

In the following graph we see the time-path for capital:



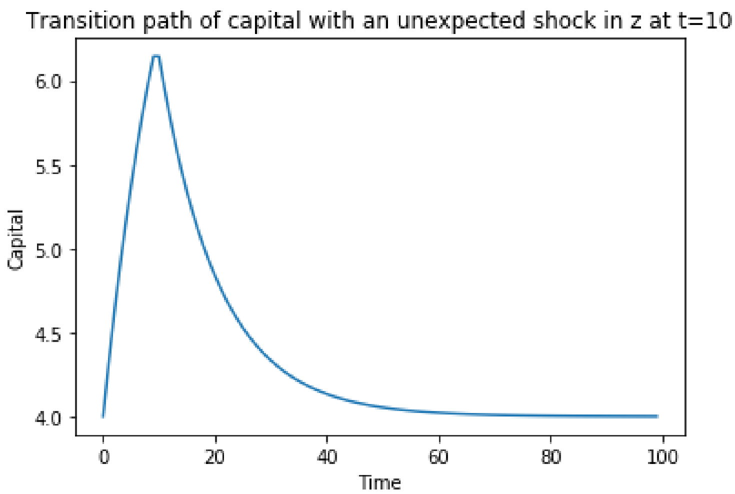
Once we have the transition path for capital, we can compute the transition paths for output, savings and consumption. The graph below shows the time-path of these three variables:



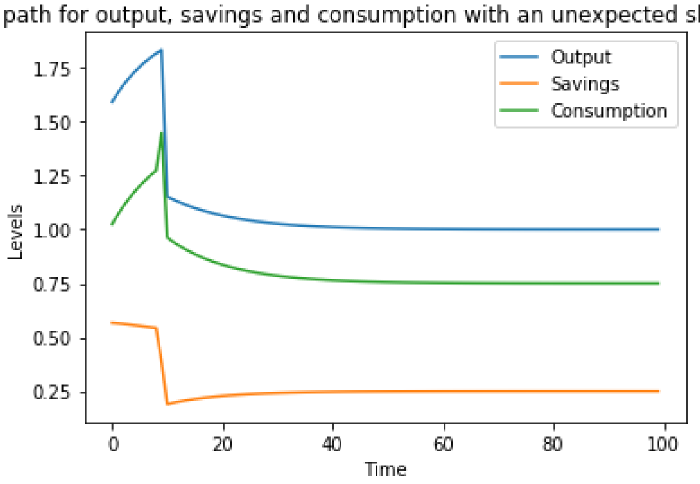
Part 1.d

In this case, the economy starts the transition to the second steady state equilibrium, since agents expect that labour productivity is doubled for all periods. However, at t=10 an unexpected shock occurs and the labour productivity goes back to the original value, so, therefore, the economy will adjust in order to obtain the first steady state equilibrium. Due to this fact, we expect that we will observe a change of trajectory from period 10.

The transition path for capital, with an unexpected labour productivity shock at period 10 is the following:



The graph below shows the transition path for output, savings and consumption:



A multicountry model with free mobility of capital and progressive labor income tax

<u>The model:</u>

- Multicountry static model, for 2 different countries: $l = \{1,2\}$.
- Agents are heterogeneous in their permanent labor productivity η and face uncertainty on their wage
- Each country produces a single good, with a representative CRS firm (competitive markets)
- Labor income taxation:

$$T_y(y) = y\left(1 - \lambda_y y^{-\phi_y}\right)$$

- Agent problem:

$$\max_{\{c_\ell, k_\ell \in [0, \bar{k}_\ell], h_\ell \in [0,1]\}} \left(\frac{(c_\ell)^{1-\sigma}}{1-\sigma} - \kappa \frac{(h_\ell)^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}} \right)$$

s. t

$$c = \lambda \left(w_\ell \left(h_\ell\right) \eta_\ell\right)^{1-\phi_\ell} + r_\ell k_\ell^\eta + r_{-\ell}\left(\bar{k}_\ell - k_\ell\right)$$

- Firm problem:

$$\max_{\{K_\ell^d, H_\ell^d\}} Z\left(K_\ell^d\right)^{1-\theta}\left(H_\ell^d\right)^\theta - w_\ell H_\ell^d - r_\ell K_\ell^d$$

For simplicity, let us assume that in each country exist a representative household with low productivity and another representative household with high productivity. Since the country capital endowment sums up to 2 in each country, I assume that each type of household in each country has an initial capital endowment of 1.

