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eISBN: 978-1-937707-30-9

Layout Design: Dan McNaney and Cathy Huang
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At Manhattan Prep, we continually aspire to provide the best instructors and resources possible. We hope that you will find our commitment manifest in this book. If you have any questions or comments, please email me at dgonzalez@manhattanprep.com. I'll look forward to reading your comments, and I'll be sure to pass them along to our curriculum team.

Thanks again, and best of luck preparing for the GRE!

Sincerely,

Dan

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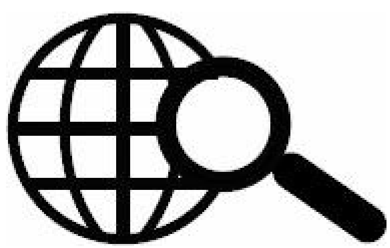

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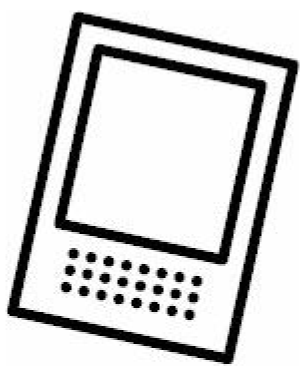

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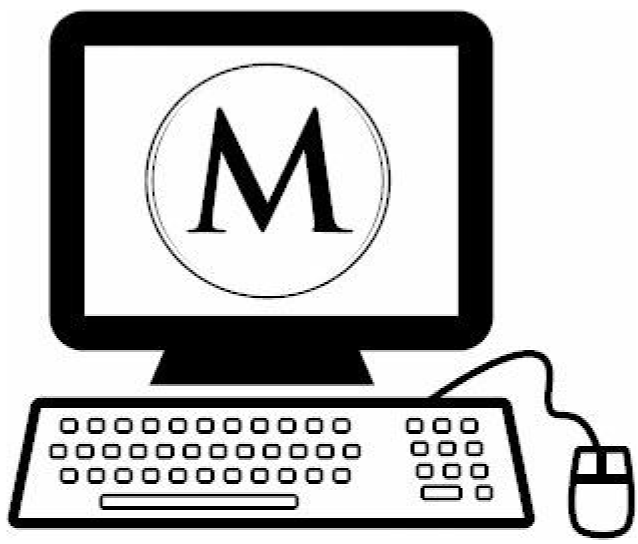

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Chapter 1
of

Verbal Diagnostic Test

In This Chapter…

Verbal Diagnostic Test

20 Questions
Time: 30 Minutes

For questions 1–6, select one entry for each blank from the corresponding column of choices. Fill in the blank in the way that best completes the text.

---

1. In interviews, despots are often surprisingly ____________; this helps to explain how seemingly awful people are able to command so many followers.

malign
indignant
forgiving
personable
munificent

2. The thriving health food company sells ___________ meat products so meat-like that vegetarians sometimes call the phone number on the box to make sure that the product is really animal-free.

mendacious
nugatory
ersatz
mimetic
clandestine

3. The successful tech company faces an ironic problem in the fall quarter; people are so excited about the next (i) _________ of its product, which will be released over the holidays, that they refuse to purchase the (ii) __________ version.

Blank (i)	Blank (ii)
ingenuity	obsolete
implication	current
iteration	practical

4. Newborn babies are perfectly (i) ______________, as yet unable to be concerned for others, or even to understand a difference between themselves and the world around them. As young children mature, they make the (ii) _______________ discovery that other people exist and have their own needs and desires — in other words, that the entire world is not about them.

Blank (i)	Blank (ii)
solipsistic	arresting
sophomoric	selfish
quixotic	undue

5. Historically, arguments against women's suffrage (i) ____________ from the claim that women would cancel out their husbands' votes to the charge that women would merely (ii) ____________ their husbands' preferences, thus making their votes redundant. Such arguments, while once convincing, today seem (iii) ___________ indeed.

Blank (i)	Blank (ii)	Blank (iii)
ran the gauntlet	override	ponderous
ran the gamut	ape	shabby
held the line	disclaim	cogent

6. His theory purported that "proper" enjoyment of art was a matter of pure aesthetics — it is surely, he says, a baser pleasure being enjoyed by the untrained (i) __________, the museum "tourist" with (ii) __________ sensibilities, and even the art theorists and art historians who simply appreciate cultural referents or narratives in art, a predilection he thinks leads to (iii) __________ view of any art that includes such elements.

Blank (i)	Blank (ii)	Blank (iii)
cabal	incendiary	a facile
literati	parochial	an urbane
hoi-polloi	dulcet	a painstaking

Question 7 is based on the following reading passage.

During an economic depression, it is common for food prices to increase even as incomes decrease. Surprisingly, however, researchers determined that during a depression, for every 5 percent increase in the cost of bread, the lowest socioeconomic class actually increases the amount of bread purchased per capita by 3 percent.

7. Which of the following hypotheses best accounts for the researchers' findings?

(A) Not all food costs increase during a depression; some food items actually become less expensive.
(B) Because bread consumption does not increase by the same percentage as the cost does, people are likely consuming more of other food items to compensate.
(C) When incomes decrease, people are typically forced to spend a larger proportion of their income on basic needs, such as food and housing.
(D) People who suddenly cannot afford more expensive foods, such as meat, must compensate by consuming more inexpensive foods, such as grains.
(E) During a depression, people in the lowest socioeconomic class will continue to spend the same amount of money on food as they did before the depression began.

Question 8 is based on the following reading passage.

Bedbug infestations have been a problem in major cities for years. The pesticide DDT has been found to be useful in killing bedbugs. However, DDT was banned in the United States and has been replaced by weaker pesticides. Thus, there is no effective means for eradicating bedbugs in the United States.

8. Which of the following, if true, most weakens the conclusion?

(A) Bedbugs resemble other small insects in their appearance and behavior.
(B) Bedbugs have largely been eradicated in other parts of the world.
(C) Some treatments that do not include DDT have proven effective against bedbugs in other parts of the world.
(D) Bedbugs are resistant to the types of pesticides used to treat cockroach and ant infestations.
(E) The number of bedbug infestations has risen significantly in the 21st century.

Questions 9–12 are based on the following reading passage.

Dan Flavin's *the alternate diagonals of March 2, 1964 (to Don Judd)*, an 8-foot-long diagonal beam of light set at a 45-degree angle, is a colorful sculpture of light that is visually arresting, even from across the room. As one approaches the work, it is difficult not to become almost blinded by the intensity of the light and the vivacity of the colors. Though it may strike one as garish on first glance, a more lengthy perusal reveals a delicate interplay between the red and yellow beams, giving the work a visual richness.

Alternate diagonals was made by Flavin in response to one of his own previous works, *the diagonal of May 25, 1963 (to Constantin Brancusi)*. His first piece composed solely of light, *the diagonal of May 25, 1963* was also an 8-foot-long fluorescent light sculpture (though Flavin never liked to call them sculptures — he referred to them as "situations") hung at a 45-degree angle, and also included a yellow fluorescent light tube. *Alternate diagonals* seems almost more of an evolution of the former work than a response to it, but regardless of the exact nature of the intended interplay between the two, it is important to frame *alternate diagonals* as a companion work.

Alternate diagonals is a kind of ready-made, entirely made of fluorescent lights that anyone could find in any hardware store and construct as Flavin has. This is precisely what is so intriguing about the work — it toys with the boundaries of what we can define as a ready-made in contemporary art and, perhaps, within the field of art production itself. It forces a spectrum to be employed instead of a black-and-white categorization of the ready-made — a spectrum stretching between the "pure" ready-made (any work that essentially could be transferred straight from anyone's garage to a gallery, such as Duchamp's *Bottle Rack*), all the way to a contemporary two-dimensional work where the artist's canvas and paints were purchased from an art supply store in an infinitely more manipulated but still semi-"readymade" fashion. Flavin's piece, it seems, is situated somewhere in the center of such a spectrum, and raises the question of where the "ready" ends and the "made" begins.

9. The main point of the passage is to

(A) assert the superiority of ready-made art
(B) decry the broadening of the definition of art

(C) discuss a work in context and its effect on the discipline
(D) explain the relationship between two works of art
(E) praise an artist and his creations

10. According to the passage, both "diagonal" works could best be described as

(A) using red and yellow light
(B) initially striking the viewer as garish
(C) toying with boundaries of art
(D) running through the plane at a particular slope
(E) identical in concept

11. The author's tone could best be described as

(A) admiring and supportive
(B) enthusiastic and fawning
(C) respectful and distant
(D) obligatory and unenthused
(E) erudite and objective

12. The passage implies which of the following?

(A) Conventional two-dimensional work is a thing of the past.
(B) Flavin is one of the most important artists of his time.
(C) *Bottle Rack* has very little artistic manipulation.
(D) Flavin disliked the word *sculpture* because of the Renaissance association.
(E) The best art work is in the center of the artistic spectrum of art production.

For questions 13–16, select the two answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole and produce completed sentences that are alike in meaning.

---

13. Many young employees actively seek out mentors, but when managers attempt to mentor young employees who have not sought out mentoring, the help often comes across as presumptuous and _______________.

☐ baneful
☐ noxious
☐ patronizing
☐ amenable
☐ pragmatic
☐ condescending

14. In previous decades, it was simply assumed that fathers of young children would work full-time and at the same intensity as they did before becoming parents, but today, increasing numbers of men — wanting to further their careers but also wanting to spend time with their children — are more ________________ about this arrangement.

☐ stolid
☐ ambiguous
☐ whimsical

☐ ambivalent
☐ equivocal
☐ officious

15. The slipstream is a partial vacuum created in the wake of a moving vehicle that allows for “drafting,” whereby a racecar can ___________ another by taking advantage of reduced wind resistance.

☐ supersede
☐ assume
☐ overtake
☐ champion
☐ collide
☐ outstrip

16. While the celebration of the artist’s work was pure paean — nothing but plaudits and tributes — many of those in the art world feel that an acknowledgment of the artist’s __________ would help to humanize the artist and make the art more accessible.

☐ faculties
☐ poultices
☐ foibles
☐ mores
☐ aptitudes
☐ peccadilloes

Questions 17 and 18 are based on the following reading passage.

In keeping with the notable incorporation of operatic elements into the rock music lexicon, the genre in the 1970’s experienced a significant shift in emphasis away from recording and toward music performance itself. Several factors effected this change. First, the extended length and the moralizing subject matter of songs of the era rendered them less appropriate for radio play and more suitable for public presentation. Additionally, the advent of the concept album, in which multiple tracks revolved around a single unifying narrative or theme, furnished a basis upon which similarly calibrated performances could be enacted. Finally, as PA system technology improved, it became possible to hold concerts with 100,000 people or more, which encouraged artists to craft concerts that diverged from the merely musical toward the experiential. Bands began conceiving of their performances as *shows*, more akin to musical theater guided by plot and setting than to the traditional concert guided by the omnipresent set-list. Instead of simply playing one song after another, therefore, bands developed full performance medleys revolving around specific motifs, and punctuated by bombastic light shows, costume changes, and other massive stunts. For example, the popular British band Pink Floyd famously built a barrier in the middle of stage during one performance then had it dramatically knocked down mid-show as a promotion for their new album *The Wall*.

17. The passage cites all of the following as reasons for the inclusion of operatic elements into the rock music genre in the 1970’s EXCEPT:

(A) Changes in concert equipment changed the way shows could be delivered.

(B) Pink Floyd's dramatic staging served to promote an influential album.
(C) Shifts in the content of the music contributed to the songs having a more "story-telling" quality.
(D) One medium through which rock songs had typically been delivered to the public became less conducive to the genre.
(E) Artists altered their performances to reflect changes in the modern concert ambiance.

18. In the context of the passage, the word *effected* (line 3) most nearly means

(A) influenced
(B) moved forward
(C) transformed
(D) determined
(E) brought about

Questions 19 and 20 are based on the following reading passage.

In 1977, the Community Reinvestment Act was passed for the purpose of reducing discriminatory credit practices in low-income neighborhoods. The act required Federal financing supervisory agencies to use their authority to encourage lending institutions to meet the credit needs of all borrowers in their communities. The CRA had little impact until 1993, when the Department of Housing and Urban Development (HUD) initiated legal proceedings against lenders who declined too many applications from minority borrowers.

Some argue that, while providing equal access to credit is an important aim, pressure on lenders from HUD led to practices that later caused those same lenders to be assailed as "predatory." In *Housing Boom and Bust*, economist Thomas Sowell wrote that the CRA, far from being as benign as it appeared, was based on a flawed assumption: that government officials were qualified to tell banks how to lend the money entrusted to them by depositors and investors.

Consider each of the answer choices separately and indicate all that apply.

---

19. It can be inferred from the passage that

☐ the CRA was perceived by some as seemingly innocuous
☐ HUD had the ability to affect banking practices
☐ Thomas Sowell has asserted that prerequisites exist for the act of directing banks' lending activities

---

Consider each of the answer choices separately and indicate all that apply.

---

20. Which of the following can be inferred about discriminatory credit practices?

☐ They are more common in middle-income neighborhoods than in high-income neighborhoods.
☐ Legislators who voted for the CRA did so because they were opposed to such practices.
☐ Critics allege that attempts to combat such practices can have unintended consequences.

Verbal Diagnostic Test: Answer Tally Sheet

Question #	Your Answer	Correct Answer	Tally your correctly answered questions. *No partial credit!*	Find more questions like this in chapter:
1		Personable		3
2		Ersatz		3
3		Iteration, current		3
4		Solipsistic, arresting		3
5		Ran the gamut, ape, shabby		3
6		Hoi-polloi, parochial, facile		3
7		(D)		6
8		(C)		6
9		(C)		5
10		(D)		5
11		(A)		5
12		(C)		5
13		Patronizing, condescending		4
14		Ambivalent, equivocal		4
15		Overtake, outstrip		4
16		Foibles, peccadilloes		4
17		(B)		5
18		(E)		5
19		I, II, and III		5
20		III only		5
	Raw Verbal Score: (# of verbal questions answered correctly)			

Verbal Diagnostic Test: Scoring Guide

Raw Verbal Score	Verbal Reasoning Diagnostic Scaled Score
20	169 – 170
19	168 – 170
18	166 – 168
17	163 – 165
16	161 – 163
15	159 – 161
14	157 – 159
13	156 – 158
12	154 – 156
11	152 – 154
10	150 – 152
9	149 – 151
8	147 – 149
7	145 – 147
6	143 – 145
5	140 – 142
4	137 – 139
3	133 – 136
2	130 – 132
1	130 – 131
0	130

NOTES:

Diagnostic Scaled Score is approximate.

(a) If your time for this diagnostic section exceeded the 30-minute guideline, this approximate score may not be indicative of your performance under standard time conditions.
(b) Scaled Score depends not only on how many ques-tions were answered correctly (Raw Score), but also on the overall difficulty of the set of questions. This diagnostic test approximates the difficulty of the official GRE® revised General Test.

Verbal Diagnostic Test Answers

1. **Personable.** Because you are told that something explains how awful people *command so many followers*, you need a positive word in the blank. While both *forgiving* and *munificent* (meaning "generous") are positive, neither of them make much sense. *Personable*, meaning "pleasant in appearance and manner," fits best.

2. **Ersatz.** It is clear from "vegetarians" and "animal-free" that the meat products are fake, or *ersatz*. Note the many trap answers: *mendacious* (lying), *nugatory* (without value), *mimetic* (copying), and *clandestine* (hidden), all of which are inappropriate to describe fake meat.

3. **Iteration, current.** This sentence is clearly comparing two "versions" of a product. For the first blank, you need a synonym of "version," such as *iteration*. For the second blank, you're contrasting something to the "next iteration," which would be the *previous* one. *Current* works well here (and *obsolete* is too judgmental; it's not *obsolete* yet!).

4. **Solipsistic, arresting.** *Solipsism* is literally the theory that only the self exists or can be known to exist (*solipsistic* is sometimes used a bit figuratively to mean "really selfish"). For the second blank, the "discovery" in question is that *the entire world is not about them*. That's a pretty earth-shaking discovery for someone who thought otherwise! *Arresting* gets this across. Note the trap answer *selfish,* which is related to the overall meaning of the sentence but does not describe the "discovery."

5. **Ran the gamut, ape, shabby.** In the first blank, you need something that means "ranged." Your clue is the two very different beliefs described — that women's votes would be the opposite of their husbands', or that the votes would be the same and therefore "redundant." *Ran the gamut* gets this across (much like the expression "from A to Z"). In the second blank, *ape* means "mimic" or "copy." In the third blank, you need something that goes against "convincing." *Shabby* means "old and worn-out." It can describe clothes or furniture, but is often used metaphorically (such as in the expression "not too shabby," which can describe any job well-done). None of the other choices oppose *convincing*.

6. **Hoi-polloi, parochial, facile.** The theory being described is a theory of incredible snobbery! First of all, the person described thinks that there is a "proper" enjoyment of art (and, presumably, an improper one as well). He thinks that untrained people viewing art enjoy only "baser pleasures," and calls other people "museum tourists." In the first blank, *hoi-polloi* matches the idea of common people, the masses. Regarding the museum "tourist," *parochial* matches the idea of ordinary, low-class, unsophisticated. Similarly, the art theorists and art historians are thought to have a *facile,* or overly simple, view. The theorist seems to think that only *aesthetic* (relating to beauty) elements should matter; the theorists and historians he disdains are those who like "cultural referents or narratives" — that is, recognizable themes or figures, or stories.

7. **(D).** This "reading comprehension" question is really a logic question. Such questions typically consist of a single paragraph with one question. First, analyze the argument: During a depression, it is normal for food prices to increase at the same time that incomes decrease. Logically, this would make it more difficult for people to afford the same food that they used to purchase prior to the depression. A study showed a surprising result, however: when the cost of bread went up during a depression, the poorest people actually bought more bread. Note that the argument doesn't say merely that more money is spent on bread; that would be expected if the price increased. The argument says that the actual amount of bread purchased increased. The correct answer will explain why people would buy more bread even though the cost has gone up and incomes have declined.

While choice (A) is likely true in the real world, it does not explain why people buy more bread when the cost of bread has increased and incomes have declined. Choice (B) is an example of faulty logic. It is true that the cost increase is a higher percentage than the consumption increase, but this does not mean that people are consuming less bread and therefore need to eat other things to compensate. In fact, the opposite is true: the argument explicitly states that people are buying more bread than they were! (C) is tempting because it talks about people spending a "larger" proportion of income on food — but "proportion" is a value relative only to the person's income level. It does not indicate that the person is spending more money on something. More importantly, though, this choice does not answer the question asked. Correct choice (D), in contrast, provides a reason why an increase in the cost of one food item might cause people to consume more of that item despite a loss of income: other food items are even more expensive and are, thus, much less affordable. The people still need some amount of food to survive, so they purchase more of the food item that does not cost as much money. This accounts for the researchers' findings. Even if (E) were true (and this would be difficult if incomes are decreasing), it would not explain why people buy more bread at a time when the bread costs more and incomes are declining.

8. **(C).** This is a logic question that asks you to weaken an argument. The author cites a problem, bedbugs, and a pesticide that is effective in killing them is no longer available for use. Thus, the author claims, there is no effective means to kill the bedbugs in the United States. The problem here is that the author makes a very large jump between *the best method* no longer being an option and there being *no effective method*. Finding something that attacks this assumption would be a good way to weaken the argument. Regarding choice (A), the behaviors of bedbugs have nothing to do with the ability to eradicate them. Choice (B) does not specify how bedbugs were eradicated in other parts of the world. It is possible that all successful eradications elsewhere relied on DDT. Correct choice (C) opens up the possibility that there might be an approved means to eradicate the dreaded bedbug in the United States. It also calls the author's conclusion about there being *no effective means in the United States* into question. Choice (D), though largely out of scope, might be viewed as strengthening the author's conclusion, as it points to more evidence that existing pesticides are not effective on bedbugs. However, eliminating one possibility is not the same as eliminating *all* possibilities, which is what the author does in the conclusion. As for choice (E), the number of infestations has nothing to do with the ability to eliminate bedbugs in the United States.

9. **(C).** After describing Flavin's work, the author places it in context to another work in the second paragraph, then goes on to discuss its effect on the definitions of a type of art. (A) is a distortion as "superiority" is not mentioned. (B), if anything, is backwards, as the author seems to approve of the broadening. Choices (D) and (E) ignore the significant part of the passage that discusses the broadening of definitions.

10. **(D).** Both works are said to be set at 45 degree angles (thus, "at a particular slope," which incidentally would happen to be 1 or -1). Choices (A), (B), and (C) are only explicitly mentioned in connection with *alternate diagonals*. (E) is incorrect, as the concept of the works is not discussed; furthermore, the author writes that *Alternate diagonals seems almost more of an evolution of the former work than a response to it*. If the second work is an "evolution," the two works cannot be "identical."

11. **(A).** The passage teems with positive words — "visually arresting," "vivacity," "intriguing," to name just a few — and this indicates the author is biased in favor of the works. Thus, the neutral aspect of choices (C), (D), and (E) is incorrect. As for (B), *fawning* has a negative connotation and is an inappropriate attitude for a GRE author.

12. **(C).** Choice (C) must be true as the author cites *Bottle Rack* as an example of the extreme end of the ready-made spectrum, and contrasts it with a type of "infinitely more manipulated" work. As for (A), the passage only mentions such conventional work in passing and does not predict its future. The passage does not rank Flavin (B), nor does it explain why he disliked the word *sculpture* for his works (D). Similarly, the passage only suggests that Flavin's work is in the center, not that such location is a virtue — the "best art work," as in choice (E).

13. **Patronizing, condescending.** The blank describes "help" offered by managers to young employees who do not want help. Furthermore, the help is "presumptuous." Giving arrogant, unwanted help is *patronizing* or *condescending*. Note that *baneful* and *noxious* are similar to each other, but much too negative.

14. **Ambivalent, equivocal.** Since the men described want to further their careers but also spend time with their children, their feelings about the traditional working arrangement are *mixed* or *complex*. *Ambivalent* and *equivocal* match the idea of mixed feelings. Note trap answer *ambiguous,* which means "unclear," and is more appropriate for describing a situation or statement than a person.

15. **Overtake, outstrip.** The slipstream allows a racecar to take advantage of "reduced wind resistance," which means the trailing car would likely go faster than the car in front of it. You need a word like *pass*. *Supersede* may look the part, but it's actually a figurative word, like *supplant* (e.g., I was *supplanted* by younger workers at my job.) Only *outstrip* and *overtake* carry the literal meaning of "physically passing something."

16. **Foibles, peccadilloes.** "Paean" is praise, as are "plaudits and tributes." You are told that the celebration was "pure" praise. The pivot word *while* tells you that the rest of the sentence should go in the opposite direction. Acknowledging flaws and mistakes — that is, *foibles* and *peccadilloes* — can indeed help to humanize an artist.

17. **(B).** While it is true that Pink Floyd's dramatic staging served to promote its album, this is not a *reason for the inclusion of operatic elements into the rock music genre in the 1970's*. It is an example, not a cause. (A) is incorrect because you are told that PA systems (a type of concert equipment) changed; (C) is incorrect because you are told that concept albums contained "narrative" themes; (D) is incorrect because you are told that rock songs were less appropriate for radio play, and (E) is incorrect because you are told that the larger concerts *encouraged artists to craft concerts that diverged from the merely musical toward the experiential*.

18. **(E).** The passage tells you about several phenomena occurring in the 1970's that were in part responsible for the incorporation of operatic elements into the rock music lexicon. These factors thus helped to "bring about," or *effect*, this change. Don't get confused by *influenced*, one synonym of which is "affected."

19. **I, II, and III.** Sowell wrote that the CRA was *far from being as benign as it appeared*. Thus, the CRA *appeared* benign, or *innocuous,* at least to some. You are told that HUD exerted pressure on lenders, which led to practices that later caused those same lenders to be assailed as "predatory." Thus, HUD had the ability to affect banking practices. Finally, Sowell argued that the idea "that government officials were qualified to tell banks how to lend" was a "flawed assumption." Since Sowell argues that government officials were not necessarily qualified, it follows that he thinks that such qualifications, or *prerequisites,* exist.

20. **III only.** You are told that discriminatory credit practices existed in low-income neighborhoods. Do not assume that there is a linear relationship wherein such practices occur most in low-income neighborhoods, less in middle-income neighborhoods, and least in high-income neighborhoods. Maybe such practices exist *only* in low-income neighborhoods. This choice is out of scope. The second choice is also out of scope — you have no way to know what legislators were thinking or feeling when they voted for the CRA. You do have good proof for the third statement — since some argue that *pressure on lenders from HUD led to practices that later caused those same lenders to be assailed as "predatory,"* you can safely conclude that some critics think that attempts to combat discriminatory credit practices (passing the CRA and enforcing it through HUD) can have unintended consequences — in this case, replacing a lack of lending with predatory lending.

Chapter 2
of

Math Diagnostic Test

In This Chapter…

Math Diagnostic Test

20 Questions
35 Minutes

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

Quantity A	**Quantity B**
$0.0\overline{1410}$	$0.\overline{0141}$

2. A certain bookstore sells only paperbacks and hardbacks. Each of the 200 paperbacks in stock sells for a price between \$8 and \$12, and each of the 100 hardbacks in stock sells for a price between \$14 and \$18.

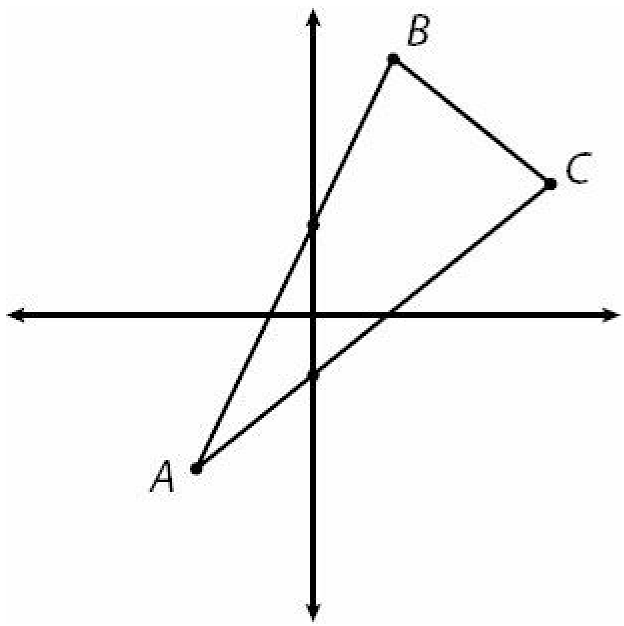
Quantity A	**Quantity B**
The average price of the books in stock at the bookstore	\$9.99

3.

$$2 < x < 4$$

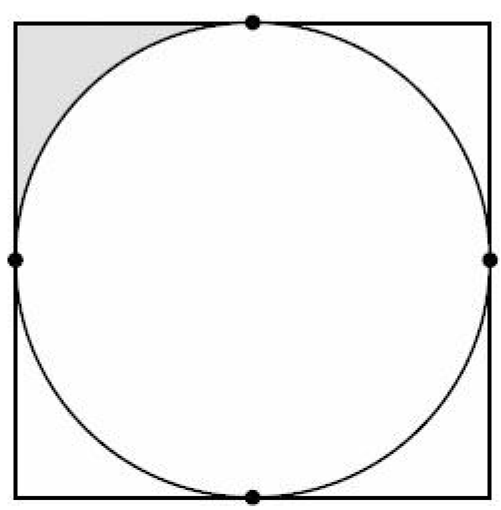
Quantity A	Quantity B
$\dfrac{x-3}{-x}$	$\dfrac{3-x}{-x}$

4.



Quantity A	Quantity B
The slope of line segment AB	The slope of line segment AC

5.



In the figure above, the circle is inscribed a square that has area 16.

Quantity A	Quantity B
The area of the shaded region	1

6.

$a > 1$

$b > 5$

Quantity A	**Quantity B**
$(5b)^a$	$(b^2)^a$

7.

Quantity A	**Quantity B**
$(5 + a)(3 + a)$	$a^2 + 2a + 15$

8.

In Triangle *ABC*, $AB = 12$, $AC = 10$, and $BC = 5$.

Quantity A	**Quantity B**
The measure of angle *A*	The measure of angle *C*

9. If $\frac{52}{x}$ is a positive integer, how many integer values are possible for x?

(A) 5
(B) 6
(C) 7
(D) 8
(E) 10

10. If $3x + 6y = 69$ and $2x - y = 11$, what is the value of y?

11. If $7^9 + 7^9 + 7^9 + 7^9 + 7^9 + 7^9 + 7^9 = 7^x$, then $x=$

(A) 9
(B) 10
(C) 12
(D) 63
(E) 9^7

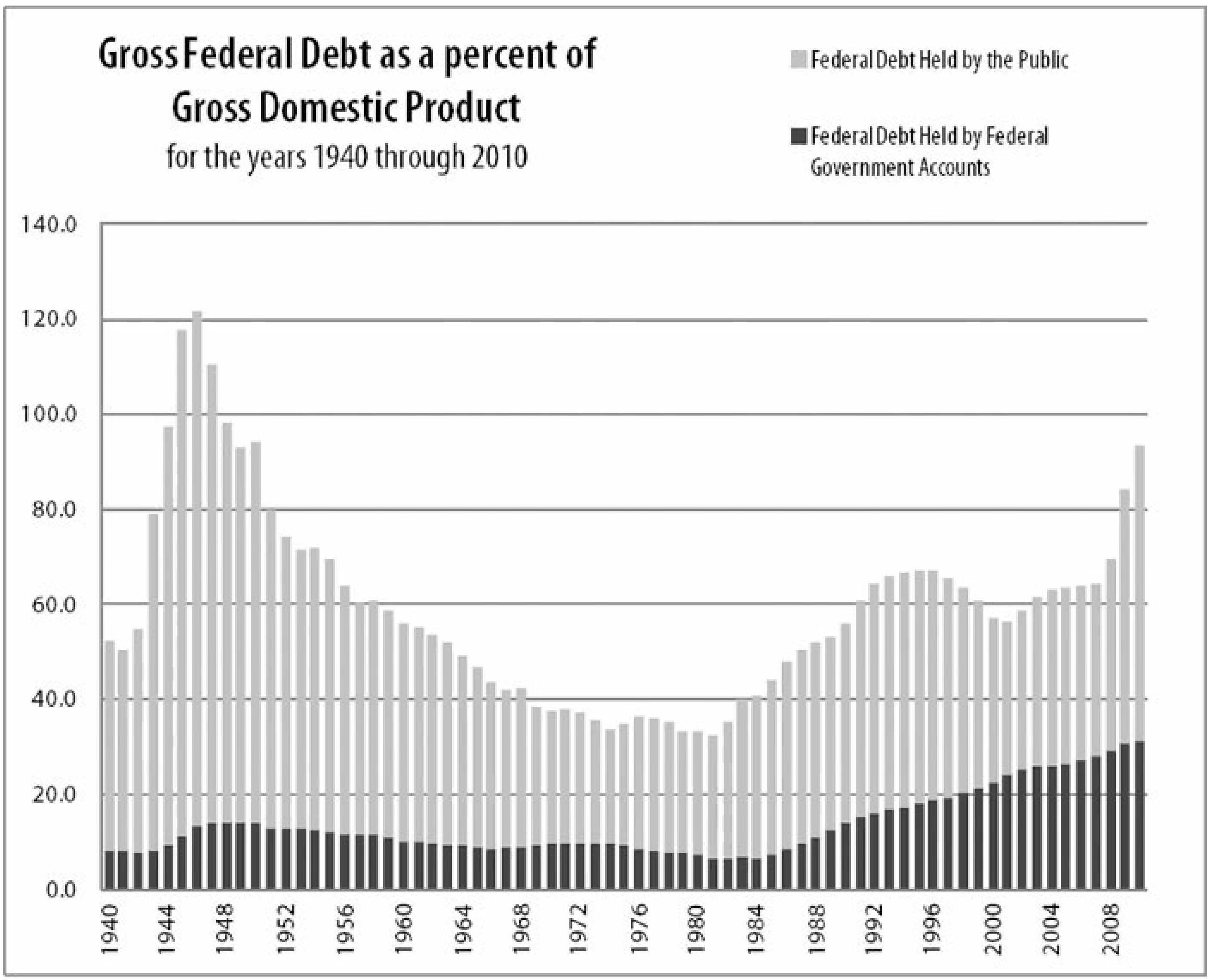
12. In a certain election race, all of the 8,400 votes were cast for either Candidate A or Candidate B. If votes for Candidate A and votes for Candidate B were cast in a 4 to 3 ratio, how many votes were cast for Candidate A?

13. What is the sum of all the integers from -457 to 459, inclusive?

14. $a^3b^4c^7 > 0$. Which of the following statements must be true?
Indicate all such statements.

☐ ab is negative
☐ abc is positive
☐ ac is positive

Questions 15–17 are based on the following chart.



15. In how many years between 1940 and 2010, inclusive, did the gross federal debt exceed the gross domestic product?

(A) 3
(B) 4
(C) 5
(D) 6

(E) more than 6

16. During which decade was federal debt held by federal government accounts closest to half of all federal debt?

(A) 1960's
(B) 1970's
(C) 1980's
(D) 1990's
(E) 2000's

17. At its highest point, what was the approximate ratio of federal debt held by the public to that held by federal government accounts?

(A) 1 : 1
(B) 2 : 1
(C) 5 : 1
(D) 8 : 1
(E) 12 : 1

18. A number x is 32% of a number y. If y is 20% of z, then, expressed in terms of x, z =

(A) $0.064x$
(B) $0.64x$
(C) $6.4x$
(D) $\frac{x}{0.064}$
(E) $\frac{x}{0.64}$

19. If $S^2 > T^2$, which of the following must be true?

(A) $S > T$
(B) $S^2 > T$
(C) $ST > 0$
(D) $|S| > |T|$
(E) $ST < 0$

20. In a certain nation, every citizen is assigned an identification number consisting of the last two digits of the person's birth year, followed by five other numerical digits. For instance, a person born in 1963 could have the identification number 6344409. How many identification numbers are possible for people born in the years 1980–1982, inclusive?

(A) 360
(B) 2,880
(C) 288,800
(D) 300,000
(E) 2,400,000

Quantitative Diagnostic Test: Answer Tally Sheet

Question #	Your Answer	Correct Answer	Tally your correctly answered questions. *No partial credit!*	Find more questions like this in chapter:
1		(C)		11
2		(A)		17
3		(D)		15
4		(A)		28
5		(B)		29
6		(B)		14
7		(D)		8
8		(B)		27
9		(B)		13
10		7		8
11		(B)		14
12		4,800		20
13		917		15
14		III only		15
15		(A)		24
16		(E)		24
17		(D)		24
18		(D)		12, 19
19		(D)		9, 15
20		(D)		23
	Raw Verbal Score: (# of quant questions answered correctly)			

Quantitative Diagnostic Test: Scoring Guide

Raw Quantitative Score	Quantitative Reasoning Diagnostic Scaled Score

20	169 – 170
19	167 – 169
18	164 – 166
17	161 – 163
16	159 – 161
15	157 – 159
14	155 – 157
13	153 – 155
12	152 – 154
11	150 – 152
10	149 – 151
9	147 – 149
8	145 – 147
7	143 – 145
6	142 – 144
5	139 – 141
4	136 – 138
3	133 – 135
2	130 – 132
1	130 – 131
0	130

NOTES:

Diagnostic Scaled Score is approximate.

(a) If your time for this diagnostic section exceeded the 35-minute guideline, this approximate score may not be indicative of your performance under standard time conditions.
(b) Scaled Score depends not only on how many ques-tions were answered correctly (Raw Score), but also on the overall difficulty of the set of questions. This diagnostic test approximates the difficulty of the official GRE® revised General Test.

Math Diagnostic Test Answers

1. **(C).** In a repeating decimal, the portion under the bar repeats without end. In Quantity A, the portion "1410" repeats, after an initial 0.0 that does not repeat. In Quantity B, the portion "0141" repeats, starting immediately after the decimal. To compare, write out more digits of each decimal:

Quantity A: 0.0141014101410…
Quantity B: 0.0141014101410…

The two numbers are identical; Quantity A and Quantity B are simply different ways of writing the same number.

2. **(A).** Because there are twice as many paperbacks as hardbacks in stock, the overall average price will be closer to the price of the paperbacks than the price of the hardbacks. However, the fact that the problem gives price *ranges* instead of prices complicates matters a bit. Calculate the lowest possible overall average and the highest possible overall average to see whether that average can be both lower and higher than $9.99.

To calculate the lowest possible overall average, assume the lowest price for all paperbacks and all hardbacks ($8 and $14, respectively). Note that there are twice as many paperbacks as hardbacks, which essentially double-weights the $8:

$$\frac{200(\$8)+100(\$14)}{200+100}=\frac{2(\$8)+1(\$14)}{3}=\frac{\$30}{3}=\$10$$

This minimum price is already greater than $9.99, so the maximum price would also be greater, and there is no need to calculate it.

Quantity A is greater.

3. **(D).** Since $2 < x < 4$, test values between 2 and 4 in both Quantities to see which Quantity is larger for each example. You must ALWAYS test more than one value to determine the answer to a Quantitative Comparison, as you should try to prove (D), or at least test whether you can.

If $x = 2.5$, Quantity $A=\frac{2.5-3}{-2.5}=\frac{-0.5}{-2.5}=\frac{5}{25}=\frac{1}{5}$ and Quantity $B=\frac{3-2.5}{-2.5}=\frac{0.5}{-2.5}=\frac{-5}{25}=\frac{-1}{5}$. In this case, Quantity A is larger.

Before just randomly trying other values between 2 and 4, try to strategize: could you find an example in which Quantity B is greater? Since the two Quantities have the same denominator, focus on the numerators. $x - 3$ is positive when $x > 3$ and negative when $x < 3$. Since the first number tested was less than 3, next try something greater than 3.

If $x = 3.5$, Quantity $A=\frac{3.5-3}{-3.5}=\frac{0.5}{-3.5}=\frac{-5}{35}=\frac{-1}{7}$ and Quantity

$B = \frac{3-x}{-x} = \frac{3-3.5}{-3.5} = \frac{-0.5}{-3.5} = \frac{5}{35} = \frac{1}{7}$. In this case, Quantity B is larger. (Note: you could have stopped calculating the exact values of Quantity A and Quantity B for this example once it became clear that Quantity A was negative and Quantity B was positive.)

Alternatively, it would also be strategic to think about what number, if any, could make the two quantities equal. If $x = 3$, Quantity A = Quantity B = 0 because both numerators will be 0 and 0 divided by any non-zero number is just 0.

Since Quantity A is greater than Quantity B for some examples of x where $2 < x < 4$, but Quantity B is greater than Quantity A for other examples where $2 < x < 4$, the correct answer is (D).

4. **(A).** While there are no numbers on the graph, both lines have clearly positive slopes (the lines rise upward when reading from left to right) and segment *AB* is clearly steeper than segment *AC*. Thus, segment *AB* has a greater slope.

While you should be cautious about making assumptions on the GRE, the fact that the two lines form two sides of a triangle and meet at vertex *A* shows that the lines are not parallel, and segment *AB*, which rises above segment *AC* to the right of their meeting point, is definitely steeper.

5. **(B).** If the area of the square is 16, then the side of the square is 4. Since the circle is inscribed in the square, its diameter is 4 and its radius is 2. Since the area of a circle is πr^2, the area of this circle is 4π. Thus, the combined area of the four "corners," outside the circle but inside the square, is $16 - 4\pi$. The shaded region is one of these four identical "corners," so the area of the shaded region is $\frac{16-4\pi}{4} = 4 - \pi \approx 4 - 3.14 = 0.86$, which is less than 1.

Quantity B is greater.

6. **(B).** Since both quantities have the same exponent as well as at least one b inside the parentheses, one way to compare the quantities is to distribute that exponent.

Quantity A: $(5b)^a = 5^a b^a$

Quantity B: $(b^2)^a = (b \times b)^a = b^a b^a$

Because b^a is positive (i.e., a positive base to a positive power) and common to both quantities, it can be ignored or canceled from both.

Quantity A: 5^a

Quantity B: b^a

Because the positive exponent is common to both quantities, a larger base indicates the greater quantity. Because $b > 5$, Quantity B is greater.

7. **(D).** FOIL the terms in Quantity A:

$$(5 + a)(3 + a) = 15 + 5a + 3a + a^2 = a^2 + 8a + 15.$$

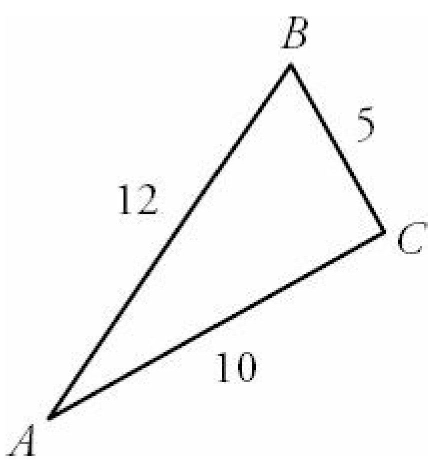
Since a^2 and 15 appear in both quantities, eliminate them.

Quantity A = $8a$
Quantity B = $2a$

If a is positive, Quantity A is greater. If a is negative, Quantity B is greater. If a is zero, the two quantities are equal. Without information about a, it cannot be determined which quantity is greater.

(Note also, because neither quantity is set equal to zero or any other number, you cannot actually "solve" either quantity to get roots or solutions for a.)

8. **(B).** Begin by drawing the triangle described by the text and labeling the sides:



According to the properties of triangles, the longer the side opposite an angle, the larger the angle itself must be. Since angle A opens to the shortest side length 5 but angle C opens to the longest side length 12, it must be true that angle $C >$ angle A. Thus, Quantity B is greater.

9. **(B).** If $\frac{52}{x}$ is a positive integer, then x is a factor of 52 (i.e., x divides evenly into 52). This question can most easily be solved by listing factor pairs for 52, as listing in pairs will help prevent omissions. Each pair multiplies to 52.

The factors of 52 are:

1 & 52
2 & 26
4 & 13

How can you be sure this list is complete? Looking down the left column, 1 and 2 made the list, then 3 didn't divide evenly into 52, then 4 did. Since 4 pairs with 13, check the other integers between 4 and 13. Since 5, 6, 7, 8, 9, 10, 11, and 12 don't divide evenly into 52, this list is complete. There are 6 factors, so the answer is (B).

10. **7.** Multiply each term of the second equation by 6: $12x - 6y = 66$. Why do this? Because the first equation has a

$6y$, and the y terms will now cancel when the equations are added.

New second equation:	$12x - 6y = 66$
+ First equation:	$3x + 6y = 69$
Sum:	$15x = 135$
	$x = 9$

Plug $x = 9$ back into an original equation to get y.

$$2x - y = 11$$
$$2(9) - y = 11$$
$$18 - 11 = y$$
$$y = 7$$

In summary, $x = 9$ and $y = 7$. Be sure to answer for y, not x.

11. **(B).** Factor 7^9 out of $7^9 + 7^9 + 7^9 + 7^9 + 7^9 + 7^9 + 7^9$ to get:

$$7^9\,(1 + 1 + 1 + 1 + 1 + 1 + 1) = 7^9\,(7)$$

Or, just count that there are seven 7^9's in the original sum, which can be written as 7×7^9.

Since $7^9\,(7)$ is the same as $7^9\,(7^1)$, simplify further: $7^9\,(7^1) = 7^{(9 + 1)} = 7^{10}$. Thus, $x = 10$.

Note: it is NOT correct to simply add the original exponents together. When adding or subtracting exponential expressions with the same base, it is not possible to directly combine exponents. Factoring out is the correct procedure.

12. **4,800.** If votes for Candidate A and votes for Candidate B were in a 4 to 3 ratio, then for every 4 votes Candidate A got, Candidate B got 3. You can think of these votes as existing in "sets" of 7 votes.

Divide 8,400 by 7 to get 1,200. Thus, the votes were cast in 1,200 "sets" of 7. In each "set," the votes went A A A A B B B (4 votes for A, 3 for B). Thus, the total number of votes for A is $1,200 \times 4 = 4,800$.

13. **917.** There must be a trick to this, as it would be impossible to sum so many numbers under GRE time constraints, even with a calculator. To see the trick, try an example with much smaller numbers. For instance, what is the sum of all the integers from -2 to 4, inclusive?

NEW EXAMPLE: $\cancel{-2} + \cancel{-1} + \cancel{0} + \cancel{1} + \cancel{2} + 3 + 4 = 7$

That is, -2 and 2 cancel, -1 and 1 cancel, and 0 has no impact on the sum. So the sum is simply the leftover numbers at the end, $3 + 4 = 7$.

Similarly, in the set -457, -456 … 0 … 456, 457, 458, 459, all the integers from -457 to 457 cancel each other out. Only 458 and 459 remain. The sum is $458 + 459 = 917$.

14. **III only.** Since b^4 must be positive, if $a^3b^4c^7$ is positive, a^3c^7 must also be positive. Since putting an *odd* exponent on a number doesn't change whether the number is positive or negative, ac must be positive, so Statement III is true. The other statements require knowledge of the sign of b, which is not known here, since the even exponent "hides" whether the underlying base is positive or negative.

15. **(A).** The chart expresses gross federal debt as a percent of gross domestic product, so federal debt exceeded gross domestic product in any year in which the value of the graph rose above 100. Only during 3 years in the 1940s does the graph extend above 100.

16. **(E).** Federal debt held by federal government accounts would be half of all federal debt in any year in which the dark, bottom portion of the bar equaled the lighter, top portion. Although it never actually reached 50% of the total federal debt, the dark portion of the bar came closest to equaling the light portion during the 2000s.

17. **(D).** The ratio of federal debt held by the public to that held by federal government accounts for any given year would be the measure of the lighter, top bar over the measure of the darker, bottom bar for that year. (To get the measure of the lighter, top bar, you must subtract the value of the darker, bottom bar.)

While the federal government accounts percentage hovered near the low teens throughout most of the graph, debt held by the public rose above 100% during portions of the 1940s. Identify the year in which the lighter, top bar is the largest in comparison to the darker, lower portion. In this chart, it happens to be the tallest bar overall, which was 1946, although it's not necessary to identify the exact year.

Total bar: 120 (actually a bit more)
Dark, bottom bar: 15 (or a bit less)
Light, top bar: 120 (or more) - 15 (or less) = 105 (or a bit more)

The ratio is thus 105 (slightly more): 15 (maybe less), which is 7 : 1 (or a bit higher). Among the given options, this ratio is closest to 8 : 1.

18. **(D).** Translate the percent relations in the problem into algebraic statements. First, "a number x is 32% of a number y" becomes

$$x = 0.32y$$

Similarly, "y is 20% of z" becomes

$$y = 0.2z$$

Substituting for y from the second equation into the first eliminates y and gives the following relationship between x and z:

$$x = 0.32(0.2z)$$
$$x = 0.064z$$

Finally, the problem asks for z in terms of x, so isolate the variable z by dividing both sides of the equation by 0.064.

$$z = \frac{x}{0.064}$$

19. **(D).** The square root of a squared variable is equal to the absolute value of that variable: $\sqrt{x^2} = |x|$, not x. So, taking the square root of both sides of this inequality results in $|S| > |T|$. Answer choice (A) does not have to be true because S could be negative while T is positive. For example, if $S = -5$ and $T = 4$, then $S^2 > T^2$. Testing fractions in answer choice (B) shows that it does not have to be true. If $S^2 = 1/9$ and $T^2 = 1/16$, then $T = 1/4$ or $-1/4$. $1/4$ is greater than $1/9$, which means that T can be greater than S^2. Answer choice (C) does not have to be true because S and T could have opposite signs. Answer choice (E) does not have to be true because S and T could have the same sign.

20. **(D).** This is a combinatorics problem in which *order matters* (as it always does with passcodes, ID codes, etc.) Since the identification numbers each have 7 digits, draw 7 slots. In each slot, list the number of possibilities for that slot.

Since the question asks only about identification numbers for people born in 1980, 1981, and 1982, the identification numbers must begin with 80, 81, or 82. Thus, only 8 (1 option) can go in the first slot. Only 0, 1, or 2 (3 options) can go in the second slot.

$$\underline{\;1\;}\quad \underline{\;3\;}\quad \underline{\quad}\quad \underline{\quad}\quad \underline{\quad}\quad \underline{\quad}\quad \underline{\quad}$$

Note that there is no rule against repeating a number in an identification number (the problem gives the example "a person born in 1963 could have the code 6344409"), so each remaining slot can each contain any of the digits 0–9 (10 options).

$$\underline{\;1\;}\quad \underline{\;3\;}\quad \underline{\;10\;}\quad \underline{\;10\;}\quad \underline{\;10\;}\quad \underline{\;10\;}\quad \underline{\;10\;}$$

This fundamental counting principle states that the total number of choices is equal to the product of the independent choices. To calculate the answer, multiply $1 \times 3 \times 10 \times 10 \times 10 \times 10 \times 10 = 300{,}000$.

Alternatively, note that for each birth year, the part of the identification number unique to the individual is a 5-digit number. These could be 00000-99999 (99,999 + 1 = 100,000 options). There are 100,000 possible numbers for each of 3 years, or 300,000 identification numbers possible.

Chapter 3
of

Text Completions

In This Chapter…

Text Completions

Select one entry for each blank from the corresponding column of choices. Fill in the blank in the way that best completes the text.

1. In Europe, football, otherwise known as soccer, it the most popular sport by several orders of magnitude, whereas in America, fandom is fairly evenly ___________ among a few different sports.

regarded
inspired
enjoyed
measured
apportioned

2. The astrophysicist argues that our books and films about interstellar space travel are a form of mass __________, and that only a miracle on a scale heretofore unseen could allow a human being to voyage to even the closest star in another solar system.

innovation
delusion
dementia
catastrophe
hysteria

3. Peculiarly enough, Shakespeare has been often (i) _______ as the best English language playwright, and often (ii) _______ as a man lacking the education to write those plays.

Blank (i)	Blank (ii)
crowned	demonized
stigmatized	dismissed
castigated	deified

4. Although it was not the university's policy to (i) __________ the authority of its faculty, the president felt that the professor's comments regarding affirmative action could not remain (ii) __________.

Blank (i)	Blank (ii)
defend	unavailed
ruminate	unchallenged

undermine	averred

5. In order to defend downloading music illegally, it's necessary to engage in a bit of ethical (i) ____________. While it is true that traditional record labels (ii) __________ their artists—demanding indefensibly large percentages of their profits—downloading a song illegally is equally (iii) __________, because it robs both the label and the artist.

Blank (i)	Blank (ii)	Blank (iii)
contortion	swindle	justifiable
leniency	disabuse	unconscionable
probity	deluge	scrupulous

6. The fact that the average life expectancy ten thousand years ago was so much shorter than it is now is often (i) _________ as evidence supporting the notion that the world always improves with time. However, if you (ii) __________ for the fact that most children in that epoch died in childbirth, it turns out that life expectancy back then was nearly the same as it is now. On a (iii) __________ note, it also turns out that many "barbaric" hunter-gatherer tribes were taller on average than the average person now living.

Blank (i)	Blank (ii)	Blank (iii)
cited	prepare	contrastive
disregarded	read	sidereal
embodied	correct	tangential

7. On an aptitude test in 1986, an argument posited that the possibility of conducting banking transactions from home was as likely as flying cars, an argument that today sounds ____________.

prescient
paradoxical
presumptuous
preposterous
pithy

8. Napoleon is of course most famous for his military triumphs, but his innovative code of law had a subtler but more ______ impact, as its principles strongly influenced legal codes well into the 20th century.

renowned
enduring
fleeting
insidious
martial

9. Many ____________ people feared for the life of Ronald Reagan because since 1840, every president elected in a year ending in zero had died in office.

knowledgeable
mathematical
superstitious
addled
conservative

10. Known for her humorous but acerbic wit, the fashion doyenne commented, in her usual, simultaneously (i) __________ and (ii) __________ manner, that in Los Angeles, "the women dressed like men and the men dressed like boys."

Blank (i)	Blank (ii)
slanderous	considerate
amusing	hysterical
serious	caustic

11. Every generation is accused of slacking by the preceding ones, before in turn calling their own progeny lackadaisical; such is the __________ of life.

vicissitude
irony
circle
serendipity
comedy

12. Although retired, the professor takes pains to remain __________ the latest developments in her field.

akimbo to
abreast of
obtuse to
subservient to
askance to

13. She was not the only (i) __________ of the long-proposed legislation, but she was the (ii) __________ who finally got the bill onto the legislative agenda.

Blank (i)	Blank (ii)
apologist	catalyst
critic	mercenary
proponent	lackey

14. Jeremy was not one to (i) __________ his success, so his family was shocked when they finally discovered that

their (ii) __________ son was a Rhodes Scholar.

Blank (i)	Blank (ii)
demarcate	prodigal
whitewash	taciturn
trumpet	dissolute

15. The elderly woman was (i) __________ to have returned to her the ring that she had lost fifty years before in the (ii) __________ waters of the Mississippi River.

Blank (i)	Blank (ii)
dumbfounded	turbid
addlepated	turgid
enervated	tepid

16. Hursthouse, (i) __________ virtue ethicists in general, argues that ethics is properly neither situational nor utilitarian and that one ought to seek out virtue and emulate it rather than base one's judgments on subjective concerns or a (ii) __________ weighing of pain and pleasure likely to result from a given action; critics, of course, tend to (iii) __________ that Hursthouse and other virtue ethicists who seek to define virtue merely seek to enshrine their own prejudices under the guise of theory.

Blank (i)	Blank (ii)	Blank (iii)
enigmatic to	pragmatic	posit
breaking away from	quixotic	deny
emblematic of	grandiloquent	cajole

17. The film was (i) __________ (ii) __________ by critics; not a single reviewer had any positive thing to say about it.

Blank (i)	Blank (ii)
warily	lauded
mendaciously	panned
roundly	venerated

18. Socrates advocated a life of moderation: live (i) __________, drink (ii) __________, and (iii) __________.

Blank (i)	Blank (ii)	Blank (iii)
apathetically	sparingly	fight for one's beliefs
lavishly	copiously	do nothing to excess
modestly	brusquely	remain inscrutable always

19. After the US Civil War, "carpetbaggers"—so-called because they carried suitcases made of inexpensive carpeting material— __________ the South, hoping to turn a quick profit.

vanquished
inundated
blacklisted
boycotted
berated

20. December's earthquake was but a __________ to a terrible year for a small island nation recently wracked by civil strife and devastating tropical storms.

prologue
catharsis
coda
homily
rampage

21. Although they had never met, the two writers felt they were of one mind, each (i) __________ anticipating the contents of the other's letters; never had two intellectuals been more (ii) __________.

Blank (i)	Blank (ii)
ominously	providential
anachronistically	shrewd
presciently	simpatico

22. After many years of war and bloodshed, some became __________ suffering, casting a blind eye to scenes of misery around them.

inured to
exempted from
dominant over
effusive towards
maudlin over

23. After a brief initial struggle over power, the group elected a leader and __________ into a surprisingly harmonious team.

fractured
syncopated
coalesced
agglomerated
amortized

24. The director inserted deliberate __________ into the play—for example, Hamlet typed his letters to Ophelia on a laptop.

platitudes
paradigms
neologisms
derivations
anachronisms

25. The ancient tablet presents a true __________: none have been able to decode it.

gem
stanza
quagmire
enigma
incentive

26. According to supporters of progressive taxation, the payment of higher tax rates by those with higher incomes, those who (i) __________ such taxation systems tend to fall into two camps: the (ii) __________, who feel it unjust that the "most industrious" among us are "penalized" for their success, and the (iii) __________, who argue that higher taxes on the rich will harm us all by dampening the entrepreneurial spirit that drives the economy.

Blank (i)	Blank (ii)	Blank (iii)
rail against	autodidacts	libertines
vote for	moralists	utilitarians
depredate	recidivists	myrmidons

27. Just as reminiscences of a childhood spent in rural Mexico color the poet's work, so too does the experience of war __________ her poetry.

inform
mimic
invalidate
defer
presage

28. The peanut is often (i) __________ referred to as a nut; (ii) __________ to such people, it is actually a legume.

Blank (i)	Blank (ii)
archaically	unbeknownst
erroneously	abhorrent

deftly	consanguineous

29. The (i) __________ child approached the diving board; (ii) __________ water in the first place, he found the prospect of jumping into it from some height even more frightening.

Blank (i)	Blank (ii)
tremulous	beguiled by
coltish	chary of
cumbersome	repulsed by

30. Soldier ants are wingless, sterile females that guard the colony and supply it with food, thus acting as both (i) __________ and laborers as the (ii) __________ queen produces enough (iii) __________ to continually populate the colony.

Blank (i)	Blank (ii)	Blank (iii)
sentries	fecund	forebears
sages	efficacious	progeny
sycophants	imperious	harbingers

31. The Paris Commune was a government that ruled France for about two months in 1871; despite its (i) __________ reign, it was at the time (ii) __________ as a sign of the emergence of a powerful working class.

Blank (i)	Blank (ii)
equivocal	discounted
ephemeral	recanted
omnipotent	heralded

32. The property, hidden by overgrown plants, had not only fallen into (i) __________, but appeared positively (ii) __________ by vermin.

Blank (i)	Blank (ii)
misanthropy	overrun
desuetude	outdone
debauchment	hampered

33. That the CEO’s son was pleasant enough was not in doubt, but the (i) __________ young man was hardly (ii) __________ enough to bargain on the company’s behalf.

Blank (i)	Blank (ii)
puissant	egoistic
restive	blithe
complaisant	artful

34. He has such a pleasingly (i) __________ personality that it’s hard to be bothered by the (ii) __________ in his past.

Blank (i)	Blank (ii)
sanguine	peccadilloes
high-handed	incendiaries
salubrious	achievements

35. The substitute French teacher accidentally walked into the wrong classroom, creating __________ situation when she began speaking French to 15 mystified physics students.

an odious
a potable
a risible
a cursory
an arch

36. The (i) __________ young employee was soon (ii) __________ for making a serious mistake that cost the company thousands of dollars.

Blank (i)	Blank (ii)
banal	enamored
ginger	castigated
verdant	deposed

37. The beauty pageant contestant told the judges she wanted world peace, but her suggestion to bring about world peace was __________ —apparently, she naively thinks everyone could just be told to “love one another” and all the world’s disagreements would fade away.

convoluted
facile
impeccable
amicable
dulcet

38. Introverts prefer a quiet, reflective lifestyle, whereas extroverts are more __________.

doctrinaire
convivial
sanguinary
jejune
modish

39. While the author's first collection of short stories presented a (i) __________ hodgepodge of voices, the second collection presents a remarkably (ii) __________ set of tales presented by a (iii) __________ narrator.

Blank (i)	Blank (ii)	Blank (iii)
motley	insightful	lonely
variegated	even	disingenuous
homogeneous	facetious	sole

40. The transformation of the NFL from a running-oriented game to a passing-oriented game demonstrates that, in any profession, when the __________ proves more successful than the time-honored, industry leaders will embrace the new with alacrity.

aerial
exciting
novel
conventional
controversial

41. After struggling to maintain her composure for hours upon hearing of her late husband's infidelities, the recently bereaved widow finally burst into __________.

hysterics
laughter
song
mirth
flames

42. The virtual (i) __________ of John F. Kennedy focuses on his presidential achievements, legend, and assassination; similarly, as a candidate, verbal and written (ii) __________ were laid at the altar of his wartime exploits.

Blank (i)	Blank (ii)
deification	calumnies
excoriation	garlands
adoration	obloquies

43. For centuries, commercial portrait painters have employed a bifurcated aesthetic: like other artists, they strive to represent the truth that gives their works life, but commerce dictates that they simultaneously employ subtle __________ that make the likeness more attractive than the sitter.

palettes
aesthetics
artifacts

artifices
sentiments

44. Dogmatic professors often alienate their classes because they fail to realize that their __________ enervates rather than inspires students.

wisdom
pedantry
parsimony
pulchritude
wit

45. General McClellan focused so intently on the petty, logistical details of the upcoming campaign that Lincoln felt that said attention to __________, however necessary, had superseded more lofty goals.

irrelevancies
tactics
minutiae
strategy
peccadilloes

46. Jimmy Stewart, the actor, spoke with an (i) __________ that (ii) __________ audiences; through hesitancy and understatement, he was at least as captivating as his flamboyant peers.

Blank (i)	Blank (ii)
awkward lisp	enthralled
overwhelming passion	repelled
appealing shyness	amused

47. The most startling aspect of Preston Brooks, the 19th century southern congressman, was his temperament, (i) __________ to the point of violence. While a man of principles, he did not merely piously pray for political converts—more likely, opposition would cause him to become (ii) __________ and fly in to a rage. Famously, he once (iii) __________ a foe into unconsciousness on the floor of the Senate.

Blank (i)	Blank (ii)	Blank (iii)
quixotic	figuratively unstrung	bored
volatile	literally insane	caned
quiescent	practically incomprehensible	lulled

48. Ironically, the commentator who so roundly condemned personal (i) __________ was (ii) __________ to the point of bankruptcy—he himself was a reflection of an aspect of the ills that, in other areas, he railed against.

Blank (i) Blank (ii)

lacunae	profligate
probity	antediluvian
dissipation	ascetic

49. Most readers of Nietzsche's *Beyond Good and Evil* probably remember it as a collection of poetic aphorisms, each standing alone; these readers missed or have forgotten the linear (i) __________ arcing through the work. Interestingly enough, it could be argued that a parallel cognitive gulf pervades the book itself. While Nietzsche rails against the (ii) "__________" of earlier philosophers and contrasts it to the ideal of the "free spirit," unencumbered by religions and prejudices, he then espouses the possibility of (iii) __________ —such an ordering of spiritual strength seemingly undercuts the notion of an independent spirit.

Blank (i)	Blank (ii)	Blank (iii)
chaos	dogmatism	conjuring seraphs
progression	hedonism	a new philosophy
stasis	morality	constructing a hierarchy

50. Just as ancient Greek culture in some ways provided the Romans with a model, the remnants of Roman culture __________ the development of medieval European mores.

duplicated
curbed
foresaw
informed
hindered

51. As an evolutionary adaptation, cats have developed a mechanism whereby their heads are measuring devices, as their bodies can fit through any space that their heads can, and this physiology is a safeguard that __________ their success as a species.

reproduces
ensures
enhances
mitigates
inundates

52. Modern political campaigns are so full of misrepresentations and worse that candor is an __________.

aphorism
anecdote
effacement
anomaly

overture

53. There are no other contemporary novelists whose works resemble those of Thomas Pynchon, but his appeal rests on more than the (i) __________ displayed in his work. In an age when novelists largely limit their observations to lightweight banter about sexuality, Pynchon still offers (ii) __________ commentary about human nature.

Blank (i)	Blank (ii)
patrimony	trenchant
eccentricity	enigmatic
orthodoxy	insipid

54. Crane Brinton argued that the middle phases of revolutions are especially (i) __________ because the unleashed force of social momentum transfers power inexorably from more stable (if oppressive) forces to less temperate ones. Yet, he then goes on to say that the excesses (ii) __________ and a more peaceful period of "Thermidor" ensues.

Blank (i)	Blank (ii)
brusque	metastasize
berserk	grow
pacific	recede

55. The Donner party made a __________ choice to try to cross the Rockies too late in the season, and they paid dearly for that dangerous decision.

prudent
parlous
suicidal
semiotic
providential

56. Struggling with the uphill descent, the ancient jalopy blew a gasket when it reached the level plateau, and then the car slowly __________ to a stop.

careened
castigated
trundled
tarried
transmuted

57. Frederick the Great of Prussia was known for his (i) __________ under fire during his military victories; however, when confronting issues of domestic policy, this equilibrium sometimes failed him. He was often (ii) __________ with his ministers, who never knew when they might be subjected to one of his tirades.

Blank (i)	Blank (ii)
intrepidity	fascist
cruelty	mercurial
sangfroid	vainglorious

58. During the mating season, mammals of many species may lie (i) __________ to their mates; yet, for the remainder of the year, some of those species lead utterly solitary and (ii) __________ existences.

Blank (i)	Blank (ii)
paramount	inconsolable
apposed	irascible
enamored	eremitic

59. At the beginning of the 20th century, wealthy business leaders piteously (i) __________ that child labor laws would ruin them and corrupt the youth; one hundred years later, members of the modern (ii) __________ similarly bemoan that any government intervention will bankrupt them and contribute to the moral (iii) __________ of the less-moneyed classes.

Blank (i)	Blank (ii)	Blank (iii)
besmirched	oligarchy	rectitude
bleated	hierarchy	scruples
postulated	monarchy	turpitude

60. The library wing was first conceived merely as (i) __________ to address the problem of book overstock until a more permanent solution could be found. Ironically, it was the flimsy nature of the wing itself that attracted such architectural interest and ultimately led to its canonization as a (ii) __________ of its kind. Now a statute exists to protect this originally transient structure in (iii) __________.

Blank (i)	Blank (ii)	Blank (iii)
a stopgap	paragon	consecration
an ornament	nadir	chronology
a modicum	catalyst	perpetuity

61. Though the negotiation was initially expected to proceed smoothly, it soon became apparent that any appearance of (i) __________ between the parties was disingenuous or, at best, a superficial adherence to certain (ii) __________.

Blank (i)	Blank (ii)
duplicity	mores
amity	politics
solace	plaudits

62. While some feel that the author’s (i) __________ late in his life (ii) __________ his reputation, others felt that his

dissolution added a certain glamour to his biography and credibility to his libertinous tales.

Blank (i)	Blank (ii)
vitiation	derogated from
debauchment	bedizened
deterioration	aggrandized

63. The children's story—seemingly a simple tale of animals gathering for a picnic in the forest—took __________ turn at the end, admonishing readers to always be honest.

a generous
a cynical
a jocular
an irate
a didactic

64. The plan, if it can be called that, has been more of a (i) __________ vision than a concrete proposal; like many similarly (ii) __________ ideas, it is unlikely to ever come to fruition.

Blank (i)	Blank (ii)
Dionysian	quixotic
protean	pragmatic
martial	unorthodox

65. It was a fact that the region was both quiet and rural, but what the typically impoverished residents considered (i) __________ refuge was considered by the well-heeled visitors to be an intolerable (ii) __________, and its residents' lifestyles unpleasantly (iii) __________.

Blank (i)	Blank (ii)	Blank (iii)
a parochial	penumbra	tony
an arcadian	utopia	spartan
a squalid	hinterland	apollonian

66. The biography, a (i) __________ account of a (ii) __________ life, was a gripping read, though hardly appropriate for young readers.

Blank (i)	Blank (ii)
faux-naif	callow
no-holds-barred	stonewalled
dyed-in-the-wool	storied

67. The club had been all male up until 1963, when it began to admit women, who now make up more than 65 percent

of the membership; hence, the female club president was both annoyed and amused at an elderly male member's (i) __________ suggestion that women be shuffled off to (ii) __________ organization where they could play bridge and drink tea without having to worry about serious issues.

Blank (i)	Blank (ii)
regressive	an incendiary
rustic	an auxiliary
prudish	a hierarchical

68. Although this historical figure had been (i) __________ politician and a brilliant inventor, the professor found himself unable to (ii) __________ the interest of his students in the career of a man with such anachronistic views.

Blank (i)	Blank (ii)
a clandestine	whet
a deft	accrue
an effete	tout

69. Though the industry leader faces prosecution while the smaller company is legally shielded from consequences due to a crafty system of hiring outside firms to do its dirty work, the smaller firm is clearly (i) __________ with the larger one, (ii) __________ the larger firm's plans to dominate the industry through illegal labor practices and offshore dumping.

Blank (i)	Blank (ii)
at loggerheads	colluding
in cahoots	stymying
preoccupied	abetting

70. The organizer of the university's series of public debates commented that it is difficult to find a suitable adjudicator for debates on certain controversial matters, as there exist topics in regards to which no one is __________.

pellucid
disinterested
solipsistic
jocular
subjective

71. It was difficult to tell whether his tirade in fact had force of fact, whether his forceful (i) __________ was mere (ii) __________, motivated by malice, or whether it ranked as reportorial (iii) __________ of which the committee ought take notice.

Blank (i)	Blank (ii)	Blank (iii)

fulmination	sophistry	muckraking
desecration	boosterism	calumny
ululation	aspersion	prattle

72. Her grandparents valued seemliness above all else, and were (i) __________ at her incorrigibly (ii) __________ behavior.

Blank (i)	Blank (ii)
loquacious	mutinous
agog	indelicate
nonchalant	tortuous

73. It was apparent to everyone present that upon receiving the medal, a decoration long past due, the superannuated veteran adopted a mood of exaggerated hilarity, almost as if the unexpected resolution of a lifetime of injustice came as a __________ surprise.

sanctimonious
risible
parsimonious
prodigal
plangent

74. There are many good reasons to construct urban traffic lanes for cyclists: city infrastructure is already __________ under the strain of excess auto traffic, and the safety advantages of limiting road-sharing between cyclists and vehicles are all too clear.

seething
waiting
groaning
baying
intensifying

75. Despite their diminutive stature, horse jockeys are far from __________ bunch: many would all-too-happily allow minor umbrage to escalate to barroom brawl.

a stygian
a farcical
an effete
an evocative
an ecumenical

76. The apparent simplicity of a cup of coffee __________ the dizzying number of hours of toil required to produce it,

from months of cultivation of the bean tree to painstaking refinement in highly sophisticated machinery.

redresses
confirms
belies
furnishes
fosters

77. Notwithstanding the mishmash of worn tools littering every surface of the artist's studio, the place exuded a certain sense of order manifest through the clutter; the decor was, if (i) __________, (ii) __________.

Blank (i)	Blank (ii)
unkempt	largely unsophisticated
dire	positively callous
arduous	surprisingly deliberate

78. Two years after the legislature's (i) __________ approval of the community edifice, construction came to an equally public standstill, largely due to the unforeseen hemorrhaging of the (ii) __________ funds at the hands of spendthrift leaders.

Blank (i)	Blank (ii)
scorned	adduced
heralded	adumbrated
ratified	appropriated

79. Incensed, and perhaps spooked, by the implications of the bureau's purportedly (i) __________ inquisitions, the Hollywood film director shuttered his studios, suspended production of numerous projects, and (ii) __________ with his wife to Europe.

Blank (i)	Blank (ii)
suspicious	immigrated
benign	absconded
risqué	pandered

80. The relationship between the two leaders has gone from positively (i) __________ to chilly at best, not least because the recent arms scandal threatens to (ii) __________ the mutual trust that has been held on both sides for years.

Blank (i)	Blank (ii)
parsimonious	bolster
reverent	erode
congenial	fester

81. One needn't resort to arrant (i) __________ in order to demonstrate that one possesses the requisite degree of deference and respect for one's elders; indeed, oftentimes such blandishments can make one appear (ii) __________ and ignorant—qualities presumably (iii) __________ with the original intent.

Blank (i)	Blank (ii)	Blank (iii)
flattery	artisan	apposite
officiousness	fatuous	incongruous
veracity	sardonic	daunting

82. Debates over free will have always focused on the extent to which humans may be said to be fully (i) __________ their actions. Dr. Wegner in his article deliberately and artfully (ii) __________ the traditional talking points of the controversy, instead asking a tangential, though possibly more (iii) __________, question: What effect does a person's belief in free will have on his or her well-being?

Blank (i)	Blank (ii)	Blank (iii)
responsible for	mitigates	fundamental
aware of	eschews	ideological
based on	contradicts	flashy

83. Once the candidate established herself as the clear frontrunner, it took but a brief interlude in the clamor for all her erstwhile (i) __________ to gather around her and to begin loudly proclaiming their (ii) __________. It seemed, in other words, to cause these newcomers not an iota of discomfort to behave in a manner that a casual observer might have characterized as (iii) __________ outright hypocrisy.

Blank (i)	Blank (ii)	Blank (iii)
factotums	reputability	tantamount to
detractors	magnanimity	reciprocal with
zealots	fealty	hinging on

84. Some substances toxic to humans induce lassitude and torpor, whereas others incite (i) __________ or (ii) __________.

Blank (i)	Blank (ii)
convulsions	complaints
contretemps	retching
apathy	drowsiness

85. Commentators and comedians had a field day with the widespread corruption in politics, so much so that one __________ defined a "gaffe" as a politician accidentally telling the truth.

wag
apologist
diplomat

egoist
transgressor

86. The seemingly pious minister, known for his (i) __________ pontifications, actually frequently indulged an unseemly (ii) __________.

Blank (i)	Blank (ii)
lascivious	ceremony
virtuous	vice
prudish	era

87. Americans who were the first to fight fascism in Spain during the Spanish Civil War were not allowed to join the US Army later, as their initiative led to the American government labeling them "__________ anti-fascists," and thus undesirable.

violent
erroneous
premature
prescient
communist

88. Paleontologists are always shown extracting dinosaur bones from pits in dusty deserts, but most of their research occurs in libraries, and most of their discoveries are ____________ from tomes every bit as dusty as the aforementioned deserts.

perused
garnished
gleaned
involved
interpreted

89. Ms. Llewellyn is known to gently ___________ students who don't do their homework, but because of her generally amiable demeanor, she refuses to punish anyone, and seldom even raises her voice.

pillory
detest
malign
penalize
chide

90. With so many of America's malls shutting down due to the financial crisis, there might soon be a significant dearth of locations in which teenagers can ___________; if we're not careful, they might soon have to start

actually doing useful things with their time.

flirt
loaf
sneak
mature
ruminate

91. The danger of giving too many long and bombastic speeches while on the campaign trail is that a politician can come off as ___________ rather than eloquent.

unscrupulous
voluble
frank
inane
glib

92. Strange to think that for the majority of the 1950's, the ____________ of nuclear holocaust was a daily part of people's lives, so much so that elementary school students ran drills in which they had to dive under their desks and curl up into a ball.

specter
annihilation
brutality
detonation
arms race

93. In their popular book *Sex at Dawn*, authors Christopher Ryan and Cacilda Jethá ____________ those who see paradigms for monogamy in nature, describing numerous animals and primitive peoples who have polyamorous relationships.

conspire with
block out
rail against
skitter around
pass over

94. The king's attendants insisted that he'd never made a bad decision during his entire reign, but the king ignored this ____________, recognizing it as the unctuous chatter of toadies.

idleness
regalia

blather
puffery
confabulation

95. The actress was desperate to (i) ____________ the part in the new Michel Gondry film. It was (ii) ___________ role, with lots of onscreen time and a number of long scenes that would allow the actress, usually cast as a vapid ingénue or bombshell, to truly show her dramatic range.

Blank (i)	Blank (ii)
land	a grave
ascertain	a plum
ensure	an agonistic

96. Those who believe that (i) _____________ is a thing of the past should remember that women were only granted (ii) ______________ in 1920, and not until 1965, with the passing of the Voting Rights Act, could African Americans feel confident that they could vote safely in US elections.

Blank (i)	Blank (ii)
disenfranchisement	hegemony
chauvanism	suffrage
plutocracy	dispensation

97. Because the superintendent was a (i) ___________ rather than a professional plumber, it's no surprise that he proceeded mostly by (ii) ______________. Nor was it particularly surprising when the new seals burst. The following week, (iii) ________ was hired to repair the superintendent's mistakes expediently.

Blank (i)	Blank (ii)	Blank (iii)
lummox	rote	a crack
bureaucrat	guesswork	an extortionate
layperson	numbers	a senescent

98. Some have argued that people who work in the service industry experience subtle psychological damage each time they (i) ___________ their true emotions in order to put on a smile for the customer. Unfortunately, such efforts are (ii) ____________ of the job, as no one wants to be served by a weepy waiter or an irate flight attendant.

Blank (i)	Blank (ii)
masquerade	the apex
simulate	the conceit
mask	part and parcel

99. Unable to raise sufficient funds for his quirky independent feature, the producer was heard to curse the "philistines" and ________ the lack of support for experimental art in this country.

debase
decry
deface
delimit
defer

100. His grandmother's house was always a bedlam of porcelain figurines, collector's spoons, and other (i) ___________ doodads. But it hardly would have been (ii) ___________ to tell her that he thought her choice of décor was vulgar; in fact, he had to think (iii) __________, because the avaricious youth was gunning for a big birthday present from her.

Blank (i)	Blank (ii)	Blank (iii)
tacky	tactful	amicably
vitreous	rancorous	tactically
grizzled	doting	duplicitously

101. The captain (i) ____________ (ii) ____________ for as long as he could, but eventually the crew became frustrated with the small portions of mead and the dearth of plunder, and decided to take matters into their own hands.

Blank (i)	Blank (ii)
dissuaded	sea change
warded off	mutiny
depreciated	putsch

102. Of course we would all like to believe that our every success is of our own manufacture, but to believe that is to neglect the (i) __________ element present in all lives, beginning with a birth lottery that assigns to some such gifts as intelligence and to others such (ii) __________ as wealth.

Blank (i)	Blank (ii)
notional	encumbrances
inchoate	dispensations
aleatory	piques

103. Anne could have made a larger salary elsewhere, but such (i) __________ as ample vacation time and excellent insurance persuaded her to (ii) __________.

Blank (i)	Blank (ii)
perquisites	negotiate
remuneration	abscond
disincentives	abide

104. Jackson's supporters praised his earthy speech as evidence of his common touch, while his (i) __________ condemned it as (ii) __________.

Blank (i)	Blank (ii)
interlocutors	vulgar
detractors	obtuse
contemporaries	genteel

105. Economists have developed such sophisticated and (i) __________ mathematical tools for modeling human behavior that other social scientists often deploy those tools to model and help (ii) __________ even decisions that have no obvious economic consequences.

Blank (i)	Blank (ii)
eclectic	interpolate
populist	replicate
versatile	explicate

106. Patients who stop taking antibiotics when symptoms subside contribute to the evolution of drug-resistant strains, because an incomplete course of treatment spares the most __________ bacteria.

widespread
immature
robust
benignant
notorious

107. Children who are recognized as preternaturally intelligent often go on to fulfill their early promise, contrary to the stereotype of maladjusted __________ wasting their gifts.

prodigies
crackpots
misanthropes
dynamos
juveniles

108. Freud's structural model of the psyche should be understood as (i) __________ device, useful for inciting and guiding discovery, rather than as an attempt to (ii) __________ physical relationships among parts of the human brain.

Blank (i)	Blank (ii)
a heuristic	dictate
a hermeneutic	ameliorate

an allegorical	represent

109. The silent-film pioneer Harold Lloyd made a virtue of the (i) __________ limits of his day, playing men so (ii) __________ it was easy to imagine it was the character rather than the medium who lacked a voice.

Blank (i)	Blank (ii)
artistic	avant-garde
commercial	diffident
technical	reluctant

110. After the séance ended, the participants were of two minds as to whether the psychic had _________ a spirit, but everyone had felt the presence of something new in the room.

convoked
evoked
elicited
excogitated
coalesced

111. When he joked about his shortcomings as a husband, his humor grew too (i) __________ for our comfort, becoming so bitter and ironic that we found ourselves (ii) __________.

Blank (i)	Blank (ii)
frantic	disquieted
mordant	antipathetic
choleric	sanguine

112. Churchill conceded that Russia, (i) __________ an ally of France and Great Britain, was in fact (ii) __________ to their security, but argued that Russian national interest would compel it to make (iii) __________ with those countries.

Blank (i)	Blank (ii)	Blank (iii)
steadfastly	pledged	common cause
nominally	indifferent	rack and ruin
sporadically	immune	vim and vigor

113. Just as Philip K. Dick's acclaimed science-fiction shows his gift for dystopian fantasy, so too his early efforts at mainstream novels (i) __________ his ability to represent the more (ii) _________ world.

Blank (i)	Blank (ii)
attest to	ebullient
belie	quotidian

gainsay	fantastical

114. Those who knew the editor as a (i) __________ and uncommunicative young man are surprised to find him so vocally dogmatic in his (ii) __________, though perhaps it is precisely because he has always been so slow to think that he has taken to the security of his (iii) __________ positions, which at least allow him to be quick to judge.

Blank (i)	Blank (ii)	Blank (iii)
costive	dotage	arbitrary
homely	jouissance	doctrinaire
cynical	quiescence	picayune

115. Always chary of doling out criticism deemed too __________, the pundit generally adhered to a more diplomatic approach.

dull
acerbic
muted
indistinct
tactful

116. Unfortunately for the young hire, the amiable, gregarious air of his boss during the initial interview belied a vastly more _________ style on the job, a fact which he learned to his chagrin within the first few days of employment.

draconian
friendly
fatuous
disconcerting
nonplussed

117. (i) ___________ comprehension of the character of Italian wine is impeded not only by labyrinthine complexities of vineyards and varietals, but also by fluctuations in environmental conditions from year to year, which render even the most reliable vintages subject to (ii) ____________.

Blank (i)	Blank (ii)
An exhaustive	efficient taxonomy
A futile	remarkable variance
An irredeemable	mitigating circumstances

118. Added to the (i) _____________ with which the clerk seems to treat his clients is what appears to be a more general lack of respect for his office; he seems to treat the whole thing as if it were some grand (ii) ________.

Blank (i)	Blank (ii)
casual nonchalance	gesture
profound meticulousness	farce
idle envy	tirade

119. Proponents of climate change theory have lamented the steady decrease in the ________ of Greenland; they fear that the melting ice will raise sea levels around the world, perhaps flooding coastal cities and much of Florida.

flukes
cads
dirges
floes
fibs

120. The more deeply one delves into the relevant literature, the more apparent it becomes that psychoanalysis is a practice (i) ____________. Even tenets that some might deem (ii) ___________ to the general philosophy, such as the notion that the human psyche is primarily governed by conflicting desires and is formed in large part by early childhood experiences, are by no means accepted as gospel, even by some of its most (iii) ____________.

Blank (i)	Blank (ii)	Blank (iii)
teeming with ridicule	critical	esteemed beneficiaries
devoid of substance	immaterial	quarrelsome factions
rife with contention	anathema	seasoned practitioners

121. The pair's apparent antagonism could easily be written off as (i) _________ pure and simple, but further scrutiny should render (ii) _________ the fact that the rivalry also confers a fair amount of (iii) __________, insofar as it provides each an opportunity to derive inspiration and motivation from the other.

Blank (i)	Blank (ii)	Blank (iii)
dissimulation	useless	worthless pride
antipathy	patent	mutual benefit
flagrancy	spurious	tacit disagreement

122. In this day and age, side show barkers, competing with the unfathomable number of spectacular oddities daily displayed on the internet for free, must increasingly lard their pitches with flights of fancy and soaring _______, arching far beyond reality, to fill the seats in their arcades.

tit-for-tat
parables
conundrums
innuendos
hyperboles

123. (i) __________ the law had little impact, but it was (ii) __________ by subsequent legislation providing funding and enforcement.

Blank (i)	Blank (ii)
Justifiably	rendered moot
Unbelievably	given teeth
Initially	kept at bay

124. Evoking both horror and joy in its audience in equal measure, the opera became an instant classic of __________ technique.

macabre
figurative
articulate
contrapuntal
contrived

125. The famous Notre Dame cathedral in Paris took almost two hundred years to complete; this immense architectural effort included the first notable use of a flying _________, but this renowned feature was not part of the original design and only employed when the walls forming the nave began to crumble and needed additional support.

ballast
albatross
hallmark
buttress
trademark

126. While no single empirical investigation can ever conclusively prove the (i) __________ of a theory, the fact that our data are (ii) __________ findings from over a dozen independent labs worldwide bodes well for our framework's resilience.

Blank (i)	Blank (ii)
rationality	consistent with
veracity	founded on
candor	antithetical to

127. A full account of the complexities of sleep, sought after by scientists, philosophers, and mystics for millennia, continues to elude us. That we are still so ignorant about a topic so (i) __________ to our daily lives is at once fascinating and (ii) __________.

Blank (i) Blank (ii)

mysterious	deeply humbling
obscure	fully impenetrable
pertinent	totally blatant

128. Mozart's brief life exemplified a discrepancy between fame and means: as his musical star (i) __________ beyond measure, his income (ii) __________.

Blank (i)	Blank (ii)
abated	grew exponentially
waxed	remained exorbitant
dwindled	barely stirred

129. Though some judges have found in the Third Amendment to the US Constitution a (i) __________ for a right to privacy, it was drafted primarily to appease opponents of the Constitution, and was particularly a (ii) __________ to those antifederalists who sought to prevent the new state from maintaining a standing army. Since Article 1, section 8 of the Constitution explicitly grants to Congress the power to raise and maintain an army, the Third Amendment was largely (ii) __________ before it was written.

Blank (i)	Blank (ii)	Blank (iii)
detriment	boon	daft
basis	sop	gauche
counterargument	bolster	moot

130. Finally, after refusing for a decade, the family patriarch, weakened by age and infirmity, surrendered to the impassioned pleas of his avaricious nieces, and gave his _________ to the risky investment stratagem.

assent
ascent
dissent
descent
asseveration

131. Even thrill-seeking visitors to amusement parks will avoid those attractions with a reputation for real (i) __________, like those at the now-shuttered Action Park. These patrons want not danger but its (ii) __________, a ride that (iii) __________ but is in fact perfectly safe.

Blank (i)	Blank (ii)	Blank (iii)
peril	complement	satisfies
titillation	simulacrum	mollifies
lavishness	abettor	terrifies

132. Desktop publishing allows (i) __________ to do for themselves the work once reserved for professionals whose (ii) __________ or other training developed design skills along with narrow technical mastery.

Blank (i)	Blank (ii)
dilettantes	sensibility
artisans	acumen
idealogues	apprenticeship

133. In order to return to the castle, the single surviving knight had to somehow get past redoubtable warriors who threatened to ______ him limb from limb.

garner
garnish
garland
rend
render

134. The American system of government is wonderfully responsive to crisis, but seemingly (i) __________ to perennial troubles, limping along with (ii) __________ solutions to (iii) __________ problems.

Blank (i)	Blank (ii)	Blank (iii)
empathetic	faddish	coarse
obliging	tendentious	chronic
indifferent	makeshift	acute

135. If impact on one's contemporaries is the test of (i) __________, Flann O'Brien's *The Third Policeman* cannot be said to be among the most significant postmodern novels, as it went unpublished and unread for twenty-seven years. The literary theorist Keith Hopper, though, appeals to standards other than (ii) __________ when he argues persuasively that *The Third Policeman* is among the most important of early postmodern works, not least because of its deep subversion of both enlightenment and modern traditions in literature.

Blank (i)	Blank (ii)
eminence	currency
modishness	influence
conversance	dissolution

136. One liberal activist asserts that politicians' tendency to (i) __________ talk of class warfare stems largely from a communal state of denial, a refusal to accept that we already occupy a highly (ii) __________ society.

Blank (i)	Blank (ii)
denigrate	socialized
besmirch	balkanized
declare	politicized

137. If anthropologists reveal too much of their culture to the subjects of their (i) __________, they risk the

corruption of the sometimes fragile societies they seek to understand, as well as other (ii) __________ that may appear incongruous with their task.

Blank (i)	Blank (ii)
objectivity	sequelae
society	inconsistencies
scrutiny	paradoxes

138. There is little agreement among specialists about whether the Second Amendment provides __________ guarantee of a right to bear arms for private citizens, or whether it was instead meant to allow the populace to protect itself in lieu of a military.

an earnest
an amended
a questionable
a defeasible
an ironclad

139. Free speech is often taken by the general population to be a sacred guarantee of the right to dissent, but this high valuation may be only (i) __________ in a society where any divergence from the general opinion leaves one labeled (ii) __________ or even an outcast.

Blank (i)	Blank (ii)
a nostrum	an assassin
a catalyst	a nuisance
an adjuvant	a miser

140. Faced with _________ job market, many young people are returning to graduate school rather than attempting to compete for the few available jobs.

a myopic
an anemic
a botched
a titular
a fissile

141. Nora Ephron's 1989 film, *When Harry Met Sally*, was more than a hit movie—for a generation, it was a cultural _________ regarding the often fraught relations between men and women.

rudder
bolster
touchstone

stanchion
cornerstone

Text Completions Answers

1. **Apportioned.** You begin by learning about football, which is by far the *most popular sport in Europe*. The word *whereas* then creates a contrast, so you need American fans to be evenly *split* between their many sports. *Measured* is close, but doesn't quite divide things up the way you need it to. *Apportioned* means "divided and allocated."

2. **Delusion.** The second half of the sentence tells you that it would take a "miracle" for a person to get to another solar system. In other words, it's more or less or *impossible*. Both *dementia* and *hysteria* are too negative and extreme. *Delusion*, meaning "a belief that is maintained despite being contradicted by reality," is the best fit.

3. **Crowned, dismissed.** "Peculiarly enough" is the clue that indicates that the blanks oppose each other, and *best English language playwright* is the clue that tells you that the first blank must be positive, so the second blank must be negative. *Crowned* is the only positive word choice for the first blank. In the second blank, *demonized* is much too strong an attitude to direct towards someone for lacking education; *dismissed* is a more appropriate word.

4. **Undermine, unchallenged.** The first word of the sentence tells you that the two parts of the sentence will oppose each other. The second suggests that the president felt that the professor's comments couldn't be allowed to stand as they were, so they could not go *unchallenged*. If the university challenges a faculty member, however, this tends to subvert or *undermine* the faculty member's authority, which is what you need for the first blank. Note that, if you picked *defend* for the first blank, you would need a word like *undefended* in the second blank — none of the choices for the second blank match *undefended*.

5. **Contortion, swindle, unconscionable.** It's easiest to start this sentence with the second blank, where record labels are described as *demanding indefensibly large percentages of [artists'] profits*. This means the second blank should be *swindle*. The third blank then uses the word *equally*, implying that downloading a song illegally is also a kind of *swindle*. Both *justifiable* and *scrupulous* are positive words, so you need *unconscionable*, meaning "not right or reasonable." Now you can return to the first blank. You know that the prompt as a whole argues that downloading music is bad, so "in order to defend" it, one must be slightly dishonest. *Probity* and *leniency* are both good things. You want *contortion*, meaning "twisting or bending out of something's normal shape."

6. **Cited, correct, tangential.** You can start with the first blank here, where a fact about life expectancy relates to the world improving with time. Clearly this would be *true* if life expectancies have increased with time, so you can simply use *cited* in the first blank. The second blank, however, says that this statistic is slightly misleading if you consider a new fact (i.e., that most children died in childbirth). This means you need to *correct* for the new piece of information. The final sentence seems a bit off-topic (suddenly you're talking about height, when you had been talking about life expectancy), so you should use the word *tangential* to set it off. Note that *sidereal* doesn't have anything to do with being "on the side" — it means "related to the stars."

7. **Preposterous.** The blank must agree with the clue that the possibility is remote. Note that *prescient* is the opposite of what you want — because the prediction about the future was entirely wrong, it did not sound *prescient.*

8. **Enduring.** The blank must agree with *strongly influenced legal codes well into the 20th century*; thus, the code of law is *enduring*. Note that incorrect choice *fleeting* is exactly the opposite of the intended meaning.

9. **Superstitious.** The pivot word *because* indicates that the blank will agree with the (interesting but meaningless) pattern that *every president elected in a year ending in zero had died in office*. Don't fall for traps like *conservative* (which would only be attractive if you were using outside information).

10. **Amusing, caustic.** The clue "usual" means that the two blanks must match "humorous but acerbic" — and in order.

11. **Circle.** The clue "in turn" leads to the correct answer. Incorrect answers *irony* and *comedy* add an idea — humor — that was not indicated by the sentence.

12. **Abreast of.** The word *although* gives you a clue that what the professor is doing is unusual or not required of a retired person. "Takes pains" also tells you that what she is doing is difficult. To keep *abreast of* a topic is to remain current.

13. **Proponent, catalyst.** You know that the woman in this sentence *finally got the bill onto the legislative agenda*, so she is definitely an advocate, or *proponent,* of the legislation. A *catalyst* — a term adopted from chemistry — is someone who creates change.

14. **Trumpet, taciturn.** You know that Jeremy is successful, but it takes his family a long time to "discover" his accomplishments. Thus, Jeremy does not brag — or even report the facts! To *trumpet* is to "talk loudly or report something to everyone." In the second blank, only *taciturn* (quiet) works. Don't be fooled by *prodigal,* which occurs in the expression "prodigal son," but actually means "wasteful." (The original *prodigal son* from the Bible ungratefully wasted his inheritance.)

15. **Dumbfounded, turbid.** In the first blank, you need a word like *amazed*. *Dumbfounded* is the only match. *Turbid* which is related to *turbulent*, is the right word to describe choppy waters in which something could easily be lost. (*Turgid* means "stiff" and *tepid* means "lukewarm.")

16. **Emblematic of, pragmatic, posit.** You learn at the end of the sentence that Hursthouse puts forth the same views as many other virtue ethicists. So in the first blank, you want something like *representative of.* The only match is *emblematic of.* You then learn that Hursthouse thinks ethics is "neither situational nor utilitarian" — the next part of the sentence will have a first part that matches up with *situational*, and a second part that matches up with *utilitarian,* (GRE sentences often use a pattern of mentioning two things, and then giving more information about those two things *in the same order.*) *Base one's judgments on subjective concerns* refers back to a *situational* approach to ethics, and *a* ___________ *weighing of pain and pleasure* refers back to a *utilitarian* (or *practical*) approach. Thus, *pragmatic*, which means "practical," matches. Finally, critics would definitely *agree* with the idea that virtue ethicists "seek to enshrine their own prejudices," so in the third blank, you just need a word like *say* or *claim. Posit* is the only match.

17. **Roundly, panned.** Wow, what a terrible movie! Not a single reviewer could think of *one* nice thing to say? You need two words that mean something like "unanimously criticized" or "100% disliked." *Roundly* means "emphatically," or "so thoroughly as to leave no doubt." *Panned* means "reviewed negatively" and is almost always used to refer to plays, movies, etc., so the word is a perfect match here.

18. **Modestly, sparingly, do nothing to excess.** The only clue you have is that Socrates advocated *a life of moderation*, so you need words that mean *moderately* in both the first and second blanks, and a phrase that means "live moderately" in the third blank.

19. **Inundated.** Upon reading this sentence, you might have filled in something like "went to" in the blank. One good

clue is that the carpetbaggers needed suitcases, so they were probably traveling. Process of elimination works well here — *vanquished, blacklisted, boycotted,* and *berated* all add ideas not indicated by the sentence. Only *inundated* (flooded, overwhelmed) makes sense.

20. **Coda.** You need a word that means something like a *bad ending.* A *coda* is an ending that sums up what came before, which, in this case, was pretty bad. A *rampage* is bad but does not fit grammatically before "to," nor does it correctly describe an earthquake — rather, murderers or mobs go on *rampages.*

21. **Presciently, simpatico.** *Presciently* means "seeing into the future," or "as though seeing into the future." *Simpatico* matches the idea of being "of one mind."

22. **Inured to.** *Casting a blind eye* to suffering means that these people ignore or pretend not to see the suffering. Thus, they are *inured to* suffering — they are hardened and desensitized as a result of seeing so much of it. Trap answer *exempted* would mean that the people described are no longer personally subject to the suffering. You don't know that, of course. You only have the clue that the people *cast a blind eye*, or *ignore* suffering. Because they *ignore* suffering, they also wouldn't be *maudlin* or *effusive,* which are about expressing strong feelings.

23. **Coalesced.** You need a word that means *came together. Coalesced* matches perfectly. *Agglomerated* is a bit of a trap — to *agglomerate* is to collect or form into a mass or group. Since the people were already in a group, the word is inappropriate (*agglomerate* is also used more for things than for people).

24. **Anachronisms.** The director inserted some *things in the wrong time period,* or *anachronisms.*

25. **Enigma.** Since no one has been able to decode the tablet, you need a word that means "mystery."

26. **Rail against, moralists, utilitarians.** You know from the definition provided of progressive taxation that those who "feel it unjust" that the "most industrious" are "penalized" would definitely be against progressive taxation, so *rail against* matches in the first blank. The word *unjust* is a good clue that *moralists* matches in the second blank (the other two words make no sense — *autodidacts* are people who teach themselves and *recidivists* are people who return to committing crimes). In the third blank, you need another group of people who are against progressive taxation but who are different from the *moralists* — in this case, because they are making a *practical* argument, not a moral one. Only *utilitarians* matches.

27. **Inform.** The poet's childhood memories *color*, or influence, her work. You need a similar word for how war *influences* her poetry. *Inform* — to give substance, character, or distinction to — can be used in this way.

28. **Erroneously, unbeknownst.** Since the peanut is not really a nut (it is a legume), *erroneously* (incorrectly) matches in the first blank. *Unbeknownst to* means "unknown by."

29. **Tremulous, chary of.** All of the clues in the sentence are about being scared, so *tremulous* matches best in the first blank. You know that the child finds diving "even more frightening" than water in general, so *chary of* (suspicious or slightly scared of), a less extreme version of *frightening,* fits in the second blank.

30. **Sentries, fecund, progeny.** You are told that solider ants *guard the colony and supply it with food, thus acting as both __________ and laborers*. In this sentence pattern, the first blank (*sentries,* or "guards") matches up with "guard the colony" and "laborers" matches up with "supply it with food." You know that the queen can *continually populate the colony*, so *fecund* (fertile) goes in the second blank and *progeny* (offspring) goes in the third.

31. **Ephemeral, heralded.** Since the government lasted only about two months, it was *ephemeral,* or "short-lived." Despite this, it was *regarded*, or *celebrated*, as the emergence of a powerful working class: only *heralded* matches.

32. **Desuetude, overrun.** *Desuetude* means "disuse," although the word is usually used in a legal context (a law that has not been enforced in a very long time may fall into *desuetude* and in some legal codes be no longer valid). *Overrun* matches the idea of having many *vermin* (rats and other undesirable animals). Don't fall for trap answer *hampered* — the house couldn't be *hampered* because it wasn't trying to accomplish anything.

33. **Complaisant, artful.** You know from the sentence that the boss's son is "pleasant" but that he is not a good person to bargain on behalf of the company. In the first blank, *complaisant* means "tractable, manageable" and in the second, *artful* means "crafty, cunning, shrewd" — qualities important for bargaining.

34. **Sanguine, peccadilloes.** You know that the man's personality is pleasing — so pleasing that it's *hard to be bothered by* something in his past. So, the first word should match *pleasing* and the second should be something negative that others might overlook. *Sanguine* means "cheerful and optimistic." (Note that *high-handed* might sound positive, but actually means "tactlessly overbearing.") *Peccadilloes* are minor faults or sins — small enough that they might be overlooked if the person has other virtues.

35. **A risible.** The substitute teacher is speaking French in the wrong room and confusing everyone? Hilarious! *Risible* means "laughable" and is the only choice that makes any sense here. Note that *arch* can mean "mischievous, playful, roguish," but is not appropriate here because the French teacher did this "accidentally."

36. **Verdant, castigated.** You know that the employee is young and makes mistakes. *Verdant* means either "green and lush" (like a forest), or "new and inexperienced" (the use of "green" as a metaphor, as in, "She's still green at her job.") Since the mistake was so expensive, it makes sense that the employee would be *castigated,* or severely criticized. (He wouldn't be *deposed* — that word is for kings, dictators, etc., forced out of power.)

37. **Facile.** The contestant's suggestion is rather childish; you also have the word *naively* as a clue. *Facile* matches well, and has the sense of "superficial; so simple it's actually stupid." Note that *convoluted* often describes an unconvincing argument, but a complicated one — this choice does not match the clue "naively."

38. **Convivial.** Extroverts are sociable, outgoing, etc. Only *convivial* matches.

39. **Motley, even, sole.** This question is an excellent lesson in the principle, "Don't add anything to your reading of the sentence that wasn't there already." What were you told? Just that the first short story collection had many diverse voices, and the second collection has "a ________ narrator" — in other words, just one speaker. Therefore, you *don't* know that the stories or narrator are *insightful, facetious, lonely,* or *disingenuous*.

40. **Novel.** The correct answer must match the clue "new" and the structure that contrasts the blank with "time-honored."

41. **Hysterics.** The blank must contrast "maintain her composure." Note that while, in casual speech, many people use *hysterical* to mean "funny," that is a metaphorical usage — *hysterical* actually relates to being very upset. The clue "bereaved" eliminates the incorrect pair of *laughter* and *mirth*. (The wrong answer *flames* might have caused you to burst into laughter or mirth, though.)

42. **Deification, garlands.** The straightforward structure and positive spin indicate positive choices for both blanks. Additionally, the clues "virtual" and "altar" suggest that the answers will be metaphorical rather than literal, thus eliminating *adoration*. Both the incorrect answers for the second blank are negative.

43. **Artifices.** The pivot means the answer must be in contrast to the clue, "truth." Trap answers *palettes* and *aesthetics* are related to painting, but don't match the meaning of the blank.

44. **Pedantry.** The correct answer should parallel "dogmatic" and match the negative spin of *enervates rather than inspires*. Only *pedantry* (being arrogant in relation to learning) and *parsimony* (stinginess) are negative, and only *pedantry* fits the idea of an uninspiring professor.

45. **Minutiae.** The correct answer must reflect the clue, "petty," and oppose the idea of "lofty goals." Lincoln felt that McClellan paid so much attention to details that he ignored the big picture.

46. **Appealing shyness, enthralled.** As is often the case, the second blank is easier; given the structure, it must match the clue "captivating" in meaning and spin. The first blank must parallel "hesitancy and understatement" and also have a positive spin.

47. **Volatile, figuratively unstrung, caned.** While the stimulus is long and a little convoluted, all three blanks agree with the clue and structure given by *to the point of violence*. Note that *literally insane* is too strong, due to the inclusion of *literally*. The word *caned* here really does mean "beat with a cane."

48. **Dissipation, profligate.** "Ironically" is a good clue here — the commentator condemned a quality that he himself actually possessed! Both words should be negative, similar in meaning, and matching the idea of "bankruptcy." *Dissipation* and *profligate* are both related to wastefulness or irresponsible living.

49. **Progression, dogmatism, constructing a hierarchy.** The first blank must contrast "standing alone" and complement the clue "linear." The second blank contrasts "free spirit." Since Nietzsche "rails against" the word in the second blank, you would also expect that word to be negative. *Dogmatism* matches perfectly. For the third blank, the dash and the use of "such" indicate that your choice must match the idea of "ordering." A *hierarchy* is "a well-defined ordering of different levels or ranks."

50. **Informed.** The clue "just as" tells you that the second part of the sentence will need to mirror the first part. Thus, the blank must match the meaning of *provided with a model*. *Informed* can be used in this way. Don't be tricked by *duplicate* (to provide a model isn't to make an exact copy) or *foresaw* (the remnants of Roman culture didn't tell the future).

51. **Enhances.** You simply need a word here that means "helps." Don't fall for *ensures,* which is too extreme. Having a head that can measure spaces is cool, but hardly guarantees the success of a species.

52. **Anomaly.** You are told that politicians lie so much that *candor (truth) is an* __________. You need a word that means "exception" or "something unusual."

53. **Eccentricity, trenchant.** The multiple negatives — "no other," "but," "more than" — make the first blank tricky. Fortunately, this blank follows "no other contemporary novelists whose works resemble [his]." Thus, you need a word that means something like "uniqueness." In the second sentence, Pynchon's work is being contrasted with "lightweight banter." *Trenchant* conveys that Pynchon's work is serious and cutting.

54. **Berserk, recede.** Transferring power to "less temperate" forces would cause chaos. You also have the clue at the end — a "more peaceful" period comes later. So you need a pretty extreme opposite of "peaceful" in the first blank. Only *berserk* works. In the second sentence, excesses *recede* (reduce, pull back), allowing a more peaceful phase.

55. **Parlous.** The correct answer must match "dangerous." Note: *suicidal* is too extreme and not indicated by the sentence. (Even if you know what eventually happened to the Donner party, don't use outside information!)

56. **Trundled.** The clue "slowly" points to the correct answer — only *trundled* can describe a slow movement from a car. *Careened* describes a fast, out-of-control movement, and *tarried* describes a person lingering.

57. **Sangfroid, mercurial.** A tricky structure — while the second blank is opposite of the clue "equilibrium" (because of the "however"), the first blank should actually have the same meaning as *equilibrium.*

58. **Apposed, eremitic.** The pivot word *yet* means that the first blank will oppose "solitary," and the second will agree with it. While the correct answers are extremely difficult words, all of the wrong answers are pretty standard GRE vocabulary words, so you should be able to answer this one by process of elimination.

59. **Bleated, oligarchy, turpitude.** "Piteously" is the clue for the first blank. *Bleated* is used metaphorically here — to *bleat* is to make the weak, wavering cry of a sheep, goat, or calf. Basically, saying that someone is bleating is calling his complaints pathetic, which matches with the clue "piteously," as well as the later "bemoan." The part of the sentence after the semicolon matches the first part in structure — the second blank parallels "wealthy business leaders," and the third mirrors "corrupt the youth" (*turpitude* is "depravity or immorality").

60. **A stopgap, paragon, perpetuity.** The implication of the phrase *a more permanent solution* is that the original building was not permanent: it was a *stopgap*, or a "temporary fix." To "canonize" something literally means to "declare to be a saint," but in its figurative sense, it means to "hold up as an exemplar," or as a *paragon*. The third blank gets its clue from "originally," indicating that you are looking for an opposite of "transient."

61. **Amity, mores.** The pivot word *though* tells you that the negotiation did not actually proceed smoothly. The first blank comes after "appearance of," which indicates fake behavior. The word *disingenuous* backs up that meaning. You need something positive — the parties are showing false *amity,* or friendliness, out of a "superficial" adherence to customs or rules — *mores.*

62. **Debauchment, derogated from.** Our clue is the author's "dissolution" — he may have turned to drugs, promiscuity, crime, or just letting his life fall apart. *Libertinous* has a similar meaning, as does the correct word *debauchment.* In the second blank, you want something negative (to contrast with the opinion of the "others" who *felt that his dissolution added a certain glamour to his biography*).

63. **Didactic.** The children's story ended with a moral — "always be honest." The word *didactic* can mean "morally instructive" or just "instructive" — this book is a *didactic* work.

64. **Protean, quixotic.** In the first blank, you need an idea that contrasts with "concrete proposal." *Protean* matches nicely. For the second blank, you need a word that also represents the idea of *not* being concrete, as well as the idea of being *unlikely to ever come to fruition*. *Quixotic* matches perfectly. Note that *pragmatic* is the exact opposite; you have no idea whether the idea is *unorthodox*.

65. **An arcadian, hinterland, spartan.** A *refuge* is positive; you also have the clue "quiet and rural." *Arcadian* conveys a positive view of rural life. In contrast, *hinterland* conveys the idea of a *backwater* or undesirable, far-out place. In the third blank, only *spartan* can be negative.

66. **No-holds-barred, storied.** You are told that the biography is gripping, but not appropriate for young people. So, it's exciting — and probably full of sex, violence, or both. *No-holds-barred* and *storied* are the only choices that could even remotely match — surely the biography isn't pretending to be naive (*faux-naif*) or unwilling to change (*dyed-in-the-wool*), and the "life" isn't immature (*callow*) or stalled (*stonewalled*).

67. **Regressive, auxiliary.** Given the lengthy explanation of the club's history, the elderly male member's suggestion was "backwards," or "hopelessly old-fashioned." Only *regressive* matches. In the second blank, you want something that reflects the idea *play bridge and drink tea without having to worry about serious issues*. *Auxiliary* has this sense of "secondary" or "on the side."

68. **A deft, whet.** Due to the "although," you can infer that the first blank must be something positive — only *deft* works. In the second blank, *whet* means "stimulate." *Tout* is a bit of a trap — the professor could *tout* the historical figure as a fun thing to learn about, but he can't *tout* someone's interest.

69. **In cahoots, abetting.** The *Though* at the beginning of the sentence tells you that the second part of the sentence will oppose the idea of the big firm (the "industry leader") being punished while the small firm gets away with something. Indeed, the small firm is working with the larger one to do wrong — it is *in cahoots*. In the second blank, you need a word that means "complicit in" or "helping (in a bad way)." Only *abetting* works. Watch out for the trap answer *colluding*. This word does mean to cooperate in order to commit a wrong, but doesn't fit for grammatical reasons — you can't *collude* plans. Rather, you collude *with* X to do Y (*collude* with a competitor to raise prices).

70. **Disinterested.** You need a word that means *unbiased* (a good quality for a debate judge or moderator). *Disinterested* does not mean *uninterested* — rather, it means not having a "personal interest" in the topic, and therefore being unbiased.

71. **Fulmination, aspersion, muckraking.** *It was difficult to tell whether his tirade in fact had force of fact* means that it was difficult tell whether someone's angry speech (*fulmination*) was actually true. It is hard to tell whether his speech is "motivated by malice" (that is, "malicious lies," or *aspersion*) or "reportorial" information (*muckraking* "exposing corruption") that should actually be listened to. Note that *calumny* is similar to *aspersion* and cannot work in the third blank — while the second blank describes negative speech meant to cause harm to others, the third blank describes negative speech that exposes the truth.

72. **Agog, indelicate.** The sentence has two good clues — the grandparents value *seemliness*, the state of being proper and appropriate. The woman's behavior is *incorrigible* (resistant to correction), which definitely has a bad connotation.

73. **Risible.** The clue phrase here is "exaggerated hilarity," letting you know that the veteran thought the situation was funny or amusing. (*Superannuated* tells you that he was extremely old, thus exaggerating the impact of *a lifetime of injustice*. *Risible*, which means "laughable," is most appropriate.

74. **Groaning.** You might feel stuck if the idiom *groan under the strain* doesn't ring familiar to you. But process of elimination will still work here: none of the other words make sense in this context.

75. **An effete.** The clue *despite their diminutive stature* means that the second part of the sentence needs to go

against what one might think about physically small people; the clue *many would all-too-happily allow minor umbrage to escalate to barroom brawl* confirms that these jockeys are a contentious bunch. Because of the expression "far from" before the blank, you need a word that expresses what the jockeys are *not*. The word *effete* means "pretentious or refined, verging on wimpy and ineffectual."

76. **Belies.** The phrase *apparent simplicity* and *dizzying number of hours* indicate that you are searching for a contrast word, specifically one indicating that there is more to coffee than meets the eye. *Belies*, one meaning of which is "to be at odds with," is most appropriate here.

77. **Unkempt, surprisingly deliberate.** The tricky blank structure masks what is ultimately a not-so-tricky word relationship: that of opposites. Clues for this relationship can be found in the words *notwithstanding* and *if*. Essentially, the right answers will be words that mean, first, "cluttered," and second, "ordered."

78. **Heralded, appropriated.** The phrase "equally public" gives you a clue that the legislator's approval was also highly publicized, or *heralded*. *Ratified*, which means "officially approved," would be redundant in this context. For the second blank, you aren't given much of a clue about what the meaning should be. When this happens, only one answer choice will make sense (and usually that correct choice won't add much to the sentence), while the other, incorrect answer choices will drastically change the meaning. Here, *appropriated*, which means "allocated" or "assigned," fits nicely and doesn't add any unintended meaning. Neither *adduced*, which means "cited as evidence," nor *adumbrated*, which means "reported in an outline," is fitting.

79. **Benign, absconded.** The first blank hinges on the meaning of the word *purport*, which means to "pretend to be," or to "masquerade as." The sentence implies that the bureau is pretending to be harmless, or *benign*. For the second blank, you are looking for a word that means to "leave hurriedly"; *absconded* fits. Notice that *immigrated*, though close in meaning, isn't quite right because it means to "arrive at" a country or region rather than to "leave from" it. *Pandered*, which means "indulged another's desire," is not appropriate in this sentence.

80. **Congenial, erode.** When a relationship goes from something to "chilly at best," you can surmise that the relationship used to be warm or cordial: *congenial* fits best here. Knowing that the relationship has devolved allows you to turn to the second blank and determine that there has been an undermining, or an *eroding*, of "mutual trust."

81. **Flattery, fatuous, incongruous.** You need to go all the way to "blandishments," a word which denotes *flattery*, to find the clue for the first blank. *Officiousness*, which is the quality of being "annoyingly, domineeringly assertive," has close to an opposite meaning. The second blank is given away by *ignorant*: it indicates you are looking for a word that carries a meaning of "silly or foolish," or *fatuous*. The third blank requires a word that means "contrary to" or "at odds with" — *incongruous*. Don't be confused by *apposite*, which means "appropriate" or "fitting."

82. **Responsible for, eschews, fundamental.** Your first clue is "free will." One can only be *responsible for* the things one decides, or freely wills, to do. For the second blank, the word *instead*, even though it occurs later in the sentence, implies that Dr. Wegner is *avoiding* the traditional talking points of the controversy, or *eschewing* them. The word *though* indicates that the third blank contrasts with the word *tangential*. *Fundamental* is thus the most appropriate answer.

83. **Detractors, fealty, tantamount to.** *Erstwhile* means "past" or "former," indicating that those who are now gathering around the candidate were once doing something different — that is, criticizing, or *detracting*. They are gathering around her proclaiming their "loyalty," or *fealty*, in a way that is deeply inconsistent with their past actions. *Tantamount to*, which means "much as the same as" or "equivalent to," is best here.

84. **Convulsions, retching.** The two blanks contrast "lassitude and torpor" because of the pivot word *whereas*. Additionally, the word *incite* indicates that the blanks will contain words that are active rather than passive. Incorrect choices *apathy* and *drowsiness* are somewhat similar to "lassitude and torpor" and are therefore the opposite of what you want.

85. **Wag.** The sentence concerns "commentators and comedians." Since defining a *gaffe* (a mistake) as a politician accidentally telling the truth is meant to be funny, you need a meaning closer to *comedian.* A *wag* is a joker, although not necessarily a professional one.

86. **Virtuous, vice.** The first blank must mirror "pious." The pivot words *seemingly* and *actually* indicate that the second blank contrasts "pious" — a *vice* is a very bad habit.

87. **Premature.** The clues are "first" and "initiative." You don't know if these fighters were violent or erroneous — avoid opinions or outside knowledge.

88. **Gleaned.** You're told that paleontologists do research in libraries, so their discoveries must *come* from tomes (i.e., books). *Gleaned* means "extracted from various sources." *Perused* is the act of reading — you can *peruse* a book, but you cannot *peruse* a discovery from a book.

89. **Chide.** The adverb *gently* is important here, because it's impossible to "gently" *detest* someone or "gently" *pillory* them (*pillory* means "attack or ridicule publicly"). *Malign* is also too negative, and you already read that Ms. Llewellyn doesn't punish anyone, so *penalize* doesn't work.

90. **Loaf.** The most important part of this sentence comes at the end. If teenagers haven't been doing anything useful with their time, you need a word that means *do nothing useful with your time*. *Flirt* is something teenagers do, but it isn't useless by definition (it can be quite fun!). *Ruminating* can also be useful, as it involves thinking. The correct answer is *loaf*. Note that the overall tone of this sentence is a bit sarcastic.

91. **Glib.** The clue in this short sentence is the word *eloquent*, which you need to flip because of the "rather than." *Unscrupulous* and *inane* are negative but don't really relate to speech. *Voluble* doesn't have any negative connotations. *Glib*, meaning "fluent and talkative, but insincere and shallow," fits perfectly.

92. **Specter.** For this sentence, it's important that you know that nuclear holocaust never actually occurred, so it was only a *threat* that was a daily part of people's lives. A secondary meaning of the word *specter* (which can also mean "ghost") is "something widely feared as a dangerous occurrence." All the other words have a kind of categorical relevance, but they don't fit the sentence.

93. **Rail against.** This sentence shows you that the authors of the book come up with many examples against *those who see a paradigm for monogamy*. Both *skitter around* and *pass over* would imply the authors fail to find examples, and *conspire with* is the opposite of what you want. *Rail* means "complain or protest strongly," and is correct here.

94. **Puffery.** The words of the attendants are described as *the unctuous chatter of toadies*, which basically means "the sucking up of people who suck up a lot." While *blather* is empty talk, and *confabulation* is invented talk, only *puffery* means "exaggerated or false praise," which is exactly what you want here.

95. **Land, a plum.** The first blank needs to be a synonym of *get*. *Ascertain* doesn't make sense, and *ensure* is used

incorrectly — the actress might want to *ensure* that she gets the part, but it doesn't make sense to say that she wants to *ensure* the part itself. The second blank should describe a role with lots of onscreen time and good scenes; you need a synonym for *good*. That's exactly what *plum* means.

96. **Disenfranchisement, suffrage.** The sentence gives you two examples involving the restriction of voting rights. *Disenfranchisement* is the deprivation of the right to vote, which fits best with the topic. Similarly, *suffrage* is the right to vote, and is the correct word for the second blank.

97. **Layperson, guesswork, a crack.** You are told that the plumber is not professional, so you need a word for the first blank that is the opposite of "professional." A *layperson* is a non-professional. For the second blank, you need a word describing how a non-professional would work. *Rote* implies experience, and *by numbers* means "following simple instructions," which is illogical because there's no reason to believe the layperson plumber has any instructions. *Guesswork* means he's just guessing what to do, which makes the most sense. Finally, since the plumber successfully and quickly fixed everything, you need a word in the third blank that means "good." *Crack* means "very good." *Extortionate* means "overly expensive"; there's no reason to believe someone is good just because they're overpriced.

98. **Mask, part and parcel.** The sentence says that people who work in the service industry *put on a smile for the customer*, implying that they *hide* their true emotions in some way. *Masquerade* is close, but it means either "to pretend to be someone" or "to be disguised as something else." Neither of these are quite right, so the correct word is *mask*. For the second blank, you need something like *a requirement*. *Apex* is a high point (i.e., "The *apex* of my career was playing for the Knicks."). *Conceit* means a fanciful notion. *Part and parcel*, meaning "an essential piece of something," is the correct phrase.

99. **Decry.** *Decry* is "to speak disparagingly" and parallels the clue, "curse." None of the other choices are justified by the meaning of the text.

100. **Tacky, tactful, tactically.** The judgmental mind behind this sentence finds his grandmother's décor "vulgar," so the first blank should be a synonym for that. *Tacky* fits the bill. For the second blank, you need a word that describes what it is *not* to tell your grandmother you find all her knick-knacks ugly. *Rancorous* is the opposite of what you want, and *doting*, meaning "be extremely fond of," goes too far. *Tactful* fits perfectly. For the third blank, you need a word that describes being nice to someone in order to get a good birthday present. *Amicable* doesn't capture the strategy of it, but *duplicitous* is way too negative (it means "deceitful," which is not really an apt description for the act of refraining from telling your grandma that she has terrible taste). *Tactically*, meaning "by way of showing adroit planning," is correct.

101. **Warded off, mutiny.** The second half of the sentence says that the crew *became frustrated* and *decided to take matters into their own hands*, implying that they took over the ship. *Mutiny* fits perfectly. (A *sea change* is "a profound transformation," while a *putsch* is "the overthrow of a government.") For the first blank, you need a word implying that the captain tried to fight off the mutiny. *Dissuading* is something you can do only to people (i.e., "I dissuaded *the pirates* from mutinying."). *Warded off* means "fended off" or "averted," which is perfect here.

102. **Aleatory, dispensations.** A "birth lottery" is offered as an example of the sort of element the first word describes. Since a lottery is random, based on chance, you would like your first word to be something like "due to luck," and that is just what *aleatory* means. The second word looks to be roughly synonymous with "gifts." *Dispensations* will work there, while *encumbrances* and *piques* are both negative.

103. **Perquisites, abide.** For the first word you want something like "non-salary rewards," and the only word that means that is *perquisites* (often informally abbreviated as "perks"). The word *but* tells you that Anne did not leave for

a larger salary, but stayed for the perquisites. For the second word you want something like "stay," and only *abide* can have that meaning. (*Abide* more commonly means "tolerate" or "wait for.")

104. **Detractors, vulgar.** *While* tells you that the clause after the comma will oppose the clause before the comma. So instead of "supporters," you want "opposers." The word *condemned* further hints that the first word will be something like "opposers/condemners." Only *detractors* works. For the second word, you want a description that a detractor would apply to "earthy speech." So you want "earthy, but in a bad way" — perhaps "coarse" or "unrefined" would serve. The word *vulgar* fits those descriptions. "common."

105. **Versatile, explicate.** For the first word, you want a word that agrees with "sophisticated," but more importantly that explains why these mathematical tools are useful to those who aren't even doing math, so you want a positive word that means "useful in many contexts." *Versatile* is the best fit. For the second word, you'd want something like the "model," in the sense of "explain," that you saw earlier in the sentence. *Explicate* is the best match.

106. **Robust.** The bacteria that survive antibiotics would presumably be those hardest to kill. You might also want a word that means something like "drug-resistant." The only word that means anything like "hard-to-kill" is *robust.*

107. **Prodigies.** The children who disprove the stereotype are the children stereotyped, so you want a word that means "children who are recognized as preternaturally intelligent." Such children are *prodigies*.

108. **Heuristic, represent.** The first word is defined in the sentence as a device *useful for inciting and guiding discovery*. That's precisely what *heuristic* means. If you are not meant to take the model of the psyche as guide to the physical structure of the brain, then you want a word that means something like "show" or "depict" for the second word. *Represent* is the closest.

109. **Technical, diffident.** Since *the medium… lacked a voice*, it was technology of the day that prevented Lloyd from speaking on film, so the first word should be something like "technological." *Technical* will do. For the second word, you'd like an adjective that would explain why a character would fail to speak, perhaps something like "mute." *Diffident* means something like "self-effacing, very shy." *Reluctant* is a bit of a trap answer — it does not mean "reluctant to speak" (the similar-sounding *reticent* does, and the two words are often confused).

110. **Evoked.** *Evoke* — to summon — is justified by the text. *Convoke* — to call a meeting to order — is not quite appropriate (maybe if there were multiple spirits — also a problem with *coalesced*). *Elicit* also has the wrong spin; it is more frequently used in the sense of eliciting information or opinions.

111. **Mordant, disquieted.** The first word should match "bitter and ironic," and the second evidently means something like "uncomfortable."

112. **Nominally, indifferent, common cause.** "In fact" suggests that Russia did not have the same concern for the security of France and Great Britain as one would expect of an ally. For the first blank, we'd like a word that describes a not-fully-committed ally, and for the second blank, we'd like a word that means "not concerned." *Nominally* works for the first word, as it means "in name (only)." *Indifferent* works for the second, as it means that Russia didn't really care. The "but" that introduces the last clause tells you that Churchill argued that Russia would act as an ally, albeit only for its own national reasons. To make *common cause* is to act as an ally.

113. **Attest to, quotidian.** *Just as… so too…* suggests that the mainstream novels show a gift, just as the science-fiction novels do. The first word should mean something like "show," or "display." Among these answers, only *attest* has that meaning. Incorrect choices *belie* and *gainsay* both indicate an opposite direction, which is incorrect, since

the *just as… so too…* structure indicates same direction. The second word should describe the world of mainstream, rather than science-fiction, novels. You want a word that means "not fantastic," "ordinary," "everyday." *Quotidian* has just that meaning.

114. **Costive, dotage, doctrinaire.** You have a couple of clues for the meaning of the first word. It is paired with "uncommunicative" and seems to mean something like "slow to think." Only *costive* among our choices means anything like that. Its original meaning, "constipated," is still current, but the word is as likely to be used metaphorically to mean "slow to form or offer opinions." Skip the middle word for a moment, and hope that you can spot some clue for it in figuring out the third word — the third word seems to mean "dogmatic," and *doctrinaire* will do. You may not have much success anticipating an answer for your second blank, but fortunately two of the available answers are exceptionally bad matches — *jouissance* means "joy," and *quiescence* means something like "rest" or "being at rest." Nothing suggests either of these answers. *Dotage* means that period of old age when one's mental faculties decline (it can also mean "foolishness," especially foolish affection, regardless of age). This is the second correct answer: the editor was young, and is now old and foolish.

115. **Acerbic.** Crucial to this question is the word *chary*, which is essentially synonymous with *wary*. If the pundit is worried about appearing something, it is probably the opposite of what he or she actually does, which is be "diplomatic." You're thus looking for an anonym for "diplomatic." *Acerbic*, which means "scathing or bitter," is best.

116. **Draconian.** *Belied* means "contradict" or "be at odds with." Thus, the boss's on-the-job style is contradicted by his "amiable," or "warm, friendly" style in the interview. You also have the clue that the young hire (the new employee) experiences "chagrin," which is "mental distress or unease." *Draconian*, which means "harsh, strict, or severe," is best. Don't be confused by *nonplussed*, the primary meaning of which is "confused" or "surprised" — it doesn't capture the boss's strictness.

117. **An exhaustive, remarkable variance.** The blank structure for this problem is tricky; in such cases you need to pay attention to the specific words in the sentence that give hints to the meanings of the blanks. For the first blank, you are looking for a modifier to "comprehension." This "comprehension," you learn, is *impeded… by labyrinthine complexities*. In other words, total, or *exhaustive*, comprehension is difficult because of these complexities. For the second blank, the key word is "reliable." The relation of this word to the blank is given by "even the most," telling you that these vintages are subject to something opposite to "reliable" — that is, *remarkable variance*.

118. **Casual nonchalance, farce.** If the clerk treats his "office" (a fancy way to say his "job") with a general lack of respect, then he treats his clients with a similarly cool and distant manner, a manner of *casual nonchalance*. Note that the other two possibilities do not fit at all (*meticulousness* would be positive, and *envy* is not indicated by the sentence). As for the second blank, knowing that "grand farce" is a common English idiom is helpful but is not essential—the word *farce*, or "mockery," is the only word that fits. A grand *gesture* would be positive (a *grand gesture* would be something like an elaborate marriage proposal, apology, etc.), and there is no clue that the clerk is making a *tirade,* or angry speech, nor does it make sense to treat one's entire job like a *tirade.*

119. **Floes.** The blank agrees with the clue "melting ice," thus *floes*, a synonym for glaciers, is correct. Trap answer *dirges* relates to "lamented" but is not what is decreasing in Greenland.

120. **Rife with contention, critical, seasoned practitioners.** This sentence is itself rife with ambiguity, so the best way to tackle it is to hunt down the most revealing clue words. Arguably the most important giveaway word in the whole two sentences is the word *gospel*, which is itself tricky because its primary definition has religious connotations. In fact, a secondary definition of *gospel* is "accepted fact," or "agreed-upon truth." Another crucial word is *tenet*, which also signifies an "accepted principle." Finally, the word *even* is an important pivot that informs you of the relationship between these words and the second blank, which is where you should start. *Even* tells you that you

should be surprised. You should be surprised that certain *tenets that some might deem* _______ are *not* accepted as gospel. In other words, the blank should signify something crucial, or *critical*, to the general philosophy of psychoanalysis. Moving to the third blank, you see another "even," once again indicating something contrary to expectation. You would *expect* the people very experienced in the field, or the most *seasoned practitioners*, to subscribe most strongly to the central tenets of the field. Now return to the first blank. You learned in the second sentence that there is disagreement in the field, even between the most experienced practitioners, about the fundamental tenets of the practice of psychoanalysis. In other words, the field is subject to great debate, or *rife with contention*.

121. **Antipathy, patent, mutual benefit.** The first and third blanks are highly related, so consider them first. You learn in the first clause of the sentence that there is "antagonism," or hostility, between two people, but from the word *apparent* you learn that there is more to this than meets the eye. More than pure "hatred," or *antipathy*, is something that allows the two individuals to "derive inspiration and motivation from the other." In other words, there is a *mutual benefit* to the seemingly hostile relationship. Now return to the second blank. Because the structure of the sentence tells you that the antagonism *could be* written off, *but* further scrutiny renders it (blank), you know you want a word that goes in the same direction (that is, you will *not* be "writing off" the antagonism). The best word here is *patent*, which, when used as an adjective, means "obvious" or "evident."

122. **Hyperboles.** The answer must match the clues "flights of fancy" and "arching far beyond reality." *Hyperbole* is exaggeration and is the only choice that fits.

123. **Initially, given teeth.** For the first blank, you need something to contrast with "subsequent." Thus, you need a word that means "at first." In the second blank, you need something that will contrast with having "little impact" and will match the idea of a law now gaining funding and enforcement. Only *given teeth* has this (obviously metaphorical) meaning.

124. **Contrapuntal.** You are looking for a word that means "having two distinct components," since the opera is capable of evoking two distinct emotions. *Contrapuntal* means "having two or more independent harmonies," and is a good fit. *Macabre* is a trap, since it goes along with horror but leaves out joy. *Articulate* is related to speech and is inappropriate here. And while the technique may be *contrived* (not natural, planned) or *figurative* (using metaphor or other non-literal device to convey meaning), nothing in the sentence suggests either.

125. **Buttress.** *Buttress*, as a noun or a verb, relates to support and is therefore justified by the clue "additional support." None of the other choices relate to the text. Note that you certainly do *not* need to memorize architectural terms for the GRE, but you should definitely know the metaphorical meaning of *buttress* (to support), and the sentence provides the clue about "needed additional support," so this question is fair game.

126. **Veracity, consistent with.** No single experiment can ever prove the "truth," or *veracity*, of a theory. *Rationality* is close but connotes the idea of the theory being logical, rather than factual. For the second blank, in order for the theory to be supported, or "resilient," the results from independent labs should agree with each other, or be *consistent with* each other.

127. **Pertinent, deeply humbling.** Sleep is important, or *relevant*, to your daily life, making *pertinent* the best word for the first blank. Don't be misled by the fact that you don't fully understand sleep into choosing *mysterious* or *obscure*; both are trap answers (being ignorant about a mysterious or obscure topic is hardly "fascinating"). It is also grammatically incorrect to call a topic *mysterious* or *obscure to your daily life.* Regarding the second blank, the expression "at once X and Y" indicates at least a loose contrast, so you need something that could contrast with *fascinating*. The clue "so ignorant" is a good indication that what you want is *deeply humbling*. Don't fall for the trap answer *fully impenetrable*. While the topic of sleep is presented as somewhat impenetrable (that is, hard to

understand), the blank is not describing sleep, but rather your ignorance about it.

128. **Waxed, barely stirred.** You've been told that there is a "discrepancy" or mismatch between Mozart's fame and his financial situation, so whatever happened to one did not happen to the other. The clue "beyond measure" tells you that his musical star must have grown, so *waxed* is the only possibility for the first blank. That means Mozart's income must have either gone down, remained low or *barely stirred*, which is the correct answer for the second blank. It could not be the case that his income "grew exponentially" or "remained exorbitant," or there would be no "discrepancy."

129. **Basis, sop, moot.** The first sentence begins with *though*, so the first part containing the blank should be opposed to the second part, saying that the Amendment was actually meant to appease opponents. So, the best choice is *basis*. Your second word should echo "appease." *Sop*, which can mean "something given to pacify," is closest. Since Article 1, section 8 apparently made the Third Amendment of only abstract concern, you want an answer that means something like "of no practical importance." *Moot*, which can mean "of merely academic interest," is your best option.

130. **Assent.** While the structure might seem tricky, the blank must oppose *refusing for a decade* since the patriarch "surrendered." Agreement would be a good focus. The other choices do not fit the text and are essentially phonetic tricks. An *asseveration* is an emphatic assertion. Note the word *avaricious* in the sentence — it means "greedy," but it turns out that you didn't need to know that to answer this question.

131. **Peril, simulacrum, terrifies.** *Even* suggests that first word describes the sort of thing you might otherwise expect a thrill seeker to find attractive. You want something bad — a meaning like, "*too* thrilling, in a bad way." You are also told that *these patrons want not danger*, so you might anticipate a word as simple as "danger." *Peril* is the best choice. Although "titillation" does describe a sort of thrill, it is neither negative, nor does it suggest danger. The third word is probably easier to anticipate than the second, so let's skip ahead. "In fact" suggests that you want a word that means "seems to be unsafe," and the earlier "thrill-seeking visitors" suggests that you want something, well, thrilling. So anticipate "seems unsafe, thrills," and take *terrifies* as your closest synonym. *Satisfies* yields a perfectly fine sentence, but there is no reason to anticipate quite that meaning. To return to your second word, you now know you want a word that means something like "imitation," and *simulacrum* is the closest synonym among your answers.

132. **Dilettantes, apprenticeship.** Your first word should indicate an opposite of the "professionals" with "training." The word that suggests itself is *amateurs*, and the closest answer to that is *dilettantes*, which is roughly synonymous with "dabblers." The second word should suggest a type of "training," and among your options only *apprenticeship* does so. *Acumen* might be the result of training, but it is not itself training.

133. **Rend.** *Rend* means "to tear apart." This question relies entirely on vocabulary. *Garner* is to gather. *Garnish* is to decorate (or to subtract from a person's wages to pay a debt). *Garland*, as a verb, is to crown. *Render* is to make, do, or provide. None matches the meaning of the text.

134. **Indifferent, makeshift, chronic.** *But* suggests that your first word should be the opposite of "wonderfully responsive," perhaps "badly unresponsive." *Indifferent*, which means "without concern or interest," is the nearest match. Your second word should take from the phrase "limping along," so you might expect something like "make-do," or "expedient." The closest synonym to those is *makeshift*. Finally, the last word describes the sort of problems that don't receive adequate responses, the very problems described earlier as "perennial," so anticipate that same meaning. *Chronic* is the best choice.

135. **Eminence, influence.** Borrowing another word from the sentence, your first word should be something like "significance." The closest option is *eminence*, which suggests great importance. If Hopper argues that the book was *among the most important* in spite of the fact that it cannot have had an impact on O'Brien's contemporaries, then he

must appeal to some standard other than "impact." *Influence* is the word most nearly synonymous with "impact." Because "currency" echoes the concern about how widely read the book was, it might appeal, but it isn't the meaning we'd anticipate here.

136. **Denigrate, balkanized.** Beginning with the second blank, the clue is "class warfare." A society with class warfare is divided into hostile groups, or *balkanized*, but it need not be either *socialized* or *politicized*. This means that politicians try to dismiss or belittle talk of class warfare, or *denigrate* it. *Besmirch* means to "attack the honor of something" — although the word is negative, it isn't quite appropriate here.

137. **Scrutiny, sequelae.** Anthropologists are attempting to understand, examine, or scrutinize other societies. (Also, both *objectivity* and *society* are traps: they may be objective scientists, but their objectivity doesn't have "subjects"; and the societies they are studying are *not* theirs.) In the second blank, you are looking for something that means unintended or secondary consequences, or *sequelae*. The other answer choices are close in meaning to "incongruous," and would be redundant in the sentence.

138. **An ironclad.** Since the final part of the sentence is meant to go against the idea that the Second Amendment provides a "guarantee" of a private right, you need to fill the blank in the first part with something that reinforces the guarantee. *Ironclad* means "rigid or fixed," and does what you need here. *Defeasible* and *questionable* would undermine the guarantee. Nothing in the passage has any bearing on whether the guarantee is *earnest*.

139. **A nostrum, a nuisance.** When you have a society in which dissenting opinions are socially stigmatized, valuing the right to dissent seems useless. A *nostrum* is "an ineffective remedy," and is appropriately used metaphorically here. A *catalyst* helps a reaction or change to occur, and an *adjuvant* is "something that assists or aids," both of which would be the opposite of what you need. In the second blank, you are looking for something that is negative but weaker than "outcast." *Nuisance* works perfectly here. *Assassin* is too strong, while *miser* is unrelated in meaning.

140. **An anemic**. While *anemia* is a medical condition, *anemic* as a metaphor means "weak or lacking vitality." *Myopia,* or "nearsightedness," also makes a good metaphor, but describes people and their decisions. The job market hasn't been *botched* (who botched it?), and the other choices are nonsensical.

141. **Touchstone**. You need a clue that matches *more than a hit movie* (and goes with the idea of having meaning for a whole generation of people). *Touchstone* matches perfectly. *Cornerstone* doesn't work — a cultural *touchstone* is a single important event, place, work of art, etc. A cultural *cornerstone* would be something the whole society is built upon (like individualism, Islam, or the monarchy, depending on the society). A *rudder* is literally part of a ship related to steering; if you are lost in life, you are "like a ship without a rudder." Note that a *bolster* is pretty similar to a *stanchion*, at least when used metaphorically to mean a support (literally, a *bolster* is a pillow and a *stanchion* is an upright bar or post), so neither could be the answer.

Chapter 4
of

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Sentence Equivalence

In This Chapter…

Sentence Equivalence

Select the two answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole and produce completed sentences that are alike in meaning.

1. While the colonists would eventually push westward, first, they were in for a long, difficult winter, and the main challenge was to __________ their existing resources.

- ☐ sell
- ☐ peddle
- ☐ steward
- ☐ upend
- ☐ husband
- ☐ procure

2. James Joyce, the author of many novels, including *Lady Chatterley's Lover*, saw deeply into the hearts of his characters, but, in a life irony as subtle yet piercing as those endured by his characters, he himself could barely __________ text well enough to proof his own galleys.

- ☐ see
- ☐ feel
- ☐ walk
- ☐ move
- ☐ distinguish
- ☐ interpret

3. Emblematic of the slaughter of the First World War, 60,000 British soldiers died for naught on the first day of the battle of the Somme, literally consumed into a stew of blood and __________.

- ☐ grief
- ☐ gore
- ☐ steel
- ☐ bullets
- ☐ carnage
- ☐ anguish

4. Forty years ago, anthropologists firmly believed that Neanderthals and modern *homo sapiens* never mated, but advances in genetic testing proved that incorrect — such is the __________ nature of science.

- ☐ fallacious
- ☐ evolving
- ☐ counterfactual
- ☐ advancing
- ☐ vacillating

- ☐ undeterminable

5. The daily routine of responding to lobbyists and exchanging polemics with intransigent political opponents seemed at odds with the president's __________ tendencies.

- ☐ tenacious
- ☐ idealistic
- ☐ dialectical
- ☐ contentious
- ☐ quixotic
- ☐ altruistic

6. A field trip was arranged so that this troupe of __________ dancers could observe the real masters of their art.

- ☐ seasoned
- ☐ fledgling
- ☐ expert
- ☐ torpid
- ☐ novice
- ☐ lithe

7. The exhibit is not so much a retrospective as a __________; the artist's weaker early work is glossed over, and any evidence of his ultimate dissolution is absent entirely.

- ☐ paean
- ☐ philippic
- ☐ tirade
- ☐ panacea
- ☐ eulogy
- ☐ crescendo

8. After a long, hard practice in the summer sun, the players were visibly __________.

- ☐ flagging
- ☐ hale
- ☐ lissome
- ☐ loathsome
- ☐ vigorous
- ☐ enervated

9. Nothing evoked memories of her grandmother's house like the __________ of scents associated with the holiday feast.

- ☐ paucity
- ☐ anomaly
- ☐ olio
- ☐ mélange
- ☐ dearth
- ☐ pilfering

10. Unlike the politician's earlier evasions and equivocations, this latest statement is a __________ lie.

- ☐ bald
- ☐ tacit
- ☐ overt
- ☐ didactic
- ☐ rhetorical
- ☐ implicit

11. Possessed of a lighthearted approach to life, Winnie thought that those who were __________ in regards to values and mores missed out on a certain liveliness and spontaneity.

- ☐ lax
- ☐ equable
- ☐ priggish
- ☐ redoubtable
- ☐ impious
- ☐ punctilious

12. After a second __________ attempt by Margulies to make a simple shot, the coach put him on the bench for the rest of the game.

- ☐ restless
- ☐ flawless
- ☐ bootless
- ☐ indefatigable
- ☐ feckless
- ☐ auspicious

13. While he says that he did not mean to kill, some feel that his actions are __________ murder.

- ☐ preceded by
- ☐ on par with
- ☐ superseded by
- ☐ an homage to
- ☐ an accrual of
- ☐ tantamount to

14. For most of the 20th century, American political contentions reflected pragmatic rather than ideological differences; candidate debates centered around whether programs were __________.

- ☐ partisan
- ☐ voluble
- ☐ feasible
- ☐ innocuous
- ☐ prejudiced
- ☐ viable

15. Though considered a somewhat somber drama at the time of its release in 1975, the film *Shampoo*, about a philandering hairdresser, now seems more the __________ comedy.

- ☐ puckish
- ☐ uninhibited

☐ prurient
☐ wry
☐ dated
☐ puritanical

16. During the German occupation of France, the draconian measures of the occupiers met armed resistance from civilian __________, inspired by desperate conditions to fight in their own streets.

☐ poets
☐ fighters
☐ writers
☐ mercenaries
☐ politicians
☐ partisans

17. Representative government arose in part from dissatisfaction with too many monarchs making __________ decisions without regard for precedents.

☐ capricious
☐ considered
☐ malicious
☐ pessimistic
☐ insidious
☐ erratic

18. Because the Lewis and Clark expedition through the West was conceived primarily as a mapping project, government officials were __________ by the wealth of information on a myriad of topics that the explorers gathered.

☐ aggravated
☐ flabbergasted
☐ crushed
☐ bedazzled
☐ bored
☐ disappointed

19. Only by ignoring Afghanistan’s long history as a military quagmire could pundits pontificate that fresh infusions of men and material would lead to __________ victory.

☐ fleeting
☐ contingent
☐ expeditious
☐ ephemeral
☐ swift
☐ pyrrhic

20. Geneticists find Iceland a living laboratory for the study of __________ because virtually all of its current 300,000 citizens descend from less than a thousand Icelanders who survived the medieval Black Death.

☐ diversity
☐ revivification

☐ therapy
☐ history
☐ mutation
☐ rejuvenation

21. Given the influx of information via social media, the only way that a person can function effectively is to __________, to metaphorically separate the wheat from the chaff.

☐ delete
☐ triage
☐ prioritize
☐ respond
☐ requite
☐ eliminate

22. Although accommodating in person, George Orwell __________ defended his political positions in print.

☐ tenaciously
☐ obsequiously
☐ inadvertently
☐ doggedly
☐ sycophantically
☐ idiosyncratically

23. Although historically, paints were often tinted with toxic elements such as lead, cadmium, and mercury, __________ number of painters lived to be seventy, eighty, and even ninety.

☐ an incomprehensible
☐ a flabbergasting
☐ an impossible
☐ a confounding
☐ a dismaying
☐ an enlightening

24. The phrase "gilding the lily" is a late 19th century expression that sneered at the ostentatious gestures of some of the newly rich, such as applying gold gilt to the carved lilies on the entrances of their Beaux Arts homes; for decades, it then served as a shorthand for any __________ and showy behavior.

☐ gauche
☐ eccentric
☐ idiosyncratic
☐ idiotic
☐ irredentist
☐ uncouth

25. In 1905, there were only fourteen automobiles in the United States, yet, __________ enough, two of them ran into each other in St. Louis.

☐ justifiably
☐ ironically
☐ inevitably

☐ deservedly
☐ ludicrously
☐ lamentably

26. A "Mycenaean waist" refers to the taut, impossibly small waists characteristic of certain ancient drawings found on Crete, and it certainly does not __________ any characteristic of modern, overweight Western societies.

☐ deify
☐ depict
☐ denigrate
☐ mirror
☐ defame
☐ distort

27. In the 19th century, Thomas Carlyle called economics "the dismal science," in part because of Malthus's theory that population growth would outpace our natural resources, causing widespread __________.

☐ dearth
☐ pulchritude
☐ rapture
☐ ravishment
☐ famine
☐ debauchery

28. In uncertain times, __________ theories often gain greater and faster adherence among the populace than proven ones do.

☐ corroborated
☐ putative
☐ conjectural
☐ assayed
☐ irrefutable
☐ irreconcilable

29. While the legend of Custer's last stand is commonly accepted and oft cited as an example of fortitude in the face of adversity, recent studies of the remains indicate that, __________, many soldiers chose suicide rather than risk capture by the Sioux.

☐ surprisingly
☐ risibly
☐ tragically
☐ remarkably
☐ amusingly
☐ horribly

30. In the early 1920's, Paul Cassirer, a Berlin art dealer, was famous not only for his clients, such as Renoir, but also for __________ a credulous art world by correctly exposing several purported Van Gogh works as forgeries.

☐ deluding
☐ apprising
☐ counseling

☐ depressing
☐ indicting
☐ misleading

31. Arthur Conan Doyle's upstanding hero Sherlock Holmes engages in just as much clever deception as his nemesis, Professor Moriarty, proving that ____________ is not inherently evil.

☐ immorality
☐ brilliance
☐ cunning
☐ subterfuge
☐ wrongdoing
☐ judgment

32. The etymologies of the words *alpha* and *omega* couldn't be more different; the former is obscure—the original symbol for *alpha* was an ox's head, and an ox is *'alp* in Phoenician — while the latter is _________, as *omega* simply means "big O."

☐ transparent
☐ complicated
☐ overt
☐ erudite
☐ abstruse
☐ scholarly

33. While the muted colors do suggest a certain sobriety, the overall effect is undeniably __________.

☐ vivacious
☐ poignant
☐ dull
☐ lackluster
☐ mirthful
☐ benign

34. Although the system's __________ is not currently in dispute at the national level, increasing local allegations of preferential treatment are threatening to change the situation.

☐ unfairness
☐ solemnity
☐ probity
☐ equity
☐ partiality
☐ solicitousness

35. Aghast at the indiscretion, the politician took a symbolic vow of __________, hoping that public attention to the matter would quickly wane.

☐ silence
☐ chastity
☐ openness
☐ transparency

- ☐ quietus
- ☐ revenge

36. A petty man obsessed with—but in perpetual doubt about—his own power, he surrounded himself with subordinates who could best be described as __________.

- ☐ sages
- ☐ myrmidons
- ☐ sycophants
- ☐ quacks
- ☐ ascetics
- ☐ sirens

37. While traveling to the spa's remote location could be hectic, visitors to the spa more than made up for the stress by unwinding in a supremely __________ environment.

- ☐ effusive
- ☐ pacific
- ☐ duplicitous
- ☐ censorious
- ☐ placid
- ☐ blithe

38. The man looked much older than his 70 years, his __________ frame looking as though it had endured at least 40 years in the desert; although the casting director had initially pictured a more physically robust Moses, the actor was, in the end, perfect for the role.

- ☐ fetid
- ☐ incendiary
- ☐ desiccated
- ☐ wizened
- ☐ arid
- ☐ hale

39. The children's attempt at a Mother's Day brunch was __________; soggy French toast, lukewarm coffee, and a syrup fight in the kitchen that would inevitably end up being cleaned up by the very recipient of the brunch.

- ☐ convivial
- ☐ amiable
- ☐ risible
- ☐ satirical
- ☐ farcical
- ☐ labile

40. In a place rife with both poverty and crime, some cautioned the young student that to __________ about with his fancy phone and designer clothing was only to invite trouble.

- ☐ sprint
- ☐ gallivant
- ☐ tergiversate
- ☐ traipse

☐ digress
☐ equivocate

41. Though the subject of the museum exhibit had been an astute politician and strategist in his day, not even his most vociferous apologists could drum up interest among a modern public for a retrospective of a career so marked by __________ views on social issues.

☐ erudite
☐ erroneous
☐ sere
☐ scholarly
☐ passé
☐ archaic

42. Just months from retirement, the disgraced executive was forced to make __________ exit from the company.

☐ a peppery
☐ a triumphant
☐ a boorish
☐ an ignominious
☐ a defiled
☐ an unseemly

43. __________ in scandal, the company could regain favor with customers only through mass firings of guilty executives.

☐ Wallowing
☐ Stoic
☐ Bogged down
☐ Resplendent
☐ Mired
☐ Besotted

44. From the battle's opening volleys to its bloody conclusion, the forces of destruction razed a path through the city, ultimately leaving behind an eerie stillness where there once had been streets and squares __________ with life.

☐ flouting
☐ assaying
☐ teeming
☐ stewing
☐ abounding
☐ vaunting

45. By framing the new law as a question of urgent safety rather than of privacy, the government obviated the need to pass through the standard channels of legislation, effectively __________ all formal dissent and relegating any would-be naysayer from a position of engaged activist to that of powerless bystander.

☐ curtailing
☐ undermining
☐ targeting
☐ lobbying

- ☐ instigating
- ☐ facilitating

46. Exactly which bird species fell victim first to the deadly virus is the subject of ongoing controversy; what is known, however, is that it took but a slight mutation in the pathogen's genetic constitution to render it lethal to __________ of related species.

- ☐ a contraband
- ☐ a surplus
- ☐ an aurora
- ☐ a myriad
- ☐ a pantheon
- ☐ a plethora

47. The coastline of the region plays home predominantly to a community of cosseted elites; aside from the tolerated presence of a smattering of __________ abodes, the area is practically inundated with opulent estates, which boast a variety of architectural styles from the neo-classical to the Gilded Age.

- ☐ ramshackle
- ☐ contiguous
- ☐ archaic
- ☐ garlanded
- ☐ underestimated
- ☐ dilapidated

48. An antiquated rifle and a revolutionary spirit were the only things in the possession of the young __________ as he forded the perilous canyon stream and wended his way up the opposite bank in hopes of joining forces with the insurgent militia.

- ☐ myrmidon
- ☐ rapscallion
- ☐ defector
- ☐ quisling
- ☐ deity
- ☐ devotee

49. After many hours of debate, things seemed to have reached ____________, as neither side was willing to give so much as an inch, and no one had anything new to offer.

- ☐ an impasse
- ☐ a pause
- ☐ a timeout
- ☐ a confrontation
- ☐ an engagement
- ☐ a stalemate

50. While kidney stones are known to produce a truly ____________ sensation, often compared to the agony of childbirth, they are almost never fatal.

- ☐ anodyne
- ☐ inoffensive

☐ painstaking
☐ tortuous
☐ excruciating
☐ torturous

51. Chris Anderson, editor of *Wired* magazine, has always been a huge ____________ of the philosophy that all information "wants to be free"; interesting, then, that his latest book retailed for $29.99, and *Wired* still charges for subscriptions.

☐ detractor
☐ champion
☐ gadfly
☐ chump
☐ critic
☐ proponent

52. The director of the musical admitted that while he was very good with characterization, scenery, lighting, and music, choreography was not at all his ___________.

☐ strong suit
☐ weakness
☐ forte
☐ hobby
☐ deficiency
☐ pastime

53. Though most technology used in the manufacture of bicycles is either decades old or adapted from other industries, the advent of carbon fiber frames brought with it genuine __________.

☐ innovation
☐ antiquity
☐ flexibility
☐ venerability
☐ novelty
☐ seriousness

54. The long history of such __________ Western religious traditions as Kaballah and theosophy suggest that it is wrong to associate mysticism only with the East.

☐ foreign
☐ devout
☐ exotic
☐ esoteric
☐ eccentric
☐ arcane

55. Sometimes __________ comes at a price; research suggests that among first-generation Chinese Americans, those who embrace the traditional Confucian values of their homeland are more likely to succeed academically than are those who do not.

☐ acculturation

☐ assimilation
☐ investiture
☐ alienation
☐ indebtedness
☐ estrangement

56. Many Enlightenment philosophers viewed Machiavelli's book as a satire meant to expose and caricature the __________ claims to power of the very figures Machiavelli pretended to endorse.

☐ sarcastic
☐ specious
☐ spurious
☐ squalid
☐ stolid
☐ stoic

57. The defendant impressed the jurors as __________; they did not believe that a woman of her education and experience could possibly be as naïve as she acted.

☐ disingenuous
☐ guileless
☐ innocent
☐ accomplished
☐ artful
☐ culpable

58. Crucial to fostering a realistic understanding of the potential boons—and perils—of the new drug will be a concerted effort to ________ the specific contexts and symptoms that render its use appropriate.

☐ furnish
☐ delineate
☐ outlaw
☐ transmute
☐ stipulate
☐ proscribe

59. Two months after the first shots of the American Revolution were fired, the British offered a pardon to any rebels willing to lay down arms, excepting only Samuel Adams and John Hancock, whose offenses the commanding British officer considered too __________ to forgive.

☐ effectual
☐ fruitful
☐ flagitious
☐ boorish
☐ heinous
☐ martial

60. Robert Gottlieb, who otherwise found much to admire in John Steinbeck, argued that he was politically __________, offering an adolescent disaffection in place of settled judgment.

☐ naïve

- ☐ perspicacious
- ☐ contemptible
- ☐ keen
- ☐ callow
- ☐ disinterested

61. Employees who demonstrate too much alacrity in the workplace often find themselves being assigned work left unfinished by their more __________ co-workers.

- ☐ torpid
- ☐ solemn
- ☐ tepid
- ☐ saturnine
- ☐ sedulous
- ☐ solicitous

62. In his writings after visiting New York, Albert Camus expressed more of an inkling rather than a ____________ understanding of what he found lacking in American culture.

- ☐ elementary
- ☐ shrewd
- ☐ penetrating
- ☐ inchoate
- ☐ sinuous
- ☐ dialectical

63. With private funding streams drying up right and left, the researcher was delighted to be suddenly freed of grant-writing responsibilities and able to pursue ______________ federally-subsidized research.

- ☐ haphazardly
- ☐ partially
- ☐ full-bore
- ☐ high-handedly
- ☐ impeccably
- ☐ wholly

64. It is in the best interest of criminal defendants to appear _________ in front of the judge, showing that not all moral sympathy is lost on them.

- ☐ callous
- ☐ vindicated
- ☐ contrite
- ☐ penitential
- ☐ messianic
- ☐ pious

65. After hours of tedious test taking, the applicant was exhausted and could feel his acuity ________.

- ☐ flag
- ☐ soar
- ☐ ebb

- ☐ energize
- ☐ metabolize
- ☐ satiate

66. Despite the blandishments of the real estate con artist, the intended mark remained ___________ about the value of the plot for sale, as, on the map, it seemed to border a swamp.

- ☐ optimistic
- ☐ enthused
- ☐ irascible
- ☐ skeptical
- ☐ jaundiced
- ☐ leery

67. Like the characters she played, the ingénue always seemed __________ when auditioning for directors, but, on the set, this almost supine attitude fell away, revealing an implacable unwillingness to compromise her artistic vision.

- ☐ sardonic
- ☐ naïve
- ☐ tractable
- ☐ innocent
- ☐ amenable
- ☐ articulate

68. In the week that followed the climber's disappearance, Internet rumor mongers blogged a myriad of ____________ reports of her demise, only to be embarrassed by the release of a dramatic video that showed her celebrating on the summit.

- ☐ apocryphal
- ☐ apocalyptic
- ☐ sentimental
- ☐ spurious
- ☐ saccharine
- ☐ scandalous

69. In agitprop theatre, actors, passing as civilians, incite others to public protest; while some consider such actions virtuous, others would label them a __________ trick.

- ☐ glib
- ☐ contemptible
- ☐ faddish
- ☐ sophisticated
- ☐ scurvy
- ☐ scintillating

70. The tragedy — and the resultant horrific loss of life and damage to property — occurred because of his ___________ approach to his duties, evinced by his slouching posture and cavalier attitude.

- ☐ murderous
- ☐ petty

- ☐ lax
- ☐ aristocratic
- ☐ barbarous
- ☐ slack

71. The doctor’s unapproved “health juice” first won him fortune, but then, after several deaths, infamy as a ___________.

- ☐ sap
- ☐ dupe
- ☐ quack
- ☐ shyster
- ☐ sage
- ☐ pundit

72. Although they were already late for the formal reception, the couple continued to ___________ because they preferred to lounge about and bask in each other’s company.

- ☐ lurk
- ☐ dally
- ☐ tarry
- ☐ skulk
- ☐ embrace
- ☐ equivocate

73. The player’s exploits both on the field and in the finest night clubs around the world earned him many ___________ from his legions of staunch admires — so many, in fact, that his given name was all but forgotten.

- ☐ similes
- ☐ appellations
- ☐ sobriquets
- ☐ misnomers
- ☐ neologisms
- ☐ aliases

74. Most people erroneously believe that the horror of biological warfare was a new and terrible invention of the late 20th century; however, 18th century British generals, with the blessings of their pastors, won a war with American Indians threatening colonial settlements by gifting them the blankets used by smallpox patients, causing a disease outbreak that virtually exterminated the __________ population.

- ☐ pandemic
- ☐ homogeneous
- ☐ indigenous
- ☐ ecumenical
- ☐ indignant
- ☐ native

75. The presidential candidate, known not only for the deeply reasoned content of his prepared speeches but also for the fiery brilliance of his delivery, badly miscalculated his ability to perform equally successfully when delivering ___________ answers to unexpected queries form the media.

☐ extemporaneous
☐ capricious
☐ lubricious
☐ disingenuous
☐ impromptu
☐ premeditated

76. Perhaps understandably because of the carnage caused by the terrorist attack, the mayor could not help but __________ over the terrible decisions to be made, even though his subordinates on the scene passionately implored him to act immediately, one going so far as to quote David Farragut: "Damn the torpedoes, full speed ahead."

☐ equivocate
☐ divagate
☐ weep
☐ pine
☐ vacillate
☐ wail

77. To avoid a lengthy prison term, the convicted financier signed a binding document delineating the millions in fines and restitution that she must pay, and another legal memorandum in which she _________ her former role and financial interest in the hedge fund she had founded.

☐ abjured
☐ jeopardized
☐ reneged
☐ deposed
☐ censured
☐ forwent

78. The remark was intended as something of a ___________, even if it sounded anodyne on the surface.

☐ compliment
☐ slight
☐ gibe
☐ profundity
☐ panegyric
☐ maxim

79. The famous bank robber John Dillinger would never have been captured but for the __________ of Ana Cumpănaș, a friend of his who had been hiding him in the brothel that she owned.

☐ assiduousness
☐ perfidy
☐ nosiness
☐ meddling
☐ treachery
☐ condemnation

80. The ____________ of the new smartphone may lead one to think that the manufacturer must be raking in the cash,

but so little profit is made on each unit that sales of the phone barely affect the bottom line.

- ☐ expenditure
- ☐ construction
- ☐ ubiquity
- ☐ aesthetics
- ☐ cost
- ☐ omnipresence

81. Though the majority of rules in sports are physically described in rulebooks, there is a _________ code of conduct that relates to sportsmanship.

- ☐ tacit
- ☐ evanescent
- ☐ incorrigible
- ☐ unambiguous
- ☐ blatant
- ☐ implicit

82. The reviewer opined that action film directors like Michael Bay seem to find elements like characterization and emotional development ___________ to a good movie, whereas more critically-acclaimed directors see them as — pun intended — “critical.”

- ☐ intrinsic
- ☐ ridiculous
- ☐ important
- ☐ extraneous
- ☐ explosive
- ☐ peripheral

83. Sandra was entirely ___________ by the crossword puzzle which, unlike the simple fill-in-the-blanks published on weekdays, was one of the more difficult cryptic crosswords only published on weekends.

- ☐ confounded
- ☐ flummoxed
- ☐ enraged
- ☐ smitten
- ☐ incensed
- ☐ impressed

84. The argument that both political parties engage in similar levels of dishonesty is a ____________ one; while you can certainly point to individual instances of chicanery on the parts of both parties, one side has built its entire platform on lies.

- ☐ sophomoric
- ☐ specious
- ☐ sweeping
- ☐ generalized
- ☐ powerful
- ☐ meretricious

85. The subject of the documentary was not bothered that the documentarian received such __________ from the critics, but that none of the money or acclaim filtered down to him.

- ☐ opprobrium
- ☐ wealth
- ☐ fulmination
- ☐ approbation
- ☐ plaudits
- ☐ capital

86. *The Thin Blue Line*, a documentary by Errol Morris, is one of a very few movies that has had a tangible effect on the real world; the film managed to _________ its subject, who had been on death row for a crime that Morris proves, fairly definitively, that the man did not commit.

- ☐ exculpate
- ☐ liberate
- ☐ inter
- ☐ excuse
- ☐ manumit
- ☐ vindicate

87. Multi-level marketing schemes prey on people ____________ promises of quick riches, and sales rewards such as fancy cars and vacations.

- ☐ aggrieved by
- ☐ beguiled by
- ☐ enamored with
- ☐ obsessed by
- ☐ vexed by
- ☐ possessed by

88. According to Buddhist thought, the mark of a truly great soul is the ability to retain one's __________ in the face of turmoil and tribulation.

- ☐ conviction
- ☐ self-esteem
- ☐ pride
- ☐ equanimity
- ☐ denomination
- ☐ sangfroid

89. In a way, the environmental movement can still be said to be __________ movement, for while it has been around for decades, only recently has it become a serious organization associated with political parties and platforms.

- ☐ an incipient
- ☐ a disorganized
- ☐ a nascent
- ☐ a nebulous
- ☐ an inconsequential
- ☐ an immaterial

90. Einstein's idea that electromagnetic radiation was divided into a finite number of "energy quanta" was purely experiential until it was theoretically ____________ by the work of physicists such as Louis de Broglie and Werner Heisenberg.

- ☐ bolstered
- ☐ undermined
- ☐ condoned
- ☐ pardoned
- ☐ sabotaged
- ☐ buttressed

91. We tried to call her over to where we were standing, down at the edge of the stage, but she couldn't hear us over the ____________.

- ☐ clamor
- ☐ music
- ☐ rabble
- ☐ amplifier
- ☐ din
- ☐ host

92. The painter was just as famous for his personality as for his work; unlike the many pretentious and pontificating men in his field, he was known to be entirely ___________.

- ☐ artless
- ☐ shrewd
- ☐ ingenuous
- ☐ klutzy
- ☐ adroit
- ☐ artful

93. The newest romantic comedy wasn't exactly bad, but simply __________; it had laughs, but they were all jokes you'd heard before.

- ☐ atrocious
- ☐ amusing
- ☐ trite
- ☐ hackneyed
- ☐ witty
- ☐ egregious

94. An obsession with aesthetics ____________ all the work of the computer company; even their unsuccessful products manage to look like pieces of modernist sculpture.

- ☐ underpins
- ☐ irradiates
- ☐ underserves
- ☐ overwhelms
- ☐ undergirds
- ☐ saturates

95. Oftentimes, when administrators force teachers to cleave too closely to a federal curriculum, those teachers feel ___________, because the mandatory curriculum curbs their sense of being creative and dynamic educators.

- ☐ crushed
- ☐ confounded
- ☐ thwarted
- ☐ undermined
- ☐ tormented
- ☐ walloped

96. The Fed knows that a certain level of financial stability can be attained by lowering interest rates, yet if it overuses this power, it risks losing its most reliable means of __________ a crisis.

- ☐ interring
- ☐ exacerbating
- ☐ annihilating
- ☐ palliating
- ☐ compounding
- ☐ assuaging

97. Even though Mariposa loved taking on roles that involved a lot of lines, she was excited to be playing a more ___________ character, which would require her to focus more on gesture and expression.

- ☐ laconic
- ☐ dramatic
- ☐ dejected
- ☐ curt
- ☐ mute
- ☐ melancholy

98. Most physics textbooks are dense and abstruse, whereas Richard Feynman can describe the most recondite mysteries with impressive ___________.

- ☐ luridness
- ☐ sagacity
- ☐ prowess
- ☐ panache
- ☐ limpidity
- ☐ lucidity

99. The beauty queen's mother could not believe her daughter had been disqualified for hitting one of the judges with her baton, and she would not be ___________ by the official's offer of a free steak dinner.

- ☐ placated
- ☐ implicated
- ☐ mollified
- ☐ pleased
- ☐ ameliorated
- ☐ deterred

100. Though Hamlet is famous for being __________, he still manages to go on something of a killing spree in

Shakespeare's play, proving that he is hardly paralyzed with depression.

- ☐ indecisive
- ☐ melancholy
- ☐ monological
- ☐ morose
- ☐ violent
- ☐ barbaric

101. It's worth wondering if the increase in diagnoses of psychological disorders has caused us to see certain behaviors that were once considered normal as ____________.

- ☐ importunate
- ☐ mythical
- ☐ unfortunate
- ☐ anomalous
- ☐ aberrant
- ☐ fabulous

102. Proust proved that the ___________ can be the domain of the novel every bit as much as the fantastical.

- ☐ mundane
- ☐ literary
- ☐ bombastic
- ☐ cosmopolitan
- ☐ belletristic
- ☐ quotidian

103. The magazine's editor was known to be a very busy woman, so it was important when speaking with her to get right to the ___________ of the issue.

- ☐ pith
- ☐ conclusion
- ☐ gist
- ☐ apex
- ☐ genesis
- ☐ culmination

104. While environmental concerns continue to __________, most climate scientists say the interest comes too late, as we've already passed the proverbial tipping point.

- ☐ disseminate
- ☐ burgeon
- ☐ mushroom
- ☐ promulgate
- ☐ concatenate
- ☐ indemnify

105. The government interpreted the enemy's decision to move their army to the borderline as __________ act of aggression.

☐ an explicit
☐ an extant
☐ an impudent
☐ a bellicose
☐ an overt
☐ a belligerent

106. The saying "Time stops for no man" also applies to rock and roll; once the rebellious sound of the young, it __________ became part of the culture of the old, just as had every preceding style of music.

☐ inevitably
☐ accidentally
☐ deliberately
☐ unavoidably
☐ resolutely
☐ painfully

107. Isherwood's sympathy for communism during the interwar period was not only a reaction against fascism, but also a mark of his fellow feeling for the laboring classes and his ___________ to engage as an equal with working people.

☐ disinclination
☐ hankering
☐ proclivity
☐ implacability
☐ unwillingness
☐ joviality

108. Academic freedom does not protect a professor's classroom remarks on matters irrelevant to his subject, though it guarantees the professor considerable liberty of speech about matters ___________ to his or her academic work.

☐ germane
☐ indifferent
☐ mimetic
☐ disinterested
☐ congruent
☐ pertinent

109. Unbridled passion, whether rage or ardor, gives way to the sort of rash declarations that too often end in ___________ and sorrow.

☐ disdain
☐ pity
☐ rue
☐ affinity
☐ remorse
☐ contempt

110. The tremendous wealth of ancient life on display as part of the Ancient Life of New York exhibit—billion-year-old blue-green bacteria from the Adirondacks, fossilized tree stumps and spiders from Schoharie County,

trilobites from Oneida County, and armored fish from throughout the state—represents only a tiny fraction of the New York State Museum's ___________ collection of over one million specimens.

- ☐ piecemeal
- ☐ voluble
- ☐ exhaustive
- ☐ evergreen
- ☐ compendious
- ☐ commanding

111. The representative's violent ascension to prominence began with a ___________ attack on the comparatively conciliatory leaders of his own party.

- ☐ truculent
- ☐ partisan
- ☐ savage
- ☐ biased
- ☐ imperious
- ☐ dissembling

112. Even those accustomed to the austere beauty of mathematics may be daunted by the ___________ with which Bourbaki conducts his proofs.

- ☐ celerity
- ☐ diligence
- ☐ alacrity
- ☐ asperity
- ☐ inscrutability
- ☐ rigor

113. Even from a distance, the man could see that the tornado had ______________ from the site all but the most basic elements of his childhood home; nothing but traces of the original foundations remained.

- ☐ tethered
- ☐ extirpated
- ☐ recapitulated
- ☐ interred
- ☐ effaced
- ☐ hallowed

114. By the third quarter, the opposing team had lost the upper hand entirely, the game becoming an utter __________, a fact made most evident, perhaps, by the escalating jeers coming from the nearby bleachers.

- ☐ debacle
- ☐ rout
- ☐ boondoggle
- ☐ forgery
- ☐ fallacy
- ☐ infirmity

115. Central to the challenger's platform was the argument that the incumbent had ultimately _________ the

agreements he had initially championed during his first stint in office.

- ☐ mustered up
- ☐ consented to
- ☐ fielded
- ☐ reneged on
- ☐ honed
- ☐ abrogated

116. The ____________ between parties is rapidly becoming insurmountable, as a lack of faith on both sides creates conditions where issues once considered beyond the pale of dispute are becoming subject to contention.

- ☐ accord
- ☐ prevalence
- ☐ rift
- ☐ breach
- ☐ travesty
- ☐ piety

117. ___________ though the family now may be, they continue to carry themselves in public with an air befitting their aristocratic heritage.

- ☐ Impecunious
- ☐ Idyllic
- ☐ Glamorous
- ☐ Patrimonial
- ☐ Bucolic
- ☐ Insolvent

118. Lie detector tests attempt to discern __________ by measuring physiological responses, including heart rate, blood pressure, and respiratory rate.

- ☐ tenacity
- ☐ perspicacity
- ☐ precocity
- ☐ improbity
- ☐ mendacity
- ☐ urbanity

119. The golden crown's dazzling _____________ is tarnished only by its torrid and bloody history.

- ☐ translucence
- ☐ variance
- ☐ evanescence
- ☐ refulgence
- ☐ tumescence
- ☐ radiance

120. The judge's keen eye for sussing out the pretension of the lawyers in her courtroom was surpassed only by the __________ wit with which she castigated them for it.

- ☐ sedulous
- ☐ mordant
- ☐ obtuse
- ☐ jurisprudent
- ☐ trenchant
- ☐ assiduous

121. The amount of self-abasement with which the inmate ___________ the probation panel to be set free verged on the humiliating; nevertheless, the judges remained unmoved and he was ultimately sent back to his cell to serve another three years.

- ☐ beseeched
- ☐ chided
- ☐ snubbed
- ☐ conceded
- ☐ drummed
- ☐ supplicated

122. The cult members treated their leader with __________ loyalty that verged on the obsessive and made them willing, should the need ever arise, to do so much as lay down their lives for him.

- ☐ a fanatical
- ☐ an arbitrary
- ☐ a fickle
- ☐ a mortifying
- ☐ a zealous
- ☐ an indeterminate

123. Jefferson regarded sumptuous living as among the most ___________ evils to threaten the young republic, more pernicious even than loyalty to the deposed empire.

- ☐ reactionary
- ☐ venerable
- ☐ epicurean
- ☐ grievous
- ☐ baneful
- ☐ fastidious

124. Theology was once regarded as the "Queen of the Sciences," because every subject eventually had to meet its demands, but two hundred years ago that honor and title fell to mathematics, which enjoys ___________ over not only physical science but social science as well.

- ☐ mayhem
- ☐ credence
- ☐ hegemony
- ☐ autonomy
- ☐ dominance
- ☐ independence

125. The new particles produced by CERN's Large Hadron Collider are ___________, lasting a millionth of a billionth of a billionth of a second before disintegrating into photons, quarks, or other particles.

- ☐ ephemeral
- ☐ infinitesimal
- ☐ myriad
- ☐ poignant
- ☐ fleeting
- ☐ countless

126. While the professor first achieved renown for the theory he devised single-handedly during the early days of his career, his later contributions were achieved in a more _________ manner.

- ☐ solitary
- ☐ collaborative
- ☐ synergetic
- ☐ exegetic
- ☐ unilateral
- ☐ collusive

127. While the author's first book was substantial and well-documented, he seems to have run out of material; the followup is mere ___________.

- ☐ pith
- ☐ bile
- ☐ marrow
- ☐ claptrap
- ☐ fettle
- ☐ dross

128. Lady Astor once commented to Winston Churchill, "If I were married to you, I'd put poison in your coffee." Churchill's famous ___________: "Nancy, if you were my wife, I'd drink it."

- ☐ anecdote
- ☐ aphorism
- ☐ retort
- ☐ recrimination
- ☐ rejoinder
- ☐ maxim

Sentence Equivalence Answers

1. **Steward, husband.** The “while” tells you that the second part of the sentence will contrast with pushing forward — due to the approaching winter, the colonists need to stay put and *conserve* what they have. Only *steward* and *husband* mean this. *Sell* and *peddle* don’t work because there’s nothing in the sentence to indicate commerce or contrast with it (and why would you sell your resources when a hard winter is coming?) Note that *procure*, which means “get” or “acquire,” doesn’t work because of the word *existing*.

2. **See, distinguish.** This sentence is worded in a tricky way, as the pivot words *but* and *barely* negate each other. Thus, the blank parallels the clue “saw deeply.” (*Proof* here means “proofread” or “edit,” and *galleys* are “drafts of a book about to be published.”) How ironic that an author who sees into the hearts of his characters is practically blind in real life! *Interpret* is an attractive trap, but Joyce had a vision problem, not an intellectual one.

3. **Gore, carnage.** While many of the words make sense, the words *slaughter* and *blood* demand the correct pair. *Grief* and *anguish* are an incorrect pair (*grief* is especially weird — generally, other people experience grief after someone dies, rather than a person feeling grief over his own violent death).

4. **Evolving, advancing.** Sometimes a seeming theme trap *(evolving)* is not a trap — this sentence is literally about evolution, and also uses *evolving* as a metaphor. *Fallacious* and *counterfactual* is an incorrect pair that reflects misunderstanding the sentence, which certainly doesn’t say that all science is wrong, just that it is its nature to constantly revise in the face of new information.

5. **Idealistic, quixotic.** Daily arguments with opponents who won’t change their mind and dealing with people who make demands are likely to make someone more realistic about the limited prospects of accomplishing something, or less *idealistic* and *quixotic*, which are synonyms. Arguing with others is not likely to make someone less *contentious* or *dialectical*, but more so. And nothing here implies that the president’s policies are *altruistic* or that he is particularly *tenacious*. Alternatively, you can look at the answer choices. *Idealistic* and *quixotic* form one pair, and *dialectical* and *contentious* form another, so one of those pairs has to be the right answer.

6. **Fledgling, novice.** You need answers that relate to being beginners (or at least not “masters”). *Fledgling* and *novice* both mean “inexperienced.”

7. **Paean, eulogy.** A *retrospective* would be an exhibit that shows the history and progression of the artist’s work, but this exhibit only shows the good parts (it “glosses over” the weaker work and omits the artist’s “dissolution”). Thus, you need a word that has something to do with “praising.” *Paean* and *eulogy* are the only matches. Note that a *paean* is generally a song or speech of praise and is being used a bit metaphorically here. Many people only know *eulogy* as a speech given at a funeral, but can actually refer to any speech (or in this case, art exhibit) of praise.

8. **Flagging, enervated.** After a long, hard practice in hot weather, naturally the players would be tired. *Flagging* and *enervated* are the only choices that mean “tired.”

9. **Olio, mélange.** If it’s a *feast,* there are many different foods involved — thus, you need a word that means “mixture” or “multitude.” *Olio* and *mélange* both mean “mixture” or “medley.”

10. **Bald, overt.** The earlier "evasions and equivocations" are ways to avoid a question or "flip-flop" on an issue. This "latest statement" is "unlike" the evasions and equivocations because it is an *explicit*, *clear,* or *obvious* lie. *Bald* and *overt* are the only matches. Note the trap answers *tacit* and *implicit,* which are synonyms that mean the exact opposite ("hinted at, unspoken") of what you need.

11. **Priggish, punctilious.** You need a word in the blank that is the opposite of the words *lighthearted* and *liveliness and spontaneity,* so something like *strict* would match. *Priggish* and *punctilious* are both related to taking the rules much too seriously.

12. **Bootless, feckless.** You now that Margulies got benched, so his attempt must have been "weak" or "unsuccessful." Both *bootless* and *feckless* match. Make sure to stick to the clues presented — you have no way to know if Margulies is *restless, indefatigable,* etc. You only know that he was pulled from the game by his coach.

13. **On par with, tantamount to.** The man's actions are being compared to murder, even though the man says the killing was unintentional (*murder* refers specifically to "deliberate killing"). Thus, you need choices that mean "equivalent to." Both *on par with* and *tantamount to* match.

14. **Feasible, viable.** You need two words that are like *pragmatic,* which means "practical." *Feasible* and *viable* relate to the practical considerations of whether the plan is workable or likely to succeed. The only other pair of similar words, *partisan* and *prejudiced*, have the exact incorrect meaning — they are more similar to "ideological" than "pragmatic."

15. **Puckish, wry.** You need two words that are the opposite of "somewhat somber." Only *puckish* and *wry* have the sense of being funny or playful.

16. **Fighters, partisans.** The correct pair must follow the clue *armed resistance from civilian*; the word *mercenaries* is an attractive trap, but a mercenary is someone who fights for money (usually for a foreign country), not someone who is "inspired by desperate conditions to fight in their own streets."

17. **Capricious, erratic.** The clue is "without regard for precedents" — that is, the monarchs made decisions based on their own whims or desires, ignoring any pre-existing standards. *Considered* is the opposite of what you are lookng for, and *malicious* and *pessimistic,* while appropriately negative, add new ideas that are not indicated by the sentence.

18. **Flabbergasted, bedazzled.** Since the expedition was *conceived primarily as a mapping project*, the officials would naturally be *amazed* or *surprised* at the *wealth of information on a myriad of topics* that the explorers gathered. Only *flabbergasted* and *bedazzled* convey any type of surprise or awe.

19. **Expeditious, swift.** The pivot indicates that the answers will oppose the clue *quagmire* — that is, Afghanistan's problems are messy and entrenched. So, what kind of victory is *not* likely? A fast or easy one.

20. **Diversity, mutation.** The geneticists mentioned are studying a population descended from a rather small number of people a rather long time ago. Geneticists — who study genes, of course — would be interested in how this population changed over time (*mutated*) and became different from one another (*diversity*). Note that none of the other answers are logical at all.

21. **Triage, prioritize.** You need two words that match the clue *to metaphorically separate the wheat from the chaff*. (*Chaff* is the "inedible part of wheat that is discarded before the wheat can be made into flour.") You can also

use the clues regarding *the only way that a person can function* given an *influx of information*. *Delete* and *eliminate* are attractive traps, but doesn't match the idea of separating *the wheat from the chaff*. Only *triage* (think of what emergency room physicians do) and *prioritize* match.

22. **Tenaciously, doggedly.** Because of the word *Although*, the correct pair must contrast "accommodating." Both *tenaciously* and *doggedly* convey the idea that Orwell refused to back down. Note that the pair *obsequiously* and *sycophantically* are the exact opposite of what you are looking for.

23. **A flabbergasting, a confounding.** The pivot *Although* indicates that the blank will convey something that contrasts with the idea of most people living only to age forty. You need two words that mean something like "surprising." Trap answer *incomprehensible* goes too far — it's interesting that many painters had long lives, but it's not beyond all comprehension.

24. **Gauche, uncouth.** The two words must match the word *ostentatious*. Both *gauche* and *uncouth* get aross the idea of showing off in a low-class way (as is often associated with the "newly rich"). Remember not to insert your own opinions — *eccentric* and *idiosyncratic* are near-synonyms, but are a trap.

25. **Ironically, ludicrously.** The pivot word *yet* indicates that a car crash in 1905 was a numerical improbability. You need two words that mean "strangely" or "ridiculously." Only *ironically* and *ludicrously* come close. (Note that the GRE tends to use the word *ironically* rather loosely.)

26. **Depict, mirror.** The images of small waists are contrasted with *overweight Western societies*. The word *not* before the blank indicates that you need two words that mean "represent" or "portray."

27. **Dearth, famine.** The correct pair must the idea of population growth outpacing natural resources — that is, having too many people to be supported by our natural resources. *Famine* and *dearth* are not synonyms (a *famine* is "a prolonged and widespread lack of food," and a *dearth* is "a lack of anything"), but are the only words that match the idea of shortages.

28. **Putative, conjectural.** The sentence indicates that the blank opposes the word *proven*. *Corroborated* and *assayed* are an incorrect pair of words that have meanings similar to *proven*. Only *putative* and *conjectural* have the desired meaning ("alleged, supposed, in theory").

29. **Surprisingly, remarkably.** The pivot word *While* indicates the answers must undermine the *commonly accepted and oft cited* legend. Note that there are two incorrect pairs: *risibly* and *amusingly*, *tragically* and *horribly*. Work off the clue and do not inject opinions or write stories.

30. **Apprising, counseling.** Cassirer "correctly" exposed forgeries to a *credulous* (that is, *gullible*) art world. Thus, he was *helping* or *advising* the art world. Only *apprising* and *counseling* match.

31. **Cunning, subterfuge.** You are told that both Sherlock Holmes and Professor Moriarty engage in "clever deception," though one of them is "upstanding" and the other is not. You simply need a synonym for "clever deception." *Immorality* and *wrongdoing* do not imply anything clever, and *brilliance* does not imply deception. *Cunning* and *subterfuge* both fit.

32. **Transparent, overt.** The etymology of "alpha" is described as "obscure," and the word *while* implies that the etymology of "omega" will be "not obscure." Both *transparent* and *overt* carry this meaning. *Complicated* and

abstruse are the opposite of what you want, and neither *erudite* nor *scholarly* are particularly relevant to the meaning of the sentence.

33. **Vivacious, mirthful.** In this case, *sobriety* means grave or serious, and you are looking for an antonym, something like *happy* or *lively*. *Vivacious* means "lively," and *mirthful* means "full of joy." These are correct answers. *Dull* and *lackluster* are the opposites of what you are looking for.

34. **Probity, equity.** There are allegations of preferential treatment or unfairness at the local level, and these are threatening to spill over into criticisms that the system is unfair or partial at the national level. But you are told that the system's fairness is *not* currently in dispute at that level, so you are looking for synonyms for *fairness* or *impartiality*. The correct answers are *equity* and *probity*.

35. **Silence, quietus.** The answer revolves around understanding the clue, "aghast," as well as the fact that the politician wants public attention to *wane* (reduce). *Openness* and *transparency* are an incorrect pair. While perhaps a policy of openness might have prevented the politician's troubles in the first place, there is no indication from the sentence that giving *more* information would cause public interest to *decrease.*

36. **Myrmidons, sycophants.** *A petty man obsessed with — but in perpetual doubt about — his own power* sounds pretty insecure. You have an additional clue about "subordinates." Correct choices *myrmidons* and *sycophants* both refer to servile followers.

37. **Pacific, placid.** You need two words that are the opposite of "hectic." You have the additional clue *make up for the stress*. *Pacific* and *placid* both mean *peaceful.*

38. **Desiccated, wizened.** The clues here are very obvious: the man looks even older than his actual age of 70, and as though he had endured 40 years in the desert. *Desiccated* means "dried out," and *wizened* means "old and withered." Note trap answer *arid,* which means "dry" — this is a good word to describe a desert, but cannot describe a person.

39. **Risible, farcical.** The children's attempt at a Mothers Day brunch went pretty badly — so badly that it actually made more work for Mom. What a joke! Correct choices *risible* and *farcical* both mean "laughable." The word *farcical* is more negative, but these two words are the only choices that fit. Note that the brunch was not *satirical* because it was not making fun of a pre-existing work of art, situation, etc.

40. **Gallivant, traipse.** You need two words that mean "to walk around in a showing-off sort of way." Both *gallivant* and *traipse* indicate this kind of carefree walking. *Sprint* is a trap answer — you certainly don't have any sense that the student was running very fast.

41. **Passé, archaic.** *Apologists* are defenders — in this case, people who are trying to make the politician and strategist seem like a good guy. However, the clues "in his day" and the lack of interest among a "modern public" indicate that the politician's views are *outdated.* Only *passé* and *archaic* match. (The accent mark on *passé* exists because the word came into English from French.) Note that trap answer *erroneous* just means "wrong" and doesn't really fit the clues, nor is it appropriate for discussing matters of culture and opinion.

42. **An ignominious, an unseemly.** Your word here is *disgraced. Ignominious* and *unseemly* match well. Note trap answer *defiled,* which does not appropriately describe an action (you could *defile* a holy place by doing something very inappropriate, but for a place or thing to be *defiled,* it should be very pure beforehand.)

43. **Bogged down, mired.** Your clue is that the scandal here is so severe that the company can save itself only through the rather extreme measure of "mass firings." *Bogged down* and *mired* have the sense of being stuck in something (a *bog* and a *mire* are both physical things — swampy, quicksand-like patches). Note trap answer *wallowing* — only a person or animal can *wallow* (and anyone *wallowing* wouldn't *want* to "regain favor" anyway).

44. **Teeming, abounding.** This terrible battle seems to have killed everything ("razed," "eerie stillness"). In contrast, the streets and squares were once "full of" or "bustling with" (that is, *teeming* or *abounding* with) life.

45. **Curtailing, undermining.** By "obviating," or "getting around," the traditional channels of legislation, the government is making formal dissent impossible. In other words, it is *curtailing* or *undermining* such dissent—words which both mean to "prevent" or "undercut."

46. **A myriad, plethora.** The virus spread to a large number (a *myriad*, a *plethora*) of bird species. Watch out for trap answer *surplus,* which means "an excess" and would not be appropriate to describe bird species.

47. **Ramshackle, dilapidated.** The key to this question is the phrase "tolerated presence," which indicates that the "abodes," or houses, are different from the "opulent estates." The pivot phrase "aside from" also provides a nice clue. *Ramshackle* and *dilapidated*, which both mean "run-down," are best. *Archaic*, meaning "old" or "old-fashioned," is close, but doesn't possess the same connotations of disrepair or neglect that the two correct answers do.

48. **Defector, quisling.** Hints to this answer are "revolutionary spirit" and "insurgent militia," both of which indicate an abandonment of a cause for the opposing one: thus *quisling* and *defector*, both of which denote traitors, are best.

49. **An impasse, a stalemate.** If neither side of the debate is willing to give an inch to the other, then it would be impossible for a solution to be found. So the debate is more or less frozen. Both *a confrontation* and *an engagement* imply some kind of clash, but the sentence implies that any kind of serious clashing is now over (*no one had anything new to offer*). *An impasse* and *a stalemate* reflect the static nature of the conflict.

50. **Excruciating, torturous.** The sensation produced by kidney stones is compared to the "agony of childbirth," so you know you're looking for a word that means "painful." *Painstaking*, however, means "done with great care," and *tortuous* means "full of twists and turns." Both *excruciating* and *torturous* correctly describe terrible pain.

51. **Champion, proponent.** The second half of this sentence (after the semicolon) tells you that Chris Anderson charges money for his products, and is introduced by an ironic "interesting." This makes it likely that he *supports* the idea of information being free. *Champion* and *proponent* both imply support (*detractor* and *critic* are a pair with the opposite meaning of what you want).

52. **Strong suit, forte.** The word *while* tells you that you're going to take the opposite of something — in this case, the fact that the director is "very good" at a lot of stuff. However, the phrase "not at all" will twist you back in the opposite direction, so you need a word for the blank that means something like "strength." *Strong suit* and *forte* both fit perfectly. *Hobby* and *pastime* are a pair, but they don't work here, as this is clearly the director's job. Similarly, *weakness* and *deficiency* are a pair, but the opposite of what you want.

53. **Innovation, novelty.** *Though* tells you that the advent of carbon fiber brought something opposite to "old" or "adapted" technology. You're looking for words that describe something new or original.

54. **Esoteric, arcane.** Whatever else Kaballah and theosophy are, they are Western religious traditions, and they must

be mystical themselves if they *suggest that it is wrong to associate mysticism only with the East*. So you want words that mean something like "mystical." *Esoteric* and *arcane* are the only good answers.

55. **Acculturation, assimilation.** The structure of this sentence demands a little extra attention. It is those who do not embrace the values of their homeland who do worse academically, so it is they who pay the price. What *comes at a price*, then, is embracing the culture of the new home. You want words that name the process of becoming part of a new land and culture.

56. **Specious, spurious.** If Machiavelli only "pretended" to endorse the claims to power, and if they were subject to exposure and caricature, they must have been not only illegitimate but ridiculous. None of the answers suggests ridiculousness, but two answers — *specious* and *spurious* — mean the claims are false. While not synonyms, both words describe claims that are superficially attractive, but in fact false.

57. **Disingenuous, artful.** You want a word that means "not as naïve as one pretends." *Disingenuous* mean precisely that, and one meaning of *artful* is "deceptive." This sentence is difficult because the word *impressed* is used to refer to making an impression in a negative way. Finally, *culpable* means "guilty" and is an attractive trap, but goes further than the clues in the sentence allow you. The clues clearly indicate that you need words that mean "not naïve."

58. **Delineate, stipulate.** A realistic understanding of the "boons and perils," or advantages and disadvantages, of the new drug requires that people know when they should and should not use it. In other words, the "contexts and symptoms" that are appropriate for taking the drug should be *delineated* or *stipulated*, both of which mean "described" or "specified."

59. **Flagitious, heinous.** First, you read that in the view of the commanding British officer, Adams and Hancock had committed "offenses," so you might anticipate a word like *offensive*. Second, they were the only rebels whom he would not pardon or forgive, so he considered their behavior "unpardonable, unforgivable." If you put these all together we'll get something like "unforgivably offensive." Both *flagitious* and *heinous* match this meaning. *Boorish* means something like "crude," but is much too weak a word to fit this sentence.

60. **Naïve, callow.** *Otherwise* suggests that Gottlieb did not find Steinbeck's political views admirable. "Adolescent disaffection" suggests that he found Steinbeck's views in some sense immature, so look for negative words that mean something like "immature." *Naïve* and *callow* are the best options. *Perspicacious* and *keen* both suggest insight, nearly opposite of what we've anticipated. *Contemptible* is negative, but with no suggestion of immaturity — make sure to use the clues provided without adding your own ideas. (Note that *naïve* is sometimes written with two dots over the "i," as in the original French, and sometimes without.)

61. **Torpid, saturnine.** The employees who *get* the work are — based on the clue *alacrity* — cheerful and ready for more. Thus, they must be getting assigned work left undone by employees who are, by contrast, either lazy or slow. Both *torpid* and *saturnine* work (*saturnine* can also mean "gloomy," but here it is being used to mean "sluggish"). Note that *sedulous* and *solicitous* are the exact opposite of what you want here.

62. **Shrewd, penetrating.** You are looking for an antonym for *inkling*, which means "only a hint of something." Instead, you want something that would make Camus's understanding complete or thorough, which is what *shrewd* or *penetrating* would mean when combined with *understanding*. *Elementary* and *inchoate* are on the same side as *inkling* in this context and are the opposite of what you want.

63. **Full-bore, wholly.** Since private funding is hard to get (the expression "right and left" means "everywhere, all around"), the researcher was happy that he or she no longer had to pursue this private funding — suddenly, the

research is being paid for — *subsidized* — by the government! *Full-bore* and *wholly* both mean "entirely." Trap answer *impeccably* (perfectly, spotlessly) is positive but inappropriate to describe the government's paying for something.

64. **Contrite, penitential.** Criminal defendants want to look good in front of the judge by showing that *not all moral sympathy is lost on them*. This implies that the defendants are not necessarily innocent, so *vindicated* doesn't work. What you want are words meaning "remorseful, apologetic." Only *contrite* and *penitential* work.

65. **Flag, ebb.** Given the clues "hours of tedious test taking" and "exhausted," you can surmise that the student's *acuity* (mental sharpness) would decrease — that is, *flag* or *ebb*.

66. **Skeptical, leery.** The word *Despite* at the beginning of the sentence means that the blank must oppose the encouragement of a "con artist" (someone who tricks people out of money) — something like *doubtful*. (A *mark* is someone who is a target of a criminal or con artist). *Skeptical* and *leery* work, although *leery* is more negative.

67. **Tractable, amenable.** The correct choices must match "agreeable" and oppose "implacable unwillingness." While *naïve* and *innocent* relate to the clue "ingénue," they do not take into account the clues in the sentence, about seeming "supine" but actually being stubborn.

68. **Apocryphal, spurious.** The clue is *rumor mongers* and the correct pair both mean "false, doubtful, or unsubstantiated." You have no way to know if the reports were *sentimental, saccharine,* or *scandalous*. Trap answer *apocalyptic* ignores the clues in the sentence, and is inappropriate for an event involving only one person, even if the outcome were tragic.

69. **Contemptible, scurvy.** The correct answers, given the pivot word *while*, must oppose the clue *virtuous*. While agitprop theater may be *faddish, sophisticated,* or *scintillating,* no clues in the sentence indicate any of these. *Glib* is negative but means "superficially fluent" and is inappropriate here (a person or remark can be *glib*).

70. **Lax, slack.** The answers must parallel *slouching posture and cavalier attitude*. Note that *cavalier* is used here to mean "offhand or disdainful." Thus, *aristocratic* is a bit of a trap answer, as are *murderous* and *barbarous,* which don't match the clues — while this person's actions caused *horrific loss of life*, it doesn't sound as though that was his intention. The clues indicate he's just a slacker.

71. Quack, shyster. A *quack* is a "fraudulent doctor" and a *shyster* is a "con man" (in medicine or any field), so both work here. *Sap* and *dupe* are synonyms (suckers, people who have been tricked) that describe the people who fell for the doctor's potion, not the doctor and his reputation.

72. **Dally, tarry.** Given the clue that they were already late, delay would be a good focus. The incorrect pair, *lurk* and *skulk*, have a negative, furtive connotation — they are related to hanging around for some bad reason. Did the couple *embrace*? You don't know. Avoid choices that make literal sense but cannot be justified by the clues.

73. **Appellations, sobriquets.** The spin of this sentence is positive — "staunch admirers" — and you learn that the player's *given name was all but forgotten* (the expression "all but" means something like "99%"). So, you need two words like *nickname. Misnomer*s and *aliases* are types of names, but false ones. *Neologism* is off because it emphasizes "new," as in a "brand new word," rather than a "nickname."

74. **Indigenous, native.** The answers must describe the American Indian population discussed in the sentence. *Native*

may be obvious; *indigenous* means the same thing. Note that smallpox could be a *pandemic,* but that word does not belong in the blank.

75. **Extemporaneous, impromptu.** "Badly miscalculated" indicates that there are two opposing parts of the sentence: the candidate's "prepared speeches" are good, but some other type of answers are bad. You need two words that mean "improvised" or "off-the-cuff." Note that *lubricious* and *disingenuous* form a pair relating to dishonesty that is unrelated to the clues and might be a trap (if you insert your own, negative ideas about politicians). *Premeditated* is the opposite of what you want.

76. **Equivocate, vacillate.** Given the pivot phrase "even though," the answers must oppose "act immediately" and the Farragut quote. *Divagate* is a trap — it means "to wander or digress," especially in speech, and so doesn't quite work here. *Weep* and *wail* are a pair, but are not justified by the clues in the sentence.

77. **Abjured, forwent.** Since the hedge fund founder is doing something to avoid a long prison term, and because of the clue "former role," you know that she "gave up" that role. *Abjured* and *forwent* both work here. Be precise — *reneged* means "to break a promise." A leader can be *deposed,* but you don't *depose* a role. The hedge fund founder may be being *censured* (probably worse), but it doesn't make sense for her to *censure* (disapprove of, reprimand) her role.

78. **Slight, gibe.** The words *even if* signal a change in the meaning of the sentence, so you need a word for the blank that is the opposite of *anodyne*, which can mean "alleviating pain," but here means "not likely to provoke offense." *Compliment* and *panegyric* both have positive connotations, so they can't work here. Only *slight* and *gibe* have the required negativity.

79. **Perfidy, treachery.** The most important word in this sentence is the word *friend*, implying that Ana Cumpănaș *betrayed* Dillinger. *Nosiness* and *meddling* are a reasonable pair, but they don't fit with the fact that Ana Cumpănaș was Dillinger's friend. Only *perfidy* and *treachery* correctly fit the sentence.

80. **Ubiquity, omnipresence.** This is a tough sentence, because it seems to be about money. But while *cost* could work if it had a pair word, *expenditure* means only "an amount of money spent," so it can't work here. You need something else for the blank that would lead you to think the company would make a lot of money. If it can't be the cost of each unit, it has to be the number of units sold. *Ubiquity* and *omnipresence* fit this meaning. *Construction* and *aesthetics* are not enough of a pair, and they don't inherently explain why the company would be raking in cash.

81. **Tacit, implicit.** The word *though* always sets up a contrast, so you need something for the blank that means the opposite of "physically described in rulebooks." Both *tacit* and *implicit* mean "implied, but not plainly expressed." *Unambiguous* and *blatant* have the opposite charge.

82. **Extraneous, peripheral.** You have the word *whereas* in this sentence, which implies a change in direction. So if the critically-acclaimed directors see characterization and emotional development as "critical," Michael Bay should find them unimportant (at least according to the reviewer). *Important* and *intrinsic* are the opposite of what you want. *Extraneous* and *peripheral* both fit the meaning you need.

83. **Confounded, flummoxed.** Because you know that the puzzle in question is *one of the more difficult* puzzles, you would expect Sandra to be, well, "puzzled" by it. *Enraged* and *incensed* are an intriguing pair, but you have no reason to believe Sandra was made angry by the puzzles. *Smitten* can't work here because the correct idiom is "smitten with." *Impressed* could work, but it doesn't have a pair to match with. *Confounded* and *flummoxed* are the best choices.

84. **Specious, meretricious.** This is a complex sentence, and the meaning of the first part is dependent on what comes after the semicolon. There you see that if you only looked shallowly at the two parties, you'd see dishonesty on both sides, but if you looked more deeply, you'd see that one side is worse. Both *specious* and *meretricious* mean "apparently attractive, but having in reality no value," which fits perfectly. *Sophomoric* means "juvenile or immature," which is not indicated by the sentence.

85. **Approbation, plaudits.** You learn in the second half of this sentence that the documentarian received "money" and "acclaim," while the subject of the documentary did not. Either one could go in the blank, but because you know it's coming "from the critics," it needs to match up with "acclaim" (critics don't give "money"). Both *approbation* and *plaudits* fit this meaning. *Opprobrium* and *fulmination* are the opposite of what you want, and *wealth* and *capital* don't fit the meaning of the sentence.

86. **Exculpate, vindicate.** This sentence tells you that Morris's film proves that its subject did not commit the crime in question. This implies that the film proved his innocence. Both *exculpate* and *vindicate* mean to "clear someone of blame or suspicion." *Liberate* and *manumit* are close, but both have the idea of freedom without freeing from blame (and *manumit* is technically only used for when someone is freed from slavery). *Excuse* means "to lessen the blame" or "forgive someone for a fault," but not to prove they were not at fault in the first place.

87. **Beguiled by, enamored with.** For the schemes to "prey" on people, those people would need to want what is being advertised. *Beguiled by* and *enamored with* mean "lured by or infatuated with." *Obsessed by* and *possessed by* go too far. *Aggrieved by* and *vexed by* are negative (if you were *vexed* by quick riches and fancy trips, you wouldn't be easy prey, and also you'd be really weird).

88. **Equanimity, sangfroid.** You need something for the blank that a "great soul" would do in the face of turmoil and tribulation. Since *in the face of* indicates opposite direction, the meaning is something like "remain calm." *Conviction* (a firmly held belief) might work, but it has no partner. *Self-esteem* and *pride* are a pair, but they don't make much sense in the sentence. *Equanimity* and *sangfroid* both imply an inner tranquility.

89. **An incipient, a nascent.** While the environmental movement is said to have been around awhile, the second half of the sentence says that it "only recently" become "a serious organization." This means that the answer choices *disorganized* and *nebulous* don't quite work. The other pair, *inconsequential* and *immaterial*, is needlessly negative. *Incipient* and *nascent* capture the idea that, while the movement has been around for a while, it is only just now becoming a serious and relevant organization.

90. **Bolstered, buttressed.** It is critical to work out here whether the physicists mentioned at the end of the sentence were for or against Einstein's theory. You know that the idea went from "purely experiential" to "theoretically" something. There's a contrast here, and the contrast is not between right and wrong. It is between something that has been shown by experiment and something that has been understood theoretically. This means the later scientists are providing support for Einstein. *Undermined* and *sabotaged* are negative, while *condoned* and *pardoned* don't make much sense. Only *bolstered* and *buttressed* correctly express the idea of support.

91. **Clamor, din.** You can assume that there would be a lot of "noise" at the edge of a stage, which would explain why someone couldn't be heard. *Clamor* and *din* both fit the sentence. *Rabble* and *host* both mean "crowd," which is something you might not be able to hear over, but they don't make nearly as good a choice. *Music* is a little too specific; don't add new ideas to the sentence.

92. **Artless, ingenuous.** The first half of this sentence tells you that you're going to focus primarily on the painter's personality. You then need something that contrasts with either "pretentious" or "pontificating." *Artless* and *ingenuous*, meaning "without effort or pretentiousness," are perfect. *Shrewd*, *adroit*, and *artful* are all positive

words, but they don’t contrast with “pretentious” or “pontificating.”

93. **Trite, hackneyed.** The beginning of the sentence says the movie “wasn’t exactly bad,” implying that you’re going to get a word that isn’t the opposite of bad, but that qualifies it in some way. Later in the sentence, you get a more specific addition: *they were all jokes you’d heard before*. You need words that match with that definition. Both *trite* and *hackneyed* fit that meaning. *Atrocious* and *egregious* mean the same thing as “bad,” so they don’t work. And while *amusing* and *witty* make a pair, they’re the opposite of what you want.

94. **Underpins, undergirds.** The second half of this sentence describes how aesthetics are a major part of the work of the company, so you need a word for the blank that means something like “permeates.” *Saturates* is close, but it has no partner (*irradiate* means either “to expose to radiation” or “to illuminate”). *Underpins* and *undergirds* fit the meaning of the sentence perfectly.

95. **Thwarted, undermined.** The end of this sentence makes it clear that the teachers in question are not happy about having to stick to a prescribed curriculum. *Crushed, confounded*, *walloped*, and *tormented* are all negative, but all of them miss the meaning of the sentence (and no two of them make a good pair). *Crushed* means “emotionally devastated” and tends to apply to things that happen all at once — a breakup, not getting a promotion, etc.

96. **Palliating, assuaging,**. The first half of the sentence explains that lowering interest rates can create financial stability, so this would allow the Fed to make a crisis “better.” *Exacerbating* and *compounding* are the opposite of what you want, and *annihilating* goes too far. Only *palliating* and *assuaging* correctly fit the meaning here.

97. **Laconic, curt.** The words *even though* set up a contrast, so you need something that contrasts with *roles that involved a lot of lines*. A character without a lot of lines would be *quiet*. *Melancholy* and *dejected* imply sadness, but not necessarily a refusal to speak (Hamlet is pretty depressed, and he talks all the time). *Mute* goes too far. *Laconic* and *curt* both match with the meaning you’re seeking. Note that *curt* has the sense of being rudely short with people, whereas *laconic* is not necessarily negative.

98. **Limpidity, lucidity.** The word *whereas* sets up a contrast with *dense* and *abstruse* (the latter of which means “difficult to understand”), so you need a word that means something like “clarity.” *Sagacity* and *prowess* both imply skill, but you need something that also implies a kind of simplicity. *Limpidity* and *lucidity* fit the bill perfectly.

99. **Placated, mollified.** For this sentence, you need to infer how the beauty queen’s mother likely felt about her daughter being disqualified—namely, badly. The conjunction “and” signals that the second half is going to match the tone of the first half (as opposed to something like “but”), so the blank needs to be something showing that the mother is still not happy. *Placated* and *mollified* both fit the meaning. *Ameliorated* is close, but only a problem can be *ameliorated*, not a person.

100. **Melancholy, morose.** The key to this sentence comes at the very end, when you learn that Hamlet is not “paralyzed with depression.” We’ve already read in the middle part of the sentence that he is not paralyzed, so the depression part must come from the blank. *Indecisive* and *monologic* both fit the famous character of Hamlet, but they don’t fit the sentence (nor do they make a pair). Similarly, *violent* and *barbaric* would be accurate in describing Hamlet, but they don’t fit the sentence. Only *melancholy* and *morose* match the meaning you want.

101. **Anomalous, aberrant.** The word *once* here signals a change in the direction of meaning. As the sentence ends with the idea that the behaviors in question were once “considered fairly normal,” you need something for the blank that means “uncommon.” *Mythical* and *fabulous* may seem to fit, but they’re too extreme. *Anomalous* and *aberrant* work much better.

102. **Mundane, quotidian.** The blank here needs to contrast with "the fantastical." *Mundane* and *quotidian* both fit. *Bombastic* and *belletristic* both may correctly describe the writing style of Proust, but they don't match the meaning of the sentence.

103. **Pith, gist.** In this sentence, the key words are "very busy woman," implying the editor doesn't have a lot of free time. That means it would be necessary to get right to the *most important part* of the issue. *Conclusion* and *culmination* are a pair, but they mean "the end," which isn't quite right. *Pith* and *gist* are a much better match for the blank.

104. **Burgeon, mushroom.** The word *while* signals a change in direction, and the second half of the sentence is negative (something is coming "too late"), so you need something in the blank that is positive. In the case of environmental concern, that would be "more of it." Both *burgeon* and *mushroom* fit that meaning. *Disseminate* is close, but it has no pair (*promulgate* means "to make something widely known," not "to become widely known"), and the concern isn't spreading out so much as growing.

105. **An explicit, an overt.** This sentence doesn't give you much to go on for meaning, but odds are good that if your enemy moves to the border, you see it as a pretty "direct act of aggression." *Explicit* and *overt* both fit this meaning. *Bellicose* and *belligerent* are a pair, but they would be redundant ("A *belligerent* act of aggression").

106. **Inevitably, unavoidably.** The clues here are pretty blatant — *Time stops for no man* and *just as had every preceding style of music*. In other words, everything always goes pretty much the same way.

107. **Hankering, proclivity.** The word *and* suggests that the phrases before and after the conjunction will agree. Isherwood has "fellow feeling for the laboring classes," that is he is sympathetic with working people, and so wants to "engage as an equal" with them. You should anticipate a word like *desire*. *Hankering* and *proclivity* are the nearest synonyms. *Disinclination* and *unwillingness* are the opposite of what you want (*implacability* is also negative). *Joviality* might seem attractive, but is more extreme than your clue of "fellow feeling."

108. **Germane, pertinent.** *Though* implies that the matters about which a professor enjoys liberty are opposite those "irrelevant to his subject." So you might anticipate an answer like "relevant." *Germane* and *pertinent* are the best synonyms. *Mimetic* and *congruent* are slightly related to the idea at hand — the former means "copying" (like a mime, for instance) and the latter means something like "standing in a similar relation."

109. **Rue, remorse.** You would like an answer very much close to "sorrow," perhaps something like "regret." *Rue* and *remorse* are the nearest synonyms.

110. **Exhaustive, compendious.** Since the *tremendous wealth… on display* represents *only a tiny fraction* of the collection, and since the collection has *over one million specimens*, it is a very large collection indeed. You might anticipate an answer like "huge." *Exhaustive*, which means "including everything," is a bit stronger even than anticipated, but it will work. The primary meaning of *compendious* is something like "synoptic, briefly stating all the essentials," and that's not what you want, but *compendious* can also mean simply "complete." The primary meaning is the one you're most likely to encounter on the GRE, but in this question, you should take the other meaning, "complete." (The meaning you rely on for this question, incidentally, is most common even in formal writing, though it's not first in the dictionary.) Imperfect though both these answers are, nothing else is even close.

111. **Truculent, savage.** You'd like a word that contrasts with *conciliatory*, since the word *comparatively* suggests an opposition. You'd also like a word that sensibly modifies *attack*. You might anticipate something as simple as

"attacking." *Truculent* and *savage* both suggest a disposition to fight.

112. **Asperity, rigor.** *Even* implies that the word is stronger than *austere*, though similar to that word. The word names something in Bourbaki's proof that daunts or intimidates. You might anticipate something like "great difficulty." *Asperity* means "severity," which will work (it especially suggests a rough or harsh surface). *Rigor* is an even closer synonym to "austerity."

113. **Extirpated, effaced.** This sentence is about as purely definition-based as you can get: the house was torn from the ground and utterly destroyed. You need words that signify "eradicated" or "obliterated"; *extirpated* and *effaced* are right.

114. **Debacle, rout.** The team once had an "upper hand" and is now losing badly. You need words that denote a "crushing failure," or a "drubbing." *Debacle* and *rout* are good, and none of the other words come close. Note that a *boondoggle* is certainly bad, but it is an unnecessary and wasteful project, and thus not relevant here.

115. **Reneged on, abrogated.** The political challenger is apparently saying that the "incumbent," or "current" politician made certain agreements during his first time in office, but didn't follow through on them. The politician, in other words, "went back on," or "failed to honor" these agreements: he *reneged on* them, or *abrogated* them.

116. **Rift, breach.** *Beyond the pale of dispute* indicates that there were issues that were once totally accepted by both parties. Now they are "subject to contention," indicating that there is a widening "gap," or "fissure," between the parties: a *rift* or a *breach* between them.

117. **Impecunious, insolvent.** This sentence trickily hinges on the idiom *though they now may be*, which indicates that whatever comes after the comma is opposite in meaning to the blank. You learn next that that family seems to act as if they are still aristocrats. The implication is that they are then *not* aristocrats, or at least don't have the same wealth that aristocrats are known to have. You're thus looking for a word that means "poor" or "impoverished." *Impecunious*, which comes from *pecunia*, Latin word for "money," and *insolvent*, which means "unable to pay debts," are best.

118. **Improbity, mendacity.** Fortunately, this sentence is pretty straightforward — you need two words that mean "lying." Only *mendacity* and *improbity* work.

119. **Refulgence, radiance.** Even though this sentence is short, its tough vocabulary makes it tricky. It is clear enough from the word *dazzling* that you're looking for a word that means "brightness" or "brilliance." *Refulgence* and *radiance* are best; none of the other words match this meaning.

120. **Mordant, trenchant.** The best clue in this sentence is its parallel structure. The judge's "keen eye" was surpassed only by her "_______ wit." *Keen* means "sharply discerning," so you should be looking for words that denote a "sharpness," or "bitingness," of wit. *Mordant* and *trenchant*, both of which are close in meaning to "incisive," are best. Don't be confused by the other set of synonyms, *assiduous* and *sedulous*, both of which have meanings close to "diligent."

121. **Beseeched, supplicated.** The inmate is "begging, pleading, imploring" the probation panel to be set free. *Beseeched* and *supplicated* are closest to this meaning.

122. **A fanatical, a zealous.** The loyalty of the cult members verged on the obsessive and thus was extremely intense.

You're looking for words similar in meaning to "extreme," or "diehard." *Fanatical* and *zealous* are best here.

123. **Grievous, baneful.** The "evils" described by this word are "more pernicious" than … well, it doesn't even really matter — you want a word like *pernicious*. Perhaps "very bad." *Grievous*, which means "causing grief," or "very harmful," will work. *Baneful* is an even closer synonym to *pernicious*. Note the trap word *epicurean,* which relates to fine food and wine (and thus to this sentence), but which is positive and does not fit the blank.

124. **Hegemony, dominance.** Just as once *every subject eventually had to meet (theology's) demands*, now *not only physical science but social science* must meet mathematics' demands. This suggests that mathematics enjoys something like "rule" over these fields; the phrase *Queen of the Sciences* suggests the same, and perhaps a sort of "autocratic rule." *Hegemony* usually describes the "dominance of one state over others," but can also mean "the dominance of one social group, political party, etc." *Dominance* will also work. If you anticipated precisely the word *autocratic*, *autonomy* might be attractive, but it doesn't actually work: *autocratic* describes government by a single person with unlimited powers, while *autonomy* means "self-governance."

125. **Ephemeral, fleeting.** Something *lasting a millionth of a billionth of a billionth of a second* is "very short-lived." *Ephemeral* and *fleeting* have just this meaning. In real life, the particles may be small (*infinitesimal),* there may be a lot of them (*myriad, countless*), and they may bring a tear to the eye of a few scientists (*poignant*), but none of these are indicated by clues in the sentence.

126. **Collaborative, synergetic.** The pivot word *while* tells you that the later contributions were achieved in a manner opposite to "single-handedly." You need two words that mean "cooperative." Note that trap answers *solitary* and *unilateral* are the opposite of what you need, and *collusive* has the wrong spin — to *collude* is to "cooperate for illegal or fraudulent purposes."

127. **Claptrap, dross.** You need two words that contrast with *substantial* and match the idea of running out of material. *Claptrap* is "pretentious but insincere language," and *dross* is "refuse, garbage." *Pith* and possibly *marrow* are a strange pair — *pith* is "the essence," and *marrow* (the substance inside of bones) can sometimes be used metaphorically in a similar way (to "suck the *marrow* out of life"), but it wouldn't make sense to say that a book lacks substance if it does, in fact, contain the most important parts.

128. **Retort, rejoinder.** You need two words describing a witty comeback. *Retort* and *rejoinder* are perfect. *Recrimination* is too strong. A witty reply is not an *anecdote,* nor is it an *aphorism* or *maxim* (which, like proverbs, contain some general truth or wisdom and do not have to be funny).

Chapter 5
of

Reading Comprehension

In This Chapter…

Reading Comprehension

Questions 1–3 are based on the following reading passage.

While new census data reveals that unemployment numbers are even direr than was previously suspected, it is not clear that the forecast for American entrepreneurship is equally alarming. An article in a major national newspaper suggests that the contraction in hiring at existing companies might result in more new companies being founded. College graduates, unable to find traditional jobs, instead opt to start their own businesses. Where a recession may seem an unpropitious time for such a historically risky endeavor, with no better options, would-be entrepreneurs have little to lose. Unfortunately, this situation does not necessarily impact the economy positively. Though the average number of new businesses started per year has been higher during the recession than it was before, the proportion of high-value businesses founded each year has declined. So even if a business manages to stay solvent, it may not bring significant returns. Also, because of an inevitable dearth of angel investors and venture capitalists, many new entrepreneurs are putting their own money on the line. In certain ways, the choice between accepting a traditional job and starting a business is not unlike the choice between renting and buying property. The latter requires a significant initial outlay and carries heavier risks, but the rewards can be equally substantial.

1. The primary purpose of the passage is to

(A) propose changes in the way the public generally interprets census data
(B) maintain that college students should form their own companies, especially during economic recessions
(C) present a nuanced view of a contemporary economic issue
(D) evaluate the viability of low-versus high-value businesses under various environmental conditions
(E) draw an analogy between career decisions and real estate decisions, specifically the choice to rent or buy property

2. According to the passage, the reason that many college graduates are choosing to launch their own companies in the present economic climate is that

(A) they are hampered by the difficulty of finding outside investors
(B) they cannot easily land positions typically open to workers of their experience
(C) the prevalence of low-value companies has increased
(D) they are forced to decide between renting and buying property
(E) forecasts of the unemployment rate are likely to become less dire in coming years

3. It can be inferred from the passage that over the course of the recent recession, the number of American high-value businesses founded per year

(A) has fallen sharply
(B) has fallen moderately
(C) has risen sharply

(D) has risen moderately
(E) may have either fallen or risen

Question 4 is based on the following reading passage.

According to Mercy Amba Oduyoye in *Daughters of Anowa: African Women and Patriarchy*, the women of the Asante people of Ghana participated in war as nurses or in providing supplies, but only those who had not yet reached or who were past childbearing age. If such women died in battle, they died "as individuals and not as potential sources of human life." As such, many old women engaged in valiant acts, sometimes sacrificing their own lives, to defend those they had given life to.

4. Which of the following can be inferred from the passage?

(A) The deaths of Asante women of childbearing age were lamented more than were the deaths of other women.
(B) Older Asante women were more courageous than younger Asante women.
(C) Some of those who worked as nurses or in providing supplies died in battle.
(D) Women of childbearing age were accorded special status above other women and men.
(E) Men could not be considered "potential sources of human life."

Questions 5–7 are based on the following reading passage.

The past decade has seen a statistically significant uptick in reports of the bacterial strains known as "super-bugs," so called not because of enhanced virulence, but because of their resistance to many antimicrobial agents. In particular, researchers have become alarmed about NDM-1 (New Delhi metallo-beta-lactamase), which is not a single bacterial species, but a transmittable genetic element encoding multiple resistance genes. A resistance "cocktail" such as NDM-1 could bestow immunity to a bevy of preexisting drugs simultaneously, rendering the bacterium nearly impregnable.

However, in spite of the well-documented dangers posed by antibiotic-resistant bacteria, many scientists argue that the human race has more to fear from viruses. Whereas bacteria reproduce asexually through binary fission, viruses lack the necessary structures for reproduction, and so are known as "intracellular obligate parasites." Virus particles called virions must marshal the host cell's ribosomes, enzymes, and other cellular machinery in order to propagate. Once various viral components have been built, they bind together randomly in the cellular cytoplasm. The newly finished copies of the virus break through the cellular membrane, destroying the cell in the process. Because of this, viral infections cannot be treated ex post facto in the same way that bacterial infections can, since antivirals designed to kill the virus could do critical damage to the host cell itself. In fact, viruses can infect bacteria (themselves complete cells), but not the other way around. For many viruses, such as that responsible for the common cold sore, remission rather than cure is the goal of currently available treatment.

While the insidious spread of drug-resistant bacteria fueled by overuse of antibiotics in agriculture is nothing to be sneezed at, bacteria lack the potential for cataclysm that viruses have. The prominent virologist

Nathan Wolfe considers human immunodeficiency virus (HIV), which has resulted in the deaths of more than thirty million people and infected twice that number, "the biggest near-miss of our lifetime." Despite being the most lethal pandemic in history, HIV could have caused far worse effects. It is only fortunate happenstance that this virus cannot be transmitted through respiratory droplets, as can the viruses that cause modern strains of swine flu (H1N1), avian flu (H5N1), and SARS.

5. The main purpose of the passage can be expressed most accurately by which of the following?

(A) To contrast the manner by which bacteria and viruses infect the human body and cause cellular damage.
(B) To explain the operations by which viruses use cell machinery to propagate.
(C) To argue for additional resources to combat drug-resistant bacteria and easily transmissible pathogenic viruses.
(D) To highlight the good fortune experienced by the human race, in that the HIV pandemic has not been more lethal.
(E) To compare the relative dangers of two biological threats and judge one of them to be far more important.

6. It can be inferred from the passage that infections by bacteria

(A) result from asexual reproduction through binary fission
(B) can be treated *ex post facto* by antimicrobial agents
(C) can be rendered vulnerable by a resistance cocktail such as NDM-1
(D) are rarely cured by currently available treatments, but rather only put into remission
(E) mirror those by viruses, in that they can both do critical damage to the host cell

7. According to the passage, intracellular obligate parasites

(A) are unable to propagate themselves on their own
(B) assemble their components randomly out of virions
(C) reproduce themselves through sexual combination with host cells
(D) have become resistant to antibiotics through the overuse of these drugs
(E) construct necessary reproductive structures out of destroyed host cells

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Questions 8–10 are based on the following reading passage.

A supernova is a brief stellar explosion so luminous that it can briefly outshine an entire galaxy. While the explosion itself takes less than fifteen seconds, supernovae take weeks or months to fade from view; during that time, a supernova can emit an amount of energy equivalent to the amount of energy the sun is expected to radiate over its entire lifespan. Supernovae generate enough heat to create heavy elements, such as mercury, gold, and silver. Although supernovae explode frequently, few of them are visible (from Earth) to the naked eye.

In 1604 in Padua, Italy, a supernova became visible, appearing as a star so bright that it was visible in daylight for more than a year. Galileo, who lectured at the university, gave several lectures widely attended by the public. The lectures not only sought to explain the origin of the "star" (some posited that perhaps it was merely "vapour near the earth"), but seriously undermined the views of many philosophers that the heavens were unchangeable. This idea was foundational to a worldview underpinned by a central and all-important Earth, with celestial bodies merely rotating around it.

8. The primary purpose of the passage is to

(A) give the history of supernovae
(B) describe a shift in thought as a result of a natural event
(C) juxtapose two opposing views about supernovae
(D) corroborate the view that the earth is not central to the universe
(E) explain how science and philosophy interrelate

Consider each of the answer choices separately and indicate all that apply.

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9. Which of the following can be inferred by the passage?

☐ Supernovae can take over a year to fade from view.
☐ Prior to 1604, no one had ever seen a supernova.
☐ Galileo convinced philosophers of the incorrectness of their views.

10. The author mentions which of the following as a result of the supernova of 1604?

(A) The supernova created and dispersed the heavy elements out of which the earth and everything on it is made.
(B) Galileo explained the origin of the supernova.
(C) The public was interested in hearing lectures about the phenomenon.
(D) Galileo's lectures were opposed by philosophers.
(E) Those who thought the supernova was "vapour" were proved wrong.

Question 11 is based on the following reading passage.

A Small Place is Jamaica Kincaid's memoir of growing up in Antigua as well as an indictment of the Antiguan government and Britain's colonial legacy in Antigua. Kincaid blames colonial rule for many of Antigua's current problems, including drug dealing and selling off land for tourist properties. Kincaid's critics question why, if the British are responsible for the Antiguan government's corruption, the British government itself isn't more corrupt. Kincaid has responded that there must have been some good people among the British, but that they stayed home.

Consider each of the answer choices separately and indicate all that apply.

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11. Based on the information in the passage, which of the following would Kincaid most likely agree with?

☐ A government can bring about a degree of corruption abroad that the government itself does not suffer from at home.
☐ Britain has caused corruption in governments throughout its former colonial empire.
☐ Selling off land for tourism-related purposes is a problem for Antigua.

Questions 12–14 are based on the following reading passage.

By 1784, Wolfgang Amadeus Mozart was internationally renowned as the composer of *The Marriage of Figaro*, and consequently received a commission from the Prague Opera House to compose another opera. The resulting product was *Don Giovanni*, which tells the tale of a criminal and seducer who nevertheless evokes sympathy from audiences, and whose behavior fluctuates from moral crisis to hilarious escapade.

While *Don Giovanni* is widely considered to be Mozart's greatest achievement, eighteenth century audiences in Vienna — Mozart's own city — were ambivalent at best. The opera mixed traditions of moralism with those of comedy — a practice heretofore unknown among the composer's works — creating a production that was not well liked by conservative Viennese audiences. Meanwhile, however, *Don Giovanni* was performed to much acclaim throughout Europe.

12. The primary purpose of the passage is to

(A) relate the story of a somewhat likable antihero
(B) discuss how a work of art has been met by diverging responses
(C) give a history of the work of Mozart
(D) make a case for the renown of *Don Giovanni*
(E) emphasize the moral aspects of a musical work

13. The author mentions the mixing of "traditions of moralism with those of comedy" primarily in order to

(A) explain a work's lackluster reception among a particular group of people
(B) remind the reader of the plot of *Don Giovanni*
(C) highlight a practice common in contemporary opera
(D) argue for an innovative approach to opera
(E) undermine a previously presented assertion

14. It can be inferred from the passage that which of the following is true about the response of Viennese audiences to *Don Giovanni*?

(A) The audiences preferred purely moralistic works.
(B) The response was unequivocally positive.
(C) They did not know that the composer was attempting to mix musical styles.
(D) The play's moral themes were offensive to Viennese audiences.
(E) To say that the response was "mixed" would be a generous interpretation.

Questions 15–17 are based on the following reading passage.

In the 1960's, Northwestern University sociologist John McKnight coined the term *redlining*, the practice of denying service, or charging more for service, to customers in particular geographic areas, areas often determined by the racial composition of the neighborhood. The term came from the practice of banks outlining certain areas in red on a map — within the red outline, banks refused to invest. With no access to mortgages, residents within the red line suffered low property values and landlord abandonment; buildings abandoned by landlords were then likely to become centers of drug dealing and other crime, thus further lowering property values.

Redlining in mortgage lending was made illegal by the Fair Housing Act of 1968, which prohibited such discrimination based on race, religion, gender, familial status, disability, or ethnic origin, and by community reinvestment legislation in the 1970's. However, redlining may have continued in less explicit ways, and can also take place in regards to constrained access to health care, jobs, insurance, and more. Even today, some credit card companies send different offers to homes in different neighborhoods, and some auto insurance companies offer different rates based on zip code.

Reverse redlining occurs when predatory businesses specifically target minority consumers for the purpose of charging them more than would usually be charged to a consumer of the majority group. Redlining can lead to reverse redlining — if a retailer refuses to serve a certain area based on the ethnic-minority composition of the area, people in that area can fall prey to opportunistic smaller retailers who sell inferior goods at higher prices.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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15. Which of the following can be inferred from the passage?

- ☐ Redlining ceased with the passing of the Fair Housing Act in 1968.
- ☐ Redlining today may be less overt than before its illegalization.
- ☐ Access to mortgages is related to higher property values.

16. Which of the following, not mentioned in the passage, would qualify as an example of reverse redlining as defined in the passage?

(A) A bank refuses to offer mortgages to consumers in certain neighborhoods.
(B) Residents of low-income neighborhoods are less likely to be hired for positions than residents of higher-income neighborhoods, even when the applicants have the same qualifications.
(C) Police respond to reports of crimes more quickly in some neighborhoods than in others.
(D) A grocery store in a low-income neighborhood sells low-quality produce for high prices, knowing that most residents do not have the ability to buy elsewhere.
(E) An auto insurance company hires an African American spokesperson in a bid to attract more African American consumers.

17. Which correctly describes a sequence of events presented in the passage?

(A) Subprime mortgages lead to widespread defaults, which lead to landlord abandonment.
(B) Reverse redlining leads to landlord abandonment, which leads to the use of buildings for crime and drug dealing.
(C) Landlord abandonment leads to redlining, which leads to crime and drug dealing.
(D) Redlining leads to reverse redlining, which leads to constrained access to health care, jobs, insurance, and more.
(E) Redlining leads to landlord abandonment, which leads to the use of buildings for crime and drug dealing.

Question 18 is based on the following reading passage.

Premastication is the practice of a mother pre-chewing food before feeding it, mouth-to-mouth, to her baby. While germophobic Western society eschews this practice, it is not only common in the developing world, but provides benefits to a developing baby. Babies are not born with digestive bacteria; they get some from passing through the birth canal, but continue to encounter the beneficial bacteria during breastfeeding and while being handled, in general, by adults. Throughout most of human history, babies also received antibodies and digestive bacteria from the mother's saliva, transmitted via premasticated food. In some cultures, fathers also premasticate food for babies; sometimes even entire family groups will do this – a toddler at a family meal might wander from person to person, being fed by many adults.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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18. Based on the information in the passage, the author of the passage would most likely agree that

- ☐ Germophobia can contribute to depriving babies of a health benefit.
- ☐ Premasticating food for babies is done only in the developing world.
- ☐ Adult saliva has benefits for babies in addition to the transmission of beneficial digestive bacteria.

Questions 19–23 are based on the following reading passage.

Matisse and Picasso; Picasso and Matisse. Throughout the twentieth century, this pairing has been touted as the quintessential artistic rivalry. In *Matisse and Picasso*, Yve-Alain Bois follows Hubert Damisch in proposing that the interaction between Picasso and Matisse should be seen as a dynamic game rather than a static conflict of artistic polarities. Bois employs the metaphor of chess, arguing that the game represents the artists' exchange as "a competitive rivalry and a complex temporality" that can be viewed both as a linear process and a simultaneous structure.

But the metaphor of a competitive sport, however complex and intellectually rich, is misleading. The two artists were engaged not just in competition (even friendly competition) but also in friendly dialogue. The two men were more than rivals: they were colleagues, critics, teachers, and occasional friends. A better model, though perhaps one with less flash, is that of a simple conversation, with all the rich variation and shifts in motivation and tone that are possible.

Picasso's *Large Nude in a Red Armchair* marks the extremes of the artist's combativeness towards Matisse. The painting is a clear parody of Matisse's earlier *Odalisque with a Tambourine*. The composition of the figures is strikingly similar: a woman lounges in an armchair at the center of the painting, arm raised above her head, decorative wallpaper behind her. Both paintings feature vivid color contrasts, with green wallpaper, vivid reds, glaring yellows, and rich browns. But Picasso's painting, finished in 1929, mocks the achievements of Matisse's earlier work. The sensuous, rich mood of Matisse's painting has been transformed in Picasso's work into something harsh and grotesque.

The other extreme of the dialogue between the two artists can be seen in Picasso's *Woman with Yellow Hair* and Matisse's response, *The Dream*. The exchange begins with Picasso's work, in 1931. The painting depicts a woman asleep on her arms, resting on a table. She is full, rich, warm, and curved, her head and arms forming a graceful arabesque. This image seems a direct attempt to master Matisse's style and to suggest to the older artist new directions for his work. While there may well be an edge of competitiveness to the painting, a sense that Picasso was demonstrating his ability to do Matisse's work, it remains in large part a helpful hint.

Matisse, nearly a decade later, continues the conversation in a similar tone. In *The Dream* of 1940, he proposes a revision of Picasso's work. Again, a woman lies asleep on a table, her arm tucked beneath her head. Matisse accepts Picasso's basic suggestions for his style: sinuous curves, volumes, and shocking uses of color to express an effect. But Matisse also modifies the earlier work significantly. Color is no longer rigidly tied to form, as bits of fuchsia seep outside the thick black line marking the outline of the table and the patch of yellow on the woman's blouse refuses to be contained by the drawn line. Matisse uses Picasso's same palette of red, purple, white, black, and yellow to create this revision, editing out only the garish green, as if to chide Picasso for the choice. The brilliant interplay of colors in Matisse's work is far more sophisticated and subtle than that offered by Picasso. "Thank you," Matisse seems to be saying, "but you missed a few spots."

19. The primary purpose of the passage is to

(A) discuss the two best painters of an epoch
(B) evaluate a theory and endorse a revision
(C) compare selected works of two masters
(D) show that Matisse's work is more sophisticated
(E) illustrate how Picasso taught Matisse

20. The author would most likely agree with which of the following statements?

(A) Artistic rivalries are more like Olympic competitions than professional sports.
(B) Artistic mastery is best demonstrated by employing multiple styles.
(C) Artists must be good conversationalists.
(D) Artistic rivalries can actually be reciprocally nourishing.
(E) Artistic rivalries generally last for decades.

21. According to the passage, which of the following describes *Woman with Yellow Hair*?

(A) It was parody of a work by Matisse.
(B) Its colors were not rigidly tied to its form.
(C) Its color palette was larger than that of *The Dream*.
(D) It was a response to a work by Matisse.
(E) It was harsh and grotesque.

22. The passage indicates that

(A) *Large Nude in a Red Armchair* conveyed a sensuous, rich mood
(B) Matisse verbally commented about *Woman with Yellow Hair*
(C) Matisse and Picasso were lifelong friends
(D) Matisse worked on *The Dream* for nearly a decade
(E) Matisse was older than Picasso

23. Which of the following, had it actually occurred during the artists' lifetimes, would further support the author's thesis?

(A) A joint exhibition of the two artists' work.
(B) A radio broadcast of the two artists discussing painting.
(C) A movie that dramatized the competition between the two artists.
(D) A play that depicted the two artists playing chess.
(E) A painting of the two artists.

Questions 24–28 are based on the following reading passage.

Often taking a prominent position over a whiteboard or across a wall, timelines are one of the most commonplace classroom tools used to teach history. They present a concise chronology with dates and events listed in a linear narrative, forming a skeletal story of history. Despite their usefulness in allowing students to gain a cursory knowledge of many key moments in the past, their bare-bones, fact-centered structure is symptomatic of the **myopic** character of curricula that emphasize the What, When, and Who and eclipse the significance of Why and How.

In the United States, by far the most common brand and format of timeline is the World Almanac for Kids US History Timeline—a banner set of 8 horizontal panels each with 8 events, beginning with Columbus's voyage in 1492 and ending with Clinton's election in 1993. This timeline has photos accompanying it—about 5–6 per panel—next to most of the dates, and below each date is a 1–2 line description of an event that took place in that year. What immediately commands one's attention when looking at this timeline are the dates themselves. Bolder and more prominently placed than anything else, they seem implied to be the most important feature of the timeline—even more so than the events' descriptions. The way the dates line up in perfect order presents the viewer with a rigid historical narrative, complete with a beginning and end.

To analyze any particular timeline, it is important to recognize what the timeline expresses implicitly. The first implicit message transmitted by the World Almanac for Kids US History Timeline is that each event listed on the timeline's face must hold some kind of particular historical significance to qualify as one of only 64 pieces of American history presented, though no event's entry gives even a vague explanation as to why it merits this. The second message the timeline conveys, simply by hanging in the classroom, is that this version of history is an "official" one. Third, that each of these events happened totally independent of one another. Fourth, that, at most, only one significant event occurred in any given year. And finally, that American history is entirely made up of wars and minor battles, punctuated by the occasional presidential election and technological innovation. Now, certainly, one can easily surmise that the timeline is not conscious of these implications, and instead assumes that the viewer will automatically acknowledge that it is not a comprehensive history but rather a simple summary of events through time. The danger of using the timeline as a teaching tool, of course, lies squarely in that assumption.

24. The author implies which of the following?

(A) Dates are not important in history.
(B) Historical events are not interconnected.
(C) Implicit messages can be as important as explicit ones.
(D) A study of American history that does not include women and minorities is incomplete.
(E) American history is best thought of as a linear continuum of events.

25. The author's attitude toward timelines can best be described as

(A) condescending and impertinent
(B) tolerant and bemused
(C) suspicious and resigned
(D) wary and portentous
(E) negative and complacent

26. The author would most likely agree with all of the following EXCEPT:

(A) There are more than 64 important events in American history.
(B) Some students ascribe importance to prominent graphic position.
(C) Timelines have some positive uses.
(D) Timelines have no subliminal effects.
(E) Unofficial histories have merit.

27. According to the passage, a problem with timelines is

(A) their prominent placement in classrooms
(B) their lack of context
(C) their infinite nature
(D) their factual inaccuracy
(E) their inclusion of photos

28. Which of the following could be substituted for the word *myopic* without changing the meaning of the passage?

(A) ignorant
(B) bigoted
(C) purblind
(D) astigmatic
(E) mordant

Questions 29–34 are based on the following reading passage.

As queen of France, Marie Antoinette suffered what were likely the harshest criticisms ever laid against any queen or mistress in France's long history. There were two major factors that combined to propagate this dark new level of acidic criticism—one was the blooming public sphere, and the second was the scandalous "Diamond Necklace Affair."

Literacy rose greatly over the course of the 18th century and, not coincidentally, the annual output of printed publications tripled by the end of Louis XV's reign and expanded exponentially throughout Louis XVI's kingship. Royal censorship had also been greatly reduced by this time, and a massive "black market" for books and extremely popular underground publications flourished during this period. Also, coffeehouse culture and print culture collided during the mid-to-late 1700's, giving Parisians open forums in which to share the gossips and criticisms circulating via the underground pamphlets.

Having endless numbers of pamphlets and an equally infinite number of readers eager to snap them up would be no good without a juicy story, however. This, of course, was exactly what the people received with the Diamond Necklace Affair. On August 11th, 1784, a social climber named Rohan and a prostitute named Nicole Leguay met in the gardens of Versailles. Nicole was a stunning look-alike of Marie Antoinette, and she was indeed believed to be the queen by Rohan that night. To get on Marie's good side—a necessary evil for anyone with social ambitions at Versailles—Rohan was led to believe that if he procured a fabulously bejeweled necklace on her behalf, it would be a great favor.

The scam was revealed when jewelers Boehmer and Bossange inquired directly to Marie Antoinette over payment for the grandiose diamond necklace. They presented her with an invoice that she had apparently signed (though it was actually a clever forgery). The queen was furious and had Rohan arrested and marched off to the Bastille. Yet, in addition to the countless aristocrats who sued to the king on Rohan's behalf, at one point over 10,000 people came to the doors of the Bastille demanding Rohan's release. He was eventually acquitted, much to the queen's dismay.

The consequences of this affair were severe for Marie Antoinette. The mere fact that the queen of France could be so easily mistaken by a nobleman for a common street prostitute—one who engages in underhanded, nocturnal dealings to obtain absurdly expensive jewelry— was incredibly damaging to her already blackened reputation. Furthermore, as this hatred of the queen began to boil over, it became inevitable that it would spill onto the monarchy itself. Though Marie would always be the ultimate villainess, she could never be completely untangled from Louis and, thus, from the monarchy itself. She was not a mistress who could be surreptitiously cut away or a political advisor who could be dismissed. She was a queen, and this fact had inescapable consequences.

29. The second paragraph of the passage serves to

(A) elucidate further the impetus for the public disdain for a public figure
(B) contrast two factors that spurred criticism of the queen
(C) explain the endemic corruption of the French court
(D) discuss the results of a famous scandal
(E) detail reasons for Marie Antoinette's unpopularity as well as the consequences

30. The passage implies that a significant proportion of the French aristocracy

(A) were jealous of the queen's riches
(B) read coffee house pamphlets
(C) proved more loyal to Rohan than to the queen
(D) were less literate than the general populace
(E) became leaders of the Revolution

31. According to the passage, readership of books and pamphlets increased in the late 18th century because

(A) the education of women nearly doubled the number of readers
(B) the literacy rate tripled during the reign of Louis XV
(C) there were more exciting scandals to write about than there had been in previous times
(D) government censorship had relaxed
(E) the number of coffee houses increased

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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32. According to the passage, Rohan

☐ committed forgery
☐ had social ambitions
☐ had support from the populace

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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33. The author suggests that Marie Antoinette

☐ undermined the monarchy
☐ never met Rohan

- ☐ had unlimited legal power

34. The author’s tone could best be described as

- ☐ arrogant and supercilious
- ☐ prim and meretricious
- ☐ thoughtful and disinterested
- ☐ sober but lascivious
- ☐ analytical but enthusiastic

Questions 35–39 are based on the following reading passage.

In John D’Emilio’s essay “Capitalism and Gay Identity,” he argues that the emergence of industrial capitalism led to new opportunities for “free laborers” in the United States, leading to various beneficial changes in social conditions. It is clear that there are two overarching themes emergent from D’Emilio’s argument about the effects of the onset of industrial capitalism: the new abundance of independence, and choice for “free laborers.” He implies throughout that these—independence and choice—are the distinct new markers of the social conditions resultant from this economic shift.

D’Emilio argues that capitalism empowers laborers as “free” in the sense that they are free to look for jobs and to negotiate contracts and terms of labor. D’Emilio’s critics suggest that he largely sidesteps the problems that confound free labor ideology and limit the ability of workers to openly negotiate contracts with employers and to accept or reject the conditions offered. The “contract negotiations” cited as a sign of freedom by D’Emilio are often hardly negotiations at all, but rather highly exploitative arrangements that workers have little ability to affect. From the first Lowell Girls all the way to the modern third world garment works described by Enloe’s “Blue Jeans and Bankers,” it is clear that for many—particularly women, minorities, and immigrants—free labor has hardly been free at all.

Such critics also suggest the D’Emilio misrepresents the historical and continued significance of the home. The shift from a home-based to an industrial economy—though indeed very drastic—was hardly as absolute as D’Emilio suggests. Indeed, from nannying, to housekeeping, to even the “home based jobs” described in “Blue Jeans and Bankers,” labor is still a very active part of the home even today.

In the essay “The Approaching Obsolesce of Housework: A Working-Class Perspective,” Davis spends a great deal of time discussing the continuation of labor in the home in stark contrast to the assertions of D’Emilio. Where D’Emilio argues that industrial capitalism equated to freedom from the home, Davis argues that it actually equated to thickening the bars that caged housewives to the home as productive and reproductive labor split further and more distinctly apart. Davis argues that women “were the losers in a double-sense: as their traditional jobs were usurped by the burgeoning factories, the entire economy moved away from the home, leaving many women largely bereft of significant economic roles.”

35. The structure of the passage is best described as:

(A) A theory is described and supported by additional studies.
(B) A theory and a screed against it are provided.
(C) A theory is presented followed by a collection of opinions that impugn it.

(D) A theory is discussed and its author's credentials questioned.
(E) A theory is presented and ridiculed.

36. The passage suggests that one argument presented against D'Emilio's theory is based on

(A) communism
(B) identity politics
(C) ethnocentrism
(D) semantics
(E) sexism

37. In the second paragraph, the author mentions the Lowell Girls in order to

(A) illustrate how industrial capitalism eroded women's economic position
(B) challenge an assertion about the importance of home based economies
(C) rebut the arguments of D'Emilio's critics
(D) favorably contrast their position to that of third world garment workers
(E) support an assertion that workers' autonomy is more myth than reality

38. The passage suggests that Davis would be most likely to agree with which of the following?

(A) Industrial capitalism leads to independence and choice.
(B) Minorities in particular had little freedom of choice.
(C) People without economic roles are disadvantaged.
(D) Home-based jobs still account for a significant percentage of the overall economy.
(E) Domestic work should be paid.

39. Which of the following would provide the best title for the passage?

(A) Industrial Capitalism and the Oppression of Labor
(B) D'Emilio, Enloe, and Davis: A Reconciliation
(C) A Rejection of D'Emilio's Account of "Free Labor"
(D) Women's Sacrifices for Industrial Capitalism
(E) Industrial Capitalism and Freedom For All

Questions 40–42 are based on the following reading passage.

Henri Matisse's *The Ochre Head* represents the artist's exuberant display of his new mastery of a technique once **peculiar** to Picasso. Matisse has learned to artfully separate color and drawing. The painting depicts a head and shoulders, a bouquet of red flowers in a dark blue vase sitting upon a bench, a framed drawing of a woman's head, and an unframed painting or drawing, also of a woman. But what is most striking about the painting is the way Matisse has begun to allow his colors and his forms to play freely, even while they are coordinated. The ochre of the head runs out past the form. Bits of blue from the bench appear in the man's neckline or along his shoulder. The colors of the various frames and surfaces of the drawing on the wall overlap and refuse to be constrained by definite lines of form. Although this technique is not given the kind of free reign Picasso allows it in his Cubist period or in works such as *Minotaur*, it is still a stunning development for Matisse. And he seems aware of this fact. The painting's composition references Picasso's *Still Life with Ancient Head* from 1925, signaling Matisse's awareness that he is borrowing from his younger colleague.

40. The author uses the word *peculiar* to mean which of the following?

(A) strange
(B) abstract
(C) unknown
(D) unique
(E) appealing

41. The main idea of the passage is to

(A) describe an artistic work and its inspiration
(B) describe how Matisse surpassed Picasso
(C) describe how Matisse developed his style
(D) describe a representative example of Matisse's work
(E) describe the influence of Picasso on the art world

42. According to the passage, all of the following are true of *The Ochre Head* EXCEPT:

(A) The artist did not color between the lines.
(B) Its arrangement is similar to that of a piece by Picasso.
(C) It is considered among the best of Matisse's work.
(D) Its colors are more constrained than those of *Minotaur*.
(E) It depicts household objects.

Questions 43–47 are based on the following reading passage.

US History textbooks are far too fact-based and even have the ability to make students feel as though all history is made up of "Quick-Facts" and diagrams, not dynamic events to be critically analyzed. Furthermore, it is often the case that textbooks are given undue authority in determining the curriculum; many teachers simply "teach the book."

This is particularly disturbing when considering the fact that state committees for choosing textbooks often treat them, in the words of Sandra Wong, "more like encyclopedias of facts than as cultural products that convey values and perspectives" when deciding which ones to pick. In her article "Evaluating the Content of Textbooks: Public Interests and Professional Authority," Wong discusses how textbook committees are rarely concerned with the actual substance of the writing in the textbooks they evaluate, and are far more interested in things like "charts, illustrations, and introductory outlines."

What, then, would be a better tool to use in the high school classroom than textbooks or timelines for creating an effective learning environment that could reflect the dynamic nature of historical study? Out of all the various alternatives—going to plays, hearing speakers, listening to music, using interactive online resources, elucidating connections to students' personal lives by going to local history museums or having students write autobiographical essays, etc—the most promising is, by far, film. Movies are a magnificent way into history for even the most resistant naysayer of historical study. Film is a hugely popular medium with endless numbers of historically based works —everything from documentaries to dramas— that not only present facts, but dramatize the human relations behind those facts.

The main critique presented against the use of historical film in the classroom is, of course, the existence of rampant inaccuracies and biases laced throughout these films, not to mention the agendas of the filmmakers themselves. However, some historians believe that these seeming flaws are actually part of the reason why film is an ideal teaching tool—not only does it allow students to see history come to life, and thus interact with it dynamically, as well as make history immediately accessible to a modern audience because of the techniques used in filmmaking, but it can also foster deep critical thinking skills if instructors lead dialogues after film viewings about the inaccuracies, the biases, and all of the things that make the film not just a record of a historical event, but also a reflection of the modern moment.

43. Which of the following is NOT cited by the passage as an alternative method for historical study?

(A) listening to music
(B) attending a public lecture
(C) volunteering at an archeology dig
(D) writing a personal statement
(E) watching a film

44. The purpose of the passage is to

(A) support the film industry
(B) criticize government education policies
(C) advocate a new process for textbook adoption
(D) propose increased use of a particular didactic tool
(E) denigrate an established philosophy

45. According to the passage, a problem with state committee textbook selection is

(A) the lack of education of the committee members
(B) misplaced priorities
(C) the dominance of larger states
(D) valuing perspectives instead of facts
(E) personal prejudices

46. The purpose of the last paragraph is to

(A) acknowledge an insurmountable obstacle
(B) raise and undermine an objection
(C) reassert the need for alternative educational tools
(D) admit a flaw in a preferred alternative
(E) advocate more interactive instruction

47. The passage implies which of the following?

(A) Students can benefit from exposure to inaccurate accounts of history.
(B) Students today prefer music to film.
(C) Students today are functional illiterates.
(D) Students today prefer charts to opinions.
(E) Students today should not be exposed to political agendas.

Questions 48–49 are based on the following reading passage.

From assemblages of found objects to bizarre video installations and digital interactive experiments, much of contemporary art has been criticized as cold, unapproachable, impersonal, and emotionless. One link between the immediately appealing, expressive paintings that are often the most popular museum attractions and the "brainy" constructivist school of art pioneered in the early twentieth century is the notion of *gesture* as an expressive tool.

Mark di Suvero's *Iroquois* (1983–1999) is composed of several industrial-sized I-beams. The materials are so heavy and large the artist used cranes and other construction tools to manipulate and connect the beams, all of which have been painted a bright red-orange. The result is an intruding work of almost architectural dimensions that one can immediately sense is terribly heavy and somewhat precarious, yet stable and balanced. As one contemplates *Iroquois*, walking in and around its structure, backing away to see it from a distance, the linear forms became considerably more complex than one might presume. The tangled steel was obviously constructed with great care, yet each piece seems to threaten the viewer with its weight and size, jutting out away from the central nexus, daring the entire form to topple over. At the same time, the piece seems to exude stability, balance, even serenity. *Iroquois* resonates with an energy born not of the physical quality of the sculpture, which is quite passive and stable, but rather of the *gestural* quality of the forms.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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48. Which of the following can be inferred from the passage?

- ☐ Some of the most popular museum attractions are contemporary art installations.
- ☐ Expressive paintings have been considered "brainy."
- ☐ Seemingly cold and cerebral art can nevertheless make use of certain expressive tools.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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49. Which of the following does the author assert about *Iroquois*?

- ☐ Paradoxically, it appears to be both stable and unstable.
- ☐ It uses gesture to evoke a sense of energy.
- ☐ Some interpret it as simpler than it really is.

Questions 50–53 are based on the following reading passage.

In his *Discourse on Inequality*, Rousseau posits that early social contract theories establish unjust social and politics arrangements that provide only the appearance of legitimacy and equality.

In Rousseau's accounting, the beginnings of the social contract lie in the fears of the rich. In a state of nature, one in which there is no government or law to control the interactions of people, the rich would have great difficulty protecting the property that they possess. Thus, the rich turn to the mechanism of the social contact to shore up the holdings Rousseau views as "hoarded." The concept of a social contract is appealing to the poor, because the poor fear death in a state of lawlessness and thus seek protection. To obtain assent to the contract, the rich focus their rhetoric on a seeming equality of obligation by creating rules that apply equally to all members of society. This system, however, simply systematizes the "theft" the rich had perpetrated on the poor in the pre-law state of nature.

Rousseau then begins to develop his own vision of a social contract, through which he attempts to right these injustices. His first departure from earlier theorists is in the formation of the sovereign. Rather than members of the state surrendering their rights to another person—an irrational course of action tantamount to surrendering oneself into slavery—they surrender their right to all members of the society and thus to no one. Rousseau refers to this sovereign as the "general will" and it has the task of legislating for the new civil society that is created in the contract.

Unlike early social contract theories, Rousseau's version conceives of property rights that allow for rights of first occupancy to justify claims, rather than rights of the strongest. In this system, property can be taken only if it has not been previously occupied and only to the degree necessary for the **subsistence** of those taking it, measures intended as a check to the hoarding of property by force enshrined in earlier contract theory.

50. Which of the following societies would Rousseau be likely to endorse?

(A) A society in which there is no government or law to control how people interact with each other.
(B) A society in which a primary leader is elected through a fair democratic process.
(C) A society in which there is only communal property, rather than private property.
(D) A society in which the social contract has been dismantled and replaced with rights of first occupancy.
(E) A society in which a homeless family could legally move into an empty house they did not purchase.

51. It can be inferred from the passage that Rousseau would believe which of the following of a society of men and women living without the primary structures of civilization?

(A) Their wealth would inevitably be equally distributed across the population.
(B) Those with more wealth would be at risk of losing it to those with less.
(C) Property would not be hoarded by those who had the most power.
(D) The social contract would not be created in order to protect the material goods of the wealthy.
(E) Property would only be taken if it had not been previously occupied and was necessary for the subsistence of those taking it.

52. Select the sentence in the second paragraph that explains how a privileged group is able to secure widespread approval for the systematized "theft" it achieves through hoarding.

53. In the context in which it appears, *subsistence* most nearly means:

(A) survival
(B) enrichment
(C) protection
(D) help
(E) opposition

Questions 54–56 are based on the following reading passage.

In the twenty-first century, *terrorism* has become a common term in the day-to-day political vernacular.

However, despite its prevalence in everyday conversation, the term *terrorism* is not easily defined.

First, there is the question of target. There is a large debate over what the target of terror must be. Many writers believe that the targeting of "innocents" is a key component of terrorism. Douglas Lackey writes, "…the common soldier is not a terrorist, because the majority of his victims are soldiers, and a minority of his victims are civilians." All bombs kill things in their target radius regardless of military status; they are indiscriminate by nature. Lackey goes on to argue that what makes a bombing campaign constitute indiscriminate killing is the targeting of an area with a high ratio of non-military units to military units. Thus, discrimination is not in the weapons used, but in the targets decided upon.

A second controversy is that of circumstance; that is, whether or not terrorist acts must be performed in times of peace, or if they can include acts perpetrated during war. Some believe that even in a war, such as World War II, terrorist actions can be undertaken. However, there is a viewpoint that in war the right to certain types of attack expands, making actions that might be considered terrorist in other contexts (such as surprise attacks or raids) justified.

The final controversy is that of perpetrator size. This debate largely manifests itself in the question of whether or not state actors can be accused of terrorist action. While Ariel Merari acknowledges the possibility of state sponsored terrorism, he decides to narrowly define terrorism as a body of violence perpetrated by sub-state insurgent groups. He does so because, "if the definition of terrorism is equally applicable to nuclear war, conventional war, and guerrilla warfare, the term loses any useful meaning." Unlike some theorists, Merari does not reject state actors because of a pro-state bias, but rather concludes that the broader the term becomes, the less useful it becomes in a dialogue.

54. Lackey and Merari would be most likely to agree on what judgment regarding terrorism?

(A) That an individual soldier in a country's military would not typically commit terrorist acts in the course of duty.
(B) That an attack that targeted primarily civilians, rather than members of a military, would necessarily be a terrorist attack.
(C) That the term *terrorism* is very difficult to define.
(D) That terrorism can only be carried out during peacetime.
(E) That a member of an elite branch of a given country's military could not be said to commit a terrorist act.

55. In the context in which it appears, *sub-state* most nearly means:

(A) operating under a government's orders
(B) terrorist
(C) insurgent
(D) anarchist
(E) nongovernmental

56. Each of the following is mentioned in the passage as a reason that the term *terrorism* is difficult to define EXCEPT:

(A) the nature of the targets of the act
(B) its prevalence in everyday conversation
(C) whether the act in question occurs during wartime or peacetime
(D) whether governmental bodies can be terrorists
(E) the fact that its usefulness as a term is inversely proportional to its inclusiveness

Questions 57–58 are based on the following reading passage.

The African American writer and social critic James Baldwin grew up as an outsider both to the language and culture of power, and yet achieved a balance between self-expression and the language of power. In the 1930's, Hollywood movies portrayed an optimistic, capitalist, white America that dealt with white issues and employed white actors. As a child, Baldwin felt acutely separated from the culture of power. In "Congo Square," Baldwin opens his essay with an important idea: "A child is far too self-centered to relate to any dilemma which does not, somehow, relate to him." Watching the films of the dominant culture, Baldwin attempted to relate the information to his own life, connecting with it however he could. Rather than blindly accept the storylines in the movies, when Baldwin saw a staircase in *A Tale of Two Cities*, he thought, "I knew about staircases."

At the same time, Baldwin distrusted the culture of power: "The civilized … do not intend to change the *status quo* … these people are not to be taken seriously when they speak of the 'sanctity' of human life, or the 'conscience' of the civilized world."

Consider each of the answer choices separately and indicate all that apply.

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57. It can be inferred from the passage that Baldwin would agree with which of the following about a 1930's film that dealt only with African American issues and employed only African American actors?

- ☐ It would be significantly better than most Hollywood movies from the 1930's.
- ☐ It would suffer from the same kind of problems as the more typical 1930's Hollywood movies.
- ☐ It would be less likely than most other 1930's Hollywood movies to reinforce the predominant culture.

58. Baldwin's quotation from "Congo Square" is used primarily in order to illuminate what aspect of 1930's America?

(A) the culture of power that Baldwin mistrusted
(B) the fact that Hollywood movies were primarily optimistic and centered on white culture
(C) the idea that America was hungry for a writer such as Baldwin, because its films focused only on white America
(D) the alienating effect of films of the era on black children
(E) the concept that children are incapable of deriving any educational value from films they can't relate to

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Questions 59–62 are based on the following reading passage.

The Parthenon has long been regarded as one of the great architectural and artistic products of the High Classical Period. Yet, scholars have struggled to reach a consensus interpretation for the meaning of the Parthenon Frieze. The study of this particular sculptural element of the Parthenon, a continuous band of sculpture that ran round the top of the building's temple-chamber, has proven quite difficult.

Today only 423 feet of the original 524 survive, and of those, 247 feet are housed in the British Museum in London. Another large section is now in the Acropolis Museum in Athens, and still other pieces reside in exhibits throughout Europe, making it a difficult task to discuss, let alone experience the unified whole the designers wished the audience to witness—a key element in deciphering any work of art.

Denied the opportunity to study the frieze as it existed in antiquity, scholars are faced with the burden of reconstructing the visual experience of the monument before they can even begin interpreting it. To do so, an inventory is taken of characters and figure-types represented on the frieze and their arrangement. Then this inventory is compared to historical precedents and placed in its contemporary context in the hopes of using prior examples to decipher its meaning. Considering the various fragments of the Parthenon frieze as a whole and comparing it to other Greek artworks, two aspects of the arrangement immediately strike the **informed** viewer.

First, it is clear that the frieze is meant to be thought of as a continuous whole. This is particularly interesting because it is completely unprecedented in Greek art. Continuous friezes on the faces of Greek temples generally depicted single subjects, but if continued over all four sides of a building, the four stretches of the frieze would generally be thematically separate.

The second unique aspect of the Parthenon frieze has to do with the fundamental nature of Greek art; namely that all works of art prior to the Parthenon frieze depicted only scenes from myth and legend. Yet, in this relief, for the first time in the history of Greek art, we find mortals, leading some scholars to the conclusion that what is depicted is a specific event that actually took place at a specific time and place.

59. In the context in which it appears, the word *informed* most nearly means:

(A) assiduous
(B) artistic
(C) unique
(D) erudite
(E) scientific

60. Which of the following would be the best title for the above passage?

(A) The Parthenon Frieze: An Insoluble Sculptural Mystery
(B) The Parthenon Frieze and Methods of Artistic Interpretation
(C) The Parthenon Frieze: Sui Generis Stonework
(D) Mortals in Greek Art
(E) The Parthenon Frieze: Continuity of Character

61. All of the following can be inferred about the Parthenon from the passage EXCEPT:

(A) It was constructed in the High Classical Period.
(B) It has been seen as a great work of art.
(C) It featured a temple-chamber.
(D) It is not known how it looked at the time of its construction.
(E) It contained one-of-a-kind sculptural elements.

62. It can be inferred from the passage that which of the following would be unique in the world of Classical Greek art and architecture if it were discovered to have been created just after the Parthenon?

(A) a painting featuring the goddess Athena springing forth from the head of her father, Zeus
(B) a room featuring a frieze around the top that depicted a single, continuous narrative on all sides of the room
(C) a drama that used the famous myth of Persephone to provide a trenchant commentary on Greek life

(D) a frieze featuring a number of different characters and figure-types
(E) a large mural depicting a beautiful Greek youth crushing grapes to make wine

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Questions 63–64 are based on the following reading passage.

A single short story can suggest a desired response from the reader. It is a difficult task, though, to create a world within a single short story and then repeat this world again in other stories while maintaining a consistent flow of ideas. Many authors prefer to use the same setting, indeed, often the same characters in each story. Isabel Allende's *Diez Cuentos' de Eva Luna* comes to mind. In these stories, Allende uses the small town of Agua Santa as the setting for the entire collection.

Woman Hollering Creek is a collection of short stories by Sandra Cisneros. Rather than using the same characters or setting throughout the collection, Cisneros takes a different approach to relating her stories to one another. Much of the burden, in fact, is placed on the reader, for the characters change each time, as does the setting. Furthermore, while the characters are largely Mexican American immigrant women, each character presents a distinct style and literary voice. What these works have in common is more intuitive; mood, circumstance, time, tone, and imagery all play a role in creating the world in which the stories take place.

63. The author of the passage would be most likely to agree with which of the following statements?

(A) Short story collections depend on the reader to find the common thread that ties the stories together.
(B) Isabel Allende uses the same setting for the majority of her fiction.
(C) It is possible to create a coherent short story collection if the stories take place in vastly different times and places.
(D) Intuition is a more important aspect of Sandra Cisneros's writing than the characters or the setting.
(E) The best short story collections feature some through line that the reader can follow and that connects the various stories.

64. According to the passage, which of the following characterizes both short stories by Isabel Allende and short stories by Sandra Cisneros?

(A) similarities in tone among the stories
(B) similarities in time among the stories
(C) similarities in characters among the stories
(D) similarities in setting among the stories
(E) none of the above

Questions 65–66 are based on the following reading passage.

Scottish economist Adam Smith's *The Wealth of Nations* heralded—or caused—the market-based economic system that has increasingly become the norm since the book's publication in 1776. Some say that Smith's magnum opus was to economics as Newton's *Principia Mathematica* was to physics or as Darwin's *On the Origin of Species* was for biology. Certainly the book made its impact in the early United States.

1776 predates wide usage of the term *capitalism*, now commonly associated with Smith, and which

Smith refers to as a “system of natural liberty.” Smith presented what we today consider Economics 101: supply and demand, and the importance of specialization and the division of labor. He also posited that individuals pursuing their own self-interest could unintentionally create a more just society by so doing—an idea sometimes referred to as the “Invisible Hand.”

Even Smith’s critics do not deny the book’s immense influence. Murray Rothbard levels the criticism that *The Wealth of Nations*, in fact, eclipsed public knowledge of all economists—better ones, he says—before Smith.

Consider each of the answer choices separately and indicate all that apply.

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65. Which of the following statements is not in contradiction with the opinions of Murray Rothbard as expressed in the passage?

☐ Smith was possibly the third best economist of all time.
☐ *The Wealth of Nations* should not have been as influential as it was.
☐ Adam Smith was not particularly influential.

66. Which of the following would be an example of the “Invisible Hand” as described by Adam Smith?

(A) A group of moviegoers who are able to get cheaper tickets for a film by buying their tickets as a group.
(B) A society in which the division of labor frees certain people to pursue careers that might seem impractical in a non-capitalist society.
(C) A university in which classes are first-come, first-served, thereby equitably distributing courses according to the passion and dedication of students.
(D) A market in which there are more buyers than sellers, thus forcing the price of goods upwards.
(E) A stock exchange in which each trader acts according to a different set of information, such that certain commodities become hyped and their prices overinflated.

Questions 67–68 are based on the following reading passage.

Fame and fortune are often associated with virtue, industry, and a host of other attributes. However, random strokes of fate often determine such success and said commonly perceived virtues prove irrelevant. For instance, decades after her death, Coco Chanel remains the epitome of French fashion and her name, as well as the company she founded, is known throughout the world. Yet, few realize that she, while undeniably talented, initially succeeded by leveraging her status as a courtesan. As a company owner, it would be an understatement to say that she was not known for her generous treatment of her employees. During the Nazi occupation, she lived a privileged life at the Hotel Ritz and her lover at the time was a German intelligence operative.

Conversely, there is the example of Madeleine Vionnet, a professional contemporary of Chanel. She is often credited with permanently transforming fashion through her use of the bias cut—cutting material against the grain of the fabric. In the 1930’s, her gowns were worn by Hollywood stars, and her standing was second to none. She was also, in many ways, an early feminist, establishing largely unheard of employee benefits—such as day care and medical care—for her largely female staff. In addition, she lobbied for fashion copyright protections. When the Second World War broke out, she closed her business and simply retired. Today, she is largely unknown outside of the fashion industry.

67. The author wrote the passage to posit which of the following?

(A) that the relationship between virtue and success is inverse
(B) that an inferior designer was more successful than a superior one
(C) to contrast the aesthetic of Chanel to that of Vionnet
(D) to suggest that no causal relationship exists between personal qualities and professional legacy
(E) to contrast the difference between transitory and lasting success

68. The author discusses the employee benefits offered by Vionnet in order to

(A) emphasize Vionnet's anti-Nazi politics
(B) illustrate Vionnet's admirable qualities
(C) highlight Chanel's reputed drug use and collaboration
(D) demonstrate the economic burden of French socialist policies
(E) provide details that bolster Vionnet's feminist credentials

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Questions 69–70 are based on the following reading passage.

During the 1960's and 70's, scientists were concerned and puzzled by a large gap in the human fossil record. The "aquatic ape theory" gained prominence as an explanation for this gap. This theory posited that primitive humans were forced toward a littoral lifestyle by competition for arboreal resources. Analogies were made to seal populations, who sleep on land at night but spend most of their days in coastal waters. Proponents pointed to various physiological human attributes, such as bipedalism and the webbing between human toes, as extant adaptations.

However, the aspect of the theory that captured the public imagination and undoubtedly boosted its standing was the point that this hypothesis explained human hairlessness; as with dolphins, this streamlining would facilitate swimming and diving. Proponents noted that the remaining body hair would match the flow of water, and extreme advocates explained the gender difference in hair by suggesting that females much more rarely ventured out of the shallows and into the putatively more dangerous forests and savannahs.

Nonetheless, despite the popular stature of the theory, the scientific community almost unanimously rejects it as mere conjecture not only because of the lack of supporting evidence but also because its claims do not withstand scrutiny. While bipedalism does facilitate swimming, it is even more of an advantage in terrestrial pursuits. Further, biomechanical analysis indicates that humans remain such inadequate swimmers that they could not so succeed. As for hairlessness, critics point out that other semi-aquatic mammals actually have dense fur and/or barrel shaped torsos for heat retention. Today, the theory, while still championed by a prominent writer but non-scientist, has no serious support among mainstream-trained paleoanthropologists.

69. The passage implies that, according to the theory, a male aquatic ape would most likely do which of the following?

(A) spend almost all of its time in the water
(B) spend its nights in the water, but its days partly on land
(C) spend its days partly in the water, but its nights partly on land
(D) spend its days mostly in the water, but its nights on land
(E) spend almost all its time on land

70. The author describes a remaining proponent of the theory as a "non-scientist" in order to do which of the following?

(A) cast doubt upon her objectivity
(B) tout the superiority of common sense over academics
(C) cast doubt upon her expertise
(D) cast doubt upon mainstream paleoanthropologists
(E) illustrate the sexism of mainstream scientists

Questions 71–72 are based on the following reading passage.

The Tokugawa period Japan (1603–1867) serves as a laboratory for organizational behavior historians for the same reason that Iceland works for geneticists—isolation removes extraneous variables. The Tokugawa shoguns brought peace to a land of warring feudal lords. To preserve that tranquility, the Tokugawa shogunate forbade contact with the outside world, allowing only a few Dutch trading ships to dock at one restricted port. Domestically, in pursuit of the same goal, the social order was fixed; there were four classes—warriors [samurai], artisans, merchants, and farmers or peasants—and social mobility was prohibited. The ensuing stability and peace brought a commercial prosperity that lasted nearly two hundred years.

However, as psychologists, social historians, and Biblical prophets have all observed, in varying ways, humans inevitably fail to anticipate unintended consequences. In the Tokugawa period, the fixed social hierarchy placed the samurai on top; they and the government were essentially supported by levies on the peasantry, as the other two classes were demographically and economically inconsequential. However, prosperity brought riches to the commercial classes and their numbers burgeoned. Eventually, their economic power dwarfed that of their supposed superiors, the samurai. Simultaneously, the increasing impoverishment of the samurai adversely affected the finances of the peasantry and the government. By the early 19th century, this imbalance between social structure and economic reality eroded the stability of the society. This condition, in conjunction with increasing pressure for access from foreigners, such as Admiral Perry in 1853, led to the collapse of the shogunate in 1867. In short, the success of this imposed order led to its undoing through consequences that were beyond the ken of the founders.

71. The primary objective of the passage is to

(A) compare the Tokugawa period to modern Iceland
(B) demonstrate the folly of imposing a social order
(C) show how American naval power ended Japan's isolation
(D) illustrate how a society can model a common human failing
(E) argue that commerce is more successful than militarization

72. Which of the following would provide further support for the main reason cited for the decline of the Tokugawa period?

(A) a samurai becomes a successful merchant
(B) a successful artisan becomes a samurai
(C) a samurai must work as a bodyguard for an artisan
(D) a peasant revolt against the samurai
(E) a military invasion by American marines

Questions 73–74 are based on the following reading passage.

The War of the Spanish Succession, 1701–14, began as a quarrel over whether an Austrian Habsburg or French Bourbon would succeed the childless Charles II of Spain. The conflict eventually embroiled most of Europe, with Austria, England, Holland, and Prussia the major powers opposing France, Spain, and Bavaria. For centuries afterward, school children learned of the Duke of Marlborough's victory at Blenheim, the military brilliance of Prinz Eugen of Savoy, an independent territory east of France, as well as the opposing brightness of Louis XIV of France, known as the Sun King, who also built the famous palace at Versailles. Today, however, virtually all those names would elicit only blank stares.

Although this war and its personages have now vanished into obscurity, its effects greatly affected the course of European and world history. The Treaty of Utrecht, which ended the war in 1714, ceded the Spanish island of Gibraltar to England. The "Rock of Gibraltar" became an invincible British fortress that controlled the Mediterranean and thus was of paramount importance in both world wars. Conversely, the same treaty elevated Prussia to a kingdom, thus setting in motion a chain of events that led to a unified Germany under a Prussian Kaiser instead of one governed by the Austrians, arguably making the bloodshed that consumed the 20th century more likely.

73. The author implies that a possibly negative aspect of the Treaty of Utrecht was

(A) awarding Gibraltar to the British
(B) reducing the Duke of Marlborough to obscurity
(C) elevating Prussia to a kingdom
(D) failing to resolve the Spanish succession
(E) unifying Germany

74. Which of the following must be true, according to the passage?

(A) Not all of the important military personages were from major powers.
(B) The battle of Blenheim was the most important engagement of the war.
(C) England was the victorious power.
(D) France was defeated in the war.
(E) The transfer of Gibraltar was the most important result of the war.

Questions 75–76 are based on the following reading passage.

Michael Lewis, the American journalist and non-fiction writer, originally studied art history as an undergraduate and worked for a prominent art dealer before obtaining a masters degree in economics at the London School of Economics. After a short stint at Salomon Brothers, he wrote *Liar's Poker*, which became a tremendous success. However, his fame and fortune continued to arc upward with the publication of *Moneyball* and *The Big Short*. Some argue that his education in the humanities, in addition to finance, greatly contributed to the popularity of his work. However, most adherents of that view are ensconced in ethereal rather than commercial disciplines.

An alternative explanation exists. Although baseball and Wall Street are not commonly associated, Lewis's works concerning both those topics share more than the common theme of lucre. All the books cited above focus on mavericks "gaming the system." Lewis's protagonists, such as Billy Beane of the small market Oakland Athletics and the small, out of the establishment traders that bet against mortgage derivatives, are smart non-conformists who rebel against monolithic systems by exploiting inherent inconsistencies. Such a theme echoes the American nation-building construct—lone, free individuals winning against great odds by employing common sense and ingenuity. This is a story line that Americans hear from birth; it is no wonder that Lewis's books are popular despite subjects that are not necessarily of universal interest. Further support for this thesis is that *Moneyball* became a major motion picture and a film of *Liar's Poker* is in development. Of course, unlike baseball games, such debates can never be definitively scored.

75. The author primarily does which of the following?

(A) contrasts two opposing theories for a process
(B) discusses two suppositions and implicitly favors the latter
(C) details two programs and definitively prefers the latter
(D) describes a writer's work and its dearth of popularity
(E) outlines the biography of a writer and critiques his work

76. The author does all of the following EXCEPT:

(A) employ an idiomatic expression
(B) describe the protagonists in question
(C) reference popular culture as evidence
(D) discuss the parallels shared by two professions
(E) mention his subject's formative influences

Questions 77–78 are based on the following reading passage.

The term *free rider* originates from the idea of someone who rides public transportation without paying the fare. The *free rider problem* is what results when too many people do this: the transit system will go bankrupt. More broadly, the *free rider* is someone who uses or enjoys the benefits of something without paying, or takes more than his or her share of a public good. *Free riders* can cause others to curtail their own contributions, not wanting to be taken advantage of.

In some cases, the *free rider problem* is viewed as a necessary cost of government. When citizens pay for national defense or environmental protection, everyone benefits, even those who evade paying taxes.

Consider each of the answer choices separately and indicate all that apply.

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77. Which of the following examples matches the definition of a *free rider problem* as described in the passage?

☐ A skateboarder grabs onto the back of a car and is able to travel effortlessly for over a mile before letting go.
☐ A senior citizen pays less for a movie ticket than do the other people in the theater.
☐ A yearly school bake sale based on the honor system is suspended when too many people take food without paying.

78. The author of the passage would be most likely to agree with which of the following statements?

(A) *Free riders* cannot be blamed for their actions, because they are an inevitable part of any government.
(B) *Free rider problems* are not worth worrying about, because they are an inevitable part of any government.
(C) There are at least some situations in which the *free rider problem* should not be viewed as an inevitable part of government.
(D) National defense is a perfect example of why *free rider problems* need to be stamped out as quickly as possible.
(E) *Free riders* are morally at fault, and ought to be punished.

Questions 79–80 are based on the following reading passage.

In 2010, a team of biologists led by Svante Paabo announced evidence that modern humans interbred with Neanderthals some 60,000–100,000 years ago. These researchers compared the full sequence of Neanderthal DNA to that of five modern humans from China, France, sub-Saharan Africa, and Papua New Guinea, and looked for DNA shared by both Neanderthals and non-African modern humans, but not by sub-Saharan Africans. Because Neanderthals and modern humans are known to have diverged hundreds of thousands of years before modern humans left Africa, Paabo attributed any such common DNA to interbreeding in Eurasia. Paabo's team announced that the modern humans from China, France, and Papua New Guinea all have the same proportion of Neanderthal DNA, and inferred that interbreeding with Neanderthals must have taken place before the ancestor population of those Eurasians divided. Paabo maintained that these two events, the migration of modern humans out of Africa and the division of the Eurasian population, mark the interval during which the interbreeding must have taken place, and that for roughly forty thousand years of that window, Neanderthals and modern humans lived near one another in the Middle East.

The team's conclusions were answered with skepticism on a number of fronts. Critics pointed out that an earlier report reached similar conclusions based on Neanderthal samples later found to be contaminated with DNA from modern humans. Paleontologists and archaeologists charged that the conclusion was unsupported by archaeological evidence. Further, Paabo's team found evidence only of Neanderthal DNA in modern humans, not of modern human DNA in Neanderthals, but critics claim that interbreeding would result in gene flow in both directions.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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79. The passage implies that which of the following claims is true?

☐ Modern humans and Neanderthals share a common ancestor.
☐ Modern humans and Neanderthals interbred.
☐ Modern humans and Neanderthals lived near one another approximately 80,000 years ago.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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80. The passage suggests which of the following is true of Paabo's critics?

☐ they doubt Paabo's integrity
☐ they ignore DNA evidence
☐ they sometimes appeal to archaeological evidence

Questions 81–82 are based on the following reading passage.

Though an echo is a fairly simple acoustic phenomenon—a reflection of sound waves off some hard surface—it occurs only under very specific circumstances. Imagine a listener standing at the sound source. The reflecting object must be more than 11.3 meters away from the sound source, or the echo will return too soon to be distinguishable from the original sound. A reflecting object more than about 170 meters, on the other hand, will rarely produce an audible echo, since sound dissipates with distance. Further, multiple surfaces each reflecting the same original sound to the same listener will likely not produce an echo, but a reverberation, a persistent sound gradually decreasing in amplitude until the listener can no longer hear it. Common though echoes are then, it is unsurprising that some sounds seem to produce no echo.

A centuries-old tradition holds that a duck's quack does not echo. Scientists in the Acoustics Department of the University of Salford set out to test and explain this claim. They recorded a duck, Daisy, first in an anechoic chamber filled with sound-absorbing fiberglass wedges, then in an echo chamber with the acoustical properties of a small cathedral. The sound of the duck quacking in the anechoic chamber was clearly different from the sound of the duck quacking in the echo chamber, but the researchers acknowledged that it would be very hard to recognize an echo in the latter recording without having very recently heard the former. Partly this is because a quack isn't a single burst of sound, but fades in and out, so that the beginning of the echo might blend with the end of the original sound. Partly it is because a quack is just not very loud. The Salford researchers also speculate that most people may simply not encounter ducks in proximity to reflectors such as buildings or mountains. A further complication, though one the researchers leave unremarked, is that people generally hear ducks in flocks, where one quack might be indistinguishable from the echo of another.

81. According to the passage, all of the following make an audible echo unlikely EXCEPT:

(A) a reflecting surface too close to the original sound
(B) a reflecting surface too far from the original sound
(C) multiple reflecting surfaces
(D) multiple listeners
(E) sound-absorbing materials

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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82. The passage suggests that which of the following would propagate echoes?

☐ an anechoic chamber
☐ a cathedral

☐ a mountain

Questions 83–85 are based on the following reading passage.

Simone de Beauvoir's feminism was heavily informed by existentialist ethics. Within this frame of thought, good and evil are expressed in human beings' transcendence and "immanence," respectively. Human existence can only be justified via continually expanding into the future by engaging in freely chosen projects—i.e., transcendence. Transcendence is thus a general goal for human beings, while its opposite—immanence—is considered a degradation of existence, from "liberty into constraint." Freely chosen, immanence is a moral fault, but when inflicted, it is described as "oppression." In a social environment where women are prevented from choosing and engaging in serious projects because of their status as women, their ability to transcend is systematically thwarted, so transcendence becomes a specifically feminist goal. De Beauvoir explains that women are viewed as intrinsically passive and immanent, in opposition to men who are meant to be active and transcendent.

In the society observed by de Beauvoir, "[women] live … attached through residence, housework, economic condition, and social standing to certain men—fathers or husbands…" It is specifically women who are attached to men—not men to women nor women and men to each other. As de Beauvoir notes, the nature of this relationship, overlapping the assumption that men are the active transcendent half of humanity, leaves women in a position of forced immanence.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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83. According to the passage, Simone de Beauvoir believed which of the following?

☐ In society, women are attached to men.
☐ Women are intrinsically passive and immanent.
☐ Self-constraint is worse than imposed constraint.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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84. According to the passage, *immanence* is always

☐ a moral fault
☐ a degradation of existence
☐ oppression

85. The passage could best be described as which of the following?

(A) an explanation of existentialist ethics and transcendence
(B) an explanation of feminist theory and female immanence
(C) a diatribe about immanence and a social injustice
(D) a description of a philosopher's influences and framework
(E) an outline of social structure and conflicts

Questions 86–87 are based on the following reading passage.

What differentiates science and non-science? According to the modern definition of science, the Ancient Greeks were not scientists but rather philosophers. Their investigations were performed in an unscientific manner, as is illustrated by Aristotle and his conclusions about the properties of water. Before studying water, Aristotle discovered that matter existed in three main categories: solid, liquid, and gas. He concluded that a solid was the least expanded of the three and verified this by seeing that a solid always sank in a liquid of the same type. However, when Aristotle encountered water, he saw that it had properties that contradicted his previous categorization. In order to reconcile this disparity, he postulated that water was an exception and that the shape of solid water caused it to stay afloat. This, of course, is incorrect.

To the contrary, the studies of Galileo Galilei followed a certain self-made doctrine for gathering data and performing scientific experiments. Galileo's method forced one to first form a hypothesis, then design an experiment to confirm or deny this hypothesis, and then accept or discard the hypothesis based on one's findings. Using this method, Galileo disproved many commonly held misconceptions about the rules of physics. In one of his more famous experiments, Galileo hypothesized that the Earth's gravitational field resulted in the same acceleration of all objects, regardless of mass. To prove this, he dropped two iron balls of different masses from an elevated place and showed that gravity pulled on both masses evenly. This experiment disproved the commonly held belief (at the time) that an object with greater mass would fall to the ground more quickly.

86. The author's primary purpose in writing this passage is to

(A) explain the modern distinction between science and non-science via historical examples
(B) criticize the research methods employed by the Ancient Greek philosophers
(C) illustrate the importance of following the scientific method as invented by Galileo Galilei
(D) compare and contrast the historical definition of science with the modern-day definition
(E) argue that the findings of Galileo are more important than those of Aristotle

87. Which of the following can be properly inferred from the passage?

(A) Solid water has the same shape as other solid forms of matter.
(B) When dropped from the same height, an object with greater mass will fall to the ground more quickly than an object with less mass.
(C) One cannot be both a scientist and a philosopher.
(D) If Aristotle had followed Galileo's method, he would have rejected his hypothesis that solids were the least expanded form of matter.
(E) In the absence of the scientific method, one cannot disprove commonly held misconceptions about the rules of physics.

Questions 88–90 are based on the following reading passage.

Lousia May Alcott's *Little Women* opens to a common scenario—the women knitting at home and waiting for news from the man of the family, who is at the war front. The family dynamics of *Little Women*, as a microcosm of the larger society, are marked by explicitly articulated male dominance. First of all, the division of labor has it so that women are confined to the domestic sphere while men step into the public sphere and engage in activities there, returning to the domestic sphere at night to be cared for by their spouse or female children. Alcott describes the character of Meg, a young wife, as “often … lonely,” with her husband “absent till night, and nothing to do but sew, or read, or potter about.”

Marmee later tells Meg that she ought to “take [her] part in the world’s work,” even though she is a woman. Ultimately, however, “taking her part in the world’s work” meant no more than talking to her husband about politics whilst remaining at home, allowing him to continue to be the mediator between Meg’s individuality and the world at large. Chapter 38 of the novel wraps up the issue by concluding that “a woman’s happiest kingdom is home, her highest honor the art of ruling it [as a] wise wife and mother,” such position being “the sort of shelf on which young wives and mothers may consent to be laid, safe from the restless fret and fever of the world.” Some have read Alcott’s romantic glorification of women’s confinement as sarcastic, but either way, her loving readers must have agreed with the statement, for the novel has never been out of print.

88. The primary purpose of the passage is to

(A) explain the continued popularity of a novel
(B) detail the domestic confinement of 19th century women
(C) analyze the sociological implications of a work of art
(D) argue for the emancipation of women
(E) indict the politics of a literary work

89. The passage implies that for a woman to effectively “take [her] part in the world’s work,” she must do which of the following?

(A) talk to her husband about politics
(B) rule her home
(C) sew, read, and potter about
(D) find a mediator to interact with the world at large
(E) leave the house

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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90. The passage implies which of the following about American society at the time that *Little Women* depicts?

☐ it was wartime
☐ it glorified women’s confinement
☐ the vast majority of public activities took place during the day

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Questions 91–92 are based on the following reading passage.

Falsifiability is the term coined by Karl Popper for the idea that a hypothesis or theory addresses the observable world only insofar as it can be found false as the result of some observation or physical experiment. For instance, the proposition “All cats have fur” can easily be proven false with the observation of a single hairless cat. The proposition “The world will end in the year 3035” is impractical to falsify, but still passes the test of falsifiability in that there exists the logical possibility that 3035 will come and go without the world ending. To the contrary, it is possible to posit that everything that happens is the will of Zeus. No matter what experiment we design—such as praying to Zeus to give us the answer or daring Zeus to strike us with lightning—we can always infer that the result is the will of Zeus. Such a proposition, as conceived here, is not falsifiable. Popper claimed that a falsifiable theory is the only kind that can truly be scientific, or at least useful to the scientific community.

By that logic, we can also say that no theory should be formed that has no chance of being true. However, seeing as that kind of theory is much less likely to be formed, it is understandable that Popper does not devote that much time to the criterion of "confirmability."

91. According to the passage, which of the following does not meet the criteria for falsifiability?

(A) All birds are black.
(B) Earth is the only planet in the universe with intelligent life.
(C) It rains on Mars every day.
(D) The sun will explode in 100,000 years.
(E) No human being lives forever.

92. To which of the following is the author most likely to agree regarding "confirmability"?

(A) It is a more important theory than *falsifiability*.
(B) It does not have much practical, scientific use.
(C) It applies to a broad range of theories.
(D) It is an unreasonable idea.
(E) Popper should have developed this idea along with *falsifiability*.

Questions 93–97 are based on the following reading passage.

A dictionary definition of the term *political* might read something like, "of or concerned with government, political parties, or politicians." Such a definition is not precisely wrong, but rather is outdated and falls short by not accounting for what Nancy Fraser calls "the shift from a repressive model of domination to a hegemonic one." If at some point we believed governments to operate exclusively through law and the threat and enforcement of concrete punishment, such as imprisonment, monetary penalties, etc., and called this and all that directly influenced it "politics," we have now acknowledged the role of hegemony, which legitimizes law and supports the exercise of power.

This is significant because, under the first definition, the only cultural products that can be said to be political must explicitly address issues of political partisanship or governance, while under the second definition, all cultural objects can be traced to a certain ideology—in accordance, negotiation, or opposition to hegemony—and therefore be political.

But we do not feel that we are discussing politics or viewing politics all the time, even if we are, according to our definition of "the political." This is because even if all subject matter is (at least potentially) political, not all talk is so. When conducting her study on political talk, Nina Eliasoph focused not as much on what people talked about, but rather on how exactly they talked about things: "whether speakers ever assume that what they say matters for someone other than themselves, ever assume that they are speaking in front of a wider backdrop." She cited Hanna Pitkin in concluding that "public-spirited conversation happens when citizens speak in terms of 'justice'." To use an example from the theater, then, we can say that when a director decides to frame her production of *A Streetcar Named Desire* as the story of a woman who is losing her mind and does not get along with her aggressive brother-in-law, she is actively depoliticizing the story, whereas she is actively politicizing it if she decides to frame the narrative as one example of the devastating effects of an old bourgeois morality, a changing economic system and the social valuing of an abusive model of masculinity.

93. The second paragraph of the passage serves to

(A) offer an alternative to the definitions previously presented
(B) discuss a revision of the definitions previously presented
(C) delineate the distinction between the definitions previously presented
(D) delineate an exception to the definitions previously presented
(E) describe the inadequacy of the definitions previously presented

94. The author cites *A Streetcar Named Desire* in order to

(A) provide a counterpoint to the thesis of the passage
(B) illustrate an aspect of the subject under discussion
(C) advocate politicizing a work of art
(D) illustrate the universality of politics
(E) illustrate a fallacy of a definition

95. All of the following provide examples of the hegemonic model of domination EXCEPT:

(A) a penitentiary sentence for assault
(B) a summary execution for treason
(C) a large fine for insider trading
(D) censorship of an art exhibit
(E) a successful military coup

96. According to a theory presented in the passage, a person is engaging in public interest conversation if that person discusses which of the following?

(A) justice
(B) theater
(C) sexism
(D) economics
(E) politicians

97. Select a sentence from the first or second paragraph that levels an explicit criticism.

Questions 98–99 are based on the following reading passage.

Explanationism is the idea that prediction is, in itself, insufficient to confirm a theory. To adequately confirm a theory, according to an explanationist, is to see how well it describes events and phenomena that have already been observed. Stephen Brush, a staunch explanationist, would say that a correct prediction does not necessarily confirm the truth of a theory; it could be the case that a theory predicts something and yet does not provide the best explanation of it. Take, for example, the difference in the perspectives of Copernicus and Brahe on the solar system. Copernicus's model of the solar system was heliocentric, positing that all of the planets revolve around the sun. Brahe's theory stated that all of the planets revolved around the sun, *except* the earth, which was immobile, and that the sun actually revolved around the earth. Even if both accurately predicted future movements of the planets, it is easy to see how Copernicus's theory has less of an 'ad hoc' quality—and, of course, provides a superior explanation of the mechanisms of the solar system. It is certainly true that a theory can successfully predict a certain event, yet fail to provide an adequate explanation for why it happened, or perhaps even stumble on the prediction more by accident than by manner of understanding the mechanism behind the event.

A predictionist would argue that while a theory can provide a perfect explanation for something happening, a theory cannot be tested for understanding or explaining the underlying mechanism of a phenomenon unless it can also predict some event that confirms that exact mechanism at work. For instance, a physicist might study the formation of solids and posit that all solids will sink if they are placed in a liquid of the same element, because the solid is denser than the liquid. Given this premise, we discover that the physicist's prediction is true, and even once he stages an experiment with ice and water, he will not be proven wrong, but rather will have discovered a unique property of solid water.

98. Which of the following best expresses the main idea of the passage?

(A) Explanationism is a superior theory to predictionism.
(B) Two very different ideas can both be used to successfully investigate scientific theories.
(C) Copernicus's model of the solar system was more accurate than Brahe's due to explanationism.
(D) One cannot posit a physical theory without predictive power or previous observations.
(E) A predictionist and an explanationist will always diverge on whether a scientific theory is correct.

99. Which of the following most accurately states the author's reason for citing the Copernicus and Brahe models of the solar system?

(A) It shows that a theory without predictive power can never be tested and verified.
(B) It reveals that some theories can have more or less of an 'ad hoc' quality.
(C) It shows that two different theories can never yield the same predictions for future events.
(D) It is used to support the idea that a more complicated model will always fail to a simpler model.
(E) It provides an example of when a theory can correctly predict future events but not offer the best explanation.

Questions 100–105 are based on the following reading passage.

Subatomic particles can be divided into two classes: fermions and bosons, terms coined by physicist Paul Dirac in honor of his peers Enrico Fermi and Satyendra Bose. Fermions, which include electrons, protons, and neutrons, obey the Pauli exclusion principle, according to which no two particles can inhabit the same fundamental state. For example, electrons cannot circle the nuclei of atoms in precisely the same orbits, loosely speaking, and thus must occupy more and more distant locations, like a crowd filling seats in a stadium. The constituents of ordinary matter are fermions; indeed, the fact that fermions are in some sense mutually exclusive is the most salient reason why two things composed of ordinary matter cannot be in the same place at the same time.

Conversely, bosons, which include photons (particles of light) and the hitherto elusive Higgs boson, do not obey the Pauli principle and in fact tend to bunch together in exactly the same fundamental state, as in lasers, in which each photon proceeds in perfect lockstep with all the others. Interestingly, the seemingly stark division between fermionic and bosonic behavior can be bridged. All particles possess "spin," a characteristic vaguely analogous to that of a spinning ball; boson spins are measured in integers, such as 0 and 1, while fermion spins are always half-integral, such as ½ and 1½. As a result, whenever an even number of fermions group together, that group of fermions, with its whole-number total spin, effectively becomes a giant boson. Within certain metals chilled to near absolute zero, for instance, so-called Cooper pairs of electrons form; these pairs flow in precise harmony and with zero resistance through the metal, which is thus said to have achieved a superconductive condition. Similarly, helium-4 atoms (composed of 2 electrons, 2 protons, and 2 neutrons) can collectively display boson-like activity when cooled to a superfluid state. A swirl in a cup of superfluid helium will, amazingly, never dissipate.

The observation that even-numbered groups of fermions can behave like bosons raises the corollary question of whether groups of bosons can ever exhibit fermionic characteristics. Some scientists argue for the existence of skyrmions (after the theorist Tony Skyrme who first described the behavior of these hypothetical fermion-like groups of bosons) in superconductors and other condensed-matter environments, where twists in the structure of the medium might permit skyrmions to form.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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100. The example of "a crowd filling seats in a stadium" (line 6) is intended to

- ☐ expand upon one consequence of the Pauli exclusion principle
- ☐ illustrate a behavior of certain fermions
- ☐ describe how electrons circle the nuclei of atoms in concentric, evenly-spaced orbits

101. The author's primary purpose in writing this passage is to

(A) explain the mechanism by which fermions can become bosons
(B) describe the two classes of subatomic particles
(C) provide examples of the different forms of matter
(D) explain the concept of particle "spin"
(E) argue that most matter is composed of one type of particle

102. Which of the following is NOT mentioned as a characteristic of bosons?

(A) They can be composed of groups of fermions.
(B) They are measured in integer spin.
(C) They are the constituents of ordinary matter.
(D) They tend to bunch together in the same fundamental state.
(E) They lead to phenomena such as superconductors and superfluids.

103. Which of the following can be properly inferred from the passage?

(A) An atom composed of two protons and a neutron would be considered a boson.
(B) Skyrmions have been discovered in superconductors and other condensed matter environments.
(C) Two electrons in an atom cannot circle the same nucleus at exactly the same distance.
(D) A current through a superconducting wire will never dissipate.
(E) Fermions cannot behave as bosons unless they are cooled to a temperature near absolute zero.

104. According to the passage, which of the following describes a difference between fermions and bosons?

(A) Fermions cannot inhabit the same fundamental state, whereas bosons bunch together in the same state.

(B) Fermions contain many more types of particles than bosons.
(C) Fermions exist in groups, but bosons do not.
(D) Fermions have integral spin values, whereas Bosons have half-integer spin.
(E) Fermions do not obey the Pauli principle, whereas bosons do.

105. Based on the information in the passage about the Pauli exclusion principle, to which one of the following situations would this principle be most relevant?

(A) Fermi Energy: The maximum energy that electrons in a solid will contain in order to avoid having identical energy levels.
(B) Particle Accelerators: Devices that will accelerate charged particles to very high speeds through the application of an external magnetic field.
(C) Quantum Entanglement: When particles interact physically and then become separated but still have interdependent properties.
(D) Double Slit Experiment: An experiment that revealed the particle and wave duality of photons.
(E) The Higgs Field: The field produced by the conjectured Higg's particle that would explain why matter has mass.

Questions 106–108 are based on the following reading passage.

Homo economicus, or economic human, denotes the idea of human beings as rational, narrowly self-interested agents who, given total information about opportunities and possible constraints, seek to obtain the highest possible well-being for themselves at the least possible cost. In the late 19th century, a host of economists built mathematical models based on conceiving of real humans as *Homo economicus.*

Exponents of *Homo economicus* tend to acknowledge that total information is not possible in the real world; thus, breakdown in models based on the concept are due to imperfect information held by the self-interested economic actors. Amartya Sen has pointed out that *Homo economicus* ignores that people can and do commit to courses of action out of morality, cultural expectations, and so forth. Veblen and Keynes allege that *Homo economicus* assumes far too great an understanding of macroeconomics on the part of humans. Tversky put forth that investors are not rational: they are unconcerned by small chances of large losses, but quite risk-averse regarding small losses. Bruno Frey points out that humans are often intrinsically motivated, and that such motivation explains heroism, craftsmanship, and other drives that do not fit neatly into the model of a narrowly focused gain-seeker. Critics of the psychoanalytic tradition point out, somewhat obviously, that humans are frequently conflicted, lazy, and inconsistent.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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106. Which of the following phenomena would exemplify Bruno Frey's critique of *Homo economicus*?

☐ A woodworker spends months on the delicate inlay of a door, knowing that his many hours of hard work will inevitably result in a higher price when he comes to sell the piece.
☐ A television journalist often travels to dangerous countries all over the world because he is contractually obligated to do so, and his ratings are higher the more dangerous his exploits appear to

be.

☐ An economist dedicates her career to illustrating a fundamental flaw in a particular theory, though she knows there will be no tangible reward for her efforts.

107. Which of the following best describes the main idea of the passage?

(A) *Homo economicus* is a useful, if theoretical, actor to use in the formation of mathematical models.
(B) *Homo economicus* is a fundamentally flawed and thus theoretically useless construction, for a host of reasons.
(C) *Homo economicus* is often criticized by those who don't fully understand its function in economic theory.
(D) *Homo economicus* is a problematic construction, because it simplifies human motivations and is overly optimistic about human understanding.
(E) *Homo economicus* fell out of favor with most economists in the 20th century due to its many incorrect assumptions about humanity.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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108. Which of the following is a complaint leveled against the theory of *Homo economicus* posited by certain economists in the late 19th century?

☐ It assumes that the average person knows a lot more about the general workings of the economy than he or she actually does.
☐ It assumes that humans experience a proportional and linear emotional response to all risks and rewards.
☐ It assumes that the primary impetus behind human decision-making is not predicated on ethics or cultural mores.

Questions 109–110 are based on the following reading passage.

While critics contend that the views expounded on in *Against Method* are tantamount to scientific anarchism, its author Paul Feyerabend maintains that his views stem not from a desire to promote scientific chaos so much as from a recognition that many of the fundamental tenets of science—rationality, empiricism, and objectivity, for example—are as seriously flawed as the "subjective" paths to truth that scientists are quick to **repudiate**. Feyerabend goes further by arguing that many methods that are now condemned in the scientific community played a critical role in historical moments of scientific progress. The fact that these methods helped science advance in the past indicates that scientists should think twice before they condemn them.

Much of *Against Method* is a case study of the events surrounding Galileo's single-handed rejection of the geocentric cosmological model in favor of the updated heliocentric model. Feyerabend goes to lengths to point out that what ultimately allowed Galileo to succeed in convincing the Western world that the earth revolved around the sun (and not the other way around) was the use of methods most modern scientists would deem highly suspect. For example, in attempting to explain why the rotation of the earth did not cause a rock dropped from a tower to follow a curved, rather than a straight, path, Galileo relied on several as-yet unproven hypotheses about the laws of motion, essentially begging the question for his own position. Additionally, his published works display a rhetorical style that reads more like propaganda than like scholarly work. By showing that these methods were critical to a crucial scientific advancement, Feyerabend casts doubt on whether these "unscientific" practices really deserve the criticism they so often garner.

109. Replacement of the word *repudiate* in line 5 of the passage with which of the following words would results in the LEAST change in meaning in the passage?

(A) overrule
(B) embrace
(C) underscore
(D) decry
(E) debate

110. The passage implies that Feyerabend makes use of a "case study" (line 10) primarily in order to:

(A) demonstrate that since a canonical example of scientific progress itself made use of practices now deemed unscientific, scientists ought to revise their account of what is and is not acceptable scientific practice
(B) show that Galileo, in his attempt to prove that a rock dropped from a tower followed a straight, not a curved path, was guilty of many of the same errors in reasoning that make science controversial today
(C) underscore the notion that if science wants to keep thinking of itself as a field that is open to "subjective," as well as "objective," paths to truth, it needs to adopt some of the techniques that were prevalent in Galileo's time
(D) back up the claim that tautological reasoning is acceptable only when used in the service of supporting hypotheses that have yet to be proven
(E) demonstrate that any endeavor in the philosophy of science that uses examples from history to support its claims is ultimately doomed to failure

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Questions 111–115 are based on the following reading passage.

In traditional theater forms, the roles of performer and audience are completely separate, so that performance space can be said to encompass an actors' sphere and a spectators' sphere. Even when performers move out into the audience or when there is scripted audience interaction, spectators do not become performers. Finally, while stories may open up the imagination or excite audiences, according to Augusto Boal, they discourage political action by providing catharsis. The passive spectator follows the play's emotional arc and, once the action concludes, finds the issue closed. Boal reminds us that our theater etiquette creates a kind of culture of apathy where individuals do not act communally, despite shared space, and remain distanced from art.

Workshop theater, such as Boal's Image Theatre and Forum Theatre, is a response to that. In the workshop form, performance space is created for a select group of people, but the performers' sphere and the audience's sphere are collapsed: everyone is at once theater maker and witness. In Image Theatre, participants will come up with a theme or issue and arrange themselves into a tableau that depicts what that issue looks like in society today, versus what the ideal situation would be. They then try to transition from the current image to the ideal image in a way that seems plausible to all the participants. Forum Theatre, on the other hand, creates a narrative skit depicting a certain problem. After the actors have gone through the action of the play once, a facilitator, known as the joker (like the one in a pack of cards), encourages those who have watched the story to watch it again and to stop it at any time to take the place of the protagonist. The aim is to find a solution to the problem, realizing along the way all of the obstacles involved. In Forum Theatre, just as in Image Theatre, there is not always a solution. The main goal of this form, then, is to engage in the action, to reflect, and to understand particular issues as being part of a larger picture, thus using art to re-cast what seem like private troubles in a public, political light.

The main reason Boal developed these workshop styles was to grant audiences **agency** so that they may create ways to free themselves of oppression. Because he found theater audiences to be locked into a passive role—just like he found the oppressed coerced into a subservient role in relation to their oppressors—he created the "spect-actor," or someone who simultaneously witnesses and creates theater.

111. The second paragraph of the passage serves to

(A) elaborate on the topic of the first paragraph
(B) provide a rationale for an artistic endeavor
(C) discuss an artistic answer to a passive culture
(D) explain the theater's lack of appeal
(E) evaluate two contrasting styles of theater

112. The author uses the word *agency* to mean

(A) profit
(B) organization
(C) publicity
(D) power
(E) hegemony

113. Which of the following would Boal consider a "spect-actor"?

(A) a person who engages in political action
(B) an audience member who finds catharsis in a play
(C) any person placed in a subservient role
(D) any actor
(E) a participant in an Image workshop

114. According to Boal, all of the following are disadvantage of traditional theater forms EXCEPT:

(A) Such productions prevent the actors from going into the audience.
(B) Such productions provide catharsis.
(C) Such productions discourage communal activity.
(D) Such productions obstruct political change.
(E) Such productions distance the audience from the art.

115. All of the following would be characteristic of a Forum workshop EXCEPT:

(A) Productions begin with a narrative script.
(B) Different people often play the protagonist.
(C) Some performances do not achieve catharsis.
(D) Participants arrange themselves into a tableau.
(E) Performances are guided by a mediator.

Questions 116–118 are based on the following reading passage.

Stars create energy through the process of fusion. When a star explodes—a phenomenon called a supernova—so much energy is released that heavy metals such as iron and gold are formed, seeding surrounding hydrogen clouds. Newer stars therefore contain more heavy elements in their atmospheres. Heavy elements form the materials that make up our planet (and even human bodies). It is believed that for a system of planets such as our solar system to form around a star during cloud contraction, the presence of these heavy elements in the cloud is a necessity.

A molecular cloud can become unstable and collapse by the force of gravity, overcoming outward thermal pressure of the constituent gases. At a given temperature and density, two critical measures of size, Jeans mass and Jeans length, can be calculated. If the size of the cloud exceeds either of these critical values, gravity will ultimately win, and the probability of eventual cloud contraction is high.

However, some outside influence is still evidently required for a theoretically unstable cloud to initiate collapse. The natural rotation of a galaxy can slowly alter the structure of a cloud, for instance. Surrounding supernovae can generate shockwaves powerful enough to affect the debris in other clouds, forcing the debris inward and possibly causing contraction to begin. One theory states that density waves propagating through spiral structures can also sufficiently stimulate clouds to cause contraction.

Consider each of the answer choices separately and indicate all that apply.

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116. The author of the passage suggests that cloud contraction may begin in which of the following ways?

☐ through the process of fusion
☐ explosions of stars within a close enough proximity generate shockwaves that prompt contraction in nearby clouds
☐ debris within a cloud is forced outwards

117. Which of the following inferences about our solar system is best supported by the passage?

(A) Life in the solar system depends on energy from the sun.
(B) When the system reaches a particular size, it can become unstable and begin to collapse.
(C) The natural rotation of the galaxy can alter the galaxy’s structure.
(D) It is believed to have been formed from materials “seeded” into hydrogen clouds.
(E) It produced so much energy when it was formed that heavy elements were generated.

118. It can be inferred from the passage that which of the following inhibits interstellar cloud collapse?

(A) supernova explosions
(B) galactic rotation
(C) thermal pressure
(D) gravitational force
(E) density waves

Question 119 is based on the following reading passage.

The Norton-Polk-Mathis House in San Antonio displays a well-integrated design well suited for the primary purpose of the building: to impress. This is evidenced by the fact that the building was designed with the street it faces in mind. Only the South façade is architecturally interesting or involved—the sides of the building are flat, featureless, and uninteresting. The house was designed not only as a living area, but also as a structure to be seen from the street and awed. This intent is expressed through the use of typical Renaissance ideals of order and weight, and, while it is asymmetrical, the building is well balanced and stable. The choice of materials also reflects the "re-discovery" of antiquity prevalent in the Italian Renaissance. The white stone lends an elegant simplicity to the building yet it radiates an air of strength and mass reminiscent of the Parthenon or the Athenian temples—especially when juxtaposed with the other, seemingly fragile brick and wood homes of the neighborhood.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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119. Which of the following can be inferred about the Norton-Polk-Mathis House?

- ☐ It was built during the Italian Renaissance.
- ☐ Its primary purpose was utilitarian in nature.
- ☐ It appears stronger than other nearby homes.

Questions 120–121 are based on the following reading passage.

After 22 years of observations in Shark Bay, Australia, behavioral biologist Janet Mann and her colleagues have discovered that certain bottlenose dolphins, known as spongers, form social networks, showing the first hints of culture among non-human animals. Spongers are dolphins that wear marine basket sponges on their beaks as hunting tools, using them to root around on deep sandy bottoms and find fish concealed below the sand. Sponging is a complex hunting technique passed on from mother to offspring. A sponger must know where the sponges grow, how to pick the right sponge, how to remove the sponge intact from the ocean floor, and how and where to properly hunt.

Spongers typically live solitary lives, but over 22 years of observation, a pattern emerged. The 28 female spongers formed cliques with other female spongers that were not necessarily genetically related to them. This behavior differs from other animal behavior where circumstances, such as genetics or food sources, dictate the formation of groups. The fact that these spongers chose to associate based upon similar, socially learned behaviors makes their cliques a cultural first among animals.

120. Which of the following expresses the main idea of the passage?

(A) Sponging is a complex behavior used by some dolphins as a hunting technique.
(B) Any study of animal behavior must take place over an extended period of time in order for patterns to emerge.
(C) A small set of non-human animals has been found to form social networks.
(D) Studying how animals form groups is important to our understanding of nature.
(E) Only humans can form social networks.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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121. Which of the following can be properly inferred from the passage?

☐ Groups formed by genetic bonds or food supplies do not qualify as social networks.
☐ All spongers of Shark Bay, Australia form social networks.
☐ Spongers can only be found in Shark Bay, Australia.

Questions 122–123 are based on the following reading passage.

For years, the idea that blind people can hear better than sighted people was considered something of an old canard. However, functional brain imaging now has allowed us to look inside the brains of blind people who possess what can only be termed cerebral superpowers—the ability to understand speech at up to 25 syllables per second, a speed that sounds like "noise" to sighted people (a typical sighted person understands closer to 10 syllables per second). As it turns out, a brain region called V1, situated at the back of the skull and which normally only responds to light has actually been rewired in the brains of blind people—and now processes auditory information. This is truly a stunning example of the brain's plasticity, a topic of cardinal importance in designing educational experiences and materials to best engage the brains of students.

Of course, in discussing the brain's amazing plasticity, modern thinkers take for granted something that would have been shocking to thinkers from Aristotle (who posited a holistic, non-corporeal mind in *De Anima* in the 4th century, BC) through Descartes (who argued, in the 17th century, for mind-body dualism)—the idea that the mind is physically located in the brain and that our intellect, personality, and selfhood are attributable to physical processes in the brain and can be altered by brain injuries.

122. According to the passage, the belief that blind people can hear better than sighted people

(A) is untrue
(B) was not a matter of contention, but was then shown to be true
(C) was, for years, thought to be true, but is now up for debate
(D) is put forth by the scientific community, but this evidence is contested by many
(E) was, for years, commonly known but thought by many to be false, and then was shown to be true

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Consider each of the answer choices separately and indicate <u>all</u> that apply.

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123. According to the passage, Aristotle would NOT have thought that

☐ the mind is separate from the body
☐ the mind exists in parts or modules, rather than as one entity
☐ blind people can hear better than sighted people

Questions 124–125 are based on the following reading passage.

The atrocities committed during the Second World War by the National Socialists are well known and have been meticulously documented by historians. Far less known, however, are the mass deportations that took place almost two years after the conclusion of the war, this time orchestrated by the Allied governments. In the years after 1945, over 12 million German-speaking citizens of Czechoslovakia, Hungary, Romania, and Poland were dispossessed, packed into trains, and left to fend for themselves in newly defeated and impoverished Germany. What allowed for this mass exile—the largest forced migration in history—was the confluence of political motivations on the part of the key players. The expelling countries of Eastern and Central Europe were especially keen on punishing Germans for the horrors of the war—though, of course, their own German-speaking populations were hardly responsible for Germany's actions—and on increasing the ethnic homogeneity within their borders. The Allied powers, too, had something to gain. The Soviet Union, intent on capitalizing politically on German's defeat, aimed to irrevocably undermine relations between Germany and Poland, especially by ceding German territory to Poland and emptying it of its inhabitants. Britain, weary from the war, hoped the resulting mass suffering would reinforce the completeness of Germany's defeat. And the United States, in turn, was attempting to cozy up to the nations of Eastern and Central Europe in the hopes of keeping them away from Soviet influence.

The result of the deportations, however, was the death of at least 500,000 people and Germany's acquisition of a homeless population far greater than that of any other industrialized country. The death toll was not far worse, furthermore, only because the Soviets' ambition to cripple Germany was unsuccessful. Following the war, Germany underwent what is known as its "economic miracle," which made it possible to house, feed, and employ the mass of exiles. That this episode is practically excised from the history books in some countries, however, is surely corroboration of the platitude that history is written by the victors.

Consider each of the answer choices separately and indicate all that apply.

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124. Which of the following can be inferred from the passage?

☐ Some events that occurred during World War II do not appear in history books.
☐ The Allied powers included the Soviet Union, Britain, the United States, and Poland.
☐ The Allied powers were not entirely unified on political matters.

Consider each of the answer choices separately and indicate all that apply.

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125. Which of the following, if true, would weaken the claim that the Allied powers had something to gain from the deportations?

☐ The United States believed that the Eastern and Central European powers hoped to create more multicultural, ethnically mixed societies.
☐ The Soviet Union wanted to profit from post-war trade between Germany and Poland.
☐ Britain, weary of the killing during the war, pledged itself to preventing suffering.

Questions 126–128 are based on the following reading passage.

Quantum mechanics is a relatively new field of physics that was developed in the early 1900's. Although we classically think of a particle as a fixed object, quantum mechanics describes particles as waves using properties such as position and energy. The quantum mechanical wave describes the probability of a particle to attain certain values of these properties, and measuring the particle samples from this probability. Take, for example, the analogy of rolling a six-sided die. For each roll there is a one-in-six chance that any single number will result. After rolling, however, only one single number will be observed. If the die is rolled enough times, one can deduce that the die has six sides and that each side is equally likely. However, one can never be completely sure, because rolling dice is probabilistic in nature. Quantum mechanics states that the same is true of the position (and other properties) of a particle. A particle trapped in a closed box has some finite probability of being at any location within the box. Open the box once and you'll find the particle at only one location. Open the box enough times and you'll see all the particle locations and the frequency at which they are achieved. From this, one can deduce the original properties of the quantum mechanical wave, just as one could deduce the properties of the die.

The counterintuitive properties of quantum mechanics, that the attributes of a particle cannot be known in advance of measurement, initially provoked many strong philosophical debates and interpretations regarding the field. In fact, Einstein was deeply troubled by the idea of nature being probabilistic and commented famously that, "God does not play dice with the universe." Over the last 70 years, however, irrefutable evidence has abounded that verifies the truth of the theory of quantum mechanics.

126. Which of the following best expresses the main idea of the passage?

(A) Particles are not a fixed object but rather waves.
(B) Controversial theories are often found to be correct.
(C) Quantum mechanics correctly postulates the probabilistic nature of particles.
(D) Many questions still exist about the nature of particles and quantum mechanics.
(E) Experiments can be designed to test the quantum mechanical nature of particles.

127. Based on the information in the passage, which of the following would best explain Einstein's motivation for stating that "God does not play dice with the universe"?

(A) Einstein did not believe that particles should be governed by probability as in a game of dice.
(B) Einstein believed that God should control the fate of the universe.
(C) Einstein was opposed to the theory of quantum mechanics on the grounds that it violated causality.
(D) Einstein's religious beliefs did not allow him to fully understand the theory of quantum mechanics.
(E) Einstein believed that God created the universe such that particles would be modeled probabilistically as in quantum mechanics.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

128. Which of the following can be properly inferred based on the information in the passage?

☐ The location of a particle within a closed box cannot be known for certain without observing the particle.

- ☐ Properties such as position and energy of a particle can never be measured.
- ☐ Particles can be properly described as quantum mechanical waves.

Questions 129–133 are based on the following reading passage.

Invisible theater and guerrilla theater are two forms of street theater with similar origins but very different approaches. Both forms take place exclusively in public places, but invisible theater conceals its performative nature whereas guerrilla theater flaunts it. While invisible theater creates a performance space unbeknownst to its audience, guerrilla theater actively seeks the attention of an audience by explicitly imposing a performance space onto a public place.

Starting in the early 1970's, Augusto Boal and fellow actors have staged scenes regarding social issues in public or semi-public places (e.g., restaurants), crafting their dialog and action to get a verbal reaction from bystanders. Because performers and non-performers remain distinct, invisible theater returns somewhat to the model set up by traditional theater. However, there are a few key differences. The performance space is created in public places without the awareness of non-performers. For non-performers, being beyond the performative space allows them to avoid the etiquette of theatergoing and removes that "lens" that unavoidably emerges when we feel we are viewing art or performance. If people do not suspect that they are viewing art, however, they are free to engage with the action and concepts of an unfolding drama as if these actions and concepts were real.

Boal has documented various successful instances of invisible theater in which non-performers actively listen, participate in public-spirited discussion, and even take unplanned public-minded action in response to the dialogue and events set up by invisible theater performers. Because onlookers think they are witnessing real life events, because the performers are bold in their statements, because the scripted characters are very vocal about what they are doing and experiencing, invisible theater is able to instigate political conversation within an everyday context; it successfully creates public forums out of thin air.

Guerrilla theater creates surprise performances in public, but is driven by the forceful imposition of "traditional" (if we can call anything about guerrilla theater "traditional") theater. One example includes two professors of Galway's University College who dressed in their robes and went out to the street, questioning pedestrians and awarding diplomas to the ones least able to provide good answers, as a way to protest their university's decision to grant Ronald Reagan an honorary doctorate in law.

A large part of the goal of guerrilla theater is to get publicized, its message echoed over and over in our ever-expanding network of technology-interface mass media. Guerrilla theater knows it may antagonize its direct audience—it often hopes to, because conflict is more likely to be broadcast, and the goal of guerrilla theater is to get people talking publicly.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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129. Which of the following can be inferred from the passage?

- ☐ When people are unaware that they are viewing a performance, they tend to act more naturally.
- ☐ Invisible theater is best described as improvisational.
- ☐ One measure of the success of a theatrical performance can be the actions taken by the audience once

the performance is over.

130. The main point of the passage could best be described as

(A) a discussion of two different but aligned artistic currents
(B) an examination of which of two art forms is more effective at prompting political action
(C) a synopsis of the evolution of theater
(D) a presentation of two theatrical concepts that conceal their performative nature
(E) an overview of artistic life in public places

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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131. Which of the following is true of both invisible theater and guerrilla theater?

☐ both have a goal of encouraging discourse
☐ both impose performance space onto public location
☐ both antagonize their audience

132. The passage implies that the Galway professors believed which of the following?

(A) Guerrilla theater was superior to invisible theater.
(B) Protesting an honor could result in the revocation of the award.
(C) Granting Ronald Reagan a degree demeaned the intellectual standard of the university.
(D) Handing out diplomas was a legal activity.
(E) Ronald Reagan’s foreign policy had deleterious effects.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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133. Which of the following, if true, would undermine the principle of invisible theater?

☐ When people knowingly view art, their heightened attention increases their perception and involvement.
☐ Audience members watch a performance and later report to others what happened, still not knowing that the event was a theater piece.
☐ A lively debate about public issues is brought to a halt by the imposition of scripted characters inserting theatrical dialogue.

Questions 134–138 are based on the following reading passage.

Cells employ many strategies to avoid genetic mutation. From the high fidelity of DNA-synthesizing enzymes to the pro-death signaling that accompanies mutagenic stimuli such as UV radiation, cellular mechanisms that stymie genetic changes are ubiquitous throughout the natural world. These mechanisms are critical because widespread genomic changes would wreak physiological havoc; indeed, malfunctions in molecular players that safeguard against mutagenesis, such as the protein p53, have been implicated in diseases such as cancer.

Yet despite the criticality of preventing and eliminating DNA mutations to avoid deleterious changes in cells, in specific contexts many organisms have also adapted beneficial mechanisms to induce genetic changes.

One such instance is observed in vertebrate immune systems: white blood cells such as T cells recognize invading pathogens through receptors on their surfaces. In order to recognize a wide variety of pathogens, these cells must generate a large repertoire of receptors. Relying only on a genetically encoded repertoire would be disadvantageously limiting—analogous to having only a few dozen language phrases with which to respond to the nearly infinite potential combinations of words in a conversation. Instead, the repertoire is generated by a process of genetic recombination, in which T cells "cut-and-paste" the DNA encoding their microbe-recognizing receptors. Many of these genetic rearrangements produce cells bearing non-functional proteins; such unproductive cells are eliminated through senescence. Nevertheless, this seemingly haphazard process of programmed genetic mutation is crucial to generating immunological diversity, as individuals with defects in this pathway exhibit clinical immunodeficiency. How this process is regulated by T cells to prevent harmful mutations remains the subject of ongoing research.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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134. Which of the following is true of genetic changes in cells?

☐ They can cause serious problems to body systems.
☐ They can provide benefits to the immune system.
☐ Some genetic mutation is regulated by T cells.