Part 2.a

In order to solve the equilibrium for a closed economy, we construct a system of equations including Euler equations, the households budget constraints and the first order conditions of the firms' problem. The Euler equation and the Budget constraint will be different for each country and for each type of household (high or low productive), whereas the FOCs for the firm problem are different for each country. Therefore, for the equilibrium of each country, we have a system of six equations for the six unknowns we need to solve: high and low consumption, high and low labor supply, rate of return of capital and wage.

In the following equations, the subindex l denotes the country A or B and the supindex *i* denotes the type of household (high or low productive).

Euler equation:

$$\kappa_l \left(h_l^i\right)^{1/v} = c^{-\sigma} \lambda_l \,(1-\varphi_l\,)(w_l \eta_l^i)(w_l\, h_l^i \eta_l^i)^{-\varphi_l}$$

Budget constraint:

$$c_l^i = \lambda_l\,(w_l\, h_l^i \eta_l^i)^{1-\varphi_l} + r_l\, k_l^{\eta_l^i}$$

Firm FOCs:

$$Z_l(1-\theta_l\,)K_l^{-\theta} H_l^{\theta} = r_l$$

$$Z_l \theta_l\, K_l^{1-\theta} H_l^{\theta-1} = w_l$$

The solution to the system requires aggregating capital and labor, as follows:

$$K_l = 2$$

$$H_l = h_l^h \eta_l^h + h_l^l \eta_l^l$$

The closed economy equilibrium of each country for the given parameters is shown in the following table:

Variable	*Country A*	*Country B*
c^L	0.483	0.849
c^H	1.476	0.999
h^L	0.151	0.309
h^H	0.355	0.314
w	0.599	0.611
r	0.403	0.384

In both countries, consumption and labor supply are higher for high productive households, but since in country B differences in productivity between agents are minor, the differences in this country in consumption and labor supply between high and low agents are less than in country A. Wages are higher than the return of capital in both countries.

Part 2.b

In order to solve the equilibrium of the union economy, we need to change the total capital endowment, since we have free mobility of capital. Therefore we need to add to the budget constraint the foreign investment on capital since this also is a source of income. This also has an effect on the aggregation of capital.

New Budget Constraint:

$$c_l^i = \lambda_l (w_l h_l^i \eta_l^i)^{1-\varphi_l} + r_l \eta k_l^{\eta_l^i} + r_{-l} (\overline{k_l} - k_l)$$

Now we will have an additional optimality condition:

$$r_l \eta_l^i k_l^{\eta_l^i - 1} = r_{-l}$$

New aggregation of capital:

$$K_l = k_l^l + k_l^h + \left(\bar{k}_{-l}^{\,l} - k_{-l}^l\right) + \left(\bar{k}_{-l}^{\,h} - k_{-l}^h\right)$$

For solve for the union equilibria, we use these equations together with the equation we have used to solve the equilibria for closed economies. That is, we have a system of 16 equations for 16 unknowns. The equation is formed by the following equations for each country: Euler low types, Euler high types, FOCs of the firm, Euler equation for capital for high and low types, and Budget Constraint for high and low types.

Variable	*Country A*	*Country B*
Consumption h-type	1.24	0.91
Consumption l-type	0.57	0.76
Labor supply h-type	0.38	0.34
Labor supply l-type	0.13	0.31
Domestic capital supply h-type	0.65	0.66
Domestic capital supply l-type	0.38	0.62
Wage	0.55	0.63
Rate of return on capital	0.45	0.37

Part 2.c

In this world government taxes labor income according to Feldestein tax function:

$$T(y) = y(1 - \lambda y^{-\varphi})$$

where the parameter φ determines the degree of progressivity.

Introducing a labor income taxation will change the agents budget constraint to the following:

$$c_l^i = \lambda_l \,(1 - T(y))(w_l\, h_l^i \eta_l^i)^{1-\varphi_l} + r_l\, \eta k_l^{\eta_l^i} + r_{-l}\, (\overline{k_l} - k_l\,)$$

Labor income is, in this case, y = $w_l\, h_l^i \eta_l^i$

In order to solve the optimal progressive taxation of labor income, we first solve the individual household's problem, for a given φ.

After solving this problem by the algorithm proposed in the above exercises, we need to solve for the optimal level of progressivity by solving the maximization problem of the government, which chooses the optimal φ by maximizing the social welfare function:

$$SWF = \omega_1\, v_A^l(c,h) + \omega_2\, v_A^h(c,h) + \omega_3\, v_B^l(c,h) + \omega_4\, v_B^h(c,h)$$

where ω are the Pareto weights and the utilities are the result of the problem of the agents.