135. The phrase "seemingly haphazard" (line 19) is meant to indicate that:

(A) the process of programmed genetic mutation deserves further study.
(B) the production by T-cells of "unproductive cells" is wasteful.
(C) genetic recombination may appear random, but is not.
(D) T-cells are essential to proper immune system functioning.
(E) programmed genetic mutation can be dangerous to an organism.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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136. Which of the following can be inferred from the first paragraph of the passage?

☐ Pro-death signaling is a mechanism that hinders genetic changes.
☐ Cellular mechanisms that safeguard against mutagenesis are very common.
☐ Protein p53 may play a role in preventing cancer from forming.

137. The analogy (lines 14–15) regarding "a few dozen language phrases … conversation" is meant to elucidate

(A) why genetic recombination is important to T cell functioning
(B) the need for numerous means of fighting cancer and other diseases caused by cell mutation
(C) why white blood cells such as T cells rely on a genetically encoded repertoire
(D) how language use is like "cutting and pasting"

(E) the mechanism by which mutagenesis can compromise physiological functioning

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Consider each of the answer choices separately and indicate <u>all</u> that apply.

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138. In the analogy in the third paragraph, the "nearly infinite potential combinations of words in a conversation," represent

☐ pathogens
☐ receptors
☐ T cells

Questions 139–140 are based on the following reading passage.

Martin Haberman pulls no punches in his scathing critique of the insensitive and unjust treatment received by children in poverty in the public school system. He focuses the brunt of his criticism on teachers who have been insufficiently trained for the realities of the modern school environment and whose prejudices, lack of deep content knowledge, and excessive focus on order and discipline profoundly limit their effectiveness. Haberman writes, "the principles and theories we call child and adolescent development were all developed to explain the middle-class experience," and that everyone else in public schools, including non-white, immigrant, or non-English-speaking children, are considered somehow anomalies, thus leading to the absurd situation in which a teacher completes teacher training and is put in front of a class of students she considers to be made entirely of "exceptions."

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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139. According to Haberman, a teacher's effectiveness can be compromised by

☐ insufficient attention to order and discipline
☐ insufficient knowledge of the material being taught
☐ personal bias

140. In the last line of the passage, the word *exceptions* is in quotes to make the point that

(A) the idea of "exceptions" is crucial to effective education
(B) the quote is taken verbatim from a teacher
(C) students who perform academically well are an aberration, not the reverse
(D) certain teachers inappropriately consider "non-white, immigrant, or non-English-speaking children" as other than the norm
(E) teachers versed in the principles and theories of child and adolescent development developed to explain the middle-class experience are actually the norm

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Questions 141–142 are based on the following reading passage.

Naturalism, arising in the 19th century as a literary response to Darwin's account of evolution, focused on describing everyday reality but differed from realism in its attempts to provide a "scientific" foundation for its depictions of characters, stressing the influence of environment and heredity upon the individual psyche. Émile Zola, in particular, saw his craft as an extension of the scientific method into the domain of art. The 19th century, perhaps in opposition to naturalism, saw the rise of the Decadent movement, embracing artifice over nature in their writing, championed by Zola's erstwhile protégé, Joris-Karl Huysmans. The protagonist of his masterpiece, *Á rebours* (literally, *Against the Grain*, but more commonly translated as *Against Nature*), removes himself from society—viewing it as the product of a nature long surpassed by human ingenuity—and surrounds himself exclusively with art, perfume, literature, and technology.

Consider each of the answer choices separately and indicate all that apply.

141. Which of the following can be inferred from the passage about perfume?

☐ Some followers of the Decadent movement considered it to be an example of human artifice.
☐ Some followers of the Decadent movement considered it to be superior to natural entities.
☐ Some followers of the Decadent movement enjoyed surrounding themselves with it.

Consider each of the answer choices separately and indicate all that apply.

142. Which of the following, if true, would undermine the claim that the Decadent movement was opposed to naturalism?

☐ Decadent authors intended to use literature as a vehicle for the scientific method.
☐ Decadent authors focused on the effects of environment on shaping character.
☐ Decadent authors elaborated on the way inherited traits influenced human behavior.

Questions 143–144 are based on the following reading passage.

For many years, biological scientists have sought to decipher cellular function by quantifying the degrees of protein and mRNA expression within populations of their cells of interest. Classically, these measurements required combining many cells into a single sample and rupturing their membranes, thus exposing pooled quantities of the target molecule for detection. One limitation of these techniques is the reliance on average measurements: it is impossible to distinguish a uniform population of cells expressing intermediate quantities of a molecule from a population composed of separate low and high expressers. The distinction has proven to be important, particularly in the context of drug targeting of cancer cells; prescribing a dose to hit the "average" cell may completely miss the more aggressive "one percent."

The advent of single-cell measurement technology such as flow cytometry and RNA FISH has made it possible to capture not only a population's average levels of a molecule, but also the distribution of the molecule's expression within the population. As a result, researchers are increasingly investigating the sources and significance of variability within populations that were previously assumed to be identical.

143. According to the passage, the limitation of combining many cells into one sample and then rupturing their membranes in order to detect a target molecule is that

(A) variability exists within cell populations
(B) some cells in the sample may contaminate others
(C) this method cannot single out the cells that express more of a certain molecule
(D) the rupture of cell membranes is implicated in the formation of cancer
(E) it is preferable to capture a population's average levels of a molecule

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Consider each of the answer choices separately and indicate <u>all</u> that apply.

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144. Which of the following can be inferred from the passage regarding flow cytometry and RNA FISH?

☐ Both technologies allow researchers to quantify properties of individual cells.
☐ Using these technologies, it is impossible to distinguish a uniform population of cells expressing intermediate quantities of a molecule.
☐ Both technologies allow researchers to measure variability of molecule expression within cell populations.

Questions 145–146 are based on the following reading passage.

The Portuguese began to enter Angola in the 16th century, and over the next three hundred years slowly pushed into the interior of the sizable nation of Southern Africa, finally achieving complete occupation by the 1920's. However, following Angolan independence in 1975, and despite a bloody civil war that lasted until 2002, the Angolan economy has grown at a double-digit pace since the 1990's, due largely to expansive mineral and petroleum reserves. Conversely, Portugal is now broke and in debt, its economy shrinking by full percentage points every year. In a grand stroke of irony, Portugal's Prime Minister Pedro Passos Coelho in 2011 suggested to Angola's President Jose Eduardo dos Santos that "We should take advantage of this moment … to strengthen our bilateral relations." President dos Santos replied, "We are aware of Portugal's difficulties and we are open and available to help." This "help" will likely come in the form of Angola's investment in Portuguese industries that the International Monetary Fund has ordered be privatized as a condition of a 78 billion dollar bailout. Already, the country that once mined Angola for slaves and raw material is now virtually helpless as Angola purchases Lisbon's prime real estate, using much of it to build luxury resorts where Angolan officials go for holidays.

Despite the stunning reversal of fortune, Angola is not without its difficulties. Corruption is rampant, and Angola has one of the highest levels of income inequality in the world—in the capital city of Luanda, hamburgers go for 50 dollars and designer jeans cost twice what they do in London or New York, while two-thirds of the population lives on less than 2 dollars a day.

145. Which of the following is NOT stated as a component of Portugal and Angola's historical relationship?

(A) Portuguese traders sold Angolan slaves.
(B) Raw material was taken from Angola by the Portuguese.
(C) The two nations were at war.
(D) The Portuguese increasingly dominated Angola over a period of several hundred years.
(E) Angola achieved independence from Portugal in the 20th century.

146. The "grand stroke of irony" (line 7) is best described as:

(A) two countries dedicated to strengthening bilateral relations share economic problems.
(B) a former colonial possession is now being called upon to assist its former possessor
(C) slavery has ended, and yet people still live in poverty
(D) the cost of living in Luanda is very high, and yet most people are poor
(E) the Portuguese economy was once thriving and is now dwindling

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Questions 147–149 are based on the following reading passage.

For as long as humans have been philosophizing about the mind, virtually every thinker has conceived of the mind as a unitary entity. In fact, such a view was crucial to both Aristotle's and Descartes's view that the mind (or the soul) survived death. Surely the self cannot be subdivided; surely one cannot have half a mind?

Indeed, the final evidence that one can, in fact, have "half a mind" came in the 1960's, from the famous studies for which Roger Sperry was awarded the Nobel Prize in 1981 for his discoveries about the functional specialization of the cerebral hemispheres. Working with epileptics who had been treated via the cutting of the *corpus callosum*, or division between the two hemispheres, Sperry was able to observe "odd behavior" in these patients—each half of the brain could gain new information, but one hemisphere was entirely unaware of what the other had learned or experienced.

Restak, in *The Modular Brain*, posits that the brain is not centrally organized (some prior theories of mind had actually posed the existence of a "director" in the brain, begging the question of who directs the director's brain) but, alternately, that different parts of the brain control different abilities, and that those "modules" can operate independently. As we can easily see from patients with brain damage, there is no "unified mind and personality"—part of ourselves, centered in different parts of the brain, can change or be obliterated entirely as a result of physical changes to the brain. Consider the case of Phineas Gage, a rail worker who, in 1848, while attempting to compress explosive powder with a tamping rod, literally blew a hole in the front of his brain. While Gage was ultimately able to function fairly normally, his personality was markedly changed; he became boorish and irresponsible. Gage's case was well documented, allowing modern reconstructions to show that his injury affected areas of the brain that we now know to be related to moral sensibilities and their expression. That is, Phineas Gage literally lost one (or more) of the modules in his modular brain system.

147. The case of Phineas Gage is presented as evidence that

(A) the modular brain system has a central "director"
(B) people who lose parts of the brain are usually able to function normally
(C) brain injury is a serious risk in certain types of work

(D) splitting the *corpus callosum* can result in marked changes in personality
(E) aspects of personality can be physically located within the brain

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Consider each of the answer choices separately and indicate <u>all</u> that apply.

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148. In lines 13–14, the phrase "begging the question of who directs the director's brain" is meant to emphasize that

- ☐ the problem of a "director" in the brain is recursive
- ☐ whether there is such a "director" of the brain is an open question
- ☐ Restak has both asked and answered a question about the brain's organization

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Consider each of the answer choices separately and indicate <u>all</u> that apply.

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149. Which of the following can be inferred about thinkers who conceive of the mind as a unitary entity?

- ☐ They believe that the mind survives death.
- ☐ Their views are incompatible with modular brain theory.
- ☐ They are unaware that certain aspects of personality are known to be controlled by certain areas of the brain.

Questions 150–152 are based on the following reading passage.

Most mental health disorders and cases of drug abuse tend to diminish a person's ability to recognize other people's feelings. A recent study in Norway suggests, however, that these effects can be bolstered by a nasal spray puff of the brain hormone oxytocin, which is known to increase feelings of calm and social bonding. Although oxytocin is already prescribed for certain disorders that affect social function, such as autism, these treatments are often tried in isolated cases, leaving the overall effects of the drug without evaluation.

The Norwegian experiment focused on 40 students, each of whom was given either a control dose of salt water or the drug oxytocin. After the nasal dose, the students were shown faces of happy, angry, or neutral expressions, some of which were subtler than others. The researchers found that after a nasal spray dose of oxytocin, the students' awareness of the expressions was intensified. Further, the experiment showed that the oxytocin had the greatest effect on those who were least able to evaluate emotions properly when given the control.

Although the results of this study seem promising, Leknes, the lead scientist in the investigation, cautions that the hormone would not be a "cure-all" for mental illness or drug addiction. Rather, he suggests, the hormone might help some individuals better interpret the social cues from the world around them.

150. Based on the information in the passage, the author of the passage would most likely agree with which of the following statements about the brain hormone oxytocin?

(A) Its overall effects require further evaluation.

(B) In the future, it will be used to cure mental illness and drug addiction.
(C) It is not useful for people who are already able to interpret social cues.
(D) Its effects on the brain are unknown.
(E) It is more effective when dosed via nasal spray than orally.

151. The passage lends the most support to which of the following conclusions about the nasal spray study of oxytocin?

(A) The results of the study are inconclusive because a sample set of 40 students is not substantial.
(B) The nasal spray of oxytocin increased feelings of calm and social bonding for the students.
(C) Many students were unable to recognize the expressions shown to them when given only the control dose of salt water.
(D) The students who might need oxytocin most are the ones who appear most responsive to the hormone.
(E) The subtler the expression, the more difficult it was for the students to identify.

152. Which of the following is NOT mentioned in the passage regarding the Norwegian study on oxytocin?

(A) The study showed that oxytocin made students more able to distinguish faces from one another.
(B) Leknes was the lead scientist in the investigation.
(C) A control dose of salt water was used to gauge normal student ability to recognize facial expressions.
(D) Students who participated in the study were shown happy, angry, or neutral expressions.
(E) Oxytocin had the greatest effect on students who were least able to evaluate emotions properly when given the control dose.

Question 153 is based on the following reading passage.

Ever-present in Jamaican folklore and storytelling is the character of the trickster Anansi, an African spider-god who regularly outsmarts other animal-god characters. Also known as Kwaku Ananse, 'Nancy Spida,' and Aunt Nancy in the Southern US, the character of Anansi originated with the Ashanti people in Ghana. In Jamaican culture, as well as throughout the Caribbean, Anansi has been a symbol of slave resistance. Just as Anansi uses cunning and subterfuge to achieve victories over his oppressors, so too did slaves employ such strategies within the power structure of the plantations.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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153. Which of the following can be inferred from the passage?

☐ Anansi originated in Jamaican folklore.
☐ Jamaican folklore features other characters that have both divine and animal characteristics.
☐ Anansi is known on at least two continents

Questions 154–156 are based on the following reading passage.

The cosmic microwave background is a uniform 2.7 Kelvin radiation that permeates the entire universe. Although it was postulated almost 50 years before, Penzias and Wilson discovered the cosmic microwave background accidentally in the 1970's. Working at Bell Labs, these two scientists were using a radio telescope to observe distant stars. They found, however, that no matter where they pointed their telescope they observed an approximately 3 Kelvin background signal. After convincing themselves that this signal was real and not some artifact of their instrument, they consulted with a team at Princeton University that had been searching for the cosmic microwave background. The Princeton team confirmed what Penzias and Wilson had found. Apparently, Penzias and Wilson had accidentally stumbled upon the oldest observable in the entire universe.

Why does the cosmic microwave background exist and permeate all of space? Just an instant after the Big Bang, all matter in the universe was so energetic, or hot, that it existed as free particles known as "quarks." In the fractions of a second following, the universe expanded and cooled until the quarks lost enough energy to form electrons, protons, and neutrons, the building blocks of ordinary matter. Photons, the smallest particles of light, also filled the universe and were so energetic that they "bounced" off electrons, keeping the electrons and protons from forming atoms. After approximately 400,000 more years, the photons lost enough energy that atoms could form readily. Without any lone electrons off of which photons could "bounce," the photons began streaming unimpeded all through the universe, mostly unchanged but for one exception. Due to the further expansion and cooling of the universe, these photons have cooled to just 2.7 degrees above absolute zero. It was these same photons that Penzias and Wilson observed approximately 13.6 billion years later here on Earth.

154. Which of the following most accurately expresses the author's intent in writing the passage?

(A) to describe the discovery and reason for the cosmic microwave background.
(B) to explain how science discoveries can be made accidentally.
(C) to argue that the cosmic microwave background is the oldest observable in the universe.
(D) to defend the work of Penzias and Wilson.
(E) to support the theory of the Big Bang using the cosmic microwave background.

155. According to the passage, which of the following events occurred first after the Big Bang?

(A) The universe expanded and cooled until atoms formed.
(B) Photons streamed unimpeded through space.
(C) All matter existed as particles known as "quarks."
(D) The cosmic microwave background cooled to 2.7 Kelvin.
(E) Atomic nucleii, composed of protons and neutrons, formed.

156. According to the passage, to which of the following would the author most likely agree regarding the discovery of Penzias and Wilson?

(A) It was not as important as the signal for which they were originally searching.
(B) The telescope belonging to Penzias and Wilson was more sensitive than that of the Princeton team.
(C) Penzias and Wilson would not have discovered the cosmic microwave background if it had been more than 3 Kelvin in temperature.
(D) Penzias and Wilson did not initially understand the implications of their results.
(E) Penzias and Wilson did not believe that their signal was real when they took their discovery to the Princeton team.

Questions 157–158 are based on the following reading passage.

American composer and conductor John Philip Sousa viewed the increasing popularity of the phonograph with deep dismay. He suggested that it would "reduce the expression of music to a mathematical system of megaphones, wheels, cogs, disks, cylinders, and all manner of revolving things, which are as like real art as the marble statue of Eve is like her beautiful, living, breathing daughters." Such "mechanical" music was not sincere, according to Sousa: "The nightingale's song is delightful because the nightingale herself gives it forth. The boy with a penny whistle and glass of water may give an excellent imitation, but let him persist, he is sent to bed as a nuisance."

Sousa further decried a "decline in domestic music," noting the decline of musical instrument purchases and predicting that when music comes so easily out of a phonograph, mothers will not bother to sing lullabies to their babies. He opined that when music is so readily playable, musical and vocal instruction as a normal part of education will fall out of fashion, the "tide of amateurism" receding, and music will become the province of machines and professional singers only. "What of the national throat?" asked Sousa. "Will it not weaken? What of the national chest? Will it not shrink?"

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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157. Which of the following, if they occurred, would contradict Sousa's arguments?

- ☐ A private school that once demanded two semesters of vocal instruction as a requirement for graduation now offers the same classes as electives.
- ☐ A young boy in an isolated rural area during the Great Depression hears a professional bluegrass band for the first time on a phonograph, and it inspires him to ask his grandfather to teach him to play the family banjo.
- ☐ A modern recording artist comments that, because of her terrible stage fright, her live performances are less genuine than the recordings she is able to produce when she feels comfortable in the studio.

158. The "national chest," as used in the passage, means:

(A) the performances of professional singers
(B) the US Treasury
(C) the phonograph
(D) the vocal abilities of amateur American singers
(E) musical instruments found in American homes

Questions 159–160 are based on the following reading passage.

In thermodynamics, an idealized blackbody is an object that reflects zero incident electromagnetic radiation, absorbing all such radiation instead and consequently warming up. The blackbody emits just as much energy per unit time as it absorbs; the electromagnetic spectrum of the emitted energy, however, is completely determined by the temperature of the blackbody and by no other properties thereof, such as material composition or structure. In contrast, reflected radiation undergoes no fundamental change in its original spectral characteristics, other than a possible Doppler shift created by the motion of the reflector relative to an observer. Researchers have recently discovered that a microscopic "forest" of vertically aligned single-wall carbon nanotubes of varying heights applied to a surface has extremely low reflectance across a wide range of wavelengths of visible light, the closest scientists have come thus far to creating a perfectly dark material.

159. Which sentence in the passage states the variables that define the electromagnetic spectrum of a blackbody?

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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160. Which of the following can be properly inferred from the passage?

- ☐ An object that reflects incident electromagnetic radiation is not an idealized blackbody.
- ☐ Reflected radiation always exactly matches the spectral characteristics of the original incident radiation.
- ☐ A microscopic "forest" of vertically aligned single-wall carbon nanotubes of varying heights applied to a surface will absorb all incident electromagnetic radiation.

Questions 161–163 are based on the following reading passage.

Universalism was most prominently set forward by the linguists Joseph Greenberg and Noam Chomsky. Chomsky, attempting to account for the celerity with which children grasp the subtle grammatical rules of their native tongues, argued that the best explanation is that the human brain has "modules" capable of generating an entire grammar on the basis of a small set of "generative rules." We should therefore expect to find grammatical features shared by all human languages. Greenberg, on the other hand, painstakingly listed the grammatical features shared by multiple languages, positing that such commonalities must reflect innate cognitive biases. Greenberg's data paid special attention to word order, yielding the hypothesis that some grammatical features of languages must be co-dependent. Chomsky's view, in turn, predicts that as languages evolve and change, the grammatical features generated by the same rule should co-vary. A team led by Russell Gray, a New Zealand psychologist, recently released the results of a massive study that they claim casts doubt on these universalist predictions. **Borrowing** the technique of phylogenetic analysis from evolutionary biology, Gray and his colleagues reconstructed four family trees containing more than two thousand languages.
They found that the co-dependencies in word-order change varied among families, suggesting that each family has evolved its own rules. Moreover, if co-dependencies were common to two families, there was evidence that they had separate origins within each family, thus yielding no evidence of family-invariant rules. Many universalists, however, were unimpressed: that languages vary widely is well-known. But given that some language is spoken by virtually all human beings, it would be strange if it did not reflect cognitive universals. It is the search for those universals, not the cataloguing of variations, that should take priority.

161. The passage most likely uses the word *borrowing* to indicate that

(A) Gray and his colleagues produced research that was not original.

(B) linguistics and evolutionary biology have many features in common.
(C) progress in linguistics cannot be carried out through the traditional methods of linguistics.
(D) research methods common to one branch of science can prove fruitful to another.
(E) facts about linguistics can shed light on our understanding of evolution.

162. Select the sentence in the passage that provides support for the thesis of universalism.

163. Which of the following can be inferred from the passage?

(A) The thesis of universalism can only be supported through empirical studies.
(B) Some researchers believe that empirical research can undermine theoretical conclusions.
(C) Grammatical features in all languages co-vary.
(D) There is no evidence of the existence of family-invariant rules.
(E) Universalism is the only way to explain the speed at which children acquire language.

Questions 164–166 are based on the following reading passage.

For many years, most physicists supported one of two cosmological theories: the steady-state universe, and the Big Bang. The theory of the steady-state universe states that the universe has always existed exactly as we observe it at present, whereas the Big Bang theory postulates that the universe was conceived from a singularity in space-time that has expanded into current universe. The validity of either theory was not tested until 1929, when Edwin Hubble famously discovered what is now known as Hubble's Law.

Hubble's experiment is now a famous benchmark in modern physics. Hubble, using the Mount Wilson Observatory, observed a class of stars known as Cephied variables, luminous stars that blink and flicker with a rate that depends on their distance from the observer. Using this relation and years of observing, Hubble calculated the distance to many of these variable stars. Milton Humason, a fellow astronomer, helped Hubble to calculate the stars' relative velocities to Earth. When Hubble combined the two data sets he found an interesting relationship: all the stars appeared to be moving away from us! In fact, the speed at which they were moving increased with an increasing distance from Earth.

Hubble realized, from this small set of data, that the earth was a part of the expanding universe. As the universe expands outward in all directions, any observer from a fixed vantage point will look out and see everything running away from them. The further away any two points are, the more the expansion affects them, and the faster they appear to be moving away from each other. Hubble's result was the first experimental proof that we do not live in a steady-state universe, but rather a dynamic and expanding one.

164. Which of the following best expresses the main idea of the passage?

(A) Edwin Hubble discovered Hubble's Law, a benchmark in modern physics.
(B) Hubble discovered that the universe is expanding, disproving the theory of the steady-state universe.
(C) Before 1929, most physicists supported one of two theories of the universe.
(D) All objects in space are receding from each other because of the expansion of the universe.
(E) Modern physics would not have progressed without Hubble's discovery of the expanding universe.

165. Which of the following is not mentioned in the passage regarding Hubble's experiment in which he deduced Hubble's Law?

(A) It used years of data on Cepheid variable stars.

(B) Hubble accumulated data using the Mount Wilson Observatory and help from a fellow astronomer.
(C) Hubble found that all the observed stars appeared to be moving away from Earth.
(D) Hubble deduced the distance to Cepheid variable stars based on the rate at which they blinked and flickered.
(E) Hubble deduced the velocity of Earth to find the stars' absolute velocities.

Consider each of the answer choices separately and indicate all that apply.

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166. Which of the following can be properly inferred from the passage?

☐ The steady-state universe theory does not allow for an expanding universe.
☐ The closer any two points in the universe are, the less expansion effects them, and the slower they appear to be moving apart.
☐ After Hubble's discovery of the expanding universe, the Big Bang was the only cosmological theory that could be valid.

Questions 167–169 are based on the following reading passage.

Homer's *The Odyssey* is an epic poem putting a popular oral myth into writing for the first time. *The Histories* is an attempt by its author Herodotus to provide an **unbiased** account of historical conflicts in the Hellenistic world. These two works share two important motifs: the interference of the gods in the events of the mortal world, and the concept of a predetermined and unavoidable destiny. One might assume that these two themes are one and the same—a predetermined fate set forth by the gods. However, Homer's and Herodotus's gods are presented as acting in a political fashion—each one acting within certain boundaries to accomplish his or her own agenda. As such, the wills of the gods do not coincide to allow for the formulation of a cohesive "master plan." Instead of destiny created by the gods, Homer and Herodotus present fate as something beyond the gods—a driving force under which the actions of gods and mortals lead to the realization of destiny. In *The Odyssey* and *The Histories*, the idea of gods with limited power leads to a conception of fate wherein the gods act not as the creators of destiny, but as agents of its fulfillment.

167. Which of the following, if true, would most strongly support the assumption rejected by the argument of the passage?

(A) The gods pursue their agendas by conferring with other gods to ensure that their agendas serve a common goal.
(B) The agendas of gods and mortals frequently coincide with the demands of fate.
(C) Homer and Herodotus disagree strongly about the motives and agendas of the gods, as well as about the nature and severity of their conflicts.
(D) Destiny would be fulfilled regardless of what activities gods and mortals engaged in.
(E) In both Homer and Herodotus, gods and mortals frequently examine their motives and goals and are capable of making their own decisions about what to do.

168. The author most likely uses the term *unbiased* to convey which of the following ideas?

(A) The historical conflicts are presented in a way that precludes religious explanation.

(B) The historical conflicts are presented in a way that does not favor any particular party to the conflicts.
(C) The subjects of the histories are not restricted to any particular ethnic, social, religious, or geographical group.
(D) The historical conflicts are explained entirely by reference to the actions of the people and states involved in them.
(E) This histories are written in such a way as to challenge the sensibilities of their readers.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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169. Which of the following can be inferred about the gods in *The Odyssey* and *The Histories*?

☐ There are limits to what the gods can accomplish.
☐ The gods, like human beings, pursue their own interests.
☐ The gods do not control the final outcomes of their actions.

Questions 170–172 are based on the following reading passage.

The goal of a sunscreen chemical is simple—to prevent harmful UVB (and in some cases UVA) radiation from penetrating the skin. There are two main ways in which this goal can be accomplished—by reflecting (physically blocking) ultraviolet light or by absorbing UV light. Sunscreen chemicals are therefore put into two groups based on which method they employ; they are classified as either *physical blockers* or *chemical absorbers*. Physical blockers, the most common of which is titanium dioxide, scatter all radiation in the UVB and UVA range. Titanium dioxide reflects light waves in the 290–770 nm range. However, the vast majority of commercial sunscreens are chemical absorbers.

Chemical absorbing sunscreens work on the principle of photo-excitation of electrons. They absorb photons of light of specific wavelengths and use the energy to promote electrons between energy levels. When the electrons later return to the ground energy state, they emit light at longer wavelengths (lower energies). Chemical species that exhibit this behavior are called chromophores. The specific wavelength absorbed by a given chromophore is determined by the discrete quantal amounts of energy that are required to excite electrons between the energy levels or its molecules. Since the primary objective of an absorbing sunscreen is to absorb UVB light (290–320 nm), the best sunscreens are those that absorb most heavily in this range. The chromophores that most readily fit this requirement are those with conjugated pi-bonding systems.

170. Which of the following best summarizes the distinction between chemical blockers and chemical absorbers?

(A) Chemical blockers darken their target light waves while chemical absorbers lighten them.
(B) Chemical blockers convert their target light waves into radiation while chemical absorbers convert them into a different kind of radiation.
(C) Chemical blockers disperse their target light waves while chemical absorbers convert them into light with a longer wavelength.
(D) Chemical blockers scatter their target light waves while chemical absorbers convert them into radiation.
(E) Chemical blockers prevent light waves from reaching the skin while chemical absorbers absorb them into the skin.

171. Based on the passage, which of the following can be inferred about the chromophores referred to in the final sentence of the passage?

(A) If exposed to light with wavelengths of approximately 300 nm, they will scatter the radiation.
(B) If exposed to light with wavelengths in the 290–320 nm range, they will lower the energy level of some of their constituent electrons.
(C) If exposed to light waves in the 290–770 nm range, they will absorb the photons and emit them as light of longer wavelengths.
(D) If exposed to light with wavelengths of approximately 300 nm, some electrons in their component molecules will switch to higher energy levels.
(E) If exposed to light waves in the 290–320 nm range, they will promote the discrete quantal amounts of energy that are required to excite electrons between energy levels.

172. Select the sentence in the second paragraph that explains the physical feature on the basis of which one could select a chromophore for a sunscreen that would protect against UVA radiation.

Questions 173–175 are based on the following reading passage.

The story of *Sir Gawain and the Green Knight* has its foundation in Arthurian legend as formulated and passed down by the pagan oral tradition. In its written form, however, the tale bears the marks of Christian influence—it contains numerous scriptural and doctrinal references to Christianity. Since the author of *Sir Gawain and the Green Knight* is unknown, it is difficult to determine with any certainty the extent to which he was responsible for the incorporation of Christianity into the legend. For all we know, the story may have been "Christianized" in its oral form long before the poet set it into writing. The poet himself supports this possibility by writing in the opening lines that he will tell "anew" the tale "as I heard it in hall." If this is the case (and even if it is not), it is distinctly possible that the heroes of the Arthurian tradition represent in the written form a pagan interpretation of Christian ideals, rather than an externally imposed Christianization of pagan codes of behavior.

While it could certainly be argued that the poet portrays Sir Gawain as a good Christian hero in an attempt to infuse the story with Christian values, the critical tone of the narrative seems to suggest a different conclusion—that by critically editorializing the paganized form of Christianity embodied by Sir Gawain, the poet is trying to correct what he sees to be the flaws of that form. From the perspective of this conclusion it is clear that the poet only "Christianizes" the traditional legend to the extent that he *criticizes* the pagan interpretation of Christianity that is inherent in the behavior of its heroes.

Those who would argue that the poet intends to portray Sir Gawain as the perfect Christian hero would point to the descriptions of his chivalric qualities. The poet does indeed describe Gawain's Christian virtues generously; he even makes a special aside early in the second fit to describe the significance of the pentangle embossed on Gawain's shield, and to explain "why the pentangle is proper to that peerless prince." The author then delves into a lengthy enumeration of Gawain's Christian virtues. What is more, the fact that he uses the pentangle—a pagan symbol—to do it would seem to suggest that the author does indeed intend to add a Christian interpretation to the pagan legend he is retelling. Taken in its larger context, however, this passage takes on a different significance. In further examination of the poet's descriptions of Sir Gawain, it becomes apparent that the knight's seemingly perfect Christian behavior is superficial. A contrast can be observed between his "Christian" words and actions and his decidedly un-Christian motives. One theory is that, by emphasizing this contrast, the poet intends to denounce the pagan "misunderstanding" of the Christian message.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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173. Which of the following can be inferred about the pagan and Christian origins of *Sir Gawain and the Green Knight*?

☐ As an orally-handed-down tale, it was pagan, but as a written tale, it was Christian.
☐ Sir Gawain was a knight in King Arthur's court.
☐ *Sir Gawain and the Green Knight* contains both Christian and pagan elements, although it is not clear that either perspective is dominant.

174. Which of the following can be inferred from the author's interpretation of the Christian aspects of the poem presented in the third paragraph?

(A) Pagans and Christians differ in their interpretations of the Christian symbolism in the story.
(B) A pagan cannot have motives that are acceptable from a Christian perspective.
(C) A pagan story cannot be used to convey a Christian attitude.
(D) Christianity was absent in Arthurian stories before such stories were written down.
(E) Being a good Christian involves having both the right actions and the right motives.

175. Which of the following, if true, would most undermine the "theory" mentioned in the final sentence of the passage?

(A) Sir Gawain is portrayed as disingenuous in his exercise of "Christian virtues."
(B) Another character in the story is also associated with pagan symbols and is praised straightforwardly for her Christian virtues.
(C) Sir Gawain, in the story, prays to God to help him in battle.
(D) Another character in the story is associated with pagan symbols but is portrayed as having no Christian virtues whatsoever.
(E) A group of people in the story are portrayed as "barbarians" who are neither pagan nor Christian.

Questions 176–177 are based on the following reading passage.

Various tales in Herodotus's *The Histories* display a circular means of the realization of fate. In one story involving the birth of Cyrus and his rise to power in Asia, Herodotus tells us that the Median king Astyages was having disturbing dreams about his daughter Mandane.

We are told that his first dream, in which Mandane's urine flooded all of Asia, was interpreted ominously by the Magi. As a consequence, when the time came to marry Mandane off, Astyages made what turned out to be a fatal mistake. While there were plenty of wealthy and powerful Medes eligible for marriage, "his fear of the dream made him refuse to marry her to any of them; instead, he gave her to a Persian called Cambyses, whom he found to be of noble lineage and peaceful behavior, although he regarded him as the social inferior by far of a Mede of the middle rank." Essentially, Astyages altered what would be a normal treatment of the marriage in order to marry his daughter to someone less threatening. This attempt to avoid the prophesy of the first dream backfired however, and when Mandane became pregnant, Astyages had another foreboding dream. This second dream was interpreted to mean that Mandane's son would rule in Astyages's place. Herodotus tells us that "[the prophecy of the second dream] was what Astyages was guarding against" when he again took action, telling his advisor Harpagus to kill the baby. This plan backfired as well since Harpagus refused to kill the baby, leading to a complicated chain of events whereby the child—later to be named Cyrus—survived and returned to conquer his grandfather's kingdom. In this story, Astyages's downfall is depicted as resulting directly from two major mistakes—marrying Mandane to Cambyses and telling Harpagus to kill their offspring. These mistakes in turn are shown to be motivated by fear of the prophesies of his downfall. Had not some divine force planted the dreams in his head, he would not have taken the steps necessary to fulfill those prophesies. Through this circular path, destiny is unavoidably realized.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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176. Which of the following can be inferred from the passage about Astyages's view of the Median socio-political structure?

☐ As a result of his first dream, Astyages believed the threat his daughter posed to him could be through her husband.
☐ Astyages believed that it is always best to observe the recommendations of the Magi.
☐ Astyages believed that a Persian noble was less of a threat to his position than a Median noble.

177. Which of the following, if true, would most strongly undermine the claim that Astyages's downfall proceeded from two major mistakes?

(A) Mandane's son would have conquered his grandfather's kingdom regardless of who his father was.
(B) Astyages's first dream was in fact a warning against allowing his daughter to marry.
(C) Harpagus would not have killed the baby regardless of whether he knew the prophesy.
(D) Mandane's husband would have deposed Astyages if he had known why his son was killed.
(E) Astyages's dreams were better interpreted as advising him not to do anything out of the ordinary.

Questions 178–180 are based on the following reading passage.

Nineteenth century painter Albert Bierstadt's view of his artistic skill as a vehicle for self-promotion is was evident in his choices of style and subject matter. From the debut of his career with the exhibition of *Lake Lucerne* (1856), he developed a fixed style that was most easily recognizable for its size—the largest of the 636 paintings on display at the exhibition, it was over three meters wide. This, coupled with the artist's ability to represent the optimistic feeling in America during the westward expansion, is what led to Bierstadt's explosive growth in popularity during the 1860's. Bierstadt deliberately appealed to those rich patrons—railroad tycoons and financiers—whose nearest substitute to making the arduous journey out West was to purchase a hyperbolized replica of a Western vista.

But trends following the Civil War produced a drastic shift away from the adventurous optimism of the pre-war era and toward a more subdued appreciation for the details of American life. In this new social context, the paintings now seemed too decadent, too gaudy, for the new philosophy taking root in the country following the horrors of war. As one commentator in 1866 put it, Bierstadt's work "may impose upon the senses, but does not affect the heart." In a sense, then, that same American pride upon which Bierstadt had capitalized to advance his success was now, in its fickleness, the source of his downfall.

178. According to the passage, the new “philosophy” taking root in America after the Civil War would be best described as

(A) justifiable pessimism
(B) somber realism
(C) restrained minimalism
(D) prideful idealism
(E) stubborn dogmatism

179. The passage makes use of the phrase in quotations primarily in order to

(A) challenge a prevailing thesis
(B) point out an erroneous assertion
(C) provide expert testimony
(D) highlight a controversy
(E) offer evidence supporting a claim

180. All of the following are mentioned as contributors to Bierstadt’s success EXCEPT:

(A) the dimensions of his paintings
(B) his ability to convey auspicious feelings
(C) subdued appreciation for the details of American life
(D) catering to the preferences of the wealthy
(E) portrayals of exaggerated landscapes

Reading Comprehension Answers

1. **(C).** This question focuses on the main idea of the passage. The reading constantly flips back and forth, taking one side at first, then another on the *forecast for American entrepreneurship*. The author is careful to present his or her arguments fairly and with some reservations (e.g., *it is not clear that…, suggests that…, may seem…, does not necessarily…, it may not bring…, can be…*). Ultimately, the view is balanced, or *nuanced.* Regarding choice (A), the author does not *propose changes.* Regarding (B), the author never argues that any group of college students should (or should not) form their own companies. If anything, the author offers both sides of the issue, leaving it up to individuals to decide. Regarding (D), the author offers one fact about the varying rates at which businesses of two types (low-value and high-value) are founded. However, the author never evaluates the viability (or likelihood of success) of these types. As for (E), the analogy drawn in the last two sentences is not the main point of the passage; it is a final observation, one that sheds light on the issue, but this analogy is not the primary purpose for which the passage was written.

2. **(B).** The question asks for a specific detail mentioned or clearly indicated in the text. You should look for very direct proof for the answer choice you pick. Specifically: *College graduates, unable to find traditional jobs, instead opt to start their own businesses.* You should look for a match for this concept, slightly reworded at most. Correct choice (B) matches the idea that college graduates are *unable to find traditional jobs*. Regarding (A), college graduates who start their own businesses may indeed be hampered by a lack of access to outside investment, but this is not offered as the reason that they start such businesses. Be careful of answer choices that restate truths from the passage but do not answer the specific question posed! As for (C), it may be true that low-value companies have become more prevalent, but again, this does not answer the specific question. Regarding (D), the choice between renting and buying property is offered only as an analogy, not as a literal choice for college graduates (let alone as the reason for their entrepreneurial decisions). Regarding choice (E), you are told nothing about how forecasts of the unemployment rate are likely to evolve.

3. **(E).** This question asks you what "can be inferred" about the number of American high-value businesses. The answer choices reveal that specifically, you should focus on how this number has changed (or may have changed) during the course of the recent recession. For an Inference question, you will need proof from the passage, but you will typically need to transform that proof — you are not simply asked to look up a fact, and you should be wary of simply matching language in the text to language in the choices. However, you should also recognize that there is no fuzziness or room for interpretation. The answer you come to must be rock-solid. In particular, when you encounter mathematical language, you should take extra care to make our argument airtight mathematically. Looking up the key phrase *high-value businesses*, you find that *the proportion of high-value businesses founded each year has declined*. Watch out! As you know from the math side of the GRE, a proportion is not the same as a number. The proportion has fallen, but you are also told that the overall number of new business starts has been higher over recent years. If the overall number is up, but the proportion is down, you cannot tell whether the absolute number of high-value businesses is up or down. (E) is correct — what has happened to the number you want is unknown.

4. **(C).** You are told that women went to war *as nurses or in providing supplies* and that *such women*, including many old women, died in battle. Therefore, *Some of those who worked as nurses or in providing supplies died in battle*. In (A), you do not know how other people felt about (*lamented*) or marked the deaths of anyone who died in battle. You also do not know anything about the courageousness of younger women (B), or anything about the status of men in Asante culture ((D) and (E)).

5. **(E).** The first paragraph introduces bacterial "super-bugs" with some alarm. The second paragraph increases the

alarm, noting how *many scientists argue that the human race has more to fear from viruses*. This paragraph describes the way in which viruses hijack the cell, in order to illustrate how tough viruses are to treat. The last paragraph continues the comparison and puts a stake in the ground: *bacteria lack the potential for cataclysm that viruses have.* This last point is illustrated by the *near-miss* you have had with the HIV pandemic. As for choice (A), you are never told how bacteria infect the body. Regarding (B), the hijacking process is certainly described, but to make a larger point: why it's hard to eradicate viruses, in comparison with bacteria. As for (C), After reading this passage, you may want to call up the CDC and donate money, but the passage itself only raises a warning, if even that — it is not a call to action. Regarding choice (D), the last paragraph does highlight your good fortune, but this is not the larger point of the whole passage. (E) is correct — the passage compares the two threats (bacteria and viruses) and judges viruses to be far more important (after all, viruses have the *potential for cataclysm*).

6. **(B).** In this Inference question, you must be careful to focus on the keywords you are given: *infections by bacteria* (which you could rearrange to *bacterial infections*). What do you know from the text about bacterial infections? Toward the end of the second paragraph, one sentence gives you a direct clue: *viral infections cannot be treated ex post facto in the way that bacterial infections can*. So you can infer correctly that bacterial infections CAN be treated "ex post facto" (it means "after the fact"). You should look for a match to this idea. Choice (B) gives you what you want. The extra phrase *by antimicrobial agents* isn't much to add; this term is used at the beginning of the passage, and all it means is "stuff that works against microbes, i.e., bacteria." Regarding (A), from the beginning of the second paragraph, you know that bacteria themselves *reproduce asexually through binary fission*, but you don't know that to be true about infections by bacteria. Regarding (C), resistance "cocktails" such as NDM-1 actually make bacteria "nearly impregnable." So an infection by bacteria that have this cocktail would be less vulnerable, not more vulnerable. Choice (D) is true about viral infections, not bacterial infections. Choice (E) makes little sense; you know only that the virus does critical damage to the host cell. In fact, it does not seem that bacteria infect organisms in the same way, so it might not even make sense to speak of the "host cell" of a bacterium.

7. **(A).** This Specific Detail question requires you to determine what is true about *intracellular obligate parasites* (or IOPs, to give them a temporary abbreviation). Going back to the passage, you read this: *Whereas bacteria reproduce asexually through binary fission, viruses lack the necessary structures for reproduction, and so are known as "intracellular obligate parasites."* The word *so* toward the end tells you that the reason viruses are called IOPs is that they *lack the necessary structures for reproduction*. You are looking for this idea, perhaps slightly restated. (A) is correct.

8. **(B).** Before reading the choices, it is helpful to determine for yourself what the main idea is. Is the passage really about supernovae, or is it about Galileo, the philosophers, and the ideas being discussed? The fact that the "twist" occurs in the second paragraph (you're not talking just about science — now you're talking about history and philosophy) supports the position that the second paragraph is more central to the main idea and that the first paragraph is just background information. Choice (A) is wrong because it does not mention the main content of the second paragraph, the ideas and assumptions that became controversial. It is also *much* too broad — giving a history of supernovae would take a lot more than two paragraphs. Choice (B) is correct — the passage does describe a "shift in thought" (from an unchangeable "heavens" to a more scientific view), and this shift (at least among intellectuals in Padua) was prompted by a "natural event" (the supernova). Choice (C) is attractive (*juxtapose* means "to put next to one another" or "to compare"), but does not describe the main idea. While Galileo and the philosophers certainly had different views about the bright light they saw in the sky, it's not clear that you can say that the philosophers had "views about supernovae" in general. Choice (D) can be stricken simply due to "corroborate" (to prove true or support with evidence). The passage is describing, not making an argument. Finally, choice (E) is *much* too broad! You could spend an entire career discussing *how science and philosophy interrelate*. This passage covers a much more narrow topic.

9. **I only.** Remember that Inference questions are *not* asking you to think for yourself. You must not make any assumptions or go beyond the information in the passage. Statement I is true because you can prove it with the first

sentence of the second paragraph. *In 1604 in Padua, Italy, a supernova became visible, appearing as a star so bright that it was visible in daylight for more than a year.* Since this supernova was visible for more than a year, it is possible for supernovae to *take more than a year to fade from view*. (Note that if Statement I said "Supernovae *always* take more than a year to fade from view," it would be wrong.) The second statement cannot be proven. Just because you are told that a supernova in 1604 caused a stir does not mean no one else had ever seen one before. The third statement also cannot be proven. You know that Galileo disagreed with the philosophers; you don't know that those particular philosophers ever changed their minds.

10. **(C).** As a result of the supernova in 1604, Galileo gave popular lectures in which he "sought to explain" the origin of the "star" and which "undermined the views" of some philosophers. That's all you know. Choice (A) is a bit silly — was the Earth created after 1604? This choice took some wording from paragraph and twisted it around to trick us. Choice (B) is too extreme — you know that Galileo *sought to explain* the origin of the supernova, but you don't know that he succeeded. Choice (C) is true — you are told that the lectures were *widely attended by the public*. (A very picky person might point out that just because people go to a lecture doesn't mean they are interested, but all of the other answers are definitely wrong, so that confirms that this is a reasonable — that is, very tiny — inferential leap.) Choice (D) is attractive, but is a trap answer. You know that Galileo, in his lectures, "undermined" (weakened) the views of the philosophers. But you don't know what the philosophers' response was, and you certainly don't know whether the philosophers were opposed to the lectures themselves (a person could be opposed to the ideas in a lecture but still think the lecturer should be allowed to lecture). Choice (E) is also attractive because you, as a modern reader, know this to be true in real life. However, the question *does not* ask, "What really happened?" It asks "*The author mentions which of the following* as a result of the supernova of 1604?" The author does not mention that the philosophers were "proved wrong." (Their views were "undermined, which is much less extreme.) You cannot validate this answer choice with evidence from the passage.

11. **I and III only.** Kincaid's critics point out that Britain's government is not as corrupt as Antigua's. Kincaid makes a witty rejoinder, but does not deny that the British government is less corrupt than Antigua's. This, coupled with her stated belief that Britain is responsible for corruption in Antigua, justifies the first statement. As for the second statement, it would not be surprising if Kincaid did think this, but no governments of the "former colonial empire" are discussed in the passage except Antigua's. Finally, the third passage refers to "Antigua's current problems, including drug dealing and selling off land for tourist properties," so the third statement is true.

12. **(B).** In main idea questions, you can often eliminate one or more answers just based on the first word or phrase: *relate, discuss, give a history, make a case, emphasize.* Does the passage "make a case"? It doesn't — like almost all GRE passages, it is merely reporting facts and/or the opinions of others. *Emphasize* is also a bit strange — usually, the purpose of a passage is something a little bigger than just "emphasizing" something. When you read the passage, you may have determined that the first paragraph simply gives background information about *Don Giovanni,* while the second paragraph gives the "twist" — even though almost the whole world loved *Don Giovanni,* people in Mozart's own hometown were less enthusiastic about it. Choice (B) matches this — the passages discusses how a work of art (the opera) has been met by diverging (different, diverse) responses, specifically those of Viennese audiences versus those of everyone else.

13. **(A).** The full sentence is, *The opera mixed traditions of moralism with those of comedy — a practice heretofore unknown among the composer's works — creating a production that was not well liked by conservative Viennese audiences*. Regarding Mozart's *mixing traditions of moralism with those of comedy*, the clue *a practice heretofore unknown among the composer's works* tells you that Mozart had not done this before. Furthermore, the clue *not well liked by conservative Viennese audiences* tells you that Viennese audiences did not like this because it was a departure from tradition. Correct answer (A) is a good match — a "lackluster reception" means that those who "received" something (the audience) were holding unimpressed. Note that the GRE is trying to hide the correct answer from you a little bit by saying "a particular group of people" for "Viennese audiences." Also note that (D) and (E) could be eliminated straightaway — the author does not "argue" or "undermine" (which would mean the author was

arguing) at any point. Obviously, talking about moralism and comedy doesn't "remind the reader of the plot" (B), and you have no idea (from the passage, at least) what is common in contemporary (modern) opera, so (C) is out of scope.

14. **(E).** You know that Viennese audiences did not like the (new) mixing of moralism and comedy because the audiences were "conservative." But there's an even better clue: the passage says *eighteenth century audiences in Vienna — Mozart's own city — were ambivalent at best*. *Ambivalent* means "having mixed feelings or undecided." *At best* is an expression meaning *or worse.* (E) is a direct match with a sentence from the passage. Note that (A) and (B) contain extreme language (*purely, unequivocally*). Choices (C) and (D) go too far. You have no indication that audiences were confused or offended, merely that they didn't like or had mixed feelings about a work of art.

15. **II and III only.** Regarding the first statement, watch out for extreme language! Redlining never "ceased"; you are told it *may have continued in less explicit ways*. The same part of the passage provides good support for the second statement — redlining has become less *explicit,* or *overt*. Finally, you are told that *With no access to mortgages, residents within the red line suffered low property values*. Thus, access to mortgages is related to higher property values.

16. **(D).** You are told that *reverse redlining occurs when predatory businesses specifically target minority consumers for the purpose of charging them more*. Choices (A) and (B) describe *regular* redlining, the practice of denying service or constraining access to something like jobs. Redlining is about businesses, not police, so (C) is out. Choice (E) does mention targeting minority consumers, but does not give an example of offering inferior products or overcharging (reverse redlining is discriminatory, and choice (E) describes a perfectly normal ad campaign). Choice (D) is a perfect match, and a good example of what is described in the third paragraph.

17. **(E).** In the first paragraph, you are told that *With no access to mortgages, residents within the red line suffered low property values and landlord abandonment; buildings abandoned by landlords were then likely to become centers of drug dealing and other crime…* The lack of access to mortgages is due to redlining. Thus, redlining causes landlord abandonment and the resultant effects — (E) is a perfect match. Note that (A) mentions "subprime mortgages" and "defaults," which are never mentioned in the passage. (B) incorrectly cites reverse redlining rather than redlining. (C) gets the relationship between redlining and landlord abandonment backwards. (D) is just wrong — "constrained access to health care, jobs, insurance, and more" is mentioned as part of the *definition* of redlining.

18. **I and III only.** You are told that "germophobic" Western society eschews premastiction, which provides "beneficial bacteria," so the first statement is true. The word *only* kills the second statement — watch out for extreme language! If just one person outside of the developing world premasticates food for a baby, this statement is false, so you cannot infer it. Finally, you are told that *babies also received antibodies and digestive bacteria from the mother's saliva*, so saliva has at least one benefit (antibodies) aside from digestive bacteria.

19. **(B).** The author discusses Bois and Damisch's metaphor ("a dynamic game"), calls it misleading, and proposes an expansion beyond the competitive aspect. (A) is incorrect, as the author does not claim that the two artists were the best. (C) only addresses the details and evidence presented in the passage but not the main point. (D) and (E) are both distortions because the passage does not rank the painters, and these issues are again not the point.

20. **(D).** This question type asks for a combination of the point, tone, and, perhaps, structure. The author expresses the point at the end of the first paragraph by suggesting that the rivalry between Picasso and Matisse was more of a dialogue or exchange. (A) is an incorrect comparison because the passage rejects the idea that such rivalries are mere competitions. (B) distorts a detail by adding an opinion not stated in the passage. The author writes about the artists employing multiple styles, but never implies they are masters *because* they employ multiple styles. (C) is a distortion based on misreading the "conversation" metaphor. (E) is incorrect, as the word *generally* is a claim that the author does not make; you are only told about one particular rivalry.

21. **(C).** In the last paragraph, the passage states that Matisse used the same palette in his work but omitted the green. (A), (D), and (E) are incorrect because they cite comments about *Large Nude in a Red Armchair*, not *Woman with Yellow Hair*. (B) is backwards — the passage states that Matisse's response, as opposed to Picasso's work, did not rigidly tie color to form.

22. **(E).** In the third paragraph, the author refers to Matisse as the older artist. Choice (A) is not true; the phrase "sensuous, rich mood" was used in the passage to describe a work by Matisse, not *Large Nude in a Red Armchair*. Choice (B) is wrong because the author, at the end of the passage, uses *seems to be saying* and that indicates that it was not actually said. (C) is incorrect — the passage states that they were "occasional friends." (D) is a distortion; the passage only says that Matisse's response came a decade later, not that he had worked on it throughout that time.

23. **(B).** The author's point was that the rivalry was more of a dialogue than a competition. (A) and (E) are not correct because those choices do not say anything about the nature of the rivalry and no more can be assumed. (C) is wrong because the author's point was that their relationship was more than a competition. (D) misconstrues the metaphor that the author rejects.

24. **(C).** The last paragraph devotes itself to highlighting the problematic implicit messages of timelines. (A) is a distortion; the author does not claim dates are unimportant but that historical study should go far beyond mere dates. (B) and (E) are backwards, as these are two of the implicit messages that the author rejects. (D), while quite likely, does not have to follow — the author implies that there is more to history than *wars and minor battles, punctuated by the occasional presidential election and technological innovation* but does not specify that content.

25. **(D).** Essentially, this question tests vocabulary. The point of the passage is the problems with timelines, and the ending is a warning — portentous can mean "ominously significant." The words in (A) would never describe a GRE author. In (B), the author is not bemused, or amused, at all. Nor is the author resigned or complacent. (Note: it would be rare for a GRE author to be called *negative*, but this passage would arguably be just such a rarity.)

26. **(D).** The last paragraph discusses the negative implicit — or subliminal — issues with timelines. In that paragraph, since the author indicates that an implicit flaw is showing "only" 64 events, it must follow that there are more. (A) is out. In the second paragraph, the author worries that the prominence of the dates will draw too much attention, so eliminate (B). In the first paragraph, the author says, *Despite their usefulness in allowing students to gain a cursory knowledge…* This means that timelines have some positive uses. cross off (C). As for (E), in the last paragraph, a flaw cited is that timelines portray an "official" history; thus, unofficial histories must have virtues.

27. **(B).** In the last paragraph, a listed implicit flaw is that the events are presented as independent — without context. The passage mentions their prominent placement (A) and use of photos (E), but does not suggest that these are the problems. (C) is false, as the passage states these timelines had beginnings and ends. Regarding (D), the author does not challenge the factual accuracy of timelines.

28. **(C).** *Myopic* means "near-sighted," and the author employs it to describe the inability of some curricula to see the big picture. Similarly, *purblind* means "partially blind or deficient in understanding." (A), (B), and (D) are not justified by evidence from the passage — ignorance and bigotry are not mentioned as problems. *Astigmatic* indicates visual distortion, but the passage spoke of missing elements. *Mordant*, meaning "caustic or corrosive," has no connection to the passage.

29. **(A).** The second paragraph further describes one of the two factors named in the first paragraph. (B) is incorrect because the passage does not contrast the factors (the second paragraph does not even mention the Diamond Necklace

Affair). (C) is out of scope and irrelevant as the passage is about Marie Antoinette's unpopularity, not the court or corruption. (D) is incorrect; the last paragraph discusses the results. (E), while arguably the main point of the passage, goes beyond that of the second paragraph, which does not discuss consequences.

30. **(C).** The third paragraph mentions *countless aristocrats who sued to the king on Rohan's behalf,* so it must be true that a significant proportion was more loyal to the accused. Nothing in the passage refers to jealousy and nothing imputes that the queen's wealth was the cause of the aristocrats' dislike. (A) can be eliminated. (B) does not have to follow; the passage only says pamphlets were popular with the general public — it could, but doesn't have to be true that a large percentage of aristocrats read them. Similarly, the passage does not give such literacy rates so they, no matter how likely, cannot be assumed. (D) is out. (E) is completely out of scope.

31. **(D).** This detail question concerns the second paragraph, which cites a reduction in royal censorship as a cause. (A) is out of scope; the education of women is never mentioned. (B) is a distortion; the passage says publications, not the literacy rate, tripled. (C) is incorrect, as the passage does not discuss the number of scandals over time. (E), however likely, is not mentioned in the passage.

32. **II and III only.** The first statement can't be proven — the passage does not indicate who the clever forger was. The second statement is correct — the passage identifies Rohan as a "social climber." In the third paragraph, the author states that *10,000 people came to the doors of the Bastille demanding Rohan's release*.

33. **I only.** The point of the final paragraph is that the queen's unpopularity caused significant problems for the monarchy. You do not know if the queen met Rohan; she wouldn't have to meet someone personally to order that he be punished. The third statement is backwards: the queen's power must have had limits since Rohan was released despite her prosecution.

34. **(C).** This is essentially a vocabulary question. *Disinterested* does not mean "uninterested" — it means "unbiased." GRE authors are never described as *arrogant* or *supercilious*, as these words are too negative (and inappropriate). It is also very unlikely that *prim*, *meretricious*, or *lascivious* would be correct for the same reason. As for (E), the author is *analytic*, but no phrases or adjectives in the passage justify *enthusiastic.*

35. **(C).** After the theory is described, the remainder of the passage cites studies and opinions that disagree in part or in whole. (A) is incorrect because no additional support is provided. (B) is incorrect, as the criticisms are not a *screed*, which is a long diatribe (and would be inappropriate from a GRE author). (D) is incorrect because it is not the author's credentials that are questioned. (E) is wrong — there is no ridicule. Note that (B), (D), and (E) all express inappropriate attitudes for a GRE author. While authors can certainly argue for or against something, or express some enthusiasm or support, GRE authors do not rant and rave, and only very, very rarely *ridicule*, or question the integrity of those with whom they disagree.

36. **(D).** In the second paragraph, D'Emilio's critics question his use of the word *free*. (A), (B), and (C) are topics that are not mentioned. (E) is at best a distortion — while one critic argues, in opposition to D'Emilio, that industrial capitalism worsened the position of women, the issue is not framed as one of sexism.

37. **(E).** The Lowell Girls are mentioned to show that historically *free labor has hardly been free at all*. (A) is incorrect — that is the subject of the last paragraph, but does not concern the Lowell Girls. (B) is incorrect — that is the subject of the third paragraph, but does not concern the Lowell Girls. The answer must come from the part of the passage referenced. (C) is backwards — this argument is put forth by D'Emlio's critics. (D) is a distortion, the reference is to show how historically normal this situation was, not to contrast two supposedly parallel groups of un-empowered workers.

38. **(C).** The answer has to be something that *must* follow from the discussion of Davis in the last paragraph, not something that he or she *could* or *likely would* agree with. (C) is correct because Davis argues that women *were the losers in a double-sense ... leaving many women largely bereft of significant economic roles*. If being "bereft" of an economic role makes one a "loser," it is not going too far (in fact, it is less extreme) to say that *People without economic roles are disadvantaged*. (A) is wrong, as it is an opinion of D'Emilio's mentioned in the first paragraph. (B) is wrong, as it is an opinion of Enloe's mentioned in the second paragraph. (D) In the last sentence of the passage, Davis actually states that the "entire economy" has left the household.

39. **(C).** A "best title" question asks for the main point. The point of this passage was to highlight trenchant criticisms of D'Emilio's work — the lack of any rebuttal of these points indicates that the author sympathizes with them. (A), (D), and (E) are incorrect because they ignore that the passage concerns scholarly opinions rather than the history itself. (B) is incorrect, as no reconciliation is attempted. Additionally, (E) is closer to D'Emilio's view than the author's.

40. **(D).** "Unique" is a secondary meaning of *peculiar*. The author uses it in the context of Matisse learning something that Picasso had done. Of the wrong answers, (B) and (E) at best come from prohibited outside knowledge, whereas (A) and (C) are the opposite of the intended meaning — anything *strange* or *unknown* to Picasso would be something he doesn't do; *peculiar to* Picasso means something that he's known for doing.

41. **(A).** The author both describes the work (*The Ochre Head*) and the inspiration for that work (*Still Life with Ancient Head*). (B) is a distortion, as the passage does not rate the painters. (C) is a distortion as the passage describes a painting but not how the artist developed his style. (D) is incorrect, as the passage indicates that this was a new direction rather than a representative work. (E) is out of scope because the passage only discusses two painters, not the art world.

42. **(C).** While the author clearly appreciates the work, its place in the hierarchy is not discussed. (A) is incorrect — the passage states that *colors ... refuse to be constrained by definite lines of form*. (B) is wrong as the end of the passage states that the painting's composition references a Picasso work. (D) is backwards — the passage says the technique is not employed with the free reign used in *Minotaur*. (E) is wrong as the passage says a vase of flowers and a bench are depicted.

43. **(C).** To answer a question of this type, one must find the other four choices in the text. In the third paragraph, (A) is mentioned verbatim. (B) is justified by "hearing speakers," and (D) by "write autobiographical essays." (E) is mentioned explicitly.

44. **(D).** The passage, after establishing problems with textbooks, proposes film as a vehicle for teaching history. Since film is mentioned as a current alternative, it is being used now to teach history, but the author would like to expand this use. *Didactic* means "intended to instruct," so films used to teach history would certainly qualify as a "didactic tool." (A) is out of scope and ignores the educational thrust. (B) and (C) do not include the bulk of the passage, which propose film as a learning tool. (E) is too negative — a GRE author would not *denigrate* — and *philosophy* is an inappropriate word for the passage (using textbooks is hardly a *philosophy* so much as just a common practice).

45. **(B).** The answer to detail questions must be found in the text, not through your own reasoning. The second paragraph quotes Wong to assert that committees wrongly value facts over perspectives, thus justifying "misplaced priorities." Leaving aside their merits, (A), (C), and (E) are not mentioned in the passage and outside knowledge is not allowed. (D) is the opposite of what you are looking for.

46. **(B).** The final paragraph begins by mentioning the main criticism of this proposal but then argues that this

supposed flaw is in fact a virtue and a golden opportunity. (A) and (D) ignore the author's rebuttal to the criticism. (C) is problematic, as the paragraph is concerned with one particular tool — film. (E) ignores both the criticism and rebuttal and merely mentions an out-of-context detail.

47. **(A).** In the last paragraph, the passage states that the main critique of the use of films to teach history is their *rampant inaccuracies and biases*. Then, the author goes on to argue that this can be a positive: *these seeming flaws are actually part of the reason why film* is *an ideal teaching tool*, because teachers can lead discussions of the film's problems and biases. Thus, the author certainly argues that students can benefit from exposure to inaccurate accounts of history (not that students would *always* benefit, but that students *can* benefit when the inaccurate film is accompanied by critical analysis). The other choices cannot be justified. (B) does not have to follow; the passage only lists both as alternatives. (C) is too extreme — preferences do not equate to capacity. (D) is a preference of the state committees, not the students. (E) is backwards — the last paragraph states that students can benefit by such exposure.

48. **III only.** The first statement is wrong, as the passage contrasts *appealing, expressive paintings that are often the most popular museum attractions* with constructivist, more recent art. The second statement is also wrong — the constructivist art, not the expressive paintings, is referred to as "brainy" in the passage. Note that these first two statements may very well be true in real life! But that is not what you are being asked. The question asks which statements can be inferred *from the passage*. The third statement is true — contemporary art is called "cold" in one sentence and "brainy" (or *cerebral*) in another sentence. In the final sentence of the paragraph, you are told that this brainy constructivist art is linked to expressive painting by using gesture as an expressive tool.

49. **I, II, and III.** The author calls *Iroquois: precarious, yet stable and balanced*, which is a good match for the first statement (*precarious* and *unstable* are reasonable synonyms). The second statement is a good match for the final sentence, Iroquois *resonates with an energy born not of the physical quality of the sculpture ... but rather of the* gestural *quality of the forms*. The third statement is a good match for the sentence, *As one contemplates* Iroquois … *the linear forms became considerably more complex than one might presume*. If the forms are more complex than one might think, it follows that some find the forms simpler than they really are.

50. **(E).** (A) is wrong because Rousseau did argue for a social contract, meaning there should be some kind of law. (B) is wrong because Rousseau did not think members of a state should surrender their rights to a single person. (C) is wrong because Rousseau did argue that people could claim property if they needed it, implying the existence of private property in his ideal society. (D) is wrong because Rousseau did not want to dismantle the social contract entirely, but to replace it with his own ideal social contract. (E) is the answer because Rousseau desired a society where *property can be taken ... to the degree necessary for the subsistence of those taking it*.

51. **(B).** (A) is incorrect because you are told *In a state of nature ... the rich would have great difficulty protecting the property that they possess*. This means the rich would still exist. (B) is correct because this is precisely what the sentence cited above says, while (C) is the opposite of what the passage says. (D) is wrong because the passage says the social contract is developed in order to protect the rich, and it must develop out of a pre-law state of nature. (E) is wrong because it comes from the fourth paragraph, which discusses Rousseau's vision for a perfect society, rather than a pre-law society.

52. *To obtain assent to the contract, the rich focus their rhetoric on a seeming equality of obligation by creating rules that apply equally to all members of society.* This is the sentence that shows how the wealthy are able to convince the poor to agree to the same social contract that will allow them to be defrauded. Be careful not to go looking in the passage for specific language mentioned in the question (*hoarding* or *systematized*) on select-the-sentence questions like these. More often than not, that will lead you to the wrong sentence.

53. **(A).** Choice (A) is correct because the passage says that people should only take something if they need it (i.e., for

survival). (B) is wrong because it is the exact opposite of what the passage says, in that people should *not* simply enrich themselves with property. (C) is wrong because though a house could be protection, that's not what *subsistence* means. (D) is wrong because it isn't specific enough. (E) is wrong because it makes no sense at all.

54. **(A).** Choice (A) is correct because Lackey believed that soldiers were not terrorists and Merari believed that state actors could not be terrorists. (B) is wrong because Merari never gave his opinion on the issue of civilian versus non-civilian attacks. (C) is wrong because while the passage argues that it is difficult to define the word *terrorism*, neither of the authors in question explicitly says it. (D) is wrong because neither author weighs in on that subject. (E) is wrong because Merari narrows the definition down to violence perpetrated by sub-state insurgent groups. An elite branch of a country's military does not match this definition.

55. **(E).** Choice (A) is wrong because it is the exact opposite of how *sub-state* is used in the passage. (B) is wrong because sub-state insurgent groups might be capable of terrorism, but they are not necessarily terrorist. (C) is wrong because it would be redundant when used in the sentence, and it has nothing to do with *states* at all. (D) is wrong because anarchists want to bring about a society without government; they do not simply operate outside of a government's jurisdiction. (E) is correct because Merari uses it to draw a contrast with governmental militaries, which he thinks are not capable of terrorist acts.

56. **(B).** Choice (A) is wrong because Lackey discusses the difference between an act that targets innocents and one that targets soldiers. (B) is correct because though the passage does say that the term is prevalent in everyday conversation, this is not a reason that it is difficult to define. Note that just because a statement is true does NOT mean it is relevant to the specific question being asked! (C) is wrong because the third paragraph discusses these issues of circumstance. (D) is wrong because Merari discusses whether state actors can commit terrorist acts. (E) is wrong because Merari says that *the broader the term becomes, the less useful it becomes in a dialogue*, implying that this needs to be taken into account when trying to define the word, in order to maintain its utility.

57. **III only.** The first statement is wrong because, while Baldwin takes issue with the average 1930's Hollywood movie for failing to represent anything other than the dominant culture, that doesn't mean he would find an individual film focused on African American culture any better. It could be bad for other reasons. The second statement is wrong because even though a film that focused only on African American issues could be just as limited as one that failed to focus on such issues, there is a significant difference: one would be reinforcing the dominant culture, while the other would be presenting an alternative culture. Baldwin would thus be unlikely to find them equally problematic. The third statement is correct because the predominant culture in the 1930's was white, so the film in question would be less likely to reinforce that culture, given that it would feature only African American issues and actors.

58. **(D).** Choice (A) is wrong because the quotation has to do with children and their ability to relate to a given work of art, rather than the culture of power. (B) is wrong because the quote does not discuss white culture specifically. (C) is wrong because the passage never says that America was hungry for a writer like Baldwin — make sure not to insert your own ideas or outside knowledge. (D) is correct because even though the quote doesn't explicitly mention black children, the point is that children in general can't relate to a dilemma that doesn't relate to them. Because the passage tells you that Hollywood films of the era dealt with *white issues and employed white actors*, you know that they would not have related to black children, who would have thus been alienated from mainstream culture. (E) is wrong because the passage doesn't say that children couldn't derive *any* educational value from films they can't relate to, only that they would feel alienated because their lives would not be represented in the films.

59. **(D).** Choice (A) is wrong because *assiduous* means "diligent," which is irrelevant here. (B) is wrong because the people studying art do not have to be artistic. (C) is totally off-base. (D) is correct because *erudite* means "knowledgeable," and knowledge would allow someone viewing the frieze to recognize its unique qualities. (E) is wrong because this is a question of knowledge, not applying technical or scientific skills.

60. **(C).** Choice (A) is wrong because many of its characteristics have in fact been worked out. (B) is wrong because the passage is not primarily about artistic interpretation. (C) is correct because *sui generis* means "unique," and the frieze is said to be unique in two ways. (D) is incorrect because only one paragraph discusses the existence of mortals in the frieze, and because, of course, this title is much too broad. (E) is wrong because none of the characters are ever said to be "continuous" (which doesn't actually mean anything).

61. **(D).** Choice (A) is incorrect because the first sentence tells you that the Parthenon was constructed in the High Classical Period. (B) is incorrect because the same sentence says that the Parthenon was regarded as a great architectural and artistic product. (C) is incorrect because the frieze is said to have come from the temple-chamber of the Parthenon. (D) is correct because the passage only says that it is difficult to reconstruct the frieze, not the Parthenon overall. (E) is incorrect because the frieze was part of the Parthenon and it is said to be unique.

62. **(E).** Choice (A) is wrong because you have been told that many Greek works of art featured scenes from myths and legends. (B) is wrong because that kind of frieze exists in the Parthenon, so it would not be unique. (C) is wrong for the same reason as (A). (D) is wrong because you are told that scientists regularly make an inventory of figure-types and characters on a given incomplete frieze to try to work out what went in the missing spaces. (E) is correct because the passage says that the frieze featured the first mortals *in the history of Greek art*. Thus, no *murals* from that time have done this (according to the passage).

63. **(C).** Choice (A) is wrong because the passage only discusses the ways in which Sandra Cisneros's work puts the burden on the reader. (B) is wrong because the passage only discusses a single one of Isabel Allende's books. (C) is correct because the second paragraph says that Sandra Cisneros's short story collection does have interrelated stories, but they do not use the same characters or setting in each story. (D) is wrong because intuition is not actually a part of Sandra Cisneros's writing. (E) is wrong because the passage does not compare short story collections with interrelated stories to short story collections without interrelated stories.

64. **(E).** Choice (A) is wrong because while tone is mentioned in relation to Sandra Cisneros's work, it is not mentioned in relation to Isabel Allende's work. (B) is wrong because while time is mentioned in relation to Sandra Cisneros's work, it is not mentioned in relation to Isabel Allende's work. (C) is wrong because while similarities in characters is mentioned in relation to Isabel Allende's work, it is directly said not to exist in Sandra Cisneros's work. (D) is wrong because while setting is mentioned in relation to Isabel Allende's work, it is directly said not to exist in Sandra Cisneros's work. (E) is correct because character and setting are mentioned in relation to Isabel Allende's work, while only mood, circumstance, time, tone, and imagery are mentioned in relation to Sandra Cisneros' work.

65. **I and II only.** Statement I is correct because Rothbard only says that there were *better [economists] than Smith*. That means that at least two were better than Smith, but Smith could still have been the third best (you certainly don't know that Rothbard actually thinks that Smith comes in third place, but the question asks for statements that simply do not contradict Rothbard's remarks). Statement II is correct because Rothbard says that Smith's book "eclipsed public knowledge" of better economists, meaning it had influences that it shouldn't have. The third statement is not correct because of the first line of the third paragraph, *Even Smith's critics do not deny the book's immense influence*.

66. **(C).** The "Invisible Hand" is described as the idea that *individuals pursuing their own self-interest could unintentionally create a more just society*. (A) is wrong because no one is acting all that selfishly here, nor is the final result particularly just — this would be a better choice if the moviegoers' actions somehow caused a benefit to people other than themselves. (B) is wrong because it has little to do with selfishness or justice, but with another aspect of Smith's writings. (C) is correct because this system promotes selfishness (each student tries to sign up as quickly as possible for each class, without thinking of others), but those classes are then said to be "equitably distributed." (D) is wrong because raising prices isn't inherently selfish, no one appears to be competing, and the end

result is not particularly just. (E) is wrong for the same reasons as (D).

67. **(D).** The passage clearly indicates that Chanel and Vionnet are examples illustrating a theory that personal virtues are "irrelevant" to material success. (A) is a distortion, as "irrelevant" indicates no relationship, not an inverse one. (B) confuses an example given in the passage with the purpose of the passage, and creates a hierarchy that the passage did not. (C) again confuses example and purpose; furthermore, the passage does not focus on aesthetics. (E) is an irrelevant comparison and also confuses example and purpose.

68. **(B).** When a question uses the phrase "in order to" the correct answer will address the author's purpose for inserting the detail rather than the literal meaning. (A) is at best unknown, as her company was in operation before the war. (B) is correct because the information supports the author's theory that personal virtue is irrelevant to material success. (C) comes from the wrong paragraph, as this information about Vionnet does not "highlight" anything about Chanel. (D) is out of scope and not supported by anything in the passage. (E) is a distortion; the author mentions that Vionnet could be so thought of, but his point is merely about her personal integrity and does not address said credentials.

69. **(D).** In the first paragraph, the passage states that seals *who sleep on land at night but spend most of their days in coastal waters* are analogous. None of the other choices accurately follows the pattern of this analogy.

70. **(C).** The author contrasts her to trained scientists and, having given criticisms of the theory credence, sides with them. (A) is nonsensical — as a "proponent," she is not objective by definition. To an extent, (B) reverses the author's point of view; it is irrelevant because the passage does not address this issue. (D) is exactly backwards. (E) brings up an issue that is not mentioned and does not have to follow from the text.

71. **(D).** The author states that the Tokugawa period in Japan was a model for patterns of organization, and writes, *as psychologists, social historians, and Biblical prophets have all observed, in varying ways, humans inevitably fail to anticipate unintended consequences*. This certainly qualifies as a "common failing." (A) is incorrect, as Iceland is only mentioned briefly as an analogy. (B) is wrong both because the author does not express a point of view and because the issue is the result rather than the imposition of a fixed order. (C) addresses a very minor detail, not the purpose. (E) is a comparison that the passage does not address and therefore cannot be inferred.

72. **(C).** The author states that the inversion of the financial and social rankings led to the decline of Tokugawa society. (A) and (B) are both incorrect because the passage states that social mobility was prohibited and that facet of the society became problematic. (D) does not necessarily support that point—for one, no reason for any such revolt can be assumed. (E) is also incorrect; while the author does mention foreign pressure as leading to the collapse of the government, the question concerns the decline of the society, which began long before Admiral Perry's arrival.

73. **(C).** The author states that unifying Germany under Prussian rather than Austrian rule made war more likely. (A) is a result of the treaty but the author does not imply that it was negative. (B) is based on information from the first paragraph and is irrelevant to this question. (D), while true, is unmentioned in the passage and thus not correct. (E) is wrong because it is a distortion to state that the author thought unifying Germany itself was the problem.

74. **(A).** The passage mentions the military brilliance of Prinz Eugen of Savoy, so (A) must be true and is correct. (B) and (E) are both incorrect because the author does not create hierarchies of importance in either case, thus no particular ranking must be true. (C) and (D) are incorrect because the passage does not indicate who won and who lost.

75. **(B).** The author discusses two ideas, but subtly dismisses the objectivity of the first's adherents in the last sentence of the first paragraph. Phrases such as *it is no wonder* indicate an implicit sympathy for the second

suggestion. (A) distorts the point, as "theories" are not the same as ideas, the two ideas are not exactly "opposing," and the issue is not a "process." (C) is incorrect — the last sentence of the passage undermines "definitively" and "programs" are not the same as ideas. (D) is wrong because *dearth* means "lack," and Lewis's work is certainly very popular. (E) is incorrect because, in addition to missing the point, there is no critique, only a very brief description.

76. **(D).** The passage only notes that Lewis's topics include both baseball and finance; the parallels mentioned are between the protagonists, not the professions. (A) is incorrect; "gaming the system" is idiomatic. (B) is incorrect; the fourth sentence of the second paragraph describes Lewis's protagonists. (C) is incorrect; movie making, used as evidence, is part of popular culture. (E) is incorrect; the introductory discussion of Lewis's education can fairly be called a mention of formative influences.

77. **III only.** Statement I is wrong because a person driving his own car isn't a public service (the skateboarder is also a pretty transparent play on the idea of a "rider"). II is wrong because the free rider problem concerns people enjoying benefits without paying for them, not people paying different amounts for the same service. III is correct because in this instance, the action of many free riders leads to a systemic problem.

78. **(C).** Choice (A) is wrong because nowhere in the passage is it stated that free riders cannot be blamed. (B) is wrong because nowhere in the passage is it stated that free rider problems are not worth worrying about. (C) is correct because the first sentence of the last paragraph says that *in some cases, the free rider problem is viewed as a necessary cost of government*, implying that in other cases, it is not. (D) is incorrect because national defense is cited as an example of the inevitability of free rider problems, not as proof that they need to be stamped out as quickly as possible. (E) is incorrect because the passage does not discuss the morality of free riders.

79. **I and III only.** Regarding the first statement, the passage claims *modern humans are known to have diverged hundreds of thousands of years before modern humans left Africa*. To say that they diverged is to say that two species share a common ancestry to that point. Regarding the second statement, whether modern humans and Neanderthals interbred is a matter of controversy (*The team's conclusions were answered with skepticism on a number of fronts*) and thus this answer cannot be definitely true. Finally, although the passage does not endorse the claim that modern humans and Neanderthals interbred between 60,000 and 100,000 years ago, it treats as given the claim that the two species lived near one another during that interval (*for roughly forty thousand years of that window Neanderthals and modern humans lived near one another in the Middle East*).

80. **III only.** The passage says that *Paleontologists and archaeologists charged that the conclusion was unsupported by archaeological evidence*, so they appeal to archaeological evidence to criticize Paabo's conclusions. Statement I is wrong because there is no suggestion that the contamination of Neanderthal DNA with modern human DNA was deliberate, nor even that it was done by Paabo, nor is any other reason offered to doubt his integrity. Statement II is wrong because there is no suggestion that they ignore DNA evidence, even if they are not as persuaded by it as Paabo and his team.

81. **(D).** The passage claims that the reflecting surface must be far enough away so that the sound of the echo is distinct from the original sound, but not so far away that the sound is completely dissipated. You can use that information to eliminate (A) and (B). The passage also claims that multiple reflecting surfaces are apt to produce a reverberation rather than an echo, so you can cross off (C). The anechoic chamber is said to prevent echoes with *sound-absorbing fiberglass wedges*, so (E) is also out.

82. **II and III only.** The echo chamber is constructed with *the acoustical properties of a small cathedral* precisely in order to create echoes. Mountains, along with buildings, are offered as an example of the sort of reflecting surface likely to bring about an echo.

83. **I only.** The first statement is justified because in the second paragraph, de Beauvoir is quoted as saying exactly so. The second statement is not justified because at the end of the first paragraph, de Beauvoir indicates that women *are viewed as intrinsically passive and immanent*. This does NOT mean that de Beauvoir believes this (in fact, the second paragraph gives good evidence that she believes precisely the reverse). The third statement is not justified because, although de Beauvoir views both self-constraint and imposed constraint (*oppression*) as negative, she does not indicate which type she considers to be worse.

84. **II only.** In the first paragraph, the passage states that immanence is considered "a degradation" before going back and forth between immanence as a freely chosen moral fault and immanence as the result of oppression. Thus, immanence is not always characterized as either.

85. **(D).** The first sentence of the passage tells you that existentialist ethics were a major influence on de Beauvoir's philosophy. Much of the rest of the passage is devoted to explaining that philosophy. (A) is too broad, as the passage doesn't explain all of existential ethics, of which transcendence is just one concept. Similarly, (B) is too broad as the passage does not discuss all of feminist theory, just de Beauvoir's. (C) is wrong — the passage is not a diatribe (the word *diatribe* is almost certainly inappropriate to describe any GRE passage). Choice (E) is again too broad and off the point.

86. **(A).** The author's primary purpose in writing this passage is to explain the distinction between science and non-science via historical examples, such as those of Aristotle and Galileo. The author begins by positing the question of differentiating the two, and then goes on to use historical examples to explain why each does or does not meet the qualification for modern science. In (B), the author never criticizes the Ancient Greek philosophers, just labels their method unscientific. In (C), nowhere does the author claim that it is important to follow Galileo's scientific method, just that this is now the modern definition of science. Regarding (D), the author never makes mention of the historical definition of science. In (E), the author never argues that the findings of Galileo are more important than those of Aristotle. Further, the author only discusses one finding of each philosopher/scientist, so this answer is out of scope.

87. **(D).** Galileo's method *forced one to first form a hypothesis, then design an experiment to confirm or deny this hypothesis, and then accept or discard the hypothesis based on one's findings*. Aristotle's hypothesis was that solids were the least expanded form of matter, and the experiment he designed to prove this was to show that solids sank within liquids of the same type. As this did not hold true for water, under the Galilean method Aristotle would be forced to discard his hypothesis based on the results of his experiment. Regarding (A), the passage does not contain any information about the shape of solid water or of other solid forms of matter, only about Aristotle's conjectures on the shape of solid water. Regarding (B), the passage claims the opposite of this fact: that an object with larger mass will not fall to the ground more quickly than an object with lesser mass. As for (C), the passage states that the ancient Greeks were more philosophers than scientists, but never prohibits one from being both. As for (E), the passage states that Galileo used his scientific method to disprove many commonly held misconceptions about the rules of physics, but never states that you cannot do this in the absence of his method.

88. **(C).** The passage describes the role of Alcott's women in their society and briefly wonders about the author's motivation. (A) concerns only the last sentence of the passage. (B) and (D) ignore that the passage discusses a novel rather than reality. (D) is also too broad and somewhat nonsensical since the passage concerns the past. (E) misses the author's tone — there is no indictment, merely a discussion.

89. **(E).** In the second paragraph, the passage dismisses Meg's *taking part* by saying it *meant no more than* talking to her husband, *remaining home*, and allowing him to be a mediator between her and the world. (A) misses the point of the passage and is almost backwards. (B) and (C) are misplaced details about the alternative to *taking part*. (D) distorts the second paragraph, which said her husband was her mediator instead of her *taking part*.

90. **I and III only.** The first statement is justified, as the first sentence states that the man of the house was at the warfront. The third statement must follow, because the passage states that men went out in public to conduct activities and returned home at night. The second statement, while quite possibly true, is not mentioned in the passage. The passage only states that Alcott glorified this condition in her novel, not that the entire society did so. Do not bring in outside information or make assumptions!

91. **(E).** A *falsifiable idea* is "one that can be shown to be false." The statement *no human being lives forever* can only be shown to be false if one observes a human being that lives forever. However, this would be impossible (because of the word *forever*), and thus the idea is not falsifiable. In addition, answer choices (A) through (D) are incorrect. The statement *all birds are black* is falsifiable by identifying a single bird that is not black. The statement *Earth is the only planet in the universe with intelligent life* can be proven false by finding intelligent life on any planet in the universe except Earth. The statement *It rains on Mars everyday* can be proven false by observing Mars on a single non-rainy day. The statement *The sun will explode in 100,000 years* can be proven false by waiting more than 100,000 years and verifying that the sun has not exploded. Note that choices (D) and (E) seem somewhat similar — however impractical it is to wait 100,000 years to falsify something, there is still a big difference between "100,000 years" and "forever."

92. **(B).** The author states in the last paragraph that a theory that is unable to be proven true is very unlikely to be formed. Therefore, it appears that he/she believes that "confirmability" lacks a practical application. The author states, *it is understandable that Popper does not devote that much time to the criterion of 'confirmability*,' inferring that confirmability is less important that falsifiability, not more. You can eliminate (A). Regarding (C), the author states that a theory that is unable to be proven true is unlikely to be formed, therefore it is unlikely he/she believes that confirmability applies to a broad range of theories. As for (D), in the first sentence of the last paragraph, the author states that confirmability follows the same logic as falsifiability ("By that logic,…") and thus it appears he/she believes that comfirmability is reasonable. As for (E), the author states, *it is understandable that Popper does not devote that much time to the criterion of 'confirmability*'. Thus, the author is unlikely to agree that Popper should have developed the idea of confirmability.

93. **(C).** The second paragraph focuses on the significance of the two definitions of 'political', as (C) states. (A) is incorrect because no alternative is offered in the second paragraph. (B) is incorrect because there is no "revision" — this choice might describe the third paragraph. Similarly, in (D), there is no "exception." (E) is closer to the point of the first paragraph.

94. **(B).** The author mentions the play as an example, or "illustration," of when speech is political, which is the aspect discussed in that paragraph. (A) is incorrect, as it is used as an example, not counterpoint. (C) is wrong because the passage does not advocate a position. (D) and (E) miss the point of the example, which is neither about universality nor a fallacy.

95. **(E).** The passage states that the hegemonic model *believed governments to operate exclusively through law and the threat and enforcement of concrete punishment, such as imprisonment, monetary penalties, etc… which legitimizes law and supports the exercise of power.* (A), (B), and (C) all exemplify a using the rule of law to exact concrete punishments. (D) does not demonstrate a clear punishment, but does represent a clear exercise of power (the ability to censor an exhibit). Only (E) does not represent the use of law and punishment, as a coup would necessarily not be legal.

96. **(A).** The answer to this type of question is always explicit in the passage. In the third paragraph, the passage cites Hanna Pitkin: *public-spirited conversation happens when citizens speak in terms of 'justice.'* None of the other choices is mentioned in this section of the passage.

97. *Such a definition is not precisely wrong, but rather is outdated and falls short…* (second sentence of first paragraph). This is the only one of the four sentences in paragraphs 1 and 2 that includes an explicit criticism.

98. **(B).** The passage describes two different ideas, explanationism and predictionism, that have both been used to verify or disprove different scientific theories. As for (A), the passage never states that either theory is superior to the other. Although (C) is true, the passage only mentions the two models of the solar system as an example of the workings of explanationism. Therefore, it cannot be the main idea of the passage. Regarding (D), the passage does not describe what is required to posit a physical theory. As for (E), a predictionist and an explanationist will always diverge on how to prove that a scientific theory is true, but they might still agree on whether or not the theory is correct.

99. **(E).** Before citing the example of the Copernicus and Brahe models of the universe, the author states, *it could be the case that a theory predicts something and yet does not provide the best explanation of it*. The author goes on to use Copernicus and Brahe as an example, stating that both of their theories have predictive power, but obviously Brahe's does not offer the best explanation for the workings of the solar system. As for (A), the author is actually arguing the opposite: that predictive power alone is never enough to verify a theory. (B) does reveal that some theories have more or less of an ad-hoc quality, but this is not the author's reason for citing this example. The main reason must be related back to explanationism. As for (C), the example showed the opposite — both theories were found to accurately predict future events, and thus they must have both made the same predictions for those future events. Although it is true that the more complicated model failed (D), the author's intent was to show that an incorrect model can still make correct predictions.

100. **I and II only.** The words *for example* at the beginning of the sentence containing the crowd analogy follow a sentence about the Pauli exclusion principle. This principle says that fermions cannot *inhabit the same fundamental state*. Electrons, which are fermions, are likened to members of a crowd in a stadium; the fact that electrons cannot *circle the nuclei of atoms in precisely the same orbits* (just as crowd members cannot sit on top of one another) is a "consequence" of the Pauli exclusion principle. Thus, the first statement is justified. These electrons *must occupy more and more distant locations*; the crowd analogy certainly illustrates that behavior, so the second statement is justified. As for the third, incorrect statement, while you do know that electrons cannot occupy the same orbits as one another and must instead occupy more and more distant locations, you do not know that those orbits are "concentric" or "evenly-spaced."

101. **(B).** The author begins by naming the two classes of subatomic particles, and then divides the remainder of the passage into descriptions of each class and their relation to each other. Regarding (A) and (D), the author explains both of these concepts within the passage, but it is subordinate to the main idea of describing the two types of subatomic particles, and thus is not the primary purpose of the passage. As for (C) and (E), the author's primary purpose in writing is not to provide examples or to argue.

102. **(C).** The author states that fermions, not bosons, are the constituents of ordinary matter. All other answers are mentioned in the second paragraph of the passage.

103. **(D).** The passage states that Cooper pairs of electrons will flow in perfect harmony and with zero resistance through the metal. As an example of the same phenomenon, you are told that a swirl in a cup of superfluid helium will never dissipate. Therefore, it is correct to infer that a current through a superconducting wire will never dissipate. If you were not certain that you could be sure of this, you can still eliminate the other choices through process of elimination. As for (A), the passage states that an even number of fermions (which, according to the first paragraph, *include electrons, protons, and neutrons*) constitute a boson, but not an odd number (1/2 integer times an odd will not give an integer). The last paragraph states that scientists argue for the existence of skyrmions in a medium that might permit them to be formed, implying that they have not yet been discovered, so eliminate (B). In (C), the author

states that two electrons cannot circle a nucleus in the same orbit, but they could spin in different orbits that are the same distance from the nucleus. Finally, in (E), the author gives two examples of fermions becoming bosons at cooled temperatures, but does not say this is the *only* situation in which this can occur.

104. **(A).** The first paragraph states that fermions obey the Pauli principle, according to which no two particles can occupy the same fundamental state. The second paragraph states that bosons tend to bunch together in exactly the same state. This is the opposite of (E), thus (E) is incorrect. (D) is also the opposite of what the passage claims. Bosons have integral spin values and fermions have half-integer spin. Answer (B) is incorrect because the passage does not discuss the total number of particle types for bosons for fermions and answer (C) is incorrect because the passage explicitly states that both fermions and bosons can exist in groups.

105. **(A).** The passage states that the Pauli principle prohibits any two particles from inhabiting the same fundamental state. Further, you know that the Pauli principle should be applied to fermions, *which include electrons, protons, and neutrons*, but not bosons (from the second paragraph). Answer choice (A) discusses electrons, which are fermions, avoiding occupation of identical energy levels, so (A) is relevant and thus the correct answer. As for (B), a charged particle in a magnetic field neither provides the criteria for a fermion nor references inhabitance of the same state. Answer (C) does not specify the type of particle. Answer (D) is about photons, which are stated in the passage as bosons, to which the Pauli principle does not apply. Regarding (E), the passage explicitly states that the Higgs particle is a boson, so the Pauli exclusion principle definitely wouldn't apply here (the beginning of the second paragraph explains this quite clearly).

106. **III only.** You are told that Frey points out that *humans are often intrinsically motivated, and that such motivation explains heroism, craftsmanship, and other drives that do not fit neatly into the model of a narrowly focused gain-seeker*. The first statement is incorrect because the craftsman in question is working for money, rather than for the inherent love of the work. The second statement is incorrect because even though the journalist may appear to act heroically, his motivations are related to his career. The third statement is correct because the economist in question is working without hope of monetary reward.

107. **(D).** (A) is incorrect because the passage never says that *Homo economicus* is a useful way to form mathematical models. (B) is incorrect because the passage never says *Homo economicus* is a theoretically useless construction, only that it is a problematic one. (C) is wrong because there's no reason to believe that the people who criticize the theory *don't fully understand its function*. (D) is correct because the second paragraph of the passage describes numerous problems with *Homo economicus*, all of which center around simplifying people's motivations and assuming they understand more than they actually do. (E) is incorrect because the passage never says that *Homo economicus* fell out of favor. You don't know that the handful of critics cited is representative of "most economists."

108. **I and III only.** Statement I is correct because of Veblen and Keynes, who allege that *homo economicus assumes far too great an understanding of macroeconomics on the part of humans*. II is incorrect (and too extreme) because, although Tversky says that people are *unconcerned by small chances of large losses, but quite risk-averse regarding small losses*, he does not imply that there is a broader pattern. Statement III is correct because of Amartya Sen, who says that people *can and do commit to courses of action out of morality, cultural expectations, and so forth*.

109. **(D).** The gist of this sentence is that while scientists condemn certain practices as flawed, the methods they themselves use are subject to many of the same flaws, according to Feyerabend. You are thus looking for a word, synonymous with *repudiate*, that means to "condemn, censure," or "denounce." *Decry* is the right word.

110. **(A).** According to the passage, Feyerabend wants to demonstrate that *historic instances of scientific progress were themselves marked by these flaws, and thus should not be seen as flaws at all*. To this end, he describes a

situation that any scientist would agree is an example of progress, and shows how it made use of practices that are now condemned by scientists, including "begging the question," the practice of using a conclusion as evidence for that same conclusion (a little bit like using a vocabulary word in the definition for that same word), and using "propaganda." Feyerabend basically implies that scientists have a choice between throwing this out as an example of good science or accepting these practices as part of good science. Since scientists are unlikely to say that the introduction of heliocentrism was a bad thing, they will be forced to *revise their account of what is and is not acceptable scientific practice*. Choice (B) is wrong because the point of the case study is that Galileo is a *good* example of science. As for choice (C), "subjectivity" is called "seriously flawed" in the first paragraph. Choice (D) is nonsensical — it basically says tautological reasoning is acceptable only when it's being tautological. It's certainly not why Feyerabend makes use of a case study. (E) misses the point completely — Feyerabend is using an example from history to defend certain ways of doing science.

111. **(C).** The first sentence of this paragraph defines Boal's work as a response to a culture of apathy. (A) is incorrect — the paragraph describes a response, not an elaboration. (B) is incorrect because it is not until the last paragraph that the author provides a rationale for the two theatres. (D) is beyond the scope of the passage. (E) is incorrect because there is no evaluation and the styles are not contrasting.

112. **(D).** This is essentially a vocabulary question. "Power" is one meaning of *agency,* and this is the only meaning that makes sense in the context of creating *ways to free themselves*. (A) and (B) are other meanings of *agency* that do not make sense in context. (C) might be related to *agent* but has no relationship to *agency* or the passage here. (E) is incorrect as it implies domination over others.

113. **(E).** The last paragraph defines a "spect-actor" as *someone who simultaneously witnesses and creates theater*. In the second paragraph, the passage states that at image events *everyone is at once theater-maker and witness*. (A) is incorrect, as theater is not mentioned. In (B), Boal specifically says that catharsis keeps people passive (also, the audience member is not acting, which is crucial to being a "spect-actor"). (C) is incorrect and, to an extent, backwards — the passage said that Boal found that position analogous to that of a passive audience. (D) is too broad, given the first paragraph about traditional theater.

114. **(A).** At the end of the first paragraph, the author paraphrases Boal: *theater etiquette creates a kind of culture of apathy where individuals do not act communally… and remain distanced from art*. (C) and (E) can be eliminated. (B) and (D) are wrong because Boal states that traditional theater discourages political action by providing catharsis. (A) is correct because Boal states that actors *do* go into the audience, so they are not prevented from doing so.

115. **(D).** This choice is a characteristic of an Image workshop, not a Forum workshop. In the second paragraph, the passage states that Forum workshops begin with a narrative skit (A) then the facilitator — or mediator (E) — encourages spectators to assume the role of the protagonist (B). Choice (C) is justified as the paragraph states that performances do not always arrive at a satisfactory solution.

116. **II only.** In the third paragraph, *the natural rotation of a galaxy*, *surrounding supernovae*, and *density waves* are listed as examples of the outside influence *evidently required for a theoretically unstable cloud to initiate collapse*. The first statement is a trap — fusion appears in the first paragraph and the author does not suggest that it leads to cloud contraction. (While the passage does suggest that fusion is related to supernovae and that supernovae can contribute to cloud collapse, don't fill in the gaps yourself — the passage simply does not provide enough information for you to infer that fusion is part of a series of events that begins cloud contraction.) The second statement is true (realizing this requires you to match up "explosions of stars" in the answer choice with "supernovae" in the third paragraph). The third statement is a distortion — forcing debris *inward,* not outward, may cause cloud contraction to begin.

117. **(D).** Choice (A) is true in real life, of course, but is not mentioned in the passage. (B) refers to the molecular clouds in the second paragraph, not to our solar system. (C) is reminiscent of this sentence in the third paragraph — *The natural rotation of a galaxy can slowly alter the structure of a cloud* — which does not refer to our solar system. Correct answer (D) reflects that *for a system of planets such as our solar system to form around a star during cloud contraction, the presence of these heavy elements in the cloud is a necessity*. (You are also told that these heavy elements make up the Earth and even human bodies — so, they sound pretty necessary!) Choice (E) gets the story backwards — heavy elements were needed to create the solar system, not vice-versa.

118. **(C).** The *outward thermal pressure of the constituent gases* (of the molecular cloud) is what is *overcome* in the process of the collapse of the cloud. This is something that *inhibits,* or holds back, cloud collapse.

119. **III only.** The Norton-Polk-Mathis House uses typical Renaissance ideals as well as materials *prevalent in the Italian Renaissance*. However, the passage certainly does not say that the house itself was built in the Renaissance. (For one, it is in San Antonio!) The first statement is not true. According to the passage, the primary purpose of the building is "to impress," so its purpose is not primarily utilitarian. The second statement is also not true. You are told that the house *radiates an air of strength*, especially when *juxtaposed with the other, seemingly fragile brick and wood homes of the neighborhood*, so you definitely can infer that the house "appears stronger than other nearby homes." The phrase "radiates an air" and the word *seemingly* in the passage indicate that you are talking about *appearances* of strength, so the word *appears* in the third statement is a very good match (you would *not* be justified in inferring that the other neighborhood homes are *actually* weak — they just look that way).

120. **(C).** The most important idea in the passage is that a small group of dolphins has been found to form social networks, which is a first among non-human animals. One indication of this is that both the first and last sentences in the passage highlight this fact. Answer choice (A) is stated in the passage, but is not the main idea; it explains the behavior of the animals that form social networks. (B) is never stated in the passage, although this study did span a relatively long time. (D) is a much bigger claim than the passage ever suggests; you are never given a motivation for this study. Finally, choice (E) goes against the main idea of the passage as the passage discusses how dolphins can form social networks.

121. **I only.** The passage states that other animals aside from the sponger dolphins often form groups based on circumstances such as genetics or food sources. As the dolphins are the first to be categorized as "social networks," it must be true that groups formed under the previous circumstances would not qualify as social networks. For the second statement, the passage does not state that all spongers of Shark Bay form social networks, only the females. As for the third statement, the passage does not comment on the location of spongers; it only mentions the spongers of Shark Bay. Thus, you cannot infer that this is true.

122. **(E).** You are told that this belief is an "old canard." A *canard* is a "rumor," or "a false or baseless story." An "old canard" is one that has been passed around for awhile — very much analogous to an "urban legend." You are then told that the blind people in the study have "cerebral superpowers" and that the discovery that blind people can hear better than sighted people is "a stunning example of the brain's plasticity." Thus, (E) is correct.

123. **II only.** You are told that Aristotle *posited a holistic, non-corporeal mind* and that he would have found "shocking" the idea that "the mind is physically located in the brain." Thus, the first statement is not correct, since Aristotle believed in a "non-corporeal" (not part of the body) mind. Since Aristotle believed in a "holistic" mind, he did NOT think that *the mind exists in parts or modules*. Thus, the second statement is correct. As for the third statement, you have no way to know what Aristotle might have thought about blind people's hearing.

124. **III only.** The last sentence tells you that the deportation of German-speaking citizens by the Allied powers is excised from history books because history is written by the victors — this makes the first statement look attractive,

but the statement says *during World War II*. The passage is clear that the deportations *took place almost two years after the conclusion of the war*. The third answer choice can be inferred from the claim that the United States hoped to keep Eastern and Central European nations away from Soviet influence, meaning that the United States and the Soviet Union were not fully aligned in their views. After you are told that *the Allied powers, too, had something to gain*, the powers listed are the Soviet Union, Britain, and the United States. Poland, however, is listed among countries *before* the "too," which means you cannot infer that it was an Allied power. (Make sure not to bring outside information into these types of questions. This is *not* a question about what you know about World War II — it's a question about whether certain things are actually written down in the passage.)

125. **I, II, and III.** The second half of the first paragraph lists the reasons why the Soviet Union, Britain, and the United States, respectively, had something to gain from the deportations. Each of the answer choices above undermines one of these reasons. Note the "if true" in the question stem. You are being asked to momentarily take each answer choice as a fact, and then determine whether that fact would weaken an argument from the passage. "If true" questions can sometimes ask you imagine that wacky or unlikely things are true — don't let that distract you from the task at hand.

126. **(C).** The passage describes the theory of quantum mechanics, how it models particles as probabilistic waves, and how it has been confirmed over the past 70 years. Answer choice (C) correctly incorporates all of these ideas into the main idea of the passage. Answer choice (A) is incorrect, as it does not address the theory of quantum mechanics, which is the overarching idea of the passage. (B) and (D) are both large claims that are out of the scope of the passage. Finally, although it is obvious from the passage that tests can and have been designed to test quantum mechanics, (E) does not express the main idea of the passage.

127. **(A).** The passage states that Einstein was deeply troubled by the theory of quantum mechanics, and thus his quote must not be in support of the theory. Further, his reference to dice is to say that he did not believe the universe should be controlled by probability, but should be set. Therefore, (A) best expresses his motivation; he argues with the theory of quantum mechanics stating that particles should not be probabilistic in nature. (B) has no bearing on quantum mechanics, and thus cannot express Einstein's motivation. (C) is out of scope because of the mention of causality. (D) both mentions Einstein's religious beliefs, which are out of scope, and confuses Einstein's being troubled by quantum mechanics with his lack of understanding. (E) makes a supposition you could not possibly know from the passage about Einstein's religious beliefs.

128. **I and III only.** The first statement is a proper inference because passage states that *A particle trapped in a closed box has some finite probability of being at any location within the box*, meaning the exact location of the particle cannot be known. You know from the next sentence that opening the box will locate the particle, and thus you can deduce that the location cannot be known for certain without observing the particle. The second statement is incorrect, as the passage actually states the opposite: that measurements of position can be taken. The third choice is a correct inference because the passage states that quantum mechanics describes particles as waves and that quantum mechanics has been verified as true.

129. **I and III only.** According to the second paragraph, not knowing that they are seeing a theater piece allows viewers to *avoid the etiquette of theatergoing* and *engage with the action and concepts of an unfolding drama as if these actions and concepts were real*. This is a good match with the first statement. The third paragraph refers to "scripted characters" in invisible theater, so you cannot infer that *invisible theater is best described as improvised*. (It may be possible in real life for scripted *characters* to engage in improvised acting, but you cannot infer this from the passage.) Another reference to *dialogue ... set up by invisible theater performers* weighs against invisible theater being improvisational. The third paragraph begins, *Boal has documented various successful instances of invisible theater in which non-performers ... take unplanned public-minded action in response to the dialogue and events set up by invisible theater performers*. The last sentence of the passage also states that *...the goal of guerilla theater is to get people talking publicly*. Thus, you can infer that *actions taken by the audience once the performance is over* can be one measure of success of a theater piece, and thus the third statement is true.

130. **(A).** In the first sentence, the author calls invisible theater and guerrilla theater *two forms of street theater with similar origins but very different approaches*. This is a good match with (A). (B) is incorrect because the passage doesn't say which form is more effective. (C) is too broad and the evolution is not the focus. (D) fails because the passage states clearly — and describes in great detail — the fact that "invisible theater conceals its performative nature *whereas guerrilla theater flaunts it*." (E) is much too broad, as the passage does not cover all artistic life in public places.

131. **I and II only.** The first statement is true, as the third paragraph goes into great detail about invisible theater's goal of encouraging public-minded talk, and the passage ends with "the goal of guerrilla theater is to get people talking publicly." The second statement is justified because you are told in the first paragraph that *Both forms take place exclusively in public places*. The third choice is only explicitly stated in regard to guerilla theater; invisible theater is only said to involve the audience.

132. **(C).** The professors gave diplomas to the people who were the least able to answer questions as a way to mock the university's decision. (A) does not have to be true because the passage merely discusses the form of the professors' protest. (B) cannot be justified since it doesn't have to be true that the professors believed that result would follow. Similarly, (D) is wrong because it is not certain that the professors considered the legality of their actions. (E) is not justifiable — no information is given about any particular polices the professors decried.

133. **I and III only.** The first statement, if true, would contradict the assumption of invisible theater that removing the boundary between performer and audience encourages involvement. The second statement describes a very likely result, and a goal, of invisible theater, and thus would not "undermine" the principle of invisible theater. The third statement describes a situation where the goal of invisible theater — a lively debate about public issues — was already happening, and invisible theater ruined it! This would definitely undermine the principle of invisible theater.

134. **I, II, and III.** You are told in this passage that *widespread genomic changes would wreak physiological havoc*, such as cancer (statement I). You are also told in the second paragraph that *many organisms have also adapted beneficial mechanisms to induce genetic change*. In short, some genetic changes are bad, but others are beneficial. The third paragraph explains how genetic change is important to immune functioning (statement II), and the last line of the passage tells you "this process is regulated by T cells to prevent harmful mutations," so the third statement is also justified.

135. **(C).** The word *seemingly* indicates that the changes are not really haphazard. "Seemingly haphazard" refers to "programmed genetic mutation." The "this" in *this seemingly haphazard process of programmed genetic mutation* harks back to "genetic recombination," so you can be sure that (C) is a match. Some of the other answers may be true — (D) certainly is — but do not answer the question.

136. **I, II, and III.** Pro-death signaling is given in a list of *cellular mechanisms that stymie genetic changes*. *Stymie* means "hinder." Statement I is true. These cellular mechanisms are called *ubiquitous*, which means "existing everywhere," so certainly they are "very common." Statement II is also true. In the final sentence of the paragraph, you are told, *malfunctions in molecular players that safeguard against mutagenesis, such as the protein p53, have been implicated in diseases such as cancer*. Since the malfunction of p53 may cause cancer, you can infer that p53, when properly functioning, may work against cancer. That means Statement III is also true.

137. **(A).** You are told that T cells need a large repertoire of receptors in order to be able to recognize a wide variety of pathogens. Then: *Relying only on a genetically encoded repertoire would be disadvantageously limiting— analogous to having only a few dozen language phrases with which to respond to the nearly infinite potential combinations of words in a conversation. Instead, the repertoire is generated by a process of genetic*

recombination… In this analogy, the language phrases are the repertoire of receptors; just as a speaker must respond to a nearly infinite body of language combinations, T cells must also have a large repertoire so they can respond to a wide variety of pathogens. You are told that the way this repertoire is increased is through genetic recombination. Note that (B) is out of scope, (C) is the exact opposite of what is being described, (D) is a distortion based on another analogy in the passage (also, an analogy on the GRE would not be "meant to elucidate" *another* analogy!), and (E) refers to the first paragraph, not the analogy in question.

138. **I only.** In the analogy referenced, the "language phrases" are receptors that can respond to various pathogens. The *nearly infinite potential combinations of words* is what a speaker must respond to — the reason a speaker needs a wide repertoire of language. Similarly, the wide variety of pathogens is the reason T cells need such a wide variety of receptors.

139. **II and III only.** You are told that Haberman *focuses the brunt of his criticism on teachers who have been insufficiently trained for the realities of the modern school environment and whose prejudices, lack of deep content knowledge, and excessive focus on order and discipline profoundly limit their effectiveness*. The word *compromise* in the question stem means something like "hurt" or "limit." Note that Haberman thinks that *too much* order and discipline is hurting teachers' effectiveness, so the first choice is the opposite of what you want. The other two choices match up with *lack of deep content knowledge* and *prejudices*, respectively.

140. **(D).** The quotes are present to make the point that the students who are referenced — *non-white, immigrant, or non-English-speaking children* — are NOT exceptions. Rather, they are the class the teacher is poorly prepared to teach. Haberman's point is that the teacher is the problem, not the students. (E) seems to reflect a truth that Haberman is railing against, but has nothing to do with the question ("exceptions" refers to students, not teachers).

141. **I and II only.** You are told that Decadent authors embraced artifice over nature, and that Huysmans's protagonist in an example of Decadent writing surrounds himself with perfume, among other items. You can thus infer that at least one follower of the Decadent movement considered perfume to be an example of artifice, and that he held it to have surpassed, or be superior to, natural entities. But you do not know that Huysmans enjoyed surrounding himself with the perfume—only that his character did so.

142. **I, II, and III.** All three of the features are listed as characteristic of the Naturalist movement. *Vehicle for the scientific method* matches *extending the scientific method*, while *focused on the effects of environment on shaping character* and *elaborated on the way inherited traits influenced human behavior* both match the passage's claim that Naturalism stressed *the influence of environment and heredity upon the individual psyche*. If Decadent authors embraced any of these literary practices, this would bring their work closer in line with Naturalism.

143. **(C).** In the first paragraph, you are told that *one limitation* of the classical method is *the reliance on average measurements: it is impossible to distinguish a uniform population of cells expressing intermediate quantities of a molecule from a population composed of separate low and high expressers*. This is a good match for choice (C). Note that (A) is precisely what you want to measure (not a limitation), and (E) is a distortion — it is *not* preferable to capture only average levels. This goes against the main point of both paragraphs.

144. **I and III only.** According to the passage, flow cytometry and RNA FISH are examples of *single-cell measurement technology*, so the first statement is true. Much of the wording in the second statement — *it is impossible to distinguish a uniform population of cells expressing intermediate quantities of a molecule* — is lifted from a sentence in the first paragraph, talking about classical methods — *not* the newer, single-celled measurement technologies. Finally, you are told that flow cytometry and RNA FISH have *made it possible to capture ... the distribution of the molecule's expression within the population*, which is a good match for the wording in the third statement.

145. **(C).** You are asked for the choice NOT in the passage. You are told that Portugal *once mined Angola for slaves and raw material*, so (A) and (B) are out. The beginning of the passage concerns Portugal taking over Angola over a period beginning in the 16th century and culminating in the 1920's, so kill (D). The passage refers to *Angolan independence in 1975* — since that date is during the 20th century (1900's = 20th century, 2000's = 21st century, etc.), kill (E). You are told that a *civil* war in Angola lasted until 2002, not a war against the Portuguese, so (C) is the answer.

146. **(B).** The *grand stroke of irony* the author refers to is Angola helping Portugal. Why is this ironic? You are told that *the country that once mined Angola for slaves and raw material is now virtually helpless…* (B) is the best match. (A) and (E) are not at all ironic. While (C) and (D) at least present some kind of contrast, they are not the *stroke of irony* to which the author refers.

147. **(E).** You are told that Gage's physical injury affected his personality, that the part of his brain that was damaged is now known to be related to morality, and that Gage *literally lost one (or more) of the modules in his modular brain system*. (E) is a good match. (A) is the opposite of what is being argued. (B) is not indicated by the passage (and really goes against common sense). (C) may be true but is not the reason the author presented the example. (D) relates to the previous paragraph, not to Phineas Gage.

148. **I only.** The author is saying that, if the brain has a director, then that director would need its own director — and, presumably, *that* director would need a director, etc. The expression "begging the question" isn't really about a question; the other two choices are traps.

149. **II only.** A "unitary entity" would NOT be split into parts, or modules. While Aristotle and Descartes believed that the mind survived death, and lived long enough ago that they couldn't have been aware *that certain aspects of personality are known to be controlled by certain areas of the brain*, the question is not only about Aristotle and Descartes — it is about all advocates of a unitary view of the mind (you are told that, historically, that includes "virtually every thinker"). You simply do not have enough information about these thinkers to know whether they think the mind survives death, or whether some of them (more modern thinkers, presumably) are aware of current research into the brain.

150. **(A).** The first paragraph of the passage stats that oxytocin treatments are often tried in isolated cases and the overall effects are without evaluation. The passage then describes a small study that seems promising, but makes no definitive claims. Therefore, it is likely that the author would agree that the effects of oxytocin require further evaluation. Answer choice (B) is incorrect, as the passage states the opposite, that oxytocin is not a "cure-all." Although the author focuses on the effects of oxytocin for those that are not able to interpret social cues, answer choice (C) is incorrect, as the author does not state that the drug would not be useful for those that can already do so. The author specifically addresses (D) in the passage, stating that the hormone oxytocin increases feelings of calm and social bonding. Finally, answer choice (E) is incorrect, as the author never addresses oxytocin as an oral treatment.

151. **(D).** The passage states that, *the experiment showed that the oxytocin had the greatest affect on those who were least able to evaluate emotions properly when given the control*. Thus, you can infer that those with the least ability to naturally infer emotions, e.g., the ones who might need it most, reaped the greatest benefits of the hormone. Additionally, (A) is incorrect, as the passage does not discuss inconclusiveness based on sample size. (B) is incorrect, as it incorrectly pairs the known effects of the hormone in the brain with the results of the student study. (C) is incorrect, as the passage does not address the ability of the students to recognize expressions, just the relative change between the controlled salt water dose and the oxytocin. Finally, (E) is incorrect for a similar reason: the passage does not state that the subtler the expression the more difficult it was for students to identify, just that some expressions that were used were subtler than others.

152. **(A).** The experiment was related to students' ability to recognize emotions from facial expressions, not their ability to tell faces apart. (B) is mentioned in the first sentence of the last paragraph of the passage. (C) is addressed throughout the second passage, first when it is stated that, *either a control dose of salt water* was given and then by comparing awareness after exposure to oxytocin to the controlled salt water dose. (D) is explicitly stated in the second sentence of the second paragraph. Finally, (E) is explicitly addressed in the last sentence of the second paragraph.

153. **II and III only.** You are told that *Anansi originated with the Ashanti people in Ghana*, so the first statement is untrue. However, you are told that in Jamaican folklore, Anansi *outsmarts other animal-god characters*, so those characters must exist. Since Anansi is called "Aunt Nancy" in the United States and is from Ghana, Anansi is known on at least two continents.

154. **(A).** In the first paragraph of the passage, the author describes the discovery of the cosmic microwave background. In the second, he or she explains why the cosmic microwave exists and its implications to science. Therefore, the author writes this passage to describe the discovery and reason for the cosmic microwave background. Answer choice (B) is incorrect, as the author cites one example of an accidental discovery, but does not explain how multiple discoveries can be made accidentally. (C) and (D) are incorrect, as the author does not argue or defend, respectively. Finally, the main theory presented in the passage is the cosmic microwave background, whereas (E) incorrectly makes it seem as though the author's intent is to defend the Big Bang and that the cosmic microwave background is only a subordinate idea.

155. **(C).** The second sentence in the first paragraph states that *just an instant after the Big Bang, all matter in the universe was so energetic, or hot, that it existed as free particles known as quarks*. The paragraph proceeds in sequential order, and thus this event happened soonest after the Big Bang. The events described in (A) and (B) are said to have happened approximately 400,000 years after the Big Bang. Answer choice (D) describes the present state of the cosmic microwave background, 13.6 billion years later. Answer choice (E) is never addressed in the passage, and thus cannot be the correct answer.

156. **(D).** The passage states that Penzias and Wilson accidentally discovered the cosmic microwave background, and did not even understand what they had found until after consulting the Princeton group. Clearly, they did not initially understand the implications of their result. (A) is incorrect, as the passage does not describe the important of the signal for which Penzias and Wilson were originally searching. (B) is incorrect, as the passage never discusses the Princeton instrumentation used for searching for the cosmic microwave background. The capabilities of the telescope used by Penzias and Wilson is never discussed, thus (C) is not supported. Finally, the opposite of (E) is stated in the passage, which says that Penzias and Wilson convinced themselves that their signal was real before approaching the Princeton team.

157. **II and III only.** Sousa argues against mechanical music based on the grounds that it is insincere, and that it will decrease music in the home and music played or sung by amateurs, as well as music instruction in education. The first statement is an example of one of the things Sousa was afraid of — vocal instruction being less a *normal part of education* — and thus does not "contradict." The second statement is an example of the phonograph *increasing* amateur music playing and "domestic music," so this does contradict Sousa's point. Finally, the third statement is an example of recorded music being *more* sincere than live music, so this definitely contradicts Sousa's point.

158. **(D).** Here, *chest* really does mean "part of the human body," not "a trunk or treasury." The statement about the "national" throat and chest comes right after a worry that *music will become the province of machines and professional singers only*. Thus, the "national chest" is a reference to regular people's singing.

159. *The blackbody emits just as much energy per unit time as it absorbs; the electromagnetic spectrum of the emitted energy, however…* (second sentence). The second sentence of the paragraph, after the semicolon, states that the electromagnetic spectrum of the emitted energy of a blackbody is completely determined by temperature and no other properties. Therefore, the only variable that defines the electromagnetic spectrum of a blackbody is temperature, as stated in the second sentence.

160. **I only.** The passage states in the first sentence that an idealized blackbody is an object that reflects zero incident electromagnetic radiation. Therefore, if an object reflects incident electromagnetic radiation, it cannot be an idealized blackbody and the first statement can be properly inferred. The second statement, however, cannot be inferred as the passage states that a possible Doppler shift can cause a fundamental change in the original spectral characteristics of reflected electromagnetic radiation. Finally, for the third statement, the passage states that any object that absorbs all incident electromagnetic radiation is a perfect blackbody. However, you are told that a microscopic "forest" of vertically aligned single-wall carbon nanotubes of varying heights applied to a surface is the *closest* that scientists have come to thus far creating a perfectly dark material, implying that this material is not a perfect blackbody. Therefore, you cannot properly infer that this object will absorb all incident radiation.

161. **(D).** Choice (A) is wrong, since there is no reason that one cannot produce original research by using techniques developed by someone else. (B) may be true, but this is not implied by the use of the word *borrowing*, since all you know is that one technique from evolutionary biology proved helpful to research in linguistics; this is compatible with the two areas having no common features. (C) is wrong, since the fact that methods from outside fields can help make progress in linguistics does not show that such progress cannot be made by other means. (D) is correct: research methods developed in evolutionary biology can lead to results in linguistics. (E) is the opposite of what the passage tells you.

162. *But given that some language is spoken by virtually all human beings, it would be strange if it did not reflect cognitive universals.* While it may be tempting to select the second sentence of the passage, this sentence does not give any support for universalism; it only tells you the grounds on which Chomsky came to a universalist view. On the other hand, the penultimate sentence of the passage does provide support for universalism by noting that it is likely that all languages reflect features innate to human thinking, since virtually all human beings speak some language.

163. **(B).** Choice (A) is wrong, since the passage gives examples of non-empirical ways to support universalism. (B) is correct: Gray's team claims their research casts doubt on Chomsky's theoretical conclusions. (C) may or may not be true, but the passage only points to evidence of features that do *not* co-vary. (D) is wrong: you know that Gray's team found no evidence of the existence of family-invariant rules, but this does not prove that there are no such rules. (E) is wrong: Chomsky argued that universalism is the "best explanation" for the speed at which children learn a language, but that does not mean that no other explanations are possible.

164. **(B).** The passage highlights two key points as they relate to each other: the two competing theories of the universe and Hubble's discovery that the universe is expanding. The correct answer, (B), highlights both of these points and their relation to each other. (A) and (C) highlight only one of these points each. (D) describes Hubble's law, which does not fully capture the main idea. (E) is not an idea presented in the passage, and therefore cannot be the main idea.

165. **(E).** The passage states that, *Milton Humason, a fellow astronomer, helped Hubble to calculate the stars' relative velocities to Earth*, but nowhere in the passage does it say that Hubble deduced the velocity of Earth or the stars' absolute velocities. (A) is mentioned at the beginning of the second paragraph when the author states that, *Using this relation and years of observing*. (B) is mentioned in the first and fourth sentences of the second paragraph. (C) is stated in the second to last sentence of the second paragraph. Finally, (D) is addressed in the second and third sentences of the second paragraph.

166. **I and II only.** The passage states that Hubble's experiment was proof that you do not live in a steady-state universe, therefore it must be true that the steady-state universe theory does not allow for an expanding universe, and the first statement can be inferred properly. The passage also states that the speed at which objects are moving away from each other in space increases with an increasing distance between the objects. From this, you can properly infer that as the distance between two objects decreases, the speed at which they move apart must decrease. Thus, the second statement can be properly inferred. Finally, the third statement incorrectly assumes that once the steady-state universe theory was disproved, the Big Bang was the only theory that remained. The first sentence of the passage states that *most* physicists believed in one of the two theories, leaving the possibility of many more theories that might still agree with Hubble's discovery.

167. **(A).** The argument of the passage dismisses the assumption that the gods determine destiny by pointing out that they act "politically," and their agendas often conflict, so that they could not formulate a "master plan." But if the gods had a common goal, then this objection would no longer apply. On the other hand, if the agendas of the gods coincided with the demands of fate, that wouldn't support the idea that the gods were in charge of fate, so (B) would not support the assumption. If Homer and Hesiod disagreed about the motives and agendas of the gods, as (C) claims, that would not undermine the view that for both authors fate is beyond the gods. Nor would the claim that destiny would be fulfilled regardless of what the gods did — choice (D) — or the idea that the gods and mortals can make their own decisions, so long as these decisions ultimately led to the fulfillment of destiny.

168. **(B).** This is the most common sense of the term *unbiased*, and is the only one that fits in this context. (A) is clearly wrong since you are told that the role of the gods is a motif in the *Histories*. Since Herodotus provides an account of conflicts in the Hellenic world, (C) is wrong. (D) is wrong since you are told that the concept of destiny is part of Herodotus's history, so the actions of people and states by themselves cannot explain the events involved. As for (E), there is no indication that the histories were meant to challenge anyone's sensibilities.

169. **I, II, and III.** The first statement paraphrases the claim that the gods act *within certain boundaries*, while the second statement paraphrases the claim that they do so *to accomplish his or her own agenda*. The third statement is the main point of the passage: that the gods act as agents of destiny which they do not themselves control.

170. **(C).** Chemical blockers scatter, or disperse, light waves. Chemical absorbers use them to promote electrons which then release them as light waves with a longer wavelength as they return to their ground energy state. (A) makes a reference to lightening and darkening light waves, neither of which is mentioned in the passage. (B) and (D) refer to converting light waves to radiation, which is impossible since light waves are already radiation. And absorbers absorb the radiation into their molecular structure, not into the skin as in (E).

171. **(D).** The chromophores absorb light in the 290–320 nm range and use it to promote (or move up) electrons between energy levels. Since light with wavelengths of 300 nm falls in this range, their electrons should move up in energy levels when exposed to it. (C) would be correct if not for the range given: you don't know how chromophores react to light above 320 nm. (B) is the exact opposite of what you are looking for. (A) applies to physical blockers but not to chemical absorbers.

172. *The specific wavelength absorbed by a given chromophore is determined by the discrete quantal amounts of energy that are required to excite electrons between the energy levels or its molecules.* In order to select a chromophore for a particular sunscreen, you would need to know which light waves the sunscreen needs to block and which chromophore would block those waves. This sentence tells you which feature of a chromophore determines which light waves it absorbs. The next sentence in the passage might seem like a good match, but it only tells you how to select a chromophore that would absorb UVB radiation, not UVA radiation.

173. **III only.** The first statement is not true, because you are told *For all we know, the story may have been Christianized in its oral form long before the poet set it into writing*. The second statement is incorrect because you cannot make a leap from *The story of* Sir Gawain and the Green Knight *has its foundation in Arthurian legend* to *Sir Gawain was a knight in King Arthur's court*. The third statement is true, and a good match for the final sentence of the passage, which tells you that you do not know whether the tale is *a pagan interpretation of Christian ideals* or *an externally imposed Christianization of pagan codes of behavior*.

174. **(E).** Choices (C) and (D) are directly contradicted in the passages. Nothing suggests that the religious outlook of the interpreter influences their interpretation of the story, so (A) is also wrong. (B) is too strong: the passage only states that, according to its interpretation of the story, Gawain's motives are not Christian. But this doesn't show that they could not be. (E), on the other hand, follows directly from the claim that while Gawain's actions and words are Christian, his motives are not.

175. **(B).** The theory in the final sentence is that the poet associates Gawain with a pagan symbol and then portrays his "Christian" virtues as superficial in order to criticize the pagan interpretation of Christianity. Choice (B), if true, would show that the poet was, at very least, inconsistent in this message (or, possibly, the part about Gawain being superficial in his virtues is really just about Gawain.) Choice (A) is true and described in the passage, so it would not "undermine" the theory. Choices (C), (D), and (E) do not address the interplay between paganism and Christianity and thus have no bearing on the theory.

176. **I and III only.** Since Astyages reacted to his first dream by altering the marriage arrangements for his daughter (in order to select a less threatening husband), you can infer he believed her husband could be a threat. And since he intentionally selected a Persian rather than a Median, you can infer that he thought a Persian would be less of a threat. However, although you are told that Astyages' dream was *interpreted ominously by the Magi* and that *as a consequence*, he pursued a course of action (indicating that *this* time, he was influenced by the Magi), you don't know that he thought it was "always" best to obey the Magi, nor is it clear that the Magi made "recommendations." Watch out for extreme language.

177. **(A).** One of the two mistakes referred to in the passage was marrying Mandane to Cambyses. But if her son would have deposed Astyages even if he had had a different father, then altering what would have been the normal treatment of her marriage was not a mistake that led to Astyages's downfall.

178. **(B).** The passage makes the case that Bierstadt's work, which is called *optimistic* and *gaudy*, was no longer suited for the prevailing trends in art in American after the war. The optimism that once characterized American preferences is now "tempered" by the "horrors of war." In other words, Americans are beginning to take a more realistic approach to life, in a way that is sadly reflective of the disastrous things that can befall a country. Their attitude, in other words, is one of *somber realism*. Don't get misled into picking *prideful idealism* by the fact that the passage mentions American pride. The passage does not imply that there was anything idealistic about the new American attitudes after the Civil War. (A) may also be tempting but the author never makes any sort of claim as to whether the new American attitudes are misguided.

179. **(E).** A central thesis of the passage is that the same elements that initially made Bierstadt's work popular eventually contributed to its downfall. These elements were, in short, an emphasis on size and quantity rather than emotionality. The quoted phrase is a criticism of his work to this effect, providing a specific example of the opinion of the time. (C) may be tempting, but this phrase may or may not be the opinion of an expert, plus "providing expert testimony" is not the best description for the purpose of the phrase.

180. **(C).** You are told that Bierstadt "developed a fixed style that was most easily recognizable for its size" (A), that he had an "ability to represent the optimistic feeling in America" (B), that he "deliberately appealed to those rich

patrons" (D), and that patrons could purchase a "hyperbolized replica of a Western vista" (E). The increasing attention to "subdued appreciation for the details of American life" is mentioned in the second paragraph as a trend that worked *against* Bierstadt.

Chapter 6
of

Logic-Based Reading Comprehension

In This Chapter…

Logic-Based Reading Comprehension

1. The school board has responded to the new school lunch guidelines by replacing fries with fruit in a standard meal option that used to consist of a hamburger, fries, and milk. However, the guidelines specifically require that vegetables, not fruits, be included in every meal.

   The information above most strongly supports which of the following conclusions?

   (A) Fruit provides just as much health value to students as vegetables.
   (B) Students are more likely to eat fruit than vegetables.
   (C) The school board is not following the new school guidelines.
   (D) The school board is responsible for the health of the student population.
   (E) The new school lunch guidelines are unnecessarily strict.

2. While many people think of the lottery as a harmless way to have fun and possibly win some money, buying lottery tickets is a form of gambling. Therefore, public officials shouldn't buy lottery tickets.

   Which of the following, if true, would most strengthen the conclusion?

   (A) Individuals who play the lottery are less likely to win a big payout than they are to be killed in a car crash.
   (B) Some public officials are guilty of much more serious offenses than gambling.
   (C) Public officials shouldn't gamble.
   (D) Many public officials are easily tempted to violate rules governing their positions.
   (E) Most lottery winners are not made as happy by their winnings as they expected.

3. Some say that Saddlebrook College provides the best value in our state. Yet, students at our state's Tunbridge College pay less, enjoy newer buildings and smaller class sizes, and earn larger incomes after graduation.

   The information above, if true, most strongly supports which of the following judgments?

   (A) Tunbridge College provides the best value in our state.
   (B) Tunbridge College has more stringent entrance requirements than Saddlebook College, and thus attracts students of a higher caliber.
   (C) It is not true that Saddlebrook College provides the best value in our state.
   (D) Student income after graduation is a valid means of judging the value of a college education.
   (E) Students at Tunbridge College report higher rates of satisfaction than students at Saddlebrook College.

4. Studies have long shown that people who drive red cars receive more speeding tickets from the police than do those who drive cars of other colors. Researchers have thus concluded that the color of a car influences its driver's behavior.

   The argument depends upon which one of the following assumptions?

   (A) Drivers of black cars receive the second-most speeding tickets.
   (B) Red cars do not attract more attention from the police than do cars of other colors.

(C) Police officers do not drive red cars.
(D) Red cars do not receive any more parking tickets, on average, than do cars of other colors.
(E) Drivers of red cars who are ticketed for speeding are able to appeal their tickets more often than drivers of other color cars.

5. The headmaster at Leawood Day School noticed that scores on math tests were lower this year than in previous years. This year, all students took math courses during the first period of the school day; in years past, they had taken math during the final period of the day. Reasoning that the students perform better on math tests when they are fully awake, the headmaster concluded that test scores would be higher if math classes were moved to the end of the day.

The headmaster's reasoning depends upon which of the following assumptions?

(A) It would be possible to reconfigure the school's schedule to accommodate having math classes in the afternoon.
(B) Several schools similar to Leawood Day School hold math classes in the afternoon.
(C) The quality of the teaching has little bearing on test scores.
(D) This year the math department started using new, unfamiliar curricular materials.
(E) Students are more likely to be fully awake during the final period of the day than they are during the first period of the day.

6. When financial services companies undertake layoffs, the most valuable employees, whom the companies want to retain, often leave the company voluntarily for new jobs not long after the layoffs. ITEL Financial Services, however, has not lost any of its most valuable employees despite multiple layoffs in the past decade. ITEL credits its success in keeping valuable employees to its high tolerance for risk when employees make investment decisions.

Which of the following, if true, mostly strongly supports ITEL's reasoning as to why the company retains its most valuable employees?

(A) People in the financial services industry are often motivated to change jobs by a significant salary increase.
(B) The way in which layoffs are handled can have a significant upward or downward impact on the morale and company loyalty of the employees who remain.
(C) Layoffs often occur during industry downturns, and it is generally much more difficult to find a new job when the industry is struggling.
(D) It is difficult to develop an objective method or definition for determining who should be considered a very valuable employee.
(E) A risk-intolerant environment impedes the entrepreneurial drive that financial services industry employees feel is a requirement to be successful.

7. <u>Exterminator</u>: Using poisoned food is the most effective tactic for combating a mouse infestation. The mouse will carry the food back to the nest, causing all of the mice to die, while a trap will kill only the one mouse that falls into it. If all signs of the mice disappear for three consecutive weeks after poisoned food is used, the homeowner can be sure that the poison was successful in eradicating the mice.

Which of the following, if true, most seriously calls into question the exterminator's claim that the absence of signs for three weeks means the mice have been eradicated by the poison?

(A) Because mice hide whenever they sense humans, it is very difficult to see or hear mice even when there is an active infestation.

(B) It is more humane to use "live-catch" traps that allow homeowners to release the still-living mice outside.
(C) In the spring, many mice that nest in houses begin foraging for food outside and do not return to the comfort of the house until the fall or winter.
(D) There are several different kinds of poison that could be used, some of which are more effective than others.
(E) It sometimes takes longer than three weeks for all of the mice to ingest and die from the poison.

8. Ammonium triiodide is a highly explosive chemical that is easy to make from only two ingredients, ammonia and concentrated iodine. However, no terrorists are known to have ever used ammonium triiodide in an attack.

Which of the following, if true, is the most likely explanation for the discrepancy described above?

(A) Ammonia can be bought in a grocery store, but concentrated iodine must be obtained from somewhat more restricted sources, such as chemical supply houses.
(B) Ammonium triiodide is only one of several powerful explosives that can be made from ammonia.
(C) Many terrorists have been more focused on acquiring weapons of mass destruction, such as nuclear or biological weapons, than on developing conventional chemical explosives.
(D) Airport security devices are typically calibrated to detect nitrogen compounds, such as ammonia and ammonium compounds.
(E) Although generally stable when wet, ammonium triiodide is unstable when dry and explodes at the slightest disturbance.

9. The recent decline in the employment rate was spurred by predictions of slow economic growth in the coming year. However, those predictions would not have affected the employment rate if it had not been for the lack of capital reserves of major industries. So if major industries increase their capital reserves, the employment rate will not decline in the future.

Which of the following, if true, casts the most doubt on the validity of the argument above?

(A) Major industry foresaw the drop in employment.
(B) Some major industries had appreciable capital reserves.
(C) An increase in labor costs could adversely affect the employment rate.
(D) The government could pass legislation mandating that major industries set aside a fixed amount as capital reserves every year.
(E) The drop in the employment rate was more severe this year than last.

10. New methods of math education in this country do a disservice to our children. In the lower grades, math instruction should focus on the basic skills that students will need in higher grades to develop the ability to solve complex problems. Learning basic math skills is like learning the scales and chords that one will later use to master complicated concertos and symphonies. Increasingly, math educators in this country seem to have it backward, emphasizing in higher grades the same narrow, skills-based approach that students learned in lower grades rather than the analytical tools they will need to solve complex math problems.

Which of the following, if true, would most seriously weaken the conclusion drawn above?

(A) While music is common in elementary school curricula, it is rarely taught in high school.
(B) On international tests of complex math skills, high-school students in this country performed no worse than did their counterparts from countries where problem solving is emphasized in higher grades.
(C) When presented with a math problem to solve, students in higher grades are more likely to arrive at

different answers than students in lower grades.

(D) Older students tend to receive higher grades in math than younger students do.

(E) Universities in this country report a steady increase in the percentage of first-year students who qualify to take courses such as engineering that require advanced math.

11. **The fight against the drug trade in Country X should focus for the time being on tightening the country's borders and targeting its major smugglers.** Wiping out poppy fields in rural areas means even greater hardship for an economically depressed farming population. **Rather, the United Nations and the government of Country X must carefully rebuild agricultural infrastructure in areas where the economy depends on these poppy fields.**

What purpose do the two boldface sentences serve in the passage?

(A) The first is the conclusion drawn by the speaker; the second is the alternative to that conclusion.

(B) The first is a short-term solution to a problem; the second is a long-term solution to the same problem.

(C) The first presents a problem; the second poses an ideal solution to the problem.

(D) The first presents a popular solution to a problem; the second presents a solution preferred by the author.

(E) The first presents an argument; the second presents evidence to support the argument.

12. In the 18th and 19th centuries, it was believed in many coastal American cities that the waterfront was an undesirable location for residential buildings. As a result, much of the waterfront in these cities was never developed aesthetically and instead was left to industry and commerce. Today, however, waterfront properties are generally seen as prestigious, as evidenced by the large sums paid for homes along the beach front. A developer who wishes to make a large profit would be wise to buy urban waterfront lots and erect residential buildings on them.

Which of the following, if true, most strongly supports the claim made about urban waterfront properties?

(A) People today have more money, relatively speaking, to spend on real estate than they did in previous centuries.

(B) Homeowners will be willing to spend large sums on residential properties in traditionally industrial or commercial districts.

(C) Many urban waterfront lots are available for purchase.

(D) Many coastal American cities are encouraging developers to rehabilitate the waterfront through tax incentives.

(E) Properties in interior residential districts in coastal American cities are significantly more expensive than those along the waterfront.

13. Psychiatric research has shown that receiving high-quality outpatient care, rather than being confined to an institution, produces the best quality of life for people who are mentally ill. Responding to this research, Congress in 1963 passed a law that released 95% of the mentally ill patients who had been confined to institutions. In 1983, however, researchers discovered that, on average, mentally ill people in America were faring worse than ever.

Which if the following, if true, best resolves the paradox in the above passage?

(A) More people were diagnosed with psychiatric disorders in 1983 than in 1963.

(B) In 1983, men who had been released from mental institutions fared worse than their female counterparts.

(C) A number of psychiatric medications were discovered between 1963 and 1983, including some early antidepressants.
(D) Congress never supplied the funding that would have been necessary to provide high-quality outpatient care to the newly-released patients.
(E) Most of the released patients who were doing badly in 1983 suffered from one of three mental illnesses.

14. Recently, some critics of the US government have pointed out that this country is the only advanced industrialized nation without a national vaccine laboratory and suggested that this lack makes the American public more vulnerable than other developed nations to infectious diseases, such as avian flu. A government official said these critics were disloyal and thus wrong about the public's vulnerability. To support his claim, the official cited the generally long life span and low infant mortality of US citizens, relative to all UN member nations. Mentioning the high quality of American hospitals, he added that all of the Europeans that he knew preferred to undergo major medical treatments in the United States rather than in the socialized medical systems in place in their home countries.

All of the following are weaknesses or potential weaknesses in the official's argument EXCEPT:

(A) The high quality of hospitals in the United States is not a factor affecting the public's vulnerability to infectious disease.
(B) Whether or not the critics are disloyal has no bearing on whether or not they are wrong.
(C) The Europeans that the official cited are a demographically narrow sample, overwhelmingly composed of wealthy males over the age of fifty.
(D) The average life span of US citizens is determined not only by deaths due to infectious diseases but also by deaths due to all other causes.
(E) Comparing the United States to all UN member nations does not address the concern that the US is behind other advanced industrialized nations in a particular way.

15. In response to the increasing cost of producing energy through traditional means, such as combustion, many utility companies have begun investing in renewable energy sources, chiefly wind and solar power, hoping someday to rely on them completely and thus lower energy costs. The utility companies claim that although these sources require significant initial capital investment, they will provide stable energy supplies at low cost. As a result, these sources will be less risky for the utilities than nonrenewable sources, such as gas, oil, and coal, whose prices can fluctuate dramatically according to availability.

The claim of the utility companies presupposes which of the following?

(A) The public will embrace the development of wind and solar power.
(B) No new deposits of gas, oil, and coal will be discovered in the near future.
(C) Weather patterns are consistent and predictable.
(D) The necessary technology for conversion to wind and solar power is not more expensive than the technology needed to create energy through combustion.
(E) Obtaining energy from nonrenewable sources, such as gas, oil and coal, cannot be made less risky.

16. Inorganic pesticides remain active on the surfaces of fruits and vegetables for several days after spraying, while organic pesticides dissipate within a few hours after application, leaving the surface of the sprayed produce free of pesticide residue. Therefore, when purchasing from a farm that uses inorganic pesticides, one must be careful to wash the produce thoroughly before eating it to prevent the ingestion of toxins. But one need not worry about ingesting pesticides when purchasing from farms that use only organic pesticides.

The argument above assumes that

(A) Consumers are aware of the origins of the produce they purchase.
(B) Produce from farms that use organic pesticides reaches the consumer within hours after it is picked or harvested.
(C) No farm uses both organic and inorganic pesticides.
(D) No pesticide is capable of penetrating the skin of a fruit or vegetable.
(E) The use of either type of pesticide does not increase the cost of produce.

17. Unlike juvenile diabetes, which is a genetic condition present from birth, type-2 diabetes is acquired in adulthood, generally as a result of obesity and inactivity. The number of cases of type-2 diabetes has been steadily increasing in the United States since 1970, indicating to many researchers that the American population is becoming increasingly heavy and sedentary. If the government wishes to stem the spread of the disease, it should educate the public about the dangers of an inactive, calorie-laden lifestyle and promote healthful diets and exercise.

Which of the following, if true, provides the strongest reason to believe that the proposed education program will NOT be effective?

(A) School health programs already educate middle-school students about the issue.
(B) The public already has access to this information through the Internet.
(C) Food companies encourage the public to indulge in unhealthful snacks.
(D) The government has not set aside money for such a program.
(E) Healthful foods and exercise programs are beyond the financial means of many people.

18. The exchange rate between the currency of Country X and that of Country Y has historically favored the currency of Country Y. Because of this, citizens of Country Y often take their vacations in Country X, where the exchange rate makes hotels and restaurants more affordable. Yet, citizens of Country Y rarely purchase clothing or electronics in Country X, despite the fact that those items are more expensive in their home country, even when sales taxes are taken into account.

Which of the following, if true, would best explain the buying habits of the citizens of Country Y?

(A) Citizens of Country Y prefer the fashions available in their own country.
(B) Stores in Country X receive the latest fashions and technology several months after they are available in Country Y.
(C) The citizens of Country X resent the buying power of the currency of Country Y.
(D) The government of Country Y imposes tariffs on imported goods.
(E) The currencies of Country X and Country Y are both weak compared to the currency of Country Z.

19. Every year, many people become ill because of airborne mold spores in their homes. After someone becomes ill, specialists are often hired to eradicate the mold. These specialists look in damp areas of the house, since mold is almost always found in places where there is substantial moisture. If one wishes to avoid mold poisoning, then, one should make sure to keep all internal plumbing in good condition to prevent leakage that could serve as a breeding ground for mold.

Which of the following is an assumption on which the argument depends?

(A) Mold itself does not create moisture.
(B) Most homeowners know enough about plumbing to determine whether theirs is in good condition.
(C) Mold cannot grow in dry areas.
(D) No varieties of mold are harmless.
(E) Mold spores cannot be filtered from the air.

20. To prevent overcrowding, last month the town zoning board limited the number of new buildings that can be constructed in the town in any given year. The board claims that doing so will preserve open spaces and lessen the strain on municipal resources such as schools and garbage disposal. Critics of the changes argue that the plan will harm the community or, at the very least, will fail in its purpose.

Which of the following most supports the claims of the critics of the plan?

(A) Other towns have had mixed success with similar zoning plans.
(B) No new schools have been built in the town in ten years.
(C) Property taxes in the town are higher than in neighboring towns.
(D) Under the new plan, developers may still erect apartment buildings.
(E) The nearest garbage dump is several miles away from the town.

21. Because of a rare type of fungus that killed off many cacao trees in Brazil, there was an unusually meager harvest of cacao beans this year. The wholesale price of cocoa solids and cocoa butter has increased significantly and is unlikely to fall in the foreseeable future. As a result, the retail price of chocolate is certain to increase within six months.

The answer to which of the following questions would provide information relevant to evaluating the argument above?

(A) Has the price of cocoa solids and cocoa butter remained steady during other periods of poor harvest?
(B) Are consumers willing to spend more for chocolate?
(C) Have the prices of other ingredients in chocolate decreased recently?
(D) What percentage of cacao trees in Brazil were affected by the fungus?
(E) Can the fungus be eliminated within the next six months?

22. Two years ago, the cost of the raw material used in a particular product doubled after an earthquake disrupted production in the region where the material is mined. Since that time, the company that makes the product has seen its profit margins decline steadily. Aiming to improve profit margins, the company's head of engineering has decided that he must find a new source for the raw material.

Which of the following, if true, would cast the most doubt on the validity of the head of engineering's decision?

(A) New competitors have entered the market every six months for the past two years, resulting in price wars that have progressively driven down revenues across the market.
(B) Although the earthquake occurred two years ago, the region's mines have still not recovered to pre-earthquake production capacity.
(C) There are several other regions in the world where the raw material is mined, but those regions do not produce as much of the raw material as the current source region.
(D) The company could use a completely different raw material to make its product.
(E) Recent advances in mining technology will make mining the raw material much more efficient and cost-effective in the future.

23. According to a recent research study, more than 90% percent of graduates of private high schools in a certain county continue their education in college. By contrast, only 65% of graduates of public high schools subsequently pursue college education. Therefore, if parents in the county wish to increase the likelihood that their children will attend college, they should send them to private rather than public schools.

Which of the following statements would most seriously weaken the argument above?

(A) Graduates of private schools typically score higher on standardized tests and other tests of academic achievement.
(B) While private schools are typically very expensive, attendance of public school is free for the residents of the county.
(C) In comparison with graduates of private schools, a substantially greater proportion of public school graduates receive need-based financial aid for their college education.
(D) In comparison with private schools, public schools provide more opportunities for student involvement in sports and other athletic activities, which almost always increase the likelihood of students' acceptance to colleges.
(E) Since most public schools are located in rural areas of the county populated primarily by farmers, nearly 30% of students from public high schools choose to pursue farming occupations rather than apply to colleges.

24. Due to high jet fuel costs, airline carriers are looking for new ways to increase revenues and thereby counteract declining profits. Airline A has proposed increasing the number of passengers that can fit on its airplanes by creating several standing room only "seats" in which passengers would be propped against a padded backboard and held in place with a harness. This proposal, since it relates to passenger safety, cannot be implemented without prior approval by the Federal Aviation Administration.

The above statements, if true, indicate that Airline A has made which of the following conclusions?

(A) The addition of standing room only "seats" will generate more revenue than the cost of ensuring that these seats meet safety standards.
(B) The Federal Aviation Administration will approve Airline A's specific proposal.
(C) The revenue generated by the addition of standing room only "seats" is greater than the current cost of jet fuel.
(D) There are no safer ways in which Airline A can increase revenues.
(E) Passenger safety is less important than increasing revenue.

25. A recent development in the marketplace for consumer technology goods has been the premium placed on design—products with innovative and appealing designs relative to competing products can often command substantially higher prices. **Because design innovations are quickly copied by other manufacturers though,** many consumer technology companies charge as much as possible when it comes to their new designs to extract as much value as possible from their new designs. But large profits generated by the innovative design will give competitors stronger incentives to copy the designs. **Therefore, the best strategy to maximize overall profit from an innovative new design is to charge less than the greatest possible price.**

In the argument above, the two portions in boldface play which of the following roles?

(A) The first is an assumption that assists a course of action criticized by the argument; the second provides a consideration to support the argument's recommended position.
(B) The first is a consideration that helps explain the appeal of a certain strategy; the second presents an alternative strategy endorsed by the argument.
(C) The first is an assumption that justifies a specific strategy; the second is that strategy.
(D) The first is a consideration presented to demonstrate that a particular strategy will not apply; the second is a factor cited to support the argument's main position.
(E) The first is a factor used to justify a particular strategy; the second is a factor against that strategy.

26. Smoking is a known cause of certain serious health problems, including emphysema and lung cancer. Now, an additional concern can be added to the list of maladies caused by smoking. A recent study surveyed both smokers and nonsmokers, and found that smokers are significantly more anxious and nervous than nonsmokers.

Which of the following is an assumption on which the argument rests?

(A) Anxiety and nervousness can lead to serious health problems.
(B) Anxiety and nervousness do not make individuals more likely to start smoking.
(C) Equivalent numbers of smokers and nonsmokers were surveyed for the study.
(D) Smokers are aware of the various health problems attributed to smoking, including lung cancer and emphysema.
(E) Smokers who had smoked a cigarette immediately before responding to the survey were more anxious and nervous than smokers who had not smoked for several hours.

27. The number of new cases of tuberculosis diagnosed in Country X increased dramatically this year. The country's news media have speculated that the sharp increase in new cases is the result of the tuberculosis outbreak that occurred in neighboring Country Y last year. Health officials in Country X have therefore proposed that all visitors from Country Y must submit to a medical examination before entering Country X.

Which of the following, if true, most strongly suggests that the proposed medical examinations will NOT help curb the spread of tuberculosis in Country X?

(A) Country Z, which also neighbors Country Y, has not experienced an increase in cases of tuberculosis.
(B) Current medical technology is not capable of detecting all carriers of tuberculosis.
(C) Country X does not have the resources to examine all visitors from Country Y.
(D) Tuberculosis is not spread through human contact.
(E) Citizens of Country Y will not travel to Country X if the proposal is implemented.

28. The people of Prohibitionland are considering banning the service of alcoholic beverages in restaurants to curb unruly behavior on the part of its residents. Proprietors of restaurants in Prohibitionland are protesting the ban on the grounds that it will reduce their revenues and profits. However, several provinces in Prohibitionland enacted restrictions on alcoholic beverages last year, and the sales taxes paid by the restaurants in those provinces rose by an average of 50 percent. In contrast, the sales taxes paid by restaurants located in areas of Prohibitionland that did not have any restrictions rose by an average of 30 percent.

Which of the following, if true, supports the restaurant proprietors' economic stance against the ban?

(A) In the provinces that restricted alcoholic beverages, there was a short-term negative impact on restaurant visitation in the beginning of last year.
(B) The sales tax in Prohibitionland is lower on food and beverages than it is on other consumer goods, such as clothing.
(C) The consumption of alcoholic beverages in Prohibitionland has been on a gradual decline the last 20 years.
(D) The restrictions on alcoholic beverages enacted last year allowed for the service of drinks beginning around dinnertime each evening.
(E) Overall sales tax revenue did not increase at a substantially higher rate in the provinces that enacted the restrictions on alcoholic beverages than in the rest of Prohibitionland last year.

29. Serious individual art collectors are usually discreet when making significant purchases or sales related to their collections. At art auctions, for example, these collectors often place anonymous bids for major artwork. Therefore, the whereabouts of most of the world's most valuable artwork is probably unknown.

Each of the following, if true, would weaken the conclusion above EXCEPT:

(A) The value of a piece of art is purely subjective.

(B) Serious art collectors usually publicize their new artwork several months after making a purchase.
(C) Museums own the vast majority of the world's most valuable artwork.
(D) Of all the individuals in the world who own extremely valuable artwork, only a few are considered serious art collectors.
(E) The private collections of most serious individual art collectors are often displayed in public settings.

30. Last January, in an attempt to lower the number of traffic fatalities, the state legislature passed its "Click It or Ticket" law. Under the new law, motorists can be pulled over and ticketed for not wearing their seat belts, even if an additional driving infraction has not been committed. Lawyers and citizens' groups are already protesting the law, saying it unfairly infringes on the rights of the state's drivers. Law enforcement groups counter these claims by stating that the new regulations will save countless additional lives.

Which of the following inferences is best supported by the passage above?

(A) Prior to the "Click It or Ticket" law, motorists could not be stopped simply for not wearing a seat belt.
(B) The "Click It or Ticket" law violates current search and seizure laws.
(C) Laws similar to "Click It or Ticket" have effectively reduced traffic fatalities in a number of states.
(D) The previous seatbelt laws were ineffective in saving lives.
(E) Law enforcement groups, rather than citizens groups, should determine how to best ensure the safety of motorists.

31. Calorie restriction, a diet high in nutrients but low in calories, is known to prolong the life of rats and mice by preventing heart disease, cancer, diabetes, and other diseases. A 6-month study of 48 moderately overweight people, who each reduced their calorie intake by at least 25 percent, demonstrated decreases in insulin levels and body temperature, with the greatest decrease observed in individuals with the greatest percentage change in their calorie intake. Low insulin level and body temperature are both considered signs of longevity, partly because an earlier study by other researchers found both traits in long-lived people.

If the above statements are true, they support which of the following inferences?

(A) Calorie restriction produces similar results in humans as it does in rats and mice.
(B) Humans who reduce their calorie intake by at least 25 percent on a long-term basis will live longer than they would have had they not done so.
(C) Calorie intake is directly correlated to insulin level in moderately overweight individuals.
(D) Individuals with low insulin levels are healthier than individuals with high insulin levels.
(E) Some individuals in the study reduced their calorie intake by more than 25 percent.

32. Ensuring that children consume less sugar is among the most effective ways to curb childhood obesity. Recently, with this goal in mind, school officials have begun to replace high calorie sugary drinks in school vending machines with bottled water, unsweetened fruit juices, and sugar-free sodas. Since students spend so much time in school, officials reason that removing access to sugary drinks during school hours will cause a dramatic reduction in the intake of sugar.

Which of the following, if true, most undermines the school officials' plan?

(A) Unsweetened fruit juices contain more sugar than does bottled water.
(B) Many students have access to sugary drinks both before and after school.
(C) Sugar-free sodas contain artificial sweeteners that some medical officials link to headaches and other health concerns.
(D) Sugary snack foods comprise the majority of sales in school vending machines.

(E) The average school-aged child consumes two twenty-ounce sugary drinks every day.

33. Recent research has indicated that married people are not only happier than unmarried people, but also healthier. This study has been widely reported by the media, with most commentators concluding that being married is good for one's health and attitude.

The conclusion of the media commentators depends on which of the following assumptions?

(A) The longer people are married, the happier and healthier they become.
(B) Married couples who had a large, extravagant wedding are happier than those who had a small, simple ceremony.
(C) Married people cannot get depressed.
(D) Single people with depression or health problems are just as likely to get married as are other single people.
(E) Some marriages are more harmonious than others.

34. For nearly a century, physiologists erroneously believed that a **buildup of lactic acid in muscle tissue was responsible for the soreness that many people experience after strenuous exercise**. The acid, they claimed, is the waste produced by metabolic activity in the muscle and reaches "threshold" levels, causing soreness, when the muscle has depleted its oxygen supply. **Researchers have recently discovered, however, that lactic acid is actually the fuel that powers muscular activity.** Therefore, the cause of muscle soreness remains unknown.

In the argument above, the portions in boldface play which of the following roles?

(A) The first is an assertion that the author accepts as true; the second is a consideration in support of that assertion.
(B) The first is an assertion that the author accepts as true; the second describes a situation that the author posits as contrary to that assertion.
(C) The first is an assertion that the author argues against; the second is evidence presented as contrary to the author's argument.
(D) The first is evidence that the author believes is no longer valid; the second is additional evidence that the author uses to support his main point.
(E) The first is a claim that the author believes to be invalid; the second is the author's main point.

35. An oil field prospector and developer reported a large oil deposit in southwestern Texas. As a result, a large oil and gas company purchased the field with the intention of drilling oil wells in the area soon afterwards. However, the company found that what had been reported to be a large oil deposit was actually much smaller than had been indicated. Thus, the methods that the prospector had used to determine the size of the oil deposit must have been inaccurate.

Which of the following is an assumption on which the argument depends?

(A) The company's methods of measuring the size of the oil deposit were determined by a third party to be more accurate than those used by the prospector.
(B) The prospector did not purposefully fabricate or misrepresent the size of the oil deposit.
(C) Though smaller than originally thought, the oil deposit contained enough oil to make drilling commercially feasible.
(D) The prospector did not explore other oil fields and use the same methods to determine the magnitude of the oil present, if any.
(E) The company had successfully drilled for oil in other large oil fields in Texas throughout the early

twentieth century.

36. According to a recent study on financial roles, one-third of high school seniors say that they have "significant financial responsibilities." These responsibilities include, but are not limited to, contributing to food, shelter, or clothing for themselves or their families. At the same time, a second study demonstrates that a crisis in money management exists for high school students. According to this study, 80% of high school seniors have never taken a personal finance class even though the same percentage of seniors has opened bank accounts and one-third of these account holders has bounced a check.

Which of the following conclusions can be properly drawn from the statements above?

(A) High schools would be wise to incorporate personal finance classes into their core curricula.
(B) At least one-third of high school seniors work part-time jobs after school.
(C) The number of high school seniors with significant financial responsibilities is greater than the number of seniors who have bounced a check.
(D) Any high school seniors who contribute to food, shelter, or clothing for themselves or their families have significant financial responsibilities.
(E) The majority of high school students have no financial responsibilities to their families.

37. Federal law prohibits businesses from reimbursing any employees for the cost of owning and operating a private aircraft that is used for business purposes. Thus, many American companies themselves purchase private aircraft. The vast majority of the business aviation fleet is owned by small and mid-size businesses, and flights are strictly for business purposes, with mostly mid-level employees on board. These companies and their boards of directors are in full compliance with the law and with what is best for their businesses.

Which of the following can be most properly inferred from the statements above?

(A) The federal law in question costs businesses money.
(B) Most executives would rather fly on company owned planes than on commercial airlines.
(C) Large businesses usually have their executives fly first or business class on commercial flights.
(D) Upper-level executives are less often in compliance with the law.
(E) By not receiving any reimbursement for these flights, the mid-level executives on board are complying with the law.

38. Experts estimate that insurance companies' tardiness in paying doctors for legitimate medical claims adds approximately 10 percent in overhead costs for physicians. Insurance companies counter that the tardiness sometimes results from billing errors made by the doctors themselves. Since dealing with these billing errors costs the insurance companies time and money, it is clear that insurance companies do not have a significant economic incentive to delay claim payments to doctors.

Which of the following pieces of information, if true, weakens the conclusion above?

(A) Some doctors who submit accurate bills to insurance companies still receive tardy payments.
(B) The cost to the insurance companies to process incorrect bills from doctors' offices is roughly equivalent to the increased costs that physicians accrue as a result of tardy payments from insurance companies.
(C) A rising proportion of medical claims submitted by doctors to insurance companies are deemed illegitimate by those insurance companies.
(D) The billing errors made by doctors' offices are typically very minor, such as the submission of a claim with an outdated patient home address.
(E) The overhead costs incurred by doctors as a result of delayed insurance payments result in an increase

in the premiums paid by consumers to health insurance companies that far exceeds any increase in the fees paid to doctors by insurance companies.

39. Farmers in developing countries claim that the US government, through farm subsidies, is responsible for the artificially low global price of wheat. Because the US government buys whatever wheat American farmers are unable to sell on the open market, American farmers have no incentive to modulate the size of their crops according to the needs of the global market. As a result, American farmers routinely produce more wheat than the global market can absorb and the global price of wheat is kept low. Without these subsidies, the farmers in developing economies claim, American farmers would produce only the amount of wheat that they could sell on the open market and the global price of wheat would rise.

Which of the following, if true, most weakens the claims of the farmers in developing countries regarding the price of wheat?

(A) Wheat that is not processed for consumption is often used for certain industrial applications.
(B) Non-governmental buyers of wheat and wheat products are able to predict how much wheat they will need several years in advance.
(C) The US government offers similar subsidies to soybean farmers, though the global price of soybeans is significantly higher than that of wheat.
(D) Other countries, such as Canada and Russia, are likely to produce more wheat if the United States were to reduce its output.
(E) The price of sorghum, a crop for which the US government offers no subsidies, is lower than that of wheat.

40. Researchers studying the spread of the Black Plague in sixteenth-century England claim that certain people survived the epidemic because they carried a genetic mutation, known as Delta-32, that is known to prevent the bacteria that causes the plague from overtaking the immune system. To support this hypothesis, the researchers tested the direct descendants of the residents of an English town where an unusually large proportion of people survived the plague. More than half of these descendants tested positive for the mutation Delta-32, a figure nearly three times higher than that found in other locations.

The researchers' hypothesis is based on which of the following assumptions?

(A) Delta-32 does not prevent a carrier from contracting any disease other than the plague.
(B) The plague is not similar to other diseases caused by bacteria.
(C) Delta-32 did not exist in its current form until the sixteenth century.
(D) No one who tested positive for Delta-32 has ever contracted a disease caused by bacteria.
(E) The plague does not cause genetic mutations such as Delta-32.

41. The state legislature has asked the public to approve a bond that would raise money for public library funding outside of the existing state budget. While building and improving libraries are certainly worthy projects, and ones that the public supports, the legislature is already wasting much of the state budget on projects that the public does not value or even want. The public should therefore reject the library bond; doing so will force legislators to cut wasteful programs from the existing budget if they wish to fund libraries.

Which of the following is most like the argument above in its logical structure?

(A) Many children would choose to eat dessert before dinner, if given the option. While this is understandable, parents should allow a child to eat dessert only if he or she first finishes the healthier dinner.
(B) While alarm systems are good for the individual property owner, homeowners do a disservice to their

neighbors by installing alarm systems because neighboring homes are then comparatively easy targets for burglars. Instead, homeowners should consider alternative home protection measures such as joining a neighborhood watch program.

(C) Some terrorist groups kidnap individuals to gain publicity for their cause. While it is understandable that the families of kidnapping victims often want to cooperate in order to get loved ones returned safely, this country should continue its policy of refusing to negotiate with terrorists or publicize the kidnapping.

(D) A labor union and a major employer are engaged in contract negotiations. While the employer proposed a pay increase matches that requested by the labor union, the contract does not include the health care coverage that the labor union also requested. The labor union should therefore reject the contract and continue negotiations.

(E) A student often asks his parents to stay up past his normal bedtime in order to finish his homework because he spends much of his afternoon playing games with friends rather than working on homework. Although his parents want him to do well in school, they should strictly enforce his bedtime; doing so will force the student to start working on his homework earlier in the evening.

42. In a certain state, hunting permits require the hunter to be at least 18 years old, possess a valid driver's license or state identification, and have completed a safety program within the past 5 years. The hunter must also sign a legal document pledging not to consume alcohol while hunting.

Which of the following can be correctly inferred from the above statements?

(A) All states have the same requirements for hunting permits.
(B) Hunters under the age of 21 don't need to sign the alcohol pledge because they are not legally allowed to drink alcohol under any circumstances.
(C) If a person last completed the safety program six years ago, he will have to complete the program again before he can be eligible for a permit in this state.
(D) Someone who isn't 18 years old cannot obtain a hunting permit in this state.
(E) Hunting is such a dangerous activity that state controls and requirements are necessary to ensure that nobody gets hurt.

43. Wide dissemination of wireless networks in cities is a practical way to meet the needs of city households, schools and businesses. Rural communities have found that wireless networks are both more reliable and cheaper than land-based networks.

Which of the following would most likely be cited by a supporter of the argument?

(A) Urban areas do not pose additional problems for the effective operation of wireless networks.
(B) Wireless networks work far better where population density is low.
(C) Iceland, a very rural country, successfully uses wireless networks.
(D) The expenses of wireless transmission in areas with large buildings is much higher.
(E) Poor neighborhoods have less access to cable internet than do educators or businesses.

44. Studies have shown that **people who keep daily diet records are far more successful at losing weight than people who don't keep track of what they eat**. Researchers believe that many weight-loss efforts fail because people eat more calories than they intend to consume. One study followed a group of patients who reported that they could not lose weight when consuming only 1,200 calories a day. **The study found that the group consumed, on average, 47% more than it claimed and exercised 51% less.** In contrast, when dieters record what they eat, their actual consumption more closely matches their reported consumption.

The two boldface portions in the argument above are best described by which of the following statements?

(A) The first is a conclusion reached by researchers; the second is evidence that that conclusion is correct.
(B) The first is an explanation of why a certain theory is thought to be true; the second is an example of research results that support this theory.
(C) The first is an example illustrating the truth of a certain theory; the second is a competing theory.
(D) The first is a premise upon which the researchers base their opinion; the second illustrates that their opinion is correct.
(E) The first introduces a theory that the researchers have disproved; the second is the basis for the researchers' argument.

45. The anticipated retirement of tens of thousands of baby boomers will create an unprecedented opportunity to move significant numbers of people into career-track jobs at family-supporting incomes. Major industries, from health care and construction to automotive repair, will soon face deep shortages of workers as a result of projected growth and boomer retirements. Fortunately, many of these jobs have relatively low barriers to entry and could be filled by out-of-work young people. To achieve this result, the city government should convene employers and educators to determine how best to create paths of upward mobility in these fields.

Which of the following, if true, most weakens the argument?

(A) Immigration reform will limit the pool of available workers.
(B) Government efforts have been shown to affect employment trends only rarely.
(C) The best available positions require skills not possessed by the vast majority of the unemployed.
(D) A small proportion of baby boomers will not retire as soon as is anticipated.
(E) Many out-of-work young people are unaware of these looming employment opportunities.

46. The "Doppler effect" refers to the perceived change in pitch that occurs when the source of a sound is in motion relative to the observer. For example, the siren on a passing police car will sound higher than its true pitch as the car approaches, sound the same as its true pitch as the car passes, and sound lower than its true pitch as the car travels away from the observer.

If two trains pass each other going opposite directions on parallel east-west tracks, which of the following observations provides another illustration of the effect described above?

(A) If the eastbound train blows its horn as they pass, passengers on the westbound train will hear a sound that decreases in pitch.
(B) If the eastbound train blows its horn as they pass, passengers on the westbound train will hear a sound that increases in pitch.
(C) If the eastbound train blows its horn as they pass, passengers on the eastbound train will hear a sound that decreases in pitch.
(D) If the eastbound train blows its horn as they pass, passengers on the eastbound train will hear a sound that increases in pitch.
(E) If the eastbound train blows its horn as they pass, passengers on the eastbound train will hear a sound that is steady in pitch.

47. The popular notion that a tree's age can be determined by counting the number of internal rings in its trunk is generally true. However, to help regulate the internal temperature of the tree, the outermost layers of wood of the Brazilian ash often peel away when the temperature exceeds 95 degrees Fahrenheit, leaving the tree with fewer rings than it would otherwise have. So only if the temperature in the Brazilian ash's environment never exceeds 95 degrees Fahrenheit will its rings be a reliable measure of the tree's age.

Which of the following is an assumption on which the argument above depends?

(A) The growth of new rings in a tree is not a function of levels of precipitation.
(B) Only the Brazilian ash loses rings because of excessive heat.
(C) Only one day of temperatures above 95 degrees Fahrenheit is needed to cause the Brazilian ash to lose a ring.
(D) The internal rings of all trees are of uniform thickness.
(E) The number of rings that will be lost when the temperature exceeds 95 degrees Fahrenheit is not predictable.

48. Celiac disease results from an inability of the digestive tract, specifically the small intestine, to absorb gluten, a protein found in wheat, barley, and certain other grains. The body's immune system attacks the gluten as if the protein were a harmful pathogen, often resulting in serious damage to the intestinal lining. People who suffer from celiac disease must eliminate gluten from their diets. Symptoms of the disease include abdominal cramps, bloating, and anemia.

If the statements above are true, which of the following assertions can be made on the basis of them?

(A) Anyone who suffers from celiac disease will experience anemia.
(B) Eliminating gluten from one's diet will cure celiac disease.
(C) People experiencing abdominal cramps, bloating, and anemia have celiac disease.
(D) Gluten is found only in grains.
(E) The human body cannot always recognize harmless substances.

49. All languages known to have exactly six basic color terms describe the same six colors—black, white, red, green, blue and yellow—corresponding to the primary neural responses revealed in studies of human color perception. In addition, all languages known to have only three basic color terms distinguish among black, white, and red. This evidence shows that the way in which the mind recognizes differences among colored objects is not influenced by culture.

Which of the following, if true, most seriously weakens the argument above?

(A) While languages differ in their number of basic color terms, no language has been conclusively determined to have more than eleven such terms.
(B) Every language contains mechanisms by which speakers who perceive subtle differences in hue can describe those differences.
(C) Among cultures employing only three color terms, the word red typically encompasses not only objects that would be called red in English but also those that would be called yellow.
(D) Several languages, such as Vietnamese and Pashto, use a single term to mean both blue and green, but speakers of such languages commonly refer to tree leaves or the sky to resolve ambiguous utterances.
(E) In a study of native speakers of Tarahumara, a language that does not distinguish between blue and green, respondents were less able to identify distinctions among blue and green chips than native speakers of Spanish, which does distinguish between blue and green.

50. In an attempt to discover the cause of malaria, a deadly infectious disease common in tropical areas, early European settlers in Hong Kong attributed the malady to poisonous gases supposed to be emanating from low-lying swampland. **Malaria, in fact, translates from the Italian as "bad air."** In the 1880s, however, doctors determined that *Anopheles* mosquitoes were responsible for transmitting the disease to humans. **The female of the species can carry a parasitic protozoa that is passed on to unsuspecting humans** when a mosquito feasts on a person's blood.

What functions do the two statements in boldface fulfill with respect to the argument presented above?

(A) The first follows from a mistaken conclusion about a topic in question; the second explicates the correct explanation of that topic.
(B) The first provides an initial conjecture; the second prevents evidence which contradicts that conjecture.
(C) The first serves to illuminate a contested assumption; the second offers confirmation of that assumption.
(D) The first identifies the cause of an erroneous conclusion; the second develops a premise to support the correct conclusion.
(E) The first provides detail about the original, and incorrect, school of thought; the second provides the judgment later found to be true.

51. Government restrictions have severely limited the amount of stem cell research American companies can conduct. Because of these restrictions, many American scientists who specialize in the field of stem cell research have signed long-term contracts to work for foreign companies. Recently, Congress has proposed lifting all restrictions on stem cell research.

Which of the following conclusions can most properly be inferred from the information above?

(A) At least some foreign companies that conduct stem cell research work under fewer restrictions than some American companies do.
(B) Because American scientists are under long-term contracts to foreign companies, there will be a significant influx of foreign professionals into the United States.
(C) In all parts of the world, stem cell research is dependent on the financial backing of local government.
(D) In the near future, American companies will no longer be at the forefront of stem cell research.
(E) If restrictions on stem cell research are lifted, many of the American scientists will break their contracts to return to American companies.

52. If life exists elsewhere in the solar system, scientists suspect it would most likely be on Europa, an ice covered moon orbiting Jupiter. However, NASA recently scrapped an unmanned science mission to Europa and reassigned most of the employees involved in the project to another project which focuses on landing an astronaut on Mars. Polls show that Americans are far more fascinated by space travel than they are by discovering life elsewhere in the universe. Critics argue that NASA's decision-making process places a greater emphasis on public interest than it does on the importance of scientific research.

Which of the following, if true, would most strengthen NASA's contention that the critics are misinformed?

(A) In 2007, NASA will spend 30% of it's total budget on developing a space shuttle that can travel to Mars. In 2013, that figure is expected to drop to 0%.
(B) Studies have shown that Congress traditionally determines NASA's budget based on its perception of public interest in NASA's projects.
(C) Some scientists are convinced that a mission to Europa would add immeasurably to our understanding of the universe; others believe that we will gain little insight from exploring Europa.
(D) A new telescope that has been developed in Tokyo allows scientists to look at Europa in ways never possible before and promises to yield more information than the planned mission was designed to provide.
(E) Most Americans feel that a shuttle to Mars is the next logical step in the development of a system that will eventually allow humans to travel to places as far away as Europa and beyond.

53. Dengue fever is a viral infection transmitted by the *Aedes* mosquito, whereas malaria is a parasitic infection transmitted by the *Anopheles* mosquito. Since both types of mosquito are prevalent on Nicaragua's "Mosquito Coast," anyone with a compromised immune system should not travel to this region, particularly during the rainy season.

Which of the following, if true, least strengthens the argument presented above?

(A) Medicines taken to prevent or treat malaria can cause serious health complications for people with weak immune systems.
(B) The Nicaraguan government is trying to reduce the number of mosquitoes on the Mosquito Coast.
(C) A compromised immune system has difficulty fighting either viral or parasitic infections.
(D) During the rainy season, the populations of both *Aedes* and *Anopheles* mosquitoes multiply in the Mosquito Coast.
(E) The most effective chemical insect repellent has been shown to cause brain damage upon repeated application to the skin.

54. In 2001, the Peruvian government began requiring tourists to buy expensive permits to hike the remote Inca Trail, which goes to the ancient city of Machu Picchu. The total number of permits is strictly limited; in fact, only 500 people per day are now allowed to hike the Inca Trail, whereas before 2001, visitors numbered in the thousands. The Peruvian government argues that this permit program has successfully prevented deterioration of archaeological treasures along the Inca Trail.

Which of the following, if true, most strengthens the argument above?

(A) Since 2001, tourist guides along the Inca Trail have received 50% to 100% increases in take-home pay.
(B) Villages near Machu Picchu have experienced declines in income, as fewer tourists buy fewer craft goods and refreshments.
(C) Many of the funds from the sale of Inca Trail permits are used to staff a museum of Incan culture in Peru's capital city, Lima, and to hire guards for archaeological sites without permit programs.
(D) Since 2001, Incan ruins similar to Machu Picchu but not on the Inca Trail have disintegrated at a significantly greater rate than those on the trail.
(E) The total number of tourists in Peru has risen substantially since 2001, even as the number of tourists hiking the Inca Trail has remained constant.

55. The Farmsley Center for the Performing Arts, designed by a world-renowned architect, was built ten years ago in downtown Metropolis. A recent study shows that, on average, a person who attends a performance at the Farmsley Center spends $83 at downtown businesses on the day of the performance. Citing this report, the chairman of the Farmsley Center's Board of Trustees claims that the Farmsley Center has been a significant source of the economic revitalization of downtown Metropolis.

Which of the following, if true, most strongly supports the chairman's contention?

(A) The Metropolis Chamber of Commerce honored the Farmsley chairman this year for his contributions to the city.
(B) Restaurants near the Farmsley Center tend to be more expensive than restaurants in outlying areas.
(C) The Farmsley Center is the only building in Metropolis designed by a world-renowned contemporary architect.
(D) For major theater companies on national tours, the Farmsley Center is the first choice among venues in downtown Metropolis.
(E) Many suburbanites visit downtown Metropolis on weekends in order to see performances at the Farmsley Center.

56. One retirement account option allows a worker to save money without paying taxes, but requires the worker to pay taxes on funds withdrawn from the account upon retirement. A second option requires the worker to pay taxes upfront, but allows the worker to withdraw funds tax-free upon retirement. Assuming that the total amount

available in the worker's retirement account at retirement is higher than the total amount contributed prior to retirement, workers can expect to pay less in taxes overall if they choose the second option.

Which of the following pieces of information would be most useful in determining whether the conclusion is valid for an individual worker?

(A) The amount of money the worker will contribute to the retirement plan over his or her career.
(B) The amount that tax rates will increase in the future.
(C) Whether or not inflation will be lower than the retirement account's annual earnings.
(D) How the worker's tax bracket in retirement compares to his or her tax bracket while still employed.
(E) The dollar value of the worker's account upon retirement.

57. In a recent poll, 71% of respondents reported that they cast votes in the most recent national election. Voting records show, however, that only 60% of eligible voters actually voted in that election.

Which of the following pieces of evidence, if true, would provide the best explanation for the apparent discrepancy?

(A) The margin of error for the survey was plus or minus five percentage points.
(B) Fifteen percent of the survey's respondents were living overseas at the time of the election.
(C) Prior research has shown that that people who actually do vote are also more likely to respond to polls than those who do not vote.
(D) Many people who intend to vote are prevented from doing so by last-minute conflicts on election day or other complications.
(E) Some people confused the national election with other recent elections when responding to the poll.

58. Scientists recently documented that influenza spreads around the world more efficiently in the modern era due to commercial air travel. Symptoms of a pandemic-level flu are severe enough that the ill would likely cancel or reschedule air travel, but an infected person can travel across the globe before the first signs appear. Further, if symptoms develop while someone is still on a plane, the infected person's cough can spread the virus easily in the enclosed and closely-packed environment.

Which of the following would best minimize the role air travel can play in the spread of influenza during a pandemic?

(A) Installing air filtration systems in the planes to kill any flu virus particles flowing through the filters.
(B) Requiring air travelers to receive flu vaccinations far enough in advance of the trip to provide protection against the disease.
(C) Refusing to allow children, the elderly, or others who are especially vulnerable to flu to travel by air during a pandemic.
(D) Requiring all air travelers to wash their hands before boarding a plane.
(E) Conducting medical examinations during the boarding process to weed out passengers with flu symptoms.

59. Male CEOs of major corporations are, on average, three inches taller than the average male. When data from the general population are analyzed and corrected for gender and age, a clear pattern emerges: for every extra inch of height, a person's annual salary increases by approximately $789. Citing these data, a prominent journalist claims that most employers have an unconscious bias in favor of tall people.

Which of the following considerations, if true, would most seriously undermine the journalist's argument?

(A) On average, a woman is shorter than her husband and earns less than he does.
(B) Socioeconomic status has been shown to have a strong positive correlation to both height and educational attainment.
(C) Professional basketball players, who are some of the tallest people in the labor force, have high incomes.
(D) Human resource professionals, who make many hiring decisions, are on average no taller than the general population.
(E) A tall person’s tenure in a paid position is typically shorter than is the tenure of a person of average height.

60. Traditionally, public school instructors have been compensated according to seniority. Recently, the existing salary system has been increasingly criticized as an approach to compensation that rewards lackadaisical teaching and punishes motivated, highly-qualified instruction. Instead, educational experts argue that, to retain exceptional teachers and maintain quality instruction, teachers should receive salaries or bonuses based on performance rather than seniority.

Which of the following, if true, most weakens the conclusion of the educational experts?

(A) Some teachers express that financial compensation is not the only factor contributing to job satisfaction and teaching performance.
(B) School districts will develop their own unique compensation structures that may differ greatly from those of other school districts.
(C) Upon leaving the teaching profession, many young teachers cite a lack of opportunity for more rapid financial advancement as a primary factor in the decision to change careers.
(D) A merit-based system that bases compensation on teacher performance reduces collaboration, which is an integral component of quality instruction.
(E) In school districts that have implemented pay-for-performance compensation structures, standardized test scores have dramatically increased.

61. Network executives have alleged that television viewership is decreasing due to the availability of television programs on other platforms, such as the internet, video-on-demand, and mobile devices. These executives claim that **declining viewership will cause advertising revenue to fall so far that networks will be unable to spend the large sums necessary to produce programs of the quality now available**. That development, in turn, will lead to a dearth of programming for the very devices which cannibalized television’s audience. However, technology executives point to research which indicates that **users of these devices increase the number of hours per week that they watch television** because they are exposed to new programs and promotional spots through these alternate platforms. This analysis demonstrates that networks can actually increase their revenue through higher advertising rates, due to larger audiences lured to television through other media.

In comparing the executives’ arguments, the portions in boldface play which of the following roles?

(A) The first is an inevitable trend that weighs against the argument; the second is that argument.
(B) The first is a prediction that is challenged by the argument; the second is a finding upon which the argument depends.
(C) The first clarifies the reasoning behind the argument; the second demonstrates why the argument is flawed.
(D) The first acknowledges a position that the technology executives accept as true; the second is a consequence of that position.
(E) The first restates the argument through an analogy; the second outlines a scenario in which that argument will not hold.

62. According to a study of more than 50,000 Norwegian smokers, smokers who reduced their nicotine intake from cigarettes, even by up to 50 percent, did not achieve significant health benefits. The mortality rate for those who cut back on cigarettes was not lower than that for heavier smokers; moreover, the rate of cardiovascular disease was similar across all subsets of smokers in the study. As a result, the sponsors of the study claim that reducing nicotine intake does not improve one's health.

Which of the following, if true, most seriously jeopardizes the findings of the study described above?

(A) The majority of study participants minimized their nicotine withdrawal symptoms through the use of skin patches and chewing gum that provide nicotine to the body.
(B) Many of the study's participants periodically dined in restaurants in which smoking was permitted.
(C) The study's participants started smoking at different ages and had varied initial nicotine intake.
(D) Quitting smoking entirely results in a marked reduction in the ill effects of smoking.
(E) Men and women who smoked pipes and cigars were excluded from the study.

63. In developed countries, such as the United States, the percentage of the population diagnosed with clinical depression is much greater than that in developing countries. Researchers hypothesize that this difference is due to the increased leisure time afforded to residents of developed countries.

The hypothesis of the researchers depends on which of the following assumptions?

(A) Clinical depression is a genetically transmitted malady.
(B) Access to accurate diagnostic procedures for depression is equal for residents of developing and developed countries.
(C) Most activities characterized as "leisure time" in developed countries are inherently boring.
(D) Certain medications that effectively treat clinical depression are not readily available in developing countries.
(E) Few residents of developing countries dedicate any of their time to leisure.

64. In an attempt to reduce misbehavior, the junior high school principal has decided to make punishments stricter, including prohibiting unruly students from having playtime outdoors. However, this action is clearly counter to the principal's goals, as studies have shown that students who frequently play outdoors are also less likely to misbehave.

Which of the following is an assumption on which the argument depends?

(A) The cited studies were conducted by academic researchers adhering to established standards of scientific research.
(B) Students who have been punished by not having playtime outdoors are less likely to misbehave again in the future.
(C) Well-behaved students do not share a particular trait, such as a desire to socialize with other well-behaved students, that leads them to play outdoors more frequently.
(D) The principal's suggested policy will not be protested vehemently by local parents who want their children to play outdoors.
(E) Playtime outdoors has not been demonstrated to have positive effects on the physical development of schoolchildren.

65. Most cable television companies currently require customers to subscribe to packages of channels, but consumer groups have recently proposed legislation that would force the companies to offer á la carte pricing as well. Subscribers would pay less, argue the consumer groups, because they could purchase only the desired channels. However, the cable industry argues that under the current package pricing, popular channels subsidize less-

popular ones, providing more options for viewers. For this reason, the industry claims that it is always cheaper for the consumer to purchase many bundled channels than to buy them individually.

Which of the following would be most important for the government to determine before deciding whether to require cable television companies to offer á la carte pricing in order to reduce consumer costs?

(A) whether the total number of channels offered to consumers would decrease, along with programming diversity, as a result of the á la carte pricing structure
(B) whether advertising revenue for the cable television companies would decrease as a result of the á la carte pricing structure
(C) whether a large number of consumers would, in fact, significantly reduce the number of channels purchased if given the option of purchasing them individually
(D) whether the number of cable television consumers has been declining as a result of new avenues of content delivery, such as the Internet
(E) whether á la carte subscribers would be required to have new television set-top boxes

66. A certain pharmaceutical firm recently developed a new medicine, Dendadrine, that provides highly effective treatment of severe stomach disorders that were previously thought to be untreatable. However, to develop the new medicine, the company spent nearly $5 billion in research and development costs. Given the size of the market for Dendadrine and the amount of the initial investment in its development, the company would need to sell Dendadrine at a price that is at least five times greater than its variable costs just to break even. Yet the company’s management claims that Dendadrine will soon become the major driver of the firm’s profits.

Which of the following statements best reconciles the management’s claim with the evidence on the expenditures associated with the development of Dendadrine?

(A) The pharmaceutical firm expects to be granted patent protection for Dendadrine; drugs under patent protection typically sell at prices that are approximately 10 times their variable costs.
(B) The development of some pharmaceutical products involves substantial initial expenditures on research, testing, and approval.
(C) In clinical tests, Dendadrine has proven far more effective at treating severe stomach disorders than any prior available treatments, without any serious side effects.
(D) No competitors are developing or planning to develop new medicines that might compete with Dendadrine in the marketplace.
(E) Millions of people suffer from severe stomach disorders, representing an estimated one to two billion dollars every year in revenue.

Logic-Based Reading Comprehension Answers

1. **(C).** You are asked to find a conclusion that the given information supports. For this sort of problem, always stick as close as you can to the passage. The guidelines call for including vegetables (not fruits) in every meal. The school board has replaced fries with fruit. While this does sound like a nutritional improvement, all you can infer for certain is that the guidelines are not being met.

(A) You know nothing from the passage about the relative health value of fruits and vegetables.

(B) Likewise, the passage provides no information about how likely students are to eat any kind of food. Avoid bringing in outside knowledge or suppositions, when you are trying to draw a proper conclusion from given information.

(C) CORRECT. This choice spells out the only inference you can legally make: the board is not following the guidelines.

(D) You do not know whether the board is responsible for the health of the students. It may be, or it may not be.

(E) This opinion may seem reasonable, but it is not at all proven by the passage, which simply presents facts. In general, avoid making value judgments when drawing a conclusion from given factual information. The premises would have to contain opinions as well.

2. **(C).** You are asked to strengthen the conclusion, which is *public officials shouldn't buy lottery tickets* (as indicated by *Therefore*). Some of the answer choices provide reasons why *no one* should buy lottery tickets. However, you are looking for a choice that supports this *specific* conclusion in the context of this argument by linking the key premise (*buying lottery tickets is a form of gambling*) to the conclusion. You need either to state the missing assumption or provide direct support for it.

(A) People who play the lottery are not likely to win, it's true. This is a reason why people in general should not buy lottery tickets. But you need a stronger link between premise and conclusion.

(B) It's irrelevant whether some public officials are guilty of more serious offenses than gambling.

(C) CORRECT. This choice provides the link you seek. Premise: *Buying lottery tickets = gambling*. Conclusion: *Public officials shouldn't buy lottery tickets*. The missing assumption, which this choice supplies word for word, is *Public officials shouldn't gamble*. If an argument says *Certain people shouldn't do* X *because* X *is* Y, then you need to assume that those people shouldn't do Y.

(D) It may be true that many officials are tempted to violate the rules, but even if so, it wouldn't strengthen a moral prohibition (the conclusion tells you what public officials *shouldn't* do).

(E) Like choice (A), this choice provides a reason why people in general shouldn't play the lottery, but it does not provide as strong support for the conclusion as choice (C) does.

3. **(C).** The question asks you to support a judgment (that is, a conclusion) on the basis of the statements in the passage. Remember to stay close to those statements— do not make additional assumptions. Look for what you can prove.

(A) Careful! You don't know whether *Tunbridge College provides the best value in the state* — the speaker has simply pointed out that Saddlebrook can't be the best value, since another college is a better value (in a variety of ways). It could be that Saddlebrook and Tunbridge are actually the 10th and 7th best values in the state, for instance. (Think about when you say things like, "Uncle Jay thinks he's the smartest person in our family? Even my eight-year-old is better at math than he is!" You're probably not arguing that your eight-year-old is the smartest person in the family — you're just pointing out that Uncle Jay can't be the best, since at least one other person is above him.)

(B) This choice is not a judgment that could be proven from premises. If you added it to the premises, it would perhaps weaken the argument that Tunbridge is a better value than Saddlebrook (for instance, higher-caliber students might be more likely to earn larger incomes after graduation, regardless of the value of the education).

(C) CORRECT. This choice takes a much safer position than choice (A). The second sentence in the passage describes various ways in which Tunbridge is a better value than Saddlebrook, and so you can conclude that Saddlebrook is *not* the best value.

(D) This choice represents an assumption in the argument — that income after graduation is part of evaluating which college provides the best value — but you are being asked here for a conclusion, not an assumption within the argument.

(E) This choice is irrelevant — *satisfaction* is not the same as *value*.

4. **(B).** This is a standard argument where the author (or, in this case, the researchers) have confused *correlation* with *causation*. If two traits X and Y are found together (in this case, red cars and speeding tickets), you do not have any particular reason to believe that X *causes* Y. After all, it could be that Y causes X, or that some third factor Z causes both (in this case, the type of person who buys a red car also tends to get speeding tickets). In addition, you should notice that the correlation is specifically observed between the *color of the car* and the *number of speeding tickets*. As most every driver knows, getting a speeding ticket depends on more than just speeding (driver behavior) — you also have to be unlucky enough to catch the attention of a police officer. Thus, the researchers have built in a second assumption — that the increased number of speeding *tickets* is the result of an increase in the number of speeding *infractions* (and not, say, the increase in *attention* that a red car might get).

(A) This choice regarding black cars is completely irrelevant to this argument.

(B) CORRECT. If the researchers believe this, they've eliminated the idea that the *reverse causation* could be possible. That is, this statement gets rid of the possibility that the color, rather than the increased speeding, is what is responsible for the increase in tickets. Note that this assumption is not the only one the argument requires, but it is certainly required by the argument.

(C) What color cars police officers drive does not matter at all.

(D) This choice focuses on an entirely different type of ticket — a parking ticket — and is thus irrelevant. You have no information regarding tendency to speed, which is necessary for the researcher's argument.

(E) This choice is outside the scope of the argument, as it deals with events after the act of speeding or the issuance of the ticket. It may give hope to drivers of red cars, but is completely irrelevant to the question at hand.

5. **(E).** The headmaster's argument explicitly relies on the idea that students perform better on math tests when they are fully awake. Noting the drop in scores, the headmaster reasons that this drop is due to the change in class time as opposed to any other factor or combination of factors. The assumption you are looking for must connect the dots between the time of the class and the waking state of the students.

(A) The logistics of the schedule are irrelevant to the headmaster's position that scores would improve if the class were moved.

(B) The time at which other schools hold math classes is irrelevant to the headmaster's argument, especially since the efficacy of that scheduling is not mentioned.

(C) The idea that the quality of teaching has little bearing on scores (whether or not you agree with this in the real world!) might provide a tiny bit of support to the headmaster's idea. After all, if the quality of teaching hardly influences test scores, another factor might be expected to have greater impact — and that factor might be time of day. However, it is a huge stretch to say that one must assume that some other factor (such as teaching quality) has little effect on test scores, in order to conclude that time of day could influence those scores.

(D) provides an alternative explanation as to why the scores might have dropped this year (the use of an unfamiliar curriculum), but this explanation is not an assumption upon which the headmaster's argument depends.

(E) CORRECT. This choice indicates the direct link needed between time of day and student wakefulness.

6. **(E).** ITEL believes that it hasn't lost its valuable employees after layoffs because it has a highly risk-tolerant environment. To further support this claim, you need information that supports the idea that *this specific reason* is why ITEL hasn't lost any of its valuable employees after the previous four layoffs.

(A) This choice offers one reason why those who do change their jobs might do so, but the conclusion in question here is why certain people would *keep* their jobs, not change their jobs.

(B) This is almost certainly true in the real world, but it does not directly address the specific question here: *why* ITEL claims it is able to retain its most valuable employees. Even if you say that this choice does offer an alternate explanation for the retention of employees (if a company handles the layoffs well, valuable employees may be more likely to stay), then this choice would actually weaken the conclusion. It would offer an alternate explanation as to why ITEL is retaining its employees, not the reason ITEL claims.

(C) This choice does the opposite of what you want: it weakens the conclusion. If people are staying with ITEL because they cannot find a new job, then ITEL is not correct in claiming that people are choosing to stay because of the highly risk-tolerant environment.

(D) Like choice (B), this choice is very likely true in the real world, but it does not address the specific question here: *why* ITEL claims it is able to retain its most valuable employees. As such, it cannot strengthen ITEL's claim that its high risk tolerance allows it to retain its employees.

(E) CORRECT. If intolerance toward risk is considered a significant detriment by employees in the industry, then having a higher risk-tolerance environment would be considered an advantage. This strengthens ITEL's claim that its high risk-tolerance helps it to retain employees.

7. **(C).** The exterminator explains the effects of using poisoned food to combat a mouse infestation: mice will pick up the food and carry it to the nest, where all of the mice will eat the food and die as a result. The exterminator then claims that *if* X happens, *then* it will be certain that Y happened first. Specifically, he claims that *if* all signs of the mice disappear for three consecutive weeks after setting out poisoned food, *then* the mice all must have died as a result of that poisoned food. The correct answer will weaken this claim, for instance by showing that if X happens, it will not necessarily mean that Y happened; perhaps there is another reason why X happened.

(A) This choice is very tempting, but the exterminator argues that "all signs" of the mice "disappear"; this choice mentions only the difficulty of seeing and hearing mice. These do not represent "all signs" of an infestation; mice may leave droppings, chew things up, and so on. Further, the mice are not harder to see and hear only after poison has been used. They are always hard to see and hear, so this does not provide an alternate explanation as to why signs might diminish or disappear after poison has been used.

(B) This choice may be true, but it does not address the exterminator's claim that if all signs disappear, then the *poison* must have killed the mice.

(C) CORRECT. This choice offers an alternate explanation for the absence of signs of an infestation. If the mice have left the house, then any signs of infestation would also cease because the mice are no longer there. However, the signs will have ceased because the mice left voluntarily, not because they have been eradicated by the poison, as claimed by the exterminator.

(D) It is likely true that different poisons have different levels of efficacy, but this argument does not distinguish between types of poison. The claim is simply that the poison must have killed the mice (as opposed to some other explanation for why the signs of mice would disappear).

(E) This is another tempting choice; however, the exterminator does not claim that the poison will work within three weeks (or within any time frame at all). He only claims that *if the signs disappear* over three consecutive weeks, then that means that the poison worked. It might be the case that the signs don't disappear until weeks 3, 4, and 5, but the exterminator's claim would still hold: because the signs have disappeared for three consecutive weeks, the poison worked.

8. **(E).** The question presents seemingly contradictory premises. On the one hand, ammonium triiodide is a powerful explosive that is easy to make from just two ingredients. On the other hand, apparently no terrorists have used this chemical in attacks. The correct answer will reconcile these premises by presenting a relevant, reasonable explanation for why terrorists have not used this explosive, despite its advantages. In other words, there must be a clear and serious obstacle preventing the use of this chemical.

(A) This answer choice does present an obstacle: one of the ingredients has limited availability. However, the obstacle is not described as serious. The ingredient can only be obtained from *somewhat more restricted sources* than a grocery store. For example, *chemical supply houses* can provide the ingredient.

(B) Other explosives can be made from ammonia, but this fact does not satisfactorily explain why terrorists have not used this explosive.

(C) This answer choice presents another seeming obstacle: many terrorists are more focused on weapons of mass destruction than on these kinds of chemical explosives. However, this does not mean that *all* terrorists are more focused on weapons of mass destruction. Moreover, even terrorists who are more focused on more devastating weapons may still be interested enough in conventional explosives to use ammonium triiodide in some attack.

(D) Airport security devices may be calibrated to detect ammonium compounds, an obstacle to their use in attacks on planes or airports. But terrorists might still use such compounds in other sorts of attacks.

(E) CORRECT. Only this answer choice presents a clear and serious obstacle to the use of ammonium triiodide in terrorist attacks. Since the chemical is unstable when dry, terrorists are unlikely to use the chemical in attacks, since some part of it might dry out and cause an early explosion.

9. **(C).** The conclusion is that "if major industries increase their capital reserves, the employment rate will not decline in the future." Why? Because major industry did not have capital reserves. The author assumes that having capital reserves is sufficient to prevent a decline in the employment rate. You are asked to cast doubt on the author's claim.

(A) Whether the drop in employment was foreseen does not relate to the core of the argument, which is that capital reserves will prevent another decline in the employment rate.

(B) The fact that some major industries had appreciable capital reserves does not contradict the claim that an increase in these reserves would prevent a future drop in employment rates.

(C) CORRECT. The author neglects to take into account the fact that other factors, such as an increase in labor costs, could adversely affect the employment rate. For example, if the cost of labor becomes prohibitively expensive, even with increased reserves, the employment rate could decline.

(D) Legislation mandating a certain level of reserves does not contradict the claim that increased reserves would prevent a drop in employment rates.

(E) The fact that the employment rate was more severe this year than last does not contradict the claim that an increase in reserves would prevent a drop in the employment rate.

10. **(E).** The conclusion of the argument is that *new methods of math education in this country do a disservice to our children*. Why? Because math teachers emphasize *in higher grades the same narrow, skills-based approach that students learned in lower grades rather than the analytical tools they will need to solve complex math problems*. In order to weaken the conclusion, you need to show that this approach has not had a negative effect on children's math skills.

(A) Music is used simply as an analogy in the argument. It is not central to the logic.

(B) The argument suggests that students are getting worse at advanced math skills due to new methods of teaching. This answer choice provides no basis for evaluating students' math performance over time; you do not know how performing "no worse than" students from other countries compares with students' performance prior to the introduction of the new teaching methods.

(C) The simple fact that older students arrive at different answers does not address the concerns of the argument: that students are not prepared for higher-level math.

(D) The fact that older students receive better grades in math does not address the concerns of the argument: that students are not prepared for higher-level math.

(E) CORRECT. This choice states that an increasing percentage of native first-year students qualify to take college courses requiring advanced math. This would seem to suggest that more children are prepared for advanced math than had previously been the case, thus weakening the conclusion of the argument.

11. **(B).** The first boldface sentence states that the fight against the drug trade in Country X should focus for the time being on tightening the country's borders and targeting its major smugglers. The second boldface sentence states that the United Nations and the government of Country X should eventually replace the poppy fields with other farming ventures ("agricultural infrastructure"). You need to find a choice that describes the relationship between these two sentences.

(A) This choice states that the first sentence is the conclusion and that the second sentence is an alternative to that conclusion. This misrepresents the relationship. The first sentence is a shorter-term conclusion, and the second sentence is a longer-term conclusion of the argument.

(B) CORRECT. This choice states that the first sentence is a short-term solution to a problem and the second a long-term solution to the same problem. This accurately describes the relationship. The first sentence states that the fight should focus *for the time being* on borders and smuggling while the second sentence states that the United Nations and the government of Country X should work to replace the poppy crop with something else.

(C) This choice states that the first sentence presents a problem. According to the text, however, the first sentence is not a problem but a solution to a problem.

(D) This choice states that the first sentence presents a popular solution to a problem. Since you have no way of knowing whether the solution is popular, this cannot be correct.

(E) This choice states that the first sentence presents an argument, and that the second sentence provides evidence to support that argument. Though the first sentence does present an argument, the second sentence, rather than providing evidence, presents a second argument.

12. **(B).** The conclusion is that a developer who wishes to make a large profit would be wise to buy urban waterfront lots and erect residential buildings on them. The basis for that claim is that people pay large sums for beach front homes. You are asked to strengthen this argument.

(A) This choice states that people have more buying power today than in previous centuries. This does not strengthen the claim that a developer will make money on urban waterfront properties.

(B) CORRECT. This choice states that homeowners will be willing to spend large sums of money on residential properties in traditionally industrial or commercial districts. Since you know from the argument that urban waterfronts have traditionally been industrial, this fact strengthens the claim that a developer can make a profit on urban waterfront properties.

(C) This choice states that many urban waterfront lots are available for purchase. This does not suggest, however, that a developer will be able to sell them after he or she builds on them.

(D) This choice states that many coastal cities are giving tax breaks to developers who rehabilitate the waterfront, but this does not suggest that anyone will buy the developed properties.

(E) This choice states that properties in the interior of cities are more expensive than those on the waterfront. Although waterfront properties are therefore cheaper to acquire, this does not necessarily mean that a developer can make a profit after buying such properties.

13. **(D).** The passage states that mentally ill people fare best when two conditions are met: (1) they are not confined to institutions, and (2) they receive high-quality outpatient care. When Congress allowed many mentally ill people to leave mental institutions, it ensured that condition (1) was met for those who were released. The passage does not tell you, however, whether Congress also ensured that condition (2) would be met. The plight of the mentally ill in 1983 would not be very surprising if those who had been released never received the high-quality outpatient care that they needed.

(A) An increase in the *number* of people diagnosed with psychiatric disorders does not explain why the average mentally ill person would be faring worse.

(B) The difference between outcomes for men and women is interesting, but without making additional assumptions, you cannot use this difference to explain the drop in average welfare for the mentally ill.

(C) The discovery of new medications does not explain why the mentally ill were doing worse in 1983; if anything, it makes their plight somewhat more surprising, since the new medications may have benefited some people with mental illness.

(D) CORRECT. This resolves the paradox by offering a plausible explanation of why the mentally ill were worse off in 1983. If Congress never supplied the funding that would have been necessary to provide high-quality outpatient care to the newly-released patients, those patients did not fulfill condition (2) for improved welfare in psychiatric patients.

(E) The specific diagnoses of those who fared worst in 1983 cannot resolve the paradox under consideration, since those individuals may well have had the same illnesses before they were released.

14. **(A).** The official's conclusion is that people who claim that the US is more vulnerable than other nations because of the country's lack of a national vaccine laboratory are disloyal and incorrect. His basis for that conclusion is that the US has generally long life span and low infant mortality relative to all UN countries. Moreover, he cites the high quality of American hospitals, and he adds that many people he knows from around the world come to the US for medical care. You are asked to find the choice that does NOT point out a weakness or potential weakness in the official's argument.

(A) CORRECT. This choice states that the high quality of hospitals is *not* a factor affecting the public's vulnerability to infectious disease. However, the quality of hospitals very arguably does affect how vulnerable the public is to such disease.

(B) This choice highlights the official's logical jump from "disloyal" to "wrong" in the phrase *these critics are disloyal and thus wrong about the public's vulnerability*. There is no necessary connection between disloyalty and wrongness.

(C) If the Europeans that the official cited overwhelmingly consisted of wealthy men over the age of fifty, then the official relied on an unrepresentative sample to justify his claim. What is true of wealthy older European men is not necessarily true of Europeans or non-Americans generally.

(D) If the average life span of Americans is determined by causes other than infectious disease, then the official is not limiting his evidence to cases relating to vaccines. Other causes of death are not relevant to the critics' argument.

(E) The ranking of the United States relative to all UN countries is misleading, since it does not compare the US to other "advanced industrialized" or "developed" nations, as the critics' claim does.

15. **(C).** The conclusion of the argument is that renewable sources of energy, chiefly solar and wind, will be less risky for certain utilities than nonrenewable sources, such as oil and gas. The basis for this claim is that the renewable sources will provide stable, low-cost supplies of energy, whereas the prices for nonrenewable sources will fluctuate according to availability. You are asked to find an assumption underlying this argument. In order for this argument to be valid, it must in fact be true that these renewable sources of energy will provide stable, low-cost supplies.

(A) The utility companies' claim has to do with the supply risk of the new energy sources, not with how these sources are received by the public.

(B) If no new supplies of traditional energy sources are found, then it is true that perhaps these nonrenewable supplies will continue to fluctuate in price in a risky manner. However, the argument does not depend upon any assumption about the future discovery of oil and gas supplies.

(C) CORRECT. If you assume that weather patterns are consistent and predictable, then with the stated premises, you can conclude that solar and wind power will be less risky than oil and gas. If, on the other hand, weather patterns are not consistent and predictable, then solar and wind power are not reliable and thus will not provide *stable energy supplies at low cost*. Thus, the argument's conclusion directly depends on this assumption.

(D) To reach the required conclusion, it is not necessary to assume that the conversion technology for new sources is not more expensive than the present technology.

(E) This choice does not directly affect the argument. Whether or not energy produced through combustion can be made less risky, the new energy sources might still be less risky than the older sources.

16. **(D).** The conclusion of the argument is that one need not worry about ingesting pesticides when purchasing produce from farms that use only organic pesticides. The basis for that claim is the fact that organic pesticides leave the surface of produce within a few hours of spraying. In order for this argument to be valid, you must assume that the organic pesticides do not harm the produce in any lasting way.

(A) This is unrelated to the argument, since the conclusion speaks about not having to worry about ingesting produce on which only organic pesticides were known to be used.

(B) If anything, this statement runs counter to what the argument is saying. If produce that has been sprayed with organic pesticide reaches the final consumer within hours after it is picked, it is possible that the consumer *does* need be concerned about the pesticides.

(C) The conclusion of the argument is already limited to those farms which use "only organic pesticides."

(D) CORRECT. If a pesticide is capable of penetrating the skin of a fruit or vegetable then, while the organic pesticide will dissipate from the surface of the fruit in a few hours, it may remain inside the fruit. The author of this argument assumes that the pesticides cannot penetrate the skin.

(E) The issue of cost is unrelated to the argument's conclusion about pesticide residues.

17. **(E).** The conclusion of the argument is that the government should educate the public about the dangers of inactivity and poor diet in order to stop the spread of type-2 diabetes. The basis for the claim is that inactivity and poor diet are the main factors in developing type-2 diabetes. You are asked to find a choice that will show that this plan likely will not work.

(A) The fact that schools educate middle school students about a disease that is generally "acquired in adulthood" does not address the effectiveness of an adult education plan sponsored by the government.

(B) The fact that the public already has access to this information through the Internet, does not say anything predictive about the effectiveness of the plan. Even with access to the information, there is a good chance that most people are not exposed to the information.

(C) Just because food companies encourage the public to indulge in unhealthful snacks, does not mean that a program that teaches them to do otherwise would not be successful.

(D) The fact that the government has not set aside money for such a program, does not say much about the projected program's effectiveness.

(E) CORRECT. The choice states that *healthful foods and exercise programs are beyond the financial means of many people*. This suggests that even with the best planning, the program might not achieve its goals simply because people cannot afford to follow the program's advice.

18. **(D).** The argument focuses on the relative cost of goods and services in Countries X and Y due to an exchange rate that has historically favored the currency of Country Y. The argument presents an apparent discrepancy: the citizens of Country Y often take their vacations in Country X, yet rarely purchase clothing or electronics in Country X, despite the fact that those items are more expensive in their home country, even when sales taxes are taken into account. The correct answer will be one that indicates some reason why it might not actually be cheaper for citizens of Country Y to purchase clothing and electronics in Country X.

(A) The fashion preferences of the citizens of Country Y do not directly explain their buying habits. To be relevant, this statement would have to go further to link fashion preferences to purchasing decisions: for example, it would have to continue …*and are therefore willing to pay more for the goods available in their own country*.

(B) The fact that stores in Country Y receive the latest fashions and technology earlier than stores in Country X does not address the buying habits of the citizens. Even if you could assume that the citizens of Country Y demand the latest goods as soon as they are available, it still would not directly explain their buying habits. To be relevant, this statement would have to go further to link consumer preferences to purchasing decisions: for example, it would have to continue …*and the citizens of Country Y are willing to pay more in their own country just to acquire the latest goods as soon as they are available*.

(C) The attitude of the citizens of Country X is irrelevant to an argument about the purchasing habits of the citizens of Country Y.

(D) CORRECT. This choice states that the government of Country Y imposes tariffs on imported goods. This suggests that perhaps items that are purchased in Country X and brought into Country Y become prohibitively expensive because of the tariffs and could explain the spending habits of the citizens of Country Y.

(E) The currency of Country Z is irrelevant to why citizens of Country Y rarely purchase clothing or electronics in Country X.

19. **(A).** The basic structure of this argument is fact that *mold is almost always found in places where there is substantial moisture*, so therefore, to avoid mold and the resultant mold poisoning, then people should take steps to prevent wet areas. This argument assumes that wet areas occur first, causing mold to grow. Conversely, this assumption requires that the mold growth itself does not occur first, creating wet areas as a result.

(A) CORRECT. The argument depends on the assumption that the reason mold and wetness are observed together is that wet areas cause mold growth. If the reverse causation (mold causes wetness) were true, then keeping all plumbing in good condition to prevent leakage would do little to prevent the growth of mold. This choice eliminates the alternate causation.

(B) If most homeowners knew enough about plumbing to determine whether theirs is in good condition, then the recommendation made in this argument would be more useful. However, this is not an assumption on which the argument depends.

(C) Even if mold could grow in dry areas, the fact that mold is almost always found in wet areas is still valid. This is the fact upon which the argument is based, so the argument does not depend on the unnecessarily absolute assertion that mold cannot grow in dry areas.

(D) Even if some varieties of mold are harmless, the conclusion of this argument, that *one should make sure to keep all internal plumbing in good condition to prevent leakage* and minimize mold growth, could still be valid. Therefore, this argument does not depend on the unnecessarily absolute assertion that no varieties of mold are harmless.

(E) Whether mold spores can be filtered from the air may be relevant to a conclusion about the health effects of mold in the home, but it is not directly relevant to this conclusion, that *one should make sure to keep all internal plumbing in good condition to prevent leakage* and minimize mold growth.

20. **(D).** This argument discusses a plan with the stated goal "to prevent overcrowding." Two points of view are represented in the argument: one is that of the town zoning board, the originators of the plan, and the other is that of critics of the plan. The question asks for information that would most support the claims of the critics of the plan. Conversely, the correct answer would most weaken the logic of the town zoning board.

(A) How other towns fared under similar zoning plans is irrelevant to this argument, unless additional information were presented to connect the situation of this town to those. You have no such connecting information, so this choice neither strengthens nor weakens either point of view.

(B) How long it has been since the construction of the last school in town is irrelevant to this argument. The argument addresses methods to prevent overcrowding, an issue that is not directly related to school construction.

(C) The argument never mentions property taxes, so how property taxes in this town compare to those in neighboring towns is irrelevant.

(D) CORRECT. The argument states that *the town zoning board limited the number of new buildings that can be constructed in the town in any given year*. The goal of this plan is to prevent overcrowding, but only does so indirectly: the town zoning board plan limits the number of new buildings, not the number of new town residents. Either an apartment building or a single family house would only count as one new building, yet the apartment building could draw many more new residents to the town than would the single family house. This plan might not be the best way to achieve the stated goal, and thus this statement supports the claims of the critics of the plan.

(E) The distance to the nearest garbage dump is irrelevant to an argument about how to prevent overcrowding.

21. **(C).** The conclusion of the argument is that *the retail price of chocolate is certain to increase within six months*. This claim is based on the fact that the wholesale price of cocoa solids and cocoa butter has increased significantly and is likely to stay high. The argument assumes that the retail price of chocolate is driven by the wholesale price of cocoa, and ignores the cost of the other ingredients in chocolate. To evaluate the conclusion of the argument, it is necessary to examine whether this assumption is valid.

(A) The price of cocoa solids and cocoa butter during other periods of poor harvest is irrelevant. There is no guarantee that the market for chocolate would respond the same way in this case.

(B) The willingness of consumers to spend more for chocolate is irrelevant when evaluating whether chocolate will be more expensive due to a shortage of cocoa beans.

(C) CORRECT. If the price of other ingredients in the chocolate has dropped, then the decrease could offset the higher price of cocoa, and the retail price of chocolate could remain steady.

(D) The percentage of cacao trees affected by the fungus is irrelevant. Regardless of the percentage, it would not change the fact stated by the argument that *there was an unusually meager harvest of cocoa beans this year*.

(E) It may seem useful to determine whether the fungus can be eliminated within the next six months. However, the conclusion was not about the fungus or the cacao crop, but rather the retail price of chocolate. The time frame for the elimination of the fungus would only be relevant to the short-term retail price of chocolate if you make several additional assumptions: that the harvest immediately increases, and that the resulting glut of cocoa beans immediately offsets the *unusually meager harvest of cocoa beans this year*, and that the wholesale price of cocoa immediately drops to its pre-fungus level, and that the retail price would not be affected in the meantime. You cannot make all of these assumptions, so the answer to this question is irrelevant to the conclusion.

22. **(A).** The head of engineering has concluded that the one-time doubling of costs for the raw material caused the steady decline, over two years, of profit margins. The engineer's conclusion rests on the assumption that there is a connection between the one-time raw material price increase and the two-year steady decline in profit margins. Alternatively, something else could have caused either a steady two-year decline in revenues or a steady two-year increase in costs; if so, this event is more likely to be the cause of a steady decline in profit margins over the same two-year period. As a result, this would weaken the engineer's conclusion that finding a new source for the raw material will improve profit margins.

(A) CORRECT. New competitors have caused a steady two-year decline in revenues. This weakens the engineer's contention that the one-time doubling of costs for the raw material is the cause of the steady two-year decline in profit margins.

(B) The fact that the region's mines are producing less than they did before the earthquake does not indicate anything about the cost of the raw material; it would be necessary to show that cost is not reduced in order to weaken the engineer's conclusion.

(C) The amount of raw material produced by other regions does not indicate anything about the cost of the raw material; it would be necessary to show that cost is not reduced in order to weaken the engineer's conclusion.

(D) The use of a different raw material does not indicate anything about the cost of that raw material; it would be necessary to show that cost is not reduced in order to weaken the engineer's conclusion.

(E) Although *mining* the raw material may become more cost-effective for the mine (that is, cheaper), this does not tell you what will happen to the *price* they charge for the material when selling to the company producing the product in question. It would be necessary to show that the company's cost is not reduced in order to weaken the engineer's conclusion.

23. **(E).** The argument concludes that children are more likely to attend college if they are sent to private high schools instead of public high schools. The basis for this claim is the higher percentage of graduates of private schools pursuing college education. It is assumed that public schools are inferior to private schools as a training ground for college. Any statement that provides an alternate explanation for the fact that public school graduates attend college at lower rates than private school graduates would weaken the argument. Another way to interpret this question involves the concepts of correlation and causation. The argument's premise states that private school attendance (versus public school attendance) is highly correlated with college attendance. The conclusion of the argument is essentially that private school attendance CAUSES college attendance (and therefore, parents ought to send their children to private schools to ensure eventual college attendance). This conclusion depends on the assumption that the causation does NOT work the other way — in other words, that "readiness or desire to attend college" does not influence the choice of public or private school. Any evidence that readiness or desire to attend college DOES influence the choice of public or private school will weaken the argument.

(A) While higher test scores might increase students' chances of admission to college, they are unrelated to whether students will actually attend college. Even if one could prove that earning higher test scores makes a student more likely to attend college, this statement would not weaken the argument, but rather strengthen it.

(B) Since the conclusion centers on the likelihood of attending college, economic and financial considerations are outside the scope of the argument.

(C) Since the amount of need-based aid is not directly related to whether a student will attend college, this statement is outside the scope of the argument.

(D) While better athletic opportunities could increase students' chances of admission to college, they are unrelated to whether students will actually attend college. Also, even though the advantages of public school mentioned in this statement were taken into account by the study, the proportion of graduates of public schools attending colleges remains substantially lower than the proportion of graduates of private schools.

(E) CORRECT. This answer choice demonstrates that the difference in the percentage of graduates attending colleges stems not from any advantage provided by private schools but from the fact that a subset of the graduates of public high schools simply choose to pursue a different career path. In other words, 30% of the graduates of public schools voluntarily choose not to pursue a college education. Yet 65% out of the 70% of graduates remaining end up in colleges. This statement indicates extremely high college matriculation rates for students who want to attend college after graduation from public high schools. Using the concepts of correlation and causation, this answer choice provides the alternative causation for the correlation observed. In other words, "desire to attend college" is *already* lower in the rural areas where public schools happen to be located. According to this evidence, attendance at private or public school is the effect, not the cause, of "desire to attend college."

24. **(A).** Only two pieces of information are given about Airline A's standing room "seats" proposal. First, that it is geared toward increasing revenue in order to counteract declining profits. And second, that, since the proposal relates to passenger safety, it must be approved by the Federal Aviation Administration. Airline A must have concluded that the cost of implementation of its proposal is less than the revenue that the new seats will generate.

(A) CORRECT. Since Airline A knows that its proposal would have to comply with safety standards, it must have concluded that the cost of compliance is worth it. In other words, the only way for Airline A to achieve its goal of increasing profit is to implement ideas that will generate more revenue than they cost. Airline A must therefore have concluded that the standing room only "seats" meet this criteria.

(B) The statements in the passage imply nothing about whether Airline A believes that the Federal Aviation Administration will approve the proposal. Although Airline A must believe that the proposal has a chance of being approved (otherwise it's unlikely to have proposed it), the airline might have proposed its specific plan knowing that it might not be approved or, that it might have to be changed in certain ways.

(C) Airline A's goal is simply to "counteract declining profits" caused by the high cost of jet fuel. This does not mean, however, that the proposal must fully mitigate the cost of jet fuel. As long as the proposal increases revenue without a corollary increase in cost, it will in some way (even if it's relatively small) counteract declining profits.

(D) The passage does not mention any other ways that Airline A has considered increasing revenue. Therefore, it is impossible to conclude anything about Airline A's perception of its standing room "seats" proposal to any other ideas.

(E) The statements in the passage do not address Airline A's view regarding the safety of the standing room only "seats." It is very possible that Airline A views its proposal as safe and sees no conflict between passenger safety and increasing revenue, much less that it has made any determination about the relative importance of these two issues.

25. **(D).** The letter writer believes that if criminal penalties for drug use are eliminated, the incidence of armed robbery and other violent crimes will decrease. In support of that belief, the letter writer offers the second boldface portion of the argument: an assertion that crimes are committed by drug users because they need money to buy expensive illegal drugs, and that if drugs were legal and therefore cheaper, the crimes would become unnecessary. The first boldface portion of the argument mentions an observed relationship between drug use and other crimes: when drug use declines, other crimes decline as well. This observation is counter to the letter writer's ultimate claim.

(A) The letter writer forecasts that violent crime will decline even if drug use is decriminalized. The first boldface portion does not offer support for that forecast, but rather evidence that violent crime decreases when anti-drug laws are enforced. The second boldface portion is not the letter writer's forecast, but rather the support given for it.

(B) The first boldface portion is an observation that violent crime decreases when anti-drug laws are enforced; that observation weighs against the letter writer's main position, but falls short of refuting his claim that violent crime will decrease as a result of decriminalizing drug use. The second boldface portion is support for the letter writer's main position, not the position itself.

(C) The first boldface portion shows a direct relationship between a decline in drug activity and a decline in violent crime, but the letter writer does not argue that future events are predicted by this relationship. In fact, the letter writer ultimately claims the opposite: that violent crime will decrease when criminal penalties for drug use are eliminated, even if drug use increases as a result.

(D) CORRECT. The first boldface portion shows a direct relationship between a decline in drug activity and a decline in violent crime, but the letter writer claims that violent crime will decrease when criminal penalties for drug use are eliminated, even if drug use increases as a result. If true, the information in the second boldface section explains why the letter writer makes the claim that the high cost of illegal drugs is the reason drug users commit violent crimes, so cheaper, legalized drugs will cause crime to decline.

(E) The first boldface portion is presented by the letter writer as true. However, the second boldface is not an inference drawn from the first boldface portion; rather, it contradicts the first boldface portion.

26. **(B).** Although the premises of this argument suggest only a correlation between smoking and anxiety or nervousness, the argument has a causal conclusion: it concludes that smoking causes individuals to be anxious and nervous (i.e., that A causes B). Any assumption in a causal argument must support the causal "direction" of the conclusion, that A causes B as opposed to some other explanation. Often, assumptions support a causal conclusion either by eliminating an alternate cause for the conclusion (that C did not cause B) or by demonstrating that the causation, if one exists, is in the proper direction (that B did not cause A).

(A) The argument concludes that smoking causes anxiety and nervousness. Whether these maladies lead to more serious health problems is not relevant to the conclusion.

(B) CORRECT. For smoking to be the cause of anxiety and nervousness (A caused B) it must be true that these individuals were not more likely to be anxious and nervous before they started smoking. If smokers had these preconditions, which contributed to their decision to begin smoking (B caused A), your conclusion — that smoking causes these maladies — would be incorrect.

(C) The argument concludes that smoking causes anxiety and nervousness. The number of survey respondents is not relevant to the conclusion.

(D) The argument concludes that smoking causes anxiety and nervousness. The awareness of the health problems related to smoking is not relevant to the conclusion.

(E) The argument is not based on the immediate impact that smoking has on anxiety and nervousness. Moreover, the argument never compares some smokers to other smokers.

27. **(D).** Because of the speculation that the tuberculosis outbreak in Country X was the result of an outbreak of tuberculosis in Country Y, health officials in Country X have proposed requiring all visitors from Country Y to undergo a medical examination. You are asked to find a choice that suggests that this proposal will not have the desired effect of curbing the spread of tuberculosis in Country X.

(A) This has no bearing on the situation between Country X and Country Y.

(B) This suggests only that the proposal would not prevent ALL cases. But even if the proposal does not prevent all cases, it could help prevent many.

(C) This suggests only that the proposal would not catch ALL carriers of the disease from Country Y. But even if the proposal does not prevent all cases, it could help prevent many.

(D) CORRECT. This suggests that the visitors from Country Y are not the source of the disease. Thus, testing them would likely do little to curb the spread of the disease.

(E) This does not suggest that the proposal will not help curb the spread of the disease. If the visitors from Country Y are indeed carriers, then their refusal to visit Country X will help curb the disease.

28. **(D).** The argument concerns the economic impact on restaurants in Prohibitionland if the service of alcoholic beverages is banned. It presents evidence that, despite restrictions on the service of alcohol in certain areas of Prohibitionland, sales taxes in restaurants in those areas rose at a higher rate than for those in other parts of Prohibitionland, suggesting that the ban would not have any adverse economic impact. You are asked to support the restaurant proprietors' claim, so the correct answer choice will call the relevance of the seemingly contradictory evidence into question.

(A). This answer choice may seem to strengthen the argument that banning the service of alcoholic beverages would have an adverse impact on restaurants. However, as the evidence involves data for the entire year, citing a short-term negative impact on restaurant visitation at the beginning of the year does not measurably strengthen the argument.

(B) The relative tax rate on food and beverages as compared to other consumer good is irrelevant here.

(C) A gradual decline in alcohol consumption over the past 20 years would suggest that over time, any ban on alcohol would have an increasingly small impact on restaurant visitation, weakening the proprietors' argument.

(D) CORRECT. This statement calls the evidence into question by indicating that any measured increase in sales taxes and, presumably, revenues for restaurants that have been operating under the restrictions last year enacted is irrelevant, as the restrictions could be argued to be completely different than the total ban that is being proposed. This answer choice substantially strengthens the proprietors' argument by threatening to make the cited evidence irrelevant.

(E) The fact that overall sales tax revenue did not increase at a higher rate in the provinces that enacted the restrictions on alcoholic beverages weakens the proprietors' argument, as it makes the cited evidence more compelling by ruling out the possibility of different growth rates in the different areas.

29. **(A).** The passage concludes that the whereabouts of most of the world's most valuable artwork is probably unknown. The basis for this claim is that serious art collectors are discreet when purchasing or selling significant pieces of art. To weaken this claim, one would need information that demonstrated that serious art collectors, while discreet in purchasing and selling their artwork, were relatively open with their art once it was in their possession. Alternatively, one could weaken this claim using information that showed that serious art collectors possessed only a small fraction of the world's most valuable art. The question asks you to find an answer choice that does NOT weaken the conclusion.

(A) CORRECT. The fact that the value of a piece of art is subjective is irrelevant to the reasoning of the argument. It does not present any information that would weaken the link drawn between the whereabouts of valuable artwork and the fact that serious art collectors are discreet when purchasing or selling such artwork. Thus, this choice does NOT weaken the conclusion.

(B) The fact that serious art collectors publicize their art shortly after purchasing it means that the whereabouts of their valuable art must be widely known. This choice makes clear that serious art collectors are discreet only in purchasing and selling their artwork, and that the whereabouts of newly purchased artwork is unknown, if at all, for only a short period of time. As such, this choice weakens the conclusion.

(C) If museums own the vast majority of the world's valuable artwork, then the practices of serious individual art collectors are essentially irrelevant to the location of most of the world's valuable artwork. Therefore, this choice weakens the conclusion since the public nature of museums means that the whereabouts of most of the world's valuable work is widely known.

(D) Since the majority of the world's valuable privately held artwork is owned by individuals who are not considered serious collectors, then the practices of serious art collectors are essentially irrelevant to the location of most of the world's valuable artwork. This choice weakens the conclusion by removing the link between serious art collectors and most of the world's valuable artwork.

(E) The fact that the collections of most serious art collectors are often displayed in public settings means that the whereabouts of their valuable art must be widely known. This choice makes clear that serious art collectors are discreet only in purchasing and selling their artwork, but that once in their possession, the artwork is typically unveiled for the public. Therefore, this choice weakens the conclusion.

30. **(A).** The argument explains that the new "Click It or Ticket" law is generating controversy. Under the new law, drivers can be cited for not wearing their seat belts, even in the absence of an additional driving infraction. Any acceptable inference must be directly supported by evidence from the text.

(A) CORRECT. The entire controversy is based on the new law that allows motorists to be cited, even in the absence of an additional infraction. Thus, it follows that prior to the passage of this law, an additional driving infraction must have been necessary in order to stop and cite an individual for not wearing a seat belt.

(B) Search and seizure laws are never mentioned in the text. This answer choice is outside the scope of the argument.

(C) Laws in other states are never mentioned in the text. This answer choice is outside the scope of the argument.

(D) Though the text states that the new regulation might save countless additional lives, the effectiveness of the previous laws are never mentioned.

(E) No preference is stated between law enforcement groups and the citizens' groups. This answer choice is simply an opinion that is unsubstantiated by the text.

31. **(E).** The passage provides some specific information about the effects of calorie restriction. In rats and mice, this diet is known to prolong life by preventing diseases. In a study of moderately overweight humans, insulin levels and body temperature decreased. A proper GRE inference will be based on the specific information provided, without relying on any significant assumptions.

(A) The passage states that calorie restriction in mice and rats prolongs life by preventing diseases. The human study had much more limited findings — that calorie restriction in moderately overweight humans decreases insulin levels and body temperature. While these traits are known to be associated with longevity, there are no data that link calorie restriction itself to prolonged human life. Additionally, calorie restriction may have other unstated effects, unrelated to longevity. There is no information in the passage that indicates whether these effects are the same in humans as in mice and rats. Finally, the use of the term *humans* is far too general; the study dealt only with moderately overweight humans and so any inference would need to be restricted to this subset of individuals.

(B) While the passage indicates that certain traits known to be associated with longevity are found in moderately overweight humans who reduce their calorie intake, this is far removed from the conclusion that calorie intake will actually increase a human's lifespan. Additionally, as with answer choice (A), the use of the term *humans* is far too general.

(C) The study observed that individuals with the greatest percentage decrease in their calorie intake demonstrated the greatest decrease in insulin levels and body temperature. This shows a strong correlation between calorie intake and insulin levels. However, this correlation is not necessarily direct. It is possible that this correlation holds, but only up to a point. For example, it might be the case that any reduction in calorie intake over 50 percent does not result in any additional insulin level decreases. Moreover, the passage only draws this correlation for individuals with the greatest percent decrease in calorie intake. It is very possible that individuals with a relatively low decrease in calorie intake exhibit the exact same decrease in insulin levels as individuals with a moderate decrease in calorie intake.

(D) The study makes no reference to the health of individuals who reduce their calorie intake. It tries to draw some connection to the longevity of those individuals, but longevity is not the same as health. An individual could live a very long, unhealthy life.

(E) CORRECT. The passage states that the greatest decrease in insulin levels was observed in individuals with the greatest percentage change in their calorie intake. This means that some individuals in the study reduced their calorie intake by a greater percentage than other individuals in the study. The passage also states that the study participants reduced their individual calorie intakes by *at least 25 percent*. Thus, one can safely infer that there were some participants who reduced their calorie intake by more than 25 percent.

32. **(D).** The argument explains that school officials plan to limit the sugar intake of students in school by replacing sugary drinks with less sugary drinks in school vending machines. To undermine this plan, the correct answer must demonstrate that that the consumption of sugar by students will not be dramatically reduced.

(A) A comparison between unsweetened fruit juices and bottled water is not relevant, as these are both less sugary alternatives to the high calorie sugary drinks.

(B) The intention of the school officials' plan is to limit the intake of sugar by students. Access to sugary drinks outside of school is certainly a factor in students' total sugar intake. However, the possibility that students might consume sugar elsewhere fails to undermine the school officials' plan to restrict access to sugary drinks at school. As a result of the restricted access during school hours, those students who have access to sugary drinks outside of school may still consume less sugar than they otherwise would.

(C) The fact that certain alternatives to the sugary drinks contain ingredients that cause health concerns is irrelevant to the argument. Students would still be consuming less sugar.

(D) CORRECT. If most of the sugar that students consume in school comes from snack foods rather than drinks, replacing the drinks with healthier alternatives will do little to curb the intake of sugar.

(E) This statistic is not directly relevant to the argument, since it is unclear whether these drinks are consumed at school. If the drinks were consumed at school, this answer choice would strengthen rather than undermine the school officials' plan.

33. **(D).** Research indicates that there is a connection between being married and being happy and healthy. Media commentators have concluded that marriage causes happiness and health. However, one could reasonably conclude from the research that the cause and effect are the reverse: being happy and healthy makes a person more likely to get married.

(A) The research compared married people to unmarried people. Neither the researchers nor the media commentators made any distinction between newlyweds and those who had been married a long time, so this assumption is not necessary.

(B) The type of wedding is outside the scope of this argument. The research compared married people to unmarried people, but made no distinction based upon the type of wedding.

(C) At first, this statement may seem necessary—after all, if the commentators conclude that marriage causes happiness, a lack of depression in married people would certainly support that conclusion. However, the statement is too extreme. One depressed married person does not invalidate the research indicating that, on average, married people are healthier and happier than non-married people.

(D) CORRECT. This statement eliminates the alternative interpretation of the research findings—that being happy and healthy makes a person more likely to get married.

(E) The research compared married people to unmarried people. Neither the researchers nor the media commentators made any distinction between harmonious marriages and combative marriages, so this assumption is not necessary.

34. **(D).** The question asks you to analyze the structure of the argument. Specifically, it asks you to determine the relationship of the two bolded sentences to the argument as a whole. To do this effectively, you need first to determine the conclusion of the argument. In this case, the conclusion is the final sentence, which follows from the evidence presented in the rest of the argument. So you know that the correct answer cannot describe either bolded portion as the author's final conclusion. The first bolded portion is presented as an earlier, erroneous belief. In evaluating the answer choices, you should look to eliminate any that suggest the author agrees with this first statement. The second bolded statement shows that the earlier theory — that lactic acid causes soreness — was incorrect, and thus also provides evidence in support of the author's conclusion. So you know that the correct answer must describe the second bolded portion as supporting the conclusion. The correct answer must describe both portions correctly, not just one or the other.

(A) This choice misrepresents the first bolded portion by claiming that the author accepts it as true, when the author actually presents it as an incorrect, outdated belief. The second portion is also incorrect because it indicates that the second bolded portion supports the first one.

(B) This choice also misrepresents the first portion, for the reason described above. It corrects the second bolded portion, however, by describing it as contradictory to the first portion.

(C) The first portion is correctly described as an assertion that the author does not believe to be true. Yet the second portion is incorrectly described as going against the author's final conclusion.

(D) CORRECT. This choice correctly represents the first portion by saying the author believes it is no longer valid. The second portion is correctly described as evidence in support of the author's conclusion.

(E) The first portion is correctly described as evidence considered by the author to be invalid. However, this choice misrepresents the second portion as the conclusion, when it is actually evidence given in support of the conclusion.

35. **(B).** The conclusion of the argument is contained in the last sentence, that *the methods that the prospector had used to determine the size of the oil deposit must have been inaccurate*. The evidence provided is that the prospector reported a large oil deposit that was later determined to be much smaller in size. You are asked to find an unstated assumption that makes the conclusion valid based upon this evidence. In order to do this, you need to assume that there is not another reason why the prospector might have reported a larger oil deposit than actually existed.

(A) It is not necessary to the conclusion that a third party affirmed the company's determination that the oil deposit turned out to be small. The conclusion accepts that the oil deposit was indeed smaller than indicated by the prospector, and focuses on the cause of the discrepancy as opposed to the discrepancy itself.

(B) CORRECT. The argument concludes that the prospector's methods resulted in inaccurate measurements of the size of the oil deposit. This assumes that the prospector did not simply misreport or misrepresent the measurements, presumably for personal gain. This answer choice addresses the most plausible alternative explanation to the one given, and is necessary for the conclusion to stand based upon the evidence presented.

(C) The commercial feasibility or profitability of the oil deposit is not integral to the argument, or its conclusion. This statement is not an assumption that would support the conclusion.

(D) Whether or not the prospector utilized the same methods in regards to measuring the oil deposits in other locations is not relevant to the argument, or the conclusion.

(E) The fact that the company had a long operating history and experience in drilling oil wells is not relevant, in that the company's measurements of the size of the oil field are accepted as given in the argument.

36. **(C).** This passage relates information from two studies concerning high school seniors: the first discusses the financial responsibilities of high school seniors, while the second explains the coursework in finance taken by typical high school seniors. On the GRE, a proper response to a draw-a-conclusion question must be directly supported by evidence from the passage.

(A) Although it might be true that schools would be wise to educate students in finance, this is an opinion; it doesn't necessarily need to follow from the given evidence. A conclusion must be directly supported by evidence from the passage without any additional information or assumptions.

(B) The fact that one-third of high school seniors claim *significant financial responsibilities* to their families does not necessarily mean that these same students work *part-time jobs after school*. There are many possible ways that these students might earn money for their families. If they do work, they might work on weekends or over the summer, for example.

(C) CORRECT. The first study states that one-third of all high school seniors have significant financial responsibilities to their families. The second study states that 80% of seniors have opened a bank account, and of this 80%, one-third has bounced a check. The number of seniors that has bounced a check (one-third of 80%) is fewer than the number of seniors with significant financial responsibilities to their families (one-third of 100%).

(D) The passage states that certain high school seniors who contribute to the food, shelter, or clothing for themselves or their families *rate themselves* as having significant financial responsibilities. This does not mean that any high school senior who contributes to these categories has significant financial responsibilities.

(E) The passage states that one-third of high school seniors say that they have *significant financial responsibilities*. This in no way indicates that the other two-thirds have "no" responsibilities. Because no information is given about the other two-thirds of the students, a reasoned conclusion cannot be drawn about them.

37. **(E).** According to the statements, the companies that own private aircraft for business use are fully in compliance with the relevant law, which is summarized. A correct inference will be a statement that must follow from at least part of the premises given.

(A) It does not have to be true that the law costs the businesses money, as no evidence about the relative costs is given.

(B) This choice is an irrelevant comparison, as the preferences of the executives are not the concern of the statements.

(C) This choice does not have to follow, as there is no information given about the travel arrangements made by large companies. The statements only indicate that the majority of private planes are not owned by large companies.

(D) There is no information given about the travel arrangements of upper level executives and no reason to believe that those with the companies discussed do not comply with their companies' policies.

(E) CORRECT. If, as the statements indicate, the companies are in full compliance with this law, it must be true that the executives following their guidelines also are.

38. **(E).** The conclusion of the argument is that insurance companies do not have a significant economic incentive to delay claim payments to doctors. To weaken this conclusion, an answer choice must provide some significant economic incentive for insurance companies to be tardy in paying doctors for legitimate medical claims.

(A) While the fact that some doctors who submit accurate bills to insurance companies still receive tardy payments seems to indicate that there must be something other than errors causing delayed payments, it fails to prove that the insurance company has an economic incentive to deliberately delay claim payments to doctors. For example, this fact could simply indicate that the insurance companies are inefficient at handling all of their paperwork.

(B) This choice compares costs insurance companies must absorb due to incorrect bills to costs physicians must absorb due to tardy payments. However, this information is irrelevant to establishing an economic incentive for insurance companies to delay claim payments to doctors.

(C) The argument is focused on the payment of legitimate claims; the rising proportion of illegitimate claims does not establish a clear economic incentive for insurance companies to delay payments of legitimate claims.

(D) The types of billing errors made by doctors' offices does not establish any economic motive for insurance companies to make a practice of delaying payments to doctors.

(E) CORRECT. This choice articulates a logical chain that establishes a clear economic motive for insurance companies to be tardy in paying doctors for legitimate medical claims. If insurance companies delay payments to doctors, this results in a 10 percent increase in overhead costs for physicians. These costs ultimately result in higher fees that doctors charge to insurance companies. Insurance companies, in turn, raise the premiums they charge consumers for health coverage. This choice states that the insurance companies increase their fees to consumers far more than the doctors increase their fees to insurance companies, enabling the insurance companies to pocket the difference; therein lies the economic motive for insurance companies to be tardy in paying doctors for legitimate medical claims.

39. **(D).** Farmers in developing countries claim that the global price of wheat is low because American farmers produce too much of the grain. They also claim that American farmers produce too much wheat because they have no incentive to manage their crops, since the US government will buy whatever wheat American farmers cannot sell on the open market. You are asked to find a choice that weakens the claims of the farmers in developing countries that removing the American subsidy would cause the price of wheat to rise.

(A) That there are uses for wheat that is not eaten is irrelevant here. This does not address any aspect of the farmers' claims.

(B) The fact that buyers of wheat can predict their needs in advance is irrelevant here, because the text indicates that American farmers do not pay attention to actual demand for wheat.

(C) In this argument, the global market for soybeans is irrelevant to the global market for wheat, which is a different commodity with different demand, supply, and pricing structures.

(D) CORRECT. The farmers assume that the sole cause of the wheat surplus is the United States. This answer choice suggests that other countries would modify their output to counterbalance any reduction on the part of the United States, keeping prices constant instead of allowing them to rise.

(E) The price of another crop is largely irrelevant. Moreover, the fact that the price of sorghum, a non-subsidized crop, is lower tends to support, rather than weaken, the claims of the farmers.

40. **(E).** The researchers claim that Delta-32 prevents its carriers from contracting the plague. They support this claim by noting that a strikingly large percentage of descendants of plague survivors carry the mutation. You are asked to find an assumption underlying the claim.

(A) The argument is specific to the relationship between Delta-32 and resistance to the plague. Other diseases are irrelevant.

(B) Again, the argument is specific to the relationship between Delta-32 and resistance to the plague. Other diseases are irrelevant.

(C) Delta-32 may have existed in its current form before the 16th century and the merit of the argument would not change.

(D) The argument does not claim that Delta-32 prevents all bacteria-caused disease.

(E) CORRECT. The researchers claim that Delta-32 prevented its carriers from contracting the plague on the basis of its presence in descendants of plague survivors. But it is theoretically possible that these descendants carry the mutation Delta-32 because the plague mutated the genes of their ancestors. In order to claim that the mutation prevented the plague, you must assume that the plague did not cause the mutation Delta-32.

41. **(E).** Consider the structure of the library funding argument. First, a proposal is made (the legislature asks the public to approve a bond to fund libraries with money outside the existing budget). Second, the argument acknowledges that the decision maker (the public) supports the intended outcome of the proposed project (building and improving public libraries). Third, the argument urges rejection of the proposal anyway (since the money could come from a better source — money in the existing budget that is currently being wasted). Finally, the argument implies that the decision maker should reject this proposal because it can achieve the intended outcome by some better means.

(A) In the library funding argument, the decision maker (the public) actually wants the outcome of the proposal (libraries) but wants to achieve that outcome via a different path. Here, in contrast, parents are the decision makers, but the proposed outcome (dessert first) is desired only by the proposal maker (children). Further, the proposed solution (dessert only if dinner is finished) is not an alternate way to achieve the original proposal (dessert first).

(B) This argument is unlike the library funding argument because it focuses on the negative consequences of a proposal and its effects on a third party (neighbors without alarm systems).

(C) In this argument, a proposal is made by terrorists: they kidnap victims in order to gain publicity. The decision maker is this country. Unlike the library funding argument, here the decision maker does not agree with the proposed outcome (increasing publicity for the terrorist group).

(D) In this argument, an employer proposes a specific pay increase. The decision maker is the labor union, and here the proposal offers the union only one part of what it wanted. In the library argument, the decision maker is urged to reject a proposal with a good outcome in order to achieve that outcome by other means. Here, the decision maker is urged to reject the proposal because it is incomplete.

(E) CORRECT. This argument follows the same format as the library funding argument. First, a proposal is made (the student wants to stay up late to finish his homework). Second, the argument acknowledges that the decision maker (his parents) supports the intended outcome of the proposed project (doing well in school). Third, the argument urges rejection of the proposal anyway (since the student wastes time earlier in the evening that could be spent on school work). Finally, the argument implies that the decision makers should reject this proposal because they can achieve the intended outcome by some better means.

42. **(C).** You are asked to determine what you can infer based upon the given information; on the GRE, whatever you infer must be based only upon the argument itself and cannot extend beyond the scope of that argument.

(A) The argument says nothing about requirements in other states; this choice is out of scope.

(B) While it may be illegal for those under 21 to consume alcohol, the argument makes no mention of special rules for this age category. Indeed, the argument says that hunters have to sign the pledge, not just hunters aged 21 and over.

(C) CORRECT. The argument says that the hunter must have completed a safety program within the past five years. If he has not, then he will have to do so before he can be eligible for a permit. (Note that this choice doesn't say he will get a permit if he completes the program again; you don't know what other requirements he may not meet. You only know that he will have to retake the program, at least, before he can become eligible.)

(D) This choice mentions someone who *isn't 18 years old*. Someone who isn't 18 can be either younger than 18 or older than 18 and, according to the argument, those older than 18 are eligible for permits if they complete all of the requirements. according to the argument, those older than 18 are eligible for permits if they complete all of the requirements.

(E) While this may be true as a general rule, it is outside of the scope of the argument, which mentions nothing about danger or why these rules were enacted. In addition, any requirements cannot *ensure that nobody gets hurt*; they can only help to minimize risks.

43. **(A).** The argument claims that wide dissemination of wireless access is now a practical way to meet urban needs, based on the evidence of its successful use in rural areas. The author then must assume that urban areas provide no additional problems for wireless use.

(A) CORRECT. This choice confirms an assumption of the argument and thus strengthens the conclusion.

(B) This choice weakens the argument because it damages the assumption that urban areas pose no extra problems for wireless use.

(C) This choice is irrelevant because it provides information about another rural area; the conclusion concerns urban areas.

(D) This choice weakens the argument because it damages the assumption that urban areas pose no extra problems for wireless use.

(E) This choice is an irrelevant distinction. The argument mentioned all three groups as in need of this service. The suggestion that one group needs it more than the others is irrelevant to the conclusion.

44. **(D).** The conclusion of this argument is that *many weight loss efforts fail because people eat more calories than they intend to consume*. The first boldface portion is a factual premise ("Studies have shown…") that there is an observed correlation between keeping a diet record and losing weight successfully. This premise (indirectly) supports the researchers' conclusion. The second boldface portion is another supporting premise, this one citing a specific study showing that dieters who do not keep a diet record eat far more than they realize.

(A) The first boldface is not the conclusion, it is an observed fact. The second boldface is evidence that the researchers' conclusion is correct, but is not evidence that the first boldface is correct.

(B) The first boldface is a fact that supports the researchers' theory, but it does not explain why their conclusion is correct—the other premises do so.

(C) The first boldface is a fact that supports the researchers' theory, but it does not illustrate the truth of that theory—the second boldface does. The second boldface is a fact that supports the researchers' theory; it is not a competing theory.

(D) CORRECT. The first boldface (diet record = diet success) is a basis for the researchers' conclusion that many weight loss efforts fail because people consume more than they intended. The second boldface directly illustrates how weight loss efforts of a certain group failed for exactly that reason.

(E) The first boldface is a factual statement, not a theory. Furthermore, the first boldface supports the theory of the researchers; it is not something they have disproved.

45. **(B).** This argument concludes that the city should convene a conference of relevant parties to create opportunities for out-of-work young people. The argument's premise is that the retirement of the baby boomers will create shortages. The argument assumes the efficacy of its conclusion — in other words, that the conference will actually be effective in creating job opportunities. Attacking an assumption is an effective way to weaken an argument.

(A) If anything, this choice strengthens the argument. If immigration does not provide a labor pool, it is more likely that a shortage will ensue.

(B) CORRECT. The argument assumes that it is feasible to affect employment patterns by government encouragement and/or action. If that assumption is denied, the conclusion is weakened, as the conference would be pointless.

(C) This choice makes an irrelevant distinction. It doesn't matter if the best positions require skills, as long as the majority are available to the unskilled unemployed in question.

(D) Knowing that a small proportion of baby boomers will not retire on schedule does not significantly weaken the argument. The argument relies on general estimates, not on exact numbers.

(E) If anything, this choice strengthens the argument. If these people are unaware of these opportunities, it would be positive to convene to plan how to reach them.

46. **(A).** The argument itself provides one example of the Doppler effect: a siren decreases in perceived pitch as it approaches, passes and then moves away from an observer. The answer choice that most closely describes a similar scenario is a correct illustration of the Doppler effect.

(A) CORRECT. The passengers on the westbound train see the eastbound train approaching, passing, then traveling away from them. The sound of the horn, therefore, will decrease in pitch for the westbound passengers.

(B) The passengers on the westbound train see the eastbound train approaching, passing, then traveling away from them. The sound of the horn, therefore, will decrease in pitch for the westbound passengers. This choice, however, says that the sound will increase in pitch.

(C) Relative to the passengers on the eastbound train, the eastbound train and its horn are in fact standing still! The Doppler effect only describes a *perceived change in pitch that occurs when the source of a sound is in motion relative to the observer,* a situation that is not illustrated by this choice.

(D) Relative to the passengers on the eastbound train, the eastbound train and its horn are in fact standing still! The Doppler effect only describes a *perceived change in pitch that occurs when the source of a sound is in motion relative to the observer,* a situation that is not illustrated by this choice.

(E) Relative to the passengers on the eastbound train, the eastbound train and its horn are in fact standing still! The Doppler effect only describes a *perceived change in pitch that occurs when the source of a sound is in motion relative to the observer,* so while it is true that the eastbound passengers would hear their train's horn at its true pitch, this situation does not illustrate the Doppler effect.

47. **(E).** The author concludes that one will only be able to determine the age of a Brazilian ash by counting its rings if the temperature in the tree's environment never exceeds 95 degrees Fahrenheit. The author bases this conclusion on the fact that the tree loses rings when the temperature exceeds that level. However, if the number of rings lost by a Brazilian ash at high temperatures can be predicted, it may be possible to determine the age of a tree even if the temperature exceeds 95 degrees.

(A) The argument says nothing about precipitation. This answer choice is out of scope since it would require a number of other assumptions to make it relevant to the argument's conclusion.

(B) Whether other trees share this feature is irrelevant; the argument focuses only on the Brazilian ash.

(C) The number of days of excessive heat needed to cause the tree to lose rings is irrelevant.

(D) The thickness of the rings is irrelevant.

(E) CORRECT. The conclusion is that the rings will be a reliable measure only if the temperature never exceeds 95 degrees. This is true only if there is no way to predict how many rings would be lost when the temperature does exceed 95 degrees. If it were possible to predict this, one might be able to assess the age of a tree using its rings even if the temperature had exceeded 95 degrees.

48. **(E).** The text tells you that celiac disease results when the body mistakes gluten for a harmful pathogen, causing damage to the intestine. You are also told that gluten is a protein found in certain grains, and that people suffering from celiac disease must eliminate it from their diets. Finally, you are told that symptoms of the disease include cramps, bloating, and anemia. You need to find an answer choice that is inferable from these facts alone.

(A) Anemia is just one of several symptoms of the disease. You do not know whether everyone who has the disease will also develop anemia.

(B) You do not know whether eliminating gluten will cure the disease, only that people with the disease must not eat gluten. Perhaps the disease will exist anyway in a latent form.

(C) You do not know whether the symptoms mentioned are also symptoms of other conditions.

(D) You do not know whether gluten is found only in grains. It may exist in other foods as well.

(E) CORRECT. If the body mistakes gluten for a harmful pathogen, then it must be true that the body cannot always recognize harmless substances.

49. **(E).** This argument proposes that culture does not influence the process by which the mind distinguishes colored objects. In support of the conclusion, the argument notes that all languages with six colors name the same colors, as

well as all languages with three colors. To weaken the conclusion, you need evidence to support the counter-argument that culture *does* influence how the mind distinguishes colored objects. Notice the logical difference between distinguishing color *names* and distinguishing colored *objects*. For instance, a speaker of English can visually distinguish two objects with different shades of red, even if he or she cannot easily name the difference. Some of the wrong answers attempt to confuse these two processes.

(A) Irrelevant. It may be interesting to observe that no language has more than eleven basic color terms. However, this observation neither weakens nor strengthens the argument that culture influences how the mind perceives color variations.

(B) Irrelevant. In fact, this statement may slightly strengthen the argument: if every language permits speakers to describe subtle color variations, then it might be argued that human color perception is independent of language, since language would then not be "boxing in" the speaker.

(C) Irrelevant. The term *red* may encompass both red and yellow, but that doesn't mean that speakers of the language can't *see* the difference between red and yellow. In the same way, the English word *blue* covers many shades that English speakers with normal vision can distinguish (sky blue, royal blue, etc.)

(D) Strengthens. If speakers of languages without a blue-green distinction refer to the sky or tree leaves to clarify their meaning, then they obviously *see* a difference between the sky and tree leaves. Using the sky or tree leaves as reference points is no different from using fruit names for colors (e.g., orange, peach). This evidence reinforces the idea that humans have a common basis for perceiving colors.

(E) CORRECT. If Tarahumara speakers are less able to identify differences between blue and green *objects* than Spanish speakers, then it can be argued that the lack of a blue-green distinction in the Tarahumara language influences how Tarahumara speakers actually perceive colors. It should be noted that this evidence does not completely prove the point — the causality could actually work the other way (e.g., Tarahumarans could share a genetic defect that causes blue-green color blindness, and so they never developed a distinction in their language). Also, if the experiment is not well designed, the difficulty in identification could simply come from the lack of a handy term to capture the difference of blue and green. However, the evidence as given does support the hypothesis that culture influences color perception.

50. **(A).** This question is an Analyze the Argument Structure question; the best approach is to identify the conclusion, and then determine how each boldface portion of the argument relates to the conclusion. The conclusion of this argument appears in the third sentence: *doctors determined that Anopheles mosquitoes were responsible for transmitting the disease to humans*. The first boldface statement tells you that malaria was named based upon the original, but incorrect, view that the disease was caused by "bad air"; the statement does not support the conclusion. The second boldface statement provides the specific mechanism by which the mosquito is responsible for human infection; it therefore supports the conclusion. More simply, the first statement does not support the conclusion, while the second statement does.

(A) CORRECT. The first statement does follow from the original, but incorrect, view that the disease was caused by "bad air." It weakens the conclusion by supporting an incorrect conclusion. The second statement is a supporting premise which explicates (or explains) why the conclusion is true.

(B) A conjecture is a hypothesis or a supposition, but the first statement is not a conjecture; it is a fact. The second statement contradicts the original conclusion, but it does not contradict the fact provided in the first statement.

(C) To illuminate a thing is to clarify or explain it. The first statement does not explain any assumption (or the original conclusion, for that matter); it merely states a fact that follows from the original, erroneous conclusion. The second statement does not confirm anything in the first half of the argument; rather, it supports the later, correct conclusion.

(D) The first statement did not cause the erroneous conclusion; rather, it is a result of that conclusion. The second statement is a premise that supports the correct conclusion.

(E) The first statement does provide additional detail about the original, erroneous conclusion. The second statement is not the actual conclusion (or judgment); it is a premise in support of the later conclusion.

51. **(A).** In this argument, a cause-and-effect relationship is presented between American scientists signing long-term contracts with foreign companies and the government's restrictions on stem cell research. This cause-and-effect relationship is the key to the correct answer.

(A) CORRECT. If American scientists signed the contracts because of US restrictions, you can infer that the new companies they signed with were under fewer restrictions. Therefore, at least *some* foreign companies *must* work under fewer restrictions than *some* American companies do.

(B) While it is possible that once the restrictions are banned, American companies will want to hire more scientists and will seek them overseas, there are too many unknowns between the premises you have have been given and this conclusion. It is doubtful that an increase in the number of immigrating stem cell research scientists would have a *significant* impact on the number of *foreign professionals* overall.

(C) This passage is about government restrictions; you are given no information about *financial backing*. Beware of extreme statements such as *in all parts of in all parts of the world*.

(D) You are not given any information regarding America's current or future position in terms of stem cell research. Though restrictions and scientists switching companies are two issues related to a company's prosperity, you are given no information about how these directly affect America's position.

(E) You are not given any information that will help you predict the behavior of the scientists in the future.

52. **(D).** In the argument, the critics make the error of mistaking correlation with causation. While it is true that most employees were reassigned to the mission to Mars, it is not established that the mission to Europa was scrapped in order to move the employees to a different project, or if there was some other motivation that caused the demise of the Europa project. The conclusion that NASA presents is that the critics, in saying that public interest is the key motivator behind NASA's actions, are misinformed.

(A) Irrelevant. The conclusion is based on the critics' opinions on causation, and this answer fails to address the issue of what motivated NASA in its decision-making process. Beware of reading too deeply into the information presented. The fact that the percentage of spending is going to go down could indicate many possible scenarios, including a) NASA is unhappy with the progress of the project and plans to cut future spending or b) they expect the development of the shuttle to be completed by 2013.

(B) Weakens. If public interest determines its budget, NASA has strong motivation to keep public interest high. Additionally, this is a highly unlikely answer because it concerns "budget," and "budget" is not mentioned in the original argument.

(C) Irrelevant. This statement differentiates between the opinions of some scientists and the opinions of others, but sheds no light on the motivations behind NASA's decisions. Not only is the answer choice only indirectly related to your statement, it also adds very little new information.

(D) CORRECT. This answer choice provides an alternate reason why NASA scrapped its plan for sending an unmanned vessel to Europa — the Tokyo telescope provides the information NASA would have attained from the mission, making the mission unnecessary.

(E) Irrelevant. The conclusion deals with NASA's motivations; this statement is about the motivations of American citizens. There is no direct relationship between what Americans see as the future of space exploration and the motivations behind NASA's decision-making.

53. **(B).** The argument claims that no one with a compromised immune system should travel to Nicaragua's "Mosquito Coast," due to the risk of contracting dengue fever or malaria. The correct answer choice is the choice that *least* strengthens the argument. Therefore, the wrong answers must each clearly strengthen the argument in one of several ways: 1) increasing the *probability* of infection, the primary negative outcome; 2) increasing the *harmful effects* of infection; or 3) increasing the harmful effects of *alternative outcomes* — for instance, demonstrating the negative side effects of prevention methods.

(A) Strengthens. Since the method of preventing or treating one of the diseases can have bad side effects, the danger is made worse — particularly for people with compromised immune systems.

(B) CORRECT. If the government were to succeed in this effort, there would be somewhat less risk to travelers with compromised immune systems. This would weaken the conclusion that such travelers should avoid the Mosquito Coast. If the effort were to fail, it would produce no change in the danger to such travelers. Either way, this answer choice does nothing to strengthen the argument that such travelers should avoid the Mosquito Coast.

(C) Strengthens. Since a compromised immune system doesn't effectively fight either dengue fever or malaria, people with such immune systems are at increased risk of infection by such diseases.

(D) Strengthens. The populations of both mosquitoes grow during rainy season; thus, the chance of contracting either disease also grows.

(E) Strengthens. The insect repellent may be the most effective available, but it can also have a terrible side effect. Using this repellent is one of the options for dealing with the risk of mosquito-borne disease. Since this option is more dangerous than before, the travel advisory is strengthened.

54. **(D).** According to the text, the Peruvian government claims that Inca Trail treasures would have deteriorated without a new permit program that has restricted the number of tourists. Supporting statements would likely emphasize one of the following two ideas. First, A high number of tourists *causes* the deterioration of Inca Trail archaeological sites, and so a reduced number reduces the deterioration. (Note that this causal connection, while reasonable, cannot be assumed. Evidence that would support this connection might include *a comparison with similar ruins* that did not benefit from a permit program and that have experienced greater deterioration, or *a comparison between two periods of time* (before and after 2001) for Inca Trail ruins.) Second, other results of the permit program (e.g., new revenue) *help prevent* archaeological damage along the Inca Trail.

(A) Irrelevant. The increase in pay may have resulted from the permit program, and it may be reasonable to assume that this increase in pay has led to greater satisfaction in the job and hence, perhaps, to greater care for historical artifacts. However, this chain of reasoning is too speculative to strengthen the argument appreciably.

(B) Irrelevant. Local villages may have seen a drop in income as a result of the restrictions on tourist numbers, but this does not strengthen or weaken the claim that the permit program prevented Inca Trail ruins from deteriorating.

(C) Irrelevant. The funds are a positive result of the permit program, but if these funds are used to protect or preserve archaeology elsewhere, then they do not impact the preservation of ruins specifically on the Inca Trail.

(D) CORRECT. The more rapid deterioration of similar ruins elsewhere supports the claim that that the permit program has helped prevent deterioration of Inca Trail ruins. Notice that this evidence does not rise to the level of absolute proof; other differences between the ruins might explain the different rates of deterioration. However, this evidence clearly supports the hypothesis that the permit program was successful.

(E) Irrelevant. Without the permit program, it is possible (though far from certain) that the number of tourists hiking the Inca Trail would have risen together with the total number of tourists in Peru. However, an increase in the number of tourists on the Inca Trail would not necessarily have led to greater deterioration of archaeological treasures on the trail.

55. **(E).** The chairman claims that same-day spending at downtown businesses by people attending performances at the Farmsley Center has contributed to the economic revitalization of downtown Metropolis. His argument depends on it being true that this spending represents an increased flow of money into the economy of downtown Metropolis. If, for example, the $83 per visitor that he cites is money that would have been spent in downtown businesses *even if the Farmsley Center had not been built*, the chairman's argument would be unsound. Therefore, you should look for an answer choice that suggests that the $83 per visitor is indeed money that probably would not have been spent in downtown Metropolis in the absence of the Farmsley Center.

(A) Irrelevant. You do not know what contributions to the city the business group has in mind. Perhaps the chairman is being honored for activities unrelated to the Farmsley Center, or perhaps he is being honored simply because Metropolis residents feel civic pride at having a grand performance space downtown. You need more information before you can say that this prize constitutes evidence that the Farmsley Center has helped to revitalize the downtown economy.

(B) Irrelevant. Expensive restaurants may be a *sign* of the economic revitalization of downtown Metropolis, but they do not tell you what causal factors led to that revitalization.

(C) Irrelevant. In the absence of information specifically relating the architecture of the Farmsley Center to spending at downtown businesses, you cannot say that the architect's international standing has helped in the economic revitalization downtown.

(D) Irrelevant. The Farmsley Center may be hosting performances that would otherwise have taken place at other downtown venues, but this does not mean that extra money is being spent downtown.

(E) CORRECT. If suburbanites are coming to Metropolis to see performances at the Farmsley Center, they are bringing money from out of town and spending it in downtown Metropolis. This inflow of money supports the idea that the Farmsley Center has contributed to the economic revitalization of downtown Metropolis. Notice, by the way, that this information about suburbanites by no means constitutes an iron-clad *proof* of the chairman's contention. (It is conceivable, for instance, that in the absence of the Farmsley Center the suburbanites would have found other reasons to visit Metropolis.) Since this is a Strengthen the Conclusion question, however, you do not need to find an answer choice that proves the conclusion — just one that makes the conclusion more likely.

56. **(D).** The argument presents two retirement account options. In the first, taxes are paid when money is withdrawn upon retirement; in the second, taxes are paid when the money is initially deposited into the account. The author assumes that it is better to pay taxes on the contributions than on the withdrawals when the total contribution amount is smaller than the total available for withdrawal. But the amount paid in taxes, whether on contributions to the fund or on withdrawals from the fund, depends on both the *amount* of money contributed or withdrawn and on the *tax rates* applied to those contributions or withdrawals. Thus, to evaluate the argument, it would be helpful for you to know the amounts of money involved as well as the relevant tax rates.

(A) It would be useful to know *both* how much is contributed *and* the value of the account upon retirement. Knowing just the first figure, however, doesn't allow the worker to figure out which retirement account option would permit him to pay less in taxes.

(B) It might be useful if you knew *when and how* the tax rates would increase, but "in the future" is too vague. The tax rates may increase while the worker is still employed, or they may not increase until the worker has retired. Without knowing, you cannot evaluate the conclusion any further.

(C) This may tell you how wisely the worker invested the funds, but it does not provide any information about the tax consequences, which are the focus of the conclusion.

(D) CORRECT. If a worker knows the relative tax brackets to expect during the different stages, he or she can better predict whether it would be less expensive to pay taxes on the contributions or on the withdrawals.

(E) It would be useful to know *both* how much is contributed *and* the value of the account upon retirement. Knowing just the second figure, however, doesn't allow the worker to figure out which retirement account option would permit him to pay less in taxes.

57. **(C).** The argument presents a discrepancy between the percentage of survey respondents who reported that they did vote in an election (71%) and the percentage of eligible voters who did vote in that election (60%). An explanation of the discrepancy will often hinge upon explaining why the apparent conflict does not apply, distinguishing the situation from others, or providing an additional premise that resolves the discrepancy.

(A) If the margin of error is ± 5%, this is the same as saying that the 71% figure could be as low as 66% (or as high as 76%). This accounts for less than half of the discrepancy between 71% and 60%.

(B) This choice does not address the stated discrepancy between those who said they voted and those who actually did vote. You have not been told that voters are required to maintain a primary residence in the home country at the time of the election, nor have you been told that voters must be physically present in the home country in order to cast a vote.

(C) CORRECT. One explanation for the discrepancy between these two results is the possibility that people who do vote will respond to surveys at a higher rate than people who do not vote — in other words, people who do vote are overrepresented in the survey's results. This is an additional premise that would explain the higher percentage of individuals polled indicating that they voted.

(D) While this may be true, the poll did not ask people if they *intended* to vote; rather, it asked people if they had already voted in a past election.

(E) While this may account for some percentage of the discrepancy, you are not given any numerical data to indicate that the entire discrepancy can be explained; "some" could mean only 2 people. In addition, the confused people wouldn't necessarily have responded that they did vote (when thinking about a different election) even though they didn't vote in the national election. They could just as easily have reported that they did not vote (when thinking about a different election) even though they did vote in the national election.

58. **(B).** You are presented with a paragraph of premises and asked to resolve the problem they present: how to minimize the spread of flu via air travel. The correct answer will need to rely specifically on the premises and will not require you to make any inferences or assumptions.

(A) The passage states that the infection can be spread by coughing. The flu virus, therefore, can reach the other passengers in the "closely-packed environment" before it enters any filters that might kill the virus.

(B) CORRECT. Vaccines provide significant protection against developing the virus (not 100% protection, but you are asked to "minimize" the impact of air travel, not eliminate it entirely). If all passengers are vaccinated against the virus, many of those who otherwise would have developed the disease will not, and, therefore, won't spread it to others.

(C) Anyone can contract the virus and subsequently spread it; the mentioned populations are merely "especially vulnerable" to it. Infected people traveling to another place can infect children, senior citizens, and others who have stayed in their home regions.

(D) The passage states that the infection can be spread by coughing; while it may be true that the virus can also spread via hand contact, this information is not stated in the passage.

(E) The passage states that people who develop symptoms before travel begins likely would not make the trip; weeding out those with observable symptoms, then, won't "minimize" the role of air travel because there aren't that many people in this category. The larger danger is those who may be infected but have not yet developed symptoms.

59. **(B).** The journalist cites data about the success of tall people, then concludes that employers have an unconscious bias in favor of tall people. The journalist is assuming that employer bias is the only explanation for the data; look for an answer choice that throws this explanation into question.

(A) Irrelevant. Gender comparisons are irrelevant in interpreting the journalist's data on CEOs, since those data are only about *male* CEOs. Likewise, gender comparisons are irrelevant in interpreting the journalist's data about the general population, since the passage says those data have been corrected for the influence of gender and age.

(B) CORRECT. If socioeconomic status is correlated to both height and educational attainment, you would expect taller people to be, on average, better educated. The economic success of tall people could then be attributed to their higher levels of educational attainment rather than to employer bias.

(C) Irrelevant or Weakens. Professional basketball players, with their above-average height and above-average pay, do account for a small part of the correlation between height and pay. And insofar as being tall is useful in the game of basketball, the high wages of tall players can be explained without reference to any unconscious bias on the part of their employers. Thus, the case of professional basketball players may weaken the journalist's argument slightly. However, basketball pros constitute a tiny fraction of the labor force, so their high pay accounts for an extremely small portion of the $789-per-inch wage advantage conferred by height. Also, basketball players tend to be very tall, so their high wages cannot explain wage differentials among short people — why, for instance, people who are 4'11" tend to earn more than do people who are 4'10". Finally, the high wages of basketball players do nothing to undermine the journalist's interpretation of the data on male CEOs, because basketball players are not CEOs.

(D) Irrelevant. An HR professional might unconsciously favor tall people (or good-looking people, or charismatic people, etc.) without herself being tall (or good-looking, or charismatic, etc.).

(E) Irrelevant. Unless you are told why a tall person tends to stay in a particular job less long than does a short person, you cannot say whether this information bolsters or undermines the journalist's argument.

60. **(D).** The argument is concerned with how public school teachers are compensated. It suggests that educational experts believe that a system of teacher compensation based on performance rather than seniority would help to retain exceptional teachers and maintain quality instruction. The correct answer is the one that most undermines this contention of the educational experts.

(A) Irrelevant. The fact that many factors contribute to job satisfaction and teaching performance neither weakens nor strengthens the argument for a performance-based pay structure for public school teachers.

(B) Irrelevant. Nothing in the argument indicates that one universal system of compensation must be adopted. It is very possible that several effective models of performance-based pay could be developed and implemented successfully.

(C) Strengthens. This choice indicates that many young teachers are extremely frustrated by the traditional pay structure, in which financial advancement is directly tied to seniority. Thus, these teachers would likely welcome a change that allows them more rapid opportunity for financial advancement.

(D) CORRECT. Weaken. This choice indicates that collaboration among teachers is integral to high-quality instruction and that a system of compensation based on teacher performance reduces collaboration. Thus, the effect of a merit-based system of pay would be to undermine quality instruction, which is one of the two stated goals of the educational experts.

(E) Strengthens. The educational experts' argument in favor of performance-based compensation is bolstered if standardized tests scores have dramatically risen in school districts that have instituted such pay structures.

61. **(B).** In order to properly evaluate the role of the two boldfaced portions, you must first identify the conclusion: *networks can actually increase their revenue through higher advertising rates, due to larger audiences lured to television through other media*. The first boldface portion opposes this position by predicting smaller audiences; the second lends support to it by citing evidence that alternate media devices lead their users to watch more television. The correct answer choice will reflect this pattern.

(A) The first boldface does weigh against the argument, but it is a prediction, rather than "an inevitable trend"; the second boldface supports the argument, but is not the conclusion itself.

(B) CORRECT. The argument about potential increased network revenue is contrary to the first boldface's prediction about shrinking audiences and falling revenue; the argument indeed depends upon the second boldface's assertion that users of alternate devices will actually watch more hours of television.

(C) The first boldface opposes the argument, rather than clarifies it; the second would suggest that the argument is sound, rather than flawed.

(D) The technology executives do not accept the prediction of the network executives; the second boldface contradicts, rather than follows as a consequence of, that prediction.

(E) The first boldface does not use an analogy; the second is in agreement with, not opposition to, the argument.

62. **(A).** This question asks you to find the answer choice that "most seriously jeopardizes the findings of the study," or, in other words, the answer choice that weakens the conclusion. The correct answer choice will cast doubt on the sponsors' claim that *reducing nicotine intake does not improve one's health*. One way in which this claim can be weakened is to demonstrate flaws in the study on which the claim is based.

(A) CORRECT. This answer choice indicates that most study participants did not actually reduce their nicotine intake; instead, they simply received nicotine from different sources. Since the data on which the conclusion is based is not reliable, the conclusion is undermined.

(B) Although this choice does indicate that a number of study participants might have consumed nicotine from secondhand smoke, it does not undermine the idea that these individuals, by cutting back their smoking, have significantly reduced their nicotine intake. Before the study, the nicotine intake of these participants was smoking plus secondhand smoke; now, the only nicotine intake is periodic secondhand smoke.

(C) This choice neither strengthens nor weakens the conclusion. The study related solely to the reduction of nicotine intake through smoking and never mentioned these other factors. The fact that the study used a diverse sample of smokers does not, in itself, undermine the validity of the conclusions.

(D) This answer choice is irrelevant. The study concerns the effects of reducing nicotine intake through smoking, not quitting smoking entirely.

(E) This choice is irrelevant. While it further details the methodology used in selecting subjects, it does not strengthen or weaken the conclusion.

63. **(B).** The passage concludes that increased leisure time in the developed world causes an increase in the percentage of people diagnosed with clinical depression. For this causal conclusion to be valid, you must eliminate alternate causes for this disparity. Note that some questions of this type focus on eliminating *reverse* causality. That is, if the conclusion is that X causes Y (because X and Y are observed together), then one assumption is that Y *does not* cause X. Other questions focus on eliminating an *outside* cause. That is, if the conclusion is X causes Y (because X and Y are observed together), then another assumption is that Z (some outside force) *does not* cause Y. As it turns out, this question is of the latter type: eliminating an *outside* cause.

(A) This statement weakens the hypothesis. If clinical depression were genetically transmitted, then the amount of leisure time would have no effect on the percentage of the population diagnosed with clinical depression.

(B) CORRECT. If individuals in the developing and developed worlds do not have equal access to accurate diagnostic procedures, it is possible that either frequent misdiagnoses or a lack of correct diagnoses causes the seeming disparity between the populations. Thus, for the argument to be valid, this assumption must hold true. Put another way, this assumption eliminates the possible *outside* cause (the difference in diagnostic techniques between the developing and the developed worlds).

(C) Nothing indicates that *most* leisure activities must be inherently boring. As long as more individuals in the developed world than in the developing world are experiencing boredom, the logic of the passage remains valid.

(D) This choice weakens the researchers' hypothesis. If fewer effective medications were available in the developing world, the incidence of clinical depression there should be *higher* than in developed countries.

(E) It is unimportant to determine whether few residents of developing countries dedicate any of their time to leisure. The argument already states that residents of developed countries have *increased* leisure time relative to residents of developing countries, so this assumption is unnecessary.

64. **(C).** The argument concludes that the principal's plan to restrict playtime outdoors as a punishment for misbehavior is counterproductive in that outdoor playtime discourages misbehavior. However, this argument assumes that the correlation between outdoor playtime and good behavior is the result of a particular cause-and-effect relationship — that is, outdoor play causes good behavior. It is possible that good behavior spurs outdoor play rather than vice versa; it is also possible that some other factor causes both good behavior and outdoor play. The correct answer will eliminate one or both of these possibilities.

(A) It is not assumed by the argument that the cited studies were conducted by academic researchers adhering to established standards of scientific research, as studies performed by other parties (i.e., consultants hired by a non-profit organization) could be equally well-constructed and valid.

(B) It is not assumed by the argument that students who have been punished will be less likely to misbehave again in the future. It is possible that the punishment is meant primarily as a deterrent. Also, the argument centers on the plan being counterproductive to the goal of reducing misbehavior; this assumption would make the principal's plan more effective.

(C) CORRECT. If students who are well-behaved are more likely to play outdoors as a result of being well-behaved, then the cause-and-effect relationship runs counter to the way it is presented in the argument. If being well-behaved causes children to play outside more, then the principal's plan would not be counterproductive. The punishment (no outdoor play) would not necessarily cause more bad behavior. It should be noted that one could still question the efficacy of the principal's plan on other grounds, but this assumption is essential to the argument that the plan is clearly counter to the principal's goals.

(D) This answer choice is not assumed by the argument. First, there is no indication that the principal will necessarily accede to the wishes of the local parents. Second, the argument concludes that the principal's plan runs counter to the principal's goals, not that implementing the plan will be unfeasible.

(E) This answer choice is incorrect in that physical development is beyond the scope of the argument. The argument focuses on behavior, not physical development.

65. **(C).** Two points of view are expressed in the argument: consumer groups claim that á la carte pricing will reduce consumer costs, while the cable television industry claims that the current package pricing structure is most cost effective for consumers. If the goal of the government is to reduce the cost of cable television for consumers, it is critical for the government to determine whether adding the option of á la carte pricing is likely to save consumers

money.

(A) According to the argument, the government's decision is to be based only on the costs to consumers, not the number of channels available to them. If there were some pricing consequences as a result of this loss of diversity, the point might be relevant, but you are given no such information.

(B) According to the argument, the government's decision is based only on the costs to consumers, not the advertising profits of the cable television companies.

(C) CORRECT. If consumers would not choose to order all of the channels they currently buy as part of a package subscription, then the television industries' claim that á la carte pricing would always be more expensive is suspect. If many consumers only watch and wish to pay for a few of their favorite channels, á la carte pricing could very well result in lower cable bills for those consumers.

(D) According to the argument, the government's decision is to be based on the costs to consumers, not the number of consumers who subscribe. If there were some pricing consequences as a result of a loss of subscribers, the point might be relevant, but you are given no such information.

(E) According to the argument, the government's decision is to be based only on the costs to consumers, not the technical equipment requirements a change in cost structure would require.

66. **(A).** The argument provides information about the substantial costs associated with the development of Dendadrine. Yet the management views Dendadrine as a highly profitable project. In order to reconcile these claims, you need to demonstrate that the drug will be able to generate profits that will more than compensate for the high initial expenditures associated with its development.

(A) CORRECT. If management expects to earn patent protection for the new drug, then the company can also expect to charge the unusually high prices that will more than compensate for the initial research and development costs. Note that the patent protection is likely to result in prices twice as high as those necessary for the company to recoup its costs, thus leading to substantial profits.

(B) This answer tells you that for some pharmaceutical products, high R&D expenditures are quite typical. While this explains the high costs associated with the development of Dendadrine, it tells you nothing about how well such products do on the market and whether they subsequently become profitable.

(C) Though this supports the idea that Dendadrine will sell well, it does not specifically support the contention that the drug will be profitable in the face of the unusually high costs associated with it.

(D) Though this supports the idea that Dendadrine will be the primary, if not only, treatment for this market segment, it does not specifically support the contention that the drug will be profitable in the face of the unusually high costs associated with it.

(E) Though this supports the idea that the market for Dendadrine is very large and will generate revenues in excess of $5 billion within a few years, it does not specifically support the contention that the drug will be profitable in the face of the unusually high costs associated with it

Chapter 7
of

Arithmetic

In This Chapter…

Arithmetic

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

Quantity A	**Quantity B**
39 - (25 - 17)	39 - 25 - 17

2.

Quantity A	**Quantity B**
14 - 3(4 - 6)	(4)(-3)(2)(-1)

3.

Quantity A	**Quantity B**
$-5 \times 1 \div 5$	$-6 \times 1 \div 6$

4. 5 - (4 - (3 - (2 - 1))) =

5.

Quantity A	**Quantity B**
17(6) + 3(6)	6(17) + 6(3)

6.

Quantity A	**Quantity B**
$-2^3/2$	$(-2)^2$

7.

Quantity A	**Quantity B**
$5^3 - 5^2$	5

8.

Quantity A	**Quantity B**
$-10 - (-3)^2$	$-[10 + (-3)^2]$

9. $32/(4 + 6 \times 2) =$

(A) 8/5
(B) 16/5
(C) 2
(D) 20
(E) 28

10.

Quantity A	**Quantity B**
(30,000,000)(2,000,000)	(15,000,000)(4,000,000)

11. What is the sum of the numbers in the grid below?

–2	–1	1	2	3	4
–4	–2	2	4	6	8
–6	–3	3	6	9	12
–8	–4	4	8	12	16
–10	–5	5	10	15	20
–12	–6	6	12	18	24

12. Mitchell plans to work at a day camp over the summer. Each week, he will be paid according to the following schedule: at the end of the first week, he will receive \$1. At the end of each subsequent week, he will receive \$1, plus an additional amount equal to the sum of all payments he's received in previous weeks. How much money will Mitchell be paid in total during the summer, if he works for the entire duration of the 8-week-long camp?

\$

13.

A book with 80,000 words costs \$24 and a short story has 1,000 words and costs \$1.

Quantity A	**Quantity B**
Price per word of the book	Price per word of the short story

14. A taxi driver makes \$50 an hour, but pays \$100 a day rent for his taxi and has other costs that amount to \$0.50 per mile. If he works three 7-hour days and one 9-hour day and drives a total of 600 miles in one week, what is his profit?

(A) \$700
(B) \$800
(C) \$1,100
(D) \$1,200
(E) \$1,500

15.

Ticket Prices at the Natural History Museum

	Weekdays	Weekends & Holidays
Child (ages 5–18)	\$7	\$9
Adult (ages 19–64)	\$14	\$16
Senior (ages 65+)	\$8	\$10
*Children under age 5 attend free		

Mr. and Mrs. Gonzales, ages 42 and 39, wish to visit the Natural History Museum with their three children (ages 4, 6, and 10), and Mr. Gonzales's 69-year-old father.

Quantity A	**Quantity B**
The cost of admission for the group on a weekday	The cost of admission for the group on a weekend after applying a coupon offering \$10 off the total purchase

16.

On a certain train, children's tickets cost \$6 and adult tickets cost \$9. Six people are charged between \$44 and \$50, total, for their tickets.

Quantity A	**Quantity B**
The number of children in the group	The number of adults in the group

17. If twice 4,632 is divided by 100, what is the tenths digit?

$\boxed{}$

18. If 617 is divided by 49, the sum of the tens digit and the tenths digit is equal to

(A) 1
(B) 5
(C) 6
(D) 7
(E) 9

19.

Quantity A	**Quantity B**
The age at death, in years and days, of a person who lived from January 31, 1817 to January 15, 1901	The age at death, in years and days, of a person who lived from January 15, 1904 to January 31, 1988

20. For the month of May, Ali's Coffee Shop is offering a "buy five drinks get one free" special, and Bob's Coffee Shop is offering 20% off all drinks. At both shops, the regular price of a coffee is \$2.25.

Quantity A	**Quantity B**
The total cost of one coffee per day from Ali's for every day of May	The total cost of one coffee per day from Bob's for every day of May

21. In a certain ancient kingdom, the length of a foot was the length of the current king's foot. If the newly crowned monarch had a 12 inch foot as opposed to the 10 inch foot of his predecessor, a formerly 300 foot fence would now have a length of how many feet? (Assume that the length of an inch remains constant.)

(A) 250
(B) 302
(C) 350
(D) 360
(E) 400

22. Mary has six more tapes than Pedro. If Pedro gives two tapes to John and then Pedro buys 5 new tapes, how many more tapes does Mary have than Pedro?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

23. 25 employees donated a total of $450 to charity. If 15 employees donated at least $12 and 9 employees donated at least $19, what is the maximum amount, in dollars, that the last employee could have donated?

$ $\boxed{}$

24. Maribel must divide 60 candies among herself and her 12 cousins, although there is no requirement that the candies be divided equally. If Maribel is to have more candies than everyone else, what is the least number of candies she could have?

$\boxed{}$

25.

Softies Facial Tissues come 84 to a box for $2.99.
Enviro Facial Tissues come 56 to a box for $1.89.

Quantity A	**Quantity B**
The positive difference between the per-tissue cost of Softies tissues and the per-tissue cost of Enviro tissues	0.2 cents

26. A tank has a capacity of 200 pints. How many gallons of water would it take to fill the tank to 3/10 of its capacity? (1 gallon = 8 pints) gallons

$\boxed{}$ gallons

27.

(1 kilogram = 2.2 pounds)

Quantity A	**Quantity B**
The number of kilograms in 44 pounds	The number of pounds in 44 kilograms

28. If the formula for converting degrees Fahrenheit to degrees Celsius is $C = \frac{5}{9}(F - 32)$, what is *F* when *C* = 30?

(A) -10/9
(B) 338/9
(C) 86
(D) 558/5
(E) 112

29.

Quantity A	**Quantity B**
The number of seconds in 12 hours	The number of minutes in 720 hours

30. Joe's car can travel 36 miles per gallon of fuel. Approximately how many kilometers can the car travel on 10 liters of fuel? (5 miles = approximately 8 kilometers; 1 gallon = approximately 4 liters)

$\boxed{}$ kilometers

31. How many 1-inch square tiles would it take to cover the floor of a closet that has dimensions 5 feet by 4 feet? (1 foot = 12 inches)

(A) 20
(B) 240
(C) 1,440
(D) 2,160
(E) 2,880

32. A pool has sprung a leak and is losing water at a rate of 5 milliliters per second. How many liters of water is this pool losing per hour? (1 liter = 1,000 milliliters)

(A) 3
(B) 6
(C) 12
(D) 18
(E) 24

33.

Child A ate 3/5 of a kilogram of chocolate and Child B ate 300 grams of chocolate. (1 kilogram = 1000 grams)

Quantity A	**Quantity B**
The weight, in grams, of the chocolate that Child A ate	Twice the weight, in grams, of the chocolate that Child B ate

34. It takes $4\frac{2}{3}$ feet of wood to make a frame for a lithograph. If wood is sold at $5 a yard and only by the yard, and a collector needs to make 4 frames, how much will the wood cost? (1 yard = 3 feet)

(A) $23.33
(B) $25
(C) $33.33
(D) $35
(E) $100

35. Out of 5.5 billion bacteria grown for an experiment, 1 in 75 million has a particular mutation. Approximately how many of the bacteria have the mutation?

(A) 7
(B) 73
(C) 733
(D) 7,333
(E) 73,333

36. A particular nation's GDP (Gross Domestic Product) is $4.5 billion. If the population of the nation is 1.75 million, what is the per capita (per person) GDP, rounded to the nearest dollar?

(A) $3
(B) $25
(C) $257
(D) $2,571
(E) $25,714

37. Global GDP (Gross Domestic Product) was $69.97 trillion in 2011. If the world population for 2011 was best estimated at 6,973,738,433, approximately what is the global GDP per person?

(A) $10
(B) $100
(C) $1,000
(D) $10,000
(E) $100,000

38. The distance between Mercury and Earth changes due to the orbits of the planets. When Mercury is at its closest point to Earth, it is 48 million miles away. When Mercury is at its furthest point from Earth, it is 138 million miles away. For a science project, Ruby calculates the maximum and minimum amount of time it would take to travel from Earth to Mercury in a spacecraft traveling 55 miles per hour. Approximately what are the times, in days?

(A) 3,636 and 10,454
(B) 14,545 and 41,818

(C) 36,364 and 104,545
(D) 87,272 and 250,909
(E) 872,727 and 2,509,091

Arithmetic Answers

1. **(A).** First simplify inside the parentheses:

$$39 - (25 - 17) =$$
$$39 - 8 =$$
$$31$$

You could also distribute the minus sign to get 39 - 25 + 17 if you prefer. Quantity B is equal to -3, so the final answer is (A). If you noticed right away that the minus sign would distribute in Quantity A but not Quantity B, you could have picked (A) without doing any arithmetic.

2. **(B).** This question is simply testing PEMDAS (Parentheses/Exponents, then Multiplication/Division, then Addition/Subtraction), at least in Quantity A. Make sure that you simplify inside the parentheses, and then multiply, before subtracting:

$$14 - 3(4 - 6) =$$
$$14 - 3(-2) =$$
$$14 + 6 =$$
$$20$$

Quantity B simply equals (4)(-3)(2)(-1) = 24.

3. **(C).** The quantities are equal. Note that

$$-5 \times 1 \div 5 =$$
$$-5 \div 5 =$$
$$-1$$

In Quantity B:

$$-6 \times 1 \div 6 =$$
$$-6 \div 6 =$$
$$-1$$

4. **3.** Make sure to begin with the innermost parentheses:

$$5 - (4 - (3 - (2 - 1))) =$$
$$5 - (4 - (3 - 1)) =$$
$$5 - (4 - 2) =$$
$$5 - (2) =$$
$$3$$

5. **(C).** According to the distributive property, the two quantities are the same. Or:

$17(6) + 3(6) =$
$102 + 18 =$
120

In Quantity B:

$6(17) + 6(3) =$
$102 + 18 =$
120

6. **(B).** In Quantity A, the exponent should be computed before taking the negative of the value — in accordance with PEMDAS.

In Quantity B:

$(-2)^2 =$
$(-2)(-2) =$
4

7. **(A).** Do not make the mistake of thinking that $5^3 - 5^2 = 5^1$. You may not simply subtract the exponents when you are subtracting two terms with the same base! Observe:

$5^3 - 5^2 =$
$125 - 25 =$
100

Obviously, Quantity A is much larger. Alternatively, you could factor out 5^2 (this is an important technique for larger numbers and exponents where pure arithmetic would be impractical):

$5^3 - 5^2 =$
$5^2(5^1 - 1) =$
$5^2(4) =$
100

8. **(C).** In Quantity A:

$-10 - (-3)^2 =$
$-10 - (9) =$
-19

In Quantity B:

$-[10 + (-3)^2]$
$-[10 + (-3)^2] =$
$-[10 + (9)] =$
-19

9. **(C).** Begin inside the parentheses and — in accordance with PEMDAS — simplify 6 × 2 first:

$32/(4 + 6 \times 2) =$
$32/(4 + 12) =$
$32/(16) =$
2

10. **(C).** The GRE calculator will not be able to handle that many zeroes. You will want to do this calculation by hand. To make things easier, you could cancel as many zeroes as you want, as long as you do the same operation to both sides. For instance, you could divide both sides by 1,000,000,000,000 (just think of this as "1 with twelve zeroes"), to get:

Quantity A	Quantity B
(30)(2)	(15)(4)

Or, just use a bit of logic: 30 million times 2 million is 60 million *million*, and 15 million times 4 million is also 60 million *million*. (A "million million" is a trillion, but this doesn't matter as long as you're sure that each Quantity will have the same number of zeroes.)

11. **147.** There are several patterns in the grid, depending on whether you look by row or by column. Within each row, there are positive and negative terms at the beginning that cancel each other. For example, in the first row, you have -2 + 2 = 0 and -1 + 1 = 0. The only terms in the first row that contribute to the sum are 3 and 4, in the far-right columns. The same is true for the other rows.

Thus, the sum of the grid is equal to the sum of only the two far-right columns. The sum in the first row in those columns is 3 + 4 = 7; the sum in the next row is 6 + 8 = 14, etc. The sum in the final row is 18 + 24 = 42. Simply add 7 + 14 + 21 + 28 + 35 + 42 in your calculator to get 147.

12. **255.** At the end of the first week, Mitchell receives $1. At the end of the second week, he gets $1, plus $1 for the total he had been paid up to that point, for a total of $2. At the end of the third week, he gets $1, plus ($1 + $2), or $3, for the total he had been paid up to that point, so this third week total is $4. Let's put this in a table:

Week #	Paid this week($)	Cumulative Pay including this week ($)
1	1	1
2	1 + 1 = 2	1 + 2 = 3
3	1 + 3 = 4	3 + 4 = 7
4	1 + 7 = 8	7 + 8 = 15
5	1 + 15 = 16	15 + 16 = 31
6	1 + 31 = 32	31 + 32 = 63
7	1 + 63 = 64	63 + 64 = 127
8	1 + 127 = 128	127 + 128 = 255

13. **(B).** In Quantity A, 24/80,000 = 0.0003, or 0.03 cents per word. In Quantity B, 1/1,000 = 0.001, or 0.1 cents per word. Quantity B is much larger. Note that your calculation was not strictly necessary — it would have been more

efficient to notice that the book costs 24 times the story but has 80 times the words. (Then remember to choose the larger amount!)

14. **(B).** The driver works 3 days × 7 hours per day, plus a 9-hour day, for a total of 30 hours. At $50 an hour, he makes $1,500 but pays $400 in rent and $300 in mileage expenses. $1,500 - $700 = $800.

15. **(C).** You don't actually need to do a lot of tedious arithmetic to answer this problem. Six people will be attending the museum, but the 4-year-old does not require a ticket (kids under 5 are free). Thus, 5 tickets need to be purchased, whether the family attends on a weekday or a weekend.

Notice that all of the weekend tickets are each $2 more expensive. Therefore, buying 5 weekend tickets will cost a total of $10 more. Thus, after the $10-off coupon, the two quantities are the same.

16. **(D).** Even though the range of costs ($44 to $50) is fairly small, there is still more than one possibility. A good way to work this out is to start with the simplest scenario: 3 adults and 3 kids. Their tickets would cost 3(9) + 3(6) = $45. That's in the range, so it's one possibility.

Since kids are cheaper, you don't want to add more kids to the mix (2 kids, 4 adults will give you too small a total), but try switching out 1 kid for 1 adult.

For 4 adults and 2 kids, tickets would cost 4(9) + 2(6) = $48. Thus, Quantity A and Quantity B could be equal, or Quantity B could be larger, so the answer is (D).

17. **6.** Multiply 4,632 by 2 to get 9,264, then divide by 100 to get 92.64. The **tenths** digit is 6. (Do not confuse **tenths** with **tens**. The tens digit is 9.)

18. **(C).** Divide 617 by 49 in your calculator to get 12.5918…. The **tens** digit is 1. The **tenths** digit is 5. The answer is 1 + 5 = 6.

19. **(B).** For Quantity A, subtract 1901 - 1817 to get 84 years. However, this person lived from January 31, 1817 to January 15 (not January 31), 1901, so you must subtract the days from January 16 to January 31, or 16 days.

For Quantity B, subtract 1988 - 1904 to get 84 years. However, the person lived from January 15, 1905 to January 31 (not January 15), 1988, so add 16 days.

The person in Quantity A lived almost 84 years. The person in Quantity B lived 84 years, plus a bit more. Quantity B is larger.

After calculating that both persons lived roughly 84 years you could also notice that all within the same month (though in different years), the person in Quantity B was born earlier in the month and died later in the month than the person in Quantity A, meaning the person in Quantity B lived for more of that month.

20. **(A).** The actual price ($2.25) of the coffee is irrelevant, and no actual calculation is required here. All that's needed to solve this problem is to realize that Bob's is a much better deal.

20% off each coffee is 1/5 off. "Buy five drinks get one free" means that, for everything six drinks one purchases, the last one is free. That's one in SIX drinks free, or 1/6 off.

So, provided that everything else (the regular price of a coffee at both shops, buying the same number of coffees in the month of May) is the same, one will pay less at Bob's than at Ali's. Thus, the total cost at Ali's is larger. By the way, remember to pick the *larger quantity* (Quantity A), NOT the "better deal"!

21. **(A).** A fence that measured 300 feet under the old king's regime would be $300 \times 10 = 3{,}000$ inches long. This same 3,000-inch fence, under the new king's regime, would be $3{,}000/12 = 250$ feet long. (Keep in mind that, if a "foot" gets bigger, fewer such "feet" fit into a fence of fixed length.)

22. **(A).** Let's make a chart for how many tapes everyone has before they start giving each other tapes or getting new ones. Since Mary has 6 more than Pedro:

Mary	Pedro	John
$P + 6$	P	?

Now, Pedro gives 2 to John:

Mary	Pedro	John
$P + 6$	$P - 2$	$? + 2$

Now, Pedro buys 5 new tapes. Note that "$P - 2$" plus 5 is $P - 2 + 5 = P + 3$.

Mary	Pedro	John
$P + 6$	$P + 3$	$? + 2$

You never learned very much about John, so the question only asks you about Mary and Pedro. Mary and Pedro each have P tapes, but Mary has 6 more than that and Pedro has 3 more than that. So, Mary has 3 more tapes than Pedro.

It also would be fairly simple to assign values to Mary and Pedro (for instance, say that Mary has 10 tapes and Pedro has 4) and proceed using a real-number example.

23. **99.** Since you want to maximize the last employee's contribution, minimize everyone else's. If 15 employees donated at least \$12 and 9 employees donated at least \$19:

$$15(12) + 9(19) = 180 + 171 = 351$$

So, the minimum that all 24 of these employees could have given is \$351. Therefore, the maximum that the 25th employee could have given is $450 - 351 = 99$, or \$99.

24. **6.** One good way to solve this problem is to first evenly divide the candies, and then give Maribel more (by taking candies away from the others) until the conditions of the problem are met. 60 candies divided by 13 people = 4.615….

So, if Maribel had 5 candies, would she have more than everyone else? Well, if the 12 cousins each had only 4 candies, that's 48 candies total plus Maribel's 5 = 53 candies. 7 candies are unaccounted for, meaning that some other cousin or cousins will have to have the same as or more than Maribel.

If Maribel had 6 candies, would she have more than everyone else? Well, if the 12 cousins each had only 5 candies, they would have 60. Since there are only 60 candies total, Maribel could have 6 and the other cousins could have 4 or 5 each. The answer is 6.

Keep in mind that when the question asks for a minimum, you can't just go messing around until you find a case that works — to find the *smallest* case that works, you need to start small and work up from there.

25. **(B).** In the calculator, divide 2.99 by 84 to get a per-tissue cost of 0.03559..., (or 3.559... cents).

Divide 1.89 by 56 to get a per-tissue cost of 0.03375... (or 3.375... cents).

Subtract the smaller number from the larger number to get 0.00184..., or 0.184... cents.

This is less than 0.2 cents. The answer is (B).

26. **7.5 gallons:**

First find out how many pints 3/10 of the capacity is:

$200 \times 3/10 = 600/10 = 60$

Now you need to convert pints to gallons:

$$60 \text{ pints} \times \frac{1 \text{ gallon}}{8 \text{ pints}} = \frac{60}{8} = 7.5 \text{ gallons}$$

27. **(B).** To compare the values, you need to convert the quantity on the left from pounds to kilograms and the quantity on the right from kilograms to pounds:

Quantity A	Quantity B
$44 \text{ pounds} \times \frac{1 \text{ kilogram}}{2.2 \text{ pounds}}$	$44 \text{ pounds} \times \frac{2.2 \text{ pounds}}{1 \text{ kilogram}}$

Before actually multiplying, notice that the quantity on the left is divided by 2.2, while the quantity on the right is multiplied by 2.2. The quantity on the right will be greater.

One could also solve this by noticing that the two quantities involve reverse calculations, with the same number of units (44). Since a kilogram is heavier than a pound, it takes more of the lighter pounds to equal 44 heavier kilograms than it takes of the heavier kilograms to equal 44 of the lighter pounds.

28. **(C).** Start by plugging 30 in for *C* in the equation:

$$30 = \frac{5}{9}(F - 32)$$

Now isolate *C*. Begin by multiplying both sides by 9/5:

$$30 = \frac{5}{9}(F - 32)$$
$$\frac{9}{5} \times 30 = F - 32$$

To multiply 30 by 9/5 quickly, reduce before multiplying:

$$\frac{9}{5} \times 30 = F - 32$$
$$\frac{9}{1} \times 6 = F - 32$$
$$54 = F - 32$$
$$86 = F$$

29. **(C).** Before doing either calculation, note that there are 60 seconds in a minute and 60 seconds in an hour. Compare the two calculations:

12 hours × 60 minutes/hour × 60 seconds/minute | 720 hours × 60 minutes/hour

Notice that 12 × 60 = 720. That means that both amounts will equal 720 × 12. The two values are equal.

30. **144 kilometers.** Convert miles per gallon to kilometers per liter by multiplying by the conversion ratios such that both the miles and gallons units are canceled out:

$$\frac{36 \text{ miles}}{1 \text{ gallon}} \times \frac{8 \text{ kilometers}}{5 \text{ miles}} = \frac{288 \text{ kilometers}}{5 \text{ gallons}}$$

$$\frac{288 \text{ kilometers}}{5 \text{ gallons}} \times \frac{1 \text{ gallon}}{4 \text{ liters}} = \frac{288 \text{ kilometers}}{20 \text{ liters}} = \frac{14.4 \text{ kilometers}}{1 \text{ liter}}$$

The car has 10 liters of fuel in the tank:

10 liters × 14.4 kilometers/liter = 144 kilometers

31. **(E).** There is a hidden trap in this question. Remember that the dimensions of this room are ft^2, not ft (because 5 feet × 4 feet = 20 square feet). To avoid this trap, you should convert the dimensions to inches first, then multiply.

$$5 \text{ feet} \times 4 \text{ feet} = 60 \text{ inches} \times 48 \text{ inches}$$

The dimensions of the closet in inches are 60 inches by 48 inches, or $60 \times 48 = 2{,}880$ square inches. Each tile is 1 square inch, so it will take 2,880 tiles to cover the floor.

32. **(D).** To answer this question, you need to convert milliliters to liters, and convert seconds to hours. The order in which you make the conversions does not matter. First, convert seconds to hours. There are 60 seconds in 1 minute, and 60 minutes in 1 hour:

$$\frac{5 \text{ milliliters}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{18{,}000 \text{ milliliters}}{1 \text{ hour}}$$

Now convert milliliters to liters:

$$\frac{18{,}000 \text{ milliliters}}{1 \text{ hour}} \times \frac{1 \text{ liter}}{1{,}000 \text{ milliliters}} = \frac{18 \text{ liters}}{1 \text{ hour}}$$

33. **(C).** 3/5 of a kilogram is 600 grams. Twice 300 grams is also 600 grams. The columns are equal.

34. **(D).** For 4 frames, the collector needs $18\frac{2}{3}$ feet or $6\frac{2}{9}$ yards of wood. Since the wood is sold "only by the yard," the collector must buy 7 yards [21 feet] at \$5 a yard. The answer is 7(5) = 35.

35. **(B).** One good way to keep track of large numbers (especially those that won't fit in the GRE calculator!) is to use scientific notation (or a loose version thereof — for instance, 5.5 billion in scientific notation is 5.5×10^9, but it would be equally correct for your purposes to write it as 55×10^8).

5.5 billion = 5,500,000,000 = 5.5×10^9
75 million = 75,000,000 = 75×10^6

Since 1 in 75 million of the bacteria have the mutation, divide 5.5 billion by 75 million:

$\frac{5.5 \times 10^9}{75 \times 10^6}$, which can also be written as $\frac{5.5}{75} \times \frac{10^9}{10^6}$. Only $\frac{5.5}{75}$. needs to go in the calculator, to yield 0.0733333….

Since $\frac{10^9}{10^6}$ is 10^3, move the decimal three places to the right to get 73.333…, or answer choice (B).

Or, write one number over the other and *cancel out the same number of zeroes from the top and bottom* before trying to use the calculator: $\frac{5{,}500{,}000{,}000}{75{,}000{,}000} = \frac{5{,}500,\cancel{000{,}000}}{75,\cancel{000{,}000}} = \frac{5{,}500}{75} = 73.333\ldots$

36. **(D).** This problem is asking you to divide \$4.5 billion by 1.75 million. When dealing with numbers that have many zeroes, you can avoid mistakes by using scientific notation or by writing out the numbers and canceling zeroes before using the calculator.

4.5 billion = 4,500,000,000 = 4.5×10^9
1.75 million = 1,750,000 = 1.75×10^6

$$\frac{4.5\times10^9}{1.75\times10^6} = 2.57142\ldots\times10^3 = 2{,}571.42\ldots$$

The answer is (D). Alternatively, write one number on top of the other in fully-expanded form, and cancel zeroes before using the calculator:

$$\frac{4{,}500{,}000{,}000}{1{,}750{,}000} = \frac{4{,}500{,}00\cancel{0{,}000}}{1{,}75\cancel{0{,}000}} = \frac{450{,}000}{175} = 2{,}571.42\ldots$$

37. **(D).** This problem is asking you to divide \$69.97 trillion by 6,973,738,433. When dealing with numbers that have many zeroes, you can avoid mistakes by using scientific notation or by writing out the numbers and canceling zeroes before using the calculator.

Before doing that, however, look at the answers — they are very far apart from one another, which gives you license to estimate. GDP is about 70 trillion. Population is about 7 billion. Thus:

$$\frac{70{,}000{,}000{,}000{,}000}{7{,}000{,}000{,}000} = \frac{70{,}000{,}\cancel{000{,}000{,}000}}{7{,}\cancel{000{,}000{,}000}} = 10{,}000$$

38. **(C).** Since Rate × Time = Distance, thus $\frac{\text{Distance}}{\text{Rate}} = \text{Time}$ = Time.

If you don't yet have that formula memorized, a little common sense will tell you that if Mercury and Earth were 110 miles apart, for instance, and you traveled at 55 mph, you would get there in two hours. Thus, the correct operation is division.

48 million miles = 48,000,000, so:

$$\frac{48{,}000{,}000}{55} = 872{,}727.2727\ldots$$

138 million miles = 138,000,000, so:

$$\frac{138{,}000{,}000}{55} = 2{,}509{,}090.909$$

Thus, it would take between 872,727 hours and 2,509,091 hours (rounded to the nearest hour) to travel to Mercury at 55 mph.

To convert to **days**, simply divide each of these numbers by 24 to get 36,364 days and 104,545 days.

Chapter 8
of

Algebra

In This Chapter…

Algebra

Algebra Answers

Algebra

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If $3x + 2(x + 2) = 2x + 16$, then $x =$

(A) 3
(B) 4
(C) 20/3
(D) 10
(E) 12

2. If $x \neq 0$ and $\frac{3x+7}{x} = 10$, what is the value of x?

$\boxed{}$

3. If $4(-3x - 8) = 8(-x + 9)$, what is x^2?

4. If $3x + 7 - 4x + 8 = 2(-2x - 6)$, what is the value of x?

5. If $2x(4 - 6) = -2x + 12$, what is the value of x?

6. If $x \neq 0$ and $\frac{3(6-x)}{2x} = -6$, what is the value of x?

7. If $x \neq -13$ and $\frac{13}{x+13} = 1$, what is the value of x?

8. If $x \neq 2$ and $\frac{10(-3x+4)}{10-5x} = 2$, what is the value of x?

9. If $x \neq 2$ and $\frac{8-2(-4+10x)}{2-x} = 17$, what is the value of x?

10.

-5 is 7 more than $-z$.

Quantity A	**Quantity B**
z	-12

11. If $(x + 3)^2 = 225$, which of the following could be the value of $x - 1$?

(A) 13
(B) 12
(C) -12
(D) -16
(E) -19

12.

$x = 2$

Quantity A	Quantity B
$x^2 - 4x + 3$	1

13.

$$p = 300c^2 - c$$
$$c = 100$$

Quantity A	Quantity B
p	$29{,}000c$

14. If $3(7 - x) = 4(1.5)$, then $x =$

$\boxed{}$

15.

$$1{,}200x + 6{,}000 = 13{,}200$$
$$12y + 60 = 132$$

Quantity A	Quantity B
x	y

16.

$$-(x)^3 = 64$$

Quantity A	Quantity B
x^4	x^5

17. If $3t^3 - 7 = 74$, what is $t^2 - t$?

(A) -3
(B) 3
(C) 6
(D) 9
(E) 18

18. If $3x + 7 - 4x + 8 = 2(-2x - 6)$, what is the value of x?

$\boxed{}$

19. If $y = 4x + 10$ and $y = 7x - 5$, what is the value of y?

20. If $2h - 4k = 0$ and $k = h - 3$, what is the value of $h + k$?

21. If $x - y = 4$ and $2x + y = 5$, what is the value of x?

22. If $x + 2y = 5$ and $x - 4y = -7$, what is the value of x?

23. $4x + y + 3z = 34$
$4x + 3z = 21$

What is the value of y?

24.

Quantity A	**Quantity B**
$(x + 2)(x - 3)$	$x^2 - x - 6$

25.

Quantity A	**Quantity B**
$(2s + 1)(s + 5)$	$2s^2 + 11s + 4$

26.

$xy > 0$

Quantity A	**Quantity B**
$(2x - y)(x + 4y)$	$2x^2 + 8xy - 4y^2$

27.

$$x^2 - 2x = 0$$

Quantity A	**Quantity B**
x	2

28.

Quantity A	**Quantity B**
$d(d^2 - 2d + 1)$	$d(d^2 - 2d) + 1$

29.

Quantity A	**Quantity B**
$xy^2z(x^2z + yz^2 - xy^2)$	$x^3y^2z^2 + xy^3z^3 - x^2y^4z$

30.

$a = 2b = 4c$ and a, b, and c are integers.

Quantity A	**Quantity B**
$a + b$	$a + c$

31.

$k = 2m = 4n$ and k, m, and n are nonnegative integers.

Quantity A	**Quantity B**
km	kn

32.

For the positive integers a, b, c, and d, a is half of b, which is one-third of c. The value of d is triple that of c.

Quantity A	**Quantity B**
$\dfrac{a+b}{c}$	$\dfrac{a+b+c}{d}$

33. If $x^2 - y^2 = 0$ and $xy \neq 0$, which of the following MUST be true?

Indicate all such statements.

☐ $x = y$

☐ $|x| = |y|$

☐ $\frac{x^2}{y^2} = 1$

34.

$$3x + 6y = 27$$
$$x + 2y + z = 11$$

Quantity A	**Quantity B**
$z + 5$	$x + 2y - 2$

35. If $(x - y) = \sqrt{12}$ and $(x + y) = \sqrt{3}$, what is the value of $x^2 - y^2$?

(A) 3
(B) 6
(C) 9
(D) 36
(E) It cannot be determined from the information given.

36.

$$a \neq b$$

Quantity A	**Quantity B**
$\frac{a-b}{b-a}$	1

37.

$$a = \frac{b}{2}$$

$$c = 3b$$

Quantity A	**Quantity B**
a	c

38. If $xy \neq 0$ and $x \neq -y$, $\frac{x^{36} - y^{36}}{(x^{18} + y^{18})(x^9 + y^9)} =$ =

(A) 1
(B) $x^2 - y^2$
(C) $x^9 - y^9$

(D) $x^{18} - y^{18}$

(E) $\dfrac{1}{x^9 - y^9}$

39. If $x \neq -y$, $\dfrac{x^2 + 2xy + y^2}{2(x+y)^2} =$

(A) 1
(B) 1/2
(C) $\dfrac{1}{x+y}$
(D) xy
(E) $2xy$

40. If $ab \neq 0$, $\dfrac{a^8 - b^8}{(a^4 + b^4)(a^2 + b^2)} =$

(A) 1
(B) $a - b$
(C) $(a + b)(a - b)$
(D) $(a^2 + b^2)(a^2 - b^2)$
(E) $\dfrac{a-b}{a+b}$

41.

$$x > y$$
$$xy \neq 0$$

Quantity A	**Quantity B**
$\dfrac{x^2}{y + \frac{1}{y}}$	$\dfrac{y^2}{x + \frac{1}{x}}$

42. If $x + y = -3$ and $x^2 + y^2 = 12$, what is the value of $2xy$?

43. If $x - y = 1/2$ and $x^2 - y^2 = 3$, what is the value of $x + y$?

44. If $x^2 - 2xy = 84$ and $x - y = -10$, what is the value of $|y|$?

45. $(x - 2)^2 + (x - 1)^2 + x^2 + (x + 1)^2 + (x + 2)^2 =$

(A) $5x^2$
(B) $5x^2 + 10$
(C) $x^2 + 10$
(D) $5x^2 + 6x + 10$
(E) $5x^2 - 6x + 10$

46. If $a = (x + y)^2$ and $b = x^2 + y^2$ and $xy > 0$, which of the following must be true?

Indicate all such statements.

- ☐ $a = b$
- ☐ $a > b$
- ☐ a is positive

47. a is directly proportional to b. If $a = 8$ when $b = 2$, what is a when $b = 4$?

(A) 10
(B) 16
(C) 32
(D) 64
(E) 128

48. a is inversely proportional to b. If $a = 16$ when $b = 1$, what is b when $a = 8$?

(A) -2
(B) -1
(C) 2
(D) 4
(E) 8

49. The time it takes to erect a bonfire is inversely proportional to the number of students doing the work. If it takes 20 students 1.5 hours to do the job, how long will it take 35 students to do the job, to the nearest minute?

(A) 51
(B) 52
(C) 53
(D) 54
(E) 55

50.

$$3a + 2b = 20 \text{ and } 2a + 3b = 5$$

Quantity A	**Quantity B**
$a + b$	a

51.

$$m + 2n = 10 \text{ and } m \text{ is 50\% of } n$$

Quantity A	**Quantity B**
m^2	n

52.

For the integers a, b, and c, the sum of a and b is 75% of c.

Quantity A	**Quantity B**
$(3/4)(a + b)$	$(4/3)(c)$

53. If $2a = 4b = 8c = 10$, then $64abc =$

(A) 64,000
(B) 16,000
(C) 8,000
(D) 4,000
(E) 1,000

54. If $4m^2 + 6n^3 - 9 = 16$, what is the value of $2m^2 + 3n^3$?

$\boxed{}$

55. If $a + b = 8$, $b + c = 11$, and $a + c = 5$, what is the value of $a + b + c$?

$\boxed{}$

Algebra Answers

1. **(B).** Distribute the 2 on the left side over the $(x + 2)$, then combine like terms and simplify:

$$3x + 2(x + 2) = 2x + 16$$
$$3x + 2x + 4 = 2x + 16$$
$$5x + 4 = 2x + 16$$
$$3x + 4 = 16$$
$$3x = 12$$
$$x = 4$$

2. **1.** First, multiply both sides by x to get:

$$3x + 7 = 10x$$
$$7 = 7x$$
$$1 = x$$

The answer is 1. By the way, "$x \neq 0$" was in the problem simply because the problem had x on the bottom of a fraction, and dividing by zero is illegal. This is just the problem writer's way of assuring you that the problem, in fact, has an answer. So, you generally don't have to worry about verbiage like "$x \neq 0$."

3. **676.** Distribute, group like terms, and solve for x:

$$4(-3x - 8) = 8(-x + 9)$$
$$-12x - 32 = -8x + 72$$
$$-32 = 4x + 72$$
$$-104 = 4x$$
$$-26 = x$$

Then, multiply 26 by 26 in your calculator (or -26 by -26, although the negatives will cancel each other out anyway) to get x^2, which is 676.

4. **-9.** $3x + 7 - 4x + 8 = 2(-2x - 6)$

$$-x + 15 = -4x - 12$$
$$3x + 15 = -12$$
$$3x = -27$$
$$x = -9$$

5. **-6.** $2x(4 - 6) = -2x + 12$

$$2x(-2) = -2x + 12$$
$$-4x = -2x + 12$$
$$-2x = 12$$
$$x = -6$$

6. **-2.** $\frac{3(6-x)}{2x}=-6$

$3(6 - x) = -6(2x)$
$18 - 3x = -12x$
$18 = -9x$
$-2 = x$

7. **0.** $\frac{13}{x+13}=1$

$13 = 1(x + 13)$
$13 = x + 13$
$0 = x$

8. **1.** $\frac{10(-3x+4)}{10-5x}=2$

$10(-3x + 4) = 2(10 - 5x)$
$-30x + 40 = 20 - 10x$
$40 = 20 + 20x$
$20 = 20x$
$1 = x$

9. **-6.** $\frac{8-2(-4+10x)}{2-x}=17$

$8 - 2(-4 + 10x) = 17(2 - x)$
$8 + 8 - 20x = 34 - 17x$
$16 - 20x = 34 - 17x$
$16 = 34 + 3x$
$-18 = 3x$
$-6 = x$

10. **(A).** Translate the question stem into an equation and solve for z:

$- 5 = -z + 7$
$-12 = -z$
$12 = z$

Because $z = 12 > -12$, Quantity A is greater.

11. **(E).** Begin by square-rooting both sides of the equation, but remember that square-rooting 225 will yield both 15 and -15 as results. (The calculator will not remind you of this! It's your job to keep this in mind.) So:

$x + 3 = 15$
$x = 12$
so, $x - 1 = 11$

OR

$x + 3 = -15$
$x = -18$
so, $x - 1 = -19$

Only -19 appears in the choices.

12. **(B).** To evaluate the expression in Quantity A, replace x with 2.

$x^2 - 4x + 3 =$
$(2)^2 - 4(2) + 3 =$
$4 - 8 + 3 = -1 < 1$

Therefore, Quantity B is greater.

13. **(A).** To find the value of p, first replace c with 100 to find the value for Quantity A:

$p = 300c^2 - c$
$p = 300(100)^2 - 100$
$p = 300(10{,}000) - 100$
$p = 3{,}000{,}000 - 100 = 2{,}999{,}900$

Since $c = 100$, the value for Quantity B is $29{,}000(100) = 2{,}900{,}000$.

Thus, Quantity A is greater.

14. **5.** Distribute the 3 on the left side and multiply 4(1.5). Feel free to use the calculator:

$3(7 - x) = 4(1.5)$
$21 - 3x = 6$
$-3x = -15$
$x = 5$

15. **(C).** First, solve for x:

$1{,}200x + 6{,}000 = 13{,}200$
$1{,}200x = 7{,}200$
$x = 6$

Now, solve for y:

$12y + 60 = 132$
$12y = 72$
$y = 6$

The quantities are equal. Alternatively, you could have noticed that dividing both sides of the first equation by 100 would yield an equation identical to the second one, except with x in place of y. Thus, without solving the equations, you could note that the two quantities must be the same.

16. **(A).** First, solve for x:

$$-(x)^3 = 64$$
$$(x)^3 = -64$$

Your calculator will not do a cube root for you. Fortunately, on the GRE, cube roots will tend to be quite small and easy to puzzle out. Ask yourself what number times itself three times equals -64? The answer is $x = -4$.

Since x is negative, Quantity A will be positive (a negative number times itself four times will be positive) and Quantity B will be negative (a negative number times itself five times will be negative). No further calculations are needed to see that Quantity A is greater.

17. **(C).** First, solve for t:

$$3t^3 - 7 = 74$$
$$3t^3 = 81$$
$$t^3 = 27$$
$$t = 3$$

Now, plug $t = 3$ into $t^2 - t$:

$$(3)^2 - 3 = 9 - 3 = 6$$

18. **-9.** First, combine like terms on each side:

$$3x + 7 - 4x + 8 = 2(-2x - 6)$$
$$-x + 15 = -4x - 12$$
$$3x + 15 = -12$$
$$3x = -27$$
$$x = -9$$

19. **30.** Since each equation is already solved for y, set the right side of each equation equal to the other.

$$4x + 10 = 7x - 5$$
$$10 = 3x - 5$$
$$15 = 3x$$
$$5 = x$$

Substitute 5 for x in the first equation and solve for y.

$$y = 4(5) + 10$$
$$y = 30$$

$x = 5$ and $y = 30$. Be sure to answer for y, not x.

20. **9.** Since the second equation is already solved for k, plug (h - 3) in for k in the first equation:

$$2h - 4k = 0$$
$$2h - 4(h - 3) = 0$$
$$2h - 4h + 12 = 0$$
$$-2h = -12$$
$$h = 6$$

Substitute 6 for h in the second equation and solve for k.

$$k = (6) - 3$$
$$k = 3$$

$h = 6$ and $k = 3$, so $h + k = 9$.

21. **3.** Notice that the first equation has the term $-y$ while the second equation has the term $+y$. While you could use the substitution method, adding the equations together will make $-y$ and y cancel, so this is the easiest way to solve for x.

$$x - y = 4$$
$$2x + y = 5$$
$$3x = 9$$
$$x = 3$$

22. **1.** Both equations have the term $+x$, so you can eliminate the variable x by subtracting the second equation from the first:

$$x + 2y = 5$$
$$-(x - 4y = -7)$$
$$6y = 12$$
$$y = 2$$

Plug this value for y into the first equation to get $x + 2(2) = 5$, or $x = 1$.

Be very careful to change the sign of each term in the second equation when subtracting. For example, $-(-4y) = +4y$ and $-(-7) = +7$.

Alternatively, you could have multiplied the entire second equation by -1 to get $-x + 4y = 7$ and then added this equation to the first. Either way, $x = 1$.

23. **13.** This question contains only two equations, but three variables. To isolate y, both x and z must be eliminated. Notice that the coefficients of x and z are the same in both equations. Subtract the second equation from the first to eliminate x and z.

$$4x + y + 3z = 34$$
$$\underline{-(4x \quad + 3z = 21)}$$

$y = 13$

24. **(C).** FOIL the terms in Quantity A:

$$(x + 2)(x - 3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

The two quantities are equal.

25. **(A).** FOIL the terms in Quantity A:

$$(2s + 1)(s + 5) = 2s^2 + 10s + s + 5 = 2s^2 + 11s + 5$$

Since $2s^2 + 11s$ appears in both quantities, eliminate it. Because 5 is greater than 4, Quantity A is greater.

26. **(B).** FOIL the terms in Quantity A:

$$(2x - y)(x + 4y) = 2x^2 + 8xy - xy - 4y^2 = 2x^2 + 7xy - 4y^2$$

Since $2x^2$ and $-4y^2$ appear in both quantities, eliminate them. Quantity A is now equal to $7xy$ and Quantity B is now equal to $8xy$. Because $xy > 0$, Quantity B is greater. (Don't assume! If xy were 0, the two quantities would have been equal. If xy were negative, Quantity A would have been greater.)

27. **(D).** Factor $x^2 - 2x = 0$:

$$x^2 - 2x = 0$$
$$x(x - 2) = 0$$
$$x = 0 \text{ OR } (x - 2) = 0$$

$x = 0$ or 2.

Thus, Quantity A could be less than or equal to Quantity B. The answer is (D).

(Note that you CANNOT simply divide both sides of the original equation by x. It is illegal to divide by a variable unless you have evidence that that variable does not equal zero.)

28. **(D).** In Quantity A, multiply d by every term in the parentheses:

$$d(d^2 - 2d + 1) =$$
$$(d \times d^2) - (d \times 2d) + (d \times 1) =$$
$$d^3 - 2d^2 + d$$

In Quantity B, multiply d by the two terms in the parentheses:

$$d(d^2 - 2d) + 1 =$$
$$(d \times d^2) - (d \times 2d) + 1 =$$

$d^3 - 2d^2 + 1$

Because $d^3 - 2d^2$ is common to both quantities, it can be ignored. The comparison is really between d and 1. Without more information about d, there is no way to know which quantity is greater.

29. **(C).** In Quantity A, the term xy^2z on the outside of the parentheses must be multiplied by each of the three terms inside the parentheses. Then simplify the expression as much as possible.

Taking one term at a time, the first is $xy^2z \times x^2z = x^3y^2z^2$, because there are three factors of x, two factors of y, and two factors of z. Similarly, the second term is $xy^2z \times yz^2 = xy^3z^3$ and the third is $xy^2z \times (-xy^2) = -x^2y^4z$. Adding these three terms together gives the distributed form of Quantity A: $x^3y^2z^2 + xy^3z^3 - x^2y^4z$.

This is identical to Quantity B, so no more work is required.

30. **(D).** Since a is common to both quantities, it can be ignored. The comparison is really between b and c. Because $2b = 4c$, it is true that $b = 2c$, so the comparison is really between $2c$ and c. Watch out for negatives. If the variables are positive, Quantity A is greater, but if the variables are negative, Quantity B is greater.

31. **(D).** If the variables are positive, Quantity A is greater. However, all three variables could equal zero, in which case the two quantities are equal. Watch out for the word "nonnegative," which means "positive or zero."

32. **(C).** The following relationships are given: $a = \frac{b}{2}$, $b = \frac{c}{3}$, and $d = 3c$. Pick one variable and put everything in terms of that variable. For instance, variable a:

$b = 2a$
$c = 3b = 3(2a) = 6a$
$d = 3c = 3(6a) = 18a$

Substitute into the quantities and simplify.

Quantity A: $\frac{a+b}{c} = \frac{a+2a}{6a} = \frac{3a}{6a} = \frac{1}{2}$

Quantity B: $\frac{a+b+c}{d} = \frac{a+2a+6a}{18a} = \frac{9a}{18a} = \frac{1}{2}$

The two quantities are equal.

33. **II and III only.** Since $x^2 - y^2 = 0$, add y^2 to both sides to get $x^2 = y^2$. It might look as though $x = y$, but this is not necessarily the case. For example, x could be 2 and y could be -2. Algebraically, when you square root both sides of $x^2 = y^2$, you do NOT get $x = y$, but rather $|x| = |y|$. Thus, statement I is not necessarily true and statement II is true. Statement III is also true and can be easily generated algebraically:

$x^2 - y^2 = 0$
$x^2 = y^2$
$$\frac{x^2}{y^2} = 1$$

34. **(C).** This question may at first look difficult, as there are three variables and only two equations. However, notice that the top equation can be divided by 3, yielding $x + 2y = 9$. This can be plugged into the second equation:

$(x + 2y) + z = 11$
$(9) + z = 11$
$z = 2$

Quantity A is simply 2 + 5 = 7.

For Quantity B, remember that $x + 2y = 9$. Thus, Quantity B is 9 - 2 = 7.

The two quantities are equal.

35. **(B).** The factored form of the Difference of Squares (one of the "special products" you need to memorize for the exam) is comprised of the terms given in this problem.

$x^2 - y^2 = (x + y)(x - y)$

Substitute the values $\sqrt{12}$ and $\sqrt{3}$ in place of $(x - y)$ and $(x + y)$, respectively:

$x^2 - y^2 = \sqrt{12} \times \sqrt{3}$

Combine 12 and 3 under the same root sign and solve:

$x^2 - y^2 = \sqrt{12 \times 3}$
$x^2 - y^2 = \sqrt{36}$
$x^2 - y^2 = 6$

36. **(B).** Plug in any two unequal values for a and b, and Quantity A will always be equal to -1. This is because you can factor a negative out of the top or bottom of the fraction to show that the top and bottom are the same, except for their signs:

$$\frac{a-b}{b-a} = \frac{a-b}{-(a-b)} = -1$$

37. **(D).** To compare a and c, put c in terms of a. Multiply the first equation by 2 to find that $b = 2a$. Substitute into the second equation: $c = 3b = 3(2a) = 6a$. If all three variables are positive, then $6a > a$. If all three variables are negative, then $a > 6a$. Finally, all three variables could equal 0, making the two quantities equal.

38. **(C).** The Difference of Squares (one of the "special products" you need to memorize for the exam) is $x^2 - y^2 = (x + y)(x - y)$. This pattern works for any perfect square minus another perfect square. Thus, $x^{36} - y^{36}$ will factor according to this pattern. Note that $\sqrt{x^{36}} = (x^{36})^{1/2} = x^{36/2} = x^{18}$, or $x^{36} = (x^{18})^2$. First, factor $x^{36} - y^{36}$ in the numerator, then cancel $x^{18} + y^{18}$ with the $x^{18} + y^{18}$ on the bottom:

$$\frac{x^{36}-y^{36}}{\left(x^{18}+y^{18}\right)\left(x^{9}+y^{9}\right)}=\frac{\cancel{\left(x^{18}+y^{18}\right)}\left(x^{18}-y^{18}\right)}{\cancel{\left(x^{18}+y^{18}\right)}\left(x^{9}+y^{9}\right)}=\frac{\left(x^{18}-y^{18}\right)}{\left(x^{9}+y^{9}\right)}$$

The $x^{18} - y^{18}$ in the numerator will also factor according to this pattern. Then cancel $x^9 + y^9$ with the $x^9 + y^9$ on the bottom:

$$\frac{\left(x^{18}-y^{18}\right)}{\left(x^{9}+y^{9}\right)}=\frac{\cancel{\left(x^{9}+y^{9}\right)}\left(x^{9}-y^{9}\right)}{\cancel{\left(x^{9}+y^{9}\right)}}=x^{9}-y^{9}$$

39. **(B)**. First, you need to know that $x^2 + 2xy + y^2 = (x + y)^2$. This is one of the "special products" you need to memorize for the exam. Factor the top, then cancel:

$$\frac{x^{2}+2xy+y^{2}}{2(x+y)^{2}}=\frac{\cancel{(x+y)^{2}}}{2\cancel{(x+y)^{2}}}=\frac{1}{2}$$

40. **(C).** The Difference of Squares (one of the "special products" you need to memorize for the exam) tells you that $x^2 - y^2 = (x + y)(x - y)$. This pattern works for any perfect square minus another perfect square. Note that $\sqrt{a^8} = (a^8)^{1/2} = a^{8/2} = a^4$, or $a^8 = (a^4)^2$. Thus, $a^8 - b^8$ will factor according to this pattern:

$$\frac{a^{8}-b^{8}}{\left(a^{4}+b^{4}\right)\left(a^{2}+b^{2}\right)}=\frac{\cancel{\left(a^{4}+b^{4}\right)}\left(a^{4}-b^{4}\right)}{\cancel{\left(a^{4}+b^{4}\right)}\left(a^{2}+b^{2}\right)}=\frac{a^{4}-b^{4}}{a^{2}+b^{2}}$$

Now, factor $a^4 - b^4$ according to the same pattern:

$$\frac{a^{4}-b^{4}}{a^{2}+b^{2}}=\frac{\cancel{\left(a^{2}+b^{2}\right)}\left(a^{2}-b^{2}\right)}{\cancel{a^{2}+b^{2}}}=a^{2}-b^{2}$$

Since $a^2 - b^2$ does not appear in the choices, factor one more time to get $(a + b)(a - b)$, which is choice (C).

41. **(D)**. You could simplify first and then plug in examples, or just plug in examples without simplifying. For instance, if $x = 2$ and $y = 1$:

Quantity A: $$\frac{x^2}{y+\frac{1}{y}} = \frac{2^2}{1+\frac{1}{1}} = \frac{4}{2} = 2$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}} = \frac{1^2}{2+\frac{1}{2}} = \frac{1}{\frac{5}{2}} = \frac{2}{5}$$

In this case, Quantity A is greater. Then, try negatives. If $x = -1$ and $y = -2$ (remember, x must be greater than y):

Quantity A: $$\frac{x^2}{y+\frac{1}{y}} = \frac{(-1)^2}{-2+\frac{1}{-2}} = \frac{1}{\frac{5}{-2}} = \frac{-2}{5}$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}} = \frac{(-2)^2}{(-1)+\frac{1}{-1}} = \frac{4}{-2} = -2$$

Quantity A is still greater. However, before assuming that Quantity A is *always* greater, make sure you have tried every category of possibilities for x and y. What if x is positive and y is negative? For instance, $x = 2$ and $y = -2$:

Quantity A: $$\frac{x^2}{y+\frac{1}{y}} = \frac{2^2}{-2+\frac{1}{-2}} = \frac{4}{-\frac{5}{2}} = 4\times-\frac{2}{5} = -\frac{8}{5}$$

Quantity B: $$\frac{y^2}{x+\frac{1}{x}} = \frac{(-2)^2}{(2)+\frac{1}{2}} = \frac{4}{\frac{5}{2}} = 4\times\frac{2}{5} = \frac{8}{5}$$

42. **-3.** One of the "special products" you need to memorize for the GRE is $x^2 + 2xy + y^2 = (x + y)^2$. Write this pattern on your paper, plug in the given values, and simplify:

$x^2 + 2xy + y^2 = (x + y)^2$
$(x^2 + y^2) + 2xy = (x + y)^2$
$(12) + 2xy = (-3)^2$
$12 + 2xy = 9$
$2xy = -3$

43. **6.** The Difference of Squares (one of the "special products" you need to memorize for the exam) is $x^2 - y^2 = (x + y)(x - y)$. Write this pattern on your paper and plug in the given values:

$x^2 - y^2 = (x + y)(x - y)$
$3 = (x + y)(1/2)$
$6 = x + y$

44. **4.** One of the "special products" you need to memorize for the exam is $x^2 - 2xy + y^2 = (x - y)^2$. Write this pattern on your paper and plug in the given values:

$x^2 - 2xy + y^2 = (x - y)^2$
$84 + y^2 = (-10)^2$
$84 + y^2 = 100$
$y^2 = 16$
$y = 4$ or -4, so $|y| = 4$.

45. **(B)**. First, multiply out (remember FOIL = First, Outer, Inner, Last) each of the terms in parentheses:

$(x^2 - 2x - 2x + 4) + (x^2 - 1x - 1x + 1) + (x^2) + (x^2 + 1x + 1x + 1) + (x^2 + 2x + 2x + 4)$

Note that some of the terms will cancel each other out (e.g., $-x$ and x, $-2x$ and $2x$):

$(x^2 + 4) + (x^2 + 1) + (x^2) + (x^2 + 1) + (x^2 + 4)$

Finally, combine:

$5x^2 + 10$

46. **II and III only.** Distribute for a: $a = (x + y)^2 = x^2 + 2xy + y^2$. Since $b = x^2 + y^2$, a and b are the same except for the "extra" $2xy$ in a. Since xy is positive, a is greater than b. Statement I is false and statement II is true.

Each term in the sum for a is positive: xy is given as positive, and x^2 and y^2 are definitely positive, as they are squared and not equal to zero. Therefore, $a = x^2 + 2xy + y^2$ is positive. Statement III is true.

47. **(B).** To answer this question, it is important to understand what is meant by the phrase "directly proportional." It means that $a = kb$, where k is a constant. In alternative form: $\frac{a}{b} = k$, where k is a constant. So, because they both equal the constant, $\frac{a_{old}}{b_{old}} = \frac{a_{new}}{b_{new}}$. Plugging in values: $\frac{8}{2} = \frac{a_{new}}{4}$. Cross multiply and solve:

$32 = 2a_{new}$
$a_{new} = 16$

48. **(C)**. To answer this question, it is important to understand what is meant by the phrase "inversely proportional." It

means that $a = \frac{k}{b}$, where k is a constant. In alternative form, $ab = k$, where k is a constant.

So, because the product of a and b is always constant: $(16)(1) = (8)(b)$, or $b = 2$.

49. **(A).** To answer this question, it is important to understand what is meant by the phrase "inversely proportional." It means that $\text{time} = \frac{k}{\text{\# of students}}$, where k is a constant. In alternative form, (time)(# of students) = k, where k is a constant.

So, because the product of (time) and (# of students) is always constant:

(1.5 hours)(20 students) = (t hours)(35 students)

$$t = \frac{(1.5)(20)}{35} = \frac{30}{35} = \frac{6}{7}$$

Remember that t is in hours, so t is $\frac{6}{7} \times 60 = \frac{360}{7} \approx 51.43$ minutes. To the nearest minute, the time is 51 minutes.

50. **(B).** Because variable a is common to both quantities, the real comparison is between $+b$ and 0. Solve the system of equations for b.

Multiply the first equation by 2: $3a + 2b = 20 \longrightarrow 6a + 4b = 40$
Multiply the second equation by 3: $2a + 3b = 5 \longrightarrow 6a + 9b = 15$

Subtract the resulting second equation from the resulting first equation, canceling the a terms:

$-5b = 25$
$b = -5$

Because b is negative, Quantity B is greater.

51. **(C).** Since $m + 2n = 10$ and $m = 0.5n$, substitute $0.5n$ for m to get:

$0.5n + 2n = 10$
$2.5n = 10$
$n = 4$

Substitute $n = 4$ back into either equation to get $m = 2$. Since $2^2 = 4$, the two quantities are equal.

52. **(D).** If a, b, and c are positive, Quantity B is greater. If the variables are negative, Quantity A is greater. For instance, $a = 1$, $b = 2$, and $c = 4$ are valid numbers to test (since $1 + 2$ is 75% of 4). In such a case, Quantity B is

obviously greater. But $a = -1$, $b = -2$, and $c = -4$ are also valid numbers to test, in which case Quantity A is greater.

53. **(E).** First, divide the entire given equation by 2 to simplify: $a = 2b = 4c = 5$

Then, break up the equation into several smaller equations, setting each variable expression equal to 5:

$a = 5$
$2b = 5$ (so $b = 2.5$)
$4c = 5$ (so $c = 1.25$)

Thus, $64abc = 64(5)(2.5)(1.25) = 1{,}000$.

54. **12.5.** You do not need to solve for m and n to answer this question. (Nor is it possible to do so!) Simplify the equation:

$4m^2 + 6n^3 - 9 = 16$
$4m^2 + 6n^3 = 25$

Now divide both sides of the equation by 2:

$2m^2 + 3n^3 = 12.5$

This is exactly the quantity the problem is asking for. No further work is required.

55. **12.** While this problem can be solved by substitution, it is much easier and faster to simply stack and add all three equations. To keep them lined up properly, insert "placeholder terms"—for instance, instead of $a + b = 8$, write $a + b + 0c = 11$ or $1a + 1b + 0c = 11$ (since this equation does not use the variable c, it has "zero c"):

$1a + 1b + 0c = 8$
$0a + 1b + 1c = 11$
$1a + 0b + 1c = 5$
$2a + 2b + 2c = 24$

Divide both sides of the equation by 2 to get $a + b + c = 12$.

Chapter 9
of

Inequalities and Absolute Values

In This Chapter…

Inequalities and Absolute Values

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

$$|3x - 18| = 9$$

Quantity A	**Quantity B**
x	6

2. If $2z + 4 \geq -18$, which of the following must be true?

(A) $z \leq -11$
(B) $z \leq 11$
(C) $z \geq -11$
(D) $z \geq -7$
(E) $z \geq 7$

3.

$$7y - 3 \leq 4y + 9$$

Quantity A	**Quantity B**
y	4

4.

$$d + \frac{3}{2} < 8$$

Quantity A	**Quantity B**
$2d$	13

5.

$$\frac{4x}{7} \le 15 + x$$

$2y - 1.5 > 7$

Quantity A	**Quantity B**
x	y

6.

$3|x - 4| = 16$

Quantity A	**Quantity B**
x	$\frac{28}{3}$

7. If $b \neq 0$ and $\frac{a}{b} > 0$, then which of the following must be true?

- ☐ $a > b$
- ☐ $b > 0$
- ☐ $ab > 0$

8. If $6 < 2x - 4 < 12$, which of the following could be a value of x?

(A) 4
(B) 5
(C) 7
(D) 8
(E) 9

9. If $y < 0$ and $4x > y$, which of the following could be equal to $\frac{x}{y}$?

(A) 0
(B) $\frac{1}{4}$
(C) $\frac{1}{2}$
(D) 1
(E) 4

10.

$$|x + 6| = 3$$
$$|2y| = 6$$

Quantity A	**Quantity B**
The greatest possible value for x	The least possible value for y

11. If $|4y + 2| = 18$, which of the following could be the value of y^2?

Indicate two such values.

☐ 2
☐ 5
☐ 16
☐ 25
☐ 36

12.

$$3(x - 7) \geq 9$$
$$0.25y - 3 \leq 1$$

Quantity A	**Quantity B**
x	y

13. If $|1 - x| = 6$ and $|2y - 6| = 10$, which of the following could be the value of xy?

Indicate all such values.

☐ -40
☐ -14
☐ -10
☐ 56

14. If $2(x - 1)^3 + 3 \leq 19$, then the value of x must be

(A) greater than or equal to 3
(B) less than or equal to 3

(C) greater than or equal to -3
(D) less than or equal to -3
(E) less than -3 or greater than 3

15. If $3P < 51$ and $5P > 75$, what is the value of the integer P?

(A) 15
(B) 16
(C) 24
(D) 25
(E) 26

16. A bicycle wheel has spokes that go from a center point in the hub to equally spaced points on the rim of the wheel. If there are fewer than six spokes, what is the smallest possible angle between any two spokes?

(A) 18 degrees
(B) 30 degrees
(C) 40 degrees
(D) 60 degrees
(E) 72 degrees

17.

$$|-x| \geq 6$$

$xy^2 < 0$ where y is an integer.

Quantity A	**Quantity B**
x	-4

18. If $\frac{|x+4|}{2} > 5$ and $x < 0$, which of the following could be the value of x?

Indicate <u>all</u> such values.

☐ -6
☐ -14
☐ -18

19.

$$|x^3| < 64$$

Quantity A	**Quantity B**
$-x$	$-|x|$

20. If $|0.1x - 3| \geq 1$, then x could be which of the following values?

Indicate <u>all</u> such values.

☐ 10
☐ 20
☐ 30
☐ 40
☐ 50
☐ 60

21. If $|3x + 7| \geq 2x + 12$, then

(A) $x \leq \frac{-19}{5}$
(B) $x \geq \frac{-19}{5}$
(C) $x \geq 5$
(D) $x \leq \frac{-19}{5}$ or $x \geq 5$
(E) $\frac{-19}{5} \leq x \leq 5$

22.

$$|3 + 3x| < -2x$$

Quantity A	**Quantity B**
$\lvert x \rvert$	4

23. If $|y| \leq -4x$ and $|3x - 4| = 2x + 6$, what is the value of x?

(A) -3
(B) -1/3
(C) -2/5
(D) 1/3
(E) 10

24.

x is an integer such that $-x|x| > 4$.

Quantity A	**Quantity B**
x	2

25.

$$|x| < 1 \text{ and } y > 0$$

Quantity A	**Quantity B**
$\lvert x \rvert + y$	xy

26.

x and y are positive numbers such that $x + y + z < 1$ and $xy = 1$

Quantity A	Quantity B
z	-1

27.

$|x| > |y|$ and $x + y > 0$

Quantity A	Quantity B
y	x

28.

x and y are integers such that $|x|(y) + 9 < 0$ and $|y| \leq 1$.

Quantity A	Quantity B
x	-9

29. If $x + y + z = 0$ and $z = 8$, which of the following must be true?

(A) $x < 0$
(B) $y < 0$
(C) $x - y < 0$
(D) $z - y > 0$
(E) $x + y < 0$

30.

$p + |k| > |p| + k$

Quantity A	Quantity B
p	k

31.

$|x| + |y| > |x + z|$

Quantity A	Quantity B
y	z

32.

$b \neq 0$

$$\frac{|a|}{b} > 1$$

$$a + b < 0$$

Quantity A	Quantity B
a	0

33. If $\frac{a}{b} > \frac{c}{d}$, which of the following statements must be true?

Indicate all such statements.

- ☐ $\frac{a}{b} - \frac{c}{d} > 0$
- ☐ $ad < bc$
- ☐ $ad > bc$

34. If $f^2g < 0$, which of the following must be true?

(A) $f < 0$
(B) $g < 0$
(C) $fg < 0$
(D) $fg > 0$
(E) $f^2 < 0$

35. $\sqrt{96} < x\sqrt{6}$ and $\frac{x}{\sqrt{6}} < \sqrt{6}$. If x is an integer, which of the following is the value of x?

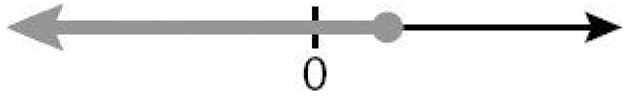
(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

36.

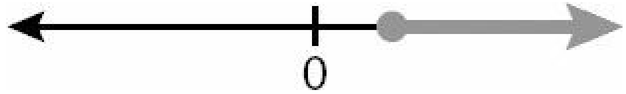
$$|x|y > x|y|$$

Quantity A	Quantity B
$(x + y)^2$	$(x - y)^2$

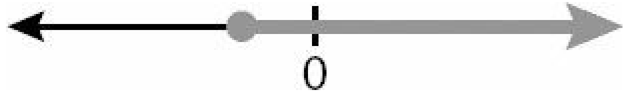
37. Which of the following could be the graph of all values of x that satisfy the inequality $4 - 11x \geq \frac{-2x+3}{2}$?



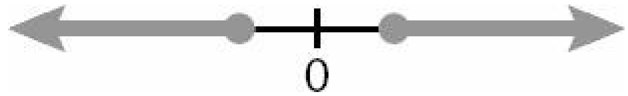
(A)



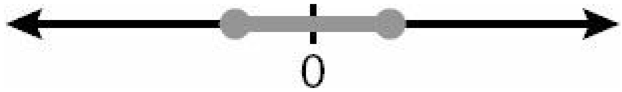
(B)



(C)



(D)



(E)

38. If $|x^2 - 6| = x$, which of the following could be the value of x?

(A) -2
(B) 0
(C) 1
(D) 3
(E) 5

39.

$$-1 < a < 0 < |a| < b < 1$$

Quantity A	**Quantity B**
$\left(\dfrac{a^2\sqrt{b}}{\sqrt{a}}\right)^2$	$\dfrac{ab^5}{\left(\sqrt{b}\right)^4}$

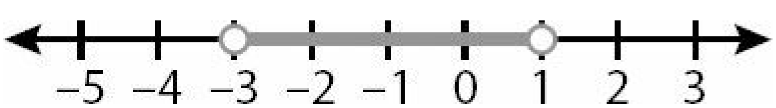
40.

$$x > |y| > z$$

Quantity A	**Quantity B**
$x + y$	$\|y\| + z$

41. The integers k, l, and m are consecutive even integers between 23 and 33. Which of the following could be the average of k, l, and m?

(A) 24
(B) 25
(C) 25.5
(D) 28
(E) 32

42. 

The number line above represents which of the following inequalities?

(A) $x < 1$
(B) $-6 < 2x < 2$
(C) $-9 < 3x < 6$
(D) $1 < 2x < 3$
(E) $x > -3$

43. For a jambalaya cook-off, there will be x judges sitting in a single row of x chairs. If x is greater than 3 but no more than 6, which of the following could be the number of possible seating arrangements for the judges?

Indicate two such numbers.

☐ 6
☐ 25
☐ 120
☐ 500
☐ 720

44. If $b \neq 0$, which of the following inequalities must be equivalent to $\frac{a}{-3b} < c$?

Indicate all such inequalities.

☐ $\frac{a}{b} > -3c$
☐ $\frac{a}{-3} < bc$
☐ $a > -3bc$

45.

$$a - b > a + b + c$$

Quantity A	Quantity B
$2b + c$	$b + c$

46.

$$|x + y| = 10$$
$$x > 0$$
$$z < y - x$$

Quantity A	Quantity B
z	10

47.

$$0 < a < \frac{b}{2} < 9$$

Quantity A	**Quantity B**
9 - *a*	$\frac{b}{2} - a$

48.

For all values of the integer p such that $1.9 < |p| < 5.3$, the function $f(p) = p^2$

Quantity A	**Quantity B**
$f(p)$ for the greatest value of p	$f(p)$ for the least value of p

49. If $\left|\frac{a}{b}\right|$ and $\left|\frac{x}{y}\right|$ are reciprocals and $\frac{a}{b}\left(\frac{x}{y}\right) < 0$ which of the following must be true?

(A) $ab < 0$
(B) $\frac{a}{b}\left(\frac{x}{y}\right) < -1$
(C) $\frac{a}{b} < 1$
(D) $\frac{a}{b} = -\frac{y}{x}$
(E) $\frac{y}{x} > \frac{a}{b}$

50. If $mn < 0$ and $\frac{k}{m} + \frac{l}{n} < mn$, which of the following must be true?

(A) $km + ln < (mn)^2$
(B) $kn + lm < 1$
(C) $kn + lm > (mn)^2$
(D) $k + l > mn$
(E) $kn > -lm$

51. Which of the following inequalities is equivalent to $|m + 2| < 3$?

(A) $m < 5$
(B) $m < 1$
(C) $-5 < m < 5$

(D) $m > -1$
(E) $-5 < m < 1$

52. If the reciprocal of the negative integer X is greater than the sum of Y and Z, then which of the following must be true?

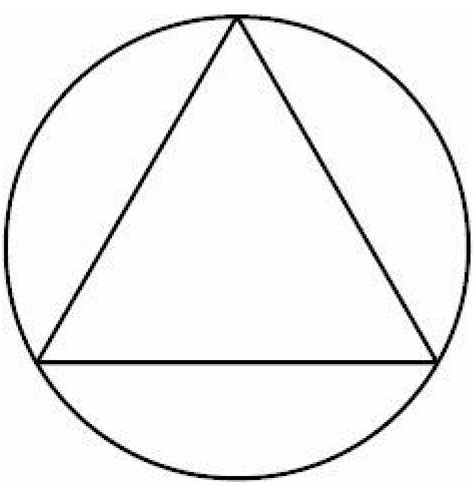
(A) $X > Y + Z$
(B) Y and Z are positive
(C) $1 > X(Y + Z)$
(D) $1 < XY + XZ$
(E) $\frac{1}{X} > Z - Y$

53. If $m + n - 2p < p + n + 4m$, which of the following inequalities must be true?

(A) $5m < 3p$
(B) $p > -m$
(C) $3m > 3p + 2n$
(D) $p > 2$
(E) $n < p$

54. If u and $-3v$ are greater than 0 and $\sqrt{u} < \sqrt{-3v}$, which of the following cannot be true?

(A) $u/3 < -v$
(B) $u/v > -3$
(C) $\sqrt{\frac{u}{-v}} < \sqrt{3}$
(D) $u + 3v > 0$
(E) $u < -3v$



55. In the figure above, an equilateral triangle is inscribed in a circle. If the arc bounded by adjacent corners of the triangle is between 4 π and 6 π long, which of the following could be the diameter of the circle?

(A) 6.5
(B) 9
(C) 11.9
(D) 15
(E) 23.5

Inequalities and Absolute Values Answers

1. **(D).** Since $3x$ - 18 is inside an absolute value, it could be either positive or negative 9 to have an absolute value of 9. Thus, solve the equation twice, once as though $3x$ - 18 is positive and once as though it is negative.

$$|3x - 18| = 9$$

$(3x - 18) = +9$	or	$(3x - 18) = -9$
$3x = 27$		$3x = 9$
$x = 9$		$x = 3$

$$x = 9 \text{ or } 3$$

Because x could be 9 or 3, x could be greater or less than 6, so the correct answer is (D).

2. **(C).** Solve the inequality algebraically:

$$2z + 4 \geq -18$$
$$2z \geq -22$$
$$z \geq -11$$

3. **(D).** Solve the inequality algebraically:

$$7y - 3 \leq 4y + 9$$
$$3y - 3 \leq 9$$
$$3y \leq 12$$
$$y \leq 4$$

Because y could be equal to 4 or greater than 4, the correct answer is (D).

4. **(B).** Solve the inequality algebraically:

$$d + \frac{3}{2} < 8$$
$$d < 8 - 1.5$$
$$d < 6.5$$

Quantity A is $2d$, so multiply both sides of the inequality by 2:

$$2d < 13$$

Quantity B is equal to 13, while $2d$ is less than 13, so the correct answer is (B).

5. **(D).** Solve algebraically for *x* and *y:*

$$\frac{4x}{7} \leq 15 + x$$
$$4x \leq 105 + 7x$$
$$-3x \leq 105$$
$$x \geq -35$$

(Remember to flip the inequality sign when dividing by -3!)

$$2y - 1.5 > 7$$
$$2y > 8.5$$
$$y > 4.25$$

Knowing that $x \geq -35$ and $y > 4.25$ is not enough to tell which is greater — the two ranges have a lot of overlap. For instance, *x* could be -30 and *y* could be 5 (Quantity B is greater), or *x* could be 100 and *y* could be 20 (Quantity A is greater). The correct answer is (D).

6. **(D).** Solve the inequality by first dividing both sides by 3 to isolate the absolute value, then solving for the positive and negative possibilities of (*x* - 4).

$$3|x-4| = 16$$
$$|x-4| = \frac{16}{3}$$
$$(x-4) = \frac{16}{3} \quad \text{or} \quad (x-4) = -\frac{16}{3}$$
$$x - 4 = \frac{16}{3} \qquad x = -\frac{16}{3} + 4$$
$$x = \frac{16}{3} + 4 \qquad x = -\frac{16}{3} + \frac{12}{3}$$
$$x = \frac{16}{3} + \frac{12}{3} \qquad x = -\frac{4}{3}$$
$$x = \frac{28}{3}$$

x could be $\frac{28}{3}$ or $-\frac{4}{3}$, making the two quantities equal or Quantity B greater, respectively. The correct answer is (D).

7. **III only.** If $\frac{a}{b} > 0$, then both a and b must have the same sign. That is, a and b are either both positive or both negative. Statement I could be true, but is not necessarily true. The relative values of a and b are not indicated by the inequality in the question stem. Statement II could be true, but is not necessarily true. If a were negative, b could be negative. Statement III must be true, as it indicates that a and b have the same sign.

8. **(C).** When manipulating a "three-sided" inequality, you must perform the same operations on all "sides." Therefore, the first step to simplify this inequality would be to add 4 to all sides: $10 < 2x < 16$. Next, divide all sides by 2. The result is $5 < x < 8$. The only answer choice that fits within the parameters of this inequality is 7. The correct answer is (C).

9. **(A).** If y is negative, then dividing both sides of the second inequality by y yields $\frac{4x}{y} < 1$. Remember, you must switch the direction of the inequality sign when multiplying or dividing by a negative (whether that negative is in number or variable form). Next, dividing both sides by 4 changes the inequality to $\frac{x}{y} < \frac{1}{4}$. The only answer choice less than $\frac{1}{4}$ is 0. The correct answer is (A).

10. **(C).** Solve each inequality, remembering that the phrase inside an absolute value can be positive or negative, so solve for each possibility:

$|x + 6| = 3$

$(x + 6) = 3$	or	$(x + 6) = -3$
$x = -3$		$x + 6 = -3$
		$x = -9$

$x = -3$ or -9

$|2y| = 6$

$(2y) = 6$	or	$(2y) = -6$
$y = 3$		$y = -3$

$y = 3$ or -3

The greatest possible value for x is -3. The least possible value for y is -3. The two quantities are equal, and the correct answer is (C).

11. **16, 25.** Solve the inequality, remembering that $4y + 2$ could be positive or negative, so solve for both possibilities:

$|4y + 2| = 18$

$(4y + 2) = 18$ or $(4y + 2) = -18$

$4y = 16$ $\quad$ $4y = -20$

$y = 4$ $\quad$ $y = -5$

$y = 4$ or -5

The value of y^2 could be 16 or 25.

12. **(D).** Solve each inequality algebraically:

$3(x - 7) \geq 9$
$x - 7 \geq 3$
$x \geq 10$

$0.25y - 3 \leq 1$
$0.25y \leq 4$
$y \leq 16$

Since the ranges for x and y overlap, either quantity could be greater. For instance, x could be 11 and y could be 15 (y is greater), or x could be 1,000 and y could be -5 (x is greater). The correct answer is (D).

13. **-40, -14, and 56 only.** Solve each absolute value:

$|1 - x| = 6$

$(1 - x) = 6$ or $(1 - x) = -6$

$-x = 5$ $\quad$ $-x = -7$

$x = -5$ $\quad$ $x = 7$

$x = -5$ or 7

$|2y - 6| = 10$

$(2y - 6) = 10$ or $(2y - 6) = -10$

$2y = 16$ $\quad$ $2y = -4$

$y = 8$ $\quad$ $y = -2$

$y = 8$ or -2

Since $x = -5$ or 7 and $y = 8$ or -2, calculate all four possible combinations for xy:

$(-5)(8) = -40$

$(-5)(-2) = 10$
$(7)(8) = 56$
$(7)(-2) = -14$

Select -40, -14, and 56. (Do NOT pick -10, as xy could be 10, but not -10).

14. **(B).** $2(x - 1)^3 + 3 \leq 19$
$2(x - 1)^3 \leq 16$
$(x - 1)^3 \leq 8$
You can take the cube root of both sides of an inequality, because cubing a number, unlike squaring it, does not change its sign.

$x - 1 \leq 2$
$x \leq 3$

This matches the language in answer choice (B).

15. **(B).** Dividing the first inequality by 3 results in $P < 17$. Dividing the second inequality by 5 results in $P > 15$. Therefore, $15 < P < 17$. Because P is an integer, it must be 16.

16. **(E).** In this scenario, if there are n spokes, there are n angles between them. Thus the measure of the angle between spokes is $\frac{360}{n}$. Since $n < 6$, you can rewrite this expression as $\frac{360}{(\text{less than } 6)}$. Dividing by a "less than" produces a "greater than" result. Therefore, $\frac{360}{(\text{less than } 6)}$ = greater than 60. The only answer that is greater than 60 is (E). To verify, note that n can be at most 5, as n is an integer. Because there are 360 degrees in a circle, a wheel with 5 spokes would have $\frac{360}{(\text{less than } 6)}$ degrees between adjacent spokes. The correct answer is (E).

17. **(B).** First, solve the inequality for x, remembering the two cases you must consider when dealing with absolute value: $-x$ is positive and $-x$ is negative.

$|-x| \geq 6$

$+(-x) \geq 6$	or	$-(-x) \geq 6$
$-x \geq 6$		$x \geq 6$
$x \leq -6$		

$x \leq -6$ or $x \geq 6$

Because $xy^2 < 0$, neither x nor y equals zero. A squared term cannot be negative, so y^2 must be positive. For xy^2 to be negative, x must be negative. This rules out the $x \geq 6$ range of solutions for x. Thus, $x \leq -6$ is the only range of valid solutions. Since all values less than or equal to -6 are less than -4, the correct answer is (B).

18. **III only.** Solve the absolute value inequality by first isolating the absolute value:

$$\frac{|x+4|}{2} > 5$$

$|x + 4| > 10$

If $(x + 4)$ is positive or zero, the absolute value bars do nothing and can be removed:

$x + 4 > 10$
$x > 6$

This is not a valid solution range, as the other inequality indicates that x is negative.

Then solve for negative case. Note that $|x + 4| > 10$ when $(x + 4)$ is more positive than 10 or *more negative* than -10.

$(x + 4) < -10$
$x < -14$

Alternatively, using the identity that $|a| = -a$ when a is negative:

$|x + 4| > 10$
$-(x + 4) > 10$ when $(x + 4)$ is negative
$-x - 4 > 10$
$-x > 14$

$x < -14$ (flip the inequality sign when multiplying both sides by -1.)

If $x < -14$, only -18 is a valid answer.

19. **(D).** First, solve the absolute value inequality, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$|x^3| < 64$

$+(x^3) < 64$ $x < 4$	or	$-(x^3) < 64$ $x^3 > -64$ (Flip the inequality sign when multiplying by -1.) $x > -4$

$-4 < x < 4$

x could be positive, negative, or zero. If x is positive or zero, the two quantities are equal. If x is negative, Quantity A is greater. The correct answer is (D).

20. **10, 20, 40, 50, 60 only.** Solve the absolute value inequality, using the identity that $|a| = a$ when a is positive or

zero and $|a| = -a$ when a is negative:

$|0.1x - 3| \geq 1$

$+(0.1x - 3) \geq 1$	or	$-(0.1x - 3) \geq 1$
$0.1x - 3 \geq 1$		$-0.1x + 3 \geq 1$
$0.1x \geq 4$		$-0.1x \geq -2$
$x \geq 40$		$x \leq 20$ (Flip the inequality sign when dividing by -0.1)

Since $x \leq 20$ or $x \geq 40$, x cannot equal 30, but it can be any of the other values from the choices.

Alternatively, plug the choices to test which values "work."

10: $|0.1(10) - 3| = |1 - 3| = |-2| = 2$, which is ≥ 1.
20: $|0.1(20) - 3| = |2 - 3| = |-1| = 1$, which is ≥ 1.
30: $|0.1(30) - 3| = |3 - 3| = |0| = 0$, which is NOT ≥ 1.
40: $|0.1(40) - 3| = |4 - 3| = |1| = 1$, which is ≥ 1.
50: $|0.1(50) - 3| = |5 - 3| = |2| = 2$, which is ≥ 1.
60: $|0.1(60) - 3| = |6 - 3| = |3| = 3$, which is ≥ 1.

21. **(D).** Solve $|3x + 7| \geq 2x + 12$, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$+(3x + 7) \geq 2x + 12$	or	$-(3x + 7) \geq 2x + 12$
$x + 7 \geq 12$		$-3x - 7 \geq 2x + 12$
$x \geq 5$		$-7 \geq 5x + 12$
		$-19 \geq 5x$
		$\frac{-19}{5} \geq x$

$x \leq \frac{-19}{5}$ or $x \geq 5$

22. **(B).** Solve the absolute value inequality, using the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

$|3 + 3x| < -2x$

$+(3 + 3x) < -2x$	or	$-(3 + 3x) < -2x$
$3 + 5x < 0$		$-3 - 3x < -2x$
$5x < -3$		$-3 < x$
$x < -\frac{3}{5}$		

$$-3 < x < -\frac{3}{5}$$

Since x is between -3 and $-\frac{3}{5}$, its absolute value is between $\frac{3}{5}$ and 3. Thus, Quantity A is less than Quantity B.

23. **(C).** The inequality is not strictly solvable, as it has two unknowns. However, any absolute value cannot be negative. Putting $0 \leq |y|$ and $|y| \leq -4x$ together, $0 \leq -4x$. Dividing both sides by -4 and flipping the inequality sign, this implies that $0 \geq x$.

Now solve the absolute value equation:

$$|3x - 4| = 2x + 6$$

$+(3x - 4) = 2x + 6$	or	$-(3x - 4) = 2x + 6$
$3x - 4 = 2x + 6$		$-3x + 4 = 2x + 6$
$x - 4 = 6$		$4 = 5x + 6$
$x = 10$		$-2 = 5x$

$$x = 10 \text{ or } -2/5$$

If $x = 10$ or $-2/5$, but $0 \geq x$, then x can only be $-2/5$.

24. **(B).** If $-x|x| \geq 4$, $-x|x|$ is positive. Because $|x|$ is positive by definition, $-x|x|$ is positive only when $-x$ is also positive. This occurs when x is negative. For example, $x = -2$ is one solution allowed by the inequality: $-x|x| = -(-2) \times |-2| = 2 \times 2 = 4$.

So, Quantity A can be -2, -3, -4, -5, -6, etc. The maximum value of Quantity A is less than 2, so Quantity B is greater.

25. **(A).** The inequality $|x| < 1$ allows x to be either a positive or negative fraction (or zero). Interpreting the absolute value sign, it is equivalent to $-1 < x < 1$. As indicated, y is positive.

When x is a negative fraction,

Quantity A: $|x| + y$ = positive fraction + positive = positive

Quantity B: xy = negative fraction × positive = negative

Quantity A is greater in these cases.

When x is zero,

Quantity A: $|x| + y$ = 0 + positive = positive

Quantity B: $xy = 0 \times \text{positive} = 0$

Quantity A is greater in this case.

When x is a positive fraction,

Quantity A: $|x| + y$ = positive fraction + y = greater than y

Quantity B: xy = positive fraction $\times$ y = less than y

Quantity A is greater in these cases.

In all cases, Quantity A is greater.

26. **(B).** Solve the inequality for z.

$x + y + z < 1$
$z < 1 - (x + y)$

Based on the facts that x and y are positive and $xy = 1$, either x and y both equal 1 or they are reciprocals (e.g., 2 and $\frac{1}{2}$, 3 and $\frac{1}{3}$, 4 and $\frac{1}{4}$, etc.). Thus, the minimum value of $x + y$ is 2. Plugging into the inequality for z:

$z < 1 - (x + y)$
$z < 1$ - (at least 2)
$z <$ at most -1

Because z cannot equal -1 (z is less than -1) Quantity B is greater.

27. **(B).** In general, there are four cases for the signs of x and y, some of which can be ruled out by the constraints of this question.

x	y	$x + y > 0$
pos	pos	true
pos	neg	true when $\|x\| > \|y\|$
neg	pos	false when $\|x\| > \|y\|$
neg	neg	false

So only the first two cases need to be considered for this question.

If x and y are both positive, $|x| > |y|$ just means that $x > y$.

If x is positive and y is negative, $x > y$ simply because positive > negative.

In both cases, $x > y$. Quantity B is greater.

28. **(D).** If y is an integer and $|y| \leq 1$, then $y = -1$, 0, or 1. The other inequality can be simplified from $|x|(y) + 9 < 0$ to $|x|(y) < -9$. In words, $|x|(y)$ is negative. Because $|x|$ cannot be negative by definition, y must be negative, so only $y = -1$ is possible.

If $y = -1$, then $|x|(y) = |x|(-1) = -|x| < -9$. So, $-|x| = -10, -11, -12, -13$, etc.

Thus, $x = \pm 10, \pm 11, \pm 12, \pm 13$, etc. Some of these x values are greater than -9 and some are less than -9.

29. **(E).** If $x + y + z = 0$ and $z = 8$, then $x + y = -8$. It is definitely true that $-8 < 0$, so $x + y < 0$ must be true.

Alternatively, find a counterexample to disprove the other choices.

(A) x could be positive: $x = 5$ and $y = -13$ make $x + y = 5 + (-13) = -8$.

(B) y could be positive: $x = -13$ and $y = 5$ make $x + y = -13 + 5 = -8$.

(C) $x - y$ could be positive: $x = 5$ and $y = -13$ make $x - y = 5 - (-13) = 18$ and $x + y + z = 5 + (-13) + 8 = 0$.

(D) $z - y$ could be positive: $z = 8$ and $y = -13$ make $z - y = 8 - (-13) = 21$ and $x = 5$ would make the sum $x + y + z = 5 + (-13) + 8 = 0$.

(E) $x + y$ CANNOT be positive or zero, as $x + y + z$ would then be at least 8, not equal to 0.

30. **(A).** In general, there are four cases for the signs of p and k, some of which can be ruled out by the constraints of this question.

p	k	$p + \|k\| > \|p\| + k$
pos	pos	Not in this case: For positive numbers, absolute value "does nothing," so both sides are equal to $p + k$.
pos	neg	True for this case: p + (a positive absolute value) is greater than p + (a negative value).
neg	pos	Not in this case: k + (a negative value) is less than k + (a positive absolute value).
neg	neg	Possible in this case: It depends on relative values. Both sides are a positive plus a negative.

Additionally, check whether p or k could be zero.

If $p = 0$, $p + |k| > |p| + k$ is equivalent to $|k| > k$. This is true when k is negative.

If $k = 0$, $p + |k| > |p| + k$ is equivalent to $p > |p|$. This is not true for any p value.

So, there are three possible cases for p and k values. For the second one, use the identity that $|a| = -a$ when a is negative.

p	k	Interpret:
pos	neg	$p = \text{pos} > \text{neg} = k$ $p > k$
neg	neg	$p + \|k\| > \|p\| + k$ $p + -(k) > -(p) + k$ $p - k > -p + k$ $2p - k > k$ $2p > 2k$ $p > k$
0	neg	$p = 0 > \text{neg} = k$ $p > k$

In all the cases that are valid according to the constraint inequality, p is greater than k. Quantity A is greater.

31. **(D).** Given only one inequality with three unknowns, solving will not be possible. Instead, test numbers with the goal of proving (D).

For example, $x = 2$, $y = 5$, and $z = 3$.

Check that $|x| + |y| > |x + z|$: $|2| + |5| > |2 + 3|$ is $7 > 5$, which is true.

In this case, $y > z$.

Try to find another example such that $y < z$. Always consider negatives in inequalities and absolute value questions.

Consider another example: $x = 2$, $y = -5$ and $z = 3$.

Check that $|x| + |y| > |x + z|$: $|2| + |-5| > |2 + 3|$ is $7 > 5$, which is true.

In this case, $z > y$.

Either statement could be greater. It cannot be determined from the information given.

32. **(B).** If $\frac{|a|}{b}$ is greater than 1, then it is positive. Because $|a|$ is nonnegative by definition, b would have to be positive. Thus, if you cross multiply, you do not have to flip the sign of the inequality:

$$\frac{|a|}{b} > 1$$

$$|a| > b$$

To summarize, $b > 0$ and $|a| > b$. Putting this together, $|a| > b > 0$.

In order for $a + b$ to be negative, a must be more negative than b is positive. For example, $a = -4$ and $b = 2$ agree with all the constraints so far. Note that a cannot be zero (because $\frac{|a|}{b} = 0$ in this case, not > 1) and a cannot be positive (because $a + b > 0$ in this case, not < 0).

Therefore, $a < 0$. Quantity B is greater.

33. **I only.**

Statement I: TRUE. Subtract $\frac{c}{d}$ from both sides of the inequality $\frac{a}{b} > \frac{c}{d}$, and you will get $\frac{a}{b} - \frac{c}{d} > 0$. It must be true.

Statement II: Maybe. This is only true if b and d have opposite signs, because it is the result of multiplying both sides by bd and flipping the inequality sign, which you would only do when bd is negative.

Statement III: Maybe. This is only true if b and d have the same sign, because it is the result of multiplying both sides by bd without flipping the inequality sign, which is only acceptable when bd is positive.

34. **(B).** Neither f nor g can be zero, or f^2g would be zero. The square of either a positive or negative base is always positive, so f^2 is positive. In order for $f^2g < 0$ to be true, g must be negative. Therefore, the correct answer is (B). Answer choices (A), (B), and (C) are not correct because f could be either positive or negative. Answer choice (E) directly contradicts the truth that f^2 is positive.

35. **(D).** Solve the first inequality:

$$\sqrt{96} < x\sqrt{6}$$

$$\frac{\sqrt{96}}{\sqrt{6}} < x$$

$$\sqrt{16} < x$$

$$4 < x$$

Solve the second inequality:

$$\frac{x}{\sqrt{6}} < \sqrt{6}$$

$$x < \sqrt{6}\sqrt{6}$$

$$x < \sqrt{36}$$

$$x < 6$$

Combining the two inequalities, $4 < x < 6$ so x must be 5. The correct answer is (D).

36. **(B).** In general, there are four cases for the signs of x and y, some of which can be ruled out by the constraint in the question stem. Use the identity that $|a| = a$ when a is positive or zero and $|a| = -a$ when a is negative:

x	y	$\|x\|y > x\|y\|$ is equivalent to:	True or False?
pos	pos	$xy > xy$	False: $xy = xy$
pos	neg	$xy > x(-y)$	False: xy is negative, and $-xy$ is positive.
neg	pos	$(-x)y > xy$	True: xy is negative, and $-xy$ is positive.
neg	neg	$(-x)y > x(-y)$	False: $-xy = -xy$

Note that if either x or y equals 0, that case would also fail the constraint.

The only valid case is when x is negative and y is positive.

Quantity A: $(x + y)^2 = x^2 + 2xy + y^2$
Quantity B: $(x - y)^2 = x^2 - 2xy + y^2$

Ignore (or subtract) $x^2 + y^2$ as it is common to both quantities. Thus,

Quantity A: $2xy$ = 2(negative)(positive) = negative
Quantity B: $-2xy$ = -2(negative)(positive) = positive

Quantity B is greater.

37. **(A).** First, solve $4 - 11x \geq \frac{-2x+3}{2}$ for x:

$$4 - 11x \geq \frac{-2x+3}{2}$$

$$8 - 22x \geq -2x + 3$$

$$5 - 22 \geq -2x$$

$$5 \geq 20x$$

$$\frac{5}{20} \geq x$$
$$\frac{1}{4} \geq x$$

Thus, the correct choice should show the black line beginning to the right of zero (in the positive zone), and continuing indefinitely into the negative zone. Even without actual values (other than zero) marked on the graphs, only (A) meets these criteria.

38. **(D).** While you could set $x^2 - 6$ equal to both x and $-x$ and then solve both equations (there is a positive and negative case because of the absolute value), it is probably easier for most people to plug in the answers:

	x	$x^2 - 6$	$x^2 - 6$
(A)	-2	$(-2)^2 - 6 = 4 - 6 = -2$	2
(B)	0	$(0)^2 - 6 = 0 - 6 = -6$	6
(C)	1	$(1)^2 - 6 = 1 - 6 = 5$	5
(D)	**3**	$\mathbf{(3)^2 - 6 = 9 - 6 = 3}$	**3**
(E)	5	$(5)^2 - 6 = 25 - 6 = 19$	19

Note that only $x = 3$ works. While this chart shows the results of trying every choice, note that if you were doing this on your own, you could stop as soon as you got a choice that worked.

39. **(A).** From $-1 < a < 0 < |a| < b < 1$, the following can be determined:

a is a negative fraction,
b is a positive fraction, and
b is more positive than a is negative. (i.e., $|b| > |a|$, or b is farther from 0 on the number line than a is.)

Using exponent rules, simplify the quantities.

Quantity A: $$\left(\frac{a^2\sqrt{b}}{\sqrt{a}}\right)^2 = \frac{(a^2)^2(\sqrt{b})^2}{(\sqrt{a})^2} = \frac{a^4 b}{a} = a^3 b$$

Quantity B: $$\frac{ab^5}{(\sqrt{b})^4} = \frac{ab^5}{(b^{1/2})^4} = \frac{ab^5}{b^{\frac{1}{2}\times 4}} = \frac{ab^5}{b^2} = ab^3$$

Dividing both quantities by b would be acceptable, as b is positive and doing so won't flip the relative sizes of the quantities. It would be nice to cancel a's, too, but it is problematic that a is negative. Dividing both quantities by a^2 would be okay, though, as a^2 is positive.

Divide both quantities by a^2b.

Quantity A: $\frac{a^3b}{a^2b} = a$

Quantity B: $\frac{ab^3}{a^2b} = \frac{b^2}{a}$

Just to make the quantities more similar in form, divide again by b, which is positive.

Quantity A: $\frac{a}{b}$

Quantity B: $\frac{b}{a}$

Both quantities are negative, as a and b have opposite signs. Remember that b is more positive than a is negative. (i.e., $|b| > |a|$, or b is farther from 0 on the number line than a is.) Thus, each fraction can be compared to -1.

Quantity A: $\frac{a}{b}$ is less negative than -1. That is, $-1 < \frac{a}{b}$.

Quantity B: $\frac{b}{a}$ is more negative than -1. That is, $\frac{b}{a} < -1$.

Quantity A is greater.

40. **(D).** Given only a compound inequality with three unknowns, solving will not be possible. Instead, test numbers with the goal of proving (D). Always consider negatives in inequalities and absolute value questions.

For example, $x = 10$, $y = -9$, and $z = 8$.

Check that $x > |y| > z$: $10 > |-9| > 8$, which is true.

In this case, $x + y = 10 + (-9) = 1$ and $|y| + z = 9 + 8 = 17$. Quantity B is greater.

Try to find another example such that Quantity A is greater.

For example, $x = 2$, $y = 1$, and $z = -3$.

Check that $x > |y| > z$: $2 > |1| > -3$, which is true.

In this case, $x + y = 2 + 1 = 3$ and $|y| + z = 1 + (-3) = -2$. Quantity A is greater.

Either statement could be greater. It cannot be determined from the information given.

41. **(D).** The values for k, l, and m, respectively, could be any of the following three sets:

Set 1: 24, 26, and 28
Set 2: 26, 28, and 30
Set 3: 28, 30, and 32

For evenly spaced sets with an odd number of terms, the average is the middle value. Therefore, the average of k, l, and m could be 26, 28, or 30. Only answer choice (D) matches one of these possibilities.

42. **(B).** The number line indicates a range between, but not including, -3 and 1. However, $-3 < x < 1$ is not a given option. However, answer choice (B) gives the inequality $-6 < 2x < 2$. Dividing all three sides of this inequality by 2 yields $-3 < x < 1$.

43. **120 and 720 only.** If x is "greater than 3 but no more than 6," then x is 4, 5, or 6. If there are 4 judges sitting in 4 seats, they can be arranged $4! = 4 \times 3 \times 2 \times 1 = 24$ ways. If there are 5 judges sitting in 5 seats, they can be arranged $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$ ways. If there are 6 judges sitting in 6 seats, they can be arranged $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ ways. Thus, 24, 120, and 720 are all possible answers. Only 120 and 720 appear in the choices.

44. **I only.** For this problem, you need to know that multiplying or dividing an inequality by a negative requires you to flip the inequality sign. Thus, multiplying or dividing an inequality by a variable should *not* be done unless you know *whether* to flip the inequality sign (i.e., whether the variable represents a positive or negative number).

Statement I: TRUE. Multiply both sides of the original inequality by -3 and flip the inequality sign.

Statement II: Maybe. Multiply both sides of the original inequality by b to get Statement II, but only if b is positive. If b is negative, the direction of the inequality sign would have to be changed.

Statement III: Maybe. Multiplying both sides of the original inequality by $-3b$ could lead to Statement III, but because the inequality sign flipped, this is only true if $-3b$ is negative (i.e., if b is positive).

45. **(D).** There are three variables in the original question, but not all of them are relevant. Simplify the original constraint:

$a - b > a + b + c$
$-b > b + 4$
$0 > 2b + c$

So, $2b + c$ is negative.

Next, notice that $b + c$ is common to both quantities, so subtracting it from both will not change their relative values:

Quantity A: $2b + c - (b + c) = 2b + c - b - c = b$
Quantity B: $b + c - (b + c) = 0$

This question is really about the sign of b!

Based on the constraint that $2b + c$ is negative, b could be positive or negative.

If $b = 2$ and $c = -6$, $2b + c = 4 - 6 = -2$, which is negative. In this case, Quantity A is greater.

If $b = -2$ and $c = 1$, $2b + c = -4 + 1 = -3$, which is negative. In this case, Quantity B is greater.

The correct answer is (D).

46. **(B).** From $z < y - x$, the value of z depends on x and y. So, solve for x and y as much as possible. There are two cases for the absolute value equation: $|x + y| = 10$ means that $(x + y) = \pm 10$. Consider these two cases separately

The positive case:

$x + y = 10$, so $y = 10 - x$.
Substitute into $z < y - x$, getting $z < (10 - x) - x$, or $z < 10 - 2x$.
Because x is at least zero, $10 - 2x \leq 10$.
Putting the inequalities together, $z < 10 - 2x \leq 10$.
Thus, $z < 10$.

The negative case:

$x + y = -10$, so $y = -10 - x$.
Substitute into $z < y - x$, getting $z < (-10 - x) - x$, or $z < -10 - 2x$.
Because x is at least zero, $-10 - 2x \leq -10$.
Putting the inequalities together, $z < -10 - 2x \leq -10$.
Thus, $z < -10$.

In both cases, 10 is greater than z. The correct answer is (B).

47. **(A).** The variable a is common to both quantities, and adding it to both quantities to cancel will not change the relative values of the quantities.

Quantity A: $(9 - a) + a = 9$

Quantity B: $\left(\frac{b}{2} - a\right) + a = \frac{b}{2}$

According to the given constraint, $\frac{b}{2} < 9$, so Quantity A is greater. The correct answer is (A).

48. **(C).** If p is an integer such that $1.9 < |p| < 5.3$, p could be 2, 3, 4, or 5, as well as -2, -3, -4, -5. The greatest value

of p is 5, for which the value of $f(p) = 5^2 = 25$. The least value of p is -5, for which the value of $f(p) = (-5)^2 = 25$.

49. **(D).** If $\frac{a}{b}\left(\frac{x}{y}\right) < 0$, then the two fractions have opposite signs. Therefore, by the definition of reciprocals, $\frac{a}{b}$ must be the negative inverse of $\frac{x}{y}$, no matter which one of the fractions is positive. In equation form, this means $\frac{a}{b} = -\frac{y}{x}$, which is choice (D). The other choices are possible but not certain.

50. **(C).** In order to get m and n out of the denominators of the fractions on the left side of the inequality, multiply both sides of the inequality by mn. The result is $kn + lm > (mn)^2$. The direction of the inequality sign changes because mn is negative. This is an exact match with (C), which must be the correct answer.

51. **(E).** When dealing with absolute values, always consider two cases.

The first case is when the expression within the absolute value signs is positive. If $m + 2 > 0$, then $|m + 2| = m + 2$, and therefore $m + 2 < 3$. Subtracting 2 from both sides, this inequality becomes $m < 1$.

The second case is when $m + 2 < 0$, so that $|m + 2| = -(m + 2)$, and therefore $-(m + 2) < 3$. Divide both sides by -1 to get $m + 2 > -3$, remembering to flip the inequality sign. Subtracting 2 from both sides, this inequality becomes $m > -5$.

Combining these two inequalities, the result is $-5 < m < 1$.

52. **(D).** The inequality described in the question is $0 > \frac{1}{X} > Y + Z$. Multiplying both sides of this inequality by X, the result is $0 < 1 < XY + XZ$. Notice that the direction of the inequality sign must change because X is negative.

(A) Maybe true: true only if X equals -1.
(B) Maybe true: either Y or Z or both can be negative.
(C) False: the direction of the inequality sign is opposite the correct direction determined above.
(D) TRUE. It is a proper rephrasing of the original inequality.
(E) Maybe true: it is not a correct rephrasing of the original inequality.

53. **(B).** The given inequality can be simplified as follows:

$m + n - 2p < p + n + 4m$
$m - 2p < p + 4m$
$-3p < 3m$
$p > -m$ (Remember to flip the inequality sign when dividing by -3.)

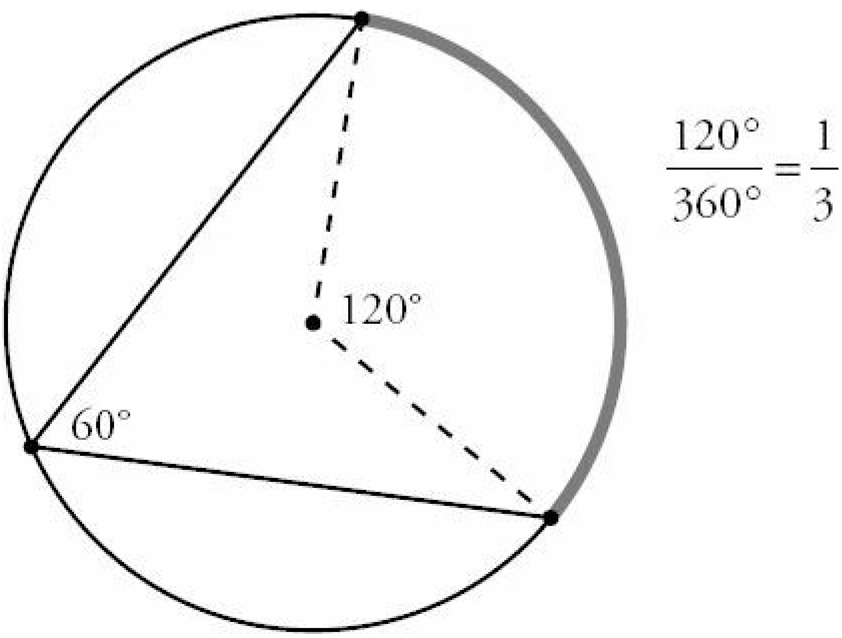
The correct answer is (B).

54. **(D).** When the GRE writes a root sign, the question writers are indicating a nonnegative root only. Therefore both sides of this inequality are positive. Thus, you can square both sides without changing the direction of the inequality

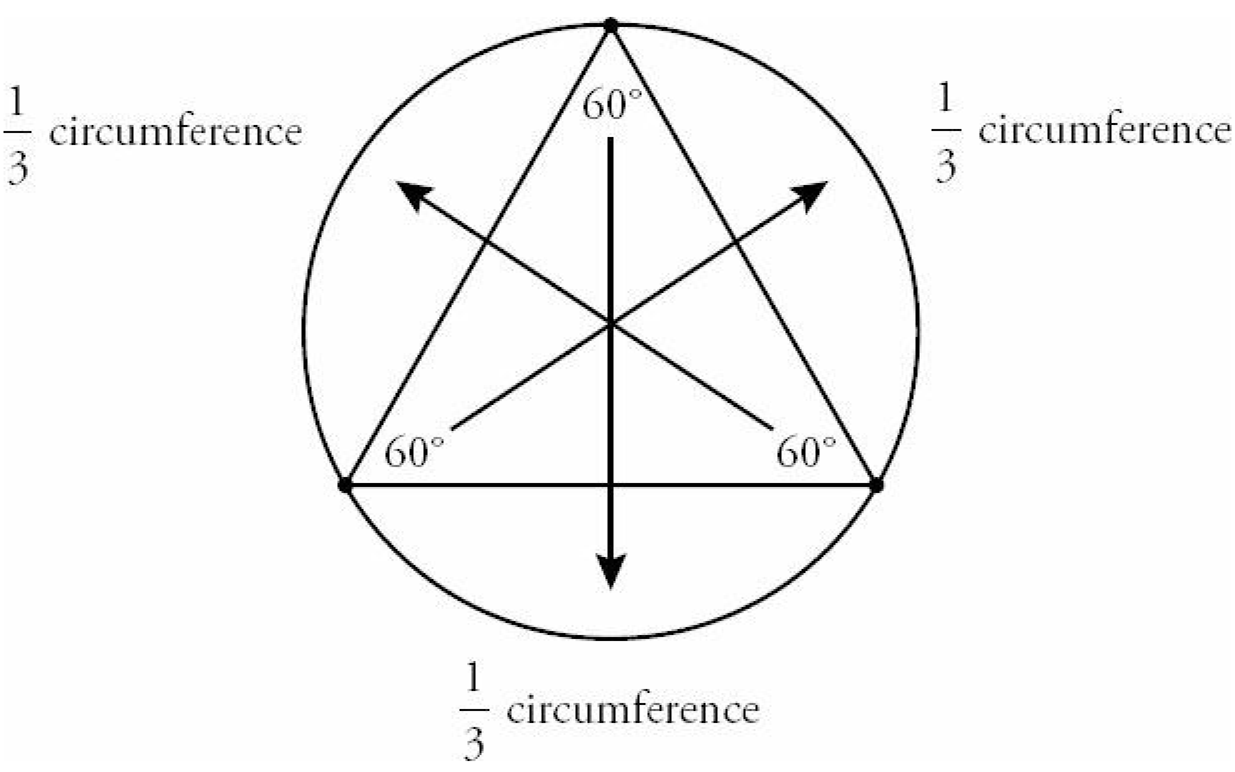
sign. So $u < -3v$. Now evaluate each answer choice:

(A) Must be true. Divide both sides of $u < -3v$ by 3.
(B) Must be true. It is given that $-3v > 0$ and therefore, $v < 0$. Then, when dividing both sides of $u < -3v$ by v, you must flip the inequality sign and get $u/v > -3$.
(C) Must be true. This is the result after dividing both sides of the original inequality by $\sqrt{-v}$.
(D) CANNOT be true. Adding $3v$ to both sides of $u < -3v$ results in $u + 3v < 0$, not $u + 3v > 0$.
(E) Must be true. This is the result of squaring both sides of the original inequality.

55. **(D).** Since each of the three arcs corresponds to one of the 60 degree angles of the equilateral triangle, each arc represents 1/3 of the circumference of the circle. The diagram below illustrates this for just one of the three angles in the triangle:



The same is true for each of the three angles:



Since each of the three arcs is between 4π and 6π, triple these values to determine that the circumference of the circle is between 12π and 18π. Because circumference equals π times the diameter, the diameter of this circle must be between 12 and 18. Only choice (D) is in this range.

Chapter 10

of

Functions, Formulas, and Sequences

In This Chapter…

Functions, Formulas, and Sequences

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If $f(x) = x^2 + 1$, what is $f(2) + f(-2)$?

(A) 0
(B) 1
(C) 4
(D) 5
(E) 10

2. If $f(x) = 2x$ and $g(x) = x^3$, what is $f(g(-3))$?

(A) -6
(B) -27
(C) 54
(D) -54
(E) -216

3. If $h(x) = 2x^3 - 3$ and $h(m) = -19$, what is the value of m?

(A) -3

(B) -2
(C) 2
(D) 6,856
(E) 6,862

4. If $f(x) = x - 3$ and $2[f(g)] = 14$, what is the value of $f(4g)$?

5. If $f(a, b) = a^2b^4$, and $f(m, n) = 5$, what is $f(3m, 2n)$?

6. If $f(x) = x^2 - 1$, what is the value of $f(y) + f(-1)$?

(A) $y^2 - 1$
(B) y^2
(C) $y^2 + 1$
(D) $y^2 - 2y$
(E) $y^2 - 2y - 1$

7. If $g(x) = 3x - 3$, what is the value of $g(3) + g\left(-\frac{1}{3}\right)$? ?

8. If $f(x) = \frac{x}{2} - 1$, what is the value of $f(f(10))$?

9. If $h(x) = 5x^2 + x$, then $h(a + b) =$

(A) $5a^2 + 5b^2$
(B) $5a^3 + 5b^3$
(C) $5a^2 + 5b^2 + a + b$
(D) $5a^3 + 10ab + 5b^3$
(E) $5a^2 + 10ab + 5b^2 + a + b$

10. If $\overline{|x} = 2x^2 + 2$, what is $\overline{|4}$?

(A) $\overline{|\overline{|-1}}$
(B) $\overline{|-2}$
(C) $\overline{|\overline{|2}}$
(D) $\overline{|17}$
(E) $\overline{|34}$

11. $\boxed{x}$ is defined as the least integer greater than x for all odd values of x, and the greatest integer less than x for all even values of x. What is $\boxed{-2} - \boxed{5}$?

(A) -12
(B) -9
(C) -8
(D) -7
(E) 3

12. $g(x) = x^2 - 4$ and $g(c) = 12$. If $c < 0$, what is $g(c - 2)$?

13. If $h(x) = 2x - 1$ and $g(x) = x^2 - 3$, what is $h(g(5))$?

14. $h(x) = 2x - 1$ and $g(x) = x^2 - 3$. If $g(m) = 61$, what is $h(m)$ if $m > 0$?

15. If &x& is defined as one-half the square of x, what is the value of $\dfrac{\&4\&}{\&6\&}$?

16. If $\sim x = |14x|$, which of the following must be true?

Indicate <u>all</u> such answers.

☐ ~2 = ~(-2)
☐ ~3 + ~4 = ~7
☐ The minimum possible value of ~x is zero

17. #x = the square of the number that is 2 less than x. What is the value of #5 - #(-1)?

☐

18. If $g(x) = \frac{x^2(4x+9)}{(3x-3)(x+2)}$, which answer represents all values of x for which $g(x)$ is undefined?

(A) 0
(B) $-\frac{9}{4}$
(C) -2, 1
(D) -2, 0, 1
(E) $-2, -\frac{9}{4}, 1$

19. If $f(x) = \frac{\sqrt{x-2}}{x}$ for all integer values of x, for how many values of x is $f(x)$ undefined?

(A) 0
(B) 1
(C) 2
(D) 3
(E) more than 3

20.

$$f(x) = 2x - 3$$
$$f(m) = -11$$

Quantity A	**Quantity B**
The value of m	Half the value of $f(m)$

21. The price of a phone call consists of a standard connection fee, which does not change, plus a per-minute charge. A 10-minute call costs \$2.90 and a 16-minute call costs \$4.40. How much does a 13-minute call cost?

(A) \$3.55
(B) \$3.57
(C) \$3.58
(D) \$3.65

(E) \$3.77

22. The first three terms in an arithmetic sequence are 30, 33, and 36. What is the 80th term?

$\boxed{}$

23. The sequence S is defined as $S_n = 2(S_{n-1}) - 4$. If $S_1 = 6$, what is S_5?

(A) -20
(B) 16
(C) 20
(D) 24
(E) 36

24. The sequence S is defined as $S_n = 3(S_{n-1}) + 1$. If $S_1 = -2$, what is S_4?

(A) -39
(B) -41
(C) -43
(D) -45
(E) -47

25. If $S_n = S_{n-1} + S_{n-2} - 3$, then what is S_6 when $S_1 = 5$ and $S_2 = 0$?

(A) -6
(B) -5
(C) -3
(D) -1
(E) 1

26. If $S_n = S_{n-1} + S_{n-2} - 1$, then what is S_4 when $S_0 = -10$ and $S_2 = 0$?

(A) -3
(B) 0
(C) 9
(D) 10
(E) 14

27. If $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$, what is S_6 when $S_1 = 4$, $S_2 = 0$, and $S_4 = -4$?

(A) -2
(B) -12
(C) -16
(D) -20
(E) -24

28. The sequence P is defined as $P_n = 10(P_{n-1}) - 2$. If $P_1 = 2$, what is P_4?

$\boxed{}$

29. The sequence S is defined as $S_{n-1} = \frac{1}{4}(S_n)$. If $S_1 = -4$, what is S_4?

(A) -256
(B) -64
(C) -1/16
(D) 1/16
(E) 256

30. In sequence A_n, $A_1 = 45$, and $A_n = A_{n-1} + 2$ for all integers $n > 1$. What is the sum of the first 100 terms in sequence A_n?

(A) 243
(B) 14,400
(C) 14,500
(D) 24,300
(E) 24,545

31. In a certain sequence, the term a_n is given by the formula $a_n = a_{n-1} + 10$. What is the positive difference between the 10th term and the 15th term?

(A) 5
(B) 10
(C) 25
(D) 50
(E) 100

32. In a certain sequence, the term a_n is given by the formula $a_n = 10(a_{n-1})$. How many times greater is a_{10} than a_8?

(A) 1
(B) 3
(C) 10
(D) 30
(E) 100

33. Ashley and Beatrice received the same score on a physical fitness test. The scores for this test, t, are determined by the formula $t = 3ps - 25m$ where s and p are the numbers of sit-ups and push-ups the athlete can do in one minute and m is the number of minutes she takes to run a mile. Ashley did 10 sit-ups and 10 push-ups and ran an 8-minute mile. Beatrice did half as many sit-ups and twice as many push-ups. If both girls received the same overall score, how many minutes did it take Beatrice to run the mile?

(A) 4
(B) 8
(C) 10
(D) 16
(E) 20

34.

The expression $a\{\}b$ is defined as $a\{\}b = (a - b)(a + b)$.

Quantity A | **Quantity B**

35. If 5||10 = 5 and 1||(-2) = 1, which of the following could define the expression $a||b$?

(A) $b - a$
(B) $\frac{a^2 - b}{3}$
(C) $-ab/4$
(D) $\frac{b+15}{a}$
(E) $a + b + 4$

36. The maximum height reached by a ball thrown straight up into the air can be determined by the formula $h = -16t^2 + vt + d$, where t is the number of seconds since it was thrown, v is the initial speed of the throw (in feet per second), d is the height (in feet) at which the ball was released, and h is the height of ball t seconds after the throw. Two seconds after a ball is thrown, how high in the air is the ball if it was released at a height of 6 feet and a speed of 80 feet per second?

(A) 96 feet
(B) 100 feet
(C) 102 feet
(D) 134 feet
(E) 230 feet

37. If $a\#b = a^2\sqrt{b} - a$, where $b \geq 0$, what is the value of (-4)#4?

(A) -36
(B) -28
(C) 12
(D) 28
(E) 36

38. The expression $x\$y$ is defined as $\frac{x^2}{y}$, where $y \neq 0$. What is the value of 9\$(6\$2)?

(A) 1/2
(B) 9/4
(C) 9/2
(D) 18
(E) 108

39. Amy deposited \$1,000 into an account that earns 8% annual interest compounded every 6 months. Bob deposited \$1,000 into an account that earns 8% annual interest compounded quarterly. If neither Amy nor Bob makes any additional deposits or withdrawals, in 6 months how much more money will Bob have in his account than Amy?

(A) \$40
(B) \$8
(C) \$4
(D) \$0.40
(E) \$0.04

40. The half-life of an isotope is the amount of time required for 50% of a sample of the isotope to undergo radioactive decay. The half-life of the carbon-14 isotope is 5,730 years. How many years must pass until a sample that starts out with 16,000 carbon-14 isotopes decays into a sample with only 500 carbon-14 isotopes?

(A) 180 years
(B) 1,146 years
(C) 5,730 years
(D) 28,650 years
(E) 183,360 years

41. $f(x) = \frac{2-x}{5}$ and $g(x) = 3x - 2$. If $f(g(x)) = 1$, what is the value of x?

(A) -5/3
(B) -1/3
(C) 2/3
(D) 1
(E) 5/3

42. $f(x) = 2x - 2$ and $g(x) = \frac{x}{2} + 2$. What is the value of $2f(g(2))$?

(A) 0
(B) 4
(C) 6
(D) 8
(E) 12

43. In a particular competition, a skater's score, s, is calculated by the formula $s = \frac{10(3t-5f)}{m}$ where t is the number of successful triple axels performed, f is the number of times the skater fell, and m is the length, in minutes, of the performance. If Seiko received a score of 6 for a 5 minute performance in which she had twice as many successful triple axels as she had falls, how many times did she fall?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

44. An investor doubles his money every 8 years. At 33 years old, he had 13 million dollars ($13m). How much money will he have when he retires at 65 years old?

(A) $26m
(B) $104m
(C) $130m
(D) $208m
(E) $260m

45. $a\sim b$ is defined as $a - \frac{5-b}{a}$. What is the value of $1\sim((-1)\sim1)$?

(A) -9
(B) -1
(C) 1
(D) 2
(E) 5

46. An archer's score is calculated by the formula $\frac{50b-10a}{10+s}$ where b is the number of bull's-eyes hit, a is the total number of arrows shot, and s is the time in seconds it took the archer to shoot. By how many points would an archer who took 10 seconds to shoot 10 arrows and hit all bull's-eyes beat an archer who shot twice as many arrows and hit half as many bull's-eyes in 15 seconds?

(A) 2
(B) 7
(C) 10
(D) 18
(E) 20

47. Each term of a certain sequence is calculated by adding a particular constant to the previous term. The second term of this sequence is 27 and the fiftjh term is 84. What is the 1st term of this sequence?

(A) 20
(B) 15
(C) 13
(D) 12
(E) 8

48. If $a\#b = \frac{1}{2a-3b}$ and $a@b = 3a - 2b$, what is the value of 1@2 - 3#4 ?

(A) -7/6
(B) -1
(C) -5/6
(D) 2/3
(E) 7/6

49. In a certain sequence, the term a_n is given by the formula $a_n = a_{n-1} + 5$ where $a_1 = 1$. What is the sum of the first 75 terms of this sequence?

(A) 10,150
(B) 11,375
(C) 12,500
(D) 13,950
(E) 15,375

50. In a certain sequence, the term a_n is given by the formula $a_n = 2 \times a_{n-1}$ where $a_1 = 1$. What is the positive difference between the sum of the first 10 terms of the sequence and the sum of the 11th and 12th terms of the same sequence?

(A) 1
(B) 1,024
(C) 1,025

(D) 2,048
(E) 2,049

51.

An operation @ is defined by the equation $a@b = (a - 1)(b - 2)$.
$x@5 = 3@x$

Quantity A	**Quantity B**
The value of x	1

52. The wait time in hours, w, for a certain popular restaurant can be estimated by the formula $w = \frac{n + ks}{10}$ where k is a constant, n is the number of parties waiting ahead of you, and s is the size of your party. If a family of 4 has a wait time of 30 minutes when 2 other parties are ahead of it, how long would a family of 6 expect to wait if there are 8 parties ahead of it?

(A) 45.5 minutes
(B) 1 hour 15 minutes
(C) 1 hour 25 minutes
(D) 1 hour 45 minutes
(E) 2 hours

53. A sequence is defined as $a_n = 5(a_{n-1}) - 3$ where $a_4 = 32$. What is the first term of the sequence, a_1?

(A) 1
(B) 7
(C) 16
(D) 128
(E) 157

54.

A certain sequence is defined by the formula $a_n = a_{n-1} - 7$.
$a_7 = 7$

Quantity A	**Quantity B**
The value of a_1	-35

55. Monthly rent for units in a certain apartment building is determined by the formula $k\left(\frac{5r^2 + 10t}{f + 5}\right)$ where k is a constant, r and t are the number of bedrooms and bathrooms in the unit, respectively, and f is the floor number of the unit. A 2-bedroom, 2-bathroom unit on the first floor is going for \$800/month. How much is the monthly rent on a 3-bedroom unit with 1 bathroom on the 3rd floor?

(A) \$825
(B) \$875
(C) \$900

(D) $925
(E) $1,000

56.

Quantity A	Quantity B
The sum of all the multiples of 3 between 250 and 350	9,990

57. Town A has a population of 160,000 and is growing at a rate of 20% annually. Town B has a population of 80,000 and is growing at a rate of 50% annually.

Quantity A	Quantity B
The number of years until Town B's population is larger than that of Town A	3

58. If $f(x) = x^2$, what is $f(m + n) + f(m - n)$?

(A) $m^2 + n^2$
(B) $m^2 - n^2$
(C) $2m^2 + 2n^2$
(D) $2m^2 - 2n^2$
(E) m^2n^2

59. S_n is a sequence such that $S_n = (-1)^n$, where $n \geq 1$. What is the sum of the first 20 terms in S_n?

$\boxed{}$

60. If //x// means "the least integer greater than or equal to x," what is //-2.5// + //3.6//?

$\boxed{}$

61. If $f(x, y) = x^2y$ and $f(a, b) = 6$, what is $f(2a, 4b)$?

$\boxed{}$

62.

$f(x) = m$ where m is the number of distinct prime factors of x.

Quantity A	Quantity B

$f(30)$ $f(64)$

63. In a particular sequence, S_n is equal to the units digit of 3^n where n is a positive integer. If $S_1 = 3$, how many of the first 75 terms of the sequence are equal to 9?

64. The sequence $a_1, a_2, a_3, \ldots, a_n$ is such that $a_n = 9 + a_{n-1}$ for all $n > 1$. If $a_1 = 11$, what is the value of a_{35}?

65. In sequence Q, the first number is 3, and each subsequent number in the sequence is determined by doubling the previous number and then adding 2. How many times does the digit 8 appear in the units digit of the first 10 terms of the sequence?

66. If $g(2m) = 2g(m)$ and $g(3) = 5.5$, what is $g(6)$?

67. For which of the following functions $f(x)$ is $f(a + b) = f(a) + f(b)$?

(A) $f(x) = x^2$
(B) $f(x) = 5x$
(C) $f(x) = 2x + 1$
(D) $f(x) = \sqrt{x}$
(E) $f(x) = x - 2$

68.

Sam invests a principal of $10,000, which earns annually compounded interest over a period of years.

Quantity A	**Quantity B**
The final value of the investment after 2 years at 8% interest, compounded annually	The final value of the investment after 4 years at 4% interest, compounded annually

69. The number of years it would take for the value of an investment to double, at 26% interest compounded annually, is approximately

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

70. The interest rate, compounded annually, that would bring a principal of $1,200 to a final value of $1,650 in 2 years is closest to

(A) 17%
(B) 18%
(C) 19%
(D) 20%
(E) 21%

71. An investment is made at 12.5% annual simple interest. The number of years it will take for the cumulative value of the interest to equal the original investment is

(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

72. If $f(2a) = 2f(a)$ and $f(6) = 11$, what is $f(24)$?

(A) 22
(B) 24
(C) 44
(D) 66
(E) 88

73. $\frac{1}{2}f(x) = f\left(\frac{1}{2}x\right)$, which is true for all values of $f(x)$?

(A) $f(x) = 2x + 2$
(B) $f(x) = 13x$
(C) $f(x) = x^2$
(D) $f(x) = x - 10$
(E) $f(x) = \sqrt{x - 4}$

Functions, Formulas, and Sequences Answers

1. **(E).** The notation "$f(x)$" and "$f(2)$" indicates that you should plug 2 in for x in the given equation:

$f(2) = (2)^2 + 1$
$f(2) = 5$

Likewise, plug -2 in for x:

$f(-2) = (-2)^2 + 1$
$f(-2) = 5$

Now add: $5 + 5 = 10$.

2. **(D).** Start with the innermost function, $g(-3)$. The notation indicates that you should plug -3 in for x in the given equation:

$g(-3) = (-3)^3$
$g(-3) = -27$

Now the problem reads "what is $f(-27)$?" Plug -27 in for x in the $f(x)$ function:

$f(-27) = 2(-27) = -54$

3. **(B).** Be careful with the notation here. The problem indicates that $h(m) = -19$, *not* that $h(-19)$ = something else. Do not plug -19 in for x; rather, plug m in for x and set the answer equal to -19:

$2m^3 - 3 = -19$
$2m^3 = -16$
$m^3 = -8$
$m = -2$

4. **37.** Be careful with the notation here. $2[f(g)] = 14$ represents some number (denoted by variable g) plugged into the function $f(g)$, and then multiplied by 2, to yield the answer 14. If $2[f(g)] = 14$, then divide both sides by 2 to get $f(g) = 7$.

The main function is $f(x) = x - 3$. The notation $f(g)$ indicates that you should plug g in for all instances of x: $f(g) = g - 3$. You also determined that $f(g) = 7$, so set the two right-hand halves of the equations equal to each other: $g - 3 = 7$. The value of g is therefore 10.

The question asks for the value of $f(4g)$. Since $g = 10$, $4g = 40$, and $f(40)$:

$f(40) = 40 - 3 = 37$

5. **720.** Plug m and n into the function in place of a and b. If $f(m, n) = 5$, then:

$$m^2n^4 = 5$$

This cannot be further simplified, so continue to the second part of the problem: plug $3m$ and $2n$ into the function for a and b:

$$f(3m, 2n) = (3m)^2(2n)^4 = 9m^2\ 16n^4 = 144m^2n^4$$

Since $m^2n^4 = 5$, $144m^2n^4 = 144(5) = 720$.

6. **(A).** The question is asking you to plug y into the function, then plug -1 into the function, then add the two answers together.

$$f(y) = y^2 - 1$$
$$f(-1) = (-1)^2 - 1$$
$$f(-1) = 0$$

Thus, $f(y) + f(-1) = y^2 - 1 + 0 = y^2 - 1$, or choice (A).

7. **2.** The question is asking you to plug 3 into the function, then plug $-\frac{1}{3}$ into the function, and then add the two answers together.

$$g(3) = 3(3) - 3$$
$$g(3) = 6$$
$$g\left(-\frac{1}{3}\right) = 3\left(-\frac{1}{3}\right) - 3$$
$$g\left(-\frac{1}{3}\right) = -1 - 3$$
$$g\left(-\frac{1}{3}\right) = -4$$

Thus, $g(3) + g\left(-\frac{1}{3}\right) = 6 + (-4) = 2.$

8. **1.** When dealing with "nested" functions, tackle the innermost function first.

$$f(10) = \frac{10}{2} - 1 = 4$$

$$f(4) = \frac{4}{2} - 1 = 1$$

Thus, $f(f(10)) = 1$.

9. **(E).** The notation $h(a + b)$ indicates that you need to replace each x with the expression $(a + b)$

$$h(a + b) = 5(a + b)^2 + (a + b)$$
$$h(a + b) = 5(a^2 + 2ab + b^2) + a + b$$
$$h(a + b) = 5a^2 + 10ab + 5b^2 + a + b$$

This is equivalent to answer choice (E).

10. **(A).** The question uses a made-up symbol in place of the traditional notation $f(x)$. The question "If $\lceil x = 2x^2 + 2$, what is $\lceil 4$?" is asking you to plug 4 into the function.

$$\lceil 4 = 2(4)^2 + 2$$
$$\lceil 4 = 34$$

Do not fall for trap answer choice (E). The correct answer is 34, which does not appear in the choices in that form.

Trap choice (E) is $\lceil 34$, which equals $2(34)^2 + 2$; this is much larger than 34.

You need to solve each answer choice until you find one that equals 34. Choice (A), $\lceil\lceil -1$, uses the function symbol twice, which requires you to plug -1 into the function, then plug your answer back into the function again:

$$\lceil -1 = 2(-1)^2 + 2 = 4$$

$\lceil 4 = 2(4)^2 + 2 = 34$ (Note: you do not need to complete this math if you notice that $\lceil 4$ must have the same value as the original $\lceil 4$ in the question stem.)

Thus, $\lceil\lceil -1 = 34$ and choice (A) is correct. It is not necessary to try the other answer choices.

11. **(B).** This problem uses a made-up symbol which is then defined verbally, rather than with a formula. $\boxed{x}$ has two different definitions:

If x is odd, $\boxed{x}$ equals the least integer greater than x (for example, if $x = 3$, then the "least integer greater than 3" is equal to 4)

If x is even, $\boxed{x}$ equals the greatest integer less than x (for example, if $x = 6$, the "greatest integer less than x" is equal to 5.

Since -2 is even, $\boxed{-2}$ = the greatest integer less than -2, or -3.

Since 5 is odd, $\boxed{5}$ = the least integer greater than 5, or 6.

Thus, $\boxed{-2} - \boxed{5} = -3 - 6 = -9$.

12. **32.** For the function $g(x) = x^2 - 4$, plugging c in for x gives the answer 12. Thus:

$c^2 - 4 = 12$
$c^2 = 16$
$c = 4$ or -4

The problem indicates that $c < 0$, so c must be -4.

The problem then asks for $g(c - 2)$. Since $c = -4$, $c - 2 = -6$. Plug -6 into the function:

$g(-6) = (-6)^2 - 4$
$g(-6) = 36 - 4 = 32$

13. **43.** The problem introduces two functions and asks for $h(g(5))$. When dealing with "nested" functions, begin with the innermost function.

$g(5) = 5^2 - 3 = 22$
$h(22) = 2(22) - 1 = 43$

Thus, $h(g(5)) = 43$.

14. **15.** The problem introduces two functions as well as the fact that $g(m) = 61$. First, solve for m:

$m^2 - 3 = 61$
$m^2 = 64$
$m = 8$ or -8

The problem indicates that $m > 0$, so m must equal 8. The problem asks for $h(m)$. Since $m = 8$, find $h(8)$:

$h(8) = 2(8) - 1$
$h(8) = 15$

15. $\frac{4}{9}$ **(or any equivalent fraction).** This function defines a made-up symbol rather than using traditional function notation such as $f(x)$. Since &x& is defined as "one-half the square of x":

$$\&x\& = \frac{1}{2}x^2$$

The problem asks for &4& divided by &6&:

$$\&4\& = \frac{1}{2}(4)^2 = 8$$

$$\&6\& = \frac{1}{2}(6)^2 = 18$$

Therefore, $\frac{\&4\&}{\&6\&} = \frac{8}{18} = \frac{4}{9}$.

16. **I, II, and III.** This function defines a made-up symbol: ~x is equivalent to $|14x|$. The question asks which statements must be true, so test each one.

~2 = ~(-2)
|14(2)| = |14(-2)|
|28| = |-28|
28 = 28

This statement must be TRUE.

Similarly test the second statement:

~3 + ~4 = ~7
|14(3)| + |14(4)| = |14(7)|
42 + 56 = 98
98 = 98

This statement must be TRUE.

Finally, the third statement is also true. Since ~x is equal to a statement inside an absolute value, this value can never be negative. If $x = 0$, then the value of $|14x|$ is also 0. The minimum possible value for ~x is 0.

17. **0.** This function defines a made-up symbol, rather than using traditional notation such as $f(x)$. First, translate the function:

$\#x = (x - 2)^2$
$\#5 = (5 - 2)^2 = 9$ and $\#(-1) = (-1 - 2)^2 = 9$.

$\#5 - \#(-1) = 9 - 9 = 0$.

18. **(C).** The term "undefined" refers to the circumstance when the solution is not a real number — for example, when a value would ultimately cause you to divide by 0, that situation is considered "undefined." There aren't many circumstances that result in an undefined answer. Essentially, you can't take the square root of a negative, and you can't divide by 0. There are no square roots in this problem, but it's possible that 0 could end up on the denominator of the fraction. Set each of the terms in the denominator equal to 0:

$3x - 3 = 0$
$3x = 3$
$x = 1$

$x + 2 = 0$
$x = -2$

Thus, if $x = 1$ or $x = -2$, then you'd have to divide by 0, making $g(x)$ undefined. All other values are acceptable.

19. **(E).** The term "undefined" refers to the circumstance when the solution is not a real number — for example, when a value would ultimately cause you to divide by 0, that situation is considered "undefined." There aren't many circumstances that result in an undefined answer. Essentially, you can't take the square root of a negative, and you can't divide by 0.

Since you can't divide by 0 and the bottom of the fraction is simply x, then x cannot be 0. So far, there is 1 "prohibited" value for x.

Since the top of the fraction is a square root and you can't take the square root of a negative, you can conclude that the quantity inside the absolute value, $x - 2$, must be positive or zero:

$x - 2 \geq 0$
$x \geq 2$

Therefore, there are an infinite number of values that x cannot be (1, 0, -1, -2, -3, -4, -5…).

20. **(A).** The problem gives a function, $f(x) = 2x - 3$, and then indicates that, when m is plugged in to the function, the answer is -11. Therefore:

$2m - 3 = -11$
$2m = -8$
$m = -4$

Quantity A is equal to -4. The problem indicates that $f(m) = -11$, so Quantity B is equal to $\frac{11}{2} = -5.5$. Quantity A is larger.

21. **(D).** Since "the price of a phone call consists of a standard connection fee, which does not change, plus a per-minute charge," you can write a formula, using variables for the unknown information. Let c equal the connection fee and r equal the per-minute rate:

$2.90 = c + r(10)$
$4.40 = c + r(16)$

Now, either substitute and solve, or stack and combine the equation. Note that there is one c in each equation, so subtracting is likely going to be fastest:

$$\begin{array}{l} 4.40 = c + 16r \\ \underline{-\ (2.90 = c + 10r)} \\ 1.50 = 6r \end{array}$$

$r = 0.25$

The calls cost 25 cents per minute. Note that most people will next plug r back into either equation to find c, but c isn't necessary to solve!

A 10-minute call costs \$2.90. That \$2.90 already includes the basic connection fee (which never changes) as well as the per-minute fee for 10 minutes. The problem asks how much a 13-minute call costs. Add the cost for another 3 minutes (\$0.75) to the cost for a 10-minute call (\$2.90): 2.90 + 0.75 = \$3.65.

In fact, if you notice earlier that both the 10-minute and 16-minute calls include the same connection fee (which never changes), you can use a shortcut to solve. The extra 6 minutes for the 16-minute call cost a total of \$4.40 - \$2.90 = \$1.50. From there, you can calculate the cost per minute ($1.5 \div 6 = 0.25$) or you can notice that 13 minutes is halfway between 10 minutes and 16 minutes, so the cost for a 13-minute call must also be halfway between the cost for a 10-minute call and the cost for a 16-minute call. Add half of \$1.50, or \$0.75, to \$2.90 to get \$3.65.

22. **267.** While the sequence is clear (30, 33, 36, 39, 42, etc.), you don't have time to count to the 80th term. Instead, find a pattern. Each new term in the list adds 3 to the previous term, so determine how many times you need to add 3. (By the way, the term "arithmetic sequence" means a sequence in which the same number is added or subtracted for each new term.)

Start with the first term, 30. To get from the first term to the second term, you start with 30 and add 3 *once*. To get from the first term to the third term, you start with 30 and add 3 *twice*. In other words, for the third term, you add one fewer instance of 3: twice rather than three times. To write this mathematically, say: $30 + 3(n\text{-}1)$, where n is the number of the term. (Note: you don't need to write that, as long as you understand the pattern.)

To get to the 80th term, then, start with 30 and add 3 exactly 79 times:

$$30 + (79 \times 3) = 267$$

23. **(E).** The sequence $S_n = 2(S_{n-1}) - 4$ can be read as "to get any term in sequence S, double the previous term and subtract 4."

The problem gives S_1 (the first term) and asks for S_5 (the fifth term):

6				
S_1	S_2	S_3	S_4	S_5

To get any term, double the previous term and subtract 4. To get S_2, double S_1 (which is 6) and subtract 4: $S_2 = 2(6) - 4 = 8$. Continue doubling each term and subtracting 4 to get the subsequent term:

6	8	12	20	36
S_1	S_2	S_3	S_4	S_5

24. **(B).** The sequence $S_n = 3(S_{n-1}) + 1$ can be read as "to get any term in sequence S, triple the previous term and add 1."

The problem gives S_1 (the first term) and asks for S_4 (the fourth term):

−2			
S_1	S_2	S_3	S_4

To get any term, triple the previous term and add 1. To get S_2, triple S_1 (which is -2) and add 1. Thus, $S_2 = 3(-2) + 1 = -5$. Continue tripling each term and adding 1 to get the subsequent term:

−2	−5	−14	−41
S_1	S_2	S_3	S_4

25. **(A).** The sequence $S_n = S_{n-1} + S_{n-2} - 3$ can be read as "to get any term in sequence S, add the two previous terms and subtract 3."

The problem gives the first two terms and asks for the sixth term:

5	0				
S_1	S_2	S_3	S_4	S_5	S_6

To get any term, add the two previous terms and subtract 3. So the third term will equal $5 + 0 - 3 = 2$. The fourth term will equal $0 + 2 - 3 = -1$. The fifth term will equal $2 + (-1) - 3 = -2$. The sixth term will equal $-1 + (-2) - 3 = -6$.

5	0	2	−1	−2	−6
S_1	S_2	S_3	S_4	S_5	S_6

26. **(C).** The sequence $S_n = S_{n-1} + S_{n-2} - 1$ can be read as "to get any term in sequence S, add the two previous terms and subtract 1."

The problem gives the zero-th term and the *second* term and asks for the fourth term:

−10		0		
S_0	S_1	S_2	S_3	S_4

Within the sequence S_0 to S_2, the problem gives two values but not the third (S_1). What version of the formula would include those three terms?

$S_2 = S_1 + S_0 - 1$
$0 = S_1 + (-10) - 1$
$0 = S_1 - 11$
$11 = S_1$

–10	11	0		
S_0	S_1	S_2	S_3	S_4

To get each subsequent term, add the two previous terms and subtract 1. $S_3 = 0 + 11 - 1 = 10$. $S_4 = 10 + 0 - 1 = 9$.

–10	11	0	10	9
S_0	S_1	S_2	S_3	S_4

27. **(E).** The sequence $S_n = S_{n-1} + S_{n-2} + S_{n-3} - 5$ can be read as "to get any term in sequence S, add the three previous terms and subtract 5."

The problem gives the first, second, and *fourth* terms and asks for the sixth term:

4	0		–4		
S_1	S_2	S_3	S_4	S_5	S_6

Within the sequence S_1 to S_4, the problem gives three values but not the fourth (S_3). What version of the formula would include those four terms?

$S_4 = S_3 + S_2 + S_1 - 5$
$-4 = S_3 + 4 + 0 - 5$
$-4 = S_3 - 1$
$-3 = S_3$

Fill in the newly-calculated value. To find each subsequent value, continue to add the three previous terms and subtract 5. $S_5 = -4 + (-3) + 0 - 5 = -12$. $S_6 = -12 + (-4) + (-3) - 5 = -24$.

4	0	–3	–4	–12	–24
S_1	S_2	S_3	S_4	S_5	S_6

28. **1,778.** The sequence $P_n = 10(P_{n-1}) - 2$ can be read as "to get any term in sequence P, multiply the previous term by 10 and subtract 2."

The problem gives the first term and asks for the fourth:

$$\begin{array}{cccc} 2 & & & \\ \hline P_1 & P_2 & P_3 & P_4 \end{array}$$

To get P_2, multiply 2 × 10, then subtract 2 to get 18. Continue this procedure to find each subsequent term ("to get any term in sequence P, multiply the previous term by 10 and subtract 2.") $P_3 = 10(18) - 2 = 178$. $P_4 = 10(178) - 2 = 1{,}778$.

$$\begin{array}{cccc} 2 & 18 & 178 & 1{,}778 \\ \hline P_1 & P_2 & P_3 & P_4 \end{array}$$

29. **(A).** The sequence $S_{n-1} = \frac{1}{4}(S_n)$ can be read as "to get any term in sequence S, multiply the term *after* that term by $\frac{1}{4}$." Since this formula is "backwards" (usually, you define later terms with regard to previous terms), you may wish to solve the formula for for S_n:

$$\begin{aligned} S_{n-1} &= \frac{1}{4}(S_n) \\ 4S_{n-1} &= S_n \\ S_n &= 4S_{n-1} \end{aligned}$$

This can be read as "to get any term in sequence S, multiply the previous term by 4."

The problem gives the first term and asks for the fourth:

$$\begin{array}{cccc} -4 & & & \\ \hline S_1 & S_2 & S_3 & S_4 \end{array}$$

To get S_2, multiply the previous term by 4: (4)(-4) = -16. Continue this procedure to find each subsequent term. $S_3 = (4)(-16) = -64$. $S_4 = (4)(-64) = -256$.

$$\begin{array}{cccc} -4 & -16 & -64 & -256 \\ \hline S_1 & S_2 & S_3 & S_4 \end{array}$$

30. **(B).** The first term of the sequence is 45, and each subsequent term is determined by adding 2. The problem asks for the sum of the first 100 terms, which cannot be calculated directly in the given time frame; instead, find the pattern. The first few terms of the sequence are 45, 47, 49, 51, ….

What's the pattern? To get to the 2nd term, start with 45 and add 2 once. To get to the 3rd term, start with 45 and add 2 twice. To get to the 100th term, then, start with 45 and add 2 ninety-nine times: 45 + (2)(99) = 243.

Next, find the sum of all odd integers from 45 to 243, inclusive. To sum up any evenly-spaced set, multiply the average by the number of elements in the set. To get the average, average the first and last terms. Since $\frac{45+243}{2} = 144$, the average is 144.

To find the total number of elements in the set, subtract 243 - 45 = 198, then divide by 2 (count only the odd numbers, not the even ones). 198/2 = 99 terms. Now, add 1 (to count both endpoints in a consecutive set, first subtract and then "add 1 before you're done"). The list has 100 terms.

Multiply the average and the number of terms:

$144 \times 100 = 14{,}400$

31. **(D).** This is an arithmetic sequence where the difference between successive terms is always +10. The difference between, for example, a_{10} and a_{11}, is exactly 10, regardless of the actual values of the two terms. The difference between a_{10} and a_{12} is $10 + 10 = 20$ or $10 \times 2 = 20$, because there are two "steps," or terms, to get from a_{10} to a_{12}. Starting from a_{10}, there is a sequence of 5 terms to get to a_{15}. Therefore, the difference between a_{10} and a_{15} is $10 \times 5 = 50$.

32. **(E).** This is a geometric sequence, in which every term is 10 times the term before. The problem does not provide the actual value of any terms in the sequence, but the sequence could be something like 10, 100, 1,000…, or 35, 350, 3,500…. Thus, any term is 100 times as large as the term that comes two before it.

Alternatively, try an algebraic approach. From the formula, $a_9 = 10a_8$ and $a_{10} = 10a_9$. Substitute for a_9 in the second equation to give: $a_{10} = 10(10a_8) = 100a_8$.

33. **(B).** First, calculate Ashley's score: $t_a = 3 \times 10 \times 10 - 25 \times 8 = 300 - 200 = 100$. If Ashley and Beatrice scored the same score and Beatrice did half as many sit-ups (5) and twice as many push-ups (20): $t_b = 3 \times 5 \times 20 - 25m = 100$. Solve for m: $300 - 25m = 100$ and $25m = 200$ so $m = 8$.

Alternatively, you could use logic. Since p and s are multiplied together in the score formula, if you multiply s by $\frac{1}{2}$ and p by 2, the two girls will have the same overall value for $3ps$. Beatrice will need the same mile time, 8 minutes, in order to achieve the same overall score as Ashley.

34. **(A).** This problem defines a function for the made-up symbol {}. To calculate the value of Quantity A, follow the rules of PEMDAS. First calculate the expressions within parentheses:

$7\{\}6 = (7 - 6)(7 + 6) = 1 \times 13 = 13$
$11\{\}11 = (11 - 11)(11 + 11) = 0$

Then substitute these values back into the original expression:

$(13)\{\}(0) = (13 - 0)(13 + 0) = 13 \times 13 = 169$. Quantity A is larger.

35. **(B).** This question provides two examples of the input and output into a made-up function, and asks which answer choice could be that function. In other words, the function in which answer choice gives the answer 5 when you evaluate 5||10, and also gives the answer 1 when you evaluate 1||(-2)?

Only one answer can work for both examples, so first test a = 5 and b = 10 to determine whether the function returns 5. If it doesn't, cross off that choice. If it does, test a = 1 and b = -2. When you find an answer that works for both, you can stop.

(A) Does 10 - 5 = 5? YES. Does (-2) -1 = 1? No. Cross off answer (A).

(B) Does $\frac{5^2-10}{3}=5$? YES. Does $\frac{1^2-(-2)}{3}=1$? YES. This is the correct answer.

It is not necessary to test the remaining answers; the work is shown below for completeness.

(C) Does $\frac{-5\times 10}{4}=5$ NO. Cross off answer (C).

(D) Does $\frac{10+15}{5}=5$? YES. Does $\frac{-2+15}{1}=1$? No. Cross off answer (D).

(E) Does 5 + 10 + 4 = 5? NO. Cross off answer (E).
The answer is (B).

36. **(C).** The question presents a formula with a number of variables and also provides values for all but one of those variables (h, the height of the ball). Solve for h by plugging in the values given for the other variables: t = 2 seconds, v = 80 feet/second, d = 6 feet.

$h = -16(2)^2 + (80)(2) + 6 = 102$ feet

37. **(E).** This problem defines a function for the made-up symbol #. In this problem a = (-4) and b = 4. Plug the values into the function: $(-4)^2\sqrt{4}-(-4) = 16\times 2+4 = 36$. Do not forget to keep the parentheses around the -4! Also note that you take only the positive root of 4 (2) because the problem has been presented in the form of a real number underneath the square root sign.

38. **(B).** This problem defines a function for the made-up symbol $. Order of operation rules (PEMDAS) stay the same even when the problem uses made-up symbols. First, calculate the value of the expression in parentheses, 6$2. Plug x = 6 and y = 2 into the function: $6^2/2 = 36/2 = 18$. Replace 6$2 with 18 in the original expression to give 9$18. Again, plug x = 9 and y = 18 into the function: 9$18 = $9^2/18 = 81/18 = 9/2$.

39. **(D).** Both Amy and Bob start with $1,000 and earn 8% interest annually; the difference is in how often this interest is compounded. Amy's interest is compounded twice a year at 4% each time (8% annual interest compounded 2 times a year means that she gets half the interest, or 4%, every six months). Bob's interest is compounded four times a year at 2% (8% divided by 4 times per year) each time. After 6 months, Amy has $1,000 × 1.04 = $1,040.00 (1 interest

payment at 4%) and Bob has $1,000 × (1.02)^2 = $1,040.40 (2 interest payments at 2%). The difference is $1,040.40 - $1,040.00 = $0.40.

For Bob's interest, you can also calculate the two separate payments. After three months, Bob will have $1,000 × 1.02 = $1,020.00. After six months, Bob will have $1,020 × 1.02 = $1,040.40.

40. **(D).** After each half-life, the sample is left with half of the isotopes it started with in the previous period. After one half-life, the sample goes from 16,000 isotopes to 8,000. After two half-lives, it goes from 8,000 to 4,000. Continue this pattern to determine the total number of half-lives that have passed: 4,000 becomes 2,000 after 3 half-lives, 2,000 becomes 1,000 after 4 half-lives, 1,000 becomes 500 after 5 half-lives. The sample will have 500 isotopes after 5 half-lives. Thus, multiply 5 times the half-life, or 5 × 5730 = 28,650 years.

Note that the answer choices are very spread apart. Once you have determined that 5 half-lives have passes, you can estimate: 5 × 5000 = 25,000 years; answer (D) is the only possible answer.

41. **(B).** The question is asking you to substitute the expression for $g(x)$ into the function for $f(x)$, and set the answer equal to 1. Since $g(x) = 3x - 2$, substitute the expression $3x - 2$ in for x in the expression for $f(x)$:

$$f(g(x)) = \frac{2 - g(x)}{5} = \frac{2-(3x-2)}{5} = \frac{4-3x}{5}$$

Since $f(g(x)) = 1$, solve the equation $\frac{4-3x}{5} = 1$: :

$$\begin{aligned} 4 - 3x &= 5 \\ -3x &= 1 \\ x &= -\frac{1}{3} \end{aligned}$$

42. **(D).** Start with the innermost portion of $2f(g(2))$:

$g(2) = 2/2 + 2 = 3$

Since $g(2) = 3$, now the expression is $2f(3)$. First evaluate $f(3)$: 2(3) - 2 = 4.

Therefore $2f(3) = 2 \times 4 = 8$.

43. **(B).** If Seiko had twice as many triple axels as falls then $t = 2f$. Substitute $2f$ in for t, along with the other given information:

$$6=\frac{10(3(2f)-5f)}{5}$$

$$6=\frac{10(6f-5f)}{5}$$

$$6=\frac{10f}{5}$$

$$30=10f$$

$$f=3$$

Note that you want to substitute 2*f* in for *t*, and not the reverse, because the problem asks for the value of *f*.

44. **(D).** The investor's amount of money doubles every 8 years. Calculate the amount of money for each 8-year period:

Age	$ (millions)
33	13
33 + 8 = 41	13 × 2 = 26
41 + 8 = 49	26 × 2 = 52
49 + 8 = 57	52 × 2 = 104
57 + 8 = 65	104 × 2 = 208

At age 65, the investor will have $208 million.

45. **(B).** This problem defines a function for the made-up symbol ~. Follow PEMDAS order and start with the inner most parentheses, (-1)~1. In this case *a* = -1 and *b* = 1: $-1-\left(\frac{5-1}{-1}\right)=-1-\left(\frac{4}{-1}\right)=-1+4=3$. (Note: it's important to insert the parentheses! Replace (-1)~1 with 3 and evaluate the next function: 1~3. Now *a* = 1 and *b* = 3:

$$1-\left(\frac{5-3}{1}\right)=1-\left(\frac{2}{1}\right)=1-2=-1$$

46. **(D).** Calculate each of the archer's scores by plugging in the appropriate values for *b*, *a*, and *s*. For the first archer, *b* = *a* = *s* = 10 and the score is $\frac{(50\times10)-(10\times10)}{10+10}=\frac{400}{20}=20$. For the second archer, *b* = half of 10 = 5, *a* = twice as many as 10 = 20, and *s* = 15. The score for the second archer is $\frac{(50\times5)-(10\times20)}{10+15}=\frac{50}{25}=2$.

The difference in scores is 20 - 2 = 18.

47. **(E).** Let k equal the constant added to a term to get the next term. If the second term = 27, then the third term = 27 + k, the 4th term = 27 + 2k, and the 5th term = 27 + 3k. The 5th term equals 84, so create an equation:

$$27 + 3k = 84$$
$$3k = 57$$
$$k = 19$$

To find the first term, subtract k from the second term. The first term = 27 - 19 = 8.

48. **(C).** This problem defines functions for the made-up symbols # and @. Substitute a = 1 and b = 2 into the function for $a@b$: 3(1) - 2(2) = -1. Substitute a = 3 and b = 4 into the function for $a\#b$: $\frac{1}{2(3)-3(4)} = \frac{1}{-6} = -\frac{1}{6}$. Subtract: $(-1) - \left(-\frac{1}{6}\right) = -1 + \frac{1}{6} = -\frac{5}{6}$.

49. **(D).** This is an arithmetic sequence: each new number is created by adding 5 to the previous number in the sequence. Calculate the first few terms of the sequence: 1, 6, 11, 16, 21, and so on. Arithmetic sequences can be written in this form: $a_n = a_1 + k(n - 1)$, where k is the added constant and n is the number of the desired term. In this case, the function is: $a_n = 1 + 5(n - 1)$. The 75th term of this sequence is $a_{75} = 1 + 5(74) = 371$.

To find the sum of an arithmetic sequence, multiply the average value of the terms by the number of terms. The average of any evenly-spaced set is equal to the midpoint between the first and last terms. The average of the 1st and 75th terms is $\frac{1+371}{2} = 186$. There are 75 terms. Therefore, the sum of the first 75 terms = 186 × 75 = 13,950.

50. **(E).** This is a geometric sequence: each new number is created by multiplying the previous number by 2. Calculate the first few terms of the series to find the pattern: 1, 2, 4, 8, 16, and so on. Geometric sequences can be written in this form: $a_n = r^{n-1}$, where r is the multiplied constant and n is the number of the desired term. In this case, the function is $a_n = 2^{n-1}$.

The question asks for the difference between the sum of the first 10 terms and the sum of the 11th and 12th terms. While there is a clever pattern at play, it is hard to spot. If you don't see the pattern, one way to solve is to use the calculator to add the first ten terms: 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1,023.

The 11th + 12th terms = 1,024 + 2,048 = 3,072

Subtract to get 2,049.

Alternatively, you may see the pattern in the first few terms (1, 2, 4, 8, 16…): Every term is equal to 1 more than the sum of the ones before it. For example, 1 + 2 = 3 and the next term is 4. 1 + 2 + 4 = 7 and the next term is 8. Thus, the sum of the first 10 terms of the sequence is 1 less than the 11th term. The 11th term = 2^{10} = 1,024, so the sum of the first 10 terms = 1,023. and the difference between the 10th and 11th terms equals 1. Add the value of the 12th term or 1 + 2,048 = 2,049.

51. **(B).** This problem defines a function for the made-up symbol @. Use the definition of the new symbol to rewrite the equation $x@5 = 3@x$ without the @ operator.

For $x@5$, $a = x$ and $b = 5$: $x@5 = (x - 1)(5 - 2) = 3x - 3$.
For $3@x$, $a = 3$ and $b = x$: $3@x = (3 - 1)(x - 2) = 2x - 4$.

Equating these two expressions gives us:

$$3x - 3 = 2x - 4$$
$$x = -1$$

Quantity B is larger.

52. **(B).** Start by solving for the constant, k. A family of 4 ($s = 4$) has a wait time of 30 minutes ($w = 0.5$ hours — don't forget that w is in hours!) when 2 parties are ahead of it ($n = 2$). Plug these values into the formula: $0.5 = \dfrac{2+4k}{10}$. Solve for k:

$$5 = 2 + 4k$$
$$3 = 4k$$
$$k = \frac{3}{4}$$

To solve for the wait time of the family of 6 with 8 parties ahead of it, plug these values into the formula along with $k = \dfrac{3}{4}$: $w = \dfrac{8+\left(\frac{3}{4}\right)6}{10} = 1.25$ hours. The answer choices are shown in hours and minutes. 0.25 hours is equal to 0.25×60 minutes = 15 minutes. The answer is 1 hour 15 minutes.

53. **(A).** The given sequence can be read as, "To get any term in a, multiply the previous term by 5 and then subtract 3." The problem indicates that $a_4 = 32$ and asks for the value of a_1.

$$\frac{}{A_1}\qquad \frac{}{A_2}\qquad \frac{}{A_3}\qquad \frac{32}{A_4}$$

Start with a_4 and find a_3:

$$32 = 5a_3 - 3$$
$$35 = 5a_3$$
$$7 = a_3$$

Use a_3 to find a_2:

$$7 = 5a_2 - 3$$

$10 = 5a_2$
$2 = a_2$

Use a_2 to find a_1:

$2 = 5a_1 - 3$
$5 = 5a_1$
$1 = a_1$

54. **(A).** The sequence $a_n = a_{n-1} - 7$ can be read as "to get any term in sequence a, subtract 7 from the previous term." The problem provides the 7th term; plug the term into the function in order to determine the pattern. Note that Quantity A asks for the value of a_1, so try to find the 6th term:

$7 = a_6 - 7$
$a_6 = 14$

In other words, each previous term will be 7 larger than the subsequent term. Therefore, $a_7 = 7$, $a_6 = 14$, $a_5 = 21$, and so on. The term a_1, then, is larger than the starting point, 7, and must also be larger than the negative value in Quantity B. Quantity A is larger. Note that the value in Quantity B is the result of incorrectly *subtracting* 7 six times, rather than adding it.

55. **(A).** First, solve for the constant k using the price information of the 2-bedroom, 2-bath unit ($m = 800$, $r = t = 2$ and $f = 1$):

$$800 = k\left(\frac{5(2)^2 + 10(2)}{1+5}\right)$$

$$800 = k\left(\frac{20+20}{6}\right)$$

$$800 = k\left(\frac{20}{3}\right)$$

$$800\left(\frac{3}{20}\right) = k$$

$$40(3) = k$$

$$120 = k$$

Next, solve for the rent on the 3-bedroom, 1-bath unit on the 3rd floor ($r = 3$, $t = 1$, and $f = 3$):

$$m = 120\left(\frac{5(3)^2 + 10(1)}{3+5}\right)$$
$$m = 120\left(\frac{45+10}{8}\right)$$
$$m = 120\left(\frac{55}{8}\right)$$
$$m = 15(55)$$
$$m = 825$$

56. **(B).** First, find the smallest multiple of 3 in this range: 250 is not a multiple of 3 (2 + 5 + 0 = 7, which is not a multiple of 3). The smallest multiple of 3 in this range is 252 (2 + 5 + 2 = 9, which is a multiple of 3). Next, find the largest multiple of 3 in this range. 350 is not a multiple of 3 (3 + 5 + 0 = 8); the largest multiple of 3 in this range is 348.

The sum of an evenly-spaced set of numbers equals the average value multiplied by the number of terms. The average value is the midpoint between 252 and 348: (252 + 348) ÷ 2 = 300. To find the number of terms, first subtract 348 - 252 = 96. This figure represents all numbers between 348 and 252, inclusive. To count only the multiples of 3, divide 96 by the 3: 96/3 = 32. Finally, "add 1 before you're done" because you do want to count both end points of the range: 32 + 1 = 33.

The sum is 300 × 33 = 9,900. Since 9,900 is smaller than 9,990, Quantity B is larger.

57. **(A).** Set up a table and calculate the population of each town after every year; use the calculator to calculate Town A's population. If you feel comfortable multiplying by 1.5 yourself, you do not need to use the calculator for Town B. Instead, add 50% each time (e.g., from 80,000, add 50% or 40,000 to get 120,000).

	Town A	**Town B**
Now	160,000	80,000
Year 1	160,000(1.2) = 192,000	80,000 + 40,000 = 120,000
Year 2	192,000(1.2) = 230,400	120,000 + 60,000 = 180,000
Year 3	230,400(1.2) = 276,480	180,000 + 90,000 = 270,000

Note that, after three years, Town A still has more people than Town B. It will take longer than 3 years, then, for Town B to surpass Town A, so Quantity A is larger.

58. **(C).** The problem provides the function $f(x) = x^2$ and asks you to evaluate $f(m + n) + f(m - n)$. Plug into this function twice — first, to insert $m + n$ in place of x, and then to insert $m - n$ in place of x.

$$f(m + n) = (m + n)^2 = m^2 + 2mn + n^2$$
$$f(m - n) = (m - n)^2 = m^2 - 2mn + n^2$$

Now add the two:

$$(m^2 + 2mn + n^2) + (m^2 - 2mn + n^2) = 2m^2 + 2n^2$$

59. **0.** Adding 20 individual terms would take quite a long time. Look for a pattern. The first several terms in $S_n = (-1)^n$, where $n \geq 1$:

$$S_1 = (-1)^1 = -1$$
$$S_2 = (-1)^2 = 1$$
$$S_3 = (-1)^3 = -1$$
$$S_4 = (-1)^4 = 1$$

The terms alternate -1, 1, -1, 1, and so on. If the terms are added, every pair of -1 and 1 will add to zero; in other words, for an even number of terms, the sum will be zero. 20 is an even number, so the first 20 terms add to zero.

60. **2.** The "least integer greater than or equal to" language is tricky because of the words "least" and "greater" in the same sentence. First, is the problem asking for a number that is larger or smaller than the starting number? "Greater than or equal to" indicates that the resulting number will be larger than the starting number. Rephrase this as "what is the next largest integer?"

The next largest integer starting from -2.5 is -2. (Remember that, for negative numbers, larger means "closer to 0.")

The next largest integer starting from 3.6 is 4.

$$-2 + 4 = 2$$

61. **96.** The problem provides the function $f(x, y) = x^2y$ and also the fact that when a and b are plugged in for x and y, the answer is 6. In other words:

$$f(x, y) = x^2y$$
$$f(a, b) = a^2b = 6$$

The problem asks for the value of $f(2a,4b)$. First, plug $2a$ in for x and $4b$ in for y:

$$f(2a, 4b) = (2a)^2(4b)$$
$$f(2a, 4b) = 4a^2(4b)$$
$$f(2a, 4b) = 16a^2b$$

The problem already provides the value for the variables: $a^2b = 6$. Therefore, $16a^2b = 16(6) = 96$.

62. **(A).** The problem indicates that $f(x) = m$ where m is the number of distinct (or different) prime factors of x. For example, if $x = 6$, 6 has two distinct prime factors: 2 and 3. Therefore, the corresponding answer (m value) would be 2.

For Quantity A $f(30)$: 30 has 3 distinct prime factors (2, 3, and 5), so $f(30) = 3$.

For Quantity B $f(64)$: 64 is made of the prime factors 2, 2, 2, 2, 2, and 2). This is only one distinct prime factor, so $f(64) = 1$. Quantity A is larger.

63. **19.** It's too much work to write out the first 75 terms of a sequence, so there must be some kind of pattern. Figure it out (the units digit is bolded):

$3^1 = \mathbf{3}$
$3^2 = \mathbf{9}$
$3^3 = 2\mathbf{7}$
$3^4 = 8\mathbf{1}$
$3^5 = 24\mathbf{3}$
$3^6 = 72\mathbf{9}$

The pattern is 3, 9, 7, 1 and repeats every 4 terms. (Note: you can memorize this pattern or recreate it when you need it. For the numbers 0 to 9, there are no more than 4 units digits in the pattern; if you remember this, you only need to test the first 4 terms.)

The 2nd term in the pattern of 4 terms is equal to 9; for each set of 4 terms, then, there will be one number with a units digit of 9. Since 75 divided by 4 = 18 remainder 3, the entire pattern will repeat 18 times, for a total of 18 terms with a units digit of 9, followed by 3 extra terms. The three extra terms (3, 9, 7) include one extra 9. Therefore, a units digit of 9 appears 18 + 1 = 19 times in this sequence.

64. **317.** Each term in the sequence is 9 greater than the previous term. To make this obvious, you may want to write a few terms of the sequence: 11, 20, 29, 38, etc.

a_{35} comes 34 terms after a_1 in the sequence. In other words, a_{35} is $34 \times 9 = 306$ greater than a_1.

Thus, $a_{35} = 11 + 306 = 317$.

65. **9.** After the first term in the sequence, every term has a units digit of 8:

$Q_1 = 3$
$Q_2 = 2(3) + 2 = 8$
$Q_3 = 2(8) + 2 = 18$
$Q_4 = 2(18) + 2 = 38$
$Q_5 = 2(38) + 2 = 78$
…

So 8 will be the units digit nine out of the first ten times.

66. **11.** This question concerns some function for which the full formula is not given. The problem indicates that $g(2m) = 2g(m)$. In other words, this function is such that plugging in $2m$ is the same as plugging in m and then multiplying by 2. Plug $g(3)$ into the equation for $g(m)$:

$g(6) = 2g(3)$
$g(6) = 2(5.5)$
$g(6) = 11$

67. **(B).** The question asks which of the functions in the answer choices is such that performing the function on $a + b$ yields the same answer as performing the function to a and b individually and then adding those answers together.

The correct answer should be such that $f(a + b) = f(a) + f(b)$ is true for any values of a and b. Test some numbers, for example $a = 2$ and $b = 3$.

	$f(a + b) = f(5)$	$f(a) = f(2)$	$f(b) = f(3)$	Does $f(a + b) = f(a) + f(b)$?
(A)	$f(5) = 5^2 = 25$	$f(2) = 2^2 = 4$	$f(3) = 3^2 = 9$	No
(B)	$f(5) = 5(5) = 25$	$f(2) = 5(2) = 10$	$f(3) = 5(3) = 15$	Yes
(C)	$f(5) = 2(5) + 1 = 11$	$f(2) = 2(2) + 1 = 5$	$f(3) = 2(3) + 1 = 7$	No
(D)	$f(5) = \sqrt{5}$	$f(2) = \sqrt{2}$	$f(3) = \sqrt{3}$	No
(E)	$f(5) = 5 - 2 = 3$	$f(2) = 2 - 2 = 0$	$f(3) = 3 - 2 = 1$	No

Alternatively, use logic — for what kinds of operations are performing the operation on two numbers and then adding them together the same as adding the original numbers together and then performing the operation? Multiplication or division would work, but squaring, square-rooting, adding, or subtracting would not. The correct function can contain ONLY multiplication and/or division.

68. **(B).** You can solve this problem by applying the compound interest formula:

$$V = P\left(1 + \frac{r}{100}\right)^t$$

Since the principal P is the same in both cases, you can leave it out and just compare the rest.

Quantity A: $\left(1 + \frac{r}{100}\right)^t = \left(1 + \frac{8}{100}\right)^2 = (1.08)^2 = 1.08 \times 1.08 = 1.1664$

Quantity B: $\left(1 + \frac{r}{100}\right)^t = \left(1 + \frac{4}{100}\right)^4 = (1.04)^4 = 1.04 \times 1.04 \times 1.04 \times 1.04 \approx 1.1699$

Quantity B is larger.

Alternatively, you can use logic. Notice that the *simple* interest in each case would be the same: 2 years of 8% *simple* interest (of an unchanging principal) is equal to 4 years of 4% *simple* interest of the same principal. Now go back to the compounded world. If the simple interest scenarios are the same, then it will always be true that the compounded scenario with *more frequent* compounding will give you a larger principal in the end, because you're earning "interest on the interest” more often.

2 periods of 8% compounded interest	$<$	4 periods of 4% compounded interest	$<$	8 periods of 2% compounded interest	$<$	16 periods of 1% compounded interest

The differences are small but real.

69. **(B).** Start with \$1, and multiply by $\left(1+\frac{26}{100}\right)$ = 1.26 for each year that passes. In order for the amount to double, it would have to reach \$2.

> End of Year 1: \$1 × 1.26 = \$1.26
> End of Year 2: \$1.26 × 1.26 = \$1.5876
> End of Year 3: \$1.5876 × 1.26 = \$2.000376 $\approx$ \$2.00

It takes 3 years for the investment to double in value. If you mistakenly thought in terms of *simple* interest, you might think it would take about 4 years (since 26% is just a tiny bit more than 25% = 1/4). In the compounded case, you’re earning “interest on the interest,” though, so the investment grows more quickly.

70. **(A).** You can try the answer choices to find the answer. Start with either choice (B) or choice (D). An 18% interest rate corresponds to multiplying by 1.18. Compounding over two years means multiplying by 1.18 twice:

> \$1,200 × 1.18 × 1.18 = \$1,670.88.

This is close to \$1,650 but 17% could be closer. Try answer choice (A):

> \$1,200 × 1.17 × 1.17 = \$1,642.68

Since this result is in fact closer to \$1,650, the interest rate must be closer to 17% than to 18%.

Alternatively, use the compound interest formula and solve for the missing interest rate:

$$V = P\left(1+\frac{r}{100}\right)^t$$

$$1{,}650 = 1{,}200\left(1+\frac{r}{100}\right)^2$$

$$\frac{1{,}650}{1{,}200} = \left(1+\frac{r}{100}\right)^2$$

$$\sqrt{1.375} = 1+\frac{r}{100}$$

$$1.172 = 1+\frac{r}{100}$$

$$(0.172)100 = r$$

$$r \approx 17$$

71. **(E).** "Simple" interest means that the interest is calculated based on the initial amount every time; the interest earned is not included in future calculations. Each year, the investment pays 12.5%, or 1/8, of the original investment as simple interest. As a result, it will take exactly 8 years for the cumulative interest to add up to the original investment.

Be careful not to apply the compound interest formula here. If the 12.5% interest is in fact compounded annually, it will take only about 6 years for the investment to double in value.

72. **(C).** This question concerns some function for which the full formula is not provided. The problem indicates that $f(2a) = 2f(a)$. In other words, this function is such that plugging in $2a$ is the same as plugging in a and then multiplying by 2. Plug $f(6) = 11$ into the equation $f(2a) = 2f(a)$:

$f(2(6)) = 2(11)$
$f(12) = 22$

Use the same process a second time. If $a = 12$ and $f(12) = 22$:

$f(2(12)) = 2(22)$
$f(24) = 44$

Alternatively, use logic. When you plug in $2a$, you'll get the same answer as when you plug in a and then multiply by 2. Plugging in 24 is the same as plugging in 6 a total of 4 times, and will give you an answer 4 times as big as plugging in 6. Since plugging in 6 yields 11, plugging in 24 yields 44.

73. **(B).** The question is asking "For which function is performing the function on x and THEN multiplying by 1/2 the equivalent of performing the function on 1/2 of x?"

The fastest method is to use logic: since the order of operations says that order does not matter with multiplication and division but DOES matter between multiplication and addition/subtraction, or multiplication and exponents, you need a function that has only multiplication and/or division. Only answer choice (B) qualifies.

Alternatively, try each choice.

	$\frac{1}{2}f(x)$	$f\left(\frac{1}{2}x\right)$	equal?
(A)	$f\left(\frac{1}{2}x\right)$	$\frac{1}{2}(2x+2)=x+1$	No
(B)	$\frac{1}{2}(13x)=\frac{13x}{2}$	$13\left(\frac{1}{2}x\right)=\frac{13x}{2}$	Yes
(C)	$\frac{1}{2}x^2$	$\left(\frac{1}{2}x\right)^2=\frac{1}{4}x^2$	No
(D)	$\frac{1}{2}(x-10)=\frac{x}{2}-5$	$\frac{1}{2}x-10=\frac{x}{2}-10$	No
(E)	$\frac{1}{2}\sqrt{x-4}$	$\sqrt{\frac{1}{2}x-4}$	No

If you aren't sure whether the two terms in choice (E) are equal, try plugging in a real number for x. If $x = 8$, then the left-hand value becomes 1 and the right-hand value becomes the square root of 0. The two values are *not* the same.

Chapter 11

of

Fractions and Decimals

In This Chapter…

Fractions and Decimals

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

---

1. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}=$

$$\frac{\boxed{}}{\boxed{}}$$

2.

Quantity A	**Quantity B**
$-\frac{3}{4}+\frac{2}{3}$	$-\frac{3}{4}\times\frac{2}{3}$

3. The temperature in Limerick is 3/4 that in Cairo, where the temperature is 8/5 that in Halifax. If the temperature in

Limerick is 60°, what is the temperature in Halifax?

(A) 50°
(B) 55°
(C) 64°
(D) 72°
(E) 75°

4. At a convention of monsters, 2/5 have no horns, 1/7 have one horn, 1/3 have two horns, and the remaining 26 have three or more horns. How many monsters are attending the convention?

(A) 100
(B) 130
(C) 180
(D) 210
(E) 260

5. One dose of secret formula is made from 1/6 ounce of Substance X and 2/3 ounce of Substance Z. How many doses are in a 30-ounce vial of secret formula?

(A) 20
(B) 24
(C) 30
(D) 36
(E) 45

6. Devora spends 1/4 of her money on a textbook, and then buys a notebook that costs 1/6 the price of the textbook. Assuming she makes no other purchases, what fraction of her money does Devora have left over?

7. 0.003482 =

Indicate <u>all</u> such statements.

☐ -0.003482×10^{-1}
☐ 0.3482×10^{-2}
☐ 34.82×10^{4}
☐ 34.82×10^{-4}
☐ $3{,}482 \times 10^{-6}$

8. 12.12×10^{-3} =

Indicate <u>all</u> such statements.

☐ -1.21×10^{3}

☐ 0.012
☐ 0.00001212×10^3
☐ 0.01212×10^3

9. 5 is how many fifths of 10?

(A) 2.5
(B) 5
(C) 10
(D) 20
(E) 50

10.

$x > 0$ and $y > 0$

Quantity A	**Quantity B**
$\frac{1}{x}+\frac{1}{y}$	$\frac{xy}{x+y}$

11.

Quantity A	**Quantity B**
$\frac{75}{4^2}\times\frac{3^2}{45}\times\frac{2^4}{45}$	$\frac{3^2}{4^2}\times\frac{2^2}{5^2}\times\frac{10}{3}$

12. 5/12 of all the students are girls and 1/4 of all the students are girls who take Spanish. What fraction of the girls take Spanish?

(A) 5/48
(B) 5/12
(C) 2/5
(D) 3/5
(E) 7/12

13. 1/5 of all the cars on a certain auto lot are red, and 2/3 of all the red cars are convertibles. What fraction of all the cars are NOT red convertibles?

$$\frac{\boxed{}}{\boxed{}}$$

14. Two identical pies are cut into a total of 16 equal parts. If each part is then split equally among three people, what fraction of a pie does each person receive?

(A) 1/48

(B) 1/24
(C) 1/16
(D) 3/16
(E) 3/8

15. Which of the following are bigger than twice 21/49?

Indicate all such values.

☐ 0.84
☐ 0.857
☐ 0.858
☐ 0.86

16.

$$xy \neq 0$$

Quantity A	**Quantity B**
$2+\frac{1}{xy}$	$\frac{2xy+1}{xy}$

17.

Quantity A	**Quantity B**
$\dfrac{\frac{1}{4}}{\frac{2}{3}-\frac{1-2}{\frac{1}{3}}}$	$\dfrac{\frac{1}{3}}{\frac{1}{4}-\frac{3-4}{\frac{2}{3}}}$

18.

At Store A, 3/4 of the apples are red.
At Store B, which has twice as many apples, 0.375 of them are red.

Quantity A	**Quantity B**
The number of red apples at Store A	The number of red apples at Store B

19.

Dweezil has one third the number of black marbles that Gina has, but he has twice as many white marbles.
Both people have only black marbles and white marbles.

Quantity A	**Quantity B**

The number of marbles Dweezil has	The number of marbles Gina has

20. A pot of soup is divided equally into two bowls. If Manuel eats 1/4 of one of the bowls of soup and 2/5 of the other bowl of soup, how much of the soup did Manuel eat?

$$\frac{\boxed{}}{\boxed{}}$$

21. What is half of $\frac{x^2}{8}$?

(A) $\frac{x}{4}$
(B) $\frac{x^2}{4}$
(C) $\frac{x}{8}$
(D) $\frac{x^2}{16}$
(E) It cannot be determined.

22. $\dfrac{\frac{ab}{c}}{\frac{cd}{a}} =$

(A) ac
(B) bd
(C) $\frac{1}{bd}$
(D) $\frac{a^2b}{c^2d}$
(E) $\frac{ab^2}{cd^2}$

23. $\left(\frac{\sqrt{12}}{5}\right)\left(\frac{\sqrt{60}}{2^4}\right)\left(\frac{\sqrt{45}}{3^2}\right) =$

(A) $\frac{1}{12}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

24. $\frac{-1}{2x} - \frac{1}{4y} + \frac{1}{xy} + \frac{1}{8} =$

(A) $\frac{(x-4)(2-y)}{8xy}$

(B) $\frac{(x-2)(y-4)}{8xy}$

(C) $\frac{(x-4)(y-2)}{8xy}$

(D) $\frac{(x+2)(4-y)}{8xy}$

(E) $\frac{(x-2)(4-y)}{8xy}$

25.

x is a digit in the decimal 12.15x9, which, if rounded to the nearest hundredth, would equal 12.16.

Quantity A	**Quantity B**
x	4

26. $\frac{\frac{a}{b}}{\frac{c}{d} + \frac{e}{f}} =$

(A) $\dfrac{acd}{bcf + def}$

(B) $\dfrac{ace}{bdf + bcd}$

(C) $\dfrac{acf}{bde + cdf}$

(D) $\dfrac{ade}{bef + cdf}$

(E) $\dfrac{adf}{bcf + bde}$

27. $\dfrac{(17^2)(22)(38)(41)(91)}{(19)(34)(123)(11)(119)(26)} =$

$$\frac{\boxed{}}{\boxed{}}$$

28. In a decimal number, a bar over one or more consecutive digits means that the pattern of digits under the bar repeats without end. As a fraction, $7.58\overline{3} =$

$$\frac{\boxed{}}{\boxed{}}$$

29.

Quantity A	**Quantity B**
$\left(\dfrac{\sqrt{25}}{\sqrt{10}}\right)\left(\dfrac{\sqrt{8}}{\sqrt{15}}\right)$	$\left(\dfrac{\sqrt{51}}{\sqrt{46}}\right)\left(\dfrac{\sqrt{23}}{\sqrt{34}}\right)$

$$\sqrt{\frac{3}{2}}-\sqrt{\frac{2}{3}}=$$

(A) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{6}}$

(B) $\frac{1}{\sqrt{6}}$

(C) $\frac{\sqrt{3}}{3}$

(D) $\frac{\sqrt{3}}{2}$

(E) $\frac{\sqrt{5}}{\sqrt{6}}$

31. If $abc \neq 0$, then $\frac{ab}{cb}+\frac{a}{c}-\frac{a^2b^3}{abc}=$

(A) $\frac{a-b^2}{c}$

(B) $\frac{a^2-2b^2}{c}$

(C) $\frac{2a^2-b^2}{c}$

(D) $\frac{a(2-b^2)}{c}$

(E) $\frac{a^2b(2-b^2)}{c}$

32. If 3/4 of all the cookies have nuts and 1/3 of all the cookies have both nuts and fruit, what fraction of all the cookies have nuts but no fruit?

(A) 1/4
(B) 5/12
(C) 1/2
(D) 7/12
(E) 5/6

33. 1/4 of all the juniors and 2/3 of all the seniors are going on a trip. If there are 2/3 as many juniors as seniors, what

fraction of the students are not going on the trip?

(A) 4/9
(B) 1/2
(C) 2/3
(D) 1/3
(E) 5/6

34. 4/5 of the women and 3/4 of the men speak Spanish. If there are 40% as many men as women, what fraction of the group speaks Spanish?

35.

$$abcd \neq 0$$

Quantity A	Quantity B
$\dfrac{a^2b}{cd^2} \times \dfrac{d^3}{abc}$	$\dfrac{d^2}{bc} \times \dfrac{ab^2}{bd}$

36.

Quantity A	Quantity B
$\dfrac{24}{3\sqrt{2}} - \dfrac{4}{\sqrt{2}}$	$\sqrt{6}$

37.

$$m \neq 0$$

Quantity A	Quantity B
$\left(\dfrac{1}{2} + \dfrac{1}{m}\right)(m+2)$	$\dfrac{(m+2)^2}{2m}$

38.

The reciprocal of x's non-integer decimal part equals $x + 1$, and $x > 0$.

Quantity A	Quantity B
x	$\sqrt{2}$

39. Which two of the following numbers have a sum between 1 and 2?

Indicate <u>both</u> of the numbers.

- ☐ $\dfrac{7(2^3)}{3^3-7}$
- ☐ $\dfrac{2^4}{1+2+3+4}$
- ☐ $\dfrac{3}{11}\div\dfrac{6}{11}$
- ☐ $\dfrac{-2^3 3^2}{2^2 5^2}$
- ☐ $\dfrac{-11^2-11^3}{(30)(44)}$

40. Which three of the following answers, when multiplied by each other, yield a product less than -1?

Indicate <u>all three</u> numbers.

- ☐ $\dfrac{-15}{17}$
- ☐ $\dfrac{-18}{19}$
- ☐ $\dfrac{23}{-22}$
- ☐ $\dfrac{17}{-16}$

41. The decimal representation of the reciprocal of integer n contains an infinitely repeating pattern of digits, expressed with a bar over the repeating digits. The minimum length of the bar (in digits) is n - 1.

Indicate <u>all</u> of the integers below that could be n.

☐ 3
☐ 5
☐ 7
☐ 9
☐ 11

42. $(3-\frac{1}{3})^2+(3+\frac{1}{3})^2=$

(A) 122/9
(B) 164/9
(C) 36
(D) 164/3
(E) 162

43. If $\dfrac{3}{\frac{m+1}{m}+1}=1$, then *m* must equal

(A) -2
(B) -1
(C) 0
(D) 1
(E) 2

44.

$$rs=\sqrt{3}$$

Quantity A	**Quantity B**
$\dfrac{2r\sqrt{12}}{r^2s\sqrt{72}}$	$\dfrac{14rs^2}{42s}$

45.

Quantity A	**Quantity B**
$\dfrac{\sqrt{10}}{\sqrt{8}}\div\dfrac{\sqrt{9}}{\sqrt{10}}$	$\dfrac{\sqrt{11}}{\sqrt{9}}\div\dfrac{\sqrt{10}}{\sqrt{11}}$

46.

$$\frac{x}{m}>0$$

Quantity A	**Quantity B**
$\dfrac{11m+17x}{11m}$	$\dfrac{17m+11x}{17m}$

47. Which of the following fractions has the greatest value?

(A) $\frac{7}{(16^2)(25)}$

(B) $\frac{5}{(32)(5^4)}$

(C) $\frac{30}{(512)(5^3)}$

(D) $\frac{5}{(4^6)(5)}$

(E) $\frac{4}{(2^{11})(5^2)}$

48.

$$\frac{m}{p} > \frac{n}{p}$$

Quantity A	**Quantity B**
m	*n*

49. If $2x \neq y$ and $5x \neq 4y$, then

$$\frac{\frac{5x-4y}{2x-y}}{\frac{3y}{y-2x}+5} =$$

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $\frac{7}{2}$

(E) $\frac{9}{2}$

50. $\frac{39^2}{2^4} \div \frac{13^3}{4^2} =$

(A) $\frac{13}{2}$

(B) $\frac{9}{2}$

(C) $\frac{3}{2}$

(D) $\frac{3}{13}$

(E) $\frac{9}{13}$

51. To the nearest integer, the non-negative fourth root of integer *n* rounds to 3. Inclusive, *n* is between

(A) 0 and 1
(B) 2 and 3
(C) 4 and 9
(D) 10 and 39
(E) 40 and 150

Fractions and Decimals Answers

1. $\frac{71}{20}$ **(or any equivalent fraction).** Add all the fractions by finding a common denominator, which is a multiple of 2, 3, 4, 5, and 6. The smallest number that will work is 60.

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}=\frac{30}{60}+\frac{40}{60}+\frac{45}{60}+\frac{48}{60}+\frac{50}{60}=\frac{30+40+45+48+50}{60}=\frac{213}{60}=\frac{71}{20}$$

2. **(A).** In Quantity A, get a common denominator and then add:

$$-\frac{3}{4}+\frac{2}{3}=-\frac{9}{12}+\frac{8}{12}=-\frac{1}{12}$$

In Quantity B, multiply across (common denominators are only needed for addition and subtraction). You can cancel the 3's first if you wish:

$$-\frac{3}{4}\times\frac{2}{3}=-\frac{\cancel{3}}{4}\times\frac{2}{\cancel{3}}=-\frac{2}{4}=-\frac{1}{2}$$

$-\frac{1}{12}$ is larger than $-\frac{1}{2}$. (Be careful with negatives! The closer to 0 a negative is, the larger it is.)

3. **(A).** When you are given two relationships in one sentence, follow the grammar carefully to make sure you produce the right equations. The first sentence of the problem gives you two relationships:

> The temperature in Limerick is 3/4 that in Cairo.
> The temperature in Cairo is 8/5 that in Halifax.

$$L=\frac{3}{4}C$$

$$C=\frac{8}{5}H$$

Replace C with $(8/5)H$ in the first equation:

$$L = \frac{3}{4}\left(\frac{8}{5}H\right)$$
$$L = \frac{24}{20}H$$
$$L = \frac{6}{5}H$$

Now plug in 60 for *L*:

$$60 = \frac{6}{5}H$$
$$\frac{5}{6} \times 60 = H$$
$$50 = H$$

4. **(D).** This is a common GRE setup—you have been given several fractions and one actual number. Once you know what fraction of the whole that number represents, you can solve for the total (call the total *m*). Notice that all of the denominators are primes, so they don't share any factors. Therefore you will have to multiply them all together to find a common denominator. 5 × 7 × 3 = 105:

$$\frac{2}{5} + \frac{1}{7} + \frac{1}{3} = \frac{42}{105} + \frac{15}{105} + \frac{35}{105} = \frac{92}{105}$$

That means that the remaining 26 monsters represent 13/105 of the total monsters at the convention:

$$26 = \frac{13}{105}m$$
$$\frac{105}{13} \times 26 = m$$
$$105 \times 2 = m$$
$$210 = m$$

5. **(D).** To find the number of doses in the vial, you need to divide the total volume of the formula in the vial by the volume of one dose.

One dose = 1/6 + 2/3 = 1/6 + 4/6 = 5/6

Now divide 30 ounces by 5/6:

30 ÷ 5/6 = 30 × 6/5 = 36

6. $\frac{17}{24}$ **(or any equivalent fraction).** The textbook costs 1/4 of Devora's money. The notebook costs 1/6 *of that amount,* or $\frac{1}{6}\left(\frac{1}{4}\right)=\frac{1}{24}$ of Devora's money. Thus, Devora has spent $\frac{1}{4}+\frac{1}{24}=\frac{6}{24}+\frac{1}{24}=\frac{7}{24}$ of her money. Subtract from 1 to get the fraction she has left: $1-\frac{7}{24}=\frac{24}{24}-\frac{7}{24}=\frac{17}{24}$.

Alternatively, pick a value for Devora's money. (Look at the denominators in the problem—4 and 6—and pick a value that both numbers go into.) For instance, say Devora has \$120. She would spend 1/4, or \$30 on the textbook. She would spend 1/6 *of that* amount, or \$5, on the notebook. She would have spent \$35 and have \$85 left, and thus $\frac{85}{120}$ of her money left. Reduce $\frac{85}{120}$ to get $\frac{17}{24}$, or simply enter $\frac{85}{120}$ in the boxes. This will work with any value you pick for Devora's total.

7. **II, IV, and V only.** Note that the first answer is negative, so it cannot be correct. For the second answer, move the decimal 2 places to the left: $0.3482 \times 10^{-2} = 0.003482$ (correct). For the third answer, move the decimal 4 places to the *right* (since the exponent is positive)—this move makes the number much larger and cannot be correct. For the fourth answer, move the decimal 4 places to the left: $34.82 \times 10^{-4} = 0.003482$ (correct). For the fifth answer, move the decimal 6 places to the left: $3{,}482 \times 10^{-6} = 0.003482$ (correct).

8. **III only.** First, simplify $12.12 \times 10^{-3} = 0.01212$. Now, test which answers are equal to this value. The first answer is negative, so it cannot be correct. The second answer is simply 0.012 and is therefore incorrect (the end has been "chopped off," so the number is not the same value). The third answer is $0.00001212 \times 10^{3} = 0.01212$ and is correct. The fourth answer is $0.01212 \times 10^{3} = 12.12$ and is not correct.

9. **(A).** Translate the words into math. If x means "how many," then "how many fifths" is $\frac{x}{5}$:

$$5=\frac{x}{5}\times 10$$

$$5=\frac{10x}{5}$$

$$25=10x$$

$$\frac{25}{10}=x$$

$$x=2.5$$

10. **(D)**. Add the fractions in Quantity A by making a common denominator (xy):

$$\frac{1}{x}+\frac{1}{y}=\frac{y}{xy}+\frac{x}{xy}=\frac{y+x}{xy}$$

Quantity B is just the reciprocal of Quantity A—one fraction is the "flipped" version of the other. You also know that both quantities are positive. However, without knowing more about x and y, you don't know whether Quantity A or Quantity B is bigger.

11. **(A).** Simplify each quantity by breaking down to primes and canceling factors:

Quantity A: $$\frac{75}{4^2}\times\frac{3^2}{45}\times\frac{2^4}{45}=\frac{3\times5\times5}{(2^2)^2}\times\frac{3^2}{3\times3\times5}\times\frac{2^4}{3\times3\times5}=\frac{2^4\times3^3\times5^2}{2^4\times3^4\times5^2}=\frac{1}{3}$$

Quantity B: $$\frac{3^2}{4^2}\times\frac{2^2}{5^2}\times\frac{10}{3}=\frac{3^2}{(2^2)^2}\times\frac{2^2}{5^2}\times\frac{2\times5}{3}=\frac{2^3\times3^2\times5}{2^4\times3\times5^2}=\frac{3}{2\times5}=\frac{3}{10}$$

Since $\frac{1}{3}>\frac{3}{10}$, Quantity A is bigger than Quantity B. You can compare these fractions by making a common denominator, by cross multiplying, or by comparing the decimal equivalents 0.333… and 0.3.

If you notice the same factors on each side in the same position (e.g., 3^2 on top or 4^2 on bottom), then you can save time by canceling those factors simultaneously from both quantities.

12. **(D).** The wording here is very confusing. The problem is *not* asking you to take 1/4 of 5/12. Rather, 1/4 and 5/12 are fractions of the same number (the number of students in the whole class). A good way to avoid this confusion is to plug in a number for the class. Pick 12, as you're asked to take 5/12ths of the class:

Class = 12
Girls = 5
Girls who take Spanish = 3 (1/4 of all the students)

You are asked for the number of girls who take Spanish over the number of girls. Thus, the answer is 3/5.

13. **13/15 (or any equivalent fraction).** If 1/5 of all the cars are red and 2/3 of THOSE are convertibles, then the fraction of all the cars that are red convertibles = (1/5)(2/3) = 2/15. Since you want all of the cars that are NOT red convertibles, subtract 2/15 from 1 to get 13/15.

14. **(B).** If *two* pies are cut into 16 parts, *each* pie is cut into eighths. Thus, 1/8 of a pie is *divided* among three people. "One third of one eighth" = (1/3)(1/8) = 1/24.

15. **III and IV only.** Simply plug 21/49 into the calculator, and then multiply by 2 to get 0.857142… You need all values larger than this number. Obviously, 0.84 is smaller. The next choice, 0.857, might seem attractive; however, it is smaller than 0.857142… You can easily see this by adding zeroes to the end of 0.857 in order to more easily compare:

Choice II: 0.857000

Your number: 0.857142…

The third and fourth choices are, of course, larger than 0.857142…

16. **(C).** Transform Quantity B by splitting the numerator:

$$\frac{2xy+1}{xy}=\frac{2xy}{xy}+\frac{1}{xy}$$

Then cancel the common factor *xy* from top and bottom of the first fraction:

$\frac{2xy}{xy}+\frac{1}{xy}=2+\frac{1}{xy}$, which is the same as Quantity A.

Alternatively, you can transform Quantity A by turning 2 into a fraction with the same denominator (xy) as the second term.

$2+\frac{1}{xy}=\frac{2xy}{xy}+\frac{1}{xy}=\frac{2xy+1}{xy}$, which is the same as Quantity B.

17. **(B).** Simplify each quantity from the inside out.

Quantity A: $$\frac{\frac{1}{4}}{\frac{2}{3}-\frac{1-2}{\frac{1}{3}}}=\frac{\frac{1}{4}}{\frac{2}{3}-\frac{-1}{\frac{1}{3}}}=\frac{\frac{1}{4}}{\frac{2}{3}-(-3)}=\frac{\frac{1}{4}}{\frac{2}{3}+3}=\frac{\frac{1}{4}}{\frac{2}{3}+\frac{9}{3}}=\frac{\frac{1}{4}}{\frac{11}{3}}=\frac{1}{4}\times\frac{3}{11}=\frac{3}{44}$$

Quantity B: $$\frac{\frac{1}{3}}{\frac{1}{4}-\frac{3-4}{\frac{2}{3}}}=\frac{\frac{1}{3}}{\frac{1}{4}-\frac{-1}{\frac{2}{3}}}=\frac{\frac{1}{3}}{\frac{1}{4}-\left(\frac{-3}{2}\right)}=\frac{\frac{1}{3}}{\frac{1}{4}+\frac{3}{2}}=\frac{\frac{1}{3}}{\frac{1}{4}+\frac{6}{4}}=\frac{\frac{1}{3}}{\frac{7}{4}}=\frac{1}{3}\times\frac{4}{7}=\frac{4}{21}$$

Since Quantity B has a larger numerator *and* a smaller denominator, it is larger than Quantity A. This rule works for any positive fractions. Of course, you can also use the calculator to compute the decimal equivalents.

18. **(C).** Whether you choose fractions or decimals, you want to make 3/4 and 0.375 the same form. Either way, you will see that 3/4 is double 0.375 (which is 3/8). Since Store B has twice as many apples, 3/8 of Store B's apples is the same value as 3/4 of Store A's apples.

Alternatively, pick numbers such that Store B has twice as many apples. If Store A has 4 apples and Store B has 8 apples, then Store A would have (3/4)(4) = 3 red apples and Store B would have (0.375)(8) = 3 red apples. The values will always be the same.

19. **(D).** To demonstrate that there is not enough information to determine who has more marbles, try extreme examples. Dweezil has 1/3 as many black marbles as Gina, but twice as many white marbles:

EXAMPLE 1: LOTS OF BLACK MARBLES

Dweezil:	1,000 black marbles	2 white marbles
Gina:	3,000 black marbles	1 white marble

In this example, Gina has more.

EXAMPLE 2: LOTS OF WHITE MARBLES

Dweezil:	1 black marble	2,000 white marbles
Gina:	3 black marbles	1,000 white marbles

In this example, Dweezil has more.

As always, when trying examples in Quantitative Comparison problems, you *must* try more than one example—with the goal of proving (D).

20. **13/40 (or any equivalent fraction).** Manuel eats 1/4 of one-half of all the soup, and then 2/5 of the other half of all the soup. As math:

$$\frac{1}{4}\left(\frac{1}{2}\right)+\frac{2}{5}\left(\frac{1}{2}\right)=\frac{1}{8}+\frac{1}{5}=\frac{13}{40}$$

Alternatively, pick numbers. Since you'll be dividing this number several times, pick a large number with many factors. For example, say there are 120 ounces of soup. Each bowl would then have 60 ounces. Manuel would then eat 1/4 of one bowl (15 ounces) and 2/5 of the other bowl (24 ounces). In total, he would eat 39 ounces out of 120. While 39/120 would be counted as correct, it is also possible to reduce 39/120 (divide both numerator and denominator by 3) to get 13/40, the answer you reached via the other method above.

21. **(D).** To take half of a number, multiply by $\frac{1}{2}$:

$$\frac{1}{2}\times\frac{x^2}{8}=\frac{x^2}{16}$$

22. **(D).** To divide by a fraction, multiply by its reciprocal:

$$\frac{\frac{ab}{c}}{\frac{cd}{a}} = \frac{ab}{c} \times \frac{a}{cd} = \frac{a^2b}{c^2d}$$

23. **(C).** Pull squares out of the square roots and cancel common factors:

$$\left(\frac{\sqrt{12}}{5}\right)\left(\frac{\sqrt{60}}{2^4}\right)\left(\frac{\sqrt{45}}{3^2}\right) = \frac{2\sqrt{3}}{5} \times \frac{2\sqrt{15}}{2^4} \times \frac{3\sqrt{5}}{3^2} = \frac{\sqrt{3}}{5} \times \frac{\sqrt{15}}{2^2} \times \frac{\sqrt{5}}{3}$$

Since $\sqrt{15} = \sqrt{3}\sqrt{5}$, you get

$$\frac{\sqrt{3}}{5} \times \frac{\sqrt{15}}{2^2} \times \frac{\sqrt{5}}{3} = \frac{\sqrt{3}}{5} \times \frac{\sqrt{3}\sqrt{5}}{2^2} \times \frac{\sqrt{5}}{3} = \frac{3 \times 5}{5 \times 2^2 \times 3} = \frac{1}{2^2} = \frac{1}{4}$$

24. **(C).** Combine the four fractions by finding a common denominator (8*xy*, which is also suggested by the answer choices):

$$\frac{-1}{2x} - \frac{1}{4y} + \frac{1}{xy} + \frac{1}{8} = \frac{-1(4y)}{2x(4y)} - \frac{1(2x)}{4y(2x)} + \frac{1(8)}{xy(8)} + \frac{1(xy)}{8(xy)}$$
$$= \frac{-4y}{8xy} - \frac{2x}{8xy} + \frac{8}{8xy} + \frac{xy}{8xy} = \frac{xy - 4y - 2x + 8}{8xy}$$

Now the key is to factor the top expression correctly.

xy-$4y$-$2x + 8 = (x - 4)(y - 2)$

You can always FOIL the expression on the right to make sure it matches the left-hand side.

So, in the end you have $\frac{xy - 4y - 2x + 8}{8xy} = \frac{(x-4)(y-2)}{8xy}$.

25. **(A).** Since the decimal rounds to 12.16, the thousandths digit x must be 5 or greater (6, 7, 8, or 9). All of these possibilities are greater than 4.

26. **(E).** Simplify from the inside out by finding a common denominator (*df*) for the two fractions "inside":

$$\frac{\frac{a}{b}}{\frac{c}{d}+\frac{e}{f}}=\frac{\frac{a}{b}}{\frac{cf}{df}+\frac{de}{df}}$$

Next, add those two inside fractions; then flip and multiply:

$$\frac{\frac{a}{b}}{\frac{cf}{df}+\frac{de}{df}}=\frac{\frac{a}{b}}{\frac{cf+de}{df}}=\frac{a}{b}\times\frac{df}{cf+de}=\frac{adf}{bcf+bde}$$

27. $\mathbf{\frac{1}{3}}$ **(or any equivalent fraction).** You could just punch the whole numerator and the whole denominator into the calculator and submit each product. If you're very careful, that will work. However, it might be wise to try canceling some common factors out of the fraction, to save time and to avoid errors. It's fine to switch to the calculator whenever the cancellations aren't obvious.

$$\frac{(17^2)\cancel{(22)}(38)(41)(91)}{(19)(34)(123)\cancel{(11)}(119)(26)}=\frac{(17^2)(2)(\cancel{38}\,2)(41)(91)}{\cancel{(19)}(34)(123)(119)(26)}$$
$$=\frac{(17^{\cancel{2}})(2)(2)(41)(91)}{(\cancel{34}\,2)(123)(119)(26)}=\frac{(17)(2)(2)\cancel{(41)}(91)}{(2)(\cancel{123}\,3)(119)(26)}$$
$$=\frac{(17)(2)(2)(\cancel{91}\,7)}{(2)(3)(119)(\cancel{26}\,2)}=\frac{\cancel{(17)}(2)(2)\cancel{(7)}}{(2)(3)(\cancel{17}\times\cancel{7})(2)}$$
$$=\frac{\cancel{(2)}\cancel{(2)}}{\cancel{(2)}(3)\cancel{(2)}}=\frac{1}{3}$$

28. $\mathbf{\frac{91}{12}}$ **(or any equivalent fraction).** First, turn the decimal into a sum of two pieces, to separate the repeating portion.

$$7.58\overline{3}=7.58+0.00\overline{3}$$

Deal with each piece in turn. Like any other terminating decimal, 7.58 can be written as a fraction with a power of 10 in the denominator.

$$7.58=\frac{758}{100}$$

Now, when you look at the repeating portion, you should be reminded that $0.\overline{3} = 0.3333... = \frac{1}{3}$

So $0.00\overline{3}$. is just $\frac{1}{3}$, moved by a couple of decimal places. $0.00\overline{3} = (0.\overline{3})(0.01) = \left(\frac{1}{3}\right)\left(\frac{1}{100}\right) = \frac{1}{300}$

Finally, you can write the original decimal as a sum of fractions, and then combine those fractions.

$$7.58\overline{3} = 7.58 + 0.00\overline{3} = \frac{758}{100} + \frac{1}{300} = \frac{758 \times 3}{300} + \frac{1}{300} = \frac{2,275}{300}$$

You can enter $\frac{2,275}{300}$, unreduced, or you can reduce it to $\frac{91}{12}$ if you want.

29. **(A).** Simplify each quantity by factoring the square roots, then canceling.

Quantity A: $\left(\frac{\sqrt{25}}{\sqrt{10}}\right)\left(\frac{\sqrt{8}}{\sqrt{15}}\right) = \left(\frac{5}{\sqrt{2}\sqrt{5}}\right)\left(\frac{2\sqrt{2}}{\sqrt{3}\sqrt{5}}\right) = \left(\frac{\cancel{5}}{\sqrt{2}\cancel{\sqrt{5}}}\right)\left(\frac{2\sqrt{2}}{\sqrt{3}\cancel{\sqrt{5}}}\right) =$

$\left(\frac{1}{\cancel{\sqrt{2}}}\right)\left(\frac{2\cancel{\sqrt{2}}}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}$

Quantity B: $\left(\frac{\sqrt{51}}{\sqrt{46}}\right)\left(\frac{\sqrt{23}}{\sqrt{34}}\right) = \left(\frac{\sqrt{3}\cancel{\sqrt{17}}}{\sqrt{2}\sqrt{23}}\right)\left(\frac{\sqrt{23}}{\sqrt{2}\cancel{\sqrt{17}}}\right) = \left(\frac{\sqrt{3}}{\sqrt{2}\cancel{\sqrt{23}}}\right)\left(\frac{\cancel{\sqrt{23}}}{\sqrt{2}}\right) = \frac{\sqrt{3}}{2}$

Since $\sqrt{3} < 2$, Quantity B is smaller than 1, whereas Quantity A is greater than 1.

Of course, you can use the calculator here, but the process would be slower and more prone to error.

30. **(B).** The square root of a fraction is the square root of the top over the square root of the bottom.

$$\sqrt{\frac{3}{2}} - \sqrt{\frac{2}{3}} = \frac{\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{3}}$$

Then make a common denominator: $\sqrt{3}\sqrt{2} = \sqrt{6}$.

$$\frac{\sqrt{3}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{3}\sqrt{3}}{\sqrt{3}\sqrt{2}} - \frac{\sqrt{2}\sqrt{2}}{\sqrt{3}\sqrt{2}} = \frac{3}{\sqrt{6}} - \frac{2}{\sqrt{6}} = \frac{1}{\sqrt{6}}$$

31. **(D).** Simplify each fraction first by canceling common terms from top and bottom.

$$\frac{ab}{cb}+\frac{a}{c}-\frac{a^2b^3}{abc}=\frac{a\cancel{b}}{c\cancel{b}}+\frac{a}{c}-\frac{a^{\cancel{2}}b^{\cancel{3}2}}{\cancel{a}\cancel{b}c}=\frac{a}{c}+\frac{a}{c}-\frac{ab^2}{c}$$

Luckily, every fraction now has the same denominator, so you can just add/subtract the numerators.

$$\frac{a}{c}+\frac{a}{c}-\frac{ab^2}{c}=\frac{2a-ab^2}{c}=\frac{a\left(2-b^2\right)}{c}$$

32. **(B).** Since 3/4 of the cookies have nuts and 1/3 of the cookies *also* have fruit, simply subtract 3/4 - 1/3 to get all the cookies with nuts but no fruit.

$$\frac{3}{4}-\frac{1}{3}=\frac{9}{12}-\frac{4}{12}=\frac{5}{12}$$

Alternatively, pick numbers. Since you will be dividing by 4 and 3, pick a number divisible by 4 and 3. If there are 12 cookies, then 9 have nuts and 4 have nuts and fruit, so 5—and thus 5/12 of the total—would have nuts but no fruit.

33. **(B).** If you have to take fractions of different numbers that are *also* related by a fraction, then you should try plugging numbers. Since there are 2/3 as many juniors as seniors, some easy numbers are:

Juniors = 20

Seniors = 30

Juniors going on trip = $\frac{1}{4}$ (20) = 5

Seniors going on trip = $\frac{2}{3}$ (30) = 20

Out of 50 total students, 25 are going on the trip, so 25 are NOT going on the trip. The answer is 25/50 = 1/2.

34. **11/14 (or any equivalent fraction).** If you have to take fractions of different numbers that are *also* related by a fraction or percent, then you should try plugging numbers. Since there are 40% as many men as women, some easy numbers are:

Men: 40

Women: 100

Women who speak Spanish = $\frac{4}{5}$ (100) = 80

Men who speak Spanish = $\frac{3}{4}$ (40) = 30

The group has 140 total people and 110 Spanish speakers. 110/140 = 11/14 (you are not required to reduce, as long as your answer is correct and fits in the box).

35. **(D).** Cancel factors on top and bottom of each product.

Quantity A: $\frac{a^2b}{cd^2} \times \frac{d^3}{abc} = \frac{a^2bd^3}{abc^2d^2} = \frac{ad}{c^2}$

Quantity B: $\frac{d^2}{bc} \times \frac{ab^2}{bd} = \frac{ab^2d^2}{b^2cd} = \frac{ad}{c}$

The two quantities differ in the denominators: A has c^2, while B has c. So you can't tell which quantity is bigger, because sometimes c^2 is greater than c, and other times c^2 is less than c.

36. **(A).** Simplify Quantity A by canceling the common factor of 3 from top and bottom of the first fraction, then subtracting the numerators.

$$\frac{24}{3\sqrt{2}} - \frac{4}{\sqrt{2}} = \frac{8}{\sqrt{2}} - \frac{4}{\sqrt{2}} = \frac{8-4}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

Now compare Quantity A $\left(\frac{4}{\sqrt{2}}\right)$ with Quantity B $(\sqrt{6})$. Multiply both quantities by $\sqrt{2}$ to eliminate the denominator on the left.

$$\text{Quantity A} = \frac{4}{\sqrt{2}}\sqrt{2} = 4, \text{ while Quantity B} = \sqrt{6}\sqrt{2} = \sqrt{12}.$$

Finally, square both quantities to get rid of the square-root sign:

Quantity A = 4^2 = 16, while Quantity B = $\left(\sqrt{12}\right)^2$ = 12.

Quantity A is obviously larger.

37. **(D).** Multiply out Quantity A by FOILing:

$$\left(\frac{1}{2} + \frac{1}{m}\right)(m+2) = \frac{1}{2}(m) + \frac{1}{2}(2) + \frac{1}{m}(m) + \frac{1}{m}(2)$$

$$= \frac{m}{2} + 1 + 1 + \frac{2}{m} = \frac{m}{2} + 2 + \frac{2}{m}$$

Make a common denominator ($2m$) to add these terms:

$$\frac{m}{2}+2+\frac{2}{m}=\frac{m(m)}{2(m)}+2\left(\frac{2m}{2m}\right)+\frac{(2)2}{(2)m}=\frac{m^2+4m+4}{2m}$$

Finally, look at Quantity B. Since $(m + 2)^2 = m^2 + 4m + 4$, you know that the quantities are the same.

38. **(C).** You can approach this problem by testing Quantity B ($\sqrt{2}$) as x. Using the calculator, you get $\sqrt{2}$... This decimal number doesn't repeat, but isolate the non-integer decimal part.

$$\sqrt{2}-1\approx 0.41421356...$$

Now take the reciprocal of both sides.

$$\frac{1}{\sqrt{2}-1}\approx\frac{1}{0.41421356...}=2.41421356...$$

The result seems to be 1 more than the original number, $\sqrt{2}$. To prove this outcome exactly, change the right side of the equation to $\sqrt{2}+1$ and rearrange. If the equation is true, you should be able to prove that the two sides are equal.

$$\frac{1}{\sqrt{2}-1}=\sqrt{2}+1?$$

Cross multiply. Notice the difference of squares:

$$1=\left(\sqrt{2}-1\right)\left(\sqrt{2}+1\right)?$$
$$1=\left(\sqrt{2}\right)^2-1^2=2-1=1$$

Since 1 = 1, the original equation is true, and $x=\sqrt{2}$.

39. **I and V only.** Compute each value. For the simple cases, practice not using the calculator:

- ☐ $\frac{7(2^3)}{3^3-7}=\frac{7\times 8}{27-7}=\frac{56}{20}=2.8$
- ☐ $\frac{2^4}{1+2+3+4}=\frac{16}{10}=1.6$
- ☐ $\frac{3}{11}\div\frac{6}{11}=\frac{3}{11}\times\frac{11}{6}=\frac{3}{6}=\frac{1}{2}=0.5$
- ☐ $\frac{-2^3 3^2}{2^2 5^2}=\frac{-8\times 9}{10^2}=\frac{-72}{100}=-0.72$
- ☐ $\frac{-11^2-11^3}{(30)(44)}=\frac{-11^2(1+11)}{(30)(44)}=\frac{-121(12)}{(30)(44)}=\frac{-1,452}{-1,320}=-1.1$

In the last case, you could cancel factors and solve without the calculator, or you could punch in the products on top and bottom as shown, then divide on the calculator.

You've converted each value to decimal form, to make them easy to add. You are looking for two values that add up to a number between 1 and 2. You know that there can only be two such values.

By inspecting the positive numbers, you can see that no two of them add up to a number between 1 and 2. So you need a positive and a negative. The only two possibilities that work are 2.8 and -1.1.

40. **II, III, and IV only.** You want the product of three of the numbers to be less than -1. You can brute-force the calculation by trying all possible products, but you can use the relative size of the numbers to reduce the effort.

Notice that the four answer choices are all very close to -1, but some are greater than -1, and others are less than -1. To get the exact order, you can use the calculator, or you can think about the difference between each fraction and -1:

$$\frac{-15}{17}=\frac{-17}{17}+\frac{2}{17}=-1+\frac{2}{17}$$

$\frac{-18}{19}=\frac{-19}{19}+\frac{1}{19}=-1+\frac{1}{19}$, which is less than the previous number (since $\frac{2}{17}>\frac{1}{19}$)

$$\frac{23}{-22}=\frac{-23}{22}=\frac{-22}{22}-\frac{1}{22}=-1-\frac{1}{22}$$

$\frac{17}{-16}=\frac{-17}{16}=\frac{-16}{16}-\frac{1}{16}=-1-\frac{1}{16}$, a greater decrease from -1 than the previous number.

So the order of the original numbers relative to each other and to -1 is this: $\frac{17}{-16} < \frac{23}{-22} < -1 < \frac{-18}{19} < \frac{-15}{17}$

Try multiplying the three lowest numbers first, since they will produce the lowest product. Only *one* product of the three numbers can be less than -1 (or there would be more than one right answer), so the three numbers must be as follows, as you can check on the calculator:

$$\frac{17}{-16} \times \frac{23}{-22} \times \frac{-18}{19} \approx -1.052... < -1$$

41. **III only.** First, eliminate any decimals that *don't* repeat. The reciprocal of 5, which is 15 $\frac{1}{5}$, equals 0.2, which doesn't repeat. Next, use your calculator to compute the repeating decimals that correspond to the other reciprocals.

$\frac{1}{3} = 0.333... = 0.\overline{3}$ The bar only has to be 1 digit long, which does not equal n - 1 (= 3 - 1 = 2).

$\frac{1}{7} = 0.14285714... = 0.\overline{142857}$ The bar is 6 digits long, which equals n - 1 (= 7 - 1 = 6).

$\frac{1}{9} = 0.111... = 0.\overline{1}$ The bar only has to be 1 digit long, which does not equal n - 1 (= 9 - 1 = 8).

$\frac{1}{11} = 0.0909... = 0.\overline{09}$ The bar only has to be 2 digits long, which does not equal n - 1 (= 11 - 1 = 10).

So, 7 is the only possibility.

42. **(B).** First, simplify inside the parentheses. Then, square and add:

$$\left(\frac{8}{3}\right)^2 + \left(\frac{10}{3}\right)^2$$
$$\frac{64}{9} + \frac{100}{9}$$

The answer is 164/9.

43. **(D).** If the left-hand side of the equation is equal to 1, then the numerator and denominator must be equal. Thus, the denominator must also be equal to 3:

$$\frac{m+1}{m}+1=3$$

$$\frac{m+1}{m}=2$$

$$m+1=2m$$

$$1=m$$

Alternatively, you can just plug in each answer choice (into both instances of *m* in the original equation), and stop when you hit a choice that works.

44. **(B).** Cancel common factors in each quantity and substitute in for *rs*.

Quantity A: $\frac{2r\sqrt{12}}{r^2s\sqrt{72}}=\frac{2\sqrt{12}}{rs\sqrt{72}}=\frac{2\sqrt{12}}{\sqrt{3}\sqrt{72}}=\frac{2\sqrt{4}}{\sqrt{72}}=\frac{2\times 2}{\sqrt{36}\sqrt{2}}=\frac{4}{6\sqrt{2}}=\frac{2}{3\sqrt{2}}$

Quantity B: $\frac{14rs^2}{42s}=\frac{14rs}{42}=\frac{14\sqrt{3}}{3\times 14}=\frac{\sqrt{3}}{3}$

At this point, you can use the calculator, or you can compare the two quantities with an "invisible inequality."

$$\frac{2}{3\sqrt{2}} \;??\; \frac{\sqrt{3}}{3}$$

Since everything is positive, you can cross multiply (be sure to do so *upward*):

$$2\times 3 \;??\; 3\times\sqrt{2}\sqrt{3}$$

Now square both sides. Since everything is positive, the invisible inequality is unaffected:

$(2 \times 3)^2$?? $3^2 \times 2 \times 3$
36 ?? 54

Since 36 < 54, Quantity B is bigger.

45. **(A).** To divide fractions, multiply by the reciprocal.

Quantity A: $\frac{\sqrt{10}}{\sqrt{8}}\div\frac{\sqrt{9}}{\sqrt{10}}=\frac{\sqrt{10}}{\sqrt{8}}\times\frac{\sqrt{10}}{\sqrt{9}}=\frac{10}{\sqrt{72}}=\frac{10}{6\sqrt{2}}=\frac{5}{3\sqrt{2}}$

Quantity B: $\dfrac{\sqrt{11}}{\sqrt{9}} \div \dfrac{\sqrt{10}}{\sqrt{11}} = \dfrac{\sqrt{11}}{\sqrt{9}} \times \dfrac{\sqrt{11}}{\sqrt{10}} = \dfrac{11}{3\sqrt{10}}$

Square both quantities to get rid of the square roots.

Quantity A: $\left(\dfrac{5}{3\sqrt{2}}\right)^2 = \dfrac{5^2}{3^2 2} = \dfrac{25}{18}$

Quantity B: $\left(\dfrac{11}{3\sqrt{10}}\right)^2 = \dfrac{11^2}{3^2 10} = \dfrac{121}{90}$

At this point, use the calculator. Quantity A is approximately 1.389, whereas Quantity B is approximately 1.344.

46. **(A).** Always start by considering the initial given(s). If $x/m > 0$, then you know the two variables have the same sign (and that neither of them are 0). Looking down at the columns, you'll notice that there are common terms in the numerators and denominators of both fractions, so it will probably pay off to separate out the fractions:

Quantity A: $\dfrac{11m+17x}{11m} = \dfrac{11m}{11m} + \dfrac{17x}{11m} = 1 + \dfrac{17x}{11m}$

Quantity B: $\dfrac{17m+11x}{17m} = \dfrac{17m}{17m} + \dfrac{11x}{17m} = 1 + \dfrac{11x}{17m}$

Now that you've rephrased your two quantities, put them in an "invisible inequality" and see what you can do (since you don't know which side is greater, use a ? instead of the < or > symbols).

$$1 + \frac{17x}{11m} \; ? \; 1 + \frac{11x}{17m}$$

$$\frac{17x}{11m} \; ? \; \frac{11x}{17m}$$

Now isolate x/m on both sides:

$$\frac{17}{11} \times \frac{x}{m} \; ? \; \frac{11}{17} \times \frac{x}{m}$$

Now, because you know that x/m is positive, you can simply divide both sides by it:

$$\frac{17}{11} \; ? \; \frac{11}{17}$$

You're left with two fractions, one of which is greater than 1, and one of which is less than 1. The answer is (A).

47. **(A).** To determine which fraction is largest, cancel common terms from all five fractions until the remaining values are small enough for the calculator. Note that every choice has at least one 5 on the bottom, so cancel 5^1 from all of the denominators.

Note also that every fraction has a power of 2 on the bottom, so convert 16^2, 32, 512, 4^6, and 2^{11} to powers of 2. Since $16 = 2^4$, $32 = 2^5$, $512 = 2^9$, and $4^6 = (2^2)^6 = 2^{12}$, so you now have:

(A) $\frac{7}{(2^4)(5)}$

(B) $\frac{5}{(2^5)(5^3)}$

(D) $\frac{30}{(2^9)(5^2)}$

(C) $\frac{5}{(2^{12})}$

(E) $\frac{4}{(2^{11})(5)}$

Since every choice has at least 2^4 on the bottom, cancel 2^4 from all 5 choices:

(A) $\frac{7}{5}$

(B) $\frac{5}{(2)(5^3)}$

(C) $\frac{30}{(2^5)(5^2)}$

(D) $\frac{5}{2^8}$

(E) $\frac{4}{(2^7)(5)}$

Note that the numerators also have some powers of 2 and 5 that will cancel out with the bottoms of each of the fractions. In choice (C), 30 = (2)(3)(5):

(A) $\frac{7}{5}$

(B) $\frac{1}{(2)(5^2)}$

(C) $\dfrac{3}{(2^4)(5)}$
(D) $\dfrac{5}{2^8}$
(E) $\dfrac{1}{(2^5)(5)}$

These values are now small enough for the calculator. Note that the GRE calculator does not have an exponent button —to get 2^8, you must multiply 2 by itself 8 times. This is why you should memorize powers of 2 up to 2^{10}, and powers of 3, 4, and 5 up to about the 4th power.

(A) 1.4
(B) 0.02
(C) 0.0125
(D) 0.01953125
(E) 0.00625

The answer is (A).

48. **(D).** Without knowing the signs of any of the variables, you cannot assume that m is larger. While it certainly *could* be (for instance, $m = 4$, $n = 2$, and $p = 1$), if p is negative, the reverse will be true (for instance, $m = 2$, $n = 4$, and $p = -1$).

49. **(A).** Since this expression is complicated, deal with the denominator first. To add 5 to the fraction, make a common denominator ($y - 2x$):

$$\frac{3y}{y-2x}+5=\frac{3y}{y-2x}+5\frac{y-2x}{y-2x}=\frac{3y}{y-2x}+\frac{5(y-2x)}{y-2x}$$
$$=\frac{3y}{y-2x}+\frac{5y-10x}{y-2x}=\frac{8y-10x}{y-2x}$$

Now put $\dfrac{8y-10x}{y-2x}$ back into the original expression. You can flip and multiply:

$$\frac{\dfrac{5x-4y}{2x-y}}{\dfrac{3y}{y-2x}+5}=\frac{\dfrac{5x-4y}{2x-y}}{\dfrac{8y-10x}{y-2x}}=\left(\frac{5x-4y}{2x-y}\right)\left(\frac{y-2x}{8y-10x}\right)$$

By looking at the answer choices, you can tell that this expression must be reducible to a number. How? Look at $y - 2x$ and $2x - y$. They are actually negatives of each other: $2x - y = -(y - 2x)$. So you can then cancel the whole expression $y - 2x$ from top and bottom:

$$\left(\frac{5x-4y}{2x-y}\right)\left(\frac{y-2x}{8y-10x}\right)=\left(\frac{5x-4y}{-(y-2x)}\right)\left(\frac{y-2x}{8y-10x}\right)=\left(\frac{5x-4y}{-1}\right)\left(\frac{1}{8y-10x}\right)$$

You can do the same thing with the remaining terms: $8y - 10x = -2(5x - 4y)$.

$$\left(\frac{5x-4y}{-1}\right)\left(\frac{1}{8y-10x}\right)=\left(\frac{5x-4y}{-1}\right)\left(\frac{1}{-2(5x-4y)}\right)=\left(\frac{1}{-1}\right)\left(\frac{1}{-2}\right)=\frac{1}{2}$$

50. **(E).** To divide fractions, multiply by the reciprocal.

$$\frac{39^2}{2^4}\div\frac{13^3}{4^2}=\frac{39^2}{2^4}\times\frac{4^2}{13^3}$$

Now break down to primes and cancel common factors.

$$\frac{39^2}{2^4}\times\frac{4^2}{13^3}=\frac{(3\times13)^2\times(2^2)^2}{2^4\times13^3}=\frac{3^2\times13^2\times2^4}{2^4\times13^3}=\frac{3^2}{13}=\frac{9}{13}$$

51. **(E).** You can take the fourth root by taking the square root *twice*. So you should expect the fourth root of an integer greater than 1 to be much smaller than the number itself (for instance, the fourth root of 16 is 2, and the fourth root of 625 is 5).

One way to approach this problem is this: what integer n would give you *exactly* 3 as its fourth root?

$$\sqrt[4]{n}=3$$

Raise each side to the fourth power:

$$n = 3^4 = 81$$

Only the interval in choice (E) contains 81.

Alternatively, take the fourth root (by using the square root button twice) of the values in the answer choices. The fourth root of 40 is 2.5148… which rounds to 3. Likewise, if you take the fourth root of 150, you get 3.4996… which also rounds to 3.

Chapter 12
of

Percents

In This Chapter…

Percents

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

Quantity A	**Quantity B**
50 as a percent of 30	The percent increase from 30 to 80

2. If Ken's salary were 20% higher, it would be 20% less than Lorena's. If Lorena's salary is $60,000, what is Ken's salary?

(A) $36,000
(B) $40,000
(C) $42,500
(D) $42,850
(E) $45,000

3. On a certain morning, Stock X is worth $$x$, where x is positive.

Quantity A	**Quantity B**
The price of Stock X if it decreases in value	The price of Stock X if it decreases in value

by 12%, then increases by 18%. by 13%, then increases by 19%.

4. Greta's salary is x thousand dollars per year, and she receives a y% raise. Annika's salary is y thousand dollars per year, and she receives an x% raise. x and y are positive integers.

Quantity A	**Quantity B**
The dollar amount of Greta's raise	The dollar amount of Annika's raise

5. Roselba's income exceeds twice Jane's income and both pay the same percentage of income in transportation fees.

Quantity A	**Quantity B**
The amount Jane pays in transportation fees	Half the amount Roselba pays in transportation fees

6. 250% of x is increased by 250% to become 350. What is the value of x?

7. An item's price is discounted by 16%. Subsequently, the discounted price is increased by 16%.

Quantity A	**Quantity B**
The original price	The price after the discount and increase

8. 12 is 5 percent of what number?

9. 7 percent of 9 is what percent of 7?

%

10. What percent of 13 is 20 percent of 195?

%

11. If 14 is added to 56, by what percent does the original number increase?

% increase

12. 25 percent of 30 is 75 percent of what number?

13. What is the percent increase from 50 to 60?

%

14. If x is reduced by 30%, the resulting number is 63. The value of x =

15. 75 reduced by x% is 54. The value of x =

%

16. What is 230% of 15% of 400?

17. 45% of 80 is x% more than 24. The value of x =

%

18. 10 percent of 30 percent of what number is 200 percent of 6?

19. If $y \neq 0$, what percent of y percent of 50 is 40 percent of y?

%

20. If $a \neq 0$, 200 percent of 4 percent of a is what percent of $a/2$?

☐ %

21. If positive integer *m* is increased by 20%, decreased by 25%, and then increased by 60%, the resulting number is what percent of *m*?

☐ %

22.

Quantity A	Quantity B
The price of an item after five consecutive 10% discounts are applied	50% of the price of the item

23. Raymond borrowed \$450 at 0% interest. If he pays back 0.5% of the total amount every 7 days, beginning exactly 7 days after the loan was disbursed, and has thus far paid back \$18, with the most recent payment made today, how many days ago did he borrow the money?

(A) 6
(B) 8
(C) 25
(D) 42
(E) 56

24. An investment loses half its value in the morning, and then increases in value by 50% that afternoon; no other changes occur to the value of the investment. (Assume the investment's original value was a positive number.)

Quantity A	Quantity B
The value of the investment before the day's changes	The value of the investment after the day's changes

25. A house valued at \$200,000 two years ago lost 40% of its value in the first year and a further 20% of that reduced value during the second year.

Quantity A	Quantity B
The current value of the house	\$100,000

26. After one year at her job, Sharon received a 50% increase on her \$1,000 weekly salary. Bob, who originally made \$1,800 a week, took a 20% percent decrease in salary.

Quantity A	Quantity B
Sharon's new salary	Bob's new salary

27. 1% of 200% of 360 is what percent of 0.1% of 60?

$\boxed{}$ %

28. If a number is increased by 20%, decreased by 15%, and increased by 7%, the overall percent change is closest to a:

(A) 2% decrease
(B) 2% increase
(C) 9% increase
(D) 12% increase
(E) 14% increase

29. If Mary has half as many cents as Nora has dollars, then Nora has what percent more cents than Mary does? (100 cents = 1 dollar)

(A) 100%
(B) 200%
(C) 1,990%
(D) 19,900%
(E) 20,000%

30. The number that is 50% greater than 60 is what percent less than the number that is 20% less than 150?

(A) 5%
(B) 10%
(C) 15%
(D) 20%
(E) 25%

31. A cockroach population doubles every three days. If there were *c* cockroaches on June 1st, what was the percent increase in the population on July 1st? (June has 30 days.)

(A) 900%
(B) 1,000%
(C) 9,999%
(D) 102,300%
(E) 102,400%

32. A computer that was discounted by 15% sold for $612. What was the price of the computer before the discount?

(A) $108.00
(B) $520.20
(C) $703.80
(D) $720.00
(E) $744.00

33.

In April, the price of fuel increased by 40%.
In May, the price rose by another 30%.

Quantity A	**Quantity B**

The price increase in April	The price increase in May

34. If 35% of the acreage of a national forest was destroyed in a wildfire, and the remainder regenerates at a rate of 10% a year, after how many years, assuming no further losses, will the forest exceed its original acreage?

(A) 10
(B) 8
(C) 5
(D) 4
(E) 3

35. Aloysius spends 50% of his income on rent, utilities, and insurance, and 20% on food. If he spends 30% of the remainder on video games and has no other expenditures, what percent of his income is left after all of the expenditures?

(A) 30%
(B) 21%
(C) 20%
(D) 9%
(E) 0%

36.

An item costs x dollars where x is a positive integer.

Quantity A	**Quantity B**
The price of the item after a 10% discount and then a 7% tax are applied	The price of the item after a 7% tax and then a 10% discount are applied

37.

An item costs x dollars where x is a positive integer.

Quantity A	**Quantity B**
The price of the item after a 10% discount and then a \$10-off coupon are applied	The price of the item after a \$10-off coupon and then a 10% discount are applied

38. Adalstein bought a bag of 15 magic beans for \$60. One-third of the beans cost \$2 each and the rest cost \$5 each. If there was a hole in the bag and all of the more expensive beans fell out, the lost beans represented approximately what percentage of the money Adalstein paid for all of them?

(A) 7%
(B) 13%
(C) 67%
(D) 83%
(E) 88%

39. Coffee formerly accounted for 5% of a family's food budget, while fresh fruits and vegetables accounted for 20%, and meat and dairy accounted for 30%. The family spent the rest of the food budget on fast food and desserts. If the price of coffee doubled and the family reduced the fruit and vegetable share to meet that expense and spend the same overall, the ratio of their new fresh fruit and vegetable expenditures to their fast food and

dessert expenditures equals which of the following?

(A) 3/20
(B) 1/5
(C) 1/3
(D) 3/8
(E) 3/5

40. J. R. weighed 200 pounds before starting a diet on January 1st. He lost 15% of his original weight. Then he went off the diet and gained 35 pounds by December 31st of the same year. From the beginning to the end of the year, J.R.'s weight changed by what percent?

(A) +5%
(B) +2.5%
(C) 0%
(D) -2.5%
(E) -5%

41. In 1970, Company X had 2000 employees, 15% of whom were women. 10% of these women were executives. In 2012, the company had 12,000 employees, 45% of whom were women. If 40% of those women were executives, what is the percent increase in the number of women executives from 1970 to 2012?

$\boxed{}$ %

42. 75% of all the boys and 48% of all the girls of Smith High School take civics. If there are 80% as many boys as girls, what percent of all the students take civics?

$\boxed{}$ %

43. Airline A and Airline B both charge $400 for a certain flight. Airline A then reduces its price by 25%. Airline B reduces its price by 55% but adds $150 in fees. Then, Airline A raises its reduced price by 10%.

Quantity A	**Quantity B**
The final price of the flight at Airline A	The final price of the flight at Airline B

44. Jake used to spend $10 for lunch at a restaurant in Chinatown. The tea served with lunch was free, and Jake left a $2 tip. However, the restaurant raised its lunch price by 20% and began to charge $1 for tea. Jake continued to order the same lunch and tea, and increased his tip so that he was still tipping the same percentage of his total bill.

Quantity A	**Quantity B**
Jake's new lunch expenditure	$15.40

45. Half of a shipment of toy trucks was left at Store W, 25% at Store X, 20% at Store Y, and the remaining 20 at Store Z.

Quantity A	**Quantity B**

The positive difference difference between the trucks left at Stores X and Y 20

46.

p is 75% of q and $p = 2r$.

Quantity A	Quantity B
$0.375q$	r

47. In a class of 40 students, exactly 90% had a lower GPA than Tom. For the new term, 60 new students join Tom's class. If Tom's GPA was higher than those of 80% of the new arrivals, what percent of the combined class now has a higher GPA than Tom?

(A) 86%
(B) 85%
(C) 16%
(D) 15%
(E) 14%

48.

$0 < x < 100$

Quantity A	Quantity B
x% of 0.5% of 40,000	0.05% of 2,000% of $40x$

Profit Per Student (in Dollars) at Dan's Dojo, 2000-2004

2000	60
2001	80
2002	80
2003	100
2004	162

49. If the percent increase from 2004 to 2005 (not shown) is the same as the percent increase from 2000 to 2001, what is the profit per student for 2005?

50. If x is 0.5% of y, then y is what percent of x?

(A) 199%
(B) 200%
(C) 2,000%
(D) 19,900%

(E) 20,000%

51. Bill pays 20% tax on his gross salary of $5,000 each month and spends 25% of the remaining amount on rent.

Quantity A	Quantity B
Bill's tax	Bill's rent

52. Four people shared a dinner with an $80 bill and tipped the waiter 15 percent of this amount. If each person contributed equally to paying the bill and tip, how much did each person pay?

(A) $20.00
(B) $23.00
(C) $23.75
(D) $24.00
(E) $25.00

53. The price of a certain stock rose by 25 percent and then decreased by y percent. After the decrease, the stock was back to its original price.

Quantity A	Quantity B
y%	25%

54. A chemist is mixing a solution of acetone and water. She currently has 30 ounces mixed, 10 of which are acetone. How many ounces of acetone should she add to her current mixture to attain a 50/50 mixture of acetone and water if no additional water is added?

(A) 2.5
(B) 5
(C) 10
(D) 15
(E) 20

55. By the end of July, a certain baseball team had played 80% of the total games to be played that season and had won 50% of those games. Of the remaining games for the season, the team won 60%.

Quantity A	Quantity B
Percentage of total games won for the season	52%

56.

Quantity A	Quantity B
0.002	0.4 percent of 4 percent of 1.25

57. Jane has a 40-ounce mixture of apple juice and seltzer that is 30% apple juice. If she pours 10 more ounces of apple juice into the mixture, what percent of the mixture will be seltzer?

(A) 33%
(B) 44%
(C) 50%
(D) 56%

(E) 67%

58. Half of the shirts in a closet are white and 30% of the remaining shirts are gray.

Quantity A	Quantity B
The percent of the shirts in the closet that are not white or gray.	20%

59. If 80 percent of the children in a certain room are more than ten years old and 20 percent of these children play an organized sport, what percent of children in the room are over ten but do not play an organized sport?

(A) 16
(B) 20
(C) 40
(D) 60
(E) 64

60. The length and width of a rectangular box are increased by 10% each.

Quantity A	Quantity B
10%	The percent increase in the volume of the box

61. The radius of a circle is doubled.

Quantity A	Quantity B
The percentage that the area of the circle has been increased	400%

62. If 35% of x equals 140, what is 20% of x?

(A) 9.8
(B) 39.2
(C) 80
(D) 320
(E) 400

63. A population of a colony of bacteria increases by 20 percent every 3 minutes. If at 9:00am the colony had a population of 144,000, what was the population of the colony at 8:54am?

(A) 100,000
(B) 112,000
(C) 120,000
(D) 121,000
(E) 136,000

64. Jane scored 15% higher on her second test than she did on her first test. Jane's score on her third test was a 25% decrease from the score on her second test. If Jane got a 69 on her third test, what was the score on her first test?

(A) 69
(B) 70
(C) 75
(D) 80

(E) 92

65. The price of an item is greater than \$90 and less than \$150

Quantity A	Quantity B
The price of the item after a 10% discount and then a \$20 off discount	The price of the item after a \$10 off discount and then a 20% off discount

66. Two classes participate in a contest stacking blocks. Each person in class 1 stacks 80 percent as many blocks as each person in class 2 and there are 25 percent more people in class 1 than class 2.

Quantity A	Quantity B
The percent of the total blocks that class 1 stacks	The percent of the total blocks that class 2 stacks

67. x is y percent of z.

Quantity A	Quantity B
The percent that z is of x.	$\frac{y}{10{,}000}$

68. The number that is 20 percent less than 300 is what percent greater than 180?

(A) 25
(B) $33^1/_3$
(C) 50
(D) $66^2/_3$
(E) 75

69. A tank that was 40% full of oil is emptied into a 20-gallon bucket. If the oil fills 35% of the bucket's volume, then what is the total capacity of the tank, in gallons?

(A) 8.75
(B) 15
(C) 16
(D) 17.5
(E) 19

70. A full glass of juice is a mixture of 20% grape juice and 80% apple juice. The contents of the glass are poured into a pitcher that is 200 percent larger than the glass. The remainder of the pitcher is filled with 16 ounces of water. What was the original volume of grape juice in the mixture?

(A) 1.6 ounces
(B) 3.2 ounces
(C) 4.8 ounces
(D) 6.4 ounces
(E) 8 ounces

71. If 150 is increased by 60% and then decreased by y percent the result is 192. What is y?

(A) 20
(B) 28
(C) 32
(D) 72
(E) 80

72. A number x is 150% greater than 200. What percent greater is x than 50% of 500?

(A) 0
(B) 20
(C) 50
(D) 100
(E) 200

73. Mixture A weighs 18 grams and is 50% aluminum. Mixture B weighs 32 grams and is 37.5% aluminum. The two mixtures are combined.

Quantity A	Quantity B
The percent of the resultant combination that is not aluminum	58%

74. A stockbroker has made a profit on 80% of his 40 trades this year.

Quantity A	Quantity B
23	The maximum number of consecutive trades that the stockbroker can lose before his profitable trades drop below 50% for the year

75. In 2011, each member of a committee voted for one of two possible candidates for president. Candidate A received 40% of the vote and Candidate B received the remainder. In 2012, the same two candidates ran for president. Candidate A received 3 more votes than the previous year, a 5% increase in his total number of votes. How many votes did Candidate B receive in 2011?

(A) 40
(B) 60
(C) 75
(D) 90
(E) 150

76. A 16 ounce jar of birdseed is 10% sesame. How much sesame must be added to make the jar 20% sesame?

(A) 1 ounce
(B) 1.6 ounce
(C) 2 ounce
(D) 2.4 ounce
(E) 4 ounce

77. a, b, and c are positive.

Quantity A	Quantity B
$(a + b)$% of c	c% of $(a + b)$

78. A certain boat sales lot sells both sailboats and boats that are not sailboats. 25% of the boats are used sailboats. Of

non-sailboats, 3/5 are new. If 33% of all boats are used, approximately what percentage of the sailboats are new?

(A) 31%
(B) 33%
(C) 67%
(D) 68%
(E) 69%

Conference Ticket Advance Discounts	
5-29 days in advance	15%
30-59 days in advance	30%
60-89 days in advance	40%

79. Helen bought a ticket for \$252. If she'd bought it 1 day later, she would have paid \$306. How many days in advance did she buy her ticket?

(A) 5
(B) 30
(C) 59
(D) 60
(E) 89

Percents Answers

1. **(C).** 50 as a percent of 30 is $\frac{50}{30} \times 100 = 166.\overline{6}\%$. (Note: it's incorrect to calculate "50 percent of 30," which is 15. This asked for 50 *as a percent* of 30, which is equivalent to asking, "What percent of 30 is 50?")

To find the percent increase from 30 to 80, use the percent change formula:

Percent Change = $\frac{Difference}{Original} \times 100$

Percent Change = $\frac{80-30}{30} \times 100 = 166.\overline{6}\%$

The two quantities are the same. Note that if you set up both quantities first, you will be able to see that the two quantities are equal without solving (because both equal $\frac{50}{30} \times 100$).

2. **(B).** The question asks for Ken's salary, so set a variable: call Ken's salary k. The problem indicates that Lorena's salary is 60,000. Now, translate the equation in the first sentence.

"If Ken's salary were 20% higher" can be translated as Ken's salary + 20% of Ken's salary, or $k + 0.2k$. "It would be 20% less than Lorena's" can be translated as Lorena's salary - 20% of Lorena's salary, or 60,000 - (0.2)(60,000). The last part is equivalent to (0.8)(60,000); use whichever form you prefer.

$$1.2k = 0.8(60{,}000)$$
$$1.2k = 48{,}000$$
$$k = 40{,}000$$

Ken's salary is $40,000.

3. **(A).** The problem never indicates an actual value for the stock, so pick your own value. Let the original price of the stock be $100. For Quantity A, if the price of the stock decreases by 12%, then the new price is 88% of the original, or $88. If that price then increases by 18%, then the final price of the stock is 88(1.18) = $103.84.

For Quantity B, if the $100 price decreases by 13%, the new price is 87% of $100, or $87. If that price then increases by 19%, then the final price of the stock is 87(1.19) = $103.53.

Quantity A is slightly larger.

4. **(C).** Because the problem never indicates real values, you can pick your own smart numbers. If $x = 100$ and $y = 50$, then:

Greta’s salary is $100,000 and she will receive a 50% raise. Greta’s raise, therefore, is $50,000.

Annika’s salary is $50,000 and she will receive a 100% raise. Annika’s raise, therefore, is $50,000.

The two quantities are the same. This will work for any numbers you choose for x and y, because x percent of y = y percent of x. This fact holds for any two numbers — just as 50% of 100 = 100% of 50, it is also true that 1% of 2,000 = 2,000% of 1, or a% of b = b% of a.

5. **(B).** Roselba’s income is more than twice as great as Jane’s income. If both pay the same percentage of income in transportation fees, that means Roselba must pay *more* than twice as much as Jane in transportation fees. Quantity B is greater.

Alternatively, you can use smart numbers. Call Jane’s income 100. Roselba’s income, then, is greater than 200. If both pay 10% in transportation fees, then Jane pays $10 and Roselba pays more than $20. Half of Roselba’s amount equals more than $10.

6. **40.** To begin, 250% of x is equal to $\frac{250}{100}x$, or $2.5x$

However, when you *increase* a number by 250%, you are NOT simply multiplying it by 2.5. Rather, you are finding 250% (or 2.5 times) that number and then *adding it back to the original.* (For example, 100% OF 15 equals 15, but INCREASING 15 by 100% means doubling it: 15 + 15 = 30.)

Thus, to increase a number by 250%, you are adding 100% to 250% for a total of 350%. Therefore, multiply by 3.5 (NOT 2.5).

$2.5x(3.5) = 350$
$8.75x = 350$
$x = 40$

40 is the final answer. If you are unsure of the logic behind these calculations, check your answer:

250% of 40 equals 2.5(40), or 100.

Next, 100 is INCREASED by 250%, so take 250% of 100, or 250, and add it back to the original 100: 100 + 250 = 350. The problem does indicate that the final answer is 350, so you have just proven that x does indeed equal 40.

7. **(A).** The problem doesn’t indicate a specific value anywhere, so you can choose your own smart number. Because this is a percent problem, call the original price $100. Quantity A is equal to $100.

Decreasing a value by 16% is the same as taking 100 - 16 = 84% of the number: so (0.84)(100) = $84. To increase the value by 16%, take 116% of the number, or multiply by 1.16: (1.16)(84) = $97.44.

Quantity A is larger.

8. **240.** Translate the question as $12 = 0.05x$ and solve on the calculator. $x = 240$. Alternatively, translate the question

as $12=\frac{5}{100}x$ and solve on paper:

$$12=\frac{1}{20}x$$
$$(12)(20)=x$$
$$x=240$$

9. **9**. Always translate the phrase "what percent" as $\frac{x}{100}$. Translate the question as:

$$0.07(9)=\frac{x}{100}(7)$$
$$0.63=\frac{7x}{100}$$
$$63=7x$$
$$9=x$$

Incidentally, the pattern "x percent of y = y percent of x" always holds true! Here, 7% of 9 = 9% of 7, but it is also true that 2% of 57 = 57% of 2, etc. This works with any two numbers. If you notice this, then you can "fill in the blank" on the answer immediately: "what percent" must be 9 percent.

Finally, notice that the answer is 9 and not 0.09 or 9%. The question asks "what percent," so the percent is already incorporated into the sentence — the "what" by itself represents only the number itself, 9.

10. **300.** Always translate the phrase "what percent" as $\frac{x}{100}$. Translate the question as:

$$\frac{x}{100}(13)=0.2(195)$$
$$\frac{13x}{100}=39$$
$$13x=3{,}900$$
$$x=300$$

Alternatively, take 20 percent of 195 (0.2 × 195 = 39) and rephrase the question: "What percent of 13 is 39?" Since 39 is three times as big as 13, the answer is 300%.

11. **25%.** In this problem, the change is 14 and the original number is 56. Use the percent change formula:

Percent Change $= \dfrac{Difference}{Original} \times 100$

14/56 = 1/4 or 0.25. Multiply by 100 to get 25%.

Notice that you never need to find the new number (14 + 56 = 70), as you have only been asked about the percent increase, not the new value.
12. **10.** Translate the question as 0.25(30) = 0.75x and solve on the calculator. x = 10.

Alternatively, write the percentages in simplified fraction form and solve on paper:

$$\frac{1}{4}(30) = \frac{3}{4}x$$

$$30 = 3x$$

$$x = 10$$

13. **20% increase.** Use the percent change formula:

Percent Change $= \dfrac{Difference}{Original} \times 100$

The difference is 60 - 50 = 10 and the original is 50. 10/50 = 1/5 or 0.2. Multiply by 100 to get 20%.

14. **90.** Because 30% less than x is the same as 70% of x, you can translate as follows: 0.7x = 63. Use the calculator to get x = 90. Alternatively, solve on paper:

$$\frac{7}{10}x = 63$$

$$x = (63)\left(\frac{10}{7}\right)$$

$$x = (9)(10)$$

$$x = 90$$

15. **28.** Reword the given information as “75 minus x percent of 75 is 54.”

$$75-\frac{75}{100}x=54$$

$$75-54=\frac{3}{4}x$$

$$21\left(\frac{4}{3}\right)=x$$

$$28=x$$

16. **138.** Translate into decimals (for the percentages, move the decimal two places to the left) and use the calculator to solve:

$x = 2.3(0.15)(400)$
$x = 138$

Alternatively, translate into fractions and solve on paper:

$$\frac{230}{100}\times\frac{15}{100}\times 400=$$

$$\frac{23}{10}\times\frac{15}{1}\times 4=$$

$$\frac{23}{2}\times\frac{3}{1}\times 4=$$

$$\frac{23}{1}\times\frac{3}{1}\times 2=138$$

17. **50.** The left-hand side of the equation is given: 45% of 80 is (0.45)(80) = 36. The problem then becomes: “36 is x% more than 24.” From this step, there are two possible approaches.

One approach is to translate the equation and solve:

$$36 = 24 + \frac{x}{100}(24) =$$

$$12 = \frac{24x}{100}$$

$$12\left(\frac{100}{24}\right) = x$$

$$50 = x$$

Alternatively, the increase (36 - 24) is 12, so rephrase the statement as "12 is x% of 24." If you recognize that 12 is half of 24, then you know x must be 50%. You can also translate and solve:

$$12 = \frac{x}{100}(24)$$

$$12\left(\frac{100}{24}\right) = x$$

$$50 = x$$

18. **400.** Translate as decimals and use the calculator to solve, keeping in mind that taking 200% of a number is the same as doubling it, or multiplying by 2:

$0.10(0.30)x = 2(6)$
$0.03x = 12$
$x = 400$

Alternatively, translate as fractions and solve on paper:

$$\left(\frac{1}{10}\right)\left(\frac{3}{10}\right)x = 2(6)$$

$$x = 12\left(\frac{100}{3}\right)$$

$$x = 400$$

19. **80.** The question already contains a variable (y). Use another variable to represent the desired value. Represent "what" with the variable x, and isolate x to solve. Notice that by the end, the y variables cancel out.

$$\left(\frac{x}{100}\right)\left(\frac{y}{100}\right)50 = \left(\frac{40}{100}\right)y$$

At this point, there are *many* options for simplifying, but do simplify before you start multiplying anything. Here is one way to simplify:

$$\left(\frac{x}{100}\right)\left(\frac{y}{2}\right)=\left(\frac{2}{5}\right)y$$

$$x=\frac{2y(100)(2)}{5y}$$

$$x=80$$

20. **16.** 200% of 4% is the same as 2 × 4% (note that 200% equals the plain number 2), or 8%. Rephrase the question as "8% of *a* is what percent of *a*/2?" Without translating to an equation, you can simplify by multiplying both sides of the "equation" by 2 (remember that "is" means "equals"):

8% of *a* is what percent of *a*/2?

16% of *a* is what percent of *a*?

The answer is 16.

Alternatively, translate the words into math; this will take longer, but this method does work if you're not comfortable "thinking through" the math as shown above:

$$\left(\frac{200}{100}\right)\left(\frac{4}{100}\right)a=\left(\frac{x}{100}\right)\left(\frac{a}{2}\right)$$

$$\left(\frac{2}{25}\right)a=\frac{xa}{200}$$

$$\left(\frac{2}{25}\right)a\left(\frac{200}{a}\right)=x$$

$$16=x$$

21. **144.** If *m* is increased by 20%, decreased by 25%, and then increased by 60%, it is being multiplied by 1.2, then 0.75, then 1.6. Since (1.2)(0.75)(1.6) = 1.44, doing these manipulations is the same as increasing by 44%, or taking 144% of a number (this is true regardless of the value of *m*).

Alternatively, pick a real value for *m*. Because this is a percent problem, 100 is a good number to pick. First, 100 is increased by 20%: (100)(1.2) = 120. Next, 120 is decreased by 25%, which is the same as multiplying by 75%: (120)(0.75) = 90. Finally, 90 is increased by 60%: (90)(1.6) = 144. The new number is 144 and the starting number was 100, so the new number is 144/100% of the original number, or 144%.

22. **(A).** Say the item costs $100. After the first 10% discount, the item costs $90. After the second, the item costs

$81 (the new discount is only $9, or 10% of 90). After the third discount, the item costs $81 - $8.10 = $72.90. What is the trend here? The cost goes down with each discount, yes, but the discount itself also gets smaller each time; it is only a $10 discount the very first time. The total of the five discounts, then, will be something less than $50.

If the item costs $100 to start, then the value for Quantity B will be $50, or a total discount of $50. This is larger than the total discount described for Quantity A.

Finally, make sure you answer (A) for the higher price — don't accidentally pick (B) for the "better deal"!

23. **(E).** 1% of $450 is $4.50, so 0.5% is $2.25. That's the amount Raymond pays back every week. Because he has paid back $18 in total, divide 18 by 2.25 to determine the total number of payments: $18/$2.25 = 8.

So Raymond has made 8 payments, once every 7 days. The payments themselves spread over only a 7-week period (in the same way that 2 payments spread over only a 1-week period). Raymond waited 1 week to begin repayment, however, so a total of 8 weeks, or 56 days, have passed since he borrowed the money. The answer is (E).

24. **(A).** The investment's value first decreases by 50%, then increases by 50%. If the investment begin at x dollars, $x(0.50)(1.5) = 0.75x$. The investment ends at 75% of its original value. The value after the changes is lower than the value before the changes.

Alternatively, choose a smart number to test. If $x = \$100$, then the investment first decreased to $50, and then increased from $50 to $75. If Quantity A = 100, then Quantity B = 75.

Finally, you could solve this question with logic. The 50% decrease is taken as a percentage of the original number. The 50% increase, however, is taken as a percentage of the new, *smaller* number. The increase, therefore, must be smaller than the decrease.

25. **(B).** To reduce $200,000 by 40%, multiply by 0.6 (reducing by 40% is the same as keeping 60%): 200,000(0.6) = 120,000.

To reduce 120,000 by 20%, multiply by 0.8 (reducing by 20% is the same as keeping 80%): 120,000(0.8) = 96,000. The answer is (B).

26. **(A).** Sharon's 50% increase raised her salary by half, to 1,000(1.5) = $1,500. A 20% decrease from $1,800 reduces Bob's salary to (1,800)(0.8) = $1,440.

27. **12,000%.** Translate the statement into an equation. Since one of the percentages is a variable, fractions are preferable to decimals:

$$\frac{1}{100}\times\frac{200}{100}\times 360=\frac{x}{100}\times\frac{0.1}{100}\times 60$$

Because 100 appears twice on the bottom of both sides of the equation, multiply each side of the equation by 10,000 (or 100 twice) to cancel the 100's out:

$$\frac{1}{\cancel{100}}\times\frac{200}{\cancel{100}}\times 360=\frac{x}{\cancel{100}}\times\frac{0.1}{\cancel{100}}\times 60$$

$$200\times 360=x(0.1)(60)$$

$$\frac{200\times 360}{60}=x\left(\frac{1}{10}\right)$$

$$200\times 6\times 10=x$$

$$x=12{,}000$$

The answer is 12,000%. (The phrase "what percent" translates into math as $\frac{x}{100}$. Additionally, $\frac{12{,}000}{100}$, is the same thing as 12,000%, just as $\frac{50}{100}$ is equal to 50%. While 12,000% may seem quite large, it is correct.)

Alternatively, you can use decimals, though you still have to write "what percent" as a fraction; also, use the calculator to solve:

$$(0.01)(2)(360)=\frac{x}{100}(0.001)(60)$$

$$7.2=\frac{x}{100}(0.06)$$

$$120=\frac{x}{100}$$

$$12{,}000=x$$

28. **(C).** Increasing by 20% is equivalent to multiplying by 1.2, decreasing by 15% is equivalent to multiplying by 1 - 0.15, or 0.85, and increasing by 7% is equivalent to multiplying by 1.07:

$$x(1.2)(0.85)(1.07)=x(1.0914)$$

Overall, these three changes are equivalent to multiplying by 1.0914, or increasing by 9.14%. Choice (C) is closest.

You *cannot* simply add 20 + 7 and subtract 15 to get a 12% increase. These percents cannot be added and subtracted because 20% is a percent of the original number, while 15% is a percent of the new, increased number, and so on.

Alternatively, you can also use a smart number. If the original number is 100, first increase 100 by 20%: 100 + 20 =120. Next, decrease 120 by 15%: 120 - 18 = 102 or 120(0.85) = 102. Finally, increase 102 by 7%. Note: the question indicates that you're trying to find the "closest" answer: a quick glance at the answers shows that they are relatively spread apart. It will be sufficient, then, to approximate the last step: 102 is almost 100 and 7% of 100 is 7.

The increase, then, is 102 + 7 = 109.

Compared to the original figure, 100, you have increased the number by about 9, or approximately 9%.

29. **(D).** You can use smart numbers to solve this problem. If Mary has half as many cents as Nora has dollars, then, as an example, if Nora had \$10, Mary would have 5 cents. Nora's \$10 equals 1,000 cents. To determine what *percent more* cents Nora has, use the percent change formula:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{1{,}000 - 5}{5} \times 100 = 19{,}900\%$$

Any example you use where "Mary has half as many cents as Nora has dollars" will yield the same result. Note that you must use the percent change formula — a percent *more* (or percent increase) is not the same as a percent *of* something.

To do the problem algebraically (which is *much* harder than using an example, as above), use M for Mary's cents and N for Nora's cents. Divide N by 100 in order to convert from cents to dollars: $\frac{N}{100}$ and set up an equation to reflect that Mary has half as many cents as Nora has dollars:

$$M = \frac{1}{2}\left(\frac{N}{100}\right)$$

$$M = \frac{N}{200}$$

$$200M = N$$

Therefore, Nora has 200 times as many cents. 200 times AS MANY is 199 times MORE. To convert 199 times MORE to a percent, add two zeros to get 19,900%.

30. **(E).** Rather than trying to write out the whole statement as math, note that "the number that is 50% greater than 60" can be calculated: 1.5(60) = 90. Similarly, "the number that is 20% less than 150" is 0.8(150) = 120. The question can be rephrased as "90 is what percent less than 120?" Use the percent change formula. Since you want a "percent *less,*" the "original" number is 120:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100 = \frac{30}{120} \times 100 = 25\%$$

31. **(D).** The percent increase is the difference between the amounts divided by the original, converted to a percentage. If the population doubles, mathematically the increase can be written as a power of 2. In the 30-day interval, if the original population is 1, it will double to 2 after three days — so, 21 represents the population after the first increase,

the second increase would then be 2^2 and so on. Since there are ten increases, the final population would be 2^{10} or 1,024. Therefore, the difference, 1024 - 1, is 1023. Use the percent change formula to calculate percent increase:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100 = \frac{1023}{1} \times 100 = 102{,}300\%$$

Note that the new number *is* 102,400% of the original, but that was not the question asked — the percent *increase* is 102,300%.

32. **(D).** Call the original price x. That price is discounted by 15% to get 612:

$0.85x = \$612$
$x = \$720$

Do not attempt to add 15% of \$612 to \$612. The 15% figure is a percentage of the unknown original number, not of \$612.

33. **(B).** Call the original price x. In April, the total price was $1.4x$. The price *increase* was $1.4x - 1x = 0.4x$.

In May, the price increased an additional 30% over April's price of $1.4x$. Thus, the total price was $(1.3)(1.4)x$, or $1.82x$. The price increase was $1.82x - 1.4x = 0.42x$.

Since $0.42x$ (42% of x) is larger than $0.4x$ (40% of x), Quantity B is larger.

Alternatively, use smart numbers. If the original price is \$100, April's increase would result in a price of \$140 and May's increase would be (1.3)(140) = \$182. Thus, April's increase was \$40 and May's increase was \$42. May's increase will be larger no matter what number you pick as the starting price (it is reasonable in GRE problems to assume that a "price" must be a positive number.)

34. **(C).** Picking numbers is a good strategy here. If the forest had 100 acres:

After the fire:	65 acres
After 1 year:	(65)(1.1) = 71.5 acres
After 2 years:	(71.5)(1.1) = 78.65 acres
After 3 years:	(78.65)(1.1) = 86.515 acres
After 4 years:	(86.515)(1.1) = 95.1665 acres
After 5 years:	(95.1665)(1.1) = 104.68315 acres

Note that each solution is multiplied by 1.1, so you can keep multiplying by 1.1 using the calculator — just be extra careful to keep track of how many times you multiply!

Alternatively, write an inequality in which a is the original acreage and y is the number of years:

$(0.65a)(1.1)^y > a$

Notice that the a values cancel out:

$$(0.65)(1.1)^y > 1$$

The GRE calculator is not equipped to solve this directly (you would need to use a logarithm, a topic not tested on the GRE), so instead plug in the answer choices for y, starting with the smallest value (3), until you find the smallest one that works. 5 is the smallest value that makes the inequality true.

35. **(B).** The 50% spent on rent, utilities, and insurance and the 20% spent on food are both percents of the total, so you can simply add the percents. 50% + 20% = 70%. After these expenditures, Aloysius has 30% left. He then spends 30% *of the remaining 30%* on video games. 30% of 30% = 0.30 × 0.30 = 0.09, or 9%. 30% - 9% = 21% remaining.

Alternatively, use smart numbers. If Aloysius's income is $100, he would spend $50 on rent, utilities, and insurance, and $20 on food, for a total of $70. Of his remaining $30, he would spend 30%, or $9, on video games, leaving $21, or 21% of the original amount.

36. **(C).** This question can be done in one of two ways. If you remember that all multiplications happen simultaneously (in terms of order of operations), you can simply look at this question and see that the order in which the discount and the tax are applied is irrelevant, so the answer must be (C). In other words, if x is the original price, $(0.9)(1.07)x = (1.07)(0.9)x$.

Alternatively, pick a real number and use the calculator. Because the problem deals with percentages, try 100. For Quantity A, first multiply by 0.9 to reflect the 10% discount: (100)(0.9) = $90. Next, multiply by 1.07 to apply the 7% tax: (90)(1.07) = $96.30

For Quantity B, first multiply by 1.07 to get $107 and then by 0.9 to get $96.30.

The answer is (C).

37. **(B).** You may be tempted to pick (C) here right away, but check the work; this problem mixes multiplication and subtraction. Pick a number to test the two quantities; because the problem deals with percentages, 100 is a good number to pick.

For Quantity A, a 10% discount would reduce the $100 price to $90, and $10 off $90 would reduce the price to $80.

For Quantity B, a $10-off coupon would reduce the price to $90, and then 10% off would reduce the price to $81, not $80! The discount is only $9 because you take 10% of 90, not 10% of 100.

Alternatively, you could derive both quantities algebraically:

Quantity A = $0.9x - 10$

Quantity B = $0.9(x - 10) = 0.9x - 9$

The answer is (B). Note that the order of the two discounts mattered here because one change was multiplication (10% off) and one was subtraction ($10 off). If both changes had been the same operation (e.g., both multiplication), the order would not have mattered.

38. **(D).** 1/3 of all the beans is 5 beans. These 5 beans each cost \$2, for a total of \$10. The remaining 10 beans cost \$5 each, for a total of \$50. If all of these more expensive beans are lost, then the lost beans represent $\frac{50}{60}$ of all the money paid. To convert to a percent: $\frac{50}{60} \times 100 = 83.\bar{3}\%$.

39. **(C).** Originally, the three figures given total 55%, so 45% was spent on fast food and desserts. If coffee doubled in price and the overall budget did not increase, coffee would then be 10% of the total, and that extra 5% would be taken from the fruit and vegetable expenditures. Thus, fruits and vegetables become 15% of the total while fast food and dessert stays at 45%. The ratio is 15/45, which reduces to 1/3.

40. **(B).** His loss was 15% of 200, or 30 pounds. If he then gained 35 pounds, he finished the year weighing 205.

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{5}{200} \times 100 = 2.5\%$$

Be sure to select (B) for a 2.5% increase, and not (D) for a 2.5% decrease.

41. **7,100%.** In 1970, Company X had 0.15(2000) = 300 female employees. Of those, 0.10(300) = 30 were female executives.

In 2012, Company X had 0.45(12,000) = 5,400 female employees. Of those, 0.40(5,400) = 2,160 were female executives.

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{2130}{30} \times 100 = 7{,}100\%$$

42. **60%.** Use smart numbers. There are 80% as many boys as girls, so choose 100 for the girls (100 is a good number to pick for percent problems). The boys, then, must be (100)(0.8) = 80. 75% of all the boys take civics, therefore there are 0.75(80) = 60 boys who take civics. 48% of all the girls take civics, therefore there are 0.48(100) = 48 girls who take civics.

60 + 48 = 108 students take civics and there are 180 total students:

$$\frac{108}{180} \times 100 = 60\%$$

43. **(C).** Airline A reduces its price by 25% to (400)(0.75) = \$300 but then raises that price by 10% to (300)(1.1) = \$330. Airline B reduces its fare to (400)(0.45) = \$180 but adds \$150 in fees, bringing the total price to 180 + 150 = \$330.

44. **(A).** The new lunch price is (10)(1.2) = \$12. With the tea charge, the new lunch bill is 12 + 1 = \$13. Since Jake leaves a 20% tip, the new tip is (0.20)(13) = \$2.60, so his new cost is 13 + 2.6 = \$15.60. This is larger than Quantity B.

45. **(C).** This problem can be solved algebraically, using t as the total. Note: because the equation requires addition, it is easier to use decimals than fractions. (Adding fractions requires finding a common denominator.)

$$0.5t + 0.25t + 0.2t + 20 = t$$
$$0.95t + 20 = t$$
$$20 = 0.05t$$
$$t = 400$$

Thus, the trucks left at Store X = 0.25(400) = 100 and the trucks left at Store Y = 0.20(400) = 80. The difference is 20.

Alternatively, Store W = 50%, Store X = 25%, and Store Y = 20%. Collectively, these three stores received 95% of the trucks, so Store Z receives 5% of the trucks. You know that Store Z receives 20 trucks, so 5% = 20 trucks. The difference between Store X (25%) and Store Y (20%) is also 5%, so that 5% difference is also 20.

46. **(C).** Write an equation from the first part of the given information: $p = 0.75q$. Since $p = 2r$, substitute $2r$ for p in the first equation:

$$2r = 0.75q$$
$$r = 0.375q$$

The two quantities are equal.

Alternatively, use smart numbers. If q is 8, then p is (8)(0.75) = 6. (Note: because you have to multiply q by 0.75, or 3/4, try to pick something divisible by 4 for q, so that p will be an integer.) Therefore, r is 6/2 = 3.

$0.375q = (0.375)(8) = 3$. The value for r is also 3, so the two quantities are equal.

47. **(D).** 90% of 40 students or 0.9(40) = 36 students had a lower GPA than Tom. Of the 60 new students, 80% or 0.80(60) = 48 had a lower GPA than Tom. Thus, 36 + 48 = 84 students in the new, larger class have GPAs lower than Tom.

The new class has 100 people, 84 of whom have lower GPAs than Tom. There are 16 people unaccounted for — don't forget that Tom is one of them! Since Tom has the lowest GPA of this group of 16 people, there are 15 people above him. Since the class has exactly 100 people, 15/100 = 15%.

48. **(A).** When a percentage contains a variable, use fractions to translate. Quantity A is equal to:

$$\frac{x}{100} \times \frac{0.5}{100} \times \frac{40{,}000}{1} = x(0.5)(4) = 2x$$

Quantity B is equal to:

$$\frac{0.05}{100} \times \frac{2{,}000}{100} \times \frac{40x}{1} = (0.05)(2)(4x) = 0.4x$$

Since x is positive, you can be sure that Quantity A is larger (this is true even if x is a fraction).

Alternatively, use smart numbers. If $x = 50$, then Quantity A equals:

$$\frac{50}{100} \times \frac{0.5}{100} \times \frac{40{,}000}{1} = (0.5)(0.5)(400) = 100$$

Quantity B equals:

$$\frac{0.05}{100} \times \frac{2{,}000}{100} \times \frac{(40)(50)}{1} = (0.05)(2)(4)(50) = 20$$

Quantity A is larger.

49. **216.** The percent increase from 2000 to 2001 is:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{20}{60} \times 100 = 33.\overline{3}\%$$

Now, apply a $33.\overline{3}\%$, or 1/3, increase to 2004's figure. You can't type a repeating decimal into the calculator; instead, multiply 162 by 1/3 to get the amount of increase, and then add to 162 for the new profit per student in 2005: (162) (1/3) + 162 = 216.

50. **(E).** First, write "x is 0.5% of y" as math. Make sure you don't accidentally interpret 0.5% as 50%!

$$x = \frac{0.5}{100} \times y$$

The question asks "y is what percent of x?", so solve for y:

$100x = 0.5y$
$200x = y$

If y is 200 times x, multiply by 100 to convert to a percent:

$$\frac{200x}{1} \times \frac{100}{100} = \frac{20{,}000x}{100}$$

The answer is 20,000%. (For reference, if one number is 2 times as big as the other, it is 200% the size — add two zeros. So, 200 times as big = 20,000%.)

Alternatively, you could use smart numbers. If $y = 100$, then $x = \frac{0.5}{100}(100) = 0.5$. Next, answer the question "100 is what percent of 0.5?" Pick a new variable to translate the "what percent" portion of the sentence:

$100 = \frac{n}{100} \times 0.5$
$10{,}000 = 0.5n$
$20{,}000 = n$

(In translating percents problems to math, always translate "what percent" as a variable over 100.)

51. **(C).** Bill's tax is (0.20)5000 = \$1,000. Thus, his remaining salary is \$4,000. His rent is therefore (0.25)4000 = \$1,000.

52. **(B).** If four people shared the \$80 bill equally, then each person paid for one-quarter of the bill, or \$80/4 = \$20.

The tip is calculated as a percentage of the bill. Because the question asks about the amount that each (one) person paid, you can calculate the 15% tip based solely on one person's portion of the bill (\$20): (0.15)(20) = \$3.

In total, each person paid \$20 + \$3 = \$23.

Alternatively, you could find the total of the bill plus tip and take one-fourth of that for the total contribution of each person. The total of bill and tip is \$80 + (0.15)(80) = \$80 + \$12 = \$92. One-fourth of this is \$92/4 = \$23.

53. **(B).** Use a smart number for the price of the stock; for a percent problem, \$100 is a good choice. The price of the stock after a 25% increase is (1.25) × \$100 = \$125.

Next, find the percent decrease (y) needed to reduce the price back to the original \$100. \$125 - \$25 = \$100, so rephrase the question: 25 is what percent of 125?

$25 = \frac{x}{100}(125)$

$\frac{2{,}500}{125} = x$

$x = 20$

You have to reduce 125 by 20% in order to get back to \$100. Therefore, Quantity A = 20% and is less than Quantity B.

Alternatively, you can use algebra, although algebra is challenging for this problem. Assign the original cost of the stock a variable, such as z. In this case, the price of the stock after a 25% increase would be $1.25z$. The percent decrease, y, is found by multiplying $1.25z$ by $1-\frac{y}{100}$ and setting the quantity equal to the original price, z.

$$z=\left(1-\frac{y}{100}\right)(1.25z)$$

$$\frac{z}{1.25z}=1-\frac{y}{100}$$

$$\frac{1}{1.25}=1-\frac{y}{100}$$

$$0.8=1-\frac{y}{100}$$

$$-0.2=-\frac{y}{100}$$

Multiply all terms by 100 in order to get rid of the fraction:

$80 = 100 - y$
$y = 100 - 80 = 20$

54. **(C).** The chemist now has 10 ounces of acetone in a 30-ounce mixture, so she must have 20 ounces of water. You want to know the amount of acetone you must add in order to make this mixture a 50% solution. No additional water is added, so the solution must finish with 20 ounces of water. Therefore, she also needs a total of 20 ounces of acetone, or 10 more ounces than the mixture currently contains.

Alternatively, you can use algebra. If the chemist adds x ounces of acetone to the mixture, then there will be $10 + x$ ounces of acetone and the total mixture will have $30 + x$ ounces. The goal is to have a mixture that is 50% acetone:

$$50\%=\frac{10+x}{30+x}$$

$$\frac{50}{100}=\frac{10+x}{30+x}$$

$$\frac{1}{2}=\frac{10+x}{30+x}$$

Cross multiply:

$30 + x = 20 + 2x$
$10 = x$

The answer is (C).

Note that one trap answer is (B), or 5. This answer is not correct because the final number of ounces in the solution is *not* 30; when the chemist adds acetone, the amount of total solution also increases — adding 5 ounces acetone would result in a solution that is 15/(30 + 5) acetone, which is not equivalent to a 50% mixture.

55. **(C).** Choose a smart number for the total number of games; for a percent problem, 100 is a good number to pick. If the total number of games for the season is 100 and the team played 80% of them by July, then the team played (100)(0.8) = 80 games. The team won 50% of these games, or (80)(0.5) = 40 games.

Next, the team won 60% of its *remaining* games. As there were 100 total games and the team has played 80 of them, there are 20 games left to play. Of these, the team won 60%, or (20)(0.6) = 12 games.

Therefore, the team has won a total of 40 + 12 = 52 games out of 100, or 52% of its total games. Quantities A and B are equal.

Alternatively, this problem could be done using weighted averages, where the total percent of games won is equal to the sum of all of the individual percentages multiplied by their weightings. In this case,

Total Percentage Won = $(50\%)(80\%) + (60\%)(100\% - 80\%) \times 100$
Total Percentage Won = $[(0.5)(0.8) + (0.6)(0.2)] \times 100$
Total Percentage Won = $[(0.4) + (0.12)] \times 100$
Total Percentage Won = 0.52×100
Total Percentage Won = 52%

56. **(A).** In order to compare, use the calculator to find 0.4 percent of 4 percent of 1.25 (be careful with the decimals!):

$0.004 \times 0.04 \times 1.25 = 0.0002$

Or, as fractions:

$$\frac{0.4}{100} \times \frac{4}{100} \times 1.25 = \frac{2}{1{,}000} = 0.0002$$

Quantity A is larger than Quantity B.

57. **(D).** Originally, Jane has a 40-ounce mixture of apple and seltzer that is 30% apple. Since 0.30(40) = 12, 12 ounces were apple and 28 ounces were seltzer.

Jane pours 10 more ounces of apple juice into the mixture, yielding a mixture that is 50 ounces total, still with 28 ounces of seltzer. Now, the percentage of seltzer in the final mixture is $\frac{28}{50} \times 100 = 56\%.$

58. **(A).** Choose a smart number for the total number of shirts in the closest; this is a percent problem, so 100 is a good number to pick. Out of 100 shirts, half, or 50, are white.

30% of the *remaining* shirts are gray. If there are 50 white shirts, there are also 50 remaining shirts and so (0.3)(50) = 15 gray shirts. Therefore, there are 50 + 15 = 65 total shirts that are white or gray, and 100 - 65 = 35 shirts that are neither white nor gray. Since 35 out of 100 shirts are neither white nor gray, exactly 35% of the shirts are neither white nor gray.

Alternatively, you can use algebra, though that is trickier on a problem such as this one. Set a variable, such as x, for the total number of shirts. The number of white shirts is $0.5x$ and the remaining shirts would equal $x - 0.5x = 0.5x$. The number of gray shirts, then, is $(0.5x)(0.3) = 0.15x$. Thus there are $0.5x + 0.15x = 0.65x$ white or gray shirts, and $x - 0.6x = 0.35x$ shirts that are neither white nor gray. $0.35x \div x = 0.35$, or 35%.

59. **(E).** As there are no amounts given in the problem, you can choose a smart number for the total number of children. On percent problems, 100 is a good choice. The problem indicates that 80% of the children, or 80 children total, are more than 10 years old.

20 percent *of these 80 children* play an organized sport. The question asks about the percentage of these children who do NOT play an organized sport. If 20% do, then the remaining 80% of the 80 children do not. (80)(0.8) = 64 children who are over 10 years old and do not play an organized sport.

Alternatively, you can use algebra; set the total number of children in the room equal to x. The problem indicates that 80%, or $0.8x$, of the children are over 10 years old. Of the $0.8x$ children, 20% do play an organized sport, so 80% do not. $(0.8x)(0.8) = 0.64x$. Therefore, 64% of the children are over 10 but do not play an organized sport.

60. **(B).** You can choose smart numbers for the dimensions of the box — for instance, length = 20, width = 10, and height = 1. (For the length and the width, pick values that will still yield an integer when increased by 10%. Since the height doesn't change, pick an easy number such as 1 to keep the overall calculations easy.)

The original volume of the box = length × width × height = 20 × 10 × 1 = 200

After a 10% increase for both the length and the width, the volume becomes 22 × 11 × 1 = 242.

The formula for percent change is:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\frac{242-200}{200} = \frac{42}{200} = \frac{21}{100} = 21\%$$

Quantity B is larger.

Alternatively, you could use algebra. Assign the variables l to the original length, w to the original width, and h to the original height of the box. The volume of the box would then be lwh. A 10% increase in the length and width changes the variables to $1.1l$ and $1.1w$ respectively. The new volume of the box would be $(1.1l)(1.1w)(h) = 1.21lwh$, which constitutes a 21% increase over lwh.

Finally, you could use logic. The formula for volume requires multiplying the length and the width. If just one side is increased by 10%, then the overall volume will increase by 10%. If two sides are increased by 10%, then the overall volume will increase by something larger than 10%.

61. **(B).** You can choose a smart number for the radius of the circle. In this case, because no restrictions are placed on the radius, choose radius = 1 for convenience. The area of a circle is πr^2 and so the area is equal to $\pi 1^2 = \pi$.

The radius of the circle then doubles from 1 to 2. The new area of the circle is $\pi(2)^2 = 4\pi$. Now calculate the percent increase in the area of the circle:

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{4\pi - \pi}{\pi} \times 100 = \frac{3\pi}{\pi} \times 100 = 300\%$$

Quantity B is larger.

Alternatively, you can use algebra. Assign the variable r, giving an original circle area of πr^2. After the radius is doubled to $2r$, the new area becomes $\pi(2r)^2 = 4\pi r^2$. Again, use the formula for percent increase:

$$\text{Percent Change} = \frac{4\pi r^2 - \pi r^2}{\pi r^2} \times 100 = \frac{3\pi r^2}{\pi r^2} \times 100 = 300\%$$

62. **(C).** Translate the given information into math:

$$\frac{35}{100}x = 140$$

$$x = 140 \times \frac{100}{35}$$

$$x = 400$$

Next, find 20% of x, or $0.20(400) = 80$.

63. **(A).** Every 3 minutes, the population increases by 20% (which is the same as multiplying by 1.2). Beginning at 8:54am, this change would occur at 8:57am and again at 9:00am. Use the variable x to represent the original quantity. Note that the 20% increase occurs twice:

$x(1.2)(1.2) = 144{,}000$
$x = 100{,}000$

Note that you cannot simply reduce 144,000 by 20% twice, because 20% is not a percentage of 144,000 — it is a percentage of the unknown, original number.

Alternatively, you could begin from 144,000 and calculate "backwards":

From 8:57am to 9:00am: $y(1.2) = 144{,}000$, so $y = 144{,}000/1.2 = 120{,}000$.

From 8:54am to 8:57am: $z(1.2) = 120{,}000$, so $z = 120{,}000/1.2 = 100{,}000$.

64. **(D).** Call the first test score x. A 15% increase and then a 25% decrease yields 69. Thus:

$x(1.15)(0.75) = 69$
$x = 80$

Alternatively, begin from the final score, 69, and solve "backwards":

25% decrease from 2nd test to 3rd test: $y(0.75) = 69$, so $y = 92$. 2nd test was 92.

15% increase from 1st test to 2nd test: $z(1.15) = 92$, so $z = 80$. 1st test was 80.

65. **(D).** Reducing a number by a percentage involves multiplication; reducing a number by a fixed amount involves subtraction. The order of operations (PEMDAS) will make a difference.

One possible value for the item is \$100. In this case, the value of Quantity A = (100)(0.9) - 20 = \$70. The value of Quantity B = (100 - 10)(0.80) = \$72. Here, Quantity B is larger.

However, a larger starting value may change the result, because a 20% discount off a larger starting value can result in a much bigger decrease. For a \$140 item, the value of Quantity A = (140)(0.9) - 20 = \$106. The value of Quantity B = (140 - 10)(0.80) = \$104. Here, Quantity A is larger. The answer is (D).

66. **(C).** You can pick smart numbers since there are no amounts specified. Each person in class 1 stacks 80 percent as many boxes as each person in class 2. You can choose 80 and 100, but it's better to pick smaller numbers to make the later math easier.

Class 1 = 8 blocks per person
Class 2 = 10 blocks per person

There are 25% more people in class 1 than class 2. If there are 4 people in class 2, then there are (4)(1.25) = 5 people in class 1.

Blocks Stacked by Class 1 = 8 people × 5 blocks per person = 40 blocks

Blocks Stacked by Class 2 = 10 people × 4 blocks per person = 40 blocks

Since each class stacks 40 blocks, each class stacks 50% of the total blocks. The quantities are equal.

Alternatively, you could use algebra.

Class 1 = $0.8x$ blocks per person
Class 2 = x blocks per person
Class 1 = $1.25y$ people
Class 2 = y people

The total blocks stacked by class 1 = $(0.8x)(1.25y) = xy$. The total blocks stacked by class 2 = xy. Since each class stacks the same number of blocks, each class stacks 50% of the total blocks.

67. **(D).** Because the problem does not specify any real values for the variables, you can test your own numbers. You might be tempted to choose 100 for z, but then x and y will be the same; for example, 60 is 60 percent of 100. It's better to pick three different values for the three variables.

One possible case: 4 is 40% of 10. In this example, $x = 4$, $y = 40$, and $z = 10$. The value of Quantity A is $z/x = 10/4 = 2.5$, or 250%. The value of Quantity B is 40/10,000, or 0.4%. In this example, Quantity A is larger.

Will that always be true? Or will a larger or smaller number change the result? In Quantity B, y is divided by a static number: 10,000 never changes. If y is a much larger number, then, perhaps Quantity B will become larger than Quantity A.

Try $y = 10{,}000$. Also change x and z to more manageable numbers. 400 is 10,000% of 4. (Note: 4 is 100% of 4. 40 is 1,000% of 4. 400 is 10,000% of 4.) In this example, $x = 400$, $y = 10{,}000$, and $z = 4$. The value of Quantity A is $z/x = 4/400 = 1/100$, or 1%. The value of Quantity B is $10{,}000/10{,}000 = 1$. In this case, the two quantities are equal. Therefore, the answer is (D).

You can also use algebra, though the algebra is challenging for this problem. Translate the given equation, x is y percent of z:

$$x = \frac{y}{100} \times z$$

Translate Quantity A, the percent that z is of x; you can rephrase as "z is what percent of x?" Note that you have to introduce a new variable:

$$z = \frac{p}{100} \times x$$

Solve for p because Quantity A is asking for the unknown percent:

$$\frac{100z}{x} = p$$

Quantity B contains an expression that uses only the variable y, while Quantity A contains 3 variables. Use the given equation to try to write the left-hand side of Quantity A only in terms of y:

Given: $x = \frac{y}{100} \times z$

$$\frac{x}{z} = \frac{y}{100}$$

$$\frac{z}{x} = \frac{100}{y}$$

Substitute for the $\frac{z}{x}$ term in the equation for Quantity A:

$$(100)\left(\frac{100}{y}\right) = p$$

$$\frac{10{,}000}{y} = p$$

Quantity A equals $\frac{10{,}000}{y}$ and Quantity B equals $\frac{y}{10{,}000}$. Without knowing the value of *y*, however, it is impossible to determine that one quantity is larger. Specifically, if *y* is less than 10,000, Quantity A is larger, but if *y* is greater than 10,000, Quantity B is larger. The answer is (D).

68. **(B).** 20% less than 300 is the same as 80% of 300, or 0.80(300) = 240. The question is "240 is what percent greater than 180?"

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{60}{180} \times 100 = 33.\overline{3}\%$$

69. **(D).** First find the volume of oil in the bucket. The oil fills 35% of the bucket's 20-gallon volume, or (20)(0.35) = 7 gallons of oil

These 7 gallons originally filled 40% of the tank. Call the volume of the tank *T*. *T*(0.4) = 7, so *T* = 17.5 gallons.

70. **(A).** "200% larger" means "*three* times as big as" the original; "200% as large as" would mean twice as big. If the pitcher is three times as big as the glass, then pouring the contents of the glass into the pitcher will make the pitcher 1/3 full. If adding another 16 ounces fills up the pitcher, the 16 ounces must be equal to the remaining 2/3 of the pitcher's capacity. 1/3 of the pitcher's capacity, then, is 16/2, or 8 ounces. The juice mixture totals 8 ounces. 20% of the juice is grape juice, so there are (8)(0.2) = 1.6 ounces of grape juice.

71. **(A).** First, find the value of 150 increased by 60%: (150)(1.6) = 240. 240 is then decreased by *y* percent to get 192. 240 - 192 = 48, so 240 is decreased by 48 to get 192. Rephrase the question: 48 represents what percentage of 240?

$$48 = \frac{x}{100}(240)$$

$$48\left(\frac{10}{24}\right) = x$$

$$x = 20$$

72. **(D).** "150% greater than 200" means 150% of 200, or 300, *added back to* 200. This is the not the same figure as 150% *of* 200. Thus, 150% greater than 200 is 200 + (200)(1.5) = 500.

50% of 500 = 250. Translate the question as "What percent greater is 500 than 250?" Since 500 is twice 250, it is 100% greater than 250.

Alternatively, use the percent change formula.

$$\text{Percent Change} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Percent Change} = \frac{500 - 250}{250} \times 100 = 100\%$$

73. **(C).** First, the question asks you to find the percent of the combination that is NOT aluminum. If Mixture A is 50% aluminum, then it is also 50% NOT aluminum. Mixture A weighs 18 grams, so the portion that is NOT aluminum is (18)(0.5) = 9 grams.

If Mixture B is 37.5% (or 3/8) aluminum, then it is 100% - 37.5% = 62.5% (or 5/8) NOT aluminum. Mixture B weighs 32 grams, so the portion that is NOT aluminum is (32)(5/8) = 20.

20 + 9 = 29 grams are NOT aluminum out of the 18 + 32 = 50 total grams. The percentage is 29/50 = 58/100 = 58%.

The two quantities are equal.

74. **(B).** The stockbroker has made a profit on 80% of his 40 trades this year, so (0.80)(40) = 32 of his trades so far have been profitable.

Quantity B asks for the maximum number of additional losses in a row he can have without dropping below 50%. If he stays at 32 profitable trades and 8 non-profitable trades, and all future trades are losses, then he can't go above 32 × 2 = 64 trades without dropping below a 50% success rate. At 64 trades exactly, he would have 32 profitable trades and 32 non-profitable trades, for a "success" percentage of 50% profitable trades. 64 - 40 = 24 trades, so he can have 24 losses in a row without dropping *below* 50%. Quantity B is larger.

Alternatively, you can use algebra. Set the number of additional losing trades (above 40) to x. Then, the number of winning trades will remain constant at 32, the total number of trades will increase to $40 + x$, and the total number of losing trades will be $8 + x$. Quantity B asks for the maximum number of additional losses in a row he can have without dropping below 50% of profitable trades, so set up an inequality. The percentage of profitable trades must be greater than or equal to 50:

$$\frac{\text{Number Profitable}}{\text{Total Number}} \times 100 \geq 50$$

$$\frac{32}{40+x} \times 100 \geq 50$$

$$\frac{32}{40+x} \geq \frac{1}{2}$$

Cross multiply (note: you know the variable represents a positive number, so you don't need to do anything to the inequality sign):

$64 \times 40 + x$

$24 \times x$

Therefore, the stockbroker can lose money on 24 trades in a row and still have 50% of trades be profitable, so Quantity B is 24. Quantity B is larger.

75. **(D).** Candidate A had a 5% increase in votes between 2011 and 2012; a percent increase is calculated based upon the "original" number, which in this case was the number of votes for Candidate A in 2011. This 5% increase was equivalent to 3 total votes. You can use algebra to solve; let x equal the number of votes for Candidate A in 2011.

$$\frac{5}{100} = \frac{3}{x}$$

$$\frac{1}{20} = \frac{3}{x}$$

$$x = 60$$

Alternatively, if 5% = 3, then 50% = 30 (multiply both sides by 10) and 100% = 60 (multiply both sides by 2).

Therefore, Candidate A received 60 votes in 2011. The problem also indicates that Candidate A received 40% of the total vote in 2011. You can solve for the total votes and subtract to find the number of votes for Candidate B: Let T equal the total number of votes in 2011.

$60 = (0.4)T$

$150 = T$

$T - A = 150 = 60 = 90$ votes for Candidate B in 2011.

Alternatively, you could set up a proportion to solve. The advantage of this method: you won't solve for the total number of votes, so you won't get distracted by trap wrong answer (E). 60 votes represent 40% of the total, and you want to solve for 60% of the total:

$$\frac{60 \text{ (A votes)}}{40 \text{ (\% of total votes)}} = \frac{x \text{ (B votes)}}{60 \text{ (\% of total votes)}}$$

$$\frac{3}{2} = \frac{x}{60}$$

$$180 = 2x$$

$$x = 90$$

76. **(C).** A 16-ounce jar that is 10% sesame has 1.6 ounces of sesame. From there, you might infer that all you need to do is add 1.6 ounces again, and the mixture will be 20% sesame. However, this is incorrect — adding 1.6 ounces of sesame will also add 1.6 ounces to the total amount of seed in the jar. 3.2 ounces sesame/17.6 ounces total = 18.18% which is NOT equal to 20%.

Instead, write an equation expressing the ratio of sesame to the total mixture, where x is the amount of sesame to add; this equals the desired 20% (or 1/5) figure:

$$\frac{1.6 + x}{16 + x} = \frac{1}{5}$$

Cross multiply:

$5(1.6 + x) = 16 + x$
$8 + 5x = 16 + x$
$4x = 8$
$x = 2$

77. **(C).** It is always the case that, for two positive quantities, M% of N = N% of M. In this case, $(a + b)$ makes the problem appear more complicated, but the principle still applies. Algebraically:

Quantity A	Quantity B
$\frac{(a+b)}{100} \times c$	$\frac{c}{100} \times (a+b)$

Both quantities can be simplified to equal $\frac{c(a+b)}{100}$.

78. **(E).** This problem is best solved with a double-set matrix. Since all figures are given in percents or fractions (no real numbers of boats), you can use any number you want for the total; 100 is the easiest choice. Since 25% of 100 = 25 boats are used sailboats and 33% of 100 = 33 boats are used, you can infer that 33 - 25 = 8 boats are used non-sailboats:

	sailboat	**non-sailboat**	**Total**
new			
used	25	8	33
Total			100

You are told that *of non-sailboats,* 3/5 (or 60%) are new. Since you don't know the number of non-sailboats:

	sailboat	**non-sailboat**	**Total**
new		$0.6x$	
used	25	8	33
Total		x	100

Since, in the non-sailboat column, new + used = total:

$0.6x + 8 = x$
$8 = 0.4x$
$20 = x$

This is enough to fill in the rest of the chart:

	sailboat	**non-sailboat**	**Total**
new	55	12	67
used	25	8	33
Total	80	20	100

You can now see that 55/80 of the sailboats are new. This is 68.75%. Rounded to the nearest percent, the answer is 69%.

79. **(B).** Helen bought a ticket for \$252; if she had bought it 1 day later, she would have paid \$54 more. There are three possibilities that represent the dividing lines between the given discount levels:

Possibility 1: She bought the ticket 60 days in advance for a 40% discount (if she'd bought it 1 day later, or 59 days in advance, she would have received a 30% discount instead).

Possibility 2: She bought the ticket 30 days in advance for a 30% discount (if she'd bought it 1 day later, or 29 days in advance, she would have received a 15% discount instead).

Possibility 3: She bought the ticket 5 days in advance for a 15% discount (if she'd bought it 1 day later, or 4 days in advance, she would not have received any kind of discount).

This question is harder than it looks, because you cannot just calculate a percent change between $252 and $306. The discounts are *percentages of the full-price ticket*, and you don't know that number. Call it x.

Do note that the only three possible answers are 5, 30, and 60 (answers (A), (B), and (D), respectively). 59 days ahead and 89 days ahead do not represent days for which the next day (58 and 88 days ahead, respectively) results in a change in the discount.

Possibility 1 (60 days in advance): $252 would represent a 40% discount from the original price, so the original price would be $252 = 0.6x$, and x would be $420.

If the full ticket price is $420, then buying the ticket 1 day later would result in a 30% discount instead, or (420)(0.7) = $294. The problem indicates, however, that Helen would have paid $306, so Possibility 1 is not correct.

Possibility 2 (30 days in advance): $252 would represent a 30% discount from the original price, so the original price would be $252 = 0.7x$ and x would be $360.

If the full ticket price is $360, then buying the ticket 1 day later would result in a 15% discount instead, or (360)(0.85) = $306. This matches the figure given in the problem, so Possibility 2 is correct; you do not need to test Possibility 3. Helen bought the ticket 30 days in advance.

Chapter 13
of

Divisibility and Primes

In This Chapter…

Divisibility and Primes

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. For how many positive integer values of x is $\frac{65}{x}$ an integer?

2. If x is a number such that $0 < x \leq 20$, for how many values of x is $\frac{20}{x}$ an integer?

(A) 4
(B) 6
(C) 8
(D) 10
(E) More than 10

3.

Quantity A	**Quantity B**
The number of even factors of 27	The number of even factors of 81

4.

Quantity A	**Quantity B**
The number of distinct factors of 10	The number of distinct <u>prime</u> factors of 210

5.

Quantity A	**Quantity B**
The least common multiple of 22 and 6	The greatest common factor of 66 and 99

6. The number of students who attend a school could be divided among 10, 12, or 16 buses, such that each bus transports an equal number of students. What is the minimum number of students that could attend the school?

(A) 120
(B) 160
(C) 240
(D) 320
(E) 480

7.

Quantity A	**Quantity B**
The number of distinct prime factors of 27	The number of distinct prime factors of 18

8.

Quantity A	**Quantity B**
The number of distinct prime factors of 31	The number of distinct prime factors of 32

9. How many factors greater than 1 do 120, 210, and 270 have in common?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 30

10. Company H distributed $4,000 and 180 pencils evenly among its employees, with each employee getting an equal integer number of dollars and an equal integer number of pencils. What is the greatest number of employees that could work for Company H?

(A) 9
(B) 10
(C) 20
(D) 40
(E) 180

11. n is divisible by 14 and 3. Which of the following statements must be true?

Indicate all such statements.

- ☐ 12 is a factor of n
- ☐ 21 is a factor of n
- ☐ n is a multiple of 42

12. Positive integers a and b each have exactly four factors. If a is a one-digit number and $b = a + 9$, what is the value of a?

☐

13. Ramon wants to cut a rectangular board into identical square pieces. If the board is 18 inches by 30 inches, what is the least number of square pieces he can cut without wasting any of the board?

(A) 4
(B) 6
(C) 9
(D) 12
(E) 15

14. If n is the product of 2, 3, and a two-digit prime number, how many of its factors are greater than 6?

☐

15.

m is a positive integer that has a factor of 8.

Quantity A	**Quantity B**
The remainder when m is divided by 6	The remainder when m is divided by 12

16. When the positive integer x is divided by 6, the remainder is 4. Each of the following could also be an integer EXCEPT

(A) $\frac{x}{2}$

(B) $\frac{x}{3}$

(C) $\frac{x}{7}$

(D) $\frac{x}{11}$

(E) $\frac{x}{17}$

17. If $x^y = 64$ and x and y are positive integers, which of the following could be the value of $x + y$?

Indicate <u>all</u> such values.

☐ 2
☐ 6
☐ 7
☐ 8
☐ 10
☐ 12

18. If k is a multiple of 24 but not a multiple of 16, which of the following cannot be an integer?

(A) $\frac{k}{8}$

(B) $\frac{k}{9}$

(C) $\frac{k}{32}$

(D) $\frac{k}{36}$

(E) $\frac{k}{81}$

19. If $a = 16b$ and b is a prime number greater than 2, how many positive distinct factors does a have?

20. If a and c are positive integers and $4a + 3 = b$ and $4c + 1 = d$, which of the following could be the value of $b + d$?

(A) 46
(B) 58
(C) 68
(D) 74
(E) 82

21. Each factor of 210 is inscribed on its own plastic ball, and all of the balls are placed in a jar. If a ball is randomly selected from the jar, what is the probability that the ball is inscribed with a multiple of 42?

(A) $\frac{1}{16}$

(B) $\frac{5}{42}$

(C) $\frac{1}{8}$

(D) $\frac{3}{16}$

(E) $\frac{1}{4}$

22. At the Canterbury Dog Fair, 1/4 of the poodles are also show dogs and 1/7 of the show dogs are poodles. What is the least possible number of dogs at the fair?

23. A "prime power" is an integer that has only one prime factor. For example, $5 = 5$, $25 = 5 \times 5$, and $27 = 3 \times 3 \times 3$ are all prime powers, while $6 = 2 \times 3$ and $12 = 2 \times 2 \times 3$ are not. Which of the following numbers is not a prime power?

(A) 49
(B) 81
(C) 100
(D) 121
(E) 243

24. If a and b are integers such that $a > b > 1$, which of the following cannot be a multiple of either a or b?

(A) a - 1

(B) $b + 1$
(C) $b - 1$
(D) $a + b$
(E) ab

25. 616 divided by 6 yields remainder p, and 525 divided by 11 yields remainder q. What is $p + q$?

26. If x is divisible by 18 and y is divisible by 12, which of the following statements must be true?

Indicate <u>all</u> such statements.

☐ $x + y$ is divisible by 6
☐ xy is divisible by 48
☐ x/y is divisible by 6

27. If p is divisible by 7 and q is divisible by 6, pq must have at least how many factors greater than 1?

(A) 1
(B) 3
(C) 6
(D) 7
(E) 8

28. If r is divisible by 10 and s is divisible by 9, rs must have at least how many factors?

(A) 2
(B) 4
(C) 12
(D) 14
(E) 16

29. If t is divisible by 12, what is the least possible integer value of a for which $\frac{t^2}{2^a}$ might not be an integer?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

30. If a, b, and c are multiples of 3 such that $a > b > c > 0$, which of the following values must be divisible by 3?

Indicate <u>all</u> such values.

☐ $a + b + c$
☐ $a - b + c$
☐ $abc/9$

31. New cars leave a car factory in a repeating pattern of red, blue, black, and gray cars. If the first car to exit the factory was red, what color is the 463rd car to exit the factory?

(A) red
(B) blue
(C) black
(D) gray
(E) It cannot be determined from the information given.

32. Jason deposits money at a bank on a Tuesday and returns to the bank 100 days later to withdraw the money. On what day of the week did Jason withdraw the money from the bank?

(A) Monday
(B) Tuesday
(C) Wednesday
(D) Thursday
(E) Friday

33. x and h are both positive integers. When x is divided by 7, the quotient is h with a remainder of 3. Which of the following could be the value of x?

(A) 7
(B) 21
(C) 50
(D) 52
(E) 57

34. a, b, c, and d are all positive integers. If $\frac{ab}{c+d} = 3.7$, which of the following statements must be true?

Indicate <u>all</u> such statements.

☐ ab is divisible by 5.
☐ $c + d$ is divisible by 5.
☐ If c is even, then d must be even.

35. When x is divided by 10, the quotient is y with a remainder of 4. If x and y are both positive integers, what is the remainder when x is divided by 5?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

36. What is the remainder when $13^{17} + 17^{13}$ is divided by 10?

☐

37. a, b, c, and d are positive integers. If $\frac{a}{b}$ has a remainder of 9 and $\frac{c}{d}$ has a remainder of 10, what is the minimum possible value for bd?

38. If n is an integer and n^3 is divisible by 24, what is the largest number that must be a factor of n?

(A) 1
(B) 2
(C) 6
(D) 8
(E) 12

39.

10! is divisible by 3^x5^y, where x and y are positive integers.

Quantity A	**Quantity B**
The greatest possible value for x	Twice the greatest possible value for y

40.

Quantity A	**Quantity B**
The number of distinct prime factors of 100,000	The number of distinct prime factors of 99,000

41. For which two of the following values is the product a multiple of 27?

Indicate two such values.

☐ 1
☐ 7
☐ 20
☐ 28
☐ 63
☐ 217
☐ 600
☐ 700

42. Which of the following values times 12 is not a multiple of 64?

Indicate all such values.

☐ 6^6
☐ 12^2
☐ 18^3
☐ 30^3
☐ 222

43. If $3^x(5^2)$ is divided by $3^5(5^3)$, the quotient terminates with one decimal digit. If $x > 0$, which of the following statements must be true?

(A) x is even
(B) x is odd
(C) $x < 5$
(D) $x \geq 5$
(E) $x = 5$

44. $\underline{abc}$ is a three-digit number in which a is the hundreds digit, b is the tens digit, and c is the units digit. Let &($\underline{abc}$)& = $(2^a)(3^b)(5^c)$. For example, &(203)& = $(2^2)(3^0)(5^3) = 500$. For how many three-digit numbers $\underline{abc}$ will the function &($\underline{abc}$)& yield a prime number?

(A) 0
(B) 1
(C) 2
(D) 3
(E) 9

Divisibility and Primes Answers

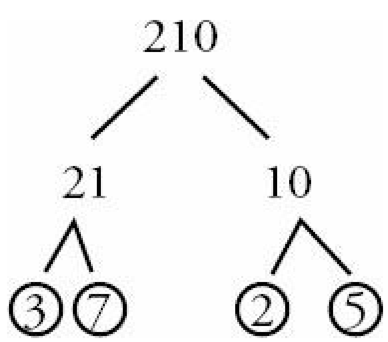
1. **4.** If x is a positive integer such that $\frac{65}{x}$ is also an integer, then x must be a factor of 65. The factors of 65 are 1, 5, 13, and 65. Thus, there are 4 positive integer values of x such that $\frac{65}{x}$ is an integer.

2. **(E).** Notice that the problem did NOT say that x had to be an integer. Therefore, the factors of 20 will work (1, 2, 4, 5, 10, 20), but so will 0.5, 0.1, 0.25, 2.5, etc. It is possible to divide 20 into fractional parts—for instance, something 20 inches long could be divided evenly into quarter inches (there would be 80 of them, as $\frac{20}{0.25} = 80$). There are an infinite number of x values that would work (it is possible to divide 20 into thousandths, millionths, etc.), so the answer is (E). It is very important on the GRE to notice whether there is an integer constraint on a variable or not! Any answer like "More than 10" should be a clue that this problem may be less straightforward than it seems.

3. **(C).** When counting factors, it helps to list them in pairs so you don't miss any. The factors of 27 are: 1 & 27, 3 & 9. The factors of 81 are: 1 & 81, 3 & 27, 9 & 9. Neither number has any even factors, so Quantity A and Quantity B are each 0 and therefore equal.

4. **(C).** The *factors* of 10 are 1 & 10, and 2 & 5. Since there are 4 factors, Quantity A is 4.

The *prime factors* of 210 are 2, 3, 5, and 7.



210 has 4 prime factors, so Quantity B is 4. Thus, the two quantities are equal.

5. **(A).** The least common multiple of 22 and 6 is 66. One way to find the least common multiple is to list the larger number's multiples (it is more efficient to begin with the larger number) until you reach a multiple that the other number goes into. The multiples of 22 are 22, 44, 66, 88, etc. The smallest of these that 6 goes into is 66.

The greatest common factor of 66 and 99 is 33. One way to find the greatest common factor is to list all the factors of one of the numbers, and then pick the greatest one that also goes into the other number. For instance, the factors of 66 are 1 & 66, 2 & 33, 3 & 22, and 6 & 11. The greatest of these that also goes into 99 is 33. Thus, Quantity A is greater.

6. **(C).** The number of students must be divisible by 10, 12, and 16. So the question is really asking, "What is the least common multiple of 10, 12, and 16?" Since all of the answer choices end in 0, each is divisible by 10. Just use your calculator to test which choices are also divisible by 12 and 16. Because you are looking for the minimum, start by checking the smallest choices. Since $\frac{120}{16}$ and $\frac{160}{12}$ are not integers, the smallest choice that works is 240.

7. **(B).** *Distinct* means *different from each other*. To find *distinct prime factors*, make a prime tree, and then disregard any repeated prime factors. The integer 27 breaks down into 3 × 3 × 3. Thus, 27 has only 1 *distinct* prime factor. The integer 18 breaks down into 2 × 3 × 3. Thus, 18 has 2 *distinct* prime factors.

8. **(C).** *Distinct* means *different from each other*. To find *distinct prime factors*, you would generally make a prime tree, and then disregard any repeated prime factors. However, 31 *is* prime, so 31 is the only prime factor of 31 and Quantity A is 1.

Any correct prime tree you make for 32 will result in five 2's, so 32 equals 2^5. Since this is the same prime factor repeated five times, 32 has only one *distinct* prime factor. Quantity B is 1, so the quantities are equal.

9. **(D).** Pick one of the numbers and list all of its factors on your paper. The factors of 120 are: 1 & 120, 2 & 60, 3 & 40, 4 & 30, 5 & 24, 6 & 20, 8 & 15, 10 & 12. Since the problem specifically asks for factors "greater than 1," eliminate 1 now. Now cross off any factors that do NOT go into 210:

~~120~~, 2 & ~~60~~, 3 & ~~40~~, ~~4~~ & 30, 5 & ~~24~~, 6 & ~~20~~, ~~8~~ & 15, 10 & ~~12~~

Now cross off any factors remaining that do NOT go into 270. Interestingly, all of the remaining factors (2, 3, 5, 6, 10, 15, 30) *do* go into 270. This is 7 shared factors.

10. **(C).** In order to distribute $4,000 and 180 pencils evenly, the number of employees must be a factor of each of these two numbers. Because you are looking for the greatest number of employees possible, start by checking the greatest choices.

(E) $4000 could not be evenly distributed among 180 employees (although 180 pencils could).
(D) $4,000 could be evenly divided among 40 people, but 180 pencils could not.
(C) is the greatest choice that works—$4,000 and 180 pencils could each be evenly distributed among 20 people.

Alternatively, find the greatest common factor (GCF) of the two numbers. Factor: 4,000 = 2 × 2 × 2 × 2 × 2 × 5 × 5 × 5 = $2^5 \times 5^3$ and 180 = 2 × 2 × 3 × 3 × 5 = $2^2 \times 3^2 \times 5$. These numbers have 2 × 2 × 5 in common, so 20 is the GCF. The correct answer is (C).

11. **II and III only.** Since n is divisible by 14 and 3, n contains the prime factors of both 14 and 3, which are 2, 7, and 3. Thus, any numbers that can be constructed using only these prime factors (no additional factors) are factors of n. Since 12 = 2 × 2 × 3, you CANNOT make 12 by multiplying the prime factors of n (you would need one more 2). However, you CAN construct 21 by multiplying two of the known prime factors of n (7 × 3 = 21), so the second statement is true. Finally, n must be at least 42 (= 2 × 7 × 3, the *least common multiple* of 14 and 3), so n is definitely a multiple of 42. That is, n can only be 42, 84, 126, etc…

12. **6.** Start by considering integer a, which is the most constrained variable. It is a positive one-digit number (between 1 and 9, inclusive) and it has four factors. Prime numbers have exactly two factors: themselves and one, so you only need to look at non-prime one-digit positive integers. That's a short enough list:

1 has just one factor!
4 has 3 factors: 1, 2, and 4
6 has 4 factors: 1, 2, 3, and 6
8 has 4 factors: 1, 2, 4, and 8
9 has 3 factors: 1, 3, and 9

So the two possibilities for a are 6 and 8. Now apply the two constraints for b. It is 9 greater than a, and it has exactly four factors. Check the possibilities:

If $a = 6$, then $b = 15$, which has 4 factors: 1, 3, 5, and 15.
If $a = 8$, then $b = 17$, which is prime, so it has only has 2 factors: 1 and 17.
Only $b = 15$ works, so a must be 6.

13. **(E).** Cutting a rectangular board into square pieces means that Ramon needs to cut pieces that are equal in length and width. "Without wasting any of the board" means that he needs to choose a side length that divides evenly into both 18 and 30. "The least number of square pieces" means that he needs to choose the largest possible squares. With these three stipulations, choose the largest integer that divides evenly into 18 and 30, or the greatest common factor, which is 6. This would give Ramon 3 pieces going one way and 5 pieces going the other. He would cut $3 \times 5 = 15$ squares of dimension 6" × 6". Note that this solution ignored squares with non-integer side length for the sake of convenience, a potentially dangerous thing to do. (After all, identical squares of 1.5" by 1.5" could be cut without wasting any of the board.) However, to cut fewer squares that are larger than 6" × 6", Ramon could only cut 2 squares of 9" or 1 square of 18" from the 18" dimension of the rectangle, neither of which would evenly divide the 30" dimension of the rectangle. The computed answer is correct.

14. **4.** Because this is a numeric entry question, you can infer that the answer will be the same regardless of which two-digit prime you pick. So for the sake of simplicity, pick the smallest and most familiar two-digit prime: 11.

If n is the product of 2, 3, and 11, n equals 66 and its factors are:

Small	Large
1	66
2	33
3	22
6	11

There are four factors greater than 6: 11, 22, 33, and 66.

Notice that because the other given prime factors of n (2 and 3) multiply to get exactly 6, you can only produce a factor greater than 6 by multiplying by the third factor, the "two-digit prime number." The right-hand column represents that third factor multiplied by all of the other factors: 11×6, 11×3, 11×2, and 11×1. If you replace 11 with any other two-digit prime, you will get the same result. (If you're not sure, try it!)

15. **(D).** Test values for m with the goal of proving (D). Because m has a factor of 8, m could equal 8, 16, 24, 32, 40, etc. If m is 24, both quantities are equal to 0. But if m is 32, Quantity A is 2 and Quantity B is 8.

16. **(B).** When dealing with remainder questions on the GRE, the best thing to do is test a few real numbers.

Multiples of 6 are 0, 6, 12, 18, 24, 30, 36, etc.

Numbers with a remainder of 4 when divided by 6 are those 4 greater than the multiples of 6:

x could be 4, 10, 16, 22, 28, 34, 40, etc.

You could keep listing numbers, but this is probably enough to establish a pattern.

(A) $x/2 \longrightarrow$ ALL of the listed x values are divisible by 2. Eliminate (A).
(B) $x/3 \longrightarrow$ NONE of the listed x values are divisible by 3, but continue checking.
(C) $x/7 \longrightarrow$ 28 is divisible by 7.
(D) $x/11 \longrightarrow$ 22 is divisible by 11.
(E) $x/17 \longrightarrow$ 34 is divisible by 17.

The question is "Each of the following could also be an integer EXCEPT." Since four of the choices could be integers, (B) must be the answer.

17. **III, IV, and IV only.** If $x^y = 64$ and x and y are positive integers, perhaps the most obvious possibility is that $x = 8$ and $y = 2$. However, "<u>all</u> such values" implies that other solutions are possible. One shortcut is noting that only an even base, when raised to a power, could equal 64. So you only have to worry about even possibilities for x. Here are all the possibilities:

$$2^6 = 64 \longrightarrow x + y = 8$$
$$4^3 = 64 \longrightarrow x + y = 7$$
$$8^2 = 64 \longrightarrow x + y = 10$$
$$64^1 = 64 \longrightarrow x + y = 65$$

The only possible values of $x + y$ listed among the choices are 7, 8, and 10.

18. **(C).** If k is a multiple of 24, it contains the prime factors of 24: 2, 2, 2, and 3. (It could also contain other prime factors, but you can only be sure of the prime factors contained in 24.)

If k were a multiple of 16, it would contain the prime factors of 16: 2, 2, 2, and 2.

Thus, if k is a multiple of 24 but NOT of 16, k must contain 2, 2, and 2, but NOT a fourth 2 (otherwise, it would be a multiple of 16).

Thus: k definitely has 2, 2, 2, and 3. It could have any other prime factors (including more 3's) EXCEPT for more 2's.

An answer choice in which the denominator contains more than three 2's would guarantee a non-integer result. Only choice (C) works. Since k has fewer 2's than 32, $k/32$ can never be an integer.

Alternatively, list multiples of 24 for k: 24, 48, 72, 96, 120, 144, 168, etc.

Then, eliminate multiples of 16 from this list: 24, ~~48~~, 72, ~~96~~, 120, ~~144~~, 168, etc.

A pattern emerges: k = (an odd integer) × 24.

(A) $k/8$ can be an integer, for example when $k = 24$.
(B) $k/9$ can be an integer, for example when $k = 72$.
(C) $k/32$ is correct by process of elimination.
(D) $k/36$ can be an integer, for example when $k = 72$.
(E) $k/81$ can be an integer, for example when $k = 81 \times 24$.

19. **10.** Because this is a numeric entry question, there can be only one correct answer. So, plugging in any prime number greater than 2 for b must yield the same result. Try $b = 3$.

If $a = 16b$ and $b = 3$, then a is 48. The factors (NOT prime factors) of 48 are: 1 & 48, 2 & 24, 3 & 16, 4 & 12, and 6 & 8. There are 10 distinct factors.

20. **(C).** The two equations are already solved for b and d, and the question is about the value of $b + d$. So, stack the equations and add:

$$\begin{array}{l} 4a + 3 = b \\ \underline{4c + 1 = d} \\ 4a + 4c + 4 = b + d \end{array}$$

Because a and c are integers, $4a + 4c + 4$ is the sum of three multiples of 4, which is a multiple of 4 itself. Therefore, the other side of the equation, $b + d$, must also equal a multiple of 4.

You could also factor out the 4:

$$4a + 4c + 4$$
$$4(a + c + 1)$$

Since a and c are integers, $a + c + 1$ is an integer, so $4(a + c + 1)$ is definitely a multiple of 4, and $b + d$ is also a multiple of 4. Only choice (C) is a multiple of 4.

21. **(C).** The factors of 210 are as follows:

1 & 210
2 & 105
3 & 70
5 & 42
6 & 35
7 & 30

10 & 21
14 & 15

Out of the list of 16 factors, there are two multiples of 42 (42 and 210).

Thus, the answer is 2/16 or 1/8.

22. **10.** If 1/4 of the poodles are also show dogs, the number of poodles must be divisible by 4. (The number of dogs is necessarily an integer.) Since the least possible number is the goal, try an example with 4 poodles.

If 1/7 of the show dogs are poodles, the number of show dogs must be divisible by 7. Since the least possible number is the goal, try an example with 7 show dogs.

So far there are:

4 poodles, 1 of which is a show dog
7 show dogs, 1 of which is a poodle

Note that the one poodle that is also a show dog *is the same dog* as the one show dog that is also a poodle! To get the total number of dogs, only count that dog *once,* not twice. In total:

3 poodles (non-show dogs)
1 dog that is both poodle and show dog
6 show dogs (non-poodles)

This equals 10 dogs in total. This example met all the constraints of the question while using minimum values at each step, so this is the least possible number of dogs at the fair.

23. **(C).** Break down each of the numbers into its prime factors.

(A) $49 = 7 \times 7$
(B) $81 = 3 \times 3 \times 3 \times 3$
(C) $100 = 2 \times 2 \times 5 \times 5$
(D) $121 = 11 \times 11$
(E) $243 = 3 \times 3 \times 3 \times 3 \times 3$

Since 100 has both 2 and 5 as prime factors, it is not a prime power. The correct answer is (C).

24. **(C).** Since a positive multiple must be equal to or larger than the number it is a multiple of, answer choice (C) cannot be a multiple of a or b, as it is smaller than both integers a and b.

You can also try testing numbers such that a is larger than b.

(A) If $a = 3$ and $b = 2$, $a - 1 = 2$, which is a multiple of b.
(B) If $a = 3$ and $b = 2$, $b + 1 = 3$, which is a multiple of a.
(C) Is the correct answer by process of elimination.
(D) If $a = 4$ and $b = 2$, $a + b = 6$, which is a multiple of b.
(E) If $a = 3$ and $b = 2$, $ab = 6$, which is a multiple of both a and b.

25. **12.** Remember, remainders are always whole numbers, so dividing 616 by 6 in your calculator won't quite give you what you need. Rather, find the largest number less than 616 that 6 *does* go into (not 615, not 614, not 613 …). That number is 612. Since 616 - 612 = 4, the remainder p is equal to 4.

Alternatively, you could divide 616 by 6 in your calculator to get 102.66…. Since 6 goes into 616 precisely 102 whole times, multiply 6×102 to get 612, then subtract from 616 to get remainder 4.

This second method might be best for finding q. Divide 525 by 11 to get 47.7272…. Since $47 \times 11 = 517$, the remainder is $525 - 517 = 8$.

Therefore, $p + q = 4 + 8 = 12$.

26. **I only.** To solve this problem with examples, make a short list of possibilities for each of x and y:

$x = 18, 36, 54\ldots$
$y = 12, 24, 36\ldots$

Now try to *disprove* the statements by trying several combinations of x and y above. In Statement I, $x + y$ could be $18 + 12 = 30$, $54 + 12 = 66$, $36 + 24 = 60$, or many other combinations. Interestingly, all those combinations are multiples of 6. This makes sense, as x and y individually are multiples of 6, so their sum is too. Statement I is true.

To test statement II, xy could be $18(12) = 216$, which is NOT divisible by 48. Eliminate statement II.

As for statement III, x/y could be 18/12, which is not even an integer (and therefore not divisible by 6), so III is not necessarily true.

27. **(D).** This problem is most easily solved with an example. If $p = 7$ and $q = 6$, then $pg = 42$, which has the factors 1 & 42, 2 & 21, 3 & 14, and 6 & 7. That's 8 factors, but read carefully! The question asks how many factors *greater than 1*, so the answer is 7. Note that choosing the smallest possible examples ($p = 7$ and $q = 6$) was the right move here, since the question asks "at least how many factors…?" If testing $p = 70$ and $q = 36$, many, many more factors would have resulted. The question asks for the minimum.

28. **(C).** This problem is most easily solved with an example. If $r = 10$ and $s = 9$, then $rs = 90$. The factors of 90 are 1 & 90, 2 & 45, 3 & 30, 5 & 18, 6 & 15, and 9 & 10. Count to get a minimum of 12 factors.

29. **(D).** If t is divisible by 12, then t^2 must be divisible by 144 or $2 \times 2 \times 2 \times 2 \times 3 \times 3$. Therefore, t^2 can be divided evenly by 2 at least four times, so a must be at least 5 before $\frac{t^2}{2^a}$ might not be an integer.

Alternatively, test values. If $t = 12$, $\frac{t^2}{2^a} = \frac{144}{2^a}$. Plug in the choices as possible a values, starting with the smallest choice and working up.

(A) Since $144/2^2 = 36$, eliminate.
(B) Since $144/2^3 = 18$, eliminate.

(C) Since $144/2^4 = 9$, eliminate.

(D) $144/2^5 = 4.5$. The first choice for which $\frac{t^2}{2^a}$ might not be an integer is (D).

30. **I, II, and III.** Since a, b, and c are all multiples of 3, $a = 3x$, $b = 3y$, $c = 3z$, where $x > y > z > 0$ and all are integers. Substitute these new expressions into the statements.

Statement I: $a + b + c = 3x + 3y + 3z = 3(x + y + z)$. Since $(x + y + z)$ is an integer, this number must be divisible by 3.

Statement II: $a - b + c = 3x - 3y + 3z = 3(x - y + z)$. Since $(x + y + z)$ is an integer, this number must be divisible by 3.

Statement III: $abc/9 = (3x3y3z)/9 = (27xyz)/9 = 3xyz$. Since xyz is an integer, this number must be divisible by 3.

31. **(C).** Pattern problems on the GRE often include a very large series of items that would be impossible (or at least unwise) to write out on paper. Instead, this problem requires you to recognize and exploit the pattern. In this case, after every 4th car, the color pattern repeats. By dividing 463 by 4, you find that there will be 115 cycles through the 4 colors of cars—red, blue, black, gray—for a total of 460 cars to exit the factory. The key to solving these problems is the remainder. Because there are 463 - 460 = 3 cars remaining, the first such car will be red, the second will be blue, and the third will be black.

32. **(D).** This is a pattern problem. An efficient method is to recognize that the 7th day after the initial deposit would be Tuesday, as would the 14th day, the 21st day, etc. Divide 100 by 7 to get 14 full weeks comprising 98 days, plus 2 days left over. For the two leftover days, think about when they would fall. The first day after the deposit would be a Wednesday, as would the first day after waiting 98 days. The second day after the deposit would be a Thursday, and so would the 100^{th} day.

33. **(D).** Division problems can be interpreted as follows: dividend = divisor × quotient + remainder. This problem is dividing x by 7, or distributing x items equally to 7 groups. After the items are distributed among the 7 groups, there are 3 things left over, the remainder. This means that the value of x must be some number that is 3 larger than a multiple of 7, such as 3, 10, 17, 24, etc. The only answer choice that is 3 larger than a multiple of 7 is 52.

34. **II and III only.** Start by rearranging the equation: $\frac{ab}{c+d} = \frac{37}{10}$ is equivalent to $10ab = 37(c + d)$. Remember that all four variables are positive integers. Because 37 and 10 have no shared factors, $(c + d)$ must be a multiple of 10 and ab must be a multiple of 37, in order to make the equation balance.

I. Could be true. All the requirements are met if $ab = (5)(37)$ and $(c + d) = 50$, so ab could be divisible by 5. But all the requirements are met if $ab = 37$ and $(c + d) = 10$, in which case ab is not divisible by 5.

II. MUST be true. Because $(c + d)$ must be divisible by 10, it must be divisible by both 5 and 2.

III. MUST be true. Because $(c + d)$ must be divisible by 10, it must be divisible by both 5 and 2. Thus, $(c + d)$ must be even, so if c were even, d would have to be even, too.

35. **(E).** This is a bit of a trick question—any number that yields remainder 4 when divided by 10 will also yield remainder 4 when divided by 5. This is because the remainder 4 is less than both divisors, and all multiples of 10 are

also multiples of 5. For example, 14 yields remainder 4 when divided either by 10 or by 5. This also works for 24, 34, 44, 54, etc.

36. **0.** The remainder when dividing an integer by 10 always equals the units digit. You can also ignore all but the units digits, so the question can be rephrased as: *What is the units digit of* $3^{17} + 7^{13}$*?*

The pattern for the units digits of 3 is [3, 9, 7, 1]. Every fourth term is the same. The 17th power is 1 past the end of the repeat: 17 - 16 = 1. Thus, 3^{17} must end in 3.

The pattern for the units digits of 7 is [7, 9, 3, 1]. Every fourth term is the same. The 13th power is 1 past the end of the repeat: 13 - 12 = 1. Thus, 7^{13} must end in 7. The sum of these units digits is 3 + 7 = 10. Thus, the units digit is 0.

37. **110.** When dividing, the remainder is always less than the divisor. If you divided a by b to get a remainder of 9, then b must have been greater than 9. Similarly, d must be greater than 10. Since b and d are integers, the smallest they could be is 10 and 11, respectively.

Thus, the minimum that bd could be is $10 \times 11 = 110$.

As an example, try $a = 19$, $b = 10$, $c = 21$, and $d = 11$ (generate a by adding remainder 9 to the value of b, and generate c by adding remainder 10 to the value of d.)

It is not possible to generate an example in which *any* of the four numbers are smaller. The least possible value of bd is 110.

38. **(C).** Start by considering the relationship between n and n^3. Because n is an integer, for every prime factor n has, n^3 must have three of them. Thus, n^3 must have prime numbers in multiples of 3. If n^3 has one prime factor of 3, it must actually have two more, because n^3's prime factors can only come in triples.

The question says that n^3 is divisible by 24, so n^3's prime factors must include at least three 2's and a 3. But since n^3 is a cube, it must contain at least three 3's. Therefore n must contain at least one 2 and one 3, or $2 \times 3 = 6$.

39. **(C).** First, expand 10! as $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.

(Do NOT multiply all of those numbers together to get 3,628,800—it's true that 3,628,800 is the value of 10!, but analysis of the prime factors of 10! is easier in the current form.)

Note that 10! is divisible by 3^x5^y, and the quantities concern the greatest possible values of x and y, which is equivalent to asking, "What is the maximum number of times you can divide 3 and 5, respectively, out of 10! while still getting an integer answer?"

In the product $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$, only the multiples of 3 have 3 in their prime factors, and only the multiples of 5 have 5 in their prime factors. Here are all the primes contained in $10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ and therefore in 10!:

$10 = 5 \times 2$

$9 = 3 \times 3$
$8 = 2 \times 2 \times 2$
$7 = 7$
$6 = 2 \times 3$
$5 = 5$
$4 = 2 \times 2$
$3 = 3$
$2 = 2$
$1 =$ no primes

There are four 3's and two 5's total. The maximum values are $x = 4$ and $y = 2$. Therefore, Quantity A and Quantity B are each 4, so the correct answer is (C).

40. **(B).** Since only the number of *distinct* prime factors matter, not what they are or how many times they are present, you can tell on sight that Quantity A has only 2 distinct prime factors, because 100,000 is a power of 10. (Any prime tree for 10, 100, or 1,000, etc. will contain only the prime factors 2 and 5, occurring in pairs.)

In Quantity B, 99,000 breaks down as $99 \times 1{,}000$. Since 1,000 also contains 2's and 5's, and 99 contains even more factors (specifically 3, 3, and 11), Quantity B has more distinct prime factors. It is not necessary to make prime trees for each number.

41. **V and VII only.** For two numbers to have a product that is a multiple of 27, the two numbers need to have at least three 3's among their combined prime factors, since 27 = 33. Only 63 and 600 are multiples of 3, so the other choices could be eliminated very quickly if you see that. There's no need to actually multiply the numbers together. Since 63 is $3 \times 3 \times 7$ and 600 is 3×200, their product will have the three 3's required for a multiple of 27.

42. **III, IV, and V only.** Because $64 = 2^6$, multiples of 64 would have at least six 2's among their prime factors.

Since 12 (which is $2 \times 2 \times 3$) has two 2's already, a number that could be multiplied by 12 to generate a multiple of 64 would need to have, at minimum, the *other* four 2's needed to generate a multiple of 64.

Since you want the choices that don't multiply with 12 to generate a multiple of 64, select only the choices that have *four or fewer 2's* within their prime factors.

6^6	$= (2 \times 3)^6$	six 2's	INCORRECT
12^2	$= (2^2 \times 3)^2$	four 2's	INCORRECT
18^3	$= (2 \times 3^2)^3$	three 2's	CORRECT
30^3	$= (2 \times 3 \times 5)^3$	three 2's	CORRECT
222	$= (2 \times 3 \times 37)$	one 2	CORRECT

43. **(D).** When a non-multiple of 3 is divided by 3, the quotient does not terminate (for instance, $1/3 = 0.333\ldots$).

Since $3^x(5^2)/3^5(5^3)$ does NOT repeat forever, x must be large enough to cancel out the 3^5 in the denominator. Thus, x must be at least 5. Note that the question asks what MUST be true. Choice (D) must be true. Choice (E), $x = 5$, represents one value that would work, but this choice does not *have* to be true.

44. **(B).** Since a prime number has only two factors, 1 and itself, $(2^a)(3^b)(5^c)$ cannot be prime unless the digits a, b and c are such that two of the digits are 0 and the third is 1. For instance, $(2^0)(3^1)(5^0) = (1)(3)(1) = 3$ is prime. Thus, the only three values of $\underline{abc}$ that would result in a prime number &($\underline{abc}$)& are 100, 010, and 001. However, only one of those three numbers (100) is a three digit number.

Chapter 14

of

Exponents and Roots

In This Chapter…

Exponents and Roots

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

Quantity A	**Quantity B**
25^7	5^{15}

2.

$$216 = 2^x 3^y$$

x and y are integers.

Quantity A	**Quantity B**
x	y

3.

Quantity A	**Quantity B**

$\sqrt{18}\sqrt{2}$	$\sqrt{6}$

4.

Quantity A	**Quantity B**
$\sqrt{3}+\sqrt{6}$	$\sqrt{9}$

5.

Quantity A	**Quantity B**
$\sqrt{7{,}777{,}777{,}777}$	88,000

6. If $5{,}000 = 2^x5^y$ and x and y are integers, what is $x + y$?

7. If $3^29^2 = 3^x$, what is x?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

8.

80 is divisible by 2^x.

Quantity A	**Quantity B**
x	3

9.

Quantity A	**Quantity B**
$(81)^2(900)^3$	270^6

10. If $17\sqrt[3]{m} = 34$, what is $6\sqrt[3]{m}$?

11. $\dfrac{1}{\dfrac{1}{\dfrac{1}{5^{-2}}}}$ is equivalent to:

(A) 1/25
(B) 1/5
(C) 1
(D) 5
(E) 25

12. If $77{,}742y^{11} = 4x^2$, what is $\dfrac{77{,}742y^{11}}{8x^2}$?

$$\frac{\boxed{}}{\boxed{}}$$

13. $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}} =$

(A) $\sqrt{2}$
(B) 2
(C) $2\sqrt{2}$
(D) 4
(E) $4\sqrt{2}$

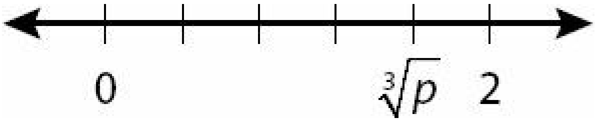
14.

<u>**Quantity A**</u>	<u>**Quantity B**</u>
$\dfrac{200}{\sqrt{200}}$	$\sqrt{200}$

15. For what positive integer is the square of the integer divided by the cube root of the integer equal to nine times the integer?

(A) 4
(B) 8
(C) 16
(D) 27
(E) 125

16.



If the hash marks above are equally spaced, what is the value of p?

(A) 3/2
(B) 8/5
(C) 24/15
(D) 512/125
(E) 625/256

17. What is the greatest prime factor of $2^{99} - 2^{96}$?

18. If $2^k - 2^{k+1} + 2^{k-1} = 2^k m$ what is m?

(A) -1
(B) -1/2
(C) 1/2
(D) 1
(E) 2

19.

Quantity A	**Quantity B**
$\frac{2}{9}(81)^{50}$	$\frac{(3^2)(9)^{99}}{2}$

20. If $5^{k+1} = 2{,}000$, what is $5^k + 1$?

(A) 399
(B) 401
(C) 1,996
(D) 2,000
(E) 2,001

21. If $3^{11} = 9^x$, what is the value of x?

22. If $x^7 = 2.5$, what is x^{14}?

23. If $\sqrt[5]{x^6} = x^{\frac{a}{b}}$, then the value of a/b =

24. $\dfrac{20^{-5}5^{10}8^{6}}{10^{8}25^{-2}} = ?$

(A) 1
(B) 4
(C) 5
(D) 6
(E) 10

25. If $\dfrac{5^7}{5^{-4}} = 5^a$ and $\dfrac{2^{-3}}{2^{-2}} = 2^b$ and $3^8(3) = 3^c$, what is the value of $a + b + c$?

26. If 12^x is odd and x is an integer, what is the value of x^{12}?

27. $\dfrac{200^{\frac{5}{2}}}{\sqrt{200}} = ?$

(A) 4
(B) 40
(C) 400
(D) 4,000
(E) 40,000

28.

$$\frac{(10^3)(0.027)}{(900)(10^{-2})} = (3)(10^m)$$

Quantity A	Quantity B
The value of m	3

29. $\frac{1}{3}(10^6 - 10^4) = ?$

(A) $33.\overline{3}$
(B) $3{,}333.\overline{3}$
(C) 33,000
(D) 330,000
(E) 333,333

30. Simplify: $\frac{2^2 + 2^2 + 2^3 + 2^4}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$

(A) 2
(B) 4
(C) 8
(D) 16
(E) 32

31. $\frac{2^{-4}3^{-20}}{4^{-1}9^{-6}} =$

(A) $2^2 3^8$
(B) $2^1 3^{12}$
(C) $\frac{1}{2^2 3^8}$
(D) $\frac{1}{2^1 3^{12}}$
(E) $\frac{1}{2^2 3^{12}}$

32. If $\frac{0.000027 \times 10^x}{900 \times 10^{-4}} = 0.03 \times 10^{11}$, what is the value of x?

(A) 13
(B) 14
(C) 15
(D) 16
(E) 17

33. $\left(\sqrt[2]{x}\right)\left(\sqrt[3]{x}\right) =$

(A) $\sqrt[5]{x}$

(B) $\sqrt[6]{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[5]{x^6}$

(E) $\sqrt[6]{x^5}$

34. $\left(\sqrt[3]{x^2}\right)\left(\sqrt[4]{x^5}\right)=$

(A) $\sqrt[7]{x^{10}}$

(B) $\sqrt[12]{x^{10}}$

(C) $\sqrt[12]{x^7}$

(D) $\sqrt[9]{x^{23}}$

(E) $\sqrt[12]{x^{23}}$

35.

$n = 0.00025 \times 10^4$ and $m = 0.005 \times 10^2$

Quantity A	**Quantity B**
$\dfrac{n}{m}$	0.5

36. $\dfrac{40^{50}-40^{48}}{2^{96}}\times 10^{-45}=$

(A) 20
(B) $10^3(1{,}599)$
(C) $10^2(1{,}601)$
(D) 200^6
(E) 200^{53}

37. Which of the following is equal to $x^{\frac{3}{2}}$??

(A) $x^2\sqrt{x}$

(B) $x\sqrt{x}$

(C) $\sqrt[3]{x^2}$

(D) $\sqrt[3]{x}$

(E) $(x^3)^2$

38. $\sqrt{(360)(240)(3)(2)} =$

(A) 180
(B) 360
(C) 720
(D) 1,440
(E) 3,600

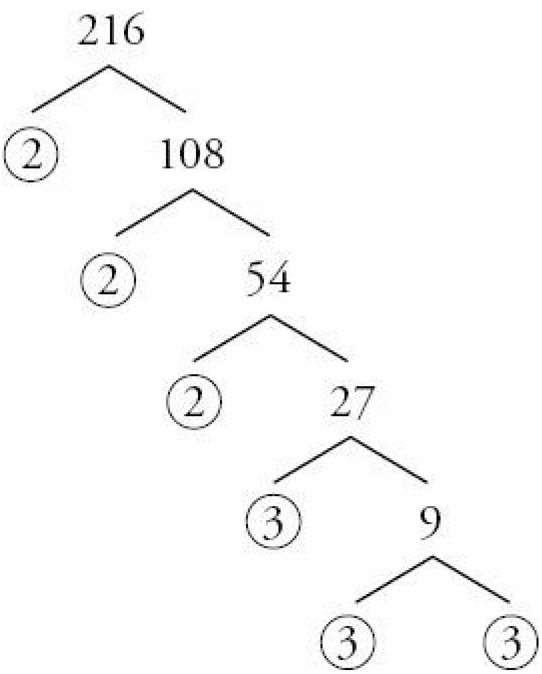
39. If $125^{14}48^{8}$ is written out as an integer, how many consecutive zeroes will that integer have at the end?

(A) 22
(B) 32
(C) 42
(D) 50
(E) 112

Exponents and Roots Answers

1. **(B).** In problems asking you to compare or combine exponents with different bases, convert to the same base when possible. Since $25 = 5^2$, Quantity A is equal to $(5^2)^7$. When raising a power to a power, multiply the exponents. Quantity A is equal to 5^{14}, so Quantity B is larger.

2. **(C).** Make a prime tree for 216:



$216 = 2^3 3^3$, so $x = 3$ and $y = 3$.

3. **(A).** In Quantity A, $\sqrt{18}\sqrt{2} = \sqrt{36} = 6$. Since 6 is greater than $\sqrt{6}$, Quantity A is larger.

4. **(A).** You may NOT add $\sqrt{3}$ and $\sqrt{6}$ to get $\sqrt{9}$, but you can simply put each value in your calculator. $\sqrt{3}$ = 1.732… and $\sqrt{6}$ = 2.449…, and their sum is about 4.18. Since Quantity B is $\sqrt{9}$ = 3, Quantity A is larger.

5. **(A).** You have a calculator with a square root button, but 7,777,777,777 is too large for the calculator.

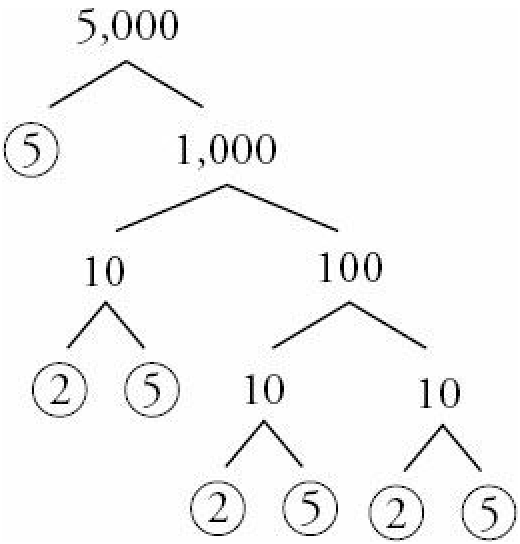
You could also square both quantities, getting 7,777,777,777 in Quantity A and $88{,}000^2$ in Quantity B. However, $88{,}000^2$ is also too large for the calculator. Here's a quick workaround. Square 88 in your calculator (there is no "squared" button on the GRE calculator — you have to type 88 × 88):

$88^2 = 7{,}744$

Therefore $88{,}000^2 = 7{,}744 \times 10^6 = 7{,}744{,}000{,}000$

Thus, Quantity A is larger.

6. **7.** Make a prime tree for 5,000:



You can see that $5{,}000 = 2^3 5^4$, therefore $x = 3$ and $y = 4$, and the answer is $3 + 4 = 7$.

7. **(E).** In problems asking you to compare or combine exponents with different bases, convert to the same base when possible. Since $9 = 3^2$:

$$3^2(3^2)^2 = 3^x$$

Multiply exponents when raising a power to a power:

$$3^2 3^4 = 3^x$$

Add exponents when multiplying with the same base:

$$3^6 = 3^x$$

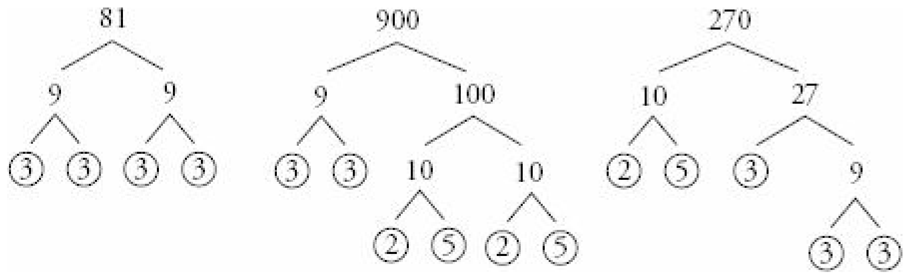
Therefore, $x = 6$.

8. **(D).** Make a prime tree for 80, or simply divide 80 by 2 in your calculator as many times as you can before you get a non-integer answer: 2 goes into 80 exactly 4 times.

That doesn't mean x *is 4, however!* The problem did NOT say "80 is *equal to* 2^x". Rather, it said "divisible by."

80 is divisible by 2^4, and therefore also by 2^3, 2^2, 2^1, and even 2^0 (anything to the 0th power = 1). Thus, x could be 0, 1, 2, 3, or 4, and could therefore be smaller than, equal to, or greater than 3.

9. **(B).** Break down 81, 900, and 270 into their prime factors:



$81 = 3^4$, $900 = 2^2 3^2 5^2$, and $270 = 2^1 3^3 5^1$. Therefore:

Quantity A = $(3^4)^2(2^2 3^2 5^2)^3 = (3^8)(2^6 3^6 5^6) = 2^6 3^{14} 5^6$

Quantity B = $(2^1 3^3 5^1)^6 = 2^6 3^{18} 5^6$

Since Quantity A and Quantity B both have 2^6 and 5^6, focus on 3^{14} vs. 3^{18}. Quantity B is larger.

10. **12.** This question looks much more complicated than it really is — note that you are not asked for *m* itself, but rather for $\sqrt[3]{m}$. Just think of $\sqrt[3]{m}$ as a very fancy variable that you don't have to break down:

$$17\sqrt[3]{m} = 34$$

$$\sqrt[3]{m} = \frac{34}{17}$$

$$\sqrt[3]{m} = 2$$

Therefore, $6\sqrt[3]{m}$.

11. **(E).** One quick trick to simplifying efficiently here is knowing that a negative exponent in the denominator turns into a positive exponent in the numerator. In other words, the lowermost portion of the fraction, $\frac{1}{5^{-2}}$, is simply equal to 5^2.

$$\frac{1}{\frac{1}{\frac{1}{5^{-2}}}} = \frac{1}{\frac{1}{5^2}}$$

Dividing by $\frac{1}{5^{-2}}$ is the same as multiplying by the reciprocal, which will leave 5^2 in the numerator:

$$\frac{1}{\frac{1}{5^2}} = 1 \times \frac{5^2}{1} = 5^2$$

The answer is simply 5^2, or 25.

12. **1/2.** This question looks much more complicated than it really is! Since $77{,}742y^{11} = 4x^2$, simply substitute $4x^2$ for $7{,}742y^{11}$ in the numerator:

$$\frac{4x^2}{8x^2} = \frac{1}{2}$$

13. **(B).** To begin solving, start at the "inner core" — that is, the physically smallest root sign:

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{4}}}} =$$
$$\sqrt{2+\sqrt{2+\sqrt{2+2}}} =$$
$$\sqrt{2+\sqrt{2+2}} =$$
$$\sqrt{2+2} = 2$$

14. **(C).** Quantity A has a root sign on the bottom of a fraction. When you see this, rationalize the denominator by multiplying the fraction by the denominator over the denominator (so that the number you are multiplying by is equal to 1, so you are not changing the value of the fraction):

$$\frac{200}{\sqrt{200}}\left(\frac{\sqrt{200}}{\sqrt{200}}\right) = \frac{200\sqrt{200}}{200} = \sqrt{200}$$

(Please note that $\sqrt{200} \times \sqrt{200}$ is simply 200. There is no need to multiply out to get $\sqrt{40{,}000} = \sqrt{200} \times \sqrt{200} = 200$. Instead, simply think of the root signs canceling out.) The two quantities are the same.

15. **(D).** To solve this question, you need to write the information from the question as an equation. Call "the square of the integer" x^2, "the cube root of the integer" $\sqrt[3]{x}$, and "nine times the integer" $9x$:

$$\frac{x^2}{\sqrt[3]{x}} = 9x$$

There are a few ways to proceed from here, but it might be most helpful to convert $\sqrt[3]{x}$ into its other form, $x^{\frac{1}{3}}$, and then subtract the exponents on the left side of the equation (always subtract exponents, of course, when dividing with the same base):

$$\frac{x^2}{x^{\frac{1}{3}}} = 9x$$

$$x^{2-\frac{1}{3}} = 9x$$

$$x^{\frac{5}{3}} = 9x$$

A good next move would be to raise both sides to the 3rd power:

$$\left(x^{\frac{5}{3}}\right)^3 = (9x)^3$$
$$x^5 = 9^3x^3$$

Now simply divide both sides by x^3:

$$x^2 = 9^3$$

Since 9 is really just 3^2:

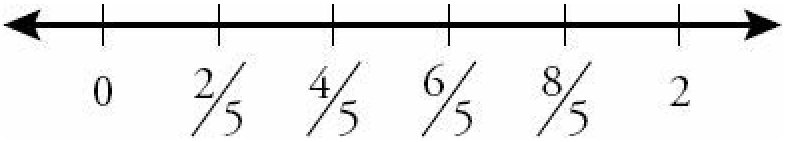
$$x^2 = (3^2)^3$$
$$x^2 = 3^6$$
$$\sqrt{x^2} = \sqrt{3^6}$$
$$x = 3^3$$
$$x = 27$$

Alternatively, you could try the answers. For instance, for choice (E):

$$\frac{27^2}{\sqrt[3]{27}} = 9(27)$$
$$\frac{27^2}{3} = 9(27)$$
$$27^2 = 9(27)(3)$$
$$27 = 9(3)$$

Thus, choice (E) is correct. While this backsolving approach works, it may be a bit slower than the algebra approach.

16. **(D).** To determine the distance between hash marks, divide 2 (the distance from 0 to 2) by 5 (the number of segments the number line has been divided into). The result is 2/5. Therefore:



Note that 2 is equal to 10/5, so you can see that the number line is labeled correctly.

Since $\sqrt[3]{p}$ marks the same hash mark on the number line as 8/5:

$$\sqrt[3]{p} = \frac{8}{5}$$

$$p = \left(\frac{8}{5}\right)^3$$

$$p = \frac{512}{125}$$

The answer is (D). Watch out for trap answer choice (B), which represents $\sqrt[3]{p}$, not *p*.

17. **7**. You cannot subtract $2^{99} - 2^{96}$ to get 2^3! You cannot directly combine, even with the same base, when adding or subtracting. (As it turns out, the difference between 2^{99} and 2^{96} is much, much larger than 2^3.) Instead, factor out the largest number 2^{99} and 2^{96} have in common:

$$2^{99} - 2^{96} = 2^{96}(2^3 - 1) = 2^{96}(7)$$

Since $2^{99} - 2^{96}$ is equal to $2^{96}7^1$, its greatest prime factor is 7.

18. **(B).** First, break down 2^{k+1} as 2^k2^1 and 2^{k-1} as 2^k2^{-1}:

$$2^k - 2^k2^1 + 2^k2^{-1} = 2^km$$

Factor out 2^k from the left, then cancel 2^k from both sides:

$$2^k(1 - 2^1 + 2^{-1}) = 2^k m$$

$$1 - 2^1 + 2^{-1} = m$$

$$1 - 2 + \frac{1}{2} = m$$

$$-\frac{1}{2} = m$$

19. **(B).** A good way to begin comparing these quantities is to look for similarities — specifically, 81^{50} and 9^{99} can each be broken down to powers of 3, as $81 = 3^4$ and $9 = 3^2$:

Quantity A: $\dfrac{2}{9}\left(3^4\right)^{50} = \dfrac{2}{9}\left(3^{200}\right)$

Quantity B: $\dfrac{\left(3^2\right)\left(3^2\right)^{99}}{2} = \dfrac{\left(3^2\right)\left(3^{198}\right)}{2} = \dfrac{3^{200}}{2}$ or $\dfrac{1}{2}\left(3^{200}\right)$

Since 3^{200} is the same on both sides, ignore it (or eliminate it by dividing both quantities by 3^{200}). Since 1/2 is greater than 2/9, Quantity B is larger.

20. **(B).** The key to solving this problem is realizing that you can split 5^{k+1} into 5^k5^1. (Exponents are added when multiplying with the same base, so the process can also be reversed; thus, any expression with the form x^{a+b} can be split into x^ax^b.)

$5^{k+1} = 2{,}000$

$5^k5^1 = 2{,}000$

Now divide both sides by 5:

$5^k = 400$

So, $5^k + 1 = 401$.

Notice that you can't solve for k itself — k is not an integer, since 400 is not a "normal" power of 5. But you don't need to solve for k. You just need 5^k.

21. **5.5**. Begin by converting 9 to a power of 3:

$3^{11} = (3^2)^x$
$3^{11} = 3^{2x}$

Thus, $11 = 2x$ and $x = 5.5$.

22. **6.25**. It is not necessary to find x to solve this problem. Simply square both sides:

$x^7 = 2.5$
$(x^7)^2 = (2.5)^2$
$x^{14} = 6.25$

23. **6/5.** Just as a square root is the same as a 1/2 exponent, so too is a fifth root the same as a 1/5 exponent. Thus:

$$\sqrt[5]{x^6} = (x^6)^{\frac{1}{5}} = x^{\frac{6}{5}}$$

Since $x^{\frac{6}{5}} = x^{\frac{a}{b}}$, $a/b = 6/5$.

24. **(C).** Since $20^{-5} = \frac{1}{20^5}$ and $25^{-2} = \frac{1}{25^2}$, one quick shortcut is to convert any term with a negative exponent to one with a positive exponent by moving it from the numerator to the denominator or vice-versa:

$$\frac{20^{-5}5^{10}8^6}{10^8 25^{-2}} = \frac{5^{10}8^6 25^2}{20^5 10^8}$$

Then, convert the non-prime terms to primes:

$$\frac{5^{10}8^6 25^2}{20^5 10^8} = \frac{5^{10}(2^3)^6(5^2)^2}{(2^2 5^1)^5(2^1 5^1)^8} = \frac{5^{10}2^{18}5^4}{2^{10}5^5 2^8 5^8} = \frac{2^{18}5^{14}}{2^{18}5^{13}} = 5$$

25. **19.** To solve this problem, you need to know that when dividing with the same base, you subtract the exponents, and when multiplying with the same base, you add the exponents. Thus:

$$\frac{5^7}{5^{-4}} = 5^{7-(-4)} = 5^{11}\text{, so } a = 11.$$

$$\frac{2^{-3}}{2^{-2}} = 2^{-3-(-2)} = 2^{-1}\text{, so } b = -1.$$

$$3^8(3) = 3^8(3^1) = 3^9\text{, so } c = 9.$$

Therefore, $a + b + c = 11 + (-1) = 9 = 19$.

26. **0.** This is a bit of a trick question. 12^x is odd? How strange! 12^1 is 12, 12^2 is 144 … it soon becomes easy to see that every "normal" power of 12 is going to be even. (An even number such as 12 multiplied by itself any number of times will yield an even answer.) These normal powers are 12 raised to a positive integer. What about negative integer exponents? They are all fractions of this form: $\frac{1}{12^{\text{positive integer}}}$.

The only way for 12^x to be odd is for x to equal 0. Any nonzero number to the 0^{th} power = 1. Since $x = 0$ and the question asks for x^{12}, the answer is 0.

27. **(E).** To solve this problem, you need to know that a square root is the same as a 1/2 exponent:

$$\frac{200^{\frac{5}{2}}}{\sqrt{200}} = \frac{200^{\frac{5}{2}}}{200^{\frac{1}{2}}} = 200^{\frac{5}{2}-\frac{1}{2}} = 200^{\frac{4}{2}} = 200^2 = 40{,}000$$

28. **(B).** Since $(10^3)(0.027)$ is simply 27 and $(900)(10^{-2})$ is simply 9:

$$\frac{27}{9} = (3)(10^m)$$

$$3 = 3(10^m)$$

$$1 = 10^m$$

You might be a little confused at this point as to how 10^m can equal 1. However, you can still answer the question correctly. If m were 3, as in Quantity B, 10^m would equal 1,000. However, 10^m actually equals 1. So m must be smaller than 3.

As it turns out, the only way 10^m can equal 1 is if $m = 0$. Any nonzero number to the 0th power is equal to 1.

29. **(D).** You CANNOT simply subtract $10^6 = 10^4$ to get 10^2. This is because you cannot do any operation directly to the exponents when subtracting with the same base. Rather, you must factor out the largest power of 10 each term has in common:

$$\frac{1}{3}\left[10^4(10^2-1)\right] = \frac{1}{3}\left[10^4(99)\right] = \frac{1}{3}[990{,}000] = 330{,}000$$

30. **(D).** You could factor 2^2 out of the top, but the numbers are small enough you might as well just say that the numerator is 4 + 4 + 8 + 16 = 32.

FOIL the denominator:

$$(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$$
$$\sqrt{25}+\sqrt{3}\sqrt{5}-\sqrt{3}\sqrt{5}-\sqrt{9}$$
$$\sqrt{25}-\sqrt{9}$$

5 - 3 = 2

32/2 = 16 is your final answer.

31. **(C).** Since $2^{-4} = \frac{1}{2^4}$, $3^{-20} = \frac{1}{3^{20}}$, etc., one quick shortcut here is to note that $\frac{2^{-4}3^{-20}}{4^{-1}9^{-6}} = \frac{4^1 9^6}{2^4 3^{20}}$, and

solve from there:

$$\frac{4^1 9^6}{2^4 3^{20}} = \frac{(2^2)^1 (3^2)^6}{2^4 3^{20}} = \frac{2^2 3^{12}}{2^4 3^{20}} = \frac{1}{2^2 3^8}$$

32. **(A).** One good approach is to convert 0.000027, 900, and 0.03 to powers of 10:

$$\frac{27 \times 10^{-6} \times 10^x}{9 \times 10^2 \times 10^{-4}} = 3 \times 10^{-2} \times 10^{11}$$

Now combine the exponents on the terms with base 10:

$$\frac{27 \times 10^{-6+x}}{9 \times 10^{-2}} = 3 \times 10^9$$

Since 27/9 = 3, cancel the 3 from both sides, then combine powers of 10:

$$\frac{10^{-6+x}}{10^{-2}} = 10^9$$

$$10^{-6+x-(-2)} = 10^9$$

$$10^{-4+x} = 10^9$$

You can now see that $-4 + x = 9$, so $x = 13$.

33. **(E).** A good first step is to convert to fractional exponents. Since a square root is the same as a 1/2 exponent and a cube root is the same as a 1/3 exponent:

$$x^{\frac{1}{2}} x^{\frac{1}{3}} = x^{\frac{1}{2}+\frac{1}{3}} = x^{\left(\frac{3}{6}+\frac{2}{6}\right)} = x^{\frac{5}{6}} = \sqrt[6]{x^5}$$

34. **(E).** A good first step is to convert to fractional exponents. Since a cube root is the same as a 1/3 exponent and a 4th root is the same as a 1/4 exponent:

$$\left(x^2\right)^{\frac{1}{3}} \left(x^5\right)^{\frac{1}{4}} = \left(x^{\frac{2}{3}}\right)\left(x^{\frac{5}{4}}\right) = x^{\left(\frac{2}{3}+\frac{5}{4}\right)} = x^{\left(\frac{8}{12}+\frac{15}{12}\right)} = x^{\frac{23}{12}} = \sqrt[12]{x^{23}}$$

35. **(A).** To simplify 0.00025×10^4, simply move the decimal in 0.00025 four places to the right to get 2.5. To simplify 0.005×10^2, move the decimal in 0.005 two places to the right to get 0.5. Thus, $n = 2.5$, $m = 0.5$, and n/m $=2.5/0.5 = 5$.

36. **(B).** Since you cannot directly combine exponential terms with the same base when you are adding and subtracting, you will need to factor out the top of the fraction:

$$\frac{40^{50}-40^{48}}{2^{96}}\times 10^{-45}$$
$$\frac{40^{48}(40^2-1)}{2^{96}}\times 10^{-45}$$
$$\frac{40^{48}(1599)}{2^{96}}\times 10^{-45}$$

Now, your goal should be to eliminate the fraction entirely by getting the denominator to cancel out. One good way to do this is to break up the 40, so as to isolate some 2's that will allow 2^{96} to be canceled out. (Note that it would also be possible to break 40 into 8 and 5, but in this particular problem, it seems wise to leave the 10 intact so it can ultimately combine with the 10^{-45}).

$$\frac{(4\times 10)^{48}(1599)}{2^{96}}\times 10^{-45}$$
$$\frac{4^{48}10^{48}(1599)}{2^{96}}\times 10^{-45}$$
$$\frac{(2^2)^{48}10^{48}(1599)}{2^{96}}\times 10^{-45}$$
$$\frac{2^{96}10^{48}(1599)}{2^{96}}\times 10^{-45}$$

2^{96} cancels! Combine 10^{48} and 10^{-45} for the final answer.

1048(1,599) × 10^{-45}
103(1,599)

37. **(B)**. Since a one-half exponent is the same as a square root, $x^{\frac{3}{2}}$ could also be written as $\sqrt{x^3}$. This, however, does not appear in the choices. Note, however, that $\sqrt{x^3}$ can be simplified a bit:

$$\sqrt{x^2\times x}$$
$$\sqrt{x^2}\times\sqrt{x}$$
$$x\sqrt{x}$$

This matches choice (B). Alternatively, convert the answer choices. For instance, in incorrect choice (A), $x^2\sqrt{x}=x^2x^{\frac{1}{2}}=x^{\frac{5}{2}}$. Since this is not equal to $x^{\frac{3}{2}}$, eliminate (A). Correct choice (B) can be converted as such:

$$x\sqrt{x} = x^1 x^{\frac{1}{2}} = x^{\frac{3}{2}}.$$

38. **(C).** The GRE calculator has a square root button, but it won't work on numbers as large as 360 × 240 × 3 × 2. Thus, you want to break this number down enough to pull out some perfect squares. You can already see that 360 is just 36 × 10. Now break it down a bit more:

$$\sqrt{(36)(10)(24)(10)(3)(2)}$$

Note that the two 10's inside can make 100, which is a perfect square:

$$\sqrt{(36)(100)(24)(3)(2)}$$

Multiply (24)(3)(2) to see if you get a perfect square. You do! It's 144:

$$\sqrt{(36)(100)(144)}$$

Since the operation inside the root sign is multiplication, it is allowable to break up the root sign into three separate root signs, as such:

$$\sqrt{36}\sqrt{100}\sqrt{144}$$

The answer is 6 × 10 × 12 = 720.

39. **(B).** Exponents questions are usually about primes, because you always want to create common bases, and the easiest common bases are primes. In order to answer this question, you have to understand what creates zeroes at the end of a number.

10 = 5 × 2
40 = 8 × 5 × 2
100 = 10 × 10 = 2 × 5 × 2 × 5
1,000 = 10 × 10 × 10 = 2 × 5 × 2 × 5 × 2 × 5

What you'll notice is that zeroes are created by 10's, each of which is created by one 2 and one 5. So to answer this question, you simply need to work out how many pairs of 2's and 5's are in the expression:

$$125^{14}48^{8} = (5^3)^{14} \times (2^4 \times 3)^8 = 5^{42} \times 2^{32} \times 3^8$$

Even though there are 42 powers of 5, there are only 32 powers of 2, so you can only make 32 pairs of one 5 and one 2. The answer is (B).

Chapter 15
of

Number Properties

In This Chapter…

Number Properties

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

On a number line, the distance from A to B is 4 and the distance from B to C is 5.

Quantity A	**Quantity B**
The distance from A to C	9

2.

$a, b, c,$ and d are consecutive integers such that $a < b < c < d$

Quantity A	**Quantity B**
The average of $a, b, c,$ and d	The average of b and c

3. $w, x, y,$ and z are consecutive odd integers such that $w < x < y < z$. Which of the following statements must be true?

Indicate all such statements.

☐ $wxyz$ is odd
☐ $w + x + y + z$ is odd
☐ $w + z = x + y$

4.

Quantity A	Quantity B
The sum of all the odd integers from 1 to 100, inclusive	The sum of all the even integers from 1 to 100, inclusive

5. If $a + b + c + d + e$ is odd, and $a, b, c, d,$ and e are integers, which of the following could be the number of integers among $a, b, c, d,$ and e that are even?

Indicate all such numbers.

☐ 0
☐ 1
☐ 2
☐ 3
☐ 4
☐ 5

6.

Quantity A	Quantity B
The least odd number greater than or equal to 5!	The greatest even number less than or equal to 6!

7. If set S consists of all positive integers that are multiples of both 2 and 7, how many numbers in set S are between 140 and 240, inclusive?

☐

8.

$$ab > 0$$
$$bc < 0$$

Quantity A	Quantity B
ac	0

9.

$$mn < 0$$
$$mp > 0$$

Quantity A	**Quantity B**
np	0

10.

$abc < 0$
$b^2c > 0$

Quantity A	**Quantity B**
ab	0

11.

a, b, and c are integers such that $a < b < c$

Quantity A	**Quantity B**
$\frac{a+b+c}{3}$	b

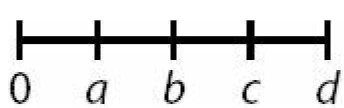
12. If $x^2 = y^2$, which of the following must be true?

- ☐ $x = y$
- ☐ $x^2 - y^2 = 0$
- ☐ $|x| - |y| = 0$

13. If p and k are even, and q is odd, which of the following cannot be even?

(A) pk
(B) pq
(C) $\frac{k}{p}$
(D) $\frac{qp}{k}$
(E) $\frac{q}{p}$

14.



Quantity A	**Quantity B**
$a \times c$	$b \times d$

15. If $a > b > c > d$ and $a = 2$, which of the following must be negative?

(A) ab
(B) ac
(C) ad
(D) bd
(E) None of the above.

16. If $y^2 = 4$ and $x^2y = 18$, $x + y$ could equal which of the following values?

Indicate <u>two</u> such values.

☐ -5
☐ -1
☐ 1
☐ 5
☐ 6

17.

Quantity A	**Quantity B**
The remainder when 10^{11} is divided by 2	The remainder when 3^{13} is divided by 3

18.

q is odd

Quantity A	**Quantity B**
$(-1)^q$	$(-1)^{q+1}$

19.

n is a positive integer

Quantity A	**Quantity B**
$(-1)^{4n} \times (-1)^{202}$	$(3)^3 \times (-5)^5$

20. If n is the smallest of three consecutive positive integers, which of the following must be true?

(A) n is divisible by 3
(B) n is even
(C) n is odd
(D) $(n)(n + 2)$ is even
(E) $n(n + 1)(n + 2)$ is divisible by 3

21. If x, y, and z are integers, $y + z = 13$, and $xz = 9$, which of the following must be true?

(A) x is even

(B) $x = 3$
(C) y is odd
(D) $y > 3$
(E) $z < x$

22.

$$abc > 0,$$
$$a < b,$$
$$\text{and } a^2(c) < 0$$

Quantity A	**Quantity B**
ab	$b(ac)^2$

23. On a number line, A is 6 units from B and B is 2 units from C. What is the distance between A and C?

(A) 4
(B) 8
(C) 12
(D) 4 or 8
(E) 4, 8, or 12

24. The average of 11 integers is 35. What is the sum of all the integers?

25. What is the sum of all the integers from 1 to 80, inclusive?

(A) 3,200
(B) 3,210
(C) 3,230
(D) 3,240
(E) 3,450

26. The average of a set of 18 consecutive integers is 22.5. What is the smallest integer in the set?

27. p is the sum of all the integers from 1 to 150, inclusive. q is the sum of all the integers from 1 to 148, inclusive. What is the value of $p - q$?

28. If m is the product of all the integers from 2 to 11, inclusive, and n is the product of all the integers from 4 to 11,

inclusive, what is the value of $\frac{n}{m}$?

Give your answer as a fraction.

29. If $\sqrt{x}$ is an integer and $xy^2 = 36$, how many values are possible for the integer y?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 8

30.

a, b, and c are positive even integers such that $8 > a > b > c$

Quantity A	**Quantity B**
The range of a, b, and c	The average of a, b, and c

31. If x is a non-zero integer and $0 < y < 1$, which of the following must be greater than 1?

(A) x
(B) $\frac{x}{y}$
(C) xy^2
(D) x^2y
(E) $\frac{x^2}{y}$

32.

a, b, and c are consecutive integers such that $a < b < c < 4$

Quantity A	**Quantity B**
The range of a, b, and c	The average of a, b, and c

33.

$x = 2y = 4z$ and x, y, and z are integers

Quantity A	**Quantity B**
The average of x and $2y$	$4z + x - 2y$

34.

$\sqrt{xy}$ is a prime number, xy is even, and $x > 4y > 0$

Quantity A	**Quantity B**
y	1

35.

$abcd$ is even and positive, and abc is odd and positive

Quantity A	**Quantity B**
1	d

36.

$b - a < 0$ and $a + 2c < 0$

Quantity A	**Quantity B**
b	$-2c$

37.

In set N consisting of n integers, the average equals the median.

Quantity A	**Quantity B**
The remainder when n is divided by 2	The remainder when $n - 1$ is divided by 2

38.

x is even, $\sqrt{x}$ is a prime number, and $x + y = 11$

Quantity A	Quantity B
x	y

39.

The product of integers f, g, and h is even and the product of integers f and g is odd

Quantity A	**Quantity B**

The remainder when f is divided by 2 | The remainder when h is divided by 2

40.

x, y, and z are integers
$xyz \geq 0$
$yz < 0$
$y < 0$

Quantity A	**Quantity B**
x	z

41.

$\sqrt{y} = 3$
$x^2 = 16$
$y - x > 10$

Quantity A	**Quantity B**
x	xy

42. If $\frac{17}{2^{10}5^{13}}$ is expressed as a terminating decimal, how many zeroes are located to the right of the decimal point before the first non-zero digit?

(A) 10
(B) 12
(C) 13
(D) 15
(E) 17

43. If x is odd, all of the following must be odd EXCEPT:

(A) $x^2 + 4x + 6$
(B) $x^3 + 5x + 3$
(C) $x^4 + 6x + 7$
(D) $x^5 + 7x + 1$
(E) $x^6 + 8x + 4$

44.

$x^2 > 25$ and $x + y < 0$

Quantity A	**Quantity B**
x	y

45.

The positive integer a is divisible by 2, and $0 < ab < 1$

Quantity A	**Quantity B**
b	$\frac{1}{2}$

46.

p and w are single-digit prime numbers
$p + w < 6$
p^2 is odd

Quantity A	**Quantity B**
3	w

47. If $\frac{x}{23}$ is a positive integer with a factor of 6, which of the following statements must be true?

Indicate <u>two</u> such statements.

- ☐ $x > 23$
- ☐ x has at least 3 prime factors
- ☐ x is odd
- ☐ x is prime
- ☐ x is not divisible by 5

48.

$x^2 > y^2$ and $x > -|y|$

Quantity A	**Quantity B**
x	y

49.

The sum of four consecutive integers is -2

Quantity A	**Quantity B**
The smallest of the four integers	-2

50. If g is an integer and x is a prime number, which of the following must be an integer?

Indicate <u>all</u> such expressions.

- [ ] $\dfrac{g^2x+5gx}{x}$
- [ ] $g^2-x^2\left(\dfrac{1}{3}\right)$
- [ ] $6\left(\dfrac{g}{2}\right)-100\left(\dfrac{g}{2}\right)^2=$

51. If $k=\dfrac{19!}{16!}$, which of the following is the smallest choice that does not have a prime factor in common with k?

(A) 19
(B) 34
(C) 77
(D) 115
(E) 133

52. If $4^6 25^5 = 10^x + k$, and x is an integer, what is the minimum positive value k could be?

(A) 0
(B) 30,000
(C) 30,000,000
(D) 10,000,000,000
(E) 30,000,000,000

53. Jose is making a necklace with beads in a repeating pattern of blue, red, green, orange, purple. If the 1st bead is blue, what color will the 234th bead be?

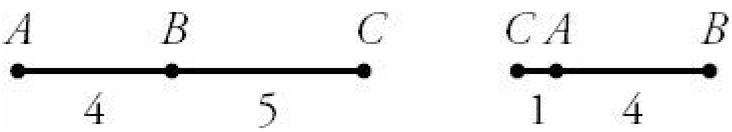
(A) blue
(B) red
(C) green
(D) orange
(E) purple

54. What is the units digit of 7^{94}?

55. What is the units digit of the sum $3^{47} + 5^{43} + 2^{12}$?

Number Properties Answers

1. **(D).** Whenever a question looks this straightforward ($4 + 5 = 9$, so the quantities initially appear equal), be suspicious. Draw the number line described. If the points *A*, *B*, and *C* are in alphabetical order from left to right, then the distance from *A* to *C* will be 9. However, alphabetical order is not required. If the points are in the order *C*, *A*, and *B* from left to right, then the distance from *A* to *C* is $5 - 4 = 1$.



2. **(C).** When integers are consecutive (or simply evenly spaced), the average equals the median. Since the median of this list is the average of the two middle numbers, Quantity A and Quantity B both equal the average of *b* and *c*. Alternatively, try this with real numbers. If the set is 2, 3, 4, 5, both quantities equal 3.5. No matter what consecutive integers you choose, the two quantities are equal.

3. **I and III only.** This question tests your knowledge of the properties of odd numbers as well as of consecutives.

Statement I is TRUE, as multiplying only odd integers together (and no evens) will always yield an odd answer.

However, when adding, the rule is "an odd number of odds makes an odd." Summing an even number of odds produces an even, so Statement II is FALSE.

Statement III is TRUE. Since *w, x, y,* and *z* are consecutive odd integers, you could define them all in terms of *w*:

$$w = w$$
$$x = w + 2$$
$$y = w + 4$$
$$z = w + 6$$

Thus, $w + z = w + (w + 6) = 2w + 6$
And $x + y = (w + 2) + (w + 4) = 2w + 6$

Therefore, $w + z = x + y$. Alternatively, try real numbers, such as 1, 3, 5, and 7. It is true that $1 + 7 = 3 + 5$. This would hold true for any set of four consecutive, ordered odd numbers you select.

4. **(B).** No math is required to solve this problem. Note that the numbers from 1 to 100 include 50 even integers and 50 odd integers. The first few odds are 1, 3, 5, etc. The first few evens are 2, 4, 6, etc. Every even is 1 greater than its counterpart (2 is 1 greater than 1, 4 is 1 greater than 3, 6 is 1 greater than 5, etc.) Not only is Quantity B greater, it's greater by precisely 50.

5. **I, III, and V only.** When adding integers, the rule is "an odd number of odds makes an odd" (you can ignore the evens for purposes of evaluating only whether the sum is even or odd — just count how many odds are being added). If 5 integers sum to an odd, the possibilities are:

1 odd, 4 evens
3 odds, 2 evens
5 odds, 0 evens

Make sure to answer for the number of *evens*. The answers are 0, 2, and 4.

6. **(B).** When multiplying integers, just one even will make the product even. Thus all factorials greater than 1! (which is just 1) are even.

$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. Since 120 is even, the "least odd number greater than or equal to 5!" is 121.

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$. Since 720 is even, the "greatest even number less than or equal to 6!" is 720.

Quantity B is greater.

7. **8.** A positive integer that is a multiple of both 2 and 7 is just a multiple of 14. Since 140 is a multiple of 14, start listing there and count the terms in the range: 140, 154, 168, 182, 196, 210, 224, 238.

Alternatively, note that 140 is the 10th multiple of 14, and $240/14 \approx 17.143$ (use the calculator). Therefore, the 10th through the 17th multiples of 14, inclusive, are in this range. The number of terms is $17 - 10 + 1 = 8$ ("add one before you are done" for an inclusive list).

8. **(B).** If $ab > 0$, then a and b have the same sign. If $bc < 0$, then b and c have opposite signs. Therefore, a and c must have opposite signs. Therefore, ac is negative, and Quantity B is greater.

If you find the logic difficult (a and b are same sign, b and c are opposite signs, therefore a and c are opposite signs), you could make a quick chart of the possibilities using plus and minus signs:

a	b	c	
+	+	−	← first possibility, a and c have different signs
−	−	+	← second possibility, a and c have different signs

9. **(B).** If $mn < 0$, then m and n have opposite signs. If $mp > 0$, then m and p have the same sign. Therefore, n and p must have opposite signs. Therefore, np is negative, and Quantity B is greater.

Alternatively, you could make a quick chart of the possibilities using plus and minus signs:

m	n	p	
+	−	+	← first possibility, n and p have different signs
−	+	−	← second possibility, n and p have different signs

10. **(B).** If abc is negative, then either exactly 1 or all 3 of the values a, b, and c are negative:

−	−	−	← first possibility, all negative
−	+	+	← second possibility, 1 negative, 2 positives (order can vary)

If b^2c is positive, then c must be positive, since b^2 cannot be negative. If c is positive, eliminate the first possibility since all 3 variables cannot be negative. Thus, only one of a, b, and c are negative, but the one negative cannot be c. Either a or b is negative, and the other is positive. It doesn't matter which one of a or b is negative — that's enough to know that ab is negative and Quantity B is greater.

11. **(D).** Note that $\frac{a+b+c}{3}$ is just another way to express "the average of a, b, and c." The average of a, b, and c would equal b if the numbers were evenly spaced (such as 1, 2, 3, or 5, 7, 9), but that is not specified. For instance, the integers could be 1, 2, 57 and still satisfy the $a < b < c$ constraint. In that case, the average is 20, which is greater than $b = 2$.

The correct answer is (D).

12. **II and III only.** When you take the square root of $x^2 = y^2$, you do NOT get $x = y$. Actually, you get $|x| = |y|$. After all, if $x^2 = y^2$, the variables could represent 2 and -2, 5 and 5, -1 and -1, etc. The information about the signs of x and y is lost when the numbers are squared; thus, taking the square root results in absolute values, which allow both sign possibilities for x and y. Thus, statement I is not necessarily true.

From $x^2 = y^2$, simply subtract y^2 from both sides to yield statement II. If you can algebraically generate a statement from the original equation $x^2 = y^2$, that statement must be true.

To generate statement III, take the square root of both sides of $x^2 = y^2$ to get $|x| = |y|$, then subtract $|y|$ from both sides.

13. **(E).** "Cannot be even" means it must be either odd or a non-integer.

(A) *must be* even, as an even times any integer equals an even.
(B) *must be* even, as an even times any integer equals an even.
(C) *can be* even (for instance, if $k = 8$ and $p = 4$).
(D) *can be* even (for instance, if $p = 16$, $k = 2$, and $q = 1$).
(E) *cannot be* even, as an odd divided by an even is *never* an integer. CORRECT.

14. **(B).** The exact values of a, b, c, and d are unknown, as is whether they are evenly spaced (do NOT assume that they are, just because the figure looks that way). However, it is known that all of the variables are positive such that $0 < a < b < c < d$.

Because $a < b$ and $c < d$ and all the variables are positive, $a \times c < b \times d$. In words, the product of the two smaller numbers is less than the product of the two greater numbers. Quantity B is greater.

You could also try this with real numbers. You could try $a = 1$, $b = 2$, $c = 3$, and $d = 4$, or you could mix up the spacing, as in $a = 0.5$, $b = 7$, $c = 11$, $d = 45$. For any scenario that matches the conditions of the problem, Quantity B is greater.

15. **(E).** Don't assume the variables are integers or that they are equally spaced. It is possible that b, c, and d are all positive (integers or fractions), so it is not true that the products in choices (A) through (D) must be negative.

16. **-1, 5.** From the first equation it seems that y could equal either 2 or -2, but if $x^2y = 18$, then y must equal only 2 (otherwise, x^2y would be negative). Still, the squared x indicates that x can equal 3 or -3. So the possibilities for $x + y$ are:

$3 + 2 = 5$
$(-3) + 2 = -1$

17. **(C).** It is not necessary to calculate 10^{11} or 3^{13}. Because 10 is an even number, so is 10^{11}, and 0 is the remainder when any even is divided by 2. Similarly, 3^{13} is a multiple of 3 (it has 3 among its prime factors), and 0 is the remainder when any multiple of 3 is divided by 3.

18. **(B).** The negative base -1 to any odd power is -1, and the negative base -1 to any even power is 1. Since q is odd, Quantity A = -1 and Quantity B = 1.

19. **(A).** Before doing any calculations on a problem with negative bases raised to integer exponents, check to see whether one quantity is positive and one quantity is negative, in which case no further calculation is necessary. Note that a negative base to an even exponent is positive, while a negative base to an odd exponent is negative.

Since n is an integer, $4n$ is even. Thus, in Quantity A, $(-1)^{4n}$ and $(-1)^{202}$ are both positive, and Quantity A is positive. In Quantity B, $(3)^3$ is positive but $(-5)^5$ is negative, and thus Quantity B is negative. Since a positive is by definition greater than a negative, Quantity A is greater.

20. **(E).** For three consecutive integers, the only possibilities are [odd, even, odd] or [even, odd, even]. Since n could be odd or even, eliminate (B) and (C). Choice (D) is true if n is even, but not if n is odd, so eliminate (D). In any set of three consecutive integers, one of the integers must be divisible by 3, but not necessarily n. Eliminate (A).

For the same reason, (E) must be true, as $n(n + 1)(n + 2)$ can be thought of as "the product of any three consecutive integers." Since one of these integers must be divisible by 3, the product of those three numbers must also be divisible by 3.

21. **(D).** If $xz = 9$ and x and z must both be integers, then they are 1 and 9 (or -1 and -9) or 3 and 3 (or -3 and -3). Therefore, they are both odd. More generally, the product of two integers will only be odd if the component integers themselves are both odd. Because z is odd, and $y + z$ equals 13 (an odd), y must be even.

Eliminate (A): x is NOT even.

Eliminate (B): x could be 3 but doesn't have to be.

Eliminate (C): y is NOT odd.

Eliminate (E): z does not have to be less than x (for instance, they could both be 3).

At this point, only (D) remains, so it must be the answer. To prove it, consider the constraint that limits the value of y: $y + z = 13$. Since z could be -1, 1, -3, 3, -9, or 9, the maximum possible value for z is 9, so y must be at least 4. All values that are at least 4 are also greater than 3, so (D) must be true.

22. **(B).** Since $a^2(c) < 0$ and it is not possible for a^2 to be negative, c must be negative.

If $abc > 0$ and c is negative, ab must be negative also, implying that a and b have opposite signs. Because $a < b$, a must be negative and b positive.

Thus, Quantity A is negative and Quantity B is pos × (neg × neg)^2 = pos × pos, which is positive.

23. **(D).** There is no rule that *A*, *B*, and *C* must occur in alphabetical order. If the points are ordered *A*, *B*, *C*, the distance from *A* to *C* is 6 + 2 = 8. But if in the order *A*, *C*, *B*, the distance from *A* to *C* is 6 – 2 = 4. Any other orders allowed by the relative distances (*C*, *B*, *A*, for instance, or *B*, *C*, *A*) will also yield either 4 or 8.

24. **385.** To find the sum of a set of numbers, given the average and number of terms, use the average formula. Average $= \frac{\text{Sum}}{\text{Number of Terms}}$, so Sum = Average × Number of Terms = 35 × 11 = 385.

25. **(D).** To find the sum of a set of evenly-spaced numbers, multiply the median (which is also the average) by the number of terms in the set. The median of the numbers from 1 to 80 inclusive is 40.5 (the first 40 numbers are 1 through 40, and the second 40 numbers are 41 through 80, so the middle is 40.5). You can also use the formula $\frac{\text{First} + \text{Last}}{2}$ to calculate the median of an evenly-spaced set: $\frac{1+80}{2}$. Multiply 40.5 times 80 to get the answer:

26. **14.** In an evenly-spaced set, the average is also the median. Thus, 22.5 is the median of the set. There are an even number of terms in the set, so the median is halfway between the two middle numbers. Thus, the 9th number is 22 and the 10th number is 23. Count backwards to get the answer, or write out the set:

First 9 integers (counting down): 22, 21, 20, 19, 18, 17, 16, 15, 14
Last 9 integers (counting up): 23, 24, 25, 26, 27, 28, 29, 30, 31

Alternatively, the 1st number is 8 "steps" down from the 9th number, so the smallest integer in the set is 22 - 8 = 14.

27. **299.** p is a large number, but it consists entirely of q + 149 + 150. Thus, p - q is what's left of p once the common terms are subtracted: 149 + 150 = 299.

28. $\mathbf{\frac{1}{6}}$. There is a trick to this problem — all of the integers in the product n will be canceled out by the same integers appearing in the product m:

$$\frac{n}{m} = \frac{\cancel{4\times5\times6\times7\times8\times9\times10\times11}}{2\times3\times\cancel{4\times5\times6\times7\times8\times9\times10\times11}} = \frac{1}{2\times3} = \frac{1}{6}$$

29. **(E).** If $\sqrt{x}$ is an integer, then x must be a perfect square. If x is a perfect square and $xy^2 = 36$, then x could actually equal *any* of the perfect square factors of 36, which are 1, 4, 9, or 36. (Only consider positive factors, because in order to have a valid square root, x must be positive.) Thus, y^2 could equal 36, 9, 4, or 1, respectively.

Of course, y^2 is positive, but y itself could be positive or negative. Thus $y = \pm 6, \pm 3, \pm 2$, or ± 1, for a total of 8 possible values.

30. **(C).** Integers a, b, and c must be 6, 4, and 2, respectively, as they are positive even integers less than 8 and ordered according to the given inequality. The range of a, b, and c is 6 - 2 = 4. The average of a, b, and c is $\frac{6+4+2}{3} = \frac{12}{3} = 4$. The two quantities are equal.

31. **(E).** Find the choices that do not have to be greater than 1. Most obviously, x could be negative, which eliminates (A), (B), and (C). For choice (D), if $x^2 = 1$, that times the positive fraction y would be less than 1. In choice (E), x^2 must be positive and at least 1, so dividing by the positive fraction y increases the value.

32. **(D).** If the variables were also constrained to be positive, they would have to be 1, 2, and 3, making the quantities both equal to 2. However, the variables could be negative, for example, $a = -10$, $b = -9$, $c = -8$. The range of a, b, and c will always be 2 because the integers are consecutive, but the average can vary depending on the specific values.

33. **(C).** Since $x = 2y$, the average of x and $2y$ is $\frac{x+2y}{2} = \frac{x+x}{2} = x$. Similarly, $4z + x - 2y$ is $x + x - x = x$.

Alternatively, pick values, such as $x = 4$, $y = 2$, and $z = 1$.

Quantity A is the average of 4 and 2(2), which is equal to 4.

Quantity B is 4(1) + 4 - 2(2) = 4.

The two quantities are equal for any set of values that conform to $x = 2y = 4z$, even negative test cases.

34. **(B).** If $\sqrt{xy}$ is a prime number, $\sqrt{xy}$ could be 2, 3, 5, 7, 11, 13, etc. Square these possibilities to get a list of possibilities for xy: 4, 9, 25, 49, 121, 169, etc. However, xy is even, so xy must equal 4.

Finally, $x > 4y > 0$, which implies that both x and y are positive. Solve $xy = 4$ for x, then substitute to eliminate the variable x and solve for y.

If $xy = 4$, then $x = \frac{4}{y}$.

If $x > 4y$, then $\frac{4}{y} > 4y$.

Because y is positive, you can multiply both sides of the inequality by y and you don't have to flip the sign of the inequality: $4 > 4y^2$

Finally, divide both sides of the inequality by 4: $1 > y^2$

Thus, y is a positive fraction less than 1 (you already know $y > 0$). Quantity B is greater.

35. **(D).** Since abc is odd and $abcd$ is even, d has to be even — if it is an integer. For instance, it could be the case that $a = 1$, $b = 1$, $c = 1$, and $d = 2$. In this case, d would be greater than 1. However, d could be a fraction — for example, $abcd$ could equal $3\times3\times3\times\frac{2}{3} = 18$. In this case, d would be $\frac{2}{3}$, which is less than 1.

36. **(B).** Solve both inequalities for a. Add a to both sides of $b - a < 0$ to get $b < a$. From $a + 2c < 0$, subtract $2c$ on both sides to get $a < -2c$. Put these together: $b < a < -2c$. Thus, b must be less than $-2c$. Quantity B is greater.

37. **(D).** "The remainder when … divided by 2" is a fancy way of asking whether a integer is odd or even (evens yield remainder 0 when divided by 2; odds yield remainder 1 when divided by 2).

If n is even, Quantity A is 0 and Quantity B is 1.

If n is odd, Quantity A is 1 and Quantity B is 0.

In this problem, n is the number of numbers in the set. So, could a set in which the average equals the median have either an even or an odd number of numbers? Absolutely. In fact, it is true of *any* evenly spaced set (and of some other sets) that the average equals the median. For instance:

1, 2, 3, 4	average and median are both 2.5	$n = 4$	$\longrightarrow$ Quantity B is greater
1, 2, 3	average and median are both 2	$n = 3$	$\longrightarrow$ Quantity A is greater

The correct answer is (D).

38. **(B).** If $\sqrt{x}$ is a prime number, $x = (\sqrt{x})(\sqrt{x})$ is the square of a prime number. Squaring a number does not change whether it is odd or even (the square of an odd number is odd and the square of an even number is even). Since x is even, it must be the square of the only even prime number. Thus, $\sqrt{x} = 2$ and $x = 4$. Since $x + y = 11$, $y = 9$ and Quantity B is greater.

39. **(A).** If fg is odd and both f and g are integers, both f and g are odd. The remainder when odd f is divided by 2 is 1. Since fgh is even and f and g are odd, integer h must be even. Thus, when h is divided by 2, the remainder is 0. Quantity A is greater.

40. **(B).** If $yz < 0$ and $y < 0$, z must be positive and y negative. If $xyz \geq 0$ and $yz < 0$, then x must be negative or 0. Quantity A is at most 0, while Quantity B is positive.

41. **(A).** If $\sqrt{y} = 3$, $y = 9$. If $x^2 = 16$, then $x = 4$ or -4. Since $y - x > 10$ and $y = 9$:

$9 - x > 10$
$-x > 1$
$x < -1$

This rules out the $x = 4$ possibility. Thus, x equals -4 and $xy = (-4)(9) = -36$. Since -4 is greater than -36, Quantity A is greater.

42. **(A).** Decimal placement can be determined by how many times a number is multiplied or divided by 10. Multiplying moves the decimal to the right, and dividing moves the decimal to the left. Look for powers of 10 in the given fraction, remembering that $10 = 2 \times 5$.

$$\frac{17}{2^{10}5^{13}} = \frac{17}{2^{10}5^{10}5^{3}} = \frac{17}{(2\times5)^{10}\,5^{3}} = \frac{17}{10^{10}5^{3}} = \frac{17/125}{10^{10}} = \frac{0.136}{10^{10}}$$

There are no zeros to the right of the decimal point before the first non-zero digit in 0.136. However, dividing by 10^{10} will move the decimal to the left 10 places, resulting in 10 zeros between the decimal and the "136" part of the number.

43. **(C).** For even and odd questions, you can either think it out logically, or plug in a number. Since one choice requires raising the number to the 6th power, pick something small! Plug in $x = 1$:

(A) $x^2 + 4x + 6 = 1 + 4 + 6 = 11$
(B) $x^3 + 5x + 3 = 1 + 5 + 3 = 9$
(C) $x^4 + 6x + 7 = 1 + 6 + 7 = 14$
(D) $x^5 + 7x + 1 = 1 + 7 + 1 = 9$
(E) $x^6 + 8x + 4 = 1 + 8 + 4 = 13$

For the logic approach, remember that an odd number raised to an integer power is always odd, an odd number multiplied by an odd number is always odd, and an odd number multiplied by an even number is always even:

(A) $x^2 + 4x + 6$ = odd + even + even = odd
(B) $x^3 + 5x + 3$ = odd + odd + odd = odd
(C) $x^4 + 6x + 7$ = odd + even + odd = even
(D) $x^5 + 7x + 1$ = odd + odd + odd = odd
(E) $x^6 + 8x + 4$ = odd + even + even = odd

44. **(D).** If $x^2 > 25$, then $x > 5$ OR $x < -5$. For instance, x could be 6 or -6.

If $x = 6$:

$6 + y < 0$
$y < -6$

x is greater than y.

If $x = -6$:

$-6 + y < 0$
$y < 6$

y could be less than x (e.g., y = -7) or greater than x (e.g., y = 4).

45. **(B).** If the positive integer a is divisible by 2, it is a positive even integer. Thus, the minimum value for a is 2. Thus, since $ab < 1$, b must be less than $\frac{1}{2}$.

46. **(A).** If the sum of two primes is less than 6, either the numbers are 2 and 3 (the two smallest unique primes), or both numbers are 2 (just because the variables are different letters doesn't mean that p cannot equal w). Both numbers cannot equal 3, though, or $p + w$ would be too great. If p^2 is odd, p is odd, and therefore $p = 3$, so w can only be 2.

47. **I and II only.** If $\frac{x}{23}$ is divisible by 6, x must have factors of 2 and 3, as well as 23. Another way to write this is $\frac{x/23}{6}$ = positive integer, so $\frac{x}{23}$ = 6 × positive integer, and finally x = 2 × 3 × 23 × positive integer.

I. TRUE. x must be greater than 23, as it is 23 × positive values.
II. TRUE. x has at least 3 prime factors, namely, 2, 3, and 23.
III. False. Because x has a factor of 2, it is not odd.
IV. False. Because it has more than one prime factor, x is definitely *not* prime.
V. Maybe. While x must have factors of 2, 3, and 23, it could also have other prime factors. For instance, x could be 2(3)(23), or it could be a very large number with more factors, such as $2^2 3^2 23^4 5^{11} 13^2$. Thus, x may or may not be divisible by 5.

48. **(A).** If $x^2 > y^2$, x must have a greater absolute value than y. For instance:

	x	y
Example 1	3	2
Example 2	-3	2
Example 3	3	-2
Example 4	-3	-2

If $x > -|y|$ must also be true, which of the examples continue to be valid?

	x	y	$x > -\|y\|$?	
Example 1	3	2	3 > -|2|	TRUE
Example 2	-3	2	-3 > -|2|	FALSE
Example 3	3	-2	3 > -|-2|	TRUE
Example 4	-3	-2	-3 > -|-2|	FALSE

Only Example 1 and Example 3 remain.

	x	y

Example 1	3	2
Example 3	3	-2

Thus, either x and y are both positive and x has a larger absolute value (Quantity A is greater) or x is positive and y is negative (Quantity A is greater). In either case, Quantity A is greater.

49. **(C).** Write an equation: $x + (x + 1) + (x + 2) + (x + 3) = -2$.

$4x + 6 = -2$
$4x = -8$
$x = -2$

Thus, the integers are -2, -1, 0, and 1. The smallest of the four integers equals -2.

50. **I and III only.** In Statement I, x can be factored out and eliminated: $\frac{g^2x+5gx}{x}=\frac{x\left(g^2+5g\right)}{x}=g^2+5g$. Since g is an integer, so is $g^2 + 5g$.

In Statement II, g^2 is certainly an integer, but $x^2\left(\frac{1}{3}\right)$ is only an integer if $x = 3$ (since 3 is the only prime number divisible by 3), so statement II does not have to be an integer.

When Statement III is simplified, $6\left(\frac{g}{2}\right)-100\left(\frac{g}{2}\right)^2=3g-\frac{100g^2}{4}=3g-25g^2$ results; since g is an integer, $3g - 25g^2$ is also an integer, albeit a negative one.

51. **(C).** $\frac{19!}{16!}=\frac{19\times18\times17\times\cancel{16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}}{\cancel{16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}}$ $= 19 \times 18 \times 17$. No need to multiply these out, because the question is only about common prime factors. If $k = 19 \times 18 \times 17$, k's prime factors are 19, 2, 3, 3, and 17 (19 and 17 are prime, and 18 breaks down into prime factors 2, 3, and 3).

Choices (A) is wrong because 19 has a prime factor in common with k (that factor is 19 itself). Choice (B) is wrong because 34 has a prime factor of 17, as does k. Choice (C) is correct. 77 has prime factors 7 and 11 and does not have a prime factor in common with k. Note that 115 also does not share any prime factors with k, but the questions asks for the *smallest* choice that works.

52. **(E).** Since $4^6 25^5$ is too big for the calculator, try another strategy: note that $4 \times 25 = 100$, and the other side of the equation involves a power of 10. Separate out "pairs" of 4 and 25 on the left:

$4^6 25^5 \quad = 10^x + k$
$4^1(4^5 25^5) \quad = 10^x + k$
$4^1(4 \times 25)^5 \quad = 10^x + k$

$4^1(100)^5 \quad = 10^x + k$

Thus, the left side of the equation is $4(100^5)$, or $4(10^{10})$, or 40,000,000,000. Thus:

$$40{,}000{,}000{,}000 = [\text{a power of } 10] + k$$

To minimize k while keeping it positive, maximize the power of 10 while keeping it less than $4^6 25^5$. The greatest power of 10 that is less than 40,000,000,000 is 10,000,000,000, or 10^{10}. Thus:

$$40{,}000{,}000{,}000 = 10{,}000{,}000{,}000 + k$$
$$30{,}000{,}000{,}000 = k$$

53. **(D).** Since the pattern has 5 elements, find the remainder when 234 is divided by 5, which is just the units digit in this case. Alternatively, 5 goes evenly into 230, and since 234 - 230 = 4, the remainder is 4. The 4th color in the pattern, orange, is the answer.

54. **9.** The units digits of 7 to positive integers create a repeating pattern (this works for digits other than 7 also). By multiplying 7 by itself repeatedly in the calculator, you can generate the pattern:

$7^1 = \mathbf{7}$
$7^2 = 4\mathbf{9}$
$7^3 = 34\mathbf{3}$
$7^4 = 2{,}40\mathbf{1}$
$7^5 = 16{,}80\mathbf{7}$
$7^6 = 117{,}64\mathbf{9}$
$7^7 = 823{,}54\mathbf{3}$
$7^8 = 5{,}764{,}80\mathbf{1}$
Pattern: 7, 9, 3, 1

The pattern for the units digits of 7 to a power is 7, 9, 3, 1. (You should know that none of the patterns ever have more than 4 elements before repeating, so you don't actually have to multiply out 8 or more times, as shown above.)

To find the 94th item in a pattern of 4 repeating items, find the remainder when 94 is divided by 4. 94 divided by 4 in the calculator is 23.5. Ignore the decimal and multiply 4 × 23 to find the largest number (less than 94) that 4 *does* go into: it's 92. 94 divided by 4 gives remainder 2 (since 94 – 92 = 2).

Thus, to get to the units digit of 7^{94}, the pattern (7, 9, 3, 1) repeats 23 times, and then continues 2 more elements into the pattern. The second element in the pattern is 9.

55. **8.** For any digit, that digit to increasing positive integer powers will end in units digits that create a repeating pattern:

$3^1 = \mathbf{3}$
$3^2 = \mathbf{9}$
$3^3 = 2\mathbf{7}$

$3^4 = 8\mathbf{1}$
$3^5 = 24\mathbf{3}$
$3^6 = 72\mathbf{9}$
$3^7 = 2{,}18\mathbf{7}$
$3^8 = 6{,}56\mathbf{1}$
Pattern: 3, 9, 7, 1

3^{47} will end in the digit that is the 47th item in the pattern 3, 9, 7, 1. Since it's a pattern of 4 elements, find the largest multiple of 4 that is still less than 47: it's 44. Since 47 - 44 = 3, the remainder when 47 is divided by 4 is 3. Thus, 3^{47} will end in the units digit that is the 3rd item in the pattern. Thus, 3^{47} ends in 7.

$5^1 = \mathbf{5}$
$5^2 = 2\mathbf{5}$
$5^3 = 12\mathbf{5}$

…this one's a "freebie," as 5 to any power ends in 5 and 5^{43} must end in 5.

$2^1 = \mathbf{2}$
$2^2 = \mathbf{4}$
$2^3 = \mathbf{8}$
$2^4 = 1\mathbf{6}$
$2^5 = 3\mathbf{2}$
$2^6 = 6\mathbf{4}$
$2^7 = 12\mathbf{8}$
$2^8 = 25\mathbf{6}$
$2^9 = 51\mathbf{2}$
$2^{10} = 1{,}02\mathbf{4}$
Pattern: 2, 4, 8, 6

2^{12} will end in the digit that is the 12th item in the pattern 2, 4, 8, 6. Since 4 goes into 12 evenly, 2^{12} will end in the last item in the pattern, which is 6. (You also could just keep going with the pattern above: 2^{11} is 2,048, and 2^{12} = 4,096, which ends in 6). Thus, 2^{12} ends in 6.

Thus, 3^{47}, 5^{43}, and 2^{12} are very large numbers that end in 7, 5, and 6, respectively. Imagine that you were adding those very large numbers by hand (the xxxx just indicates the unknown and unimportant parts of these numbers):

```
  xxxxxxx7
  xxxxxxx5
+ xxxxxxx6
```

To begin adding these numbers, you would add 7 + 5 + 6 = 18. You would put 8 in the units place of your answer, and carry the 1:

```
         1
  xxxxxxx7
```

```
  xxxxxxx5
+ xxxxxxx6
        8
```

The units digit of the answer is 8.

Chapter 16
of

Word Problems

In This Chapter…

Word Problems

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If a taxi charges $8.00 for the first mile, and $1.00 for each additional quarter mile, how much does the taxi charge for a 4.5 mile ride?

(A) $16.00
(B) $18.00
(C) $22.00
(D) $24.00
(E) $26.00

2. If Nash had 12 grandchildren and three times as many granddaughters as grandsons, how many granddaughters did he have?

(A) 3
(B) 4
(C) 6
(D) 8
(E) 9

3. If Deepak pays 30% of his income in taxes and his take-home pay after taxes is $2,800 per month, how much does Deepak make per month?

$ $\boxed{}$

4. Movie theater X charges $6 per ticket, and each movie showing costs the theatre $1,750. How many people need to see a movie so that the theater makes $1 of profit per customer?

(A) 300
(B) 325
(C) 350
(D) 375
(E) 400

5. Arnaldo earns $11 for each ticket that he sells, and a bonus of $2 per ticket for each ticket he sells over 100. If Arnaldo was paid $2,400, how many tickets did he sell?

(A) 120
(B) 160
(C) 180
(D) 200
(E) 250

6. Attendees at a charity dinner each gave at least $85 to the charity. If $6,450 was collected, what is the maximum number of people who could have attended?

(A) 73
(B) 74
(C) 75
(D) 76
(E) 77

7. Eva meditates for 20 minutes at a time, with a 5-minute break in between sessions. If she begins meditating at 10:10, what time will it be when she completes her third session?

(A) 11:20
(B) 11:25
(C) 11:50
(D) 11:55
(E) 12:25

8. A washing machine takes 35 minutes to wash one load of laundry, and in between washing different loads of laundry it takes Derek 2 minutes to unload and another 4 minutes to reload the machine. If the washing machine begins washing one load of laundry at 12:30pm, how many loads of laundry can Derek wash and unload before 6:35pm?

(A) 8
(B) 9
(C) 10
(D) 14
(E) 15

9.

Kendra is more than 5 years old.

Quantity A	Quantity B
Five years less than twice Kendra’s age	Twice what Kendra’s age was five years ago

10.

Six years ago, Billy was twice as old as Allie.

Quantity A	Quantity B
The difference between their ages today	2 years

11. Every day the drama club has a car wash, the club pays a fixed amount for supplies. If the club charges \$12 for a car wash and earned a total profit of \$190 in one day by giving 20 car washes, how much did the club pay for supplies, assuming that there are no other expenses?

12. A store owner pays her assistant \$22 per hour for every hour the store is open. If all other expenses for the store work out to \$160 per day, and the store is open for 8 hours on Monday and sells \$720 worth of merchandise on that day, what is the store’s profit for the day?

(A) 384
(B) 396
(C) 530
(D) 538
(E) 560

13. Regular gas costs \$3.00 a gallon and is consumed at a rate of 25 miles per gallon. Premium costs \$4.00 a gallon and is consumed at a rate of 30 miles per gallon. How much more will it cost to use premium rather than regular for a 300-mile trip?

(A) \$1
(B) \$4
(C) \$5
(D) \$36
(E) \$40

14. A toy retailer buys toys from the toy company and marks the price up 25% to sell to customers. If the retailer has a sale of 80% off retail price, what is its percent loss in terms of the original amount it paid for the toys?

(A) 25%
(B) 30%
(C) 40%
(D) 75%
(E) 80%

15. Mr. Choudury’s fourth-grade class consists of 20 students: 12 boys and 8 girls. If the boys weigh an average of 80 pounds each, and the girls weigh an average of 70 pounds each, what is the average weight in pounds of all 20 students?

(A) 71
(B) 74
(C) 75
(D) 76
(E) 79

16. It costs a certain bicycle factory $11,000 to operate for one month, plus $300 for each bicycle produced during the month. Each of the bicycles sells for a retail price of $700. What is the minimum number of bicycles that the factory must sell in one month to make a profit?

(A) 26
(B) 27
(C) 28
(D) 29
(E) 30

17. The yoga company Yoga for Life offers 45-minute classes at $12 per class. If the number of minutes Randolf spent doing yoga this month was 132 greater than the number of dollars he paid, how many classes did he attend?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 8

18. When Mark fills his car with regular gasoline, he gets 20 miles/gallon. When he fills his car with premium gasoline, he gets 25 miles/gallon. If the price of regular gasoline is $4.00 per gallon and the price of premium gas is $6.25 per gallon, then the cost efficiency of regular gasoline, in miles/dollar, is what percent greater than the cost efficiency of premium gasoline?

(A) 4%
(B) 10%
(C) 20%
(D) 25%
(E) 50%

19. An online merchant sells wine for $20 a bottle or $220 for a case of 12. Either way, his cost for the wine is $10 per bottle. Shipping costs him $5 for a bottle and $40 for a case. If, in a month, he sells 12 cases and 60 bottles and has no other revenue or expenses, his profit is equal to which of the following?

(A) $780
(B) $1,020
(C) $2,160
(D) $2,640
(E) $3,840

20. If everyone who contributed to a charity drive paid at least $14 and $237 was collected, what is the maximum number of people who could have contributed?

(A) 13
(B) 14
(C) 15
(D) 16
(E) 17

21.

In a certain barter system, one sack of rice can be traded for two and a half chickens or a third of a medallion.

Quantity A	**Quantity B**
The value of a chicken in sacks of rice	The value of a medallion in sacks of rice

22. A hunting lodge has enough fuel to keep 20 rooms heated for fourteen days. If the lodge decides to save fuel by turning off the heat in 5 unoccupied rooms, and each room requires the same amount of fuel to heat it, how many extra FULL days will the fuel supply last?

(A) 3
(B) 4
(C) 5
(D) 18
(E) 19

23. Last year, a magazine charged a $50 subscription fee. This year, the price will be increased by $10. If the magazine could lose 4 subscribers this year and still collect the same revenue as it did last year, how many subscribers did the magazine have last year?

(A) 20
(B) 21
(C) 22
(D) 23
(E) 24

24. Store Y sells two brands of socks, one at $3 a pair and the other at $4 a pair. If Janine must spend exactly $29 on socks and there is no sales tax, how many pairs of socks can she buy?

Indicate <u>all</u> such values:

☐ 6
☐ 7
☐ 8
☐ 9
☐ 10

Word Problems Answers

1. **(C).** Break the trip into two parts: the first mile and the final 3.5 miles. The first mile costs \$8, and the final 3.5 miles cost \$1 per 1/4 mile, or \$4 per mile. $8 + 3.5(4) = 8 + 14 = 22$.

2. **(E).** Rather than assigning separate variables to the granddaughters and grandsons, define them both in terms of the same unknown multiplier, based on the ratio given:

Number of granddaughters = $3m$
Number of grandsons = m

Note that you are solving for $3m$, not simply for m!

$$3m + m = 12$$
$$4m = 12$$
$$m = 3$$
$$3m = 9$$

SHORTCUT: Another method depends on the same underlying logic, but forgoes the algebra. Suppose that Nash had exactly one grandson and three granddaughters. That would add to four grandchildren altogether. Triple the number of grandsons and granddaughters to triple the number of grandchildren.

Further, note that only (D) and (E) are greater than half of 12, and that 8 isn't a multiple of 3.

3. **4,000.** If Deepak pays 30% in taxes, his take-home pay after taxes is 70%. Since this amount is equal to \$2,800:

$$0.70x = 2{,}800$$
$$x = 4{,}000$$

4. **(C).** This problem requires you to know that profit equals revenue minus cost. You should memorize the formula Profit = Revenue - Cost (or Profit = Revenue - Expenses), but you could just think about it logically — of course a business has to pay its expenses out of the money it makes: the rest is profit.

The question tells you that the cost is \$1,750. If the theater charges \$6 per ticket, the revenue will be equal to 6 times the number of customers. Let c be the number of customers. The revenue is $6c$. Lastly, the question asks for the number of customers so that the profit is \$1 per customer. That means that the profit will be \$1 times the number of customers, or c. Plug these values into the equation Profit = Revenue - Cost:

$$c = 6c - 1750$$
$$-5c = -1750$$
$$c = 350$$

5. **(D).** Let x = the total number of tickets sold. Therefore, $(x - 100)$ = the number of tickets Arnaldo sold beyond the first 100

$11x + 2(x - 100) = 2,400$
$11x + 2x - 200 = 2,400$
$13x = 2,600$
$x = 200$

6. **(C).** Divide \$6,450 by \$85 to get 75.88…. But don't just round up! You are told that each person gave at least \$85. If 76 people attended and each gave the minimum of \$85, then \$6,460 would have been collected. Since only \$6,450 was collected, that 76th person could not have attended. You need to round down to 75. (This means at least one person gave more than the minimum.)

7. **(A).** Simply list Eva's meditation sessions and breaks:

10:10 - 10:30	session 1
10:30 - 10:35	break
10:35 - 10:55	session 2
10:55 - 11:00	break
11:00 - 11:20	session 3

Note that you are asked for the time when she completes her third session, so do not add a third break!

A quicker way to do this problem would be to add 20(3) + 5(2) to get 70 minutes. 70 minutes after 10:10 is 11:20.

8. **(B).** You *could* simply list Derek's activities:

12:30 - 1:05	load 1
1:05 - 1:11	unload/reload
1:11 - 1:46	load 2
1:46 - 1:52	unload/reload
1:52 - 2:27	load 3
Etc.	

However, completing this rather tedious list all the way up to 6:35pm is not a good expenditure of time on the GRE. A better approach would be to determine how many minutes are available for Derek to do laundry. From 12:30 to 6:35 is 6 hours and 5 minutes, or 365 minutes. (Many students make silly mistakes here by calculating as though there were 100 rather than 60 minutes in an hour!)

It takes 41 minutes to do one load of laundry and then switch to the next one (34 + 4 + 2 minutes).

Divide 365 minutes by 41 to get 8.9…. So, Derek can definitely do 8 total loads of laundry plus switching time.

What about that extra 0.9…? You need to figure out whether Derek can fit in one more laundry load. Importantly, for this last load he needs only 2 extra minutes to unload, since he will not be reloading the machine.

Multiply 8 (the total number of loads Derek can definitely do) by 41 minutes to get 328 minutes. Subtract 328 from the 365 available minutes to get 37 minutes. Amazingly, that is *exactly* how much time it takes Derek to do one load of laundry (35 minutes) and then unload it (2 minutes). So, Derek can wash and unload 9 total loads of laundry.

9. **(A).** This is an algebraic translation, meaning you need to translate the text into algebra. Use k to represent Kendra's

age, and you know $k > 5$ (they only tell you this because Quantity B requires you to consider Kendra's age five years ago, and if she were younger than five years old, that would create an impossible negative age!).

Quantity A becomes $2k - 5$, while Quantity B becomes $2(k - 5)$ or $2k - 10$. From here, the easiest thing is to manipulate the columns as if they were a giant inequality:

$$2k - 5 > 2k - 10$$

(The direction of the inequality sign in the center doesn't matter.)

The $2k$ on either side cancels, and you're left with -5 on the left and -10 on the right. -5 is bigger than -10, so Quantity A is bigger.

You could also solve this question by plugging in values, but there's no need, as the algebra is nice and simple. The answer is (A).

10. **(D).** This is an algebraic translation question, so you should start by translating the given information into algebra. Start by creating variables, A for Allie and B for Billy. Notice that the question is asking about six years ago, so you need to subtract six from both Billy and Allie's ages:

$$B - 6 = 2(A - 6)$$
$$B - 6 = 2A - 12$$
$$B = 2A - 6$$

That's as far as you can take the algebra. From here, Quantity A asks you for $B - A$ (we know that Billy is older from the given information). You can't solve for that with the equation you've been given, so you should quickly look at some values:

$$B = 2A - 6$$

Notice that you can't pick whatever values you want here. You need both Billy and Allie to be older than 6, or else the issue of six years ago won't make a lot of sense (you don't want negative ages!). Start out with Allie at 7 years old. That would make Billy 8 years old.

$$8 - 7 = 1$$

In this case, Quantity B is bigger, so the answer has to be (B) or (D).

Because you've minimized A and B, you should try something big next. If A were 100, B would be 194.

$$B - A = 194 - 100 = 94$$

In this case, Quantity A is bigger, so the answer is (D).

11. **50.** Since Profit = Revenue - Expenses, and \$12 for a car wash multiplied by 20 car washes = \$240:

$$190 = 240 - E$$

Subtract 240 from each side, then multiply each side by -1 to flip the signs:

$190 = 240 - E$
$-50 = -E$
$50 = E$

12. **(A).** Since Profit = Revenue - Expenses, and you are told that revenue = $720:

$P = 720 - E$

Expenses are equal to $22 per hour times 8 hours, plus a fixed $160, or 22(8) + 160 = $336. Thus:

$P = 720 - 336$
$P = 384$

13. **(B).** 12 gallons of regular are needed to go 300 miles (300 divided by 25 miles per gallon), costing $36 (12 gallons × $3 per gallon). 10 gallons of premium would be needed to go 300 miles (300 divided by 30 miles per gallon), costing $40 (10 gallons × $4 per gallon). You need the difference, so $40 - $36 = $4.

14. **(D).** For problems like this that ask for percents and use no real numbers, it is almost always easy and possible to use 100 as a starting number. Suppose the toy store buy the toys for $100, and marks them up to $125. An 80% sale will drop the price down to $25. The percent loss is the amount it loses per toy expressed as a percentage of the original price per toy:

$(100 - 25)/100 \times 100 = 75\%$

15. **(D).** The most straightforward approach is to determine the total weight of all 20 students, and divide that total by 20.

12 boys × 80 pounds per boy = 960 pounds
8 girls × 70 pounds per girl = 560 pounds
Total = 1,520 pounds

1520/20 = 76

SHORTCUT: Pretty much all GRE multiple-choice weighted average problems have the same five answers:

Much closer to the lesser value
A little closer to the lesser value
The unweighted average of the two values
A little closer to the greater value
Much closer to the greater value

Any of these five choices *could* be correct, but the correct answer is usually “a little closer to the lesser value” or “a little closer to the greater value.” In this case, because there are a few more boys than girls, the weighted average will be a little closer to the boys’ average than to the girls’.

16. **(C).** The question asks how many bicycles the factory must sell to make a profit. One way of phrasing that is to say

the profit must be greater than 0. Since Profit = Revenue - Cost, you can rewrite the equation to say:

Revenue - Cost > 0

Let b equal the number of bicycles sold. Each bike sells for \$700, so the total revenue is $700b$. The cost is equal to \$11,000 plus \$300 for every bicycle sold.

$(700b) - (11{,}000 + 300b) > 0$

Isolate b on one side of the inequality:

$700b - 11{,}000 + 300b > 0$
$400b - 11{,}000 > 0$
$400b > 11{,}000$
$b > 27.5$

If b must be greater than 27.5, then the factory needs to sell at least 28 bicycles to make a profit.

17. **(B).** The normal way to do this problem would be to assign variables and set up equations, using X to represent the number of classes Randolf took, $12X$ to represent the amount he paid, and $45X$ to represent the number of minutes he spent.

A quicker way might be to notice that with every class Randolf takes, the difference between the number of minutes he spends and the amount he pays increases by 33. If Randolf takes 1 class, then the number of minutes he spends is 33 greater than the number of dollars he pays. If he takes 2 classes, the number of minutes is 66 greater than the number of dollars, and so on. $132 = 4 \times 33$, so Randolf must have taken 4 classes.

18. **(D).** In order to solve this problem, you first need to convert each of the gasoline types into miles/dollar. To do this, you take the miles/gallon and divide it by dollars/gallon. Thus, for regular gasoline, if it gets 20 miles/gallon and each gallon costs \$4.00, then the miles/dollar is $\frac{20}{4} = 5$. Similarly, the miles/dollar of premium gasoline is $\frac{25}{6.25} = 4$.

To express this as a percentage, use the equation:

$$\text{Percent Increase} = \frac{\textit{Difference}}{\textit{Original}} \times 100$$

$$\text{Thus, } \frac{5-4}{4} \times 100 = 25\%.$$

19. **(B).** Profit is equal to Revenue - Expenses. First, calculate revenue:

12 cases sold for \$220 each = \$2,640
60 bottles sold for \$20 each = \$1,200

TOTAL REVENUE = $3,840

Now, calculate expenses. How many total bottles of wine were sold? 12 cases × 12 bottles, plus 60 individual bottles = 204 bottles. Note that the bottles sold individually versus those sold in cases have the same inventory cost ($10), but different shipping costs. Thus:

204 bottles at $10 each = $2,040
Shipping on bottles = 60 × $5 = $300
Shipping on cases = 12 × $40 = $480
TOTAL EXPENSES = $2,820

Profit = Revenue - Expenses
Profit = $3,840 - $2,820
Profit = $1,020

20. **(D).** This is a maximization question. To solve maximization questions, you often have to minimize something else. In order to get the most people involved here, you need the donation per person to be as small as possible. In this case, everyone could pay exactly $14:

237/14 = 16.92

You can't round up to 17, because it is not possible that 17 people donated $14 each (you would end up with something bigger than $237). The answer is (D), or 16.

21. **(B).** If one sack of rice is worth one-third of a medallion, buying the whole medallion would require three sacks of rice. Quantity B is equal to 3. The math is a bit tougher in Quantity A, but no calculation is really required — if a sack of rice gets you 2.5 chickens, a single chicken is worth less than a sack of rice. Quantity A is less than 1.

22. **(D).** The lodge has 20(14) = 280 "fuel-days" of fuel. (A "fuel-day" is enough fuel for 1 room for 1 day.) If the lodge only needs to heat 15 rooms instead of 20, divide 280 by 15 to get 18.666…. You are asked for FULL days, so round down to get 18.

23. **(E).** Assign the variable s for subscribers.

Last year:
$50 per subscription
s subscribers

This year:
$60 per subscription
$s - 4$ subscribers

You are told that the magazine "could" lose 4 subscribers and that the magazine would then collect the same revenue as last year — don't let the "could" throw you off. You are being asked to calculate using this hypothetical situation:

$50s = 60(s - 4)$
$50s = 60s - 240$
$-10s = -240$
$s = 24$

24. **III and IV only.** Let's say that Janine buys x of the first pair of socks and y of the second. This means that she spends $3x$ on the first and $4y$ on the second. You can rephrase the question as follows: if $3x + 4y = 29$, what could be the value of $x + y$? (Keep in mind that x and y must be integers.)

Consider all multiples of 4 that are less than 29, and find which of those multiples, when subtracted from 29, will leave you with a multiple of 3. Here are all the multiples of 4 under 29: 4, 8, 12, 16, 20, 24, 28. Only 8 and 20 can leave you with a multiple of 3 when subtracted from 29. If Janine spends $8 on the second brand of socks, she buys 2 pairs. She then has $21 to spend on the first brand of socks, meaning that she can buy 7 pairs of the first brand. If you add up the pairs of socks, she buys 2 + 7 = 9 pairs. If she spends $20 on the second brand of socks, which gets her 5 pairs, then she has $9 left to spend on the first brand, at $3 per pair, which lets her buy 3 pairs. In this scenario she can buy 3 + 5 = 8 pairs. The correct answers are 8 and 9 pairs.

Chapter 17

of

Two-Variable Word Problems

In This Chapter…

Two-Variable Word Problems

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box , you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes , you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. There are five more computers in the office than employees. If there are 10 employees in the office, what is the ratio of computers to employees in the office?

(A) 2 : 3
(B) 2 : 5
(C) 3 : 2
(D) 3 : 5
(E) 5 : 2

2. Two parking lots can hold a total of 115 cars. The Green lot can hold 35 fewer cars than the Red lot. How many cars can the Red lot hold?

(A) 35
(B) 40
(C) 70
(D) 75
(E) 80

3. Three friends sit down to eat 14 slices of pizza. If two of the friends eat the same number of slices, and the third eats two more slices than each of the other two, how many slices are eaten by the third friend?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

4.

In 8 years, Polly's age (which is currently p) will be twice Quan's age (which is currently q).

Quantity A	**Quantity B**
$p - 8$	$2q$

5. Pens cost 70 cents each, and pencils cost 40 cents each. If Iris spent \$5.20 on 10 pens and pencils, how many pencils did she purchase? (\$1 = 100 cents)

(A) 4
(B) 6
(C) 8
(D) 10
(E) 13

6. Jack downloaded ten songs and two books for \$48, Jill downloaded fifteen songs and one book for \$44. How much did Jack spend on books, if all songs are the same price and all books are the same price?

(A) \$14
(B) \$20
(C) \$28
(D) \$29
(E) \$30

7. Marisa has \$40 more than Ben, and Ben has one-third as much money as Marisa. How many dollars does Ben have?

\$

8. Norman is 12 years older than Michael. In 6 years, he will be twice as old as Michael. How old is Michael now?

(A) 3
(B) 6
(C) 12
(D) 18
(E) 24

9. 3 people split a \$100 stereo but pay different amounts. If A pays \$5 less than B, C pays more than \$35, and all 3 people pay integer amounts, what is the most A could pay?

(A) 29
(B) 29.5
(C) 30
(D) 33

(E) 34

10. Krunchy Kustard sells only two kinds of doughnuts, glazed and cream-filled. A glazed doughnut has 200 calories, and a cream-filled doughnut has 360 calories. If Michael ate 5 doughnuts totaling 1,640 calories, how many were glazed?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

11. Olympic lifting consists of two disciplines, the Snatch, and the Clean and Jerk. Halil's best Snatch and best Clean and Jerk sum to 295 kilograms. If his best Clean and Jerk was 25 kilograms heavier than his best Snatch, what was the weight of his best Clean and Jerk?

(A) 135
(B) 142.5
(C) 145
(D) 147.5
(E) 160

12. The "aspect ratio" of a computer monitor is the ratio of the monitor's width to its height. If a particular monitor has an aspect ratio of 16 : 9, and a perimeter of 100 inches, how many inches wide is the monitor?

(A) 18
(B) 25
(C) 32
(D) 36
(E) 64

13. Cindy bought 48 containers of soda, all either 12-ounce cans or 20-ounce bottles. If the number of ounces she purchased in cans was equal to the number of ounces she purchased in bottles, how many bottles of soda did Cindy buy?

(A) 18
(B) 21
(C) 24
(D) 27
(E) 30

14. Red chips all have the same value as one another, blue chips all have the same value as one another, and yellow chips also all have the same value as one another. If the value of a red chip plus a blue chip is 4.25, the value of a blue chip plus a yellow chip is 2.75, and the value of a red chip plus a blue chip plus a yellow chip is 4.5, what is the value of a red chip plus a yellow chip?

(A) 0.25
(B) 2
(C) 2.25
(D) 2.75
(E) 3

15. Two runners' race times add to 170 seconds and one of the race times is ten seconds less than twice the other. What is the faster race time, in seconds?

(A) 40
(B) 50
(C) 60
(D) 70
(E) 110

16. Beth is twelve years younger than Alan. In 20 years, Beth will be 80% of Alan's age. How old is Beth now?

[ ] years old

17. Rey is 12 years younger than Sebastian. Five years ago, Rey was half Sebastian's age. How old will Sebastian be next year?

(A) 15
(B) 20
(C) 25
(D) 30
(E) 35

18. During a sale, the local outlet of the Chasm sold three times as many jeans as chinos. If they made twice as much profit for a pair of chinos as for a pair of jeans, and sold no other items, what percent of their profits during the sale came from chinos?

(A) $16\frac{2}{3}\%$
(B) 20%
(C) 40%
(D) 60%
(E) $83\frac{1}{3}\%$

19. Marisol is twice as old as Vikram. Eight years ago, Marisol was 6 years younger than three times Vikram's age at that time. How old will Marisol be in 5 years?

[ ]

20. Mark is twice as old as Vicky. Four years ago, Mark was 6 years younger than three times Vicky's age at that time. How old will Mark be in 2 years?

[ ]

21. The length of a rectangle is two more than twice the width, and the area of the rectangle is 40. What is the rectangle's perimeter?

[ ]

22. Marcy bought one pair of jeans at 70% off and one blouse at 40% off. If she paid $12 more for the blouse than for the jeans, and she spent a total of $84, what was the original price of the jeans?

(A) 76
(B) 96
(C) 100
(D) 120
(E) 124

23. Cranwell Golf Course offers two different pricing packages for golf lessons. Under the "Sapphire" pricing plan, lessons can be bought for a flat rate of $80 per hour. Under the "Diamond" pricing plan, for an initial fee of $495, lessons can be bought for a rate of $15 per hour. If Jeanie buys the "Diamond" pricing plan, how many golf lessons does she need to take in order to have spent exactly 40% less than she would have under the "Sapphire" plan?

(A) 10
(B) 12
(C) 15
(D) 18
(E) 20

24. Wall-to-wall carpeting is installed in a certain hallway. The carpeting costs $4.25 per square foot. If the perimeter of the hallway (in feet) is equal to 44% of the area of the hallway (in square feet) and the hallway is 50 feet long, how much did it cost to install the carpeting?

(A) $182.50
(B) $212.50
(C) $505.25
(D) $1,062.50
(E) $1,100.00

25. Jamal gets three monthly credit card statements over the course of three months. If his average monthly statement over these three months is $44 more than the median amount, and the sum of the largest and the smallest statement is $412, what is the total amount that Jamal spent over these three months?

(A) $456
(B) $552
(C) $600
(D) $824
(E) $1,000

26. A certain kennel houses only collies, labs, and golden retrievers. If the ratio of collies to labs is 5 : 9, there are 66 golden retrievers, and 12 more golden retrievers than labs, what percent of the total number of dogs in the kennel are collies?

(A) 5%
(B) 9%
(C) 12%
(D) 20%
(E) 25%

27. If Mason is now twice as old as Gunther was 10 years ago, and G is Gunther's current age in years, which of the following represents the sum of Mason and Gunther's ages 4 years from now?

(A) $\frac{3G}{2}+3$
(B) $3G + 28$
(C) $3G - 12$
(D) $8 - G$
(E) $14-\frac{3G}{2}$

28. A baker makes a combination of chocolate chip cookies and peanut butter cookies for a school bake sale. His recipes only allow him to make chocolate chip cookies in batches of 7, and peanut butter cookies in batches of 6. If he makes exactly 95 cookies for the bake sale, what is the minimum number of chocolate chip cookies that he makes?

(A) 7
(B) 14
(C) 21
(D) 28
(E) 35

29. Janie has 5 fewer candies than Mark. If Janie gives Mark 5 candies, Mark will then have 4 times as many candies as Janie. How many candies does Janie have?

(A) 5
(B) 10
(C) 15
(D) 20
(E) 25

30. If Standard Jeans cost $60 and Designer Jeans cost 150% more, and 29 total pairs of jeans are sold for a total of $3,540, how many pairs were Designer Jeans?

(A) 2
(B) 9
(C) 18
(D) 20
(E) 23

31. Lou has three daughters: Wen, Mildred, and Tyla. Three years ago, when Lou was twice as old as Tyla, he was thirty years older than Mildred. Now, he is forty-seven years older than Wen. In four years, Wen will be half as old as Tyla. What is Lou's, Wen's, Mildred's and Tyla's combined age?

(A) 138
(B) 144
(C) 154
(D) 166
(E) 181

32. A farmer has exactly 1,000 square feet of farmland, on which he can grow both soy and corn. Every square foot can produce either one pound of soy or three pounds of corn. If soy can be sold on the market for $12/pound for the first hundred pounds and then $6 per pound after that, and corn can be sold on the market for $10/pound, what is the number of square feet of farmland that the farmer should devote to soy to make a profit of exactly $13,080?

(A) 30
(B) 100
(C) 270
(D) 600
(E) 730

33. Dwayne planted 70 acres with two types of field beans, navy beans, and pinto beans. Each acre of navy beans yielded 27 bushels, and each acre of pinto beans yielded 36 bushels. If Dwayne grew twice as many bushels of pinto beans as navy beans, how many acres of pinto beans did he plant?

(A) 28
(B) 30
(C) 35
(D) 40
(E) 42

Two-Variable Word Problems Answers

1. **(C).** Let c = number of computers. Let e = number of employees

There are 5 more computers than employees. You can translate that into an equation:

$c = e + 5$
If $e = 10$, then $c = (10) + 5$
$c = 15$

The ratio of computers to employees is 15 : 10, which can be reduced to 3 : 2.

2. **(D).** Let g = the number of cars that the Green lot can hold. Let r = the number of cars that the Red lot can hold

The first two sentences can be translated into two equations:

$g + r = 115$
$g = r - 35$

You want to solve for r, so you should substitute (r - 35) for g in the first equation:

$(r - 35) + r = 115$
$2r - 35 = 115$
$2r = 150$
$r = 75$

3. **(D).** Let P = the number of slices of pizza eaten by each of the two friends who eat the same amount. Let T = the number of slices of pizza eaten by the third friend.

$T = P + 2$
$P + P + T = 14$

Substitute (P + 2) for T in the second equation:

$P + P + (P + 2) = 14$
$3P + 2 = 14$
$3P = 12$
$P = 4$

You can use the value of P to solve for T:

$T = P + 2 = 4 + 2 = 6$

4. **(C).** This is an algebraic translation question, so you should start by translating the given information into equations.

Remember to add eight to both Polly and Quan's ages, because they will *both* be eight years older in eight years!

$p + 8 = 2(q + 8)$
$p + 8 = 2q + 16$
$p = 2q + 8$

Looking at the two columns, you can see that it would be helpful to manipulate the equation one last time:

$p - 8 = 2q$

You can see from that equation that the two columns are equal. The answer is (C).

5. **(B).** Many questions that involve two unknowns (e.g., the number of pens and the number of pencils) can be translated either as one equation involving one variable, or two equations involving two variables each.

It's a bit more work to *translate* using just one variable than using two, but generally much less work to *solve* one equation with one variable than to solve a system of two equations with two variables.

With One Variable

First, assign one variable to the pencils, and then define the pens in terms of the pencils:

Number of pencils = x
Number of pens = $10 - x$

Since this problem describes a real-life situation, it is not too difficult to write the formula:

(Cost per pen × number of pens) + (cost per pencil × number of pencils) = total cost

Plugging in the numbers from the problem (70 cents per pen and 40 cents per pencil):

$70(10 - x) + 40x = 520$
$700 - 70x + 40x = 520$
$700 - 30x = 520$
$180 = 30x$
$x = 6$

With Two Variables

First, assign one variable to the pencils, and another variable to the pens.

Number of pencils = x
Number of pens = y

$x + y = 10$
$70y + 40x = 520$

Next, isolate y (from the first, simpler equation) so you can substitute:

$y = 10 - x$

Substitute $10 - x$ for y:

$$70(10 - x) + 40x = 520$$
$$700 - 70x + 40x = 520$$
$$700 - 30x = 520$$
$$180 = 30x$$
$$x = 6$$

SHORTCUT: This question allows backsolving, choosing an answer choice to see whether it works. Generally, you'll want to start with answer choice (C); if it turns out to be too large, you can eliminate it and the two answer choices larger than it; if it turns out to be too little, you can eliminate it and the two answer choices less than it. (C) is not always the most efficient answer choice to test first, though. Here, for instance, (D) and (E) are implausibly large, so start with (B), the middle value of the remaining answers.

But proceed without that insight. Assume that Iris bought 8 pencils, and therefore 2 pens. $8 \times 40 + 2 \times 70 = 320 + 140 = 460$. That's 60 cents too little, so Iris must have bought fewer pencils and more pens. Try 6 pencils and 4 pens. $6 \times 40 + 4 \times 70 = 240 + 280 = 520$. (You might also have noticed that every time Iris swaps a pencil for a pen, she spends an extra 30 cents.)

6. **(C).** The equations are $10s + 2b = 48$ and $15s + b = 44$. The easiest next move would be to solve the second equation for b:

$$b = 44 - 15s$$

Substitute that into the first equation:

$$10s + 2(44 - 15s) = 48$$
$$10s + 88 - 30s = 48$$
$$-20s + 88 = 48$$
$$-20s = -40$$
$$s = 2$$

Plug $s = 2$ back into whichever equation you prefer to get that $b = 14$, and thus TWO books cost \$28.

7. **20.** Write both facts from the problem as simple equations:

$$M = B + 40$$
$$B = \frac{1}{3}M$$

Since you want the dollar amount for Ben, substitute for the other variable, M. Since M equals $B + 40$, write $B + 40$ in parentheses in place of M in the second equation:

$$B = \frac{1}{3}(B + 40)$$

Now distribute:

$$B = \frac{B}{3} + \frac{40}{3}$$

Subtract $\frac{B}{3}$ from both sides (note that $\frac{B}{3}$ is the same as $\frac{1}{3}B$, so subtracting $\frac{1}{3}B$ from B will give you $\frac{2}{3}B$, or $\frac{2B}{3}$.

$$\frac{2B}{3} = \frac{40}{3}$$
$$2B = 40$$
$$B = 20$$

The final answer is \$20.

Alternatively, you could reason that if Ben has one-third what Marisa does, then he's *missing* two-thirds of her amount. Since that two-thirds turns out to equal \$40, then two-thirds of Marisa's money is \$40. Thus, her total amount is \$60. Divide by 3 to get Ben's amount, \$20.

8. **(B).** Let N = Norman's age now $(N + 6)$ = Norman's age in 6 years.
Let M = Michael's age now $(M + 6)$ = Michael's age in 6 years.

Translate the first two sentences into equations. Note that the second equation deals with Norman and Michael's ages in 6 years:

$$N = M + 12$$
$$(N + 6) = 2(M + 6)$$

You want to solve for N, so substitute $(M + 12)$ for N in the second equation:

$$(M + 12) + 6 = 2(M + 6)$$
$$M + 18 = 2M + 12$$
$$M + 6 = 2M$$
$$6 = M$$

9. **(A).** This is a maximization question (what is the *most* A could pay). In order to solve maximization questions, you often have to minimize the other terms. In this case, you need to minimize B and C in order to maximize A.

The minimum possible C is \$36, leaving \$64 for A and B to pay. Since B pays \$5 more than A:

$$B = A + 5$$
$$A + (A + 5) = 64$$
$$2A = 59$$
$$A = 29.5$$

Of course, this isn't possible, because the three people have to pay integer values. So you need to move C up to \$37, leaving \$63 for A and B to pay.

$$A + (A + 5) = 63$$
$$2A = 58$$
$$A = 29$$

The correct answer is (A), or \$29.

10. **(A).** Many questions that involve two unknowns (e.g., the number of glazed doughnuts and the number of cream-filled doughnuts) can be translated either as one equation involving one variable, or two equations involving two variables each.

It's a bit more work to *translate* using just one variable than using two, but generally much less work to *solve* one equation with one variable than to solve a system of two equations with two variables.

With One Variable

Michael ate 5 doughnuts.
Number of glazed = x
Number of cream-filled = $5 - x$

(number of glazed × calories per glazed) + (number of cream-filled × calories per cream-filled) = total calories

$$200x + 360(5 - x) = 1{,}640$$
$$200x + 1{,}800 - 360x = 1{,}640$$
$$-160x = -160$$
$$x = 1$$

With Two Variables

Number of glazed = x
Number of cream-filled = y

$x + y = 5$
$200x + 360y$ Isolate y in first equation to allow substitution.

$y = 5 - x$ Substitute $5 - x$ for y in second equation.

$$200x + 360(5 - x) = 1{,}640$$
$$200x + 1{,}800 - 360x = 1{,}640$$
$$-160x = 160$$
$$x = 1$$

Alternative Method

This question allows backsolving, choosing an answer choice to see whether it works. Generally, you'll want to start with answer choice (C); if it turns out to be too great, you can eliminate it and the two answer choices less than it; if it turns out to be too little, you can eliminate it and the two answer choices less than it.

Assume that Michael ate 3 glazed and therefore 2 cream-filled doughnuts. $3(200) + 2(360) = 600 + 720 = 1{,}320$. That's 320 calories too few. You might notice that every time Michael swaps a glazed for a cream-filled, he consumes another 160 calories. Since $2(160) = 320$, he needs to swap 2 of his 3 glazed for cream-filled. If you don't notice that, just try (B) 2 glazed and 3 cream-filled, and see that yields only 1,480 calories.

11. **(E).** Many questions that involve two unknowns (e.g., the weight of the Snatch and the weight of the Clean and Jerk) can be translated either as one equation involving one variable, or two equations involving two variables each.

It's a bit more work to *translate* using just one variable than using two, but generally much less work to *solve* one equation with one variable than to solve a system of two equations with two variables.

With One Variable

Halil's best Snatch and best Clean and Jerk sum to 295 kilograms.

Weight of best Clean and Jerk = x
Weight of best Snatch = $295 - x$

His best Clean and Jerk was 25 kilograms heavier than his best Snatch.

$$x = (295 - x) + 25$$
$$x = 320 - x$$
$$2x = 320$$
$$x = 160$$

With Two Variables

Weight of best Clean and Jerk = x
Weight of best Snatch = y

$x + y = 295$ Isolate y to allow substitution.

$y = 295 - x$ Substitute $295 - x$ for y.

$$x = (295 - x) + 25$$
$$x = 320 - x$$
$$2x = 320$$
$$x = 160$$

SHORTCUT: You could also subtract the difference (25) from the sum (295), then divide the result by 2. This will give you the weight of the *lighter* lift. Or, you might notice simply that the Clean and Jerk must be more than half of the total of 295. Half of 295 is 147.5, and only one answer is greater than that.

12. **(C).** Rather than assigning separate variables to the width and height, define them both in terms of the same unknown multiplier, based on the ratio given:

Width = $16m$
Height = $9m$

Note that, since you want the width, YOU ARE SOLVING FOR $16m$, NOT SIMPLY FOR m!

The perimeter of a rectangle = 2(length + width), or in this case 2(width + height)

$100 = 2 \times (16m + 9m)$
$100 = 50m$
$m = 2$
$16m = 32$

SHORTCUT: Another method depends on the same underlying logic, but forgoes the algebra.

Suppose the dimensions were simply 16 inches and 9 inches. This would yield a perimeter of 50 inches. Double the width and height to double the perimeter.

13. **(A).** Many questions that involve two unknowns (e.g., the number of bottles and the number of cans) can be translated either as one equation involving one variable, or two equations involving two variables each.

With One Variable

Cindy bought 48 containers of soda
Number of bottles = x
Number of cans = $48 - x$

The number of ounces she purchased in cans was equal to the number of ounces she purchased in bottles
(ounces/can)(number of cans) = (ounces/bottle)(number of bottles)

$12(48 - x) = 20x$
$576 - 12x = 20x$
$576 = 32x$
$x = 18$

With Two Variables

Number of bottles = x
Number of cans = y
Cindy bought 48 containers of soda

$x + y = 48$

the number of ounces she purchased in cans was equal to the number of ounces she purchased in bottles

(ounces/can)(number of cans) = (ounces/bottle)(number of bottle)

$12y = 20x$ Isolate y in first equation to allow substitution.

$y = 48 - x$ Substitute into second equation.

$$12(48 - x) = 20x$$
$$576 - 12x = 20x$$
$$576 = 32x$$
$$x = 18$$

Alternative Method

This question allows backsolving, choosing an answer choice to see whether it works. Generally, you'll want to start with answer choice (C); if it turns out to be too large, you can eliminate it and the two answer choices larger than it; if it turns out to be too little, you can eliminate it and the two answer choices less than it. There are exceptions, though. Here, for instance, you notice that (C), (D) and (E) are implausibly large. There must be fewer bottles than cans, if the total volumes of the (large) bottles is to equal the total volume of the (small) cans, which means that the answer must be either (A) or (B).

Start with (B). Assume that Cindy bought 21 bottles, and therefore 27 cans. $21 \times 20 = 420$, $27 \times 12 = 324$. These numbers aren't equal. Cindy must have bought fewer than 21 bottles, and more than 27 cans, and only (A) matches that.

SHORTCUT: Because (ounces/can)(number of cans) = (ounces/bottle)(number of bottles), the ratio of (number of bottles) to (number of cans) will be the inverse of the ratio of (ounces per bottle) to (ounces per can). So the ratio of bottles to cans is 12 : 20, or 3 : 5. This means that only 3/8 of all the containers are bottles, 3/(5 + 3). 3/8 of 48 is 18. This shortcut requires very little math, but a great deal of mathematical sophistication. It comes in handy, but don't worry if you didn't spot it. There are many valid ways to solve problems like this one.

14. **(B).** First, write all the statements as equations:

$$r + b = 4.25$$
$$b + y = 2.75$$
$$r + b + y = 4.5$$

Also write "$r + y$?" on your paper, as a reminder of what you are looking for.

If $r + b = 4.25$, then $r + b + y = 4.5$ could be rewritten as:

$$4.25 + y = 4.5$$
$$y = 0.25$$

Since $b + y = 2.75$ and $y = 0.25$:

$$b + 0.25 = 2.75$$
$$b = 2.5$$

Since $r + b = 4.25$ and $b = 2.5$:

$$r + 2.5 = 4.25$$

$r = 1.75$

Therefore, $r + y = 1.75 + 0.25 = 2$.

15. **(C).** Call the race times x and y. Since you are told a sum:

$$x + y = 170$$

One of the race times is ten seconds less than twice the other

$$x = 2y - 10$$

Since the second equation is already solved for x, plug $(2y - 10)$ in for x in the first equation:

$$2y - 10 + y = 170$$
$$3y - 10 = 170$$
$$3y = 180$$
$$y = 60$$

If $y = 60$ and the times add to 170, then $x = 110$.

Note that you are asked for the *faster* race time — that means the smaller number! The answer is 60.

16. **28.** Since Beth is twelve years younger than Alan:

$$B = A - 12$$

To translate, *In 20 years, Beth will be 80% of Alan's age*, make sure that Beth becomes $B + 20$ and Alan becomes $A + 20$ (if you will be doing other operations to these values, put parentheses around them to make sure the rules of PEMDAS are not violated):

$$B + 20 = 0.8(A + 20)$$
$$B + 20 = 0.8A + 16$$
$$B + 4 = 0.8A$$

Since the first equation is already solved for B, plug $(A - 12)$ into the simplified version of the second equation in place of B.

$$B + 4 = 0.8A$$
$$(A - 12) + 4 = 0.8A$$
$$A - 8 = 0.8A$$
$$0.2A - 8 = 0$$
$$0.2A = 8$$
$$A = 40$$

Alan is 40. Since $B = A - 12$, Beth is 28. Since you are answering for Beth, the answer is 28.

17. **(D).** Many questions that involve two unknowns (e.g., Rey's age and Sebastian's age) can be translated either as one

equation involving one variable, or two equations involving two variables each.

It's a bit more work to *translate* using just one variable than using two, but generally much less work to *solve* one equation with one variable than to solve a system of two equations with two variables.

With One Variable

Rey is 12 years younger than Sebastian

s = Sebastian's age NOW

$s - 12$ = Rey's age NOW

In problems that involve values that change over time (ages, prices, number of employees, etc.), make sure to fix the variable to some particular time.

How old will Sebastian be next year?

Solve for $s + 1$

Five years ago, Rey was half Sebastian's age

Five years ago, Rey was $(s - 12) - 5$, and Sebastian was $(s - 5)$, so…

$(s - 12) - 5 = (s - 5)/2$

$s - 17 = (s - 5)/2$

Multiply both sides by 2 so you no longer have to deal with a fraction:

$2(s - 17) = s - 5$

$2s - 34 = s - 5$

$2s = s + 29$

$s = 29$

$s + 1 = 30$

With Two Variables

r = Rey's age NOW

s = Sebastian's age NOW

How old will Sebastian be next year?

Solve for $s + 1$

Rey is 12 years younger than Sebastian

$r = s - 12$

Five years ago, Rey was half Sebastian's age

$r - 5 = (s - 5)/2$

Substitute $s - 12$ for r.

$(s - 12) - 5 = (s - 5)/2$
$s - 17 = (s - 5)/2$

Multiply both sides by 2 to clear the fraction.

$2s - 34 = s - 5$
$2s = s + 29$
$s = 29$
$s + 1 = 30$

18. **(C).** If all the values given in a problem and its answers are *percents*, *ratios*, or *fractions of* some unknown, then the problem will probably be easiest to solve by stipulating values for the unknowns. In this problem, the two ratios given are 3 : 1 (jeans sold : chinos sold) and 2 : 1 (profits per pair of chinos : profits per pair of jeans). The easiest number to stipulate are:

3 pairs of jeans sold
1 pair of chinos sold
\$2 profit/pair of chinos
\$1 profit/pair of jeans

This yields \$2 profit from the chinos, out of a total \$5 in profit. 2/5 = 40%

Notice that this relatively simple problem is surprisingly complicated with variables. Even if you're clever enough to use just two variables rather than four, it's still a bit of work.

c = number of chinos sold
$3c$ = number of jeans sold
p = profit per jeans sold
$2p$ = profit per chinos sold

solve for (profit for chinos)/[(profit for chinos) + (profit for jeans)]

$(c \times 2p)/(c \times 2p + 3cp)$
$2pc/(5pc) = 2/5 = 40\%$

If you use four variables, you'll almost certainly make a translation error.

19. **49.** Write each sentence as its own equation:

Equation 1: $M = 2V$
Equation 2: $(M - 8) = 3(V - 8) - 6$

Next, simplify Equation 2 and substitute. Since Equation 1 is already solved for M, the easiest way to substitute is to plug in $2V$ in place of M in Equation 2.

$2V - 8 = 3V - 24 - 6$
$2V - 8 = 3V - 30$
$2V + 22 = 3V$
$22 = V$

Thus, $M = 44$, and Marisol in 5 years, or $M + 5$, is equal to 49.

20. **30.** Translate the first two sentences in the problem into two separate equations:

Equation 1: $M = 2V$
Equation 2: $(M - 4) = 3(V - 4) - 6$

Next, simplify Equation 2 and substitute. Since Equation 1 is already solved for M, the easiest way to substitute is to plug in $2V$ in place of M in Equation 2.

$2V - 4 = 3V - 12 - 6$
$2V - 4 = 3V - 18$
$2V + 14 = 3V$
$14 = V$

Thus, $V = 14$, $M = 28$, and Mark in 2 years, or $M + 2$, is equal to 30.

21. **28.** Convert this word problem into two equations with two variables. "The length is two more than twice the width" can be written as:

$L = 2W + 2$

Since the area is 40 and area is length × width:

$LW = 40$

Since the first equation is already solved for L, plug $(2W + 2)$ in for L into the second equation:

$(2W + 2)W = 40$
$2W^2 + 2W = 40$

Since you now have a quadratic on your hands (you have both a W^2 and a W term), get all terms on one side to set them to zero:

$2W^2 + 2W - 40 = 0$

Simplify as much as possible — in this case, divide the entire equation by 2 — before trying to factor:

$W^2 + W - 20 = 0$
$(W - 4)(W + 5) = 0$
$W = 4$ or -5

Since a width cannot be negative, the width = 4. Since $LW = 40$, the length must be 10.

Perimeter = $2L + 2W$
Perimeter = $2(10) + 2(4)$
Perimeter = 28

Note that it might have been possible for you to puzzle out that the sides were 4 and 10 just by trying values. However, if you did this, you got lucky — no one said that the values even had to be integers! The ability to translate into equations and solve is very important for the GRE.

22. **(D).** To solve this problem, establish the following variables:

J = original jean price
B = original blouse price

Then establish a system of equations, keeping in mind that "70% off" is the same as 100% - 70% = 30%, or 0.3, of the original price:

$0.3J + 12 = 0.6B$
$0.3J + 0.6B = 84$

Now simply use whatever strategy you're most comfortable with to solve a system of equations — for example, aligning the equations and then subtracting them:

$$\begin{array}{l} 0.3J + 12 = 0.6B \\ 0.3J - 84 = -0.6B \\ \hline 0 + 96 = 1.2B \end{array}$$

$B = 96/1.2$
$B = 80$

And you can plug the price of the blouse back into the original equation to get the price of the jeans:

$0.3J + 12 = 0.6B$
$0.3J + 12 = 48$
$0.3J = 36$
$J = 120$

Alternatively, you could first figure out the price of the discounted jeans with this equation:

$x + (x + 12) = 84$
$2x + 12 = 84$
$2x = 72$
$x = 36$

Then plug that discounted price into the equation *discounted price = original price × (100% - percent discount):*

$36 = 0.3P$

$360 = 3P$
$120 = P$

23. **(C).** Start by assigning variables:

D = price under the Diamond plan
S = price under the Sapphire plan
x = the number of lessons Jeanie takes

…then establish equations:

$D = 495 + 15x$
$S = 80x$
$0.6S = D$

…then solve by substitution:

$0.6S = 495 + 15x$
$0.6(80x) = 495 + 15x$
$48x = 495 + 15x$
$33x = 495$
$x = 15$

24. **(D).** The equation for the perimeter of a space is $2W + 2L = P$, where W is width and L is length.

The equation for the area is $A = W \times L$. Thus,

$0.44(W \times L) = 2W + 2L$
$0.44(50W) = 2W + 2(50)$
$22W = 2W + 100$
$20W = 100$
$W = 5$

If $W = 5$ and $L = 50$, then the total square footage of the room is 250, and the total cost is:

$\$4.25 \times 250 = \$1{,}062.50$

25. **(B).** Call the smallest statement S, the middle statement M, and the largest statement L.

From your knowledge of medians, you should know that M is the same as the median, since there are only three values. The equation for averages is:

$$\frac{\text{Sum of \#'s}}{\text{Number of \#'s}} = \text{Average}$$

Incorporate the equation for averages into the following equation:

$$\frac{S+M+L}{3} = M + 44$$

$S + M + L = 3M + 132$
$S + L = 2M + 132$

While you don't know the actual quantities of S and L, you know their sum:

$412 = 2M + 132$
$280 = 2M$
$140 = M$

Finally, add M to the sum of S and L:

$140 + 412 = 552$

26. **(D).** Start by assigning variables:

C = Number of collies
L = Number of labs
G = Number of golden retrievers

…and work from what you know.

$G = 66$
$L = 66 - 12$
$L = 54$

Ratios work like fractions, and you can set them up accordingly.

$$\frac{5}{9} = \frac{C}{54}$$

Cross multiplying and simplifying, you get:

$C = 30$

Now take the number of collies and express it as a percentage of the total number of dogs:

Total # of Dogs = 30 + 54 + 66 = 150

$$\frac{30}{150} \times 100 = 20\%$$

27. **(C).** The sum of Mason and Gunther's ages 4 years from now requires adding 4 to BOTH ages.

First equation (what you are looking for):

$(M + 4) + (G + 4) = ?$
$M + G + 8 = ?$

Since Mason is twice as old as Gunther was 10 years ago, put (G - 10) in parentheses and build the second equation from there (the parentheses are crucial).

Second equation:

$M = 2(G - 10)$
$M = 2G - 20$

Note that the answer choices ask for the sum of the ages 4 years from now, in terms of G, so substitute for M (the variable you substitute for is the one that drops out).

Substituting from the second equation into the first:

$(2G - 20) + G + 8 = ?$
$3G - 12 = ?$

This matches choice (C).

Alternatively, you could write the second equation, $M = 2(G - 10)$, and then come up with two values that "work" in this equation for M and G. The easiest way to do this is to make up G, which will then tell you M. For instance, set $G = 12$ (use any number you want, as long as it's over 10, since the problem strongly implies that Gunther has been alive for more than 10 years).

$M = 2(12 - 10)$
$M = 4$

If Gunther is 12, then Mason is 4. In four years, they will be 16 and 8, respectively. Add these together to get 24.

Now, plug $G = 12$ into each answer choice to see which yields the correct answer (for this example), 24. Only choice (C) works.

28. **(E).** The equation for the situation described is $7x + 6y = 95$, where x stands for the number of batches of chocolate chip cookies and y stands for the number of batches of peanut butter cookies.

It looks as though this equation is not solvable, because you have two variables and only one equation. However, since the baker can only make whole batches, x and y must be integers, which really limits the possibilities.

Furthermore, you want the *minimum* number of chocolate chip cookies the baker could have made. So, try 1 for x and see if you get an integer for y (use your calculator when needed!):

$7(1) + 6y = 95$
$6y = 88$
$y = 14.6\ldots$

Since this did not result in an integer number of batches of peanut butter cookies, this situation doesn't work. Try 2, 3, 4, etc. for x. (Don't try values out of order — remember, there might be more than one x value that works, but you need to be sure that you have the smallest one!)

You will see that the smallest value that works for x is 5:

$7(5) + 6y = 95$
$6y = 60$
$y = 10$

Remember that you need the minimum number of chocolate chip *cookies*, not *batches of cookies*. Since the minimum number of batches is 5 and there are 7 cookies per batch, the minimum number of chocolate chip cookies is 35.

29. **(B).** First, translate the problem into two equations, writing "Janie after she gave Mark 5 candies" as (J - 5) and "Mark after receiving 5 more candies" as (M + 5).

$J = M - 5$
$4(J - 5) = M + 5$

Since $J = M - 5$, plug $M - 5$ in for J in the second equation:

$4(M - 5 - 5) = M + 5$
$4(M - 10) = M + 5$
$4M - 40 = M + 5$
$4M = M + 45$
$3M = 45$
$M = 15$

If $M = 15$, then, since Janie has 5 fewer candies, $J = 10$.

Alternatively, you could backsolve from the answers. Start with choice (C). If Janie had 15 candies, Mark would have 20. If Janie gave Mark 5, she would have 10 and he would have 25. Since 25 is NOT 4 times 10, this answer is not correct. Since you want Mark to have more and Janie to have less, you might intuit that you should try a smaller answer. Try choice (B). If Janie had 10 candies, Mark would have 15. If Janie then gave Mark 5, she would have 5 and he would have 20. Since 20 is 4 times more than 5, this is the answer.

30. **(D).** This is an algebra question with two unknowns: the number of Standard jeans sold and the number of Designer jeans sold.

First, you're told that 29 pairs of jeans are sold altogether: $s + d = 29$

You also know that the cost of Standard Jeans is \$60 and the cost of Designer Jeans is \$150 (Be careful! The question doesn't say Designer Jeans cost 150% *as much* as Standard Jeans, but 150% *more*. If one thing costs 100% more than another thing, it's twice as much, so something that costs 150% more is 2.5 times as much). You've been given the total cost of the jeans, so you can write a second equation:

$60s + 150d = 3{,}540$

Now, look at your two equations:

$s + d = 29$
$60s + 150d = 3{,}540$

The easiest way to solve from here is to multiply the top equation by 60, then combine the two through elimination.

$$\begin{array}{l} 60s + 150d = 3{,}540 \\ \underline{-\ 60s + 60d = 1{,}740} \\ 90d = 1{,}800 \end{array}$$

Therefore, $d = 20$.

31. **(A).** The key to this tricky-sounding problem is setting up variables correctly and ensuring that you subtract or add appropriately for these variables when representing their ages at different points in time.

L = Lou's age now
W = Wen's age now
M = Mildred's age now
T = Tyla's age now

Represent the second sentence of the problem:

Equation 1: $(L - 3) = 2(T - 3)$
Equation 2: $(L - 3) = (M - 3) + 30$

Next:

Equation 3: $L = W + 47$
Equation 4: $(W + 4) = (T + 4)/2$

In order to solve this problem effectively, look for ways that you can get two of the equations to have the same two variables in them. If you have two equations with only two variables, you can solve for both of those variables. Equation 4 has a W and a T; the only other equation with a T is Equation 1. If you substitute the L in Equation 1 with the W from Equation 3, you will have two equations with just W's and T's.

Equation 1: $(L - 3) = 2(T - 3)$

$(W + 47) - 3 = 2(T - 3)$
$W + 44 = 2T - 6$
$W + 50 = 2T$

Equation 4: $(W + 4) = (T + 4)/2$

$2W + 8 = T + 4$
$2W + 4 = T$

Now combine the equations to solve for W.

$W + 50 = 2(2W + 4)$
$W + 50 = 4W + 8$
$W + 42 = 4W$
$42 = 3W$
$14 = W$

Now that you know Wen's age, you can solve for the rest.

Equation 3: $L = W + 47$

$L = 14 + 47$
$L = 61$

Equation 1: $(L - 3) = 2(T - 3)$

$(61 - 3) = 2(T - 3)$
$58 = 2T - 6$
$64 = 2T$
$32 = T$

Equation 2: $(L - 3) = (M - 3) + 30$

$61 - 3) = (M - 3) + 30$
$58 = M + 27$
$31 = M$

Now that you know that $L = 61$, $W = 14$, $M = 31$, and $T = 32$, add them together to find the answer.

$61 + 14 + 31 + 32 = 138$

32. **(C).** First, assign variables:

S = number of square feet devoted to soy
C = number of square feet devoted to corn

…and set up the equation:

$[(12)100 + (6)(S - 100)] + [(10)(3)(C)] = 13{,}080$

This means that the farmer gets \$12 per pound for the first 100 pounds, or (12)100, and then \$6 per pound for each pound after 100, or $(6)(S - 100)$. He also gets \$10 per pound for corn, but you have to also account for the fact that he can grow 3 pounds of corn per acre, not just one. Hence, $(10)(3)(C)$.

Simplify:

$[1{,}200 + 6S - 600] + [30C] = 13{,}080$
$600 + 6S + 30C = 13{,}080$

$6S + 30C = 12{,}480$
$S + 5C = 2{,}080$

You also know that:

$S + C = 1{,}000$
$S = 1{,}000 - C$

Combining, you get:

$(1{,}000 - C) + 5C = 2{,}080$
$1{,}000 + 4C = 2{,}080$
$4C = 1{,}080$
$C = 270$
$S = 1{,}000 - 270$
$S = 730$

33. **(E).** This question is difficult to translate. Begin by finding two things that are equal, and build an equation around that equality. *Dwayne grew twice as many bushels of pinto beans as navy beans*:

2(bushels of navy beans) = (bushels of pinto beans)

Break that down further:

bushels of navy beans = acres of navy beans × bushels per acre of navy beans
bushels of pinto beans = acres of pinto beans × bushels per acre of pinto beans

So:

2(acres of navy beans × bushels per acre of navy beans) =
(acres of pinto beans × bushels per acre of pinto beans)

Each acre of navy beans yielded 27 bushels, and each acre of pinto beans yielded 36 bushels

2 × 27 × (acres of navy beans) = 36 × (acres of pinto beans)

You can finish your translation with one variable or with two.

One Variable

Number of acres planted with pinto beans = p
Number of acres planted with navy beans = $70 - p$

$2 \times 27(70 - p) = 36p$
$54(70 - p) = 36p$
$3{,}780 - 54p = 36p$
$3{,}780 = 90p$
$p = 42$

Two Variables

Number of acres planted with pinto beans = p
Number of acres planted with navy beans = n

$$2 \times 27n = 36p$$
$$n + p = 70$$

Isolate n to allow substitution:

$$n = 70 - p$$

Substitute:

$$2 \times 27(70 - p) = 36p$$
$$54(70 - p) = 36p$$
$$3{,}780 - 54p = 36p$$
$$3{,}780 = 90p$$
$$p = 42$$

SHORTCUT: Notice that the ratio of (bushels of pinto beans produced) to (bushels of navy beans produced) is 2 : 1, greater than the ratio of (bushels of pinto beans produced per acre) to (bushels of navy beans produced per acre), 36 : 27. This means that a greater number of acres must have been planted with pinto beans than with navy beans, so the answer must be more than half of the 70 total, so either (D) or (E). You could simply backsolve the easier answer choice, (D).

40 acres of pinto beans × 36 bushels per acre = 1,440 bushels
30 acres of navy beans × 27 bushels per acre = 810 bushels

1440 isn't quite twice 810, so 40 isn't quite enough. The answer must be (E).

Chapter 18
of

Rates and Work

In This Chapter…

Rates and Work

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Roger took 2 hours to walk from his home to a store 3 miles away, and then returned home along the same path. If Roger's average rate for the round trip was 2 miles per hour, at what rate, in miles per hour, did Roger return home?

(A) $\frac{10}{3}$
(B) 3
(C) $\frac{5}{2}$
(D) 2
(E) 1

2. Running on a 10-mile loop in the same direction, Sue ran at a constant rate of 8 miles per hour and Rob ran at a constant rate of 6 miles per hour. If they began running at the same point on the loop, how many hours later did Sue complete exactly 1 more lap than Rob?

(A) 3
(B) 4

(C) 5
(D) 6
(E) 7

3. Svetlana ran the first 5 kilometers of a 10-kilometer race at a constant rate of 12 kilometers per hour. If she completed the entire 10-kilometer race in 55 minutes, at what constant rate did she run the last 5 kilometers of the race, in kilometers per hour?

(A) 15
(B) 12
(C) 11
(D) 10
(E) 8

4. A standard machine fills paint cans at a rate of 1 gallon every 4 minutes. A deluxe machine fills gallons of paint at twice the rate of a standard machine. How many hours will it take a standard machine and a deluxe machine, working together, to fill 135 gallons of paint?

(A) 1
(B) 1.5
(C) 2
(D) 2.5
(E) 3

5. Wendy builds a birdhouse in 15 hours and Michael builds an identical birdhouse in 10 hours. How many hours will it take Wendy and Michael, working together at their respective constant rates, to build a birdhouse? (Assume that they can work on the same birdhouse without changing each other's work rate.)

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9

6. Machine A, which produces 15 golf clubs per hour, fills a production lot in 6 hours. Machine B fills the same production lot in 1.5 hours. How many golf clubs does Machine B produce per hour?

$\boxed{}$ golf clubs per hour

7. Davis drove from Amityville to Beteltown at 50 miles per hour, and returned by the same route at 60 miles per hour.

Quantity A	**Quantity B**
Davis' average speed for the round trip, in miles per hour	55

8. If a turtle traveled $\frac{1}{30}$ of a mile in 5 minutes, what was its speed in miles per hour?

(A) 0.02
(B) $0.1\overline{6}$

(C) 0.4
(D) 0.6
(E) 2.5

9. Akilah traveled at a rate of x miles per hour for $2x$ hours

Quantity A	**Quantity B**
The number of miles Akilah traveled	$3x$

10. Claudette travels the first $\frac{2}{3}$ of a 60-mile trip at 20 miles per hour (mph) and the remainder of the trip at 30 mph. How many minutes later would she have arrived if she had completed the entire trip at 20 mph?

$\boxed{}$ minutes

11. Rajesh traveled from home to school at 30 miles per hour, then returned home at 40 miles per hour to retrieve a forgotten item, and finally returned back to school at 60 miles per hour, all along the same route. What was his average speed for the entire trip, in miles per hour?

(A) 32
(B) 36
(C) 40
(D) 45
(E) 47

12. Jack traveled the first 75% of an 80-mile trip at 45 miles per hour and the remainder at 30 miles per hour. What was Jack's average speed for the entire 80-mile trip, in miles per hour?

(A) 37.5
(B) 38.25
(C) 40
(D) 41.25
(E) 42.5

13. Lamont traveled 80 miles in 2.5 hours, at a constant rate. He then decreased his speed by 25% and traveled 120 additional miles at the new constant rate. How many hours did the entire journey take?

(A) 6.25
(B) 7.5
(C) 8.75
(D) 10
(E) 11.25

14. Twelve workers pack boxes at a constant rate of 60 boxes in 9 minutes. How many minutes would it take 27 workers to pack 180 boxes, if all workers work at the same constant rate?

(A) 12
(B) 13
(C) 14
(D) 15
(E) 16

15. Four editors can proofread 4 documents in 4 hours. How many editors would be required to proofread 80 documents in 2 hours, if all editors proofread all documents at the same constant rate?

(A) 120
(B) 130
(C) 140
(D) 150
(E) 160

16. To service a single device in 12 seconds, 700 nanorobots are required, with all nanorobots working at the same constant rate. How many hours would it take for a single nanorobot to service 12 devices?

(A) 7/3
(B) 28
(C) 108
(D) 1,008
(E) 1,680

17. Working at a constant rate, Sarita answered x verbal test questions in 3 hours. Separately, she solved math problems at a constant rate of y math problems every 30 minutes.

Quantity A	**Quantity B**
The number of verbal test questions Sarita answered in 1 hour	The number of math problems Sarita solved in 1 hour

18. If 45 people built $\frac{1}{2}$ of a pyramid in 288 days, how many days did it take 65 people to build the next $\frac{1}{6}$ the pyramid, rounded to the nearest integer, assuming each person works at the same constant rate?

days

19. A machine purifies 100 cubic feet (ft^3) of water in 4 minutes. How many minutes will it take the machine to purify the contents of a 15 foot × 15 foot × 10 foot tank that is $\frac{1}{2}$ of full of water?

(A) 20
(B) 30
(C) 45
(D) 60
(E) 75

20. If a baker made 60 pies in the first 5 hours of his workday, by how many pies per hour did he increase his rate in the last 3 hours of the workday in order to complete 150 pies in the entire 8-hour period?

(A) 12
(B) 14
(C) 16
(D) 18
(E) 20

21. A stockbroker worked 10 hours a day on Monday, Wednesday, and Friday, 11 hours a day on Tuesday and Thursday, and 8 hours on Saturday. She earned \$600 each weekday and \$300 on Saturday.

Quantity A	**Quantity B**
The stockbroker's average earnings, in dollars per hour, over the 6-day period.	50

22. Two coal carts, A and B, started simultaneously from opposite ends of a 400-yard track. Cart A traveled at a constant rate of 40 feet per second; Cart B traveled at a constant rate of 56 feet per second. After how many seconds of travel did the two carts collide? (1 yard = 3 feet)

(A) 75
(B) 48
(C) $23\frac{1}{3}$
(D) $12\frac{1}{2}$
(E) $4\frac{1}{6}$

23. Nine identical machines, each working at the same constant rate, can stitch 27 jerseys in 4 minutes. How many minutes would it take 4 such machines to stitch 60 jerseys?

(A) 8
(B) 12
(C) 16
(D) 18
(E) 20

24. Brenda walked a 12-mile scenic loop in 3 hours. If she then reduced her walking speed by half, how many hours would it take Brenda to walk the same scenic loop two more times?

(A) 6
(B) 8
(C) 12
(D) 18
(E) 24

25. A gang of criminals hijacked a train heading due south. At exactly the same time, a police car located 50 miles north of the train started driving south toward the train on an adjacent roadway parallel to the train track. If the train traveled at a constant rate of 50 miles per hour, and the police car traveled at a constant rate of 80 miles per hour, how long after the hijacking did the police car catch up with the train?

(A) 1 hour
(B) 1 hour and 20 minutes
(C) 1 hour and 40 minutes
(D) 2 hours
(E) 2 hours and 20 minutes

26. Each working at a constant rate, Rachel assembles a brochure every 10 minutes and Terry assembles a brochure every 8 minutes.

Quantity A	Quantity B
The number of minutes it will take Rachel and Terry, working together, to assemble 9 brochures	40

27. With 4 identical servers working at a constant rate, a new Internet search provider processes 9,600 search requests per hour. If the search provider adds 2 more identical servers, and server work rate never varies, the search provider can process 216,000 search requests in how many hours?

(A) 15
(B) 16
(C) 18
(D) 20
(E) 24

28. A pipe siphons ink from an 800-liter drum at a rate of r liters per minute. If two such pipes were used, the drum could be emptied 100 minutes faster than when one pipe is used.

Quantity A	Quantity B
r	5

29. If Sabrina can assemble a tank in 8 hours, and Janis can assemble a tank in 13 hours, then Sabrina and Janis working together at their constant respective rates can assemble a tank in approximately how many hours?

(A) 21
(B) 18
(C) 7
(D) 5
(E) 2

30. Etienne began to eat 20 cookies at exactly the same time Jacques began making more cookies, one at a time, at a constant rate of 16 cookies per hour. If Etienne ate 20 cookies per hour, after how many hours were there no cookies?

$\boxed{}$ hours

31. Phil collects virtual gold in an online computer game, and then sells the virtual gold for real dollars. After playing 10 hours a day for 6 days, he collected 540,000 gold pieces. If he immediately sold this virtual gold at a rate of $1 per 1,000 gold pieces, what were his average earnings per hour, in real dollars?

(A) 5
(B) 6
(C) 7
(D) 8
(E) 9

32. After completing a speed training, Alyosha translates Russian literature into English at a rate of 10 more than twice as many words per hour as he was able to translate before the training. If he was previously able to translate 10 words per minute, how many words can he now translate in an hour?

(A) 30

(B) 70
(C) 610
(D) 1,210
(E) 1,800

33. Jenny takes 3 hours to sand a picnic table; Laila can do the same job in $\frac{1}{2}$ hour. Working together at their respective constant rates, Jenny and Laila can sand a picnic table in how many hours?

(A) 1/6
(B) 2/9
(C) 1/3
(D) 3/7
(E) 5/6

34.

One worker strings 2 violins in 3 minutes. All workers string violins at the same constant rate.

Quantity A	**Quantity B**
The number of minutes required for 12 workers to string 720 violins	The number of violins that 5 workers can string in 24 minutes

35. Riders board the Jelly Coaster in groups of 4 every 15 seconds. If there are 200 people in front of Kurt in line, in approximately how many minutes will Kurt board the Jelly Coaster?

(A) 5
(B) 8
(C) 10
(D) 13
(E) 20

36. Machines A and B both shrink-wrap CDs continuously, each working at a constant rate, but Machine B works 50% faster than Machine A. If Machine B shrink-wraps 48,000 more CDs in a 24-hour period than Machine A does, what is Machine A’s shrink-wrapping rate in CDs per hour?

(A) 4,000
(B) 6,000
(C) 8,000
(D) 12,000
(E) 16,000

37. A team of 8 chefs produce 3,200 tarts in 5 days. All chefs produce tarts at the same constant rate.

Quantity A	**Quantity B**
The number of chefs needed to produce 3,600 tarts in 3 days	The number of days that 4 chefs need to produce 4,800 tarts

38. Working together at their respective constant rates, robot A and robot B polish 88 pounds of gemstones in 6 minutes. If robot A’s rate of polishing is 3/5 that of robot B, how many minutes would it take robot A alone to

polish 165 pounds of gemstones?

(A) 15.75
(B) 18
(C) 18.75
(D) 27.5
(E) 30

39. Car A started driving north from point X, traveling at a constant rate of 40 miles per hour. One hour later, car B started driving north from point X at a constant rate of 30 miles per hour. Neither car changed direction of travel. If each car started with 8 gallons of fuel, which is consumed at a rate of 30 miles per gallon, how many miles apart were the two cars when car A ran out of fuel?

(A) 30
(B) 60
(C) 90
(D) 120
(E) 150

40. A population of bacteria doubled at a constant rate, increasing from 50 to 3,200 bacteria in exactly two days.

Quantity A	**Quantity B**
Twice the population of bacteria after 16 more hours	The population of bacteria after 32 more hours

41. One robot, working independently at a constant rate, can assemble a doghouse in 12 minutes. What is the maximum number of complete doghouses that can be assembled by 10 such identical robots, each working on separate doghouses at the same rate for $2\frac{1}{2}$ hours?

(A) 20
(B) 25
(C) 120
(D) 125
(E) 150

42. A semiconductor company predicts that it will be able to double the density of transistors on its circuits (measured in transistors per square mm) every 18 months. If this prediction holds true, and the company's circuits currently have a density of 5 million transistors per square mm, what will be the density of transistors on the company's circuits, measured in millions of transistors per square mm, exactly 30 years from now?

(A) 5×2^{18}
(B) 5×2^{20}
(C) 5×2^{26}
(D) 5×2^{36}
(E) 5×2^{45}

43. Working continuously 24 hours a day, a factory bottles Soda Q at a rate of 500 liters per second and Soda V at a rate of 300 liters per second. If twice as many bottles of Soda V as of Soda Q are filled at the factory each day, what is the ratio of the volume of a bottle of Soda Q to a bottle of Soda V?

(A) 3/10
(B) 5/6
(C) 6/5
(D) 8/3
(E) 10/3

44. Working alone at their respective constant rates, Audrey can complete a certain job in 4 hours, while Ferris can do the same job in 3 hours. Audrey and Ferris worked together on the job and completed it in 2 hours, but while Audrey worked this entire time, Ferris worked for some of the time and took 3 breaks of equal length. How many minutes long was each of Ferris's breaks?

(A) 5
(B) 10
(C) 15
(D) 20
(E) 25

45. A turtle climbed to the top of a plateau at a rate of 4 miles an hour, crossed the plateau at a rate of x miles per hour, and descended the other side of the plateau at a rate of x^2 miles per hour. If each portion of the journey was equal in distance, what was the turtle's average speed for the entire trip, in terms of x?

(A) $\dfrac{2x}{x+2}$

(B) $\dfrac{(x+2)^2}{3}$

(C) $(x+2)^2$

(D) $\dfrac{4x^2}{(x+2)^2}$

(E) $\dfrac{12x^2}{(x+2)^2}$

Rates and Work Answers

1. **(B).** The average rate at which Roger travels is the total distance traveled divided by the total time spent traveling. In this case, Roger traveled 3 miles to and 3 miles back from a store, covering a total of 6 miles. The average rate for the whole trip is given as 2 miles per hour. Solve for the total time, using the variable *t*:

$$\text{average rate} = \frac{\text{total distance}}{\text{total time}}$$
$$2 = \frac{6}{t}$$
$$2t = 6$$
$$t = 3$$

The total time that Roger spent traveling was 3 hours. Since he took 2 hours to walk to the store, he only took 3 - 2 = 1 hour returning from the store. Roger traveled the 3 miles back in 1 hour, so he traveled at a rate of 3 miles per hour on the return trip.

2. **(C).** If Sue completed exactly one more lap than Rob, she ran 10 more miles than Rob. If Rob ran *d* miles, then Sue ran *d* + 10 miles. Rob and Sue began running at the same time, so they ran for the same amount of time. Let *t* represent the time they spent running. Fill out a chart for Rob and Sue:

	D (miles)	=	R (miles/hour)	×	T (hours)
Rob	d	=	6	×	t
Sue	$d + 10$	=	8	×	t

There are two equations:

$d = 6t$ $\qquad\qquad d + 10 = 8t$

Substitute $6t$ for d in the second equation, and then solve for t:

$$6t + 10 = 8t$$
$$10 = 2t$$
$$5 = t$$

3. **(D).** To calculate Svetlana's speed during the second half of the race, first calculate how long it took her to run the first half of the race. Svetlana ran the first 5 kilometers at a constant rate of 12 kilometers per hour. These values can be used in the $D = RT$ formula.

D (km)	=	R (km/hr)	×	T (hr)
5	=	12	×	t

Svetlana's time for the first part of the race is 5/12 hours, or 25 minutes.

She completed the entire 10-kilometer race in 55 minutes, so she ran the last 5 kilometers in 55 - 25 = 30 minutes, or 0.5 hours.

D (km)	=	R (km/hr)	×	T (hr)
5	=	r	×	0.5

$5 = 0.5r$
$10 = r$

Svetlana ran the second half of the race at a speed of 10 kilometers per hour.

4. **(E).** The question asks for the amount of time in hours, so re-express the work rates in gallons per hour, not gallons per minute. First, calculate the rate of the standard machine:

$$\frac{1 \text{ gallon}}{4 \text{ minutes}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} = \frac{60 \text{ gallons}}{4 \text{ hours}} = 15 \text{ gallons/hour}$$

Since the deluxe machine's rate is twice the standard machine's rate, the deluxe machine can fill 15 × 2 = 30 gallons of paint per hour. Together, the machines can fill 15 + 30 = 45 gallons of paint per hour. Now apply $W = R \times T$:

$135 = 45 \times T$
$3 = T$

5. **(B).** Use two separate lines in a $W = RT$ chart, one for Wendy and one for Michael, to calculate their respective rates. Building 1 birdhouse equals doing 1 unit of work:

	W (birdhouses)	=	R (birdhouses/hour)	×	T (hours)
Wendy	1	=	R_W	×	15
Michael	1	=	R_M	×	10

Thus, Wendy's rate is 1/15 birdhouses per hour, and Michael's rate is 1/10 birdhouses per hour. Since Wendy and Michael are working together, add their rates:

	W (birdhouses)	=	R (birdhouses/hour)	×	T (hours)
Wendy + Michael	1	=	$\frac{1}{15}+\frac{1}{10}$	×	t

Now solve for t by first combining the fractions:

$$1=\left(\frac{1}{15}+\frac{1}{10}\right)t$$
$$1=\left(\frac{2}{30}+\frac{3}{30}\right)t$$
$$1=\left(\frac{5}{30}\right)t$$
$$\frac{30}{5}=t$$
$$6=t$$

6. **60 golf clubs per hour.** First, calculate the size of a production lot. Machine A works at a rate of 15 golf clubs per hour and completes a production lot in 6 hrs. Plug this information into the $W = RT$ formula.

W (clubs)	=	R (clubs/hour)	×	T (hours)
w	=	15	×	6

w = (15 clubs per hour)(6 hours) = 90 clubs

Therefore, a production lot consists of 90 golf clubs. Since Machine B can complete the lot in 1.5 hours, use the $W = RT$ chart a second time to calculate the rate for Machine B.

W (clubs)	=	R (clubs/hour)	×	T (hours)
90	=	r	×	1.5

Make the calculation easier by converting 1.5 hours to 3/2 hours.

$$90 = \frac{3}{2}r$$

$$\frac{2}{3} \times 90 = r$$

$$2 \times 30 = r$$

$$60 = r$$

7. **(B).** Never take an average speed by simply averaging the two speeds (50 mph and 60 mph). You must use the formula Average Speed = Total Distance/Total Time. Fortunately, for Quantitative Comparisons, you can often sidestep actual calculations.

Davis' average speed can be thought of as an average of the speed he was traveling at every single moment during his journey — for instance, say Davis wrote down the speed he was going during every second he was driving, then he averaged all the seconds. Since Davis spent more *time* going 50 mph than going 60 mph, the average speed will be closer to 50 than 60, and Quantity B is larger. If the distances are the same, average speed is always weighted towards the *slower* speed.

If you want to actually do the math, pick a convenient number for the distance between Amityville and Beteltown — for instance, 300 miles (divisible by both 50 and 60). If the distance is 300 miles, it took Davis 6 hours to drive there at 50 mph, and 5 hours to drive back at 60 mph. Using Average Speed = Total Distance/Total Time (and a total distance of 600 miles, for both parts of the journey):

Average Speed = 600 miles/11 hours
Average Speed = 54.54…

You will get the same result with any value you choose for the distance. Thus, Quantity B is greater.

8. **(C).** The turtle traveled 1/30th of a mile in 5 minutes, which is 1/12 of an hour. Using the $D = RT$ formula, solve for r:

D (mile)	=	R (miles/hour)	×	T (hours)
$\frac{1}{30}$	=	r	×	$\frac{1}{12}$

$$\frac{1}{30} = \frac{1}{12}r$$

$$\frac{12}{30} = r$$

$$0.4 = r$$

9. **(D)** Use $D = RT$:

Distance = $x(2x)$
Distance = $2x^2$

Which is greater, $2x^2$ or $3x$? If $x = 1$, $3x$ is greater. But if $x = 2$, $2x^2$ is greater.

Without information about the value of x, the relationship cannot be determined.

10. **20 minutes.** First, figure out how long it took Claudette to travel 60 miles under the actual conditions. The first leg of the trip was 2/3 of 60 miles, or 40 miles. To travel 40 miles at a rate of 20 miles per hour, Claudette spent 40/20 = 2 hours = 120 minutes. The second leg of the trip was the remaining 60 - 40 = 20 miles. To travel that distance at a rate of 30 miles per hour, Claudette spent 20/30 = 2/3 hour = 40 minutes. In total, Claudette traveled for 120 + 40 = 160 minutes.

Now consider the hypothetical trip. If Claudette had traveled the whole distance of 60 miles at 20 miles per hour, the trip would have taken 60/20 = 3 hours = 180 minutes.

Finally, compare the two trips. The real trip took 160 minutes, so the hypothetical trip would have taken 180 - 160 = 20 minutes longer.

11. **(C).** Do not simply average the three speeds. You will always get the wrong answer that way. To compute the average speed for a trip, figure out the total distance and divide by the total time.

Pick a convenient distance from home to school, one that is divisible by 30, 40, and 60—say 120 miles (tough for Rajesh, but easier for you).

The first part of the journey (from home to school) takes 120/30 = 4 hours. The second part of the journey takes 120/40 = 3 hours. The third part of the journey takes 120/60 = 2 hours.

The total distance Rajesh travels is 120 + 120 + 120 = 360 miles. The total time is 4 + 3 + 2 = 9 hours. Finally, his average speed for the entire trip was 360/9 = 40 miles per hour.

12. **(C).** To find the average speed, divide the total distance by the total time. You have to figure out the time for each part of the journey separately. First, figure out the miles traveled for each part of the journey.

First part: 75% of 80 miles = $\left(\frac{3}{4}\right)(80) = 60 \text{ miles}$

Second part: 80 - 60 = 20 miles.

Now use $D = RT$ for each part of the journey. The two rates are 45 miles per hour and 30 miles per hour.

First part: $60 = 45t$, which gives $t = 4/3$ for this part
Second part: $20 = 30t$, which gives $t = 2/3$ hour for the second part

So the total time is 4/3 + 2/3 = 6/3 = 2 hours. The total distance is 80 miles, so the average speed was 80/2 = 40 miles per hour.

13. **(B).** First, determine Lamont's rate for the first part of the journey. Since he traveled 80 miles in 2.5 hours, his rate was $\frac{80}{2.5}$ miles per hour. This fraction can be reduced: $\frac{80}{2.5}=\frac{800}{25}=\frac{8\times100}{25}=8\times4=32$ miles per hour. Now, for the second part of the journey, Lamont decreased his speed by 25%. In other words, his new speed was 100% - 25% = 75% of the original speed.

New speed = $75\% \times 32=\left(\frac{3}{4}\right)(32)=24$ miles per hour.

He traveled 120 miles at 24 miles per hour, so the time for the second part of the journey was 120/24 = 5 hours.

Finally, the entire journey took 2.5 + 5 = 7.5 hours.

14. **(A).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Use the first sentence to solve for an individual worker's rate. Plug in the fact that 12 workers pack boxes at a constant rate of 60 boxes in 9 minutes:

Work = *Individual Rate* × *Number of Workers* × *Time*

60 = (R)(12)(9 minutes)

R = 5/9 boxes per minute

In other words, each worker can pack 5/9 of a box per minute. Plug that rate back into the formula, but use the details from the second sentence in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*
180 = (5/9)(27)(T)
180 = 15T
12 = T

15. **(E).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual worker's rate, using the fact that 4 editors can proofread 4 documents in 4 hours:

Work = *Individual Rate* × *Number of Workers* × *Time*
4 documents = (R)(4)(4 hours)
R = 1/4 document per hour

So each editor can proofread 1/4 of a document per hour.

Note that it is NOT correct to infer that if 4 editors can proofread 4 documents in 4 hours, then 1 editor can proofread 1 document in 1 hour. (After all, if 4 editors can proofread 4 documents in 4 hours, then each editor proofreads one of the documents over the whole 4 hours, not in 1 hour.)

Plug the 1/4 rate back into the formula, but using the details from the second sentence in the problem (using E for the unknown number of editors):

$$80 = (1/4)(E)(2)$$
$$E = 160$$

160 editors are required. Alternatively, you could reason that, since it takes an editor 4 hours to proofread one document, you could get 80 documents proofread by 80 editors in that same period of time (4 hours). To get the job done in half the time, you need twice as many editors, or 160 of them.

16. **(B).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual nanorobot's rate, using the fact that 700 nanorobots can service 1 device in 12 seconds. Notice that the "work" here is 1 device:

Work = *Individual Rate* × *Number of Workers* × *Time*
1 device = $(R)(700)$(12 seconds)
$R = 1/8{,}400$ devices per second

That is, each nanorobot can service $\frac{1}{8,400}$ of a device in 1 second. Plug that rate back into the formula, but using the details from the second sentence in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*
$$12 = (1/8{,}400)(1)(T)$$
$$T = 100{,}800$$

The answer is 100,800 seconds. Divide by 60 to convert this time to 1,680 minutes; divide by 60 again to get 28 hours.

17. **(D).** Since Sarita answered x verbal test questions in 3 hours, she answered $x/3$ verbal test questions in 1 hour. So Quantity A is $x/3$.

Thirty minutes is 1/2 an hour. If Sarita can do y math problems in ½ an hour, then she can do $2y$ math problems in an hour. So Quantity B is $2y$.

Without more information about x and y, it cannot be determined whether $x/3$ or $2y$ is a greater number. The correct answer is (D).

18. **66 days.** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

> *Work* = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual worker's rate, plugging in the fact that 45 people built ½ of a pyramid in 288 days:

> *Work* = *Individual Rate* × *Number of Workers* × *Time*
> 1/2 pyramid = (R)(45)(288 days)
> R = 1/25,920 pyramid per day (don't be shy about using the calculator here)

That is, each person can build $\frac{1}{25{,}920}$ of a pyramid in 1 day. Now put this rate into the work equation, together with the remaining details in the problem:

> *Work* = *Individual Rate* × *Number of Workers* × *Time*
> 1/6 = (1/25,920)(65)(T)
> $T \approx 66.46$ days

Rounded to the nearest integer, the answer is 66 days.

19. **(B).** Since the question asks for time, you need: the rate and the amount of work. To find the rate, divide the total work the purifier can do by the time required. 100 cubic feet ÷ 4 minutes = 25 cubic feet per minute.

Now find the amount of work required. The tank has a total volume of (15)(15)(10) = 2,250 cubic feet, but is only 1/2 full of water. Thus, the volume of water to be purified is 1,125 cubic feet.

Plug these numbers back into the $W = RT$ formula and solve.

> $W = RT$
> $(1{,}125) = (25)T$
> $T = 1{,}125 \div 25 = 45$ minutes

20. **(D).** First identify what you are looking for. To find the amount by which the baker's rate of pie-making increased, so you need both his rate for the first 5 hours and his rate in the last 3 hours. The difference is the ultimate answer:

> Rate for last 3 hours - Rate for first 5 hours = Increase

The rate for the first 5 hours was 60 pies ÷ 5 hours = 12 pies per hour.

In the last 3 hours, the baker made 150 - 60 = 90 pies. The rate in the last 3 hours of the workday was thus 90 pies ÷ 3 hours = 30 pies per hour.

Now find the difference between the two rates of work:

> 30 pies per hour - 12 pies per hour = 18 pies per hour

21. **(A).** To find average earnings per hour, divide the total earnings by the total time (in hours). So compute the total earnings and the total time.

5 week days × \$600 = \$3,000
1 Saturday × \$300 = \$300
Total earnings = \$3,300

3 days × 10 hours = 30
2 days × 11 hours = 22
1 day × 8 hours = 8
Total hours = 60

The broker's average earnings per hour = \$3,300 ÷ 60 = \$55 per hour. Since 55 is greater than 50, Quantity A is greater.

22. **(D).** This is a classic combined rates problem. Since the carts are moving directly toward each other, add their rates together. Remember that when two objects are moving in opposite directions — either toward each other or away from each other — add their rates to find how fast the gap is closing (or opening up).

To avoid any unit conversion trap (answer choice (E)), do the yards-to-feet conversion up front: 400 yards × 3 ft/yard = 1,200 feet. The combined rate of the two carts is 96 ft/sec. Therefore the time it takes for them to meet is 1,200 feet ÷ 96 feet/second = $12\frac{1}{2}$ seconds.

	Distance	=	Rate	×	Time
Cart A	$40t$	=	40 ft/sec	×	t
Cart B	$56t$	=	56 ft/sec	×	t
Combined Rate	1200 feet	=	96 ft/sec	×	$12\frac{1}{2}$ sec

23. **(E).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual machine's rate, using the fact that 9 machines can stitch 27 jerseys in 4 minutes.

Work = *Individual Rate* × *Number of Workers* × *Time*
27 jerseys = (R)(9)(4 minutes)
R = 3/4 jersey per minute

That is, each machine can stitch 3/4 of a jersey in 1 minute. Plug that rate back into the formula, but using the details from the second sentence in the problem:

Work = *Individual Rate* × *Number of Workers* × *Time*
60 = (3/4)(4)(T)

$T = 20$

24. **(C).** In this problem, you must compare an actual scenario with a hypothetical one. Start by figuring out the rate (speed) for Brenda's actual walk. Since she walked 12 miles in 3 hours, she walked at a rate of 12/3 = 4 miles per hour.

Now, in the hypothetical situation, she would walk the loop twice, for a total distance of 12 × 2 = 24 miles. Her hypothetical speed would be 1/2 of 4 miles per hour, or 2 miles per hour.

Walking 24 miles at a rate of 2 miles per hour would take Brenda 24/2 = 12 hours.

Alternatively, you might note that both of the changes—doubling the distance and halving the rate—have the same effect: Each change makes the trip take twice as long as it would have before. So the time required for this hypothetical situation is multiplied by four: 3 × 2 × 2 = 12 hours.

25. **(C).** In this "chase" problem, the two vehicles are moving in the same direction, with one chasing the other. To determine how long it will take the rear vehicle to catch up, *subtract* the rates to find out how quickly the rear vehicle is gaining on the one in front.

The police car gains on the train at a rate of 80 - 50 = 30 miles per hour. Since the police car needs to close a gap of 50 miles, plug into $D = RT$ to find the time:

$50 = 30t$
$5/3 = t$

The time it takes to catch up is 5/3 hours, or 1 hour and 40 minutes.

26. **(C).** "Cheat" off the easy quantity. In 40 minutes (from Quantity B), Rachel would assemble 40/10 = 4 brochures and Terry would assemble 40/8 = 5 brochures, for a total of 4 + 5 = 9 brochures. Thus, Quantity A is also 40, and the two quantities are equal.

27. **(A).** If the search provider adds 2 identical servers to the original 4, there are now 6 servers. Because 6/4 = 1.5, the rate at which all 6 servers work is 1.5 times the rate at which 4 servers work:

9,600 searches per hour × 1.5 = 14,400 searches per hour

Now apply this rate to the given amount of work (216,000 searches), using the $W = RT$ formula.

$216{,}000 = (14{,}400)T$
$216{,}000 \div 14{,}400 = 15$ hours

28. **(B).** Start by filling in a $W = RT$ chart with the pieces you know. Remember, two identical pipes would siphon ink at double the rate of a single pipe.

	Work (liters)	=	Rate (liters/min)	×	Time (min)
1 pipe	800	=	r	×	**$800/r$**

2 pipes	800	=	$2r$	×	**800/(2*r*)**

One pipe does the work in $800/r$ minutes, and two pipes do the work in $800/2r$ minutes.

The questions states that "If two such pipes were used, the drum could be emptied 100 minutes faster than when one pipe is used. Express this as a skeleton equation:

Time for 2 pipes + 100 = Time for 1 pipe

(You can check that you've written this correctly by noting that the time for 2 pipes should be shorter, so it is necessary to add 100 to that side to make it equal the longer time for 1 pipe.)

Now plug in expressions for the times and solve for r:

$$\frac{800}{2r} + 100 = \frac{800}{r}$$
$$800 + 200r = 1600$$
$$200r = 800$$
$$r = 4$$

1 pipe siphons 4 liters of ink per minute. Thus, Quantity A equals 4, which is less than Quantity B.

29. **(D).** Since Sabrina and Janis are working together, add their rates. Sabrina completes 1 tank in 8 hours, so she works at a rate of $\frac{1}{8}$ tank per hour. Likewise, Janis works at a rate of $\frac{1}{13}$ tank per hour. Now, add these fractions:

$$\frac{1}{8} + \frac{1}{13} = \frac{13}{104} + \frac{8}{104} = \frac{21}{104}$$ tanks per hour, when working together

Now plug this combined rate into the $W = RT$ formula to find the time. You might also notice that since the work is equal to 1, the time will just be the reciprocal of the rate.

Sabrina & Janis:	1 tank	=	21/104 tank/hr	×	**104/21 minutes**

At this point, you do not need to do long division or break out the calculator! Just approximate: 104/21 is about 100/20 = 5. The answer is (D).

You might also use some intuition to work the answer choices and avoid setting up this problem at all! You can immediately eliminate (A) and (B), since these times exceed either worker's individual time. Also, since Sabrina is the faster worker, Janis's contribution will be less than Sabrina's. The two together won't work twice as fast as Sabrina, but they will work *more* than twice as fast as Janis. Therefore, the total time should be more than half of Sabrina's individual time, and less than half of Janis's individual time. $4 < t < 6.5$, which leaves (D) as the only possible answer.

30. **5 hours.** Etienne and Jacques were working at cross-purposes (although perhaps Etienne didn't mind), so *subtract* their rates. Usually you add work rates, but this situation is just like a car chase: when one car (or person) gains on another, you subtract rates.

20 cookies/hour - 16 cookies/hour = 4 cookies/hour, so the quantity of cookies decreased by 4 per hour. Since Etienne began with a pile of 20 cookies, it took him 20/4 = 5 hours to eat all the cookies.

Note that Etienne ate a lot more than 20 cookies. In 5 hours he ate 100 cookies—the initial 20, plus the 80 that Jacques made in the 5 hours.

31. **(E).** To solve for average earnings, fill in this formula:

Total earnings/Total hours = Average earnings per hour

Since the gold-dollar exchange rate is \$1 per 1,000 gold pieces: Phil's real dollar earnings for the 6 days were 540,000/1,000 = \$540. His total time worked was 10 hours/day × 6 days = 60 hours. Therefore, his average hourly earnings were \$540/60 = \$9/hour.

32. **(D).** To find the new rate in words per hour, start by setting up an equation to find this value:

New words/hr = 10 + 2(Old words/hr)

The old rate was given in words per minute, so convert to words per hours:

10 words/min × 60 min/hr = 600 words/hr.

Now plug into the equation:

New words/hr = 10 + 2(600) = 1,210

Note that you would not want to start by working with the rate per minute. If you did so, you'd get 10 + 2(10) = 30 words/minute, then 30 × 60 = 1,800 words/hr. You would get this inflated number because you added an additional 10 words per minute instead of per hour. This is another reason to perform your conversions right away!

33. **(D).** Since the two women are working together, add their rates. To find their individual rates, divide work by time. Never divide time by work! (Also, be careful when dividing the work by 1/2. The rate is the reciprocal of 1/2, or 2 tables/hour.)

Once you find Jenny and Laila's combined rate, divide the work required (1 table) by this rate. 1 table ÷ 7/3 table/ hour = 3/7 hour. s

	Work (tables)	=	Rate (table/hour)	×	Rate (table/hour)
Jenny	1	=	**1/3**	×	3
Laila	1	=	**2**	×	1/2
Jenny & Laila	1	=	**1/3 + 2 = 7/3**	×	3/7

34. **(A).** First, figure out the individual rate for 1 worker: 2 violins/3 minutes = 2/3 violin per minute. (Always divide work by time to get a rate.) Now apply $W = RT$ separately to Quantity A and Quantity B.

Quantity A:

$R = 12 \times$ the individual rate $= 12 \times 2/3 = 8$ violins per minute.
$W = 720$ violins

Solve for T in $W = RT$:

$720 = 8T$
$90 = T$

Quantity B:

$R = 5 \times$ the individual rate $= 5 \times 2/3 = 10/3$ violins per minute.
$T = 24$ minutes

Solve for W in $W = RT$:

$W = (10/3)(24)$
$W = 80$

Since $90 > 80$, Quantity A is greater.

35. **(D).** To find Kurt's wait time, determine how long it will take for 200 people to board the Jelly Coaster. The problem states that 4 people board every 15 seconds. Since there are four 15-second periods in one minute, this rate converts to 16 people/minute. To find the time, divide the "work" (the people) by this rate.

200 people ÷ 16 people/minute = 200/16 = 12.5 minutes. The question asks for an approximation, and this is now close enough to answer (D). In theory there may be an additional 15 seconds while Kurt's group is boarding (the problem doesn't really say), but Kurt's total wait time would still be approximately 13 minutes.

36. **(A).** Machine B is 50% faster than A, so relate their rates with the equation $b = 1.5a$. Put the data into a chart to compare work each machine does in 24 hours.

	Work (CDs)	=	Rate (CDs/hour)	×	Time (hour)
Machine A	$24a$	=	a	×	24
Machine B	$24b$	=	b	×	24

Machine B's work in a 24-hour period exceeds Machine A's work by 48,000 CDs. That is to say:

$36a - 24a = 48{,}000$
$12a = 48{,}000$
$a = 4{,}000$

Machine A shrink-wraps 4,000 CDs per hour.

Another way to solve this problem is to notice that since B is 50% faster than A, the quantity by which its work exceeds A's in an hour will equal 50%, or half, of A's hourly rate. Since B shrink-wraps 48,000/24 = 2,000 more CDs per hour than A, Machine A wraps 2,000 × 2 = 4,000 CDs per hour.

37. **(C).** To solve a Rates & Work problem with multiple workers, modify the standard formula *Work* = *Rate* × *Time* to this:

Work = *Individual Rate* × *Number of Workers* × *Time*

Solve for an individual chef's rate, using the fact that 8 chefs produce 3,200 tarts in 5 days.

Work = *Individual Rate* × *Number of Workers* × *Time*
3,200 tarts = (R)(8)(5 days)
R = 80 tarts per day

That is, each chef can produce 80 tarts per day. Plug that rate back into the formula for each of the quantities.

<u>Quantity A</u>
Work = *Individual Rate* × *Number of Workers* × *Time*
3,600 = (80)(*Number of Workers*)(3)
Number of Workers = 15

<u>Quantity B</u>
Work = *Individual Rate* × *Number of Workers* × *Time*
4,800 = (80)(4)(Time)
Time = 15 days

The number of chefs in Quantity A equals the number of days in Quantity B.

38. **(E).** When rate problems involve multiple situations, it can help to set up an initial "skeleton" $W = RT$ chart for the solution. That way, you can easily determine what data is needed, and fill in that data as you find it. Since you want to know how long Robot A will take alone, the chart will look like this:

	Work (pounds)	=	Rate (pounds/min)	×	Rate (min)
Robot A	165	=	A's rate	×	t

You know the work and you want to know the time, so you just need A's rate. Call the rates a and b. Now set up another chart representing what you know about the two robots working together.

	Work (pounds)	=	Rate (pounds/min)	×	Rate (min)
Robot A	$6a$	=	a	×	6
Robot B	$6b$	=	b	×	6

A & B together	$6(a + b) = 88$	=	$a + b$	×	6

Now that you know that $6(a + b) = 88$, just apply the other piece of information you know: robot A's rate is 3/5 of B's rate. This can be written as $a = (3/5)b$. Since you are looking for a, substitute for b:

$a = (3/5)b$
$(5/3)a = b$

$6(a+(5/3)a) = 88$
$6(8/3)a = 88$
$(48/3)(a) = 88$
$a = 88(3/48)$
$a = 88(1/16) = 88/16 = 11/2$

So A's rate is 11/2 pounds per minute. Now just plug into the original chart.

	Work (pounds)	=	Rate (pounds/min)	×	Rate (min)
Robot A	165	=	11/2	×	**30**

The time Robot A takes to polish 165 pounds of gems is 165/(11/2) = 330/11 = 30 minutes.

39. **(C).** No distances are given in this problem, so you need to determine how far the two cars end up traveling before finding the distance between them. Since the cars go in the same direction, the skeleton equation is as follows:

Car A's distance - Car B's distance = distance between cars
(all distances refer to the time when car A ran out of fuel).

Since the limiting factor in this case is A's fuel supply, you must calculate how far the car is able to drive before running out of fuel. This in itself is a rate problem of sorts:

30 miles per gallon × 8 gallons = 240 miles

So Car A will end up 240 miles north of its starting point, which happens 240/40 = 6 hours after it started. What about Car B? It started an hour later and thus traveled (30 miles per hour)(6 hours - 1 hour) = 180 miles by that time.

Therefore the two cars were 240 - 150 = 90 miles apart when car A ran out of fuel.

40. **(B).** First, build a chart to see how many doubling periods occured to grow the population from 50 to 3,200. Most problems of this sort rely on converting any growth rate to a doubling period.

50
100
200

400
800
1,600
3,200

The population doubled 6 times in 2 days. 48 hours/6 doubling periods = 8 hours per doubling period. Therefore the population doubles every 8 hours.

Quantity A:
After 16 more hours, the population has gone through 2 doubling periods, so it has quadrupled (that is, it has increased by a factor of 4) from the final level of 3,200. Since Quantity A is actually twice that population, the quantity is 8 times 3,200.

Quantity B:
After 32 more hours, the population has gone through 4 doubling periods, so it has gone up by a factor of $2 \times 2 \times 2 \times 2 = 2^4 = 16$. Quantity B is 16 times 3,200.

Quantity B is greater. Notice that you don't need to actually figure out 8 × 3,200 or 16 × 3,200.

41. **(C).** (D) is a trap. This issue is relatively rare, but it's worthwhile to be able to recognize it if you see it. Note that in this case, each robot is *independently* assembling complete doghouses. Since the question asks for the number of *completed* doghouses after $2\frac{1}{2}$ hours, you need to remove any *incomplete* doghouses from the calculations.

Since one robot completes a doghouse in 12 minutes, the individual hourly rate is 60/12 = 5 doghouses per hour. Therefore, each robot produces 5 × 2.5 = 12.5 doghouses in $2\frac{1}{2}$ hours. (You could also simply divide the 150 total minutes by 12 minutes per doghouse to get the same result.)

However, since you are interested only in *completed* doghouses, and the robots are working independently, drop the decimal. Each robot completes only 12 doghouses in the time period, for a total of 12 × 10 = 120 doghouses.

42. **(B).** The question gives you the initial value and the rate of doubling, so you can set up the solution in a straightforward manner. The hardest part is cutting through all the language to find what you need. The initial value is stated outright: 5 million transistors/square mm. To find the number of doubling periods, first convert from years to months:

$$\frac{30 \text{ years} \times 12 \text{ months per year}}{18 \text{ months per doubling period}} = \frac{360}{18} = 20 \text{ doubling periods}$$

So there is an initial density of 5 million, doubled 20 times. Thankfully, you don't have to calculate that number! Since the answer choices are in exponential form, set things up that way. Also, note that the question asks for the density in millions of transistors/square mm, so you need to use 5, rather than 5,000,000, in the calculation. 5 doubled 20 times = 5×2^{20}

43. **(E) 10/3.** If twice as many bottles of Soda V as of Soda Q are filled at the factory each day, then twice as many bottles of Soda V as of Soda Q are filled at the factory each second.

Use smart numbers for the number of bottles filled each second. Since twice as many bottles of Soda V are produced, so the output in one second could be 100 bottles of V and 50 bottles of Soda Q. Using these numbers, the volume of the Q bottles is 500 liters / 50 bottles = 10 liters/bottle and the volume of the V bottles is 300 liters/100 bottles = 3 liters/bottle. The ratio of the volume of a bottle of Q to a bottle of V is 10 liters/3 liters = 10/3.

44. **(B).** To determine how long Ferris' breaks were, you need to know the difference between the amount of work the two *should* have completed in two hours, and the amount they actually *did* complete (that is, one full job).

Audrey and Ferris are working together, so first find each worker's individual rate, and then add them together to get the combined rate.

	Work (jobs)	=	Rate (jobs/hour)	×	Time (hours)
Audrey	1	=	**1/4**	×	4
Ferris	1	=	**1/3**	×	3
Audrey & Ferris	**7/6**	=	**1/4 + 1/3 = 3/12 + 4/12 = 7/12**	×	2

Combining the two workers' rates, together they complete 7/12 job per hour, so they should have completed 14/12 = 7/6 job in two hours. Therefore, Ferris' breaks cost them 7/6 - 1 = 1/6 job worth of productivity.

How long was Ferris on break? The amount of time it would have taken him to do 1/6 of the job.

Ferris' breaks	1/6 job	=	1/3 job/hour	×	**1/2 hour**

At the rate of 1/3 job/hour, Ferris must have spent 1/2 hour on break to miss 1/6 job. Therefore, each of his 3 breaks was 30 minutes ÷ 3 = 10 minutes long. The answer is (B).

Alternatively, use smart numbers to eliminate the fractions. Define the job as, say, making 12 toys. Now you can say that Audrey makes 3 toys/hour and Ferris makes 4 toys/hour. The work is all the same as above, but when you get to the point of figuring out Ferris' missed time, the work becomes much easier. Together, the two should produce 14 toys, but they produced only 12. Thus, Ferris' slacking costs them 2 toys. Since his rate is 4 toys/hour, he must have missed 1/2 hour of work (by taking 3 breaks of 10 minutes each).

45. **(E).** To find the turtle's average speed for the trip, divide total distance by total time.

Since the distance has not been defined, call each equal leg of the trip D. Therefore, the turtle's total distance is $3D$. (Note that the D must cancel out before you are done.) Find the turtle's total time by calculating the time for each leg of the journey. In each case, the time is equal to D/rate.

	Distance	=	Rate	×	Time
Up	D miles	=	4 mi/hr	×	**D/4 hr**
Across	D miles	=	x mi/hr	×	**D/x hr**
Down	D miles	=	x^2 mi/hr	×	**D/x^2 hr**

Now find the total time by adding up the separate legs, using a common denominator:

$$\frac{D}{4}+\frac{D}{x}+\frac{D}{x^2}=$$

$$\frac{Dx^2}{4x^2}+\frac{4Dx}{4x^2}+\frac{4D}{4x^2}=$$

$$\frac{Dx^2+4Dx+4D}{4x^2}=$$

$$\frac{D\left(x^2+4x+4\right)}{4x^2}$$

All that's left to do is plug this total time, along with the total distance of 3D, into the formula for average rate (= Total distance/Total time):

$$\frac{3D}{\frac{D\left(x^2+4x+4\right)}{4x^2}}=$$

$$\frac{3D\times 4x^2}{D\left(x^2+4x+4\right)}=$$

$$\frac{12x^2}{x^2+4x+4}$$

Notice that the D's cancel out, as predicted. To match this to answer choice (E), you need to recognize the denominator as a special product: $(x + 2)^2$.

Alternatively, you can solve this problem with smart numbers. Make $x = 5$. Since $x^2 = 25$, you want a distance for each leg that is divisible by 4, 5, and 25. A distance of 100 will work nicely. Now find the time for each leg of the trip.

	Distance (miles)	=	Rate (miles/hour)	×	Time (hours)
Up	100	=	4	×	**25**
Across	100	=	$x = 5$	×	**20**
Down	100	=	$x^2 = 25$	×	**4**
Entire Trip	300	=			**49 hr**

The turtle's average rate is 300/49 mi/hr. There is no need to simplify, as you just need to plug in for x in each answer choice and see whether the result matches. When you plug $x = 5$ into the answer choices, only (E) produces 300/49.

Chapter 19

of

Variables-in-the-Choices Problems

In This Chapter…

Variables-in-the-Choices Problems

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. If Josephine reads b books per week and each book has, on average, 100,000 words, which best approximates the number of words Josephine reads per day?

(A) $100{,}000b$

(B) $\dfrac{100{,}000b}{7}$

(C) $\dfrac{700{,}000}{b}$

(D) $\dfrac{7b}{100{,}000}$

(E) $\dfrac{100{,}000b}{(7)(24)}$

2. A rectangle's width w is twice its length. Which of the following expresses the rectangle's area in terms of w?

(A) w

(B) $2w^2$
(C) $3w^2$
(D) $\frac{w^2}{2}$
(E) $\frac{w^2}{4}$

3. A clothing store bought a container of 100 shirts for \$$x$. If the store sold all of the shirts at the same price for a total of \$50, what is the store's profit per shirt, in dollars, in terms of x?

(A) $50 - \frac{x}{100}$
(B) $50 - x$
(C) $5 - x$
(D) $0.5 - x$
(E) $0.5 - \frac{x}{100}$

4. There are two trees in the front yard of a school. The trees have a combined height of 60 feet, and the taller tree is x times the height of the shorter tree. How tall is the shorter tree, in terms of x?

(A) $\frac{60}{1+x}$
(B) $\frac{60}{x}$
(C) $\frac{30}{x}$
(D) $60 - 2x$
(E) $30 - 5x$

5. Louise is three times as old as Mary. Mary is twice as old as Natalie. If Louise is L years old, what is the average age of the three women, in terms of L?

(A) $L/3$
(B) $L/2$
(C) $2L/3$
(D) $L/4$
(E) $L/6$

6. Toshi is four times as old as Kosuke. In x years Toshi will be three times as old as Kosuke. How old is Kosuke, in terms of x?

(A) $2x$
(B) $3x$
(C) $4x$
(D) $8x$
(E) $12x$

7. A shirt that costs k dollars is increased by 30%, then by an additional 50%. What is the new price of the shirt in

dollars, in terms of k?

(A) $0.2k$
(B) $0.35k$
(C) $1.15k$
(D) $1.8k$
(E) $1.95k$

8. Carlos runs a lap around the track in x seconds. His second lap is five seconds slower than the first lap, but the third lap is two seconds faster than the first lap. What is Carlos's average race time, in *minutes*, in terms of x?

(A) $x - 1$
(B) $x + 1$
(C) $\frac{x-1}{60}$
(D) $\frac{x+1}{60}$
(E) $\frac{x+3}{60}$

9. Andrew sells vintage clothing at a flea market at which he pays $150 per day to rent a table plus $10 per hour to his assistant. He sells an average of $78 worth of clothes per hour. Assuming no other costs, which of the functions below best represents profit per day P in terms of hours h that the flea market table is open for business?

(A) $P(h) = 238 - 10h$
(B) $P(h) = 72 - 10h$
(C) $P(h) = 68h - 150$
(D) $P(h) = 78h - 160$
(E) $P(h) = -160h + 78$

10. If $a, b, c,$ and d are consecutive integers and $a < b < c < d$, what is the average of a, b, c, and d in terms of d?

(A) $d - \frac{5}{2}$
(B) $d - 2$
(C) $d - \frac{3}{2}$
(D) $d + \frac{3}{2}$
(E) $\frac{4d-6}{7}$

11. A cheese that costs c cents per ounce costs how many dollars per pound? (16 ounces = 1 pound and 100 cents = 1 dollar)

(A) $4c/25$
(B) $25c/4$
(C) $25/4c$
(D) $c/1,600$

(E) 1,600c

12. A bag of snack mix contains 3 ounces of pretzels, 1 ounce of chocolate chips, 2 ounces of mixed nuts, and x ounces of dried fruit by weight. What percent of the mix is dried fruit, by weight?

(A) $\frac{x}{600}$

(B) $\frac{100}{6x}$

(C) $\frac{100x}{6}$

(D) $\frac{100x}{6+x}$

(E) $\frac{x}{100(6+x)}$

13. At her current job, Mary gets a 1.5% raise twice per year. Which of the following choices represents Mary's current income y years after starting the job at a starting salary of s?

(A) $s(1.5)^{2y}$
(B) $s(0.015)^{2y}$
(C) $s(1.015)^{2y}$
(D) $s(1.5)^{y/2}$
(E) $s(1.015)^{y/2}$

14. Phone Plan A charges \$1.25 for the first minute and \$0.15 for every minute thereafter. Phone Plan B charges a \$0.90 connection fee and \$0.20 per minute. Which of the following equations could be used to find the length, in minutes, of a phone call that costs the same under either plan?

(A) $1.25 + 0.15x = 0.90x + 0.20$
(B) $1.25 + 0.15x = 0.90 + 0.20x$
(C) $1.25 + 0.15(x - 1) = 0.90 + 0.20x$
(D) $1.25 + 0.15(x - 1) = 0.90 + 0.20(x - 1)$
(E) $1.25 + 0.15x + 0.90x + 0.20 = x$

15. If powdered drink mix costs c cents per ounce and p pounds of it are purchased by a supplier who intends to resell it, what will be the total revenue, in dollars, in terms of c and p if all of the drink mix is sold at a price per ounce equivalent to three times what the supplier paid? (16 ounces = 1 pound and 100 cents = 1 dollar)

(A) 48cp

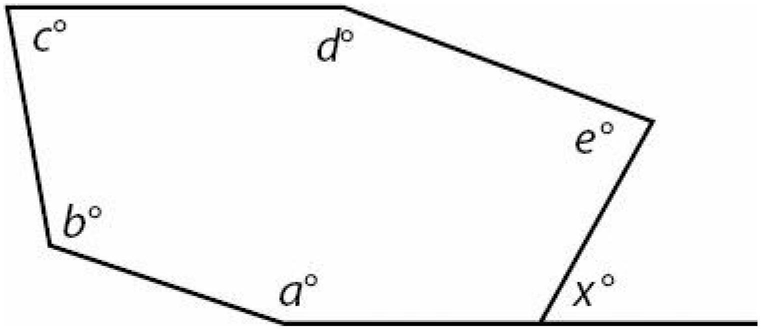
(B) $\frac{32cp}{100}$

(C) $\frac{100(32)}{cp}$

(D) $\frac{12cp}{25}$

(E) $\dfrac{25cp}{12}$

16.



16. If $d = 2c$ and $e = \dfrac{1}{2}a$, what is x in terms of a, b, and c?

(A) $\dfrac{3}{2}a + b + 3c - 540$
(B) $\dfrac{3}{2}a + b + 3c$
(C) $720 - \dfrac{3}{2}a - b - 3c$
(D) $720 - \dfrac{1}{2}a - b - 2c$
(E) $540 - \dfrac{1}{2}a - b - \dfrac{3}{2}c$

17. a, b, and c are 3 consecutive odd integers such that $a < b < c$. If a is halved to become m, b is doubled to become n, c is tripled to become p, and $k = mnp$, which of the following is equal to k in terms of a?

(A) $3a^3 + 18a^2 + 24a$
(B) $3a^3 + 9a^2 + 6a$
(C) $\dfrac{11}{2}a + 16$
(D) $6a^2 + 36a + 24$
(E) $a^3 + 6a^2 + 4a$

18. If m pencils cost the same as n pens, and each pencil costs 20 cents, what is the cost, in dollars, of 10 pens, if each pen costs the same amount (100 cents = 1 dollar)?

(A) $\dfrac{200n}{m}$
(B) $\dfrac{2n}{100m}$

(C) $\frac{2m}{n}$

(D) $\frac{2n}{m}$

(E) $200mn$

19. Randi sells forklifts at a dealership where she makes a base salary of $2,000 per month, plus a commission equal to 5% of the selling price of the first 10 forklifts she sells that month, and 10% of the value of the selling price of any forklifts after that. If all forklifts have the same sale price, *s,* which of the choices below represents Randi's monthly pay, *P*, as a function of number of forklifts sold, *f*, in months in which she sells more than 10 forklifts? (Assume Randi's pay is made up entirely of base salary and commission, and no deductions are taken from this pay.)

(A) $P = 2{,}000 + 0.05sf + 0.10sf$
(B) $P = 2{,}000 + 0.05sf + 0.10s(f - 10)$
(C) $P = 2{,}000 + 0.05s + 0.10s(f - 10)$
(D) $P = 2{,}000 + 0.5s + 0.10sf - 10$
(E) $P = 2{,}000 + 0.5s + 0.10s(f - 10)$

20. If the width of a rectangle is w, the length is l, the perimeter is p, and $w = 2l$, what is the area in terms of p?

(A) $\frac{p^2}{18}$

(B) $\frac{p^2}{36}$

(C) $\frac{p}{9}$

(D) $\frac{p^2}{9}$

(E) $\frac{p}{6}$

Variables-in-the-Choices Problems Answers

1. **(B).** Since Josephine reads *b* books per week and each book has an average of 100,000 words, she reads 100,000*b* words per week. However, the question asks for words per *day,* so divide this quantity by 7.

Alternatively, you could try picking numbers. If $b = 2$, for instance, then Josephine would read 2 books per week and thus 200,000 words per week. Divide by 7 to get 28,571.42… words per day. Plug 2 into each answer choice in place of *b,* and pick the answer that gives you 28,571.42… Only (B) works.

2. **(D).** Since width is twice length, write $W = 2L$. However, you want your answer in terms of *w,* so solve for *L*:

$L = 1/2\ W$ or $\frac{w}{2}$

Since area is $L \times W$ and $L = \frac{w}{2}$:

$$A = \frac{w}{2} \times W$$

$A = \frac{w^2}{2}$, or choice (D).

Alternatively, pick values. If width were 4, length would be 2. The area would therefore be $4 \times 2 = 8$. Plug in 4 for *w* to see which answer choice yields 8. Only (D) works.

3. **(E).** This problem requires you to know that profit equals revenue minus cost. You could memorize the formula Profit = Revenue - Cost (or Profit = Revenue - Expenses), or you could just think about it logically — of course a business has to pay its expenses out of the money it makes: the rest is profit.

The revenue for all 100 shirts was \$50, and the cost to purchase all 100 shirts was \$*x*. Therefore:

Total profit = $50 - x$

The question does not ask for the total profit, but for the profit per shirt. The store sold 100 shirts, so divide the total profit by 100 to get the profit per shirt:

Profit per shirt = $\frac{50 - x}{100}$

None of the answer choices match this number, so you need to simplify the fraction. Split the numerator into two separate fractions:

$$\frac{50-x}{100}=\frac{50}{100}-\frac{x}{100}=0.5-\frac{x}{100}$$

4. **(A).** Let s = the height of the shorter tree. Let t = the height of the taller tree.

If the combined height of the tress is 60 feet, then:

$s + t = 60$

You also know that the height of the taller tree is x times the height of the shorter tree:

$t = xs$

You need to solve for the height of the shorter tree, so substitute (xs) for t in the first equation:

$s + (xs) = 60$

You need to isolate s, so factor s out of the left side of the equation:

$$s(1+x)=60$$

$$s=\frac{60}{1+x}$$

5. **(B).** First, you need to express all three women's ages in terms of L. If Louise is three times as old as Mary, then Mary's age is $L/3$.

You also know that Mary is twice as old as Natalie. If Mary's age is $L/3$, then Natalie's age is 1/2 of that, or $L/6$.

Now you can plug those values into the average formula. The average of the three ages is:

$$\text{average}=\frac{L+\frac{L}{3}+\frac{L}{6}}{3}$$

To get rid of the fractions in the numerator, multiply the entire fractions by 6/6:

$$\frac{6}{6}\times\left(\frac{L+\frac{L}{3}+\frac{L}{6}}{3}\right)=\frac{6L+2L+L}{18}=\frac{9L}{18}=\frac{L}{2}$$

6. **(A).** Let T = Toshi's age $(T+x)$ = Toshi's age in x years

Let K = Kosuke's age $(K+x)$ = Kosuke's age in x years

If you know that Toshi is four times as old as Kosuke, then you know that:

$T = 4K$

To translate the second sentence correctly, remember that you need to use $(T + x)$ and $(K + x)$ to represent their ages:

$(T + x) = 3(K + x)$

You need to solve for Kosuke's age in terms of x, so replace T with $(4K)$ in the second equation:

$(4K) + x = 3K + 3x$
$K + x = 3x$
$K = 2x$

7. **(E).** If the cost of the shirt is increased 30%, then the new price of the shirt is 130% of the original price. If the original price was k, then the new price is $1.3k$.

Remember that it is this new price that is increased by 50%. You need to multiply $1.3k$ by 1.5 (150%) to get the final price of the shirt:

$1.3k \times 1.5 = 1.95k$

8. **(D).** Carlos's race times can be expressed as x, $x + 5$, and $x - 2$. (Remember, SLOWER race times are LARGER numbers, so "five seconds slower" means *plus* 5, not *minus* 5!) Average the race times:

$$\frac{x+(x+5)+(x-2)}{3} = \frac{3x+3}{3} = x+1$$

His average time is $x + 1$ *seconds*. But you need *minutes*. Since there are 60 seconds in a minute, divide by 60 to get $\frac{x+1}{60}$, or choice (D).

Alternatively, pick values. If x were 60 seconds, for example, Carlos's race times would be 60, 65, and 58. His average time would be 61 seconds, or 1 minute and 1 second, or $1\frac{1}{60}$ minutes, or $\frac{61}{60}$ minutes. Plug in $x = 60$ to see which value yields $\frac{61}{60}$. Only (D) works.

9. **(C).** For every hour Andrew's business is open, he sells \$78 worth of clothes but pays \$10 to his assistant. Thus, he is making \$68 an hour after paying the assistant. He also must pay \$150 for the whole day.

So, the formula for his daily profit, using Revenue - Expenses = Profit and h for hours he is open:

$68h - 150$

Written as a function of profit in terms of hours, this is $P(h) = 68h - 150$, or choice (C).

Be careful that you are reading the answer choices as *functions*. P is not a variable that is being multiplied by h! P is the *name* of the function, and h is the variable on which the output of the function depends.

Note that (D) is a very good trap — this formula represents what the profit would be if Andrew only had to pay the assistant \$10 *total*. However, he pays the assistant \$10 *per hour*.

Alternatively, you could pick numbers. If Andrew were open for an 8-hour day (here, you are testing out $h = 8$), he would make \$68 an hour (\$78 of sales minus \$10 to the assistant), or \$544 total. Subtract the \$150 rental fee to get \$394.

Then, plug 8 into the answer choices in place of h to see which answer yields 394. Only (C) works.

10. **(C).** Since *a, b, c,* and *d* are consecutive and *d* is largest, you can express c as $d - 1$, b as $d - 2$, and a as $d - 3$. Therefore, the average is:

$$\frac{(d-3)+(d-2)+(d-1)+d}{4} = \frac{4d-6}{4} = d - \frac{6}{4} \text{ or } d - \frac{3}{2},$$ which matches choice (C).

Alternatively, plug in numbers. Say *a, b, c,* and *d* are simply 1, 2, 3, and 4 (generally, you want to avoid picking the numbers 0 and 1, lest *several* of the choices appear to be correct and you have to start over, but since only *d* appears in the choices, it's no problem that *a* is 1 in this example).

Thus, the average would be 2.5. Plug in 4 for *d* to see which choice yields an answer of 2.5. Only (C) works.

11. **(A).** If a cheese costs *c* cents per ounce, it costs $16c$ cents per pound. To convert from cents to dollars, divide by 100:

$$\frac{16c}{100} = \frac{4c}{25},$$ or choice (A).

Alternatively, pick numbers. If $c = 50$, a cheese that costs 50 cents per ounce would cost 800 cents, or \$8, per pound. Plug in $c = 50$ and select the answer that gives the answer 8. Only (A) works.

12. **(D).** To figure out what *fraction* of the mix is fruit, put the amount of fruit over the total amount of the mix: $\frac{x}{6+x}$. To convert a fraction to a percent, simply multiply by 100: $\frac{x}{6+x}(100) = \frac{100x}{6+x}$, or answer choice (D).

Alternatively, pick smart numbers. For instance, say $x = 4$. In that case, the total amount of mix would be 10 ounces, 4 of which would be dried fruit. Since 4/10 = 40%, the answer to the question for your example would be 40%. Now, plug $x = 4$ into each answer choice to see which yields 40%. Only choice (D) works: $\frac{100(4)}{6+(4)} = \frac{400}{10} = 40$. This will work for any number you choose for x, provided that you correctly calculate what percent of the mix would be dried fruit in your particular example.

13. **(C).** To increase a number by 1.5%, first convert 1.5% to a decimal by dividing by 100 to get 0.015.

Do NOT multiply the original number by 0.015 — this approach would be very inefficient, because multiplying by 0.015 would give you only the increase, not the new amount (you would then have to add the increase back to the original amount, a process so time-wasting and inefficient that it would not likely appear in a formula in the answer choices).

Instead, multiply by 1.015. Multiplying by 1 keeps the original number the same; multiplying by 1.015 gets you the original number plus 1.5% more.

Finally, if you want to multiply by 1.015 twice per year, you will need to do it $2y$ times. This $2y$ goes in the exponent spot, to give you $s(1.015)^{2y}$, or choice (C).

14. **(C).** A formula to find the cost of a call under Plan A, using x as the number of minutes:

Cost = $1.25 + 0.15(x - 1)$

Note that you need to use $x - 1$ because the caller does *not* pay \$0.15 for every single minute — the first minute was already paid for by the \$1.25 charge.

A formula to find the cost of a call under Plan B, using x as the number of minutes:

Cost = $0.90 + 0.20x$

Note that here you do *not* use $x - 1$ because the connection fee does not "buy" the first minute — you still have to pay \$0.20 for every minute.

To find the length of a call that would cost the same under either plan, set the two formulas equal to one another:

$1.25 + 0.15(x - 1) = 0.90 + 0.20x$

This is choice (C). Note that you are not required to solve this equation, but you might be required to solve a similar equation in a different problem on this topic:

$1.25 + 0.15x - 0.15 = 0.90 + 0.20x$
$1.1 + 0.15x = 0.90 + 0.20x$
$0.20 = 0.05x$
$20 = 5x$
$4 = x$

A 4-minute call would cost the same under either plan. To test this, calculate the cost of a 4-minute call under both plans: it's $1.70 either way.

15. **(D).** The mix costs *c* cents per ounce. Since you want your final answer in dollars, convert right now:

$$c \text{ cents per ounce} = \frac{c}{100} \text{dollars per ounce}$$

The supplier then purchases *p* pounds of mix. You cannot simply multiply *p* by $\frac{c}{100}$, because *p* is in pounds and $\frac{c}{100}$ is in dollars per OUNCE. You must convert again. Since there are 16 ounces in a pound, it makes sense that a pound would cost 16 times more than an ounce:

$$\frac{c}{100} \text{ dollars per ounce} = \frac{16c}{100} \text{ dollars per pound}$$

Reduce to get $\frac{4c}{25}$ dollars per pound.

Multiply by *p*, the number of pounds, to get what the supplier paid: $\frac{4cp}{25}$ dollars.

Now, the supplier is going to sell the mix for three times what he or she paid. (Don't worry that the problem says three times the "price per ounce" — whether you measure in ounces or pounds, this stuff just got three times more expensive.)

$\frac{4cp}{25} \times 3 = \frac{12cp}{25}$, or answer choice (D).

Note: Make sure you were calculating for revenue, not profit! You were not asked to subtract expenses (what the supplier paid) from the money he or she will be making from selling the mix.

An alternative solution is to plug in smart numbers. An easy number to pick when working with cents is 50 (or 25 — whatever is easy to think about and convert to dollars). Write a value on your paper along with what the value means in words:

$c = 50$ mix costs 50¢ per ounce

Now, common sense (and the fact that 16 ounces = 1 pound) will easily allow you to convert:

50¢ per ounce = $8.00 per pound

The supplier bought *p* pounds. Pick any number you want. For example:

$p = 2$ bought 2 pounds, so spent $16

Notice that no one asked you for this $16 figure, but when calculating with smart numbers, it's best to write down obvious next steps in the reasoning process.

Finally, the supplier is going to sell the mix for three times what he or she paid, so the supplier will sell for $48.

Plug in $c = 50$ and $p = 2$ to see which answer choice generates 48. Only (D) works.

16. **(A).** Since the figure has six sides, use the formula $(n - 2)(180)$, where n is the number of sides, to figure out that the sum of the angles inside the figure = $(6 - 2)(180) = 720$.

The angle supplementary to x can be labeled on your paper as $180 - x$ (since two angles that make up a straight line must sum to 180). Thus:

$$a + b + c + d + e + 180 - x = 720$$
$$a + b + c + d + e - x = 540$$

You are asked to solve for x. Since x is being subtracted from the left side, it would be easiest to add x to both sides, and get everything else on the opposite side.

$$a + b + c + d + e - x = 540$$
$$a + b + c + d + e = 540 + x$$
$$a + b + c + d + e - 540 = x$$

Since $d = 2c$ and $e = \frac{1}{2}a$ and the answers are in terms of a, b, and c, you need to make the d and e drop out of $a + b + c + d + e - 540 = x$.

Fortunately $d = 2c$ and $e = \frac{1}{2}a$ are already solved for d and e, the variables you want to drop out. Substitute:

$$a + b + c + 2c + \frac{1}{2}a - 540 = x$$
$$\frac{3}{2}a + b + 3c - 540 = x$$

This is a match with answer choice (A).

Alternatively, pick numbers. To do this, use the formula $(n - 2)(180)$, where n is the number of sides, to figure out that the sum of the angles inside the figure = $(6 - 2)(180) = 720$. Then, pick values for a, b, c, d, and e, so that $d = 2c$ and $e = \frac{1}{2}a$.

$a = 100$

$b = 110$
$c = 120$
$d = 240$ (this is twice the value picked for c)
$e = 50$ (this is 1/2 the value picked for a)

Subtract all of these values from 720 to get that the unlabeled angle, for this example, is equal to 100. This makes x equal to 180 - 100 = 80.

Now plug $a = 100$, $b = 110$, and $c = 120$ into the answers to see which formula yields a value of 80. (A) is the correct answer.

17. **(A).** One algebraic solution involves defining all three terms in terms of a. Since the terms are consecutive odd integers, they are 2 apart from each other, as such:

a
$b = a + 2$
$c = a + 4$

Then, a is halved to become m, b is doubled to become n, and c is tripled to become p, so:

$$\frac{1}{2}a = m$$

$2b = n$
$2(a + 2) = n$
$2a + 4 = n$

$3c = p$
$3(a + 4) = p$
$3a + 12 = p$

Since $k = mnp$, multiply the values for m, n, and p:

$$k = \left(\frac{1}{2}a\right)(2a+4)(3a+12)$$
$$k = \left(\frac{1}{2}a\right)(6a^2 + 24a + 12a + 48)$$
$$k = \left(\frac{1}{2}a\right)(6a^2 + 36a + 48)$$
$$k = 3a^3 + 18a^2 + 24a$$

This is a match with answer choice (A).

A "smart numbers" solution would be to pick three consecutive odd integers for a, b, and c. When picking numbers for a Variables in the Choices problem, avoid picking 0, 1, or any of the numbers in the problem (this can sometimes cause more than one answer to appear to be correct, thus necessitating starting over with another set of numbers). So:

$a = 3$
$b = 5$
$c = 71$

Then, a is halved to become m, b is doubled to become n, and c is tripled to become p, so:

$1.5 = m$
$10 = n$
$21 = p$

Since $k = mnp$, multiply the values for m, n, and p:

$k = (1.5)(10)(21)$
$k = 315$

Now, plug $a = 3$ (the value originally selected) into the answer choices to see which choice equals 315. Only (A) works.

Because the correct answer is simply a mathematical way of writing the situation described in the problem, this will work for any value you pick for a, provided that a, b, and c are consecutive odd integers and you calculate k correctly.

18. **(C).** The phrase "m pencils cost the same as n pens" can be written as an equation, using x for the cost per pencil and y for the cost per pen:

$$mx = ny$$

Keep in mind here that m stands for the NUMBER of pencils and n for the NUMBER of pens (not the cost). Now, since pencils cost 20 cents or \$0.2 (the answer needs to be in dollars, so convert to dollars now), substitute in for x:

$$0.2m = ny$$

Solve for y to get the cost of 1 pen:

$$y = \frac{0.2m}{n}$$

Since y is the cost of 1 pen and $y = \frac{0.2m}{n}$, multiply by 10 to get the cost of 10 pens:

$$10y = 10\left(\frac{0.2m}{n}\right)$$

$$10y = \frac{2m}{n}$$

Thus, the answer is $\frac{2m}{n}$, or (C).

Alternatively, plug in smart numbers. Since pencils cost 20 cents, maybe pens cost 40 cents (you can arbitrarily pick this number). You are told "*m* pencils cost the same as *n* pens"—pick a number for one of these variables, and then determine what the other variable would be for the example you've chosen. For instance, if $m = 10$, then 10 pencils would cost $2.00. Since 5 pens can be bought for $2.00, *n* would be 5. Now, answer the final question as a number: the cost of 10 pens in this example is $4.00, so the final answer is 4. Plug in $m = 10$ and $n = 5$ to all of the answer choices to see which yields an answer of 4. Only (C) works. For any working system you choose in which "*m* pencils cost the same as *n* pens," choice (C) will work.

19. **(E).** One way to do this problem is simply to construct a formula. Randi's pay is equal to $2,000 plus commission:

$P = 2000 + \dots$

You are only asked to construct a formula for months during which she sells more than 10 forklifts, so you know that she will definitely be receiving 5% commission on 10 forklifts that each cost *s*. Since the revenue from the forklifts would then be $10s$, Randi's commission would be $0.05(10s)$, or $0.5s$.

$P = 2{,}000 + 0.5s + \dots$

Now, you must add the commission for the forklifts she sells above the first 10. Since these first 10 forklifts are already accounted for, you can denote the forklifts at this commission level by writing $f - 10$. Since each forklift still costs *s,* the revenue from these forklifts would be $s(f - 10)$. Since Randi receives 10% of this as commission, the amount she receives would be $0.10s(f - 10)$.

$P = 2{,}000 + 0.5s + 0.10s(f - 10)$

It is possible to simplify further by distributing $0.10s(f - 10)$, but before doing more work, check the answers — you have an exact match already, answer choice (E).

Alternatively, plug in numbers. Say forklifts cost $100 (so, $s = 100$). Randi makes $5 each for the first ten she sells, so $50 total. Then she makes $10 each for any additional forklifts. Pick a value for *f* (make sure the value is more than 10, since the question asks for a formula for months in which Randi sells more than 10 forklifts). So, in a month in which she sells, for example, 13 forklifts (so, $f = 13$), she would make $2,000 + $50 + 3($10) = $2,080.

In this example:

$s = 100$
$f = 13$

Plug in these values for *s* and *f* to see which choice yields $2,080. Only choice (E) works:

$P = 2{,}000 + 0.5(100) + 0.10(100)(13 - 10)$
$P = 2{,}000 + 50 + 10(3)$
$P = 2{,}080$

20. **(A).** This question can be solved either with smart numbers or algebra. First, consider plugging in smart numbers.

Set $l = 2$, so $w = 4$. The perimeter will be $2l + 2w = 2(2) + 2(4) = 12$. The answer is the area, which is $wl = (2)(4) = 8$ based on these numbers. Now plug $p = 12$ into the choices to see which choice equals 8:

(A) $144/18 = 8$
(B) $144/36 = 4$
(C) $12/9 = 4/3$
(D) $144/9 = 16$
(E) $12/6 = 2$

The correct answer is (A).

Though smart numbers are easier and faster here, you could also solve with algebra. If $w = 2l$:

$$a = l \times w = l \times 2\,l = 2l^2$$
$$p = 2l + 2w = 2l + 4l = 6l$$

Solve the second equation for l:

$$l = p/6$$

And plug back into the first equation:

$$2\left(\frac{p}{6}\right)^2 = 2\left(\frac{p^2}{36}\right) = \frac{2p^2}{36} = \frac{p^2}{18}$$

This method is much more difficult than plugging in numbers, but you can still get to the correct answer, (A).

Chapter 20
of

Ratios

In This Chapter…

Ratios

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

The ratio of men to women in a senior citizen garden club is 5 to 4.

Quantity A	**Quantity B**
The smallest possible number of members in the garden club	20

2.

In a certain children's class, there is a 2 to 3 ratio of boys to girls. The ratio of students from the north side of town to students from the south side of town is 4 to 3, and no student is from anywhere else.

Quantity A	**Quantity B**
The smallest possible number of students in the class	35

3. A pantry holds x cans of beans, twice as many cans of soup, and half as many cans of tomato paste as there are cans of beans. If there are no other cans in the pantry, which of the following could be the total number of cans in the pantry?

Indicate <u>two</u> such numbers.

- ☐ 6
- ☐ 7
- ☐ 36
- ☐ 45
- ☐ 63

4. If there are 20 birds and 6 dogs in a park, which of the following represents the ratio of dogs to birds in the park?

(A) 3 : 13
(B) 3 : 10
(C) 10 : 3
(D) 13 : 3
(E) 1 : 26

5. Of the 24 children in a classroom, 12 are boys. Which of the following is the ratio of boys to girls in the classroom?

(A) 1 : 1
(B) 1 : 2
(C) 2 : 1
(D) 1 : 3
(E) 3 : 1

6. If there are 24 white marbles and 36 blue marbles in a bag, what is the ratio of blue to white marbles?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

7. If there are 7 whole bananas, 14 whole strawberries, and no other fruit in a basket, what is the ratio of strawberries to the total pieces of fruit in the basket?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

8. The ratio of cheese to sauce for a single pizza is 1 cup to 1/2 cup. If Bob used 15 cups of sauce to make a number of pizzas, how many cups of cheese did he use on those pizzas?

cups

9. Laura established a new flower garden, planting 4 tulip plants to every 1 rose plant, and no other plants. If she planted a total of 50 plants in the garden, how many of those plants were tulips?

tulip plants

10. The ratio of oranges to peaches to strawberries in a basket containing no other kinds of fruit is 2 : 3 : 4. If there are 8 oranges in the basket, a total of how many pieces of fruit are in the basket?

(A) 16
(B) 32
(C) 36
(D) 48
(E) 72

11. A certain automotive dealer sells only cars and trucks, and the ratio of cars to trucks on the lot is 1 to 3. If there are currently 51 trucks for sale, how many cars does the dealer have for sale?

(A) 17
(B) 34
(C) 68
(D) 153
(E) 204

12. Last season, Arjun’s tennis record was 3 matches won for every 2 he lost. If he played 30 matches last season, how many did he win?

(A) 10
(B) 12
(C) 18
(D) 20
(E) 50

13. A steel manufacturer combines 98 ounces of iron with 2 ounces of carbon to make one sheet of steel. How many ounces of iron were used to manufacture 1/2 of a sheet of steel?

(A) 1
(B) 49
(C) 50
(D) 198
(E) 200

14. Maria uses a recipe for 36 cupcakes that requires 8 cups of flour, 12 cups of milk, and 4 cups of sugar. How many cups of milk would Maria require for a batch of 9 cupcakes?

(A) 2
(B) 3

(C) 4
(D) 6
(E) 8

15. In a certain orchestra, each musician plays only one instrument and the ratio of musicians who play either the violin or the viola to musicians who play neither instrument is 5 to 9. If 7 members of the orchestra play the viola and four times as many play the violin, how many play neither?

(A) 14
(B) 28
(C) 35
(D) 63
(E) 72

16. The ratio of 0.4 to 5 equals which of the following ratios?

(A) 4 to 55
(B) 5 to 4
(C) 2 to 25
(D) 4 to 5
(E) 4 to 45

17. At an animal shelter, the ratio of cats to dogs is 4 to 7. If there are 27 more dogs than cats, how many cats are at the shelter?

(A) 12
(B) 16
(C) 24
(D) 28
(E) 36

18. On a wildlife preserve, the ratio of giraffes to zebras is 37 : 43. If there are 300 more zebras than giraffes, how many giraffes are on the wildlife preserve?

(A) 1,550
(B) 1,850
(C) 2,150
(D) 2,450
(E) 2,750

19. On a youth soccer team, the ratio of boys to girls is 6 to 7. If there are 2 more girls than boys on the team, how many boys are on the team?

(A) 12
(B) 18
(C) 24
(D) 30
(E) 36

20. At a certain company, the ratio of male to female employees is 3 to 4. If there are 5 more female employees than male employees, how many male employees does the company have?

(A) 12
(B) 15

(C) 18
(D) 21
(E) 24

21. On Monday, a class has 8 girls and 20 boys. On Tuesday, a certain number of girls joined the class just as twice that number of boys left the class, changing the ratio of girls to boys to 7 to 4. How many boys left the class on Tuesday?

(A) 5
(B) 6
(C) 11
(D) 12
(E) 18

22. If a dak is a unit of length and 14 daks = 1 jin, how many squares with a side length of 2 daks can fit in a square with a side length of 2 jin?

(A) 14
(B) 28
(C) 49
(D) 144
(E) 196

23.

In a group of adults, the ratio of women to men is 5 to 6, while the ratio of left-handed people to right-handed people is 7 to 9. Everyone is either left- or right-handed; no one is both.

Quantity A	**Quantity B**
The number of women in the group	The number of left-handed people in the group

24.

Party Cranberry is 3 parts cranberry juice and 1 part seltzer. Fancy Lemonade is 1 part lemon juice and 2 parts seltzer. One glass of Party Cranberry is mixed with an equally sized glass of Fancy Lemonade.

Quantity A	**Quantity B**
The fraction of the resulting mix that is cranberry juice	The fraction of the resulting mix that is seltzer

25.

The ratio of 16 to g is equal to the ratio of g to 49.

Quantity A	**Quantity B**
g	28

26. In a parking lot, 1/3 of the vehicles are black and 1/5 of the remainder are white. How many vehicles could be parked on the lot?

(A) 8
(B) 12
(C) 20
(D) 30
(E) 35

27. Three friends divided a bag of chocolates so that David received a fifth the number of chocolates that Fouad did, and Stina received 80 percent of the total number of chocolates. What is the ratio of the number of chocolates Stina received to the number that David received?

(A) 4 : 3
(B) 8 : 5
(C) 8 : 1
(D) 24 : 1
(E) 80 : 1

28. A new sport is played with teams made up of 2 forwards, 3 guards, and 1 goalie. There are 23 players available to play forward, 21 other players available to play guard, and 9 other players available to play goalie. If the maximum possible number of complete teams are formed, how many of the available players will not be on a team?

(A) 7
(B) 9
(C) 11
(D) 13
(E) 15

29. Oil, vinegar, and water are mixed in a 3 to 2 to 1 ratio to make salad dressing. If Larry has 8 cups of oil, 7 cups of vinegar, and access to any amount of water, what is the maximum number of cups of salad dressing he can make with the ingredients he has available, if fractional cup measurements are possible?

(A) 12
(B) 13
(C) 14
(D) 15
(E) 16

30. With y dollars, 5 oranges can be bought. If all oranges cost the same, how many dollars do 25 oranges cost, in terms of y?

(A) $y/5$
(B) y
(C) $y + 5$
(D) $5y$
(E) $25y$

31. A certain drawer contains only black and white socks. If the ratio of black socks to white socks is 3 : 4 and there are 15 black socks in the drawer, how many socks total are in the drawer?

(A) 15
(B) 20
(C) 30
(D) 35
(E) 45

32.

A tree grows taller at a constant rate. The ratio of its growth in feet to the time spent growing in years is 4 : x.

Quantity A	**Quantity B**
The number of feet the tree grows taller in 10 years	40

33. Ten robots painted 3 identical houses in 5 hours, working at a constant rate. How many hours would it take 20 identical robots to paint 12 such houses, working at the same constant rate?

(A) 2.5
(B) 5
(C) 10
(D) 15
(E) 20

34. Dick takes twice as long as Jane to run any given distance. Starting at the same moment, Dick and Jane run towards each other from opposite ends of the schoolyard, a total distance of x, at their respective constant rates until they meet.

Quantity A	**Quantity B**
The fraction of the total distance x that is covered by Jane	$\frac{2}{3}x$

35. One robot can pack a box in 15 minutes. Working together at the same constant rate, how many boxes can 16 robots pack in 1 hour?

(A) 4
(B) 16
(C) 24
(D) 64
(E) 256

36. A woman spent $\frac{5}{8}$ of her weekly salary on rent, and $\frac{1}{3}$ of the remainder on food, leaving \$40 available for other expenses. What is the woman's weekly salary?

(A) \$160
(B) \$192
(C) \$216
(D) \$240
(E) \$256

37. A mixture contains nothing but water and acetone in a ratio of 1 : 2. After 200 mL of water is added to the mixture, the ratio of water to acetone is 2 : 3.

Quantity A	**Quantity B**
The original volume of the mixture	1,800 mL

38. In a certain rectangle, the ratio of length to width of a rectangle is 3 : 2 and the area is 150 square centimeters. What is the perimeter of the rectangle, in centimeters?

(A) 10
(B) 15
(C) 25
(D) 40
(E) 50

39. At a certain college, the ratio of students to professors is 8 : 1 and the ratio of students to administrators is 5 : 2. No person is in more than one category (for instance, there are no administrators who are also students).

Quantity A	**Quantity B**
The fractional ratio of professors to administrators	$\frac{5}{8}$

40. Mary prepared x pounds of pasta for the y people expected to attend a banquet. If only z of these y people actually attend, such that $z < y$, how many pounds of pasta will be left over if Mary serves the originally intended portion to each of the guests in attendance?

(A) $\frac{x(y-z)}{y}$
(B) $\frac{zx}{y}$
(C) $\frac{x-z}{y}$
(D) $\frac{x(z-y)}{y}$
(E) $\frac{x(y+z)}{y}$

41. Sara purchased a number of wrenches and hammers from a hardware store, such that the ratio of wrenches to hammers purchased was 5 : 4 and she purchased 10 more wrenches than hammers.

Quantity A	**Quantity B**
The number of hammers Sara purchased from the hardware store	50

42. A family drove from home to a vacation destination 100 miles away, driving the first half of the distance at a constant speed of 50 miles per hour and the second half of the distance at a constant speed of 20 miles per hour. Returning home by the same route, they traveled at a constant speed of 30 miles per hour for the whole trip.

Quantity A	**Quantity B**
The number of hours it took to drive from home to the vacation destination	The number of hours it took to drive from the vacation destination back home

43. A hose is filling a large bucket with water at a constant rate of 3 gallons per minute. The bucket is losing water through a leak at a constant rate of 1 gallon per minute. If the bucket can hold a total volume of 8 gallons, how many minutes are required to fill the bucket to capacity, starting from empty?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

44. If Dan can make 10 widgets every 15 seconds, how many widgets can Dan make in 1 hour, working at this constant rate?

(A) 40
(B) 240
(C) 600
(D) 2,400
(E) 4,000

45. In a certain country, 8 rubels are worth 1 schilling, and 5 schillings are worth 1 lemuw. In this country, 6 lemuws are equivalent in value to how many rubels?

(A) 20/3
(B) 30
(C) 40
(D) 48
(E) 240

46. The ratio of Kim's time to paint a house to Jane's time to paint a house is 3 : 5. If Kim and Jane work together at their respective constant rates, they can paint a house in 10 hours.

Quantity A	**Quantity B**
The number of hours it takes Kim to paint the house alone	16

47. Team A and Team B are raising money for a charity event. The ratio of money collected by Team A to money collected by team B is 5 : 6. The ratio of the number of students on Team A to the number of students on Team B is 2 : 3. What is the ratio of money collected per student on team A to money collected per student on team B?

(A) 4 : 5
(B) 5 : 4
(C) 5 : 6
(D) 5 : 9
(E) 9 : 5

48. Ketchup, soy sauce, and mayonnaise are mixed together in a ratio of 3 : 2 : 5 to make Mr. Anderson's special sauce. If Mr. Anderson prepared 25 ounces of special sauce for his upcoming barbeque, how many ounces of soy sauce did he use?

(A) 2.5
(B) 5
(C) 7.5
(D) 10

(E) 12.5

49. Saul ran from point A to point B and then back again by the same route in 63 minutes. It took Saul $\frac{3}{4}$ as much time to run from point A to point B as it took him to run from point B to point A.

Quantity A	Quantity B
The number of minutes Saul's point A to point B run took	30

50. Jarod needs 2/3 of an ounce of vinegar for every 2 cups of sushi rice that he prepares. To prepare 7 cups of sushi rice in the same proportion, how many ounces of vinegar does Jarod need?

(A) 3/2
(B) 4/3
(C) 7/3
(D) 7/2
(E) 14/3

51 Joe drove from Springfield to Shelbyville at x miles per hour. He then drove from Shelbyville to Bakersfield at $(1.5)x$ miles per hour. If the distance between Springfield and Shelbyville is twice the distance between Shelbyville and Bakersfield, what was Joe's average speed for the entire trip?

(A) $\frac{9}{8}x$
(B) $\frac{6}{5}x$
(C) $\frac{5}{4}x$
(D) $\frac{7}{4}x$
(E) $\frac{9}{4}x$

52. The total cost of 3 bananas, 2 apples, and 1 mango is \$3.50. The total cost of 3 bananas, 2 apples, and 1 papaya is \$4.20. The ratio of the cost of a mango to the cost of a papaya is 3 : 5.

Quantity A	Quantity B
The cost of a papaya	\$2.00

53. In a certain town, $\frac{2}{5}$ of the total population is employed. Among the unemployed population, the ratio of males to females is 5 : 7. If there are 40,000 employed people in the town, how many females are unemployed?

(A) 16,000
(B) 25,000
(C) 35,000

(D) 65,000
(E) 75,000

54.

Quantity A	Quantity B
The ratio of $2\frac{11}{12}$ to $1\frac{3}{4}$	$\frac{5}{3}$

55. On a certain map of the United States, 3/5 of an inch represents a distance of 400 miles. If Oklahoma City and Detroit are separated on the map by approximately 3/2 of an inch, what is the approximate distance between them in miles?

(A) 240
(B) 360
(C) 600
(D) 800
(E) 1,000

56. A machine can manufacture 20 cans per hour, and exactly 10 such cans fit into every box. Maria packs cans in boxes at a constant rate of 3 boxes per hour. If the machine ran for 2 hours and was then turned off before Maria started packing the cans in boxes, how many minutes would it take Maria to pack all the cans that the machine had made?

(A) 40
(B) 45
(C) 80
(D) 160
(E) 800

57. At Company X, a person's pay is increased by the same dollar amount every year that he or she works for Company X. Since Joe started with Company X, he has seen 10 such raises, and the ratio of his pay now to his pay when he started is 5 : 2. What is the ratio of Joe's yearly pay increase to his starting pay?

(A) 1/4
(B) 3/20
(C) 5/3
(D) 4/1
(E) 20/3

58. If Beth has 1/4 more money than Ari, and each person has an integer number of dollars, which of the following could be the combined value of Beth and Ari's money?

Indicate <u>all</u> such values.

☐ $12
☐ $54
☐ $72
☐ $200

59. If salesperson A sold 35% more motorcycles than salesperson B, which of the following could be the total

number of motorcycles sold by both salespeople?

Indicate <u>all</u> such total numbers of motorcycles.

☐ 47
☐ 70
☐ 135
☐ 235

60. A zoo has twice as many zebras as lions and four times as many monkeys as zebras. Which of the following could be the total number of zebras, lions, and monkeys at the zoo?

Indicate <u>all</u> such totals.

☐ 14
☐ 22
☐ 28
☐ 55
☐ 121

61. In Nation Z, 10 terble coins equal 1 galok. In Nation Y, 6 barbar coins equal 1 murb. If a galok is worth 40% more than a murb, what is the ratio of the value of 1 terble coin to the value of 1 barbar coin?

(A) $\frac{3}{5}$
(B) $\frac{11}{13}$
(C) $\frac{3}{7}$
(D) $\frac{21}{23}$
(E) $\frac{21}{25}$

62. Autolot has a 2 : 1 ratio of blue cars to red cars and a 6 : 1 ratio of red cars to orange cars on the lot. What could be the total number of blue, red and orange cars on the lot?

(A) 38
(B) 39
(C) 40
(D) 41
(E) 42

63. Originally, 70% of the clients at Bob's Dating Bistro were male. After z of the female clients left, the service still had 74 clients. Which of the following could be the value of z?

Indicate <u>all</u> such values.

☐ 4
☐ 6
☐ 12
☐ 16
☐ 18

64. Beaker B has a volume of b, which is twice the volume c of Beaker C. The volume of Beaker C is one third the volume g of Beaker G.

Quantity A	**Quantity B**
$\frac{b+c}{g}$	1

Ratios Answers

1. **(B).** The ratio of men to women is 5 to 4. Since both 5 and 4 are whole numbers, they could actually *be* the number of men and women, respectively.

5 and 4 are also the *lowest* possible numbers of men and women, because reducing the ratio of 5 to 4 any further is impossible without making one part a non-integer (e.g., 2.5 to 2) or both parts negative, and the numbers of men and women must be positive.

So the smallest number of people who could be in the garden club is $5 + 4 = 9$, which is less than 20. Quantity B is greater.

2. **(C).** When a ratio includes things you can count (such as people), each group must be a positive multiple of the numbers in the ratio. For instance, the ratio of boys to girls is 2 to 3. So the number of boys is $2n$ and the number of girls is $3n$, where n is a positive integer. The total number of students is $2n + 3n = 5n$, which is a positive multiple of 5.

Using the second ratio, write the number of students from the north side as $4m$ and the number of students from the south side as $3m$, where m is a positive integer. The total number of students is $4m + 3m = 7m$, which is a positive multiple of 7.

The smallest number that is a positive multiple of both 5 and 7 is 35. The two quantities are equal.

3. **II and V only.** Write the number of each type of can.

Cans of beans = x Cans of soup = $2x$ Cans of tomato paste = $0.5x$

Since each number of cans must be an integer, you know that $0.5x = n$, where n is some positive integer. This means $x = 2n$. (In other words, x is even.)

The total number of cans is $x + 2x + 0.5x = 3.5x$. Now substitute in $2n$:

Total = $3.5x = 3.5(2n) = 7n$. So the total number of cans is a multiple of 7.

Of the answer choices, only 7 and 63 are multiples of 7.

4. **(B)** If there are 6 dogs and 20 birds in the park, the ratio of dogs to birds is 6 : 20, which reduces to 3 : 10.

5. **(A).** If 12 of 24 children in the classroom are boys, then the remaining $24 - 12 = 12$ children are girls. Therefore the ratio of boys to girls in the classroom is 12 : 12. Reduce this ratio by dividing both sides by the common factor of 12. The reduced ratio is $\frac{12}{12} : \frac{12}{12}$ or 1 : 1.

Be sure to answer the right question on a quick problem such as this one. The correct ratio of boys to all children is 1 : 2, which is also the correct ratio of girls to all children; however, the question asks for the ratio of boys to girls, which is 1 : 1.

6. $\frac{3}{2}$ **or any equivalent fraction.** If there are 36 blue marbles and 24 white marbles, the ratio of blue to white marbles is 36 : 24. Write this ratio as a fraction and cancel the common factor of 12 from top and bottom. $\frac{36}{24}=\frac{3\times12}{2\times12}=\frac{3}{2}$. The the original ratio of $\frac{36}{24}$ would also be counted as correct if entered as-is.

7. $\frac{2}{3}$ **or any equivalent fraction.** If there are 7 bananas and 14 strawberries, then there are 7 + 14 = 21 total pieces of fruit. The ratio of strawberries to the total is therefore 14 : 21. Write this ratio as a fraction and cancel the common factor of 7 from top and bottom. $\frac{14}{21}=\frac{2\times7}{3\times7}=\frac{2}{3}$. The original ratio of $\frac{14}{21}$ would also be counted as correct if entered as-is.

8. **30 cups.** To solve for the actual amount of cheese Bob used, work with the Part : Part ratio. The ratio of cheese to sauce is $1:\frac{1}{2}$, or $1x:\frac{1}{2}x$ (putting in an unknown multiplier). When the actual amount of sauce is $\frac{1}{2}x$, the actual amount of cheese is 1x, or simply x. Bob used 15 cups of sauce. Solve for x:

$$\frac{1}{2}x = 15$$
$$x = 15(2)$$
$$x = 30$$

Since x also indicates the actual amount of cheese, Bob used 30 cups of cheese. Don't go so fast on this sort of problem that you make a silly mistake, such as reversing the ratio and getting $\frac{1}{2}$ of 15, or 7.5 cups, as the incorrect answer.

9. **40 tulip plants.** To solve for the number of tulips, work with the Part : Part : Whole ratio. The ratio of tulips to roses is 4 : 1, so the Tulip : Rose : Total relationship is 4 : 1 : 5. This ratio can be written as $4x : 1x : 5x$, with x as the unknown integer multiplier. There are 50 total plants in the garden, so set $5x$ equal to 50 and solve for x:

$$5x = 50$$
$$x = 10$$

Now plug this value into the expression for the actual number of tulips: $4x = 4(10) = 40$. Laura planted 40 tulip plants in the garden.

10. **(C).** Work with the Part : Whole ratio to solve for the total number of pieces of fruit. The ratio of oranges to

peaches to strawberries is 2 : 3 : 4, so the Part : Whole relationship would include the total of $2 + 3 + 4 = 9$, or 2 : 3 : 4 : 9 (three parts and a whole). This four-way ratio can be written as $2x : 3x : 4x : 9x$, with x as the unknown multiplier. There are 8 oranges in the basket, so set the oranges part ($2x$) equal to 8 and solve for x.

$$2x = 8$$
$$x = 4$$

Now plug this value into the expression for the whole: $9x = 9(4) = 36$. There are 36 pieces of fruit total.

11. **(A).** Focus on the given Part : Part ratio. The ratio of cars to trucks is 1 : 3, or $x : 3x$ with x as the unknown multiplier. Since there are 51 trucks for sale, set $3x$ equal to 51 and solve for x.

$$3x = 51$$
$$x = 17$$

Since x also represents the number of cars, the dealer has 17 cars for sale.

12. **(C).** Work with the Part : Part : Whole ratio to solve for the number of wins. The ratio of matches won to matches lost is 3 to 2, so the Wins : Losses : Total ratio is 3 : 2 : 5. This ratio can be written as $3x : 2x : 5x$, with x as the unknown multiplier. Arjun played 30 matches in all, so set $5x$ equal to 30 and solve.

$$5x = 30$$
$$x = 6$$

Now plug this value into the expression for the number of wins: $3x = 3(6) = 18$. Arjun won 18 matches.

13. **(B).** Iron and carbon combine to make steel in a specific given ratio. The ratio of iron (ounces) to carbon (ounces) to steel (sheets) is 98 : 2 : 1. Because there are different units (ounces and sheets), the Part numbers do not add to the Whole number as they typically do, but don't be concerned.

This ratio can be written as $98x : 2x : x$, with x as the unknown multiplier, which is also the number of sheets. To make 1/2 a sheet of steel, set x equal to 1/2.

Now plug this value into the expression for the number of iron ounces: $98x = (98)(1/2) = 49$. To make 1/2 a sheet of steel, 49 ounces of iron are required.

14. **(B).** As a ratio, Flour : Milk : Sugar : Cupcakes is equal to 8 : 12 : 4 : 36, where the first 3 numbers are in cups. Because there are different units (cups and cupcakes), the Part numbers do not add to the Whole number, but don't be concerned.

This ratio can be written as $8x : 12x : 4x : 36x$, with x as the unknown multiplier. To make 9 cupcakes, set $36x$ equal to 9 and solve for x.

$$36x = 9$$
$$x = 1/4$$

In words, for a batch of 9 cupcakes, Maria would make 1/4 of the original recipe.

Now plug this value into the expression for cups of milk: $12x = (12)(1/4) = 3$. Maria would need 3 cups of milk.

15. **(D).** Since 7 members of the orchestra play the viola and four times as many play the violin, $(7)(4) = 28$ people must play the violin. All together, $7 + 28 = 35$ musicians in the orchestra play either the viola or the violin.

The ratio of *either* to *neither* is $5 : 9$, or $5x : 9x$ using the unknown multiplier. Since 35 people play either instrument, set $5x$ equal to 35 and solve for x.

$5x = 35$
$x = 7$

Now plug this value into the expression for *neither*: $9x = 9(7) = 63$. There are 63 people in the orchestra who play neither instrument.

16. **(C).** You can rewrite ratios as fractions and then multiply or divide top and bottom by the same number, keeping the ratio (or fraction) the same.

First, multiply top and bottom by 10, to remove the decimal. $\frac{0.4}{5} = \frac{0.4 \times 10}{5 \times 10} = \frac{4}{50}$

Next, cancel the common factor of 2. $\frac{4}{50} = \frac{2 \times 2}{25 \times 2} = \frac{2}{25}$

Finally, the fraction $\frac{2}{25}$ is the same as the ratio of 2 to 25, which is therefore equivalent to the original ratio of 0.4 to 5.

17. **(E).** The ratio of cats to dogs is $4 : 7$. Because the number of cats or dogs in this question can only be positive integers, introduce the unknown multiplier x: the number of cats is $4x$, and the number of dogs is $7x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. “There are 27 more dogs than cats” becomes Dogs - Cats = 27, or $7x - 4x = 27$. Solve for x:

$7x - 4x = 27$
$3x = 27$
$x = 9$

Finally, substitute into the expression for the number of cats: $4x = 4(9) = 36$. There are 36 cats.

18. **(B).** The ratio of giraffes to zebras is $37 : 43$. Introduce the unknown multiplier x: the number of giraffes is $37x$, and the number of zebras is $43x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. “There are 300 more zebras than giraffes” becomes Zebras - Giraffes = 300, or $43x - 37x = 300$. Solve for x:

$$43x - 37x = 300$$
$$6x = 300$$
$$x = 50$$

Finally, substitute into the expression for the number of giraffes: $37x = 37(50) = 1{,}850$. There are 1,850 giraffes.

In a pinch, here’s a shortcut: the right answer must be a multiple of 37, because the giraffe number in the ratio is 37, and you need a positive whole number of giraffes. If you test the answer choices, only 1,850 is divisible by 37. This shortcut doesn’t always work this well, of course!

19. **(A).** The ratio of boys to girls is 6 : 7. If you introduce the unknown multiplier x, the number of boys is $6x$, and the number of girls is $7x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. “There are 2 more girls than boys” becomes Girls - Boys = 2, or $7x - 6x = 2$. Solve for x:

$$7x - 6x = 2$$
$$x = 2$$

Finally, substitute into the expression for the number of boys: $6x = 6(2) = 12$. There are 12 boys on the youth soccer team.

20. **(B).** The ratio of male to female employees is 3 : 4. If you introduce the unknown multiplier x, the number of males is $3x$, and the number of females is $4x$, where x is a positive integer.

Now translate the second sentence of the problem into algebra. “There are 5 more female employees than male employees” becomes Females – Males = 5, or $4x - 3x = 5$. Solve for x:

$$4x - 3x = 5$$
$$x = 5$$

Finally, substitute into the expression for the number of male employees: $3x = 3(5) = 15$. There are 15 male employees.

21. **(D).** Call the number of girls who joined the class x, so the new number of girls in the class was $8 + x$. Twice as many boys left the class, so the number of boys who left the class is $2x$, and the new number of boys in the class was $20 - 2x$.

The resulting ratio of boys to girls was 7 to 4. Since there is already a variable in the problem, don’t use an unknown multiplier. Rather, set up a proportion and solve for x.

$$\frac{\text{Girls}}{\text{Boys}} = \frac{8+x}{20-2x} = \frac{7}{4}$$

$4(8+x) = 7(20-2x)$
$32+4x = 140-14x$
$18x = 108$
$x = 6$

Finally, the question asks for the number of boys who left the class. This is $2x = 2(6) = 12$ boys.

Check: There were 8 girls in the class, then 6 joined for a total of 14 girls. There were 20 boys in the class until 12 left the class, leaving 8 boys in the class. The resulting ratio of girls to boys was $\frac{14}{8} = \frac{7\times2}{4\times2} = \frac{7}{4}$, as given.

22. **(E).** Since 14 daks = 1 jin, a length measured in daks is 14 times the same length measured in jins. In other words, the ratio of the length in daks to the length in jins is 14 to 1.

Write this relationship as a fraction: $\frac{14\text{ daks}}{1\text{ jin}}$. You can also write $\frac{1\text{ jin}}{14\text{ daks}}$. You can convert a measurement from one unit to the other by multiplying by one of these unit conversion factors.

Side of big square: $(2\text{ jins})\left(\frac{14\text{ daks}}{1\text{ jin}}\right) = 28$. Since the small square has a side length of 2 daks, the number of small sides that will fit along a big side is $28 \div 2$, or 14.

However, 14 is not the right answer. 14 is the number of small squares that will fit along *one wall* of the big square, in one row. There will be 14 rows, so in all there will be (14)(14) = 196 small squares that fit inside the big square.

23. **(A).** Write two different Part : Part : Whole relationships. In each relationship, the two parts sum to the whole.

Women : Men : Total = 5 : 6 : 11, so Women : Total = 5 : 11.

Left-handed : Right-handed : Total = 7 : 9 : 16, so Left-handed : Total = 7 : 16.

In other words, women account for $\frac{5}{11} = 45.\overline{45}\%$ of the group, left-handed people for $\frac{7}{16} = 43.75\%$ of the group.

Since the total number of people is the same (it's the same group, whether divided by gender or handedness), the percents can be compared directly. There must be more women than left-handed people in the group. Quantity A is greater.

24. **(B).** Be careful — don't just add the "parts" from the different glasses, because the parts will generally not be the same size! Start by writing Part : Part : Whole relationships for each glass. In each relationship, the whole is the sum of the parts.

For Party Cranberry, *Cranberry* : *Seltzer* : *Whole* = 3 : 1 : 4

For Fancy Lemonade, *Lemon* : *Seltzer* : *Whole* = 1 : 2 : 3

Since the two glasses that are mixed are the same size, you can choose a smart number to represent the volume of a glass. This number should be a multiple of both 4 and 3, according to the ratios above, so it is convenient to say that a glass is 12 ounces. Multiply the Party Cranberry ratio by 3 and the Fancy Lemonade ratio by 4, in both cases to get 12 total ounces.

For Party Cranberry, *Cranberry* : *Seltzer* : *Whole* = 9 : 3 : 12

For Fancy Lemonade, *Lemon* : *Seltzer* : *Whole* = 4 : 8 : 12

Finally, when the two glasses are mixed, the resulting total is 24 ounces, of which 9 ounces are cranberry juice but 3 + 8 = 11 ounces are seltzer. There is more seltzer in the resulting mix, so its fraction of the mix is greater than cranberry juice's fraction of the mix. Quantity B is greater.

25. **(D).** Write the ratios as fractions and set them equal to each other.

$$\frac{16}{g}=\frac{g}{49}$$

Cross multiply to get $16 \times 49 = g^2$.

Remember that when you "unsquare" an equation, you must account for the negative possibility. The value of g could be *either* $4 \times 7 = 28$ *or negative* 28. Nothing in the problem indicates that g must be positive. Since Quantity A might equal Quantity B *or* be less than Quantity B, the relationship cannot be determined from the information given.

26. **(D) 30.** Since vehicles must be counted with whole numbers and 1/3 of the cars are black, the total number of cars must be divisible by 3. Otherwise, 1/3 of the total would not be a whole number. The answer must be (B) or (D).

The remainder of the cars is 1 – 1/3 = 2/3 of the total. Of these, 1/5 are white, so 1/5 of 2/3, or $\frac{2}{15}$ of the total number of vehicles are white. Again, because the white cars must be countable with whole numbers, $\frac{2}{15}$ of the total must be an integer. You can write the equation using fractions:

$$\left(\frac{2}{15}\right)(\text{Total})=\text{Integer}$$

To get an integer outcome, the total must be divisible by 15. Of the answer choices, only (D) is divisible by 15.

27. **(D).** The ratio of David's chocolates to Fouad's chocolates is 1 : 5, which you can represent as x and $5x$, respectively, using an unknown multiplier. Together, David and Fouad received $x + 5x = 6x$ chocolates.

Since Stina received 80% of the total, David and Fouad received 20% of the total, or 1/5 of the total. Thus, the total is 5 times what David and Fouad received, or $5(6x) = 30x$. Stina received 80% of this total, or $24x$. You could get to Stina's number directly — since she gets 4/5 of the total and the others get 1/5, she gets 4 times as many chocolates as David and Fouad together, or $4(6x) = 24x$.

Finally, the ratio of Stina's chocolates to David's is $24x$ to x, or 24 : 1.

28. **(C).** To figure out the "limiting factor," take the number of players available for each position and figure out how many teams could be formed in each case, if there were more than enough players in all the other positions.

Forwards: 23 players available ÷ 2 forwards needed per team = 11.5 teams (if there could be partial teams) = 11 complete teams, rounding down.

Guards: 21 players available ÷ 3 guards needed per team = 7 complete teams.

Goalies: 9 players available ÷ 1 goalie needed per team = 9 complete teams.

The guards are the limiting factor, because the fewest complete teams can be formed with them. Only 7 complete teams can be formed, using all of the available guards and some of the other players. A total of 7 × 2 = 14 forwards are required, leaving 23 - 14 = 9 unused forwards. Likewise, 7 × 1 = 7 goalies are required, leaving 9 - 7 = 2 unused goalies. In all, there are 9 + 2 = 11 unused players, who will not be on a team.

29. **(E).** Since the ratio of ingredients is 3 : 2 : 1 in the recipe, imagine that Larry works in cups. Then a recipe makes 3 + 2 + 1 = 6 cups of dressing. To figure out the "limiting factor," take each available amount of ingredient and figure out how many times he could make the recipe, permitting fractions, if he had more than enough of the other ingredients.

Oil: 8 cups available ÷ 3 cups needed per recipe = 8/3 recipes (in other words, $2\frac{2}{3}$ times the recipe). There is no need to round down, because fractional cups of ingredients are allowed.

Vinegar: 7 cups available ÷ 2 cups needed per recipe = 7/2 recipes (in other words, $3\frac{1}{2}$ times through the recipe).

Water availability is not limited, so ignore it.

Oil is the limiting factor, because Larry can make the fewest recipes with it. Thus, he can only make 8/3 recipes. To find the total cups of salad dressing, multiply this fraction by the total number of cups that a recipe makes:

$\frac{8}{3}$ recipe × 6 cups per recipe = 16 cups

30. **(D).** Create a unit conversion factor, using the given ratio of oranges to dollars. The conversion factor will look

like either $\frac{5 \text{ oranges}}{y \text{ dollars}}$ or $\frac{y \text{ dollars}}{5 \text{ oranges}}$. Which one you use depends on how you want to convert the units.

You are given 25 oranges and asked how many dollars, in terms of y, these oranges will cost. Since you are starting with oranges and want to get to dollars, choose the conversion unit that cancels oranges and leaves dollars on top: $\frac{y \text{ dollars}}{5 \text{ oranges}}$. Then multiply:

$$(25 \text{ oranges})\left(\frac{y \text{ dollars}}{5 \text{ oranges}}\right) = \frac{25y}{5} \text{ dollars} = 5y \text{ dollars}$$

Intuitively, a total of 25 oranges is the same as 5 sets of 5 oranges each. Each set costs y dollars. Therefore, the total cost for 5 sets of oranges is $5 \times y = 5y$.

31. **(D).** The ratio of black socks to white socks is 3 : 4, so you can represent the number of black socks as $3x$ and the number of white socks as $4x$, with x as the unknown multiplier.

You are told that there are 15 black socks, so set that equal to the expression for black socks and solve for x:

$$3x = 15$$
$$x = 5$$

The total number of socks is $3x + 4x = 7x$, so there are $7(5) = 35$ socks total in the drawer.

32. **(D).** Translate the second sentence from a ratio to a rate. The growth of the tree, divided by the time it spends growing, is $4/x$. In other words, the constant rate of growth is $4/x$ feet per year.

Now apply the formula Distance = Rate × Time.

$$\text{Distance} = \left(\frac{4 \text{ feet}}{x \text{ years}}\right)(10 \text{ years}) = \frac{40}{x} \text{ feet}$$

Without knowing x, it cannot be determined whether Quantity A is greater or less than Quantity B. For example, if the tree grows 4 feet every year ($x = 1$), the two quantities are equal. If the tree grows 4 feet every 10 years ($x = 10$), then Quantity B is greater than Quantity A (which would be 4).

33. **(C).** Work problems can be solved in various ways; this explanation takes an approach involving ratios. Start with what you know—10 robots can paint 3 houses in 5 hours. Twice as many robots can paint twice as many houses in the same amount of time. In other words, the ratio 10 robots : 3 houses = 20 robots : 6 houses. That is, the 20 robots in question can paint 6 houses in 5 hours.

But 12 houses need painting, not 6. That’s twice as many houses to paint. Since the rate at which the robots work is constant, a given set of robots will take twice as long to paint twice as many houses. So, to paint 12 houses, the 20 robots need 2×5 hours = 10 hours.

34. **(C).** Rate problems can be solved in various ways; this explanation takes an approach involving ratios. If Dick takes twice as long as Jane to run any distance, then he must run half as fast. So the ratio of Dick's speed to Jane's speed is 1 : 2. Because *Rate* × *Time* = *Distance*, this also means that for a fixed period of time, the ratio of the distances they run is also 1 : 2. That is, Dick only covers half the distance Jane covers in the same amount of time. They start running at the same moment and stop running when they meet, so they do run for the same amount of time.

Now use a Part : Part : Whole relationship. Since together, Dick and Jane run the whole distance, Dick's distance : Jane's distance : Total distance ratio must be 1 : 2 : 3. Dick runs $\frac{1}{3}x$, Jane runs $\frac{2}{3}x$, and the total distance is $\frac{3}{3}x$. Thus, Quantity A equals Quantity B.

35. **(D).** First find how many boxes 1 robot can pack in 1 hour. Since it is given that 1 robot can pack a box in 15 minutes, multiply by the conversion ratio of 60 minutes to 1 hour:

$$\left(\frac{1\text{ box}}{15\text{ minutes}}\right)\left(\frac{60\text{ minutes}}{1\text{ hour}}\right)=\frac{4\text{ boxes}}{1\text{ hour}}$$

This result should make intuitive sense. Each hour contains four 15-minute segments. A robot can build 1 box in each of these 4 segments, so 4 boxes per hour are packed by one robot.

The rate is the same for all robots, so if 1 robot can build 4 boxes in 1 hour, then 16 robots can build 16 times as many boxes in the same amount of time. 16×4 boxes per hour = 64 boxes in one hour.

36. **(A).** The total amount of money left over after paying rent and buying food is \$40. From this number, you can find her total weekly salary by determining what fraction this is of her total salary.

Since the woman spent 5/8 of her salary on rent, she had 1 - 5/8 = 3/8 of her salary remaining. Of the remainder, she spent 1/3 on food and had 2/3 left over. So, 2/3 of 3/8 of her total weekly salary was left over for other expenses.

$$\left(\frac{2}{3}\right)\left(\frac{3}{8}\right)=\frac{2}{8}=\frac{1}{4}$$

One quarter of her salary was the \$40 left over. If *T* is her total weekly salary, then

$$\left(\frac{1}{4}\right)T=\$40$$

$$T=\$160$$

37. **(C).** Use the unknown multiplier, x, for the first ratio given. Since the ratio of water to acetone is 1 : 2, make the amount of water x and the amount of acetone $2x$. After 200 mL of water are added to the mixture, there will be x + 200 mL of water. Now write the new ratio:

$$\frac{\text{Water}}{\text{Acetone}} = \frac{x + 200}{2x} = \frac{2}{3}$$

Cross multiply and solve for x:

$3x + 600 = 4x$
$x = 600$

Finally, compute the original volume of the mixture. The original volume of water is $x = 600$ mL, while the original volume of acetone is $2x = 2(600) = 1{,}200$ mL. Therefore, the total original volume is $600 + 1200 = 1800$ mL.

38. **(E).** Rewrite the given ratio using the unknown multiplier x, so that the length of the rectangle is $3x$, while the width is $2x$. Now express the area of the rectangle in these terms, set it equal to 150 square centimeters, then solve for x:

Area = (Length)(Width)
$150 = (3x)(2x)$
$150 = 6x^2$
$25 = x^2$
$x = 5$ cm

In this case, you don't need to worry about the negative possibility for the square root, since lengths cannot be less than zero. The length is $3x = 15$ centimeters, while the width is $2x = 10$ centimeters.

Finally, the perimeter of a rectangle is twice the length, plus twice the width:

Perimeter = 2 × length + 2 × width
Perimeter = 2 × 10 cm + 2 × 15 cm
Perimeter = 20 cm + 30 cm
Perimeter = 50 cm

39. **(B).** One way to approach this problem is to pick a smart number for the number of students, which shows up in both ratios. In the first ratio, students are represented by 8, so you want the smart number of students to be a multiple of 8. Likewise, in the second ratio, students are represented by 5, so you want the smart number of students to be a multiple of 5 as well. So pick 40 for the number of students.

From here, solve for the number of professors.

$$\frac{\text{Students}}{\text{Professors}} = \frac{40}{\text{Professors}} = \frac{8}{1}$$

$40 = 8 \times \text{Professors}$
$5 = \text{Professors}$

Likewise, solve for the number of administrators.

$$\frac{\text{Students}}{\text{Administrators}} = \frac{40}{\text{Administrators}} = \frac{5}{2}$$

$40 \times 2 = 5 \times \text{Administrators}$
$16 = \text{Administrators}$

Therefore, the ratio of professors to administrators is 5 : 16. In fractional ratio form, this is $\frac{5}{16}$. Comparing the two quantities, both have the same numerator, but the denominator of Quantity A is greater, making it the smaller value. In fact, Quantity B is exactly twice as great as Quantity A.

40. **(A).** To determine the number of pounds of leftover pasta, find the amount of pasta actually served to the guests who actually attended, and subtract from the total amount of pasta originally prepared. Originally, Mary prepared x pounds of pasta for y people. Create a ratio of pasta to people: each person was supposed to receive $\frac{x}{y}$ pounds of pasta. Since z people attended, with the same $\frac{x}{y}$ portion for each, you can write this equation:

Total Pasta Served = (z guests)($\frac{x}{y}$ pounds per guest) = $\frac{zx}{y}$ pounds of pasta served

Finally, compute the excess. Extra Pasta = Original Amount Prepared – Amount Served $= x - \frac{zx}{y}$

No answer choice matches this expression, so use a common denominator to combine terms and then reduce:

$$\text{Extra Pasta} = \frac{xy}{y} - \frac{zx}{y} = \frac{xy - zx}{y} = \frac{x(y-z)}{y}$$

41. **(B).** Let x be the number of hammers that Sara purchased from the store. Since there were 10 more wrenches than hammers, she received $x + 10$ wrenches. The ratio of wrenches to hammers is 5 : 4, so you can write a proportion:

$$\frac{\text{Wrenches}}{\text{Hammers}} = \frac{x+10}{x} = \frac{5}{4}$$

Cross multiply and solve for x:

$(4)(x + 10) = 5x$
$4x + 40 = 5x$
$x = 40$

Quantity A is 40, which is less than 50.

Quantity B is greater.

42. **(A).** To make the comparison, you must determine the time it takes for the family to drive both to their vacation destination and back home. Recall the formula for rates problems:

Distance = Rate × Time. This can be rearranged to = $\frac{\text{Distance}}{\text{Rate}}$.

The drive is 100 miles each way, so half the distance is 50 miles. The first half of the drive to

the vacation destination was covered at 50 miles per hour:

$$\frac{\text{Distance}}{\text{Rate}} = \frac{50 \text{ miles}}{50 \text{ miles per hour}} = 1 \text{ hour}$$

The second half of the drive was covered at 20 miles per hour:

$$\frac{\text{Distance}}{\text{Rate}} = \frac{50 \text{ miles}}{20 \text{ miles per hour}} = 2.5 \text{ hours}$$

Thus, the total time for the drive to the vacation destination is 1 hour + 2.5 hours = 3.5 hours. Quantity A is 3.5.

The time for the drive home is $\frac{\text{Distance}}{\text{Rate}} = \frac{100 \text{ miles}}{30 \text{ miles per hour}} = 3.333...$ hours. Quantity B is $3.\overline{3}$.

Thus, Quantity A is greater.

43. **(C).** In order to solve this rates problem, remember the formula for work: Work = Rate × Time.

The total work is 8 gallons (a fully filled bucket). The hose is working against the leak, so the effective total rate will be the difference of the two rates:

Effective rate of filling = 3 gallons per minute in – 1 gallon per minute out = 2 gallons per minute

Therefore, by Work = Rate × Time:

$$\frac{8 \text{ gallons}}{2 \text{ gallons per minute}} = \text{Time to fill}$$

Time to fill = 4 minutes

44. **(D).** To solve this problem, convert Dan's rate from widgets per second to widgets per hour. Conceptually, there are two steps: first convert seconds to minutes, then convert minutes to hours. The fast way to do this two-step conversion is to multiply the rate by the right conversion factors, which express identities (such as 60 minutes = 1

hour) in the form of ratios: $\frac{60 \text{ minutes}}{1 \text{ hour}}$ or $\frac{1 \text{ hour}}{60 \text{ minutes}}$. If you make sure that the units cancel correctly, then you can always be sure under pressure whether to multiply or divide by 60.

Here is the conversion, done all in one line:

$$\left(\frac{10 \text{ widgets}}{15 \text{ seconds}}\right)\left(\frac{60 \text{ seconds}}{1 \text{ minute}}\right)\left(\frac{60 \text{ minutes}}{1 \text{ hour}}\right) = \frac{2{,}400 \text{ widgets}}{1 \text{ hour}}$$

Notice that seconds and minutes both cancel on the left. In 1 hour, Dan can make 2,400 widgets.

45. **(E).** You want to convert an amount of money in "lemuws" to "rubels." Conceptually, there are two steps: first convert lemuws to schillings, then convert schillings to rubels. The fast way to do this two-step conversion is to multiply the money by the right conversion factors, which express identities (such as 8 rubels = 1 schilling) in the form of $\frac{8 \text{ rubels}}{1 \text{ schilling}}$ or $\frac{1 \text{ schilling}}{8 \text{ rubels}}$. If you make sure that the units cancel correctly, then you can always be sure under pressure whether to multiply or divide by 8.

Here is the conversion, done all in one line:

$$(6 \text{ lemuws})\left(\frac{5 \text{ schillings}}{1 \text{ lemuw}}\right)\left(\frac{8 \text{ rubels}}{1 \text{ schilling}}\right) = 240 \text{ rubels}$$

Both lemuws and schillings cancel on the left, leaving rubels. 6 lemuws are worth 240 rubels.

46. **(C).** To compute the time it takes Kim to paint the house alone, use an unknown multiplier. Since the ratio of Kim's time to Jane's time is 3 : 5, Kim's time can be written as $3x$, while Jane's time is $5x$. The work in both cases is 1 house. Since Rate = $\frac{\text{Work}}{\text{Time}}$, you can write Kim's rate as $\frac{1}{3x}$ and Jane's rate as $\frac{1}{5x}$.

To find the speed at which the two people work together, add their rates. This combined rate must equal the rate at which they paint 1 house. Since it takes them 10 hours working together to paint a house, the combined rate is $\frac{1}{10}$.

Kim's rate + Jane's rate = Combined rate

$$\frac{1}{3x} + \frac{1}{5x} = \frac{1}{10}$$

Solve for x by finding a common denominator on the left ($15x$) and adding:

$$\frac{5}{15x}+\frac{3}{15x}=\frac{1}{10}$$

$$\frac{8}{15x}=\frac{1}{10}$$

$$80=15x$$

$$16=3x$$

$$\frac{16}{3}=x$$

Finally, Kim's time is not *x* but $3x=(3)\left(\frac{16}{3}\right)$. The two quantities are equal.

47. **(B).** To solve this ratios problem, choose smart numbers for the money collected for each team and the number of students on each team. Choose multiples of the ratios given, such as the following:

Money collected by Team A = \$10
Money collected by Team B = \$12
Number of students in Team A = 2
Number of students in Team B = 3

Then compute the money per student:

Money per student in Team A = \$10 / 2 = \$5 per student
Money per student in Team B = \$12 / 3 = \$4 per student

Thus, the ratio of money per student in Team A to money per student in Team B is 5 : 4.

Alternatively, you could solve this problem algebraically by creating unknown multipliers that must eventually cancel, but this method is more work.

As a shortcut, you could express each ratio as a fraction, then divide the fractions:

$$\frac{\text{Ratio of money collected}}{\text{Ratio of students}}=\frac{5/6}{2/3}=\frac{5}{6}\times\frac{3}{2}=\frac{5}{4}$$, which is 5:4.

48. **(B).** If ketchup, soy sauce, and mayonnaise are mixed together in a ratio of 3 : 2 : 5, then there are 2 parts of soy sauce for every 3 + 2 + 5 = 10 parts total. In other words, soy sauce comprises $\frac{2}{10}$ of the total mixture. Thus, if the

total mixture volume is 25 ounces, $\frac{2}{10}\times 25=5$ ounces of that is soy sauce.

49. **(B).** Begin by computing the time spent running from point A to point B. Define the variable x as the time Saul spent running from B to A. The trip from A to B took $\frac{3}{4}$ as much time, so the time spent from A to B is $\left(\frac{3}{4}\right)x$. The total time was 63 minutes:

$$63=\left(\frac{3}{4}\right)x+x$$

$$63=\left(\frac{7}{4}\right)x$$

$$63\left(\frac{4}{7}\right)=x$$

$$9\times 4=x$$

$$x=36 \text{ minutes}$$

The time for the trip from A to B was $\left(\frac{3}{4}\right)x=\left(\frac{3}{4}\right)(36)=27$ minutes

Notice that you never need the typical rate equation (Distance = Rate × Time), as the Part : Part : ratio given for running times and the Whole trip time were enough to solve for time directly.

Quantity B is greater.

50. **(C).** To find how much vinegar Jarod needs, think about how many multiples of his original recipe Jarod wants to make. The original recipe makes 2 cups of sushi rice, so 7 cups of rice is $\frac{7}{2}$ times his original recipe.

Since Jarod is scaling proportionally, to make $\frac{7}{2}$ times the usual amount of rice, he must also use $\frac{7}{2}$ times as much vinegar. Therefore, Joe must use:

$$\left(\frac{7}{2}\right)\left(\frac{2}{3}\text{ ounces}\right)=\frac{7}{3}$$

Alternatively, you can start with 7 cups of rice and multiply by the recipe's ratio of vinegar to rice, cancelling cups of rice and producing ounces of vinegar:

$$(7 \text{ cups of rice})\left(\frac{\frac{2}{3} \text{ ounces of vinegar}}{2 \text{ cups of rice}}\right) = \frac{7}{3} \text{ ounces of vinegar}$$

51. **(A).** You are looking for the average rate for the entire trip. Use Distance = Rate × Time, rearranging to get $\frac{\text{Total Distance}}{\text{Total Time}}$.

Define the distance from Shelbyville to Bakersfield as y. This way, the distance from Springfield to Shelbyville, which is twice as far, is $2y$. (Notice that y will have to vanish by the end of the problem.)

Now compute the time from Springfield to Shelbyville, given that the rate of travel is x miles per hour.

Time from Springfield to Shelbyville = $\frac{\text{Distance}}{\text{Rate}} = \frac{2y}{x}$

Similarly, compute the time from Shelbyville to Bakersfield:

Time from Shelbyville to Bakersfield = $\frac{\text{Distance}}{\text{Rate}} = \frac{y}{1.5x}$

Thus, the total travel time is the sum of the two times: Total Time = $\frac{2y}{x} + \frac{y}{1.5x}$

Using a common denominator ($3x$), add the two fractions: Total Time = $\frac{6y}{3x} + \frac{2y}{3x} = \frac{8y}{3x}$

Compute the total distance, which equals $2y + y = 3y$.

Now you can figure out the average rate:

Average Rate = $\frac{\text{Total Distance}}{\text{Total Time}} = \frac{3y}{8y/3x} = \frac{9xy}{8y} = \frac{9}{8}x$

Alternatively, since the variable x appears in the answer choices, you could pick numbers. However, because of the complexity of this problem, it is difficult to pick numbers that both fit the constraints and don't yield messy fractions.

52. **(B).** Solve for the cost of a papaya by translating the information given into mathematical statements. The first sentence tells you that 3 bananas, 2 apples, and 1 mango cost \$3.50. Letting B represent the cost of a banana, A the cost of an apple, and M the cost of a mango, write

$3B + 2A + M = \$3.50$

Similarly, for the second sentence write

$3B + 2A + P = \$4.20$ (where P is the cost of a papaya)

The problem asks for the cost of a papaya and provides the ratio of the costs of a mango and papaya. To use this information, you must remove bananas and apples from the list of unknowns. Here's how: try elimination. Specifically, subtract the first equation from the second:

$$\begin{array}{l} 3B + 2A + P = \$4.20 \\ \underline{-\ (3B + 2A + M = \$3.50)} \\ P - M = \$0.70 \end{array}$$

Now, since the ratio of the cost of a mango to a papaya is 3 : 5, write a proportion:

$\frac{M}{P} = \frac{3}{5}$, which becomes $M = \frac{3}{5}P$ if you isolate M. Now substitute back into the equation above, to eliminate M and solve for P:

$$P - M = \$0.70$$

$$P - \frac{3}{5}P = \$0.70$$

$$\frac{5}{5}P - \frac{3}{5}P = \$0.70$$

$$\frac{2}{5}P = \$0.70$$

$$P = \frac{5}{2}(\$0.70) = \$1.75$$

Quantity B is greater.

53. **(C).** To solve for the number of unemployed females, first compute the total number of people who are unemployed. You need to represent the total number of people in the town. Call this number x. Since 2/5 of the town is employed, a total of 40,000 people, write the ratio

$$\frac{\text{Employed}}{\text{Total population}} = \frac{40{,}000}{x} = \frac{2}{5}$$

Cross multiply and solve for x:

$5(40{,}000) = 2x$
$200{,}000 = 2x$
$100{,}000 = x$

40,000 people in the town are employed, so 100,000 - 40,000 = 60,000 people are unemployed.

Finally, the ratio of unemployed males to females is 5 : 7. In other words, out of every 5 + 7 = 12 unemployed people, there are 7 unemployed females. Therefore, the fraction of unemployed females in the total unemployed population is 7 out of 12, or 7/12. Setting y as the number of unemployed females, write

$$\frac{\text{Unemployed females}}{\text{Total unemployed}} = \frac{y}{60{,}000} = \frac{7}{12}$$

Solve for y: $y = \dfrac{7 \times 60{,}000}{12} = 7 \times 5{,}000 = 35{,}000$ unemployed females.

54. **(C).** Reduce the ratio in Quantity A to lowest form. To do so, convert both numbers to improper fractions first:

$$2\frac{11}{12} = 2 + \frac{11}{12} = \frac{2 \times 12}{12} + \frac{11}{12} = \frac{2 \times 12 + 11}{12} = \frac{24 + 11}{12} = \frac{35}{12}$$
$$1\frac{3}{4} = 1 + \frac{3}{4} = \frac{4}{4} + \frac{3}{4} = \frac{4 + 3}{4} = \frac{7}{4}$$

Now compute the ratio by dividing the two numbers and simplifying:

$$\text{Ratio} = \frac{2\frac{11}{12}}{1\frac{3}{4}} = \frac{35/12}{7/4} = \frac{35}{12} \times \frac{4}{7} = \frac{5}{3}$$

The two quantities are equal.

55. **(E).** According to the problem, 3/5 of an inch on the map is equivalent to 400 miles of actual distance. So you can set up a ratio of these two measurements to use as a conversion factor: $\dfrac{3/5 \text{ inch}}{400 \text{ miles}}$ or $\dfrac{400 \text{ miles}}{3/5 \text{ inch}}$. Which one you use depends on which way you're converting: from miles to inches, or vice versa.

You are told that Oklahoma City is separated from Detroit by approximately 3/2 inches on the map, and you are asked how many real miles, approximately, lie between the two cities. Since you want to go from inches to miles, multiply the given measurement (3/2 inches) by the conversion factor that will cancel out inches and give you miles:

$$\left(\frac{3}{2}\text{ inches}\right)\left(\frac{400\text{ miles}}{3/5\text{ inch}}\right)=\left(\frac{3}{2}\right)(400)\left(\frac{5}{3}\right)\text{ miles}=1{,}000\text{ miles}$$

56. **(C).** First, figure out how many boxes worth of cans the machine produced in the 2 hours that it was on. The first step is to find the number of cans produced in 2 hours. Use the formula Work = Rate × Time. 20 cans per hour is the rate, and 2 hours is the time:

Work = (20 cans per hour) × (2 hours) = 40 cans

Now, since there are 10 cans per box, compute the number of boxes:

Number of boxes = 40 cans × $\left(\frac{1\text{ box}}{10\text{ cans}}\right)$ = 4 boxes

So Maria must pack 4 whole boxes to accommodate all the cans that the machine had made.

One more time, use the formula Work = Rate × Time. Maria's rate is 3 boxes per hour, while the total work as 4 boxes. Rearrange and plug in:

$$\text{Time}=\frac{\text{Work}}{\text{Rate}}=\frac{4\text{ boxes}}{3\text{ boxes per hour}}=\frac{4}{3}\text{ hours}$$

Finally, convert from hours to minutes as the question demands.

$$\text{Time}=\frac{4}{3}\text{ hours}\times\left(\frac{60\text{ minutes}}{1\text{ hour}}\right)=80\text{ minutes}$$

57. **(B).** First, assign variables to unknown quantities. Let y be Joe's starting salary with Company X and z be the amount of money that Joe's pay is increased each year. Joe has seen 10 raises of z dollars each, so Joe has had a total raise since he started of $10z$ dollars.

Joe's pay now is his starting pay plus the ten raises, or $y + 10z$.

Also, the ratio of his pay now to his pay when he started is 5 : 2. Write this relationship as a proportion:

$$\frac{y+10z}{y}=\frac{5}{2}$$

Cross multiply and simplify:

$2(y + 10z) = 5y$
$2y + 20z = 5y$

$3y = 20z$

Finally, the problem asks for the ratio of Joe's yearly increase, z, to his starting pay, y. Rearrange the equation to put $\frac{z}{y}$ by itself:

$$\frac{z}{y} = \frac{3}{20}$$

58. **II and III only.** If Beth has 1/4 more money than Ari, their money is in a ratio of 5 : 4 (because 5 is 1/4 more than 4). Another way to see this result is with algebra:

$$B = A + \frac{1}{4}A = \frac{5}{4}A \text{, so } \frac{B}{A} = \frac{5}{4}.$$

As a result, for every \$9 total, Beth has \$5 and Ari has \$4. To keep both Ari and Beth in integer dollar values, the answer needs to be a multiple of 9. Among the answer choices, only 54 and 72 are multiples of 9.

59. **I and IV only.** Since salesperson A sold 35% more motorcycles than salesperson B, their sales are in a ratio of 135 : 100. You can reduce this ratio to 27 : 20 by cancelling a common factor of 5.

As a result, for every 47 motorcycles sold, salesperson A sold 27 and salesperson B sold 20. The number of motorcycles sold must be integer multiples of these numbers (because you can't sell partial motorcycles — not legally anyway), so the total needs to be a multiple of 47. Among the answer choices, only 47 and 235 are multiples of 47.

60. **II, IV, and V only.** First, figure out which animal there are fewest of. "Twice as many zebras as lions" means Zebras > Lions, and "four times as many monkeys as zebras" means Monkeys > Zebras. So lions are found at the zoo in smallest numbers. To make the calculation straightforward, pick 1 lion as a smart number to start with. Since there are twice as many zebras, there are 2 zebras. Finally, there are four times as many monkeys as zebras, so there are 4 × 2 = 8 monkeys. Putting all that together:

Lions : Zebras : Monkeys = 1 : 2 : 8

So, for every 11 animals (1 + 2 + 8), there are 1 lion, 2 zebras, and 8 monkeys. To preserve integer numbers of lions, zebras, and monkeys, the total number of animals could only be a multiple of 11. Among the answer choices, only 22, 55, and 121 fit the bill.

61. **(E).** To tackle this question, rewrite all these ridiculously named currencies in terms of just one currency, ideally a real currency. Use whatever real currency you like, but here's an example with dollars.

Say that 1 murb is worth \$1.

A galok is worth 40% more than a murb, or 40% more than \$1. A galok is worth \$1.40.

10 terble coins equal 1 galok, so 10 terble coins are worth a total of \$1.40. Each terble coin is worth \$0.14 or 14

cents.

6 barbar coins equal 1 murb, so 6 barbar coins equal \$1. Each barbar coin is worth $\$\frac{1}{6}$ or $\frac{100}{6}$ cents.

The ratio of the value of 1 terble coin to the value of 1 barbar coin:

$$\frac{1 \text{ terble}}{1 \text{ barbar}} = \frac{14 \text{ cents}}{100/6 \text{ cents}} = 14 \times \frac{6}{100} = \frac{21}{25}$$

62. **(A).** Manipulate the given ratios to create one ratio that includes all three colors. You might use a table:

R	B	O
1	2	
6		1

The problem here is the red car: that column contains both a 1 and a 6. In order to fix this issue, create a common term. Multiply the entire first ratio (the first row) by 6:

R	B	O
6	12	
6		1

Now that the same number is in both rows of the red column, you can combine the two rows into a single ratio:

R : B : O = 6 : 12 : 1

For every 19 cars (6 + 12 + 1), there are 6 red cars, 12 blue cars, and 1 orange car. To maintain whole numbers of cars in each color, the correct answer has to be a multiple of 19. Only 38 is a multiple of 19.

63. **II and IV only.** Since 70% of the clients were originally male, the other 30% were female. So the ratio of men to women was 7 : 3. In other words, Bob's Bistro had $7x$ male and $3x$ female clients, for a total of $10x$ clients. Notice that x must be an integer. Thus, the original number of clients MUST have been a multiple of 10.

After z women leave, the total number of people is equal to $10x - z$, which is given as 74. Since $10x$ is a multiple of 10, it ends in 0. What has to be true about z, so that when you subtract it from a number ending in 0, you get 74? The restriction is that the units digit has to be 6. For instance, $10x$ could be 80, and $z = 6$ to make $10x - z = 74$. Or $10x$ could be 90, in which case z would be 16.

Answer choices 6 and 16 work, as proven above. No other choices have 6 as their units digit.

64. **(C).** The ratio of b to c is 2 : 1, while the ratio of g to c is 3 : 1. Since the variable common to both ratios (c) has

the same number (1) in both ratios, you can just combine to make a three-part ratio:

$$b : c : g = 2 : 1 : 3$$

Put in an unknown multiplier: $b = 2x$, while $c = x$ and $g = 3x$.

Thus, $\frac{b+c}{g} = \frac{2x+x}{3x} = \frac{3x}{3x} = 1$ The two quantities are equal.

Chapter 21
of

Averages, Weighted Averages, Median, and Mode

In This Chapter…

Averages, Weighted Averages, Median, and Mode

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Husain and Dino have an average of \$20 each. Dino wins a cash prize, which raises their average to \$80. Assuming no other changes occurred, how many dollars did Dino win?

\$

2.

Janani is 6 centimeters taller than Preeti, who is 10 centimeters taller than Rey.

Quantity A	**Quantity B**
The average height of the three people	The median height of the three people

3. The average of Joelle's five quiz scores is 88. What score does Joelle need to get on a sixth quiz to raise her average for all six quizzes to 90?

(A) 88

(B) 94
(C) 98
(D) 100
(E) 102

4.

The average of x and y is 55. The average of y and z is 75.

Quantity A	**Quantity B**
$z - x$	40

5. In Clarice’s class, each test weights her overall grade average three times as much as each quiz does. If Clarice scored 88 and 94 on two quizzes, respectively, and she scored 90 on the only test, what is her current overall grade average?

6. What is the average of x, x - 6, and x + 12?

(A) x
(B) $x + 2$
(C) $x + 9$
(D) $3x + 6$
(E) It cannot be determined from the information given.

7. The average of four numbers is 12. If the set of numbers includes 9, 11, and 12, what is the fourth number?

(A) 12
(B) 14
(C) 16
(D) 20
(E) 24

8.

For a set of 30 integers, the average is 30 and none of the integers are greater than 60.

Quantity A	**Quantity B**
The range of the set	30

9. If x is negative, what is the median of the list 20, x, 7, 11, 3?

(A) 3
(B) 7
(C) 9
(D) 11
(E) 15.5

10. If the average of n and 11 is equal to $2n$, then what is the average of n and $\frac{13}{3}$?

(A) 4
(B) 8
(C) 11
(D) 14
(E) 19

11.

Quantity A	**Quantity B**
The average of $x - 3, x, x + 3, x + 4$, and $x + 11$	The median of $x - 3, x, x + 3, x + 4$, and $x + 11$

12. John buys 5 books with an average price of \$12. If John then buys another book with a price of \$18, what is the average price of the 6 books?

(A) \$12.50
(B) \$13
(C) \$13.50
(D) \$14
(E) \$15

13. Every week, Renee is paid 40 dollars per hour for the first 40 hours she works, and 80 dollars per hour for each hour she works after the first 40 hours. How many hours would Renee have to work in one week to earn an average of 60 dollars per hour that week?

(A) 60
(B) 65
(C) 70
(D) 75
(E) 80

14.

At a certain school, the 118 juniors have an average final exam score of 88 and the 100 seniors have an average final exam score of 92.

Quantity A	**Quantity B**
The average final exam score for all of the juniors and seniors combined.	90

15. Last year a car dealership sold 640 cars over the entire year. This year, the dealership has sold an average of 32 cars per month for the first four months. What is the average number of cars sold per month over the entire 16-month period?

(A) 43
(B) 44
(C) 48
(D) 51
(E) 64

16.

Quantity A	Quantity B
The average (arithmetic mean) of x, y, and z	The average (arithmetic mean) of $0.5x$, $0.5y$, and $0.5z$

17. Balpreet's quiz scores in English are 80, 82, 79 and 84. Her quiz scores in History are 90 and 71. What is the sum of the scores she would need to get on her next English quiz and her next History quiz to raise each class' quiz score average to 85?

(A) 109
(B) 192
(C) 194
(D) 198
(E) 218

18. Aaron's first three quiz scores were 75, 84, and 82. If his score on the fourth quiz reduced his average quiz score to 74, what was his score on the fourth quiz?

19. Paco's practice test scores are 650, 700, 630 and 640. What score on the 5th test would result in an average score of 660 for all 5 tests?

20. A quiz is scored from 0 to 110. JaeHa has 5 quiz scores: 90, 95, 88, 84, 92. What does the average on her next 2 quizzes need to be in order to bring her average for all 7 quizzes up to 95?

21.

The integer ages of the three children in the Chen family range from 2 to 13, and no two children are the same age.

Quantity A	Quantity B
The average age of all three children in the Chen family	10

22.

Four people have an average age of 18, and none of the people are older than 30.

Quantity B

Quantity A	
The range of the four people's ages	25

23.

Set A consists of 5 numbers, which have an average value of 43. Set B consists of 5 numbers.

Quantity A	**Quantity B**
The value of x if the average of x and the 5 numbers in Set A is 46	The average of Set B if the average of the 10 numbers in Sets A and B combined is 52

24. The average of 7 numbers is 12. The average of the 4 smallest numbers in this set is 8, while the average of the 4 greatest numbers in this set is 20. How much greater is the sum of the 3 greatest numbers than the sum of the 3 smallest numbers?

(A) 4
(B) 14
(C) 28
(D) 48
(E) 52

25. If the average of a, b, c, 5, and 6 is 6, what is the average of a, b, c, and 13?

(A) 8
(B) 8.5
(C) 9
(D) 9.5
(E) It cannot be determined from the information given.

26. The average (arithmetic mean) of 8 numbers is 42. One of the numbers is removed from the set, and the resulting average (arithmetic mean) of the remaining numbers is 40. What number was removed from the set?

(A) 26
(B) 28
(C) 50
(D) 54
(E) 56

27. The average of 13 numbers is 70. If the average of 10 of these numbers is 90, what is the average of the other 3 numbers?

(A) -130
(B) $\frac{10}{3}$
(C) 30
(D) 90
(E) 290

28. Town A has 6,000 citizens and an average (arithmetic mean) of 2 radios per citizen. Town B has 10,000 citizens and an average (arithmetic mean) of 4 radios per citizen. What is the average number of radios per citizen in both towns combined?

Give your answer as a fraction.

$$\frac{\boxed{}}{\boxed{}}$$

29.

Joe’s quiz scores are 80, 82, 78, 77, and 83. Dave’s quiz scores are 60, 90, and 80.

Quantity A	**Quantity B**
The score Joe would need on his next quiz to increase his overall average score to 82.	The score Dave would need on his next quiz to increase his overall average score to 82.

30. Fiber X Cereal is 55% fiber. Fiber Max Cereal is 70% fiber. Sheldon combines an amount of the two cereals in a single bowl of mixed cereal that is 65% fiber. If the bowl contains 12 ounces of cereal, how much of the cereal, in ounces, is Fiber X?

(A) 3
(B) 4
(C) 6
(D) 8
(E) 9

31. The average population in Town X was recorded as 22,455 during the years 2000–2010, inclusive. However, an error was later uncovered: the figure for 2009 was erroneously recorded as 22,478, but should have been correctly recorded as 22,500. What is the average population in Town X during the years 2000–2010, inclusive, once the error is corrected?

$\boxed{}$

Liquor Brand	**Percent Alcohol**
Delicate Flower	4%
Fine and Dandy	6%
Monster Smash	12%

32. Based on the chart above, which of the following statements must be true?

Indicate all such statements.

☐ A cocktail made with one part Fine and Dandy and two parts Monster Smash (and no other ingredients) will be more than 9% alcohol.

☐ A cocktail made with 11.5 grams each of all three liquors (and no other ingredients) will be more than

7% alcohol.

☐ A cocktail made with one part Delicate Flower, two parts Fine and Dandy, and one part alcohol-free mixer (and no other ingredients) will be more than 7% alcohol.

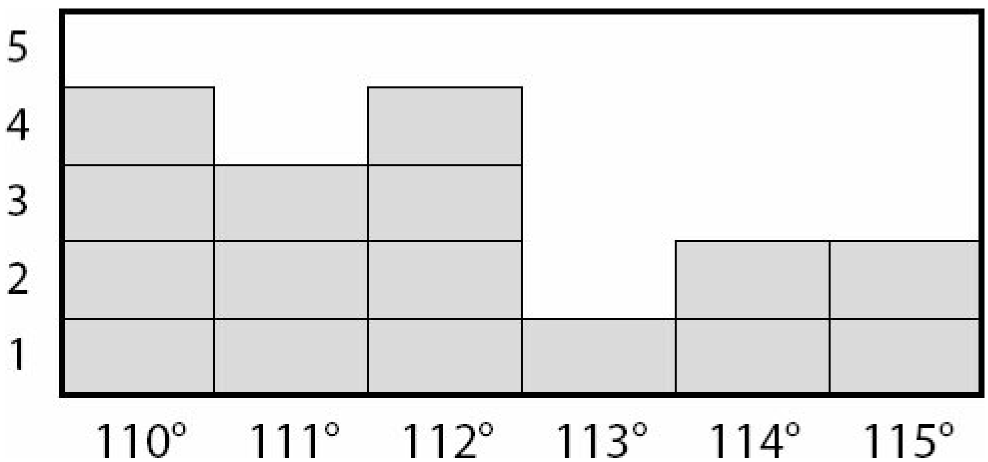
33.

$$S_n = 3n + 3$$

Sequence S is defined for all integers n such that $0 < n < 10{,}000$

Quantity A	**Quantity B**
The median of sequence S	The mean of sequence S

34. The bar graph below displays the number of temperature readings at each value from a sample, measured in degrees Fahrenheit. What was the average temperature reading?



degrees Fahrenheit

35. In a certain dance troupe, there are 55 women and 33 men. If all of the women are 62 inches tall and all of the men are 70 inches tall, what is the average height of the dancers in the troupe?

inches

36. Set A: 1, 3, 5, 7, 9
Set B: 6, 8, 10, 12, 14

For the sets of numbers above, which of the following statements are true?

Indicate all such statements.

☐ The mean of Set B is greater than the mean of Set A.
☐ The median of Set B is greater than the median of Set A.
☐ The standard deviation of Set B is greater than the standard deviation of Set A.
☐ The range of Set B is greater than the range of Set A.

37. Three people have \$32, \$72, and \$98, respectively. If they pool their money then redistribute it among them, what is the maximum value for the median amount of money?

(A) \$72
(B) \$85
(C) \$98
(D) \$101
(E) \$202

38.

Weekly Revenue Per Product Category at Office Supply Store *X*

Product Category	**Weekly Revenue in Dollars**
Pens	164
Pencils	111
Legal Pads	199
Erasers	38
Average of Categories above	128

According to the chart above, the average revenue per week per product category is \$128. However, there is an error in the chart; the revenue for Pens is actually \$176, not \$164. What is the new, correct average revenue per week per product category be, in dollars?

(A) 130
(B) 131
(C) 132
(D) 164
(E) 176

39.

A set of 7 integers has a range of 2, an average of 3, and a mode of 3.

Quantity A	**Quantity B**
The third number in the set when the numbers are arranged in ascending order	The fifth number in the set when the numbers are arranged in ascending order

40.

Set *S* consists of the first 500 positive, even multiples of 7.

Quantity A	**Quantity B**
The average of the set	The median of the set

41.

The average of $3x$, x, and y is equal to $2x$

Quantity A	**Quantity B**
$2x$	y

42. The average age of the buildings on a certain city block is greater than 40 years old. If four of the buildings were built two years ago and none of the buildings are more than 80 years old, which of the following could be the number of buildings on the block?

Indicate all such numbers.

☐ 4
☐ 6
☐ 8
☐ 11
☐ 40

43. Four students contributed to a charity drive, and the average amounts contributed by each student was \$20. If no student gave more than \$25, what is the minimum amount that any student could have contributed?

\$ ☐

44.

The average of 7 distinct integers is 12, and the least of these integers is -15.

Quantity A	**Quantity B**
The greatest that any of the integers could be	84

45.

Set N consists of the first 9 positive multiples of 3

Quantity A	**Quantity B**
The average of the first and last terms in the set	The average of the third and seventh terms in the set

46. The average of 15 consecutive integers is 88. What is the greatest of these integers?

☐

47.

The average of 5 integers is 10 and the range of the 5 integers is 10.

Quantity A	**Quantity B**
The median of the 5 integers	10

48.

3 numbers have a range of 2 and a median of 4.4

Quantity A	**Quantity B**
The greatest of the numbers	5.4

Averages, Weighted Averages, Median, and Mode Answers

1. **$120.** If the two people had an average of \$20 each, they held a sum of 2(\$20) = \$40. After Dino wins a cash prize, the new sum is $40 + p$ and the new average is 80. Plug into the average formula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$80 = \frac{40 + p}{2}$$

$$160 = 40 + p$$

$$120 = p$$

Dino won \$120.

2. **(B).** Pick numbers that agree with the given height constraints. Rey is the shortest person, and if Rey is 100 cm tall, Preeti is 110 cm tall, and Janani is 116 cm tall. The average height is $\frac{100 + 110 + 116}{3} = 108.67$ (rounded to nearest 0.01). The median height is the middle height, which is 110. Quantity B is greater.

Alternatively, note that Preeti's height is the median. Preeti's height is closer to Janani's than to Rey's. Since the average of Janani's and Rey's heights would be midway between those heights, and Preeti's height is higher than that middle, the median is greater than the average.

3. **(D).** There are two ways to solve this question. The first involves using the average formula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

If the average of Joelle's 5 quiz scores is 88, plug these numbers in and solve for the sum of the scores:

$$88 = \frac{\text{Sum}}{5}$$

$$88 \times 5 = \text{Sum}$$

$$440 = \text{Sum}$$

Use the average formula again to solve for the sixth quiz score, x, keeping in mind that the new average is 90 and the new number of quizzes is 6:

$$90 = \frac{440 + x}{6}$$

$90 \times 6 = 440 + x$
$540 = 440 + x$
$x = 100$

The other way to solve the question is by using the concept of residuals, or differences from the average. If 5 scores of 88 need to be brought up to the new average of 90, there are 5(2) = 10 points needed in the sum. So the sixth quiz score should be high enough to "give away" those 10 points to the sum, while still retaining 90 points for itself. That number is 100.

4. **(C).** Since the average of x and y is 55, $\frac{x+y}{2} = 55$, and $y + z = 150$.

Since the average of y and z is 75, $\frac{y+z}{2} = 75$, and $y + z = 150$.

Stack the two equations and subtract to cancel the y's and get $z - x$ directly:

$z + y = 150$
$\underline{-(x + y = 110)}$
$z - x = 40$

5. **90.4.** To account for the fact that tests weight the grade three times as much as quizzes, include each test score as if it were three identical quizzes. So, 2 quizzes and 1 test = 2 + 3 = 5 quizzes.

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$
Average = (88 + 94 + 90 + 90 + 90)/5
Average = 452/5 = 90.4

6. **(B).** The average formula is just as easily applied to algebraic expressions as arithmetic ones:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$
$$\text{Average} = \frac{(x)+(x-6)+(x+12)}{3}$$
$$\text{Average} = \frac{3x+6}{3} = x+2$$

The correct answer is (B).

7. **(C).** There are two ways to solve this question. The first involves using the average formula, plugging in 12 for the average and 4 for the number of terms.

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$12 = \frac{\text{Sum}}{4}$$

$$48 = \text{Sum}$$

The sum of the three known terms is 9 + 11 + 12 = 32. If the total of all four numbers must be 48, then the missing fourth number is 48 - 32 = 16.

The other way to solve uses the concept of residuals, or differences from the average. The average of 12 is the "balance point" of the four numbers. The 9 is -3 from this balance point on the number line, 11 is -1 from this balance point, and 12 is on this balance point, because it's equal to the average. Thus, the three known terms are weighted -3 + (-1) = -4 from the average, so the fourth terms needs to be 12 + 4 = 16 in order for the set to balance.

8. **(D).** There are infinite possibilities for a set of 30 integers less than or equal to 60, with an average of 30. For instance:

Example 1: The set consists of fifteen 0's and fifteen 60's
In this example, the range is 60 and the average is 30.

Example 2: The set consists of fifteen 14's and fifteen 16's.
In this example, the range is 2 and the average is 30.

The range could be greater or less than 30.

9. **(B).** The easiest way to start thinking about a question like this is to plug in a value and see what happens. If $x = -1$, the list looks like this when ordered from least to greatest:

-1, 3, 7, 11, 20

The median is 7. Because any negative x you pick will be the least term in the list, the order of the list won't change, so the median will always be 7.

10. **(A).** This question can be quickly solved with the average formula:

$$\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$$

$$2n = \frac{n+11}{2}$$

$$4n = n + 11$$

$$3n = 11$$

$$n = \frac{11}{3}$$

Since $n = 11/3$, the average of n and $\frac{13}{3}$ is:

$$\frac{\frac{11}{3}+\frac{13}{3}}{2}=\frac{\frac{24}{3}}{2}=\frac{8}{2}=4$$

Or just notice that the midpoint between $\frac{11}{3}$ and $\frac{13}{3}$ is $\frac{12}{3}$, just as 12 is the midpoint between 11 and 13. The average is $\frac{12}{3}=4$.

11. **(C).** To find the median of the numbers, notice that they are already in order from least to greatest: $x - 3$, x, $x + 3$, $x + 4$, $x + 11$

The median is the middle, or third, term: $x + 3$.

Now find the average of the numbers:

$$\frac{(x-3)+(x)+(x+3)+(x+4)+(x+11)}{5}=\frac{5x+15}{5}=x+3$$

The median and the mean are both $(x + 3)$.

12. **(B).** First, calculate the cost of the first 5 books.

Sum = (Average cost)(Number of books) = ($12)(5) = $60

Total cost of all 6 books = $60 + $18 = $78
Total number of books = 6

Average = $78/6 = $13 per book.

13. **(E).** Let h = number of hours Renee would have to work. The average rate Renee gets paid is equal to the total wages earned divided by the total number of hours worked. Rene earns $40 per hour for the first 40 hours, so she makes 40 × 40 = $1,600 in the first 40 hours. She also earns $80 for every hour after 40 hours, for additional pay of $80($h$ - 40). The total number of hours worked is h.

$$\frac{1{,}600+80(h-40)}{h}=60$$

$1{,}600 + 80h - 3{,}200 = 60h$

Now isolate h:

$80h - 1{,}600 = 60h$
$-1{,}600 = -20h$
$80 = h$

You could also notice that 60 is exactly halfway between 40 and 80. Therefore, Renee needs to work an equal number of hours at $40 per hour and $80 per hour. If she works 40 hours at $40 per hour, she also needs to work 40 hours at $80 per hour.

14. **(B).** This is a weighted average problem. Because the number of juniors is greater than the number of seniors, the overall average will be closer to the juniors' average than the seniors' average. Since 90 is halfway between 88 and 92, and the weighted average will be closer to 88, Quantity B is larger.

It is not necessary to do the math because this is a Quantitative Comparison question with a very convenient number as Quantity B. However, you can actually calculate the overall average by summing up all 218 scores and dividing by the number of people: $\frac{118(88)+100(92)}{118+100} = 89.83...$

15. **(C).** The dealership sold 640 cars last year. This year, the dealership has sold 32 cars per month for the first 4 months of this year, which is a total of 4(32) = 128 cars. Now, if you're thinking that there's a difference between selling an average of 32 cars per month and selling exactly 32 cars per month, you're right. However, it won't make a difference to the total sum of cars sold over the whole period, which is all that is needed to calculate the average.

Over the entire 16-month period, the dealership sold 640 + 128 = 768. Now use the average formula to calculate the answer:

Average number of cars sold per month = $\frac{\text{Sum (= \# of cars sold)}}{\text{Number of Terms (= \# of months)}}$

Average number of cars sold per month = $\frac{\text{768 total cars}}{\text{16 months}}$ = 48 cars/month

16. **(D).** The average of x, y, and z is $\frac{x+y+z}{3}$. Calculated similarly, the average of $0.5x$, $0.5y$, and $0.5z$ is exactly half that. If the sum of the variables is positive, Quantity A is greater. However, if the sum of the variables is negative, Quantity B is greater. If the sum of the variables is zero, the two quantities are equal.

17. **(C).** For Balpreet to raise her English average to 85:

$\frac{80+82+79+84+x}{5} = 85$
$325 + x = 425$
$x = 100 +$

For Balpreet to raise her history average to 85:

$$\frac{90+71+y}{3} = 85$$

$161 + y = 255$
$y = 94$
$x + y = 100 + 94 = 194$

The correct answer is (C).

18. **55.** To find Aaron's fourth quiz score:

$$\frac{75+84+82+x}{4} = 74$$

$241 + x = 296$
$x = 55$

19. **680.** To find the score Paco would need on his 5th test:

$$\frac{650+700+630+640+x}{5} = 660$$

$2{,}620 + x = 3{,}300$
$x = 680$

20. **108.** To find the score JaeHa would need to average on her next 2 quizzes to bring her total average up to a 95:

$$\frac{90+95+88+84+92+x+y}{7} = 95$$

$449 + x + y = 665$
$x + y = 216$

Note that it is not necessary to determine x and y individually. Since the two new quiz scores sum to 216, their average is $\frac{216}{2} = 108$.

21. **(B).** Since there are only three children and the range is from 2 to 13, one child must be 2, one must be 13, and the other child's age must fall somewhere in the middle.

If the average of the children's ages were 10, as in Quantity B, the sum of the three ages would be 30. Subtract 2 and 13 from 30 to get that the third child would need to be 15. This is not possible, because the middle child cannot be older than the 13-year-old. Since this age is too great, the true average age must be less, and Quantity B is greater.

Alternatively, since no two children are the same age, the middle child has a maximum age of 12. If that child were 12, the average age would be $\frac{2+12+13}{3} = \frac{27}{3} = 9$. Thus, the true average age must be 9 or less, and Quantity B is greater.

22. **(D).** If 4 people have an average age of 18, then the sum of their ages is $4 \times 18 = 72$. Since the question is about range, try to minimize and maximize the range. Minimizing the range is easy — if everyone were exactly 18, the average age would be 18 and the range would be 0. So clearly, the range can be smaller than 25.

To maximize the range, make the oldest person the maximum age of 30, and see whether the youngest person could be just 1 year old while still obeying the other rules of the problem: the sum of the ages is 72 and, of course, no one can be a negative age.

One such set: 1, 20, 21, 30

This is just one example that would work. In this case, the range is 30 - 1 = 29, which is greater than 25.

The correct answer is (D).

23. **(C).** If the average of the 5 numbers in Set A is 43, the sum of Set A is $(5)(43) = 215$.

For Quantity A, use the Average Formula. Sum all 6 numbers, and divide by 6:

$$\frac{\text{sum of the 5 numbers in Set A} + x}{6} = 46$$

$$\frac{215 + x}{6} = 46$$

$$215 + x = 276$$

$$x = 61$$

For Quantity B, use the Average Formula again:

$$\frac{\text{sum of Set A} + \text{sum of Set B}}{10} = 52$$

$$215 + \text{sum of Set B} = 520$$

$$\text{sum of Set B} = 305$$

The average of the 5 numbers in Set B is thus $\frac{305}{5} = 61$.

Alternatively, you could note that each set of 5 numbers has the same "weight" in the average of all 10 numbers. The average of Set A is 43, which is 52 - 43 = 9 below the average of all 10 numbers. The average of Set B must be 9 above the average of all 10 numbers: 52 + 9 = 61.

24. **(D).** Using the average formula, $\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$, build three separate equations:

All 7 numbers:

$$12 = \frac{\text{Sum of all 7 numbers}}{7}$$

Sum of all 7 numbers = 84

The 4 smallest numbers:

$$8 = \frac{\text{Sum of the 4 smallest numbers}}{4}$$

Sum of the 4 smallest numbers = 32

The 4 greatest numbers:

$$20 = \frac{\text{Sum of the 4 greatest numbers}}{4}$$

Sum of the 4 greatest numbers = 80

There are only 7 numbers, yet information is given about the 4 smallest and the 4 greatest, which is a total of 8 numbers! The middle number has been counted twice—it is included in both the 4 greatest and the 4 smallest.

The sum of all 7 numbers is 84, but the sum of the 4 greatest and 4 smallest is 80 + 32 = 112. The difference can only be attributed to the double counting of the middle number in the set of 7: 112 - 84 = 28.

The middle number is 28, so subtract it from the sum of the 4 smallest numbers to get the sum of the 3 smallest numbers: 32 - 28 = 4.

Now subtract the middle number from the sum of the 4 greatest numbers to get the sum of the 3 greatest numbers: 80 - 28 = 52.

The difference between the sum of the 3 greatest numbers and the sum of the 3 smallest numbers is 52 - 4 = 48.

25. **(A).** Since $\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$:

$$6 = \frac{a+b+c+5+6}{5}$$

$30 = a + b + c + 11$
$19 = a + b + c$

It is not necessary, or possible, to determine the values of a, b, and c individually. The second average includes all three variables, so the values will be summed again anyway.

$$\text{Average} = \frac{a+b+c+13}{4}$$

$$\text{Average} = \frac{19+13}{4}$$

$$\text{Average} = \frac{32}{4} = 8$$

26. **(E).** There are two ways to solve this question. The first involves using the average formula: Average = $\frac{\text{Sum}}{\text{Number of Terms}}$ or Sum = Average × Number of Terms.

If the average of 8 numbers is 42, the sum of all 8 numbers = 42 × 8 = 336.

After removing one number, the new average of the remaining 7 numbers is 40. So, the sum of the remaining 7 numbers = 40 × 7 = 280.

The number that was removed accounts for the difference in these sums, so the number that was removed is 336 - 280 = 56.

The other way to solve the question is by using the concept of residuals, or differences from the average. A number was removed, and it caused the average of the remaining 7 numbers to drop by 2 (from 42 to 40). That requires a 7 × 2 = 14 point drop in the sum. The removed number must be 14 more than the preexisting average. That would be 42 + 14 = 56.

27. **(B).** $\text{Average} = \frac{\text{Sum}}{\text{Number of Terms}}$ or Sum = Average × Number of Terms.

The average of 13 numbers is 70, so:

Sum of all 13 terms = 70 × 13 = 910

The average of 10 of these numbers is 90, so:

Sum of 10 of these numbers = 90 × 10 = 900

Subtract to find the sum of "the other 3 numbers": 910 - 900 = 10

Average of the other 3 numbers $\frac{\text{Sum}}{\text{Number of Terms}} = \frac{10}{3}$.

28. $\frac{13}{4}$ **(or any equivalent fraction).** To find this weighted average, you must find the sum of all the radios in Towns A and B, and divide by the total number of people in both towns:

$$\text{Average} = \frac{6{,}000(2)+10{,}000(4)}{16{,}000}$$

Cancel three zeros from each term:

$$\text{Average} = \frac{6(2)+10(4)}{16}$$

$$\text{Average} = \frac{52}{16}$$

This reduces to $\frac{13}{4}$, although you are not required to reduce.

29. **(B).** Average = $\frac{\text{Sum}}{\text{Number of Terms}}$. Set the unknown final test score as x for Joe and y for Dave.

Quantity A:

$$82 = \frac{80+82+78+77+83+x}{6}$$

$492 = 400 + x$
$x = 92$

Quantity B:

$$82 = \frac{60+90+80+y}{4}$$

$328 = 230 + y$
$y = 98$

30. **(B).** Use the weighted average formula to get the ratio of Fiber X to Fiber Max:

$\frac{0.55x+0.70m}{x+m} = 0.65$, where x is the amount of Fiber X and m is the amount of Fiber Max.

This is not that different from the regular average formula—on the top, there is the total amount of fiber (55% of Fiber X and 70% of Fiber Max), which is divided by the total amount of cereal ($x + m$) to get the average. Simplify by multiplying both sides by ($x + m$):

$0.55x + 0.70m = 0.65(x + m)$
$0.55x + 0.70m = 0.65x + 0.65m$

If you wish, you can multiply both sides of the equation by 100 to eliminate all the decimals:

$55x + 70m = 65x + 65m$
$55x + 5m = 65x$
$5m = 10x$

$\frac{m}{x} = \frac{10}{5}$ or $\frac{2}{1}$

Since m and x are in a 2 to 1 ratio, 2/3 of the total is m and 1/3 of the total is x. Since the total is 12 ounces, Fiber X accounts for $\frac{1}{3}(12) = 4$ ounces of the mixed cereal.

One shortcut to this procedure is to note that the weighted average (65%) is 10% away from Fiber X's percent and 5% away from Fiber Max's percent. Since 10 is twice as much as 5, the ratio of the two cereals is 2 to 1. However, it is a 2 to 1 ratio of Fiber Max to Fiber X, not the reverse! Whichever number is closer to the weighted average (in this case, 70% is closer to 65%) gets the larger of the ratio parts. Since the ratio is 2 to 1 (Fiber Max to Fiber X), again, 1/3 of the cereal is Fiber X and $\frac{1}{3}(12) = 4$.

31. **22,457.** There is a simple shortcut for a change to an average. The figure for 2009 was recorded as 22,478, but actually should have been recorded as 22,500. Thus, 22 people in that year were not counted. Thus, the sum should have been 22 higher when the average was originally calculated.

2000–2010, inclusive, is 11 years (subtract low from high and then add 1 to count an inclusive list of consecutive numbers). When taking an average, you divide the sum by the number of things being averaged (in this case, 11). So the shortcut is to take the change to the sum and "spread it out" over all of the values being averaged by dividing the change by the number of things being averaged.

Divide 22 by 11 to get 2. The average should have been 2 higher. Thus, the correct average for the 11 year period is 22,457.

Alternatively, the traditional method: 22,455 × 11 years = 247,005, the sum of all 11 years' recorded populations. Add the 22 uncounted people: the corrected sum would be 247,027. Divide by 11 to get the real average: 22,457. (Note that while the traditional method is faster to explain, the shortcut is faster to actually execute!)

32. **I and II only.** This is a weighted average problem. Consider the statements individually:

I. TRUE. A cocktail made with one part Fine and Dandy and two parts Monster Smash will have a percent alcohol equal to the average of 6%, 12%, and 12%, or $\frac{6\% + 12\% + 12\%}{3} = \frac{30\%}{3} = 10\%$. (Count the Monster Smash twice, since the cocktail contains twice as much of it.)

II. TRUE. The 11.5 grams is irrelevant—all that matters is that equal amounts of each liquor were used, so there is no need to weight the average. Take an ordinary average of 4, 6, and 12 percent. The average is 7.333…%.

III. FALSE. To find the percent alcohol for a cocktail made with one part Delicate Flower (4% alcohol), two parts Fine and Dandy (6% alcohol) and one part mixer (0% alcohol), average 4, 6, 6, and 0. (Count the 6 twice since two parts Fine and Dandy went into the cocktail versus one part of each of the other components.) The average of 4, 6, 6, and 0 is 4. If you got this wrong, you may have ignored the 0% alcohol mixer. You cannot ignore the effect of a zero in an average—a zero can often lower an average considerably. Alternatively, note that none of the components of this cocktail have more than 6% alcohol. The resulting drink cannot have a greater concentration of alcohol than any of its components.

33. **(C).** Sequence S is an evenly spaced set, which can be seen by plugging in a few n values:

$S_1 = 3(1) + 3 = 6$
$S_2 = 3(2) + 3 = 9$
$S_3 = 3(3) + 3 = 12\ldots$

Terms increase by three every time n increases by 1; this meets the definition of an evenly spaced set. For ANY evenly spaced set, the median equals the mean.

34. **112.** This is a weighted average problem. You CANNOT simply average 110, 111, 112, 113, 114, and 115. You must take into account how many times each number appears. The chart is really another way of writing:

110, 110, 110, 110
111, 111, 111
112, 112, 112, 112
113
114, 114
115, 115

In other words, the average temperature reading is really an average of 16 numbers. The easiest way to do this is:

$$\frac{4(110)+3(111)+4(112)+1(113)+2(114)+2(115)}{16}$$

Use your calculator—the correct answer is 112.

35. **65.** This is a weighted average problem. You CANNOT simply average 62 and 70. You must take into account how many times each number appears (55 and 33 times, respectively). You are actually averaging 88 numbers:

$$\frac{55(62)+33(70)}{88}=$$

Use your calculator. The correct answer is 65.

36. **I and II only.** In both sets, the numbers are evenly spaced. Moreover, both sets are evenly spaced by the same amount (adjacent terms increase by 2) and have the same number of terms (5 numbers in each set). The difference is that each term in Set B is 5 greater than the corresponding term in Set A (i.e., 6 - 1 = 5, 8 - 3 = 5, etc.)

In evenly spaced sets, the mean = median. Also, if an evenly spaced set has an odd number of numbers, the mean and median both equal the middle number. (When a set has an even number of numbers, the mean and median both equal the average of the 2 middle numbers).

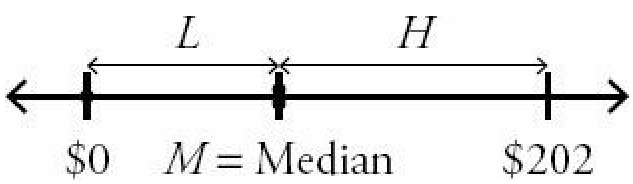
So, Set A has mean and median of 5 and Set B has mean/median = 10. Statement I and Statement II are true.

Since Sets A and B are equally spaced and have the same number of elements, their standard deviations are equal (that is, Set A is exactly as spread out from its own mean as Set B is from its own mean), so Statement III is false.

Since 9 - 1 = 8 and 14 - 6 = 8, the ranges are equal and Statement IV is false.

37. **(D).** The pool of money is \$32 + \$72 + \$98 = \$202. After the redistribution, each person will have an amount between \$0 and \$202, inclusive. Call the amounts *L*, *M*, and *H* (low, median, high). To maximize *M*, minimize *L* and *H*.

The minimum value for *H* is, in fact, *M*. The "highest" of the three values can actually be equal to the median (if *H* were lower than *M*, the term order and therefore which number is the median would change, but if *H* = *M*, *M* can still be the median).



Minimum *L* = \$0
Minimum *H* = *M*

Maximum *M* = Total pool of money - Minimum *L* - Minimum *H*
M = \$202 - \$0 - *M*
2*M* = \$202
M = \$101

The correct answer is (D).

38. **(B).** The chart provides the average and the number of product categories. If the incorrectly-calculated average was \$128 for the 4 categories, then the sum was 4 × 128 = \$512. Since the revenue for Pens was actually \$176, not \$164, the sum should have been \$12 higher. Thus, the correct sum is \$524. Divide by 4 to get \$131, the answer.

Alternatively, notice that the \$128 average given in the question stem actually does a lot of work for you. If \$164 jumps up to \$176, that's an increase of \$12. Distributed over the four categories, it will bring the overall average up by \$3, from \$128 to \$131.

39. **(C).** A set of 7 integers with a range of 2 and an average of 3 could consist of only these possibilities:

Example 1: 2, 2, 2, 3, 4, 4, 4

Example 2: 2, 2, 3, 3, 3, 4, 4

Example 3: 2, 3, 3, 3, 3, 3, 4

However, the mode of the set must be 3 for this set. The mode is the most common number in the set. Example 1 has two modes: 2 and 4, so Example 1 is invalid for this question. Only Examples 2 and 3 remain.

In both valid examples, the third and fifth numbers in each set are 3. The two quantities are equal.

40. **(C).** The set begins 14, 28, 42, etc. However, the specific numbers—and even the number of elements—in the set are irrelevant, since the average equals the median for any evenly spaced set. Consecutive multiples (in this case, of 14) are evenly spaced.

41. **(C).** Write "the average of 3x, x, and y is equal to 2x" as an equation:

$$\frac{3x + x + y}{3} = 2x$$

$$4x + y = 6x$$

$$y = 2x$$

The two quantities are equal.

42. **8, 11, 40.** Because Average = $\frac{\text{Sum}}{\text{Number of Terms}}$, this question about averages depends both on x, the total number of buildings on the block, and on the sum of the building ages. The 4 buildings that are 2 years old have a total age of 4(2), and the (x - 4) other buildings have a total age of (x - 4)(no more than 80).

$$\text{Average age} = \frac{4(2)+(x-4)(\text{no more than } 80)}{x}$$

Having many 80 year old buildings on the block would raise the average much closer to 80. (For instance, if there were a million 80-year-old buildings and four 2-year-old buildings, the average would be very close to 80 years old.) So, there is some minimum number of older buildings that could raise the average above 40.

Ignore the "greater than" 40 years old constraint on the average building age for a moment. What is the minimum x needed to be to make the average age exactly 40 when the age of the other buildings is maximized at 80?

$$40x = 8 + (x - 4)(80)$$

$$40x = 8 + 80x - 320$$

$$-40x = -312$$

$$x = \frac{312}{40} = 7.8$$

Because there can't be a partial building and the age of the buildings can't be greater than 80, x must be at least 8 to bring the average age up over 40. (You would need even more buildings to bring the average above 40 if those older buildings were only between 50 and 70 years old, for example.)

Alternatively, test the answer choices. Try the first choice, 4 buildings. Since 4 of the buildings on the block are only 2 years old, this choice can't work—the average age of the buildings would be 2.

Try the second choice. With 6 total buildings, there would be four 2-year-old buildings, plus two others. To maximize the average age, maximize the ages of the two other buildings by making them both 80 years old.

$$\frac{4(2)+2(80)}{6} = 28$$

Since the average is less than 40 years old, this choice is not correct.

Try the third choice. With 8 total buildings, there would be the four 2-year-old buildings, plus four others. To maximize the average age, maximize the ages of the four other buildings by making them each 80 years old:

$$\frac{4(2)+4(80)}{8} = 41$$

Since the average age is greater than 40 years old, this choice is correct. Since the other, greater choices allow the possibility of even more 80-year-old buildings, increasing the average age further, those choices are also correct.

43. **$5.** The average of four values is $20. Thus, the sum of the four values is $80. To determine the minimum contribution one student could have given, maximize the contributions of the other three students. If the three other students each gave the maximum of $25, the fourth student would only have to give $5 to make the sum equal to $80.

44. **(A).** If the average of 7 integers is 12, then their sum must be $7 \times 12 = 84$. To maximize the largest of the numbers, minimize the others.

The smallest number is -15. The integers are distinct (that is, different from each other), so the minimum values for the smallest 6 integers are -15, -14, -13, -12, -11, and -10. To find the maximum value for the 7th integer, sum -15, -14, -13, -12, -11, -10, and x, while setting that sum equal to 84:

$$-15 + (-14) + (-13) + (-12) + (-11) + (-10) + x = 84$$
$$-75 + x = 84$$
$$x = 159$$

Quantity A is greater.

45. **(C).** In an evenly spaced set, the middle number is also the average. Numbers equally spaced on opposite sides of the middle will also average to the average of the whole set. Thus, the answer is (C). Here is the set written out, with the median underlined:

3 6 9 12 <u>15</u> 18 21 24 27

In an evenly spaced set, the median is equal to the average. Thus, 15 is the average. It is also the case that 12 and 18 average to 15. So do 9 and 21. So do 6 and 24. And so do 3 and 27.

Both quantities are equal to 15.

46. **95.** In any evenly spaced set, the average equals the median. Thus, 88 is the middle number in the set. Since the set has 15 elements, the 8th element is the middle one.

Lowest 7 integers:	81	82	83	84	85	86	87
Middle integers:	88						
Greatest 7 integers:	89	90	91	92	93	94	95

The largest integer in the list is 95. If you were confident about the process, you could skip listing the integers. Instead you could reason that to go from 8th integer to the 15th integer, you must add 7: 88 + 7 = 95.

47. **(D).** If the average of 5 integers is 10, their sum must be 50. The range is given as 10. Try to make two examples where this is true, but where the medians are as different as possible.

Example 1: 5, 10, 10, 10, 15

In this case, Quantity A is equal to Quantity B.

Is there a list such that the average is still 10 and the range is still 10, but the median is something else? Try adjusting the three middle numbers while keeping 5 as the least integer and 15 as the greatest integer. To adjust the numbers without disturbing the average, anything you subtract from one number should be added to another number, so the sum stays constant.

Example 2: 5, 6, 12, 12, 15

Here, 4 was subtracted from the second term, and then 2 was added to each of the third and fourth terms. The average and range still each equal 10, but now the median is 12. The answer is (D).

48. **(D).** If the set has an odd number of terms, then the median is the middle number. So, the middle number is 4.4. The set has a range of 2. The other two numbers could be 2 apart and also equally distributed around 4.4:

Example 1: 3.4, 4.4, 5.4

Here, the two quantities are equal.

Or, the two other numbers could be 2 apart but both a bit higher, or both a bit lower.

Example 2: 4.3, 4.4, 6.3

Example 3: 2.5, 4.4, 4.5

Thus, Quantity A could be equal to, less than, or greater than Quantity B. The correct answer is (D).

Chapter 22
of

Standard Deviation and Normal Distribution

In This Chapter…

Standard Deviation and Normal Distribution

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. Set *S* {5, 10, 15}

If the number 15 were removed from Set *S* and replaced with the number 1,000, which of the following would change?

Indicate all such statements.

☐ The mean
☐ The median
☐ The standard deviation

2.

Set *W*: -9, -3, 3, 9
Set *X*: 2, 4, 6, 8
Set *Y*: 100, 101, 102, 103
Set *Z*: 7, 7, 7, 7

Which of the following choices lists the four sets above in order from smallest standard deviation to greatest standard deviation?

(A) W, X, Y, Z
(B) W, Y, X, Z
(C) W, X, Z, Y
(D) Z, Y, X, W
(E) Z, X, Y, W

3.

Set N is a set of x distinct positive integers where $x > 2$.

<u>Quantity A</u>	**<u>Quantity B</u>**
The standard deviation of Set N	The standard deviation of Set N if every number in the set is multiplied by -3

4. Set S is a set of distinct positive integers. The standard deviation of Set S must increase if which of the following were to occur?

Indicate <u>all</u> such statements.

☐ Each number in the set is multiplied by 1/2.
☐ The smallest number is increased to become equal to the median.
☐ The smallest number is increased to become larger than the current largest number.
☐ The largest number is doubled.

5. The 75th percentile on a test corresponded to a score of 700, while the 25th percentile corresponded to a score of 450.

<u>Quantity A</u>	**<u>Quantity B</u>**
800	A 95th percentile score

6. Set X consists of 9 total terms, but only two different terms. Six of the terms are each equal to twice the value of each of the remaining 3. Which of the following would provide sufficient additional information to determine the average of the set?

Indicate <u>all</u> such statements.

☐ The smaller number is positive and is 3 less than the larger number.
☐ The standard deviation of the set is equal to $2\sqrt{3}$.
☐ The biggest term in the set is 6.

7. Set $S = \{2, 5, 7, 11, 16, 24, 28, 50, 52, 101, 120, 130\}$

What is the average of the first quartile ("Q1") and the third quartile ("Q3") of set S?

(A) 9

(B) 26
(C) 42.75
(D) 76.5
(E) 85.5

8. The test scores at Millbrook High School are normally distributed, and the 60th percentile is equal to a score of 70.

Quantity A	**Quantity B**
The 30th percentile score	35

9. The lengths of a certain population of earthworms are normally distributed with a mean length of 30 centimeters and a standard deviation of 3 centimeters. One of the worms is picked at random.

Quantity A	**Quantity B**
The probability that the worm is between 24 and 30 centimeters, inclusive	The probability that the worm is between 27 and 33 centimeters, inclusive

10. The hourly wage paid to working adults in Maplewood is normally distributed around a mean of \$18 per hour with a standard deviation of \$3.50.

Quantity A	**Quantity B**
The percent of working adults in Maplewood who are paid between \$18 and \$25 per hour, inclusive	40%

11. Home values among the 8,000 homeowners of Town X are normally distributed, with a standard deviation of \$11,000 and a mean of \$90,000.

Quantity A	**Quantity B**
The number of homeowners in Town X whose home value is above \$112,000	300

12. Exam grades among the students in Ms. Harshman's class are normally distributed, and the 50th percentile is equal to a score of 77.

Quantity A	**Quantity B**
The number of students who scored less than 80 on the exam	The number of students who scored greater than 74 on the exam

13. The length of bolts made in factory *Z* is normally distributed, with a mean length of 0.1630 meters and a standard deviation of 0.0084 meters. The probability that a randomly selected bolt is between 0.1546 meters and 0.1756 meters long is between

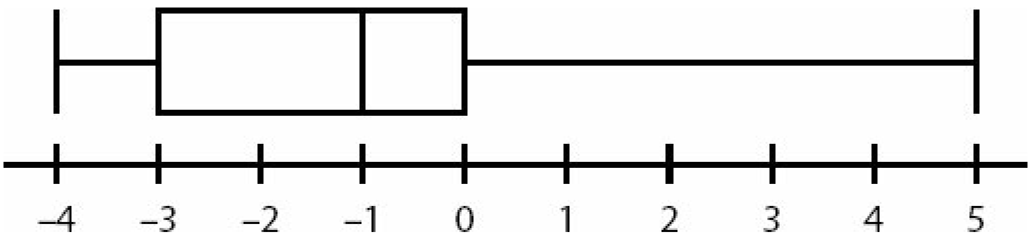
(A) 54% and 61%
(B) 61% and 68%
(C) 68% and 75%
(D) 75% and 82%
(E) 82% and 89%

14. Birth weight of babies born at City Hospital is normally distributed. A baby 2 standard deviations above the mean birth weight weighs 10.8 pounds, and a baby 1 standard deviation below the mean birth weight weighs 5.85

pounds.

Quantity A	**Quantity B**
Twice the weight of a baby 2 standard deviations below the mean	The weight of a baby 1 standard deviation above the mean

15. Which of the following sets of data applies to this graph?



(A) -4, -4, -2, 0, 0, 5
(B) -4, 1, 1, 3, 4, 4
(C) -4, -4, -3, 1, 5
(D) -5, 3, 4, 5
(E) -4, -4, -2, -2, 0, 0, 0, 5

16. If a set of data consists of only the first ten positive multiples of 5, what is the interquartile range of the set?

(A) 15
(B) 25
(C) 27.5
(D) 40
(E) 45

17. On a given math test out of 100 points, the vast majority of the 149 students in a class scored either a perfect score or a zero, with only one student scoring within 5 points of the mean. Which of the following logically follows about Set *T*, made up of the scores on the test?

Indicate <u>all</u> such statements.

☐ Set *T* will not be normally distributed.
☐ The range of Set *T* would be significantly smaller if the scores had been more evenly distributed.
☐ The mean of Set *T* will not equal the median.

18. If Set *X* is a normally distributed set of numbers with a mean of 4 and a standard deviation of 4, approximately what is the probability that a number chosen at random from the set will be negative?

(A) 1/10
(B) 1/6
(C) 1/4
(D) 1/3
(E) 1/2

19. Jane scored in the 68th percentile on a test, and John scored in the 32nd percentile.

Quantity A	**Quantity B**

Quantity A	Quantity B
The proportion of the class that received a score less than John's score	The proportion of the class that scored as high as or higher than Jane

20. If a set of data has a mean of 4.2 and a standard deviation of 7.1, what is the range of values that lie within 2 standard deviations of the mean?

(A) -2.9 to 11.3
(B) -2.9 to 12.6
(C) -10 to 12.6
(D) -10 to 18.4
(E) 4.2 to 18.4

21. If octiles divide up a set of data into 8 ordered groups, each with the same number of terms, what is the median of the sixth octile group of the set of data composed of the integers from 25 to 48, inclusive?

22. In a class with 20 students, a test was administered, scored only in whole numbers from 0 to 10. At least one student got every possible score, and the average was 7.

Quantity A	**Quantity B**
4	The lowest score that two students could have received

23.

Frequency	6	5	5
Result	4	6	8

Quantity A	**Quantity B**
The mode of this data set	5

24. A test is scored out of 100 and the scores are divided into five quintile groups. Students are not told their scores, but only their quintile group.

Quantity A	**Quantity B**
The scores of two students in the bottom quintile group, chosen at random and added together	The score of a student in the top quintile group, chosen at random

25. In a set of 10 million numbers, one percentile would represent what percent of the total number of terms?

(A) 1,000,000
(B) 100,000
(C) 10,000
(D) 100

(E) 1

26. What is the range of the set of numbers comprised entirely of $\{1, 6, x, 17, 20, y\}$ if all terms in the set are positive integers and $xy = 18$?

(A) 16
(B) 17
(C) 18
(D) 19
(E) Cannot be determined from the information given.

27. On a particular test whose scores are distributed normally, the 2nd percentile is 1720, while the 84th percentile is 1990. What score, rounded to the nearest 10, most closely corresponds to the 16th percentile?

(A) 1,750
(B) 1,770
(C) 1,790
(D) 1,810
(E) 1,830

28. A data set contains at least two different integers.

Quantity A	**Quantity B**
The range of the data set	The interquartile range of the data set

29. In a normally distributed set of data, one standard deviation above the mean is 77 and the standard deviation is 10. What is the mean of the data?

30. Some rock samples are weighed, and their weights are determined to be normally distributed. One standard deviation below the mean is 250 grams and one standard deviation above the mean is 420 grams.

Quantity A	**Quantity B**
335 grams	The median weight, in grams

31. In a normally distributed set of data, the mean is 12 and the standard deviation is less than 3.

Quantity A	**Quantity B**
Number of data points in the set located between 9 and 15	60% of the total number of data points

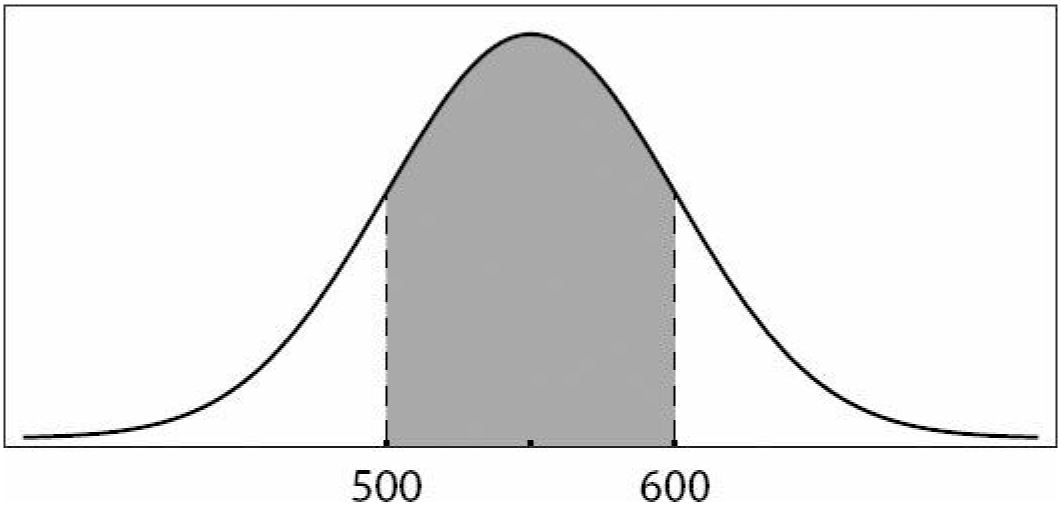
32.

Quantity A	**Quantity B**
The standard deviation of the set 10, 20, 30	The standard deviation of the set 10, 20, 20, 20, 20, 20, 30

33.

Quantity A	Quantity B
The standard deviation of a set of numbers with a range of 8	The standard deviation of a set of numbers consisting of 3 consecutive multiples of 3

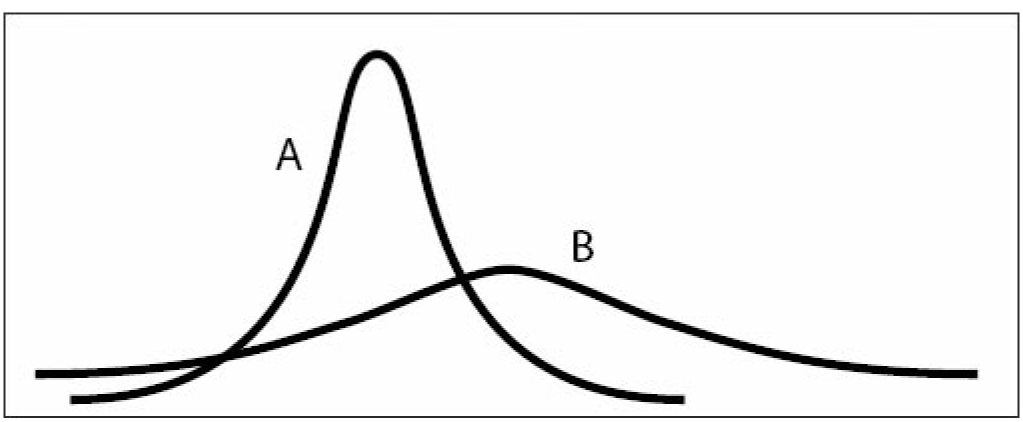
34.



The graph represents the normally distributed scores on a test. The shaded area represents approximately 68% of the scores.

Quantity A	Quantity B
The mean	550

35.



A and B are graphical representations of normally distributed random variables *X* and *Y*, respectively, with relative positions, shapes, and sizes as shown. Which of the following must be true?

Indicate all such statements.

- ☐ *Y* has a larger standard deviation than *X*.
- ☐ The probability that *Y* falls within 2 standard deviations of its mean is larger than the probability that *X* falls within 2 standard deviations of its mean.

☐ *Y* has a larger mean than *X*.

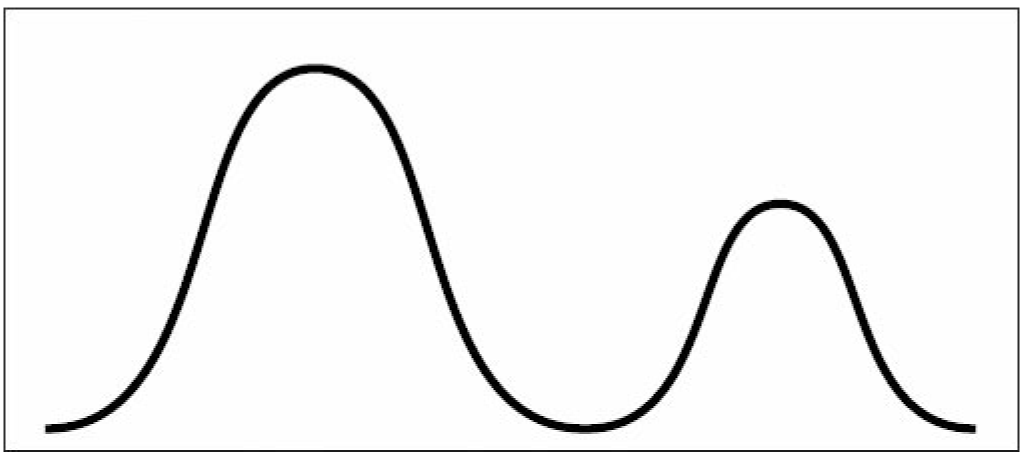
36. 300 test results are integers ranging from 15 to 75, inclusive. Dominick's result is clearly in the 80th percentile of those results, not the 79th or the 81st.

Quantity A	**Quantity B**
Number of other test results in the same percentile as Dominick's	Maximum number of other test-takers with the same result as Dominick

37. The outcome of a standardized test is an integer between 151 and 200, inclusive. The percentiles of 400 test scores are calculated, and the scores are divided into corresponding percentile groups.

Quantity A	**Quantity B**
Minimum number of integers between 151 and 200, inclusive, that include more than one percentile group	Minimum number of percentile groups that correspond to a score of 200

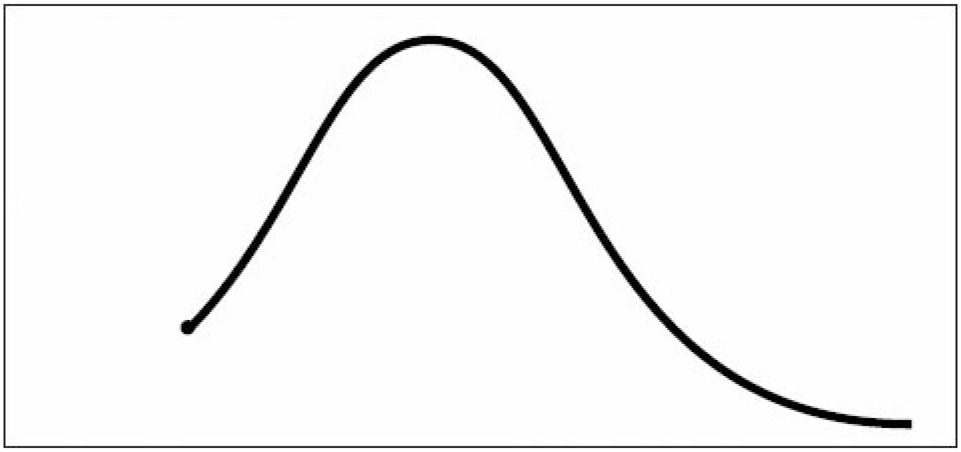
38.



Which of the following would the data pattern shown best describe?

(A) The number of grams of sugar in a selection of drinks is normally distributed.
(B) A number of male high school principals and a larger number of female high school principals have normally distributed salaries, distributed around the same mean.
(C) A number of students have normally distributed heights, and a smaller number of taller, adult teachers also have normally distributed heights.
(D) The salary distribution for biologists skews to the left of the median.
(E) The maximum-weight bench presses for a number of male athletes are normally distributed, and the maximum-weight bench presses for a smaller number of female athletes are also normally distributed, although around a smaller mean.

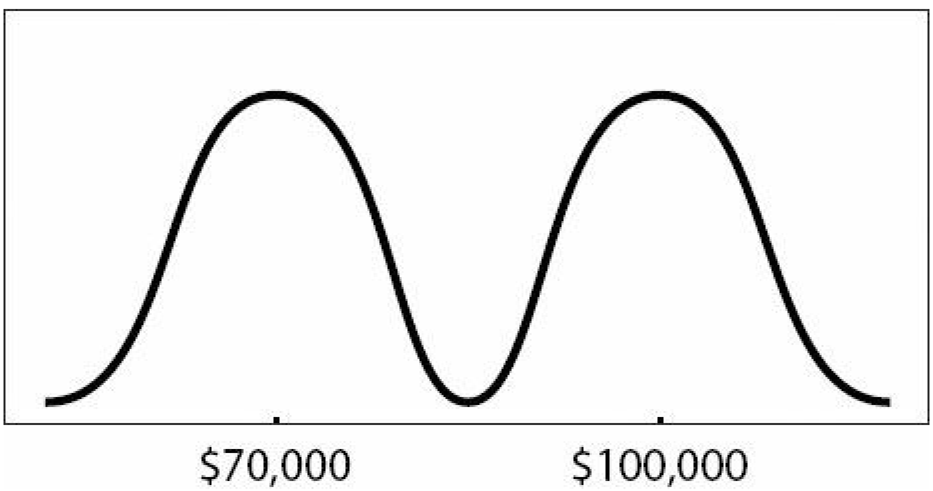
39.



Which of the following would the data pattern shown best describe?

(A) The weights of raccoons in a population are normally distributed.
(B) Salaries in a certain field appear normally distributed, except that salaries cannot dip below the limits of a minimum-wage law.
(C) The fraction of people at a certain age in a certain population is inversely proportional to age.
(D) a set of consecutive integers
(E) a set with a standard deviation of zero

40.

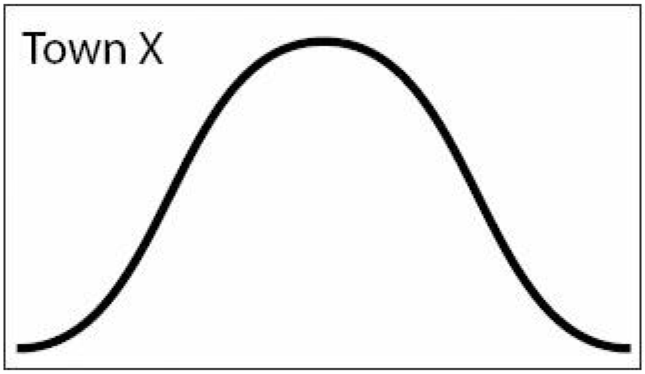


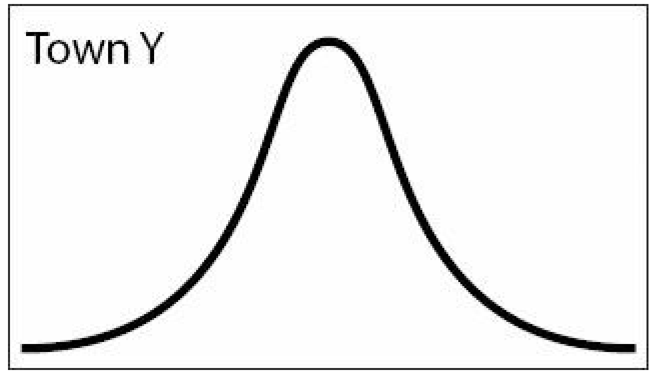
A number of scientists' salaries were reported; physicists' salaries clustered around a mean of $100,000, and biologists' clustered around a mean of $70,000. Which of the following could be true, according to the graph above?

Indicate all such statements.

☐ Some biologists earn more than some physicists.
☐ Both biologists' and physicists' salaries are normally distributed.
☐ The range of salaries is greater than $150,000

41.





The graph on the left represents the number of family members per family in Town X, while the graph on the right represents the number of family members per family in Town Y. The median family size for Town X is equal to the median family size for Town Y. The horizontal and vertical dimensions of the boxes above are identical and correspond to the same measurements. Which of the following must be true?

Indicate all such statements.

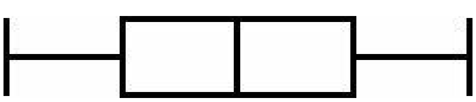
- ☐ The range of family sizes measured as the number of family members is larger in Town X than in Town Y.
- ☐ Families in Town Y are more likely to have sizes within 1 family member of the mean than are families in Town X.
- ☐ The data for Town X has a larger standard deviation than the data for Town Y.

42.

The box-and-whisker plot shown could be a representation of which of the following?

(A) a data set with a range of 100, symmetrically distributed around its median
(B) a data set with a range of 10 and an interquartile range of 6
(C) a data set in which the median of the upper half of the data is closer to the lowest value in the set than to the highest value
(D) a set of consecutive integers
(E) a normal distribution

43.



The box-and-whisker plot shown could be a representation of which of the following sets?

(A) -2, 0, 2, 4
(B) 3, 3, 3, 3, 3, 3
(C) 1, 25, 100
(D) 2, 4, 8, 16, 32
(E) 1, 13, 14, 17

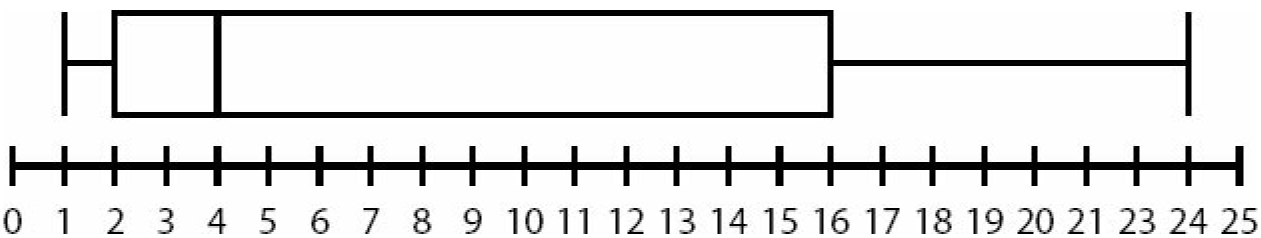
44.

Which of the following must be true about the data described by the box-and-whisker plot above?

Indicate all such statements.

- ☐ The median of the whole set is closer to the median of the lower half of the data than it is to the median of the upper half of the data.
- ☐ The data is normally distributed.
- ☐ The set has a standard deviation greater than zero.

45.



The box-and-whisker plot above represents a data set with:

(A) a mean of 4 and a range of 14
(B) a mean of 4 and a range of 23
(C) a median of 4 and a range of 14
(D) a median of 4 and a range of 23
(E) a median of 4 and a range of 24

46. The earthworms in Sample A have an average length of 2.4 inches, and the earthworms in Sample B have an average length of 3.8 inches. The average length of the earthworms in both samples is 3.0 inches. Which of the following must be true?

Indicate all such statements.

- ☐ There are more earthworms in Sample A than in Sample B.
- ☐ The median length of the earthworms is 3.2 inches.
- ☐ The range of lengths of the earthworms is 1.4.

Standard Deviation and Normal Distribution Answers

1. **I and III only.** The word *mean* is a synonym for the average. Because an average is calculated by taking the sum of the numbers in the set and dividing by the number of numbers in the set, changing *any* one number in a set (without adjusting the others) will change the sum and, therefore, the average. The median is the middle number in a set, so making the biggest number even bigger won't change that (the middle number is still 10). Standard deviation is a measure of how *spread out* the numbers in a set are — the more spread out the numbers, the larger the standard deviation — so making the biggest number *really far away* from the others would greatly increase the standard deviation.

2. **(D).** Standard deviation is a measure of how "spread out" the numbers in a set are — in other words, how far are the individual data points from the average of all of the data points? The GRE will not ask you to calculate standard deviation — in problems like this one, you will be able to eyeball which sets are more spread out and which are less spread out.

Since Set *Z*'s members are identical, the standard deviation is zero. Zero is the smallest possible standard deviation for any set, so it must be the smallest here. You can eliminate answer choices (A), (B), and (C). Set *Y*'s members have a *spread* of 1 between each number, Set *X*'s members are two away from each other, and *W*'s members are six away from each other, so Set *Y* has the next-smallest standard deviation (note that this is enough to eliminate answer choice (E) and choose answer choice (D)). The correct answer is (D) *Z*, *Y*, *X*, *W*.

3. **(B).** "Set N is a set of x distinct positive integers where $x > 2$" just means that the members of the set are all positive integers different from each other, and that there are at least 3 of them. You don't know anything about the standard deviation of the set other than that it is not zero. (Because the numbers are different from each other, they are at least a little spread out, which means the standard deviation must be greater than zero. The only way to have a standard deviation of zero is to have a set of identical numbers).

In Quantity B, multiplying each of the distinct integers by -3 would definitely spread out the numbers and thus increase the standard deviation. For instance, if the set had been 1, 2, 3, it would now be -3, -6, -9. The negatives are irrelevant — multiplying any set of *different* integers by 3 will spread them out more.

Thus, whatever the standard deviation is for the set in Quantity A, Quantity B must represent a larger standard deviation because the numbers in that set are more spread out.

4. **IV only.** In a set of distinct (different) integers, if each number is multiplied by 1/2, the numbers will get closer together (for instance, 2, 4, 6 would become 1, 2, 3), so the standard deviation would *decrease.*

If the smallest number in a set became equal to the median, then two numbers in the set would now be the same. The set would become *less* spread out, not more.

If the smallest number were increased to become larger than the current largest number, the standard deviation *could* increase, but wouldn't have to. For instance, if the set were 1, 2, 3, and the 1 were increased to become 100, the standard deviation would increase. But if the set were 1, 100, 101, and the 1 were increased to become 102, the set would get closer together.

Finally, if the largest number were doubled, the standard deviation would have to increase. For instance, if the set were 1, 2, 3 and the largest number doubled to 6, then the set would become more spread out. Because only the largest number is changing, and because the largest number becomes even larger when doubled, the numbers in the set will always become more spread out, thus increasing the standard deviation.

5. **(D).** Scoring scales on a test are not necessarily linear, so while it may look like you can line up the difference in percentiles with the difference in score, you cannot make any predictions about *other* percentiles. For all you know, 750 could be the 95th percentile score — or 963 could be. All you know is that 25% of the scores are $\leq$ 450, while 50% of the scores are > 450 and $\leq$ 700, and 25% of the scores are > 700.

6. **I and III only.** For the first statement, if the smaller number is positive and 3 less than the larger number AND one term is twice the other, the two terms have to be 3 and 6. (If you couldn't work out the numbers, you could write this as two equations: $S + 3 = L$ and $L = 2S$.) If you know the numbers, you can calculate the average.

The second statement is NOT sufficient. The standard deviation tells you how far all the terms are from the mean — but knowing *how spread out* the numbers are doesn't tell you what they're spread out *from.* For instance, the sets [3, 3, 3, 6, 6, 6, 6, 6, 6] and [-3, -3, -3, -6, -6, -6, -6, -6, -6] have the same standard deviation and meet the other constraints of the problem — each set consists of 9 terms, but only two different numbers, and 6 of the terms are each twice the value of each of the remaining 3. However, the two sets would have different averages (without calculating, you can easily see that one average would be positive and one would be negative).

The third statement works — if the biggest term in the set is 6, the smallest would, of course, be 3, allowing you to calculate the average.

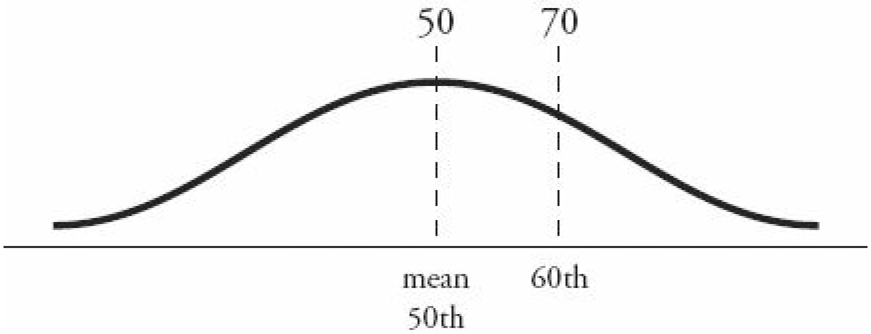
7. **(C).** A quartile is defined as the median of half of a set of data. The first quartile (or Q1) of a set of data is the median of the lower half of the data. For the first half, {2, 5, 7, 11, 16, 24}, the median is (7 + 11)/2 = 9 = Q1.

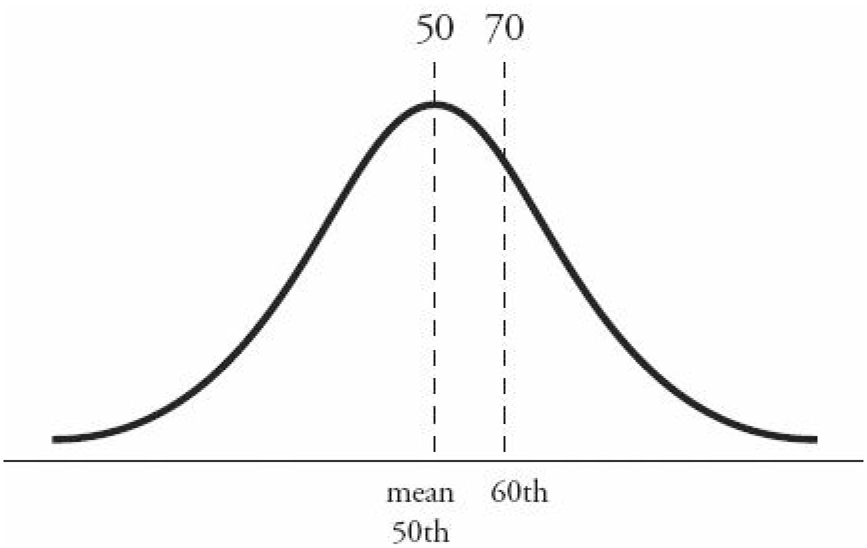
The third quartile (or Q3) of a set of data is the median of the upper half of the data. For the second half, {28, 50, 52, 101, 120, 130}, the median is (52 + 101)/2 = 76.5 = Q3.

Now find the average of Q1 and Q3 = (9 + 76.5) /2 = 42.75.

8. **(D).** The test scores are distributed normally — that is, in a specific hump-shaped pattern. That pattern is determined by two numbers: the mean (where the peak of the hump is) and the standard deviation (a measure of the width of the hump). To know the exact shape of the distribution, you need to know *both* numbers.

It's given that the 60th percentile is equal to a score of 70. The 60th percentile corresponds to a specific point on the right side of the normal distribution's hump—the point where 60% of the area under the curve is to the left and 40% is to the right. However, you don't know the width of the curve. It could be low and flat (imagine a lower mean, such as 50, and a high standard deviation, which allows the 60th percentile score of 70 to fall far from the mean), or it could be high and narrow (with a mean of, say, 68 and a tiny standard deviation). So a score of 35 (Quantity B) could correspond to *any* percentile below 60th, in fact—either less than or greater than the 30th percentile score, which is Quantity A.





9. **(B).** Normal distributions are always centered on and symmetrical around the mean, so the chance that the worm's length will be within a certain 6-centimeter range (or any specific range) is highest when that range is centered on the mean, which in this case is 30 centimeters.

More specifically, Quantity A equals the area between -2 standard deviations and the mean of the distribution. In a normal distribution, roughly 34 + 34 + 14 + 14 = 96% of the sample will fall within 2 standard deviations above or below the mean. If you limit yourself only to the 2 standard deviations below the mean, then half of that, or 96%/2 = 48%, will fall in this range. In contrast, Quantity B equals the area between -1 standard deviation and +1 standard deviation. In a normal distribution, roughly 34 + 34 = 68% of the sample will fall within 1 standard deviation above or below the mean. 68% is larger than 48%, so Quantity B is larger than Quantity A.

But you don't need these exact figures to answer this question! Picture any bell curve — the area under the "hump" (that is, centered around the middle) is bigger! Thus, it has more members of the set (in this case, worms) in it.

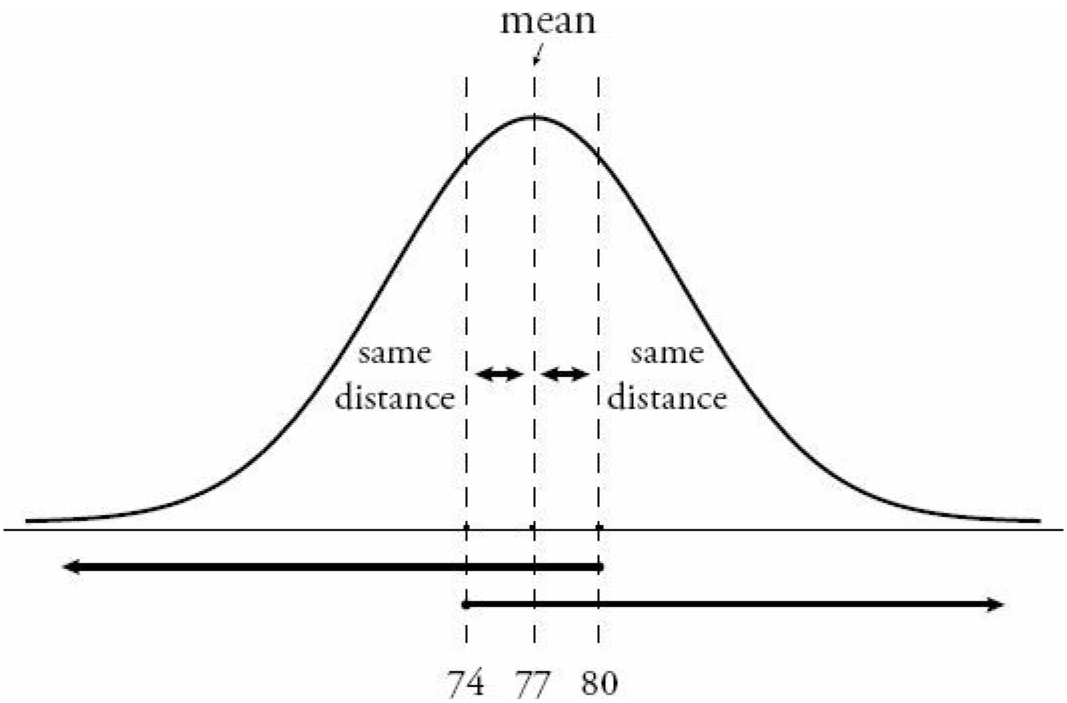
10. **(A).** The area under a normal distribution between the mean (the center) and +1 standard deviation represents approximately 34% of the total area (this is just a fact to memorize for the GRE), while the area between +1 and +2 standard deviations represents approximately 14%. The sum of those areas is 34 + 14 = 48%, so the percent of wages that fall between the mean ($18) and +2 standard deviations ($18 + $3.50 + $3.50 = $25) is 48%. Thus Quantity A is larger than Quantity B.

11. **(B).** How many standard deviations above $90,000 is $112,000? The difference between the two numbers is $22,000, which is 2 times the standard deviation of $11,000. So Quantity A is really the number of home values greater than 2 standard deviations above the mean.

In any normal distribution, roughly 2% will fall more than 2 standard deviations above the mean (this is something to memorize). The value of Quantity A is roughly (8,000)(0.02) = 160, which is definitely smaller than Quantity B

(300).

12. **(C).** The normal distribution is symmetrical around the mean. For any symmetrical distribution, the mean equals the median (also known as the 50th percentile). Even though you don't know the standard deviation, the number of students who scored *less* than 3 points *above* the mean (77 + 3 = 80) must be the same as the number of students who scored *greater* than 3 points *below* the mean (77 - 3 = 74). As long as the boundary scores (80 and 74) are placed symmetrically around the mean, you will have equal proportions. Draw this if it is at all confusing:



Notice that the two conditions overlap and are perfectly symmetrical. Each number consists of a short segment between it and the 50th percentile mark, as well as half of the students (either above or below the 50th percentile mark). That is, the "less than 80" category consists of the segment between 80 and 77, as well as all students below the 50th percentile mark (below 77). The "greater than 74" category consists of the segment between 74 and 77, as well as all students above the 50th percentile mark (above 77).

13. **(D).** First, make the numbers easier to use. You can multiply every number by the same constant, or move the decimal the same number of places for each number. If you move the decimal four places, the mean becomes 1630, the standard deviation becomes 84, and the two other numbers become 1,546 and 1,756.

Next, "normalize" the boundaries you care about. That is, take 1,546 meters (the lower boundary) and 1,756 meters (the upper boundary) and convert each of them to a number of standard deviations away from the mean. To do so, subtract the mean, then divide by the standard deviation.

Lower boundary: 1546 - 1630 = -84

$-84 \div 84 = -1$

So the lower boundary is -1 standard deviation (that is, 1 standard deviation less than the mean).

Upper boundary: 1756 - 1630 = 126

$126 \div 84 = 1.5$

So the upper boundary is 1.5 standard deviations above the mean.

You need to find the probability that a random variable distributed according to the standard normal distribution falls between -1 and 1.5.

Use the approximate areas under the normal curve. Approximately 34 + 34 = 68% will fall within 1 standard deviation above or below the mean, so 68% accounts for the -1 to 1 portion of the standard deviation. What about the portion from 1 to 1.5?

Approximately 14% will fall between 1 and 2 standard deviations above the mean. You are not expected to know the exact area between 1 and 1.5; however, since a normal distribution has its hump around 0, *more* than half of the area between 1 and 2 must fall closer to 0 (between 1 and 1.5). So the area under the normal curve between 1 and 1.5 must be greater than half of the area, or greater than 7%, but less than the full area, 14%.

Put it all together. The area under the normal curve between -1 and 1.5 is approximately 68% + (something between 7% and 14%). The lower estimate is 68% + 7% = 75%, and the upper estimate is 68% + 14% = 82%.

14. **(B).** First, compute the standard deviation from the information given. 10.8 pounds is 3 standard deviations more than 5.85 pounds, since 10.8 is 2 standard deviations *more* than the mean, and 5.85 is 1 standard deviation *less* than the mean.

> 10.8 - 5.85 = 4.95 pounds = 3 standard deviations.
> 4.95 ÷ 3 = 1.65 pounds = 1 standard deviation.

Now compute the mean. Since 5.85 pounds is 1 standard deviation below the mean, 5.85 plus 1 standard deviation equals the mean:

> 5.85 + 1.65 = 7.5 pounds = mean.

Quantity A: *Twice the weight of a baby 2 standard deviations below the mean.* Note that you already know 5.85 is one standard deviation below the mean; subtract another standard deviation (1.65) in order to reach 2 standard deviations below the mean, then multiply by 2.

> = 2 (5.85 - 1.65) = 2 (4.2) = 8.4 pounds.

Quantity B: *The weight of a baby 1 standard deviation above the mean.* Add 1 standard deviation to the mean.

> = 7.5 + 1.65 = 9.15 pounds.

Quantity B is larger than Quantity A.

15. **(E).** The smallest number in the set is -4, so you can eliminate (D). The largest number in the set is 5, so you can eliminate (B).

The median is -1; now check the medians of the remaining answer choices. The median of (A) is between 0 and -2, which is -1; (A) could be the right answer. The median of (C) is -3, which is wrong. The median of (E) is between -2 and 0, which is -1; (E) could also be the right answer.

Q1 is -3; check Q1 for both (A) and (E). The median of the smaller three numbers (-4, -4, -2) for (A) is -4, which is wrong; you want Q1 to be -3. (E) is the only answer choice left and you can pick it without checking if you're confident in your previous work. Here's the actual proof: the median of the smaller four numbers (-4, -4, -2, -2) is -3.

16. **(B).** The interquartile range of a set of data is the distance between Q1 (quartile marker 1, the median of the first half of the set) and Q3 (quartile marker 3, the median of the second half of the set).

The first ten positive multiples of 5 are: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50. Q1 is the median of the first 5 terms, or 15. Q3 is the median of the last 5 terms, or 40.

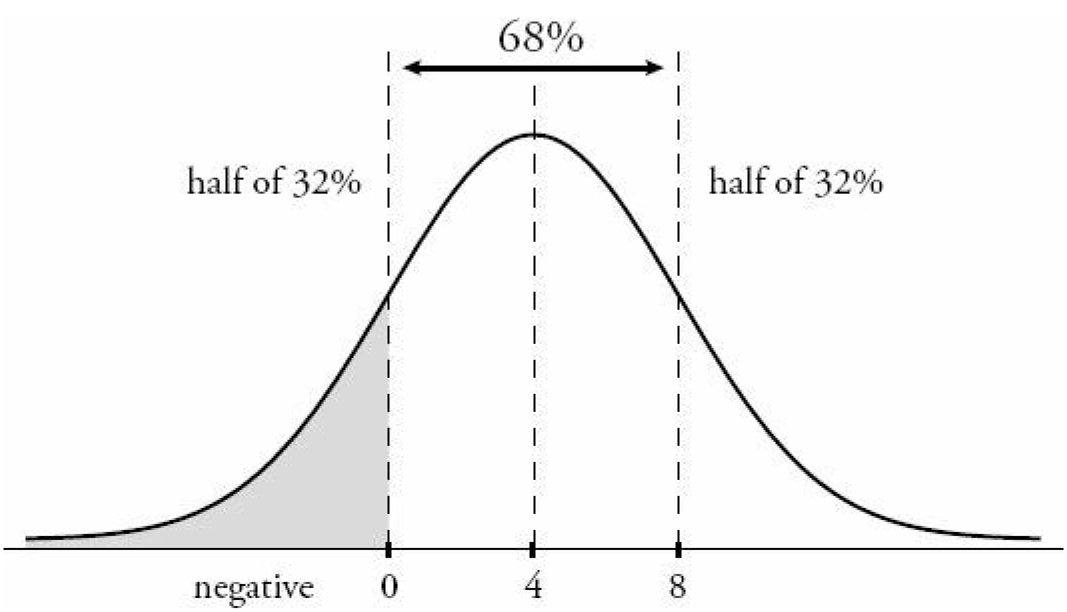
Take the difference between Q3 and Q1: 40 - 15 = 25.

17. **I only.** The definition of a normally distributed set is that about two-thirds of the data falls within one standard deviation of the mean. If only one person scored close to the mean (and most people were at the top or bottom of the curve), that set of data is not normally distributed, so the first statement is true.

The second statement is false—the range of the data would not necessarily change if the set were more evenly distributed. For instance, as long as one person still had a zero and one person still had a score of 100, the other scores could fall anywhere without changing the range.

The third statement is also false. The mean of Set *T* might or might not be equal to the median. For instance, the one student within five points of the mean could have a score actually equal to the mean; of the remaining 148 students, half could have scores of 0 and half could have scores of 100. In this case, the mean would equal the median. However, the same scenario with *unequal* numbers of students scoring 0 and 100 would result in the mean *not* equaling the median.

18. **(B).** In a normally distributed set of data, roughly 34 + 34 = 68% of the data fall within one standard deviation of the mean. This means about 100 - 68 = 32% of the data fall outside of one standard deviation, with HALF of that 32% falling one standard deviation to the RIGHT of the mean, and HALF of that 32% falling to the left. 68%



A negative number would be more than one standard deviation to the left of the mean (4 - 4 = 0), meaning that 1/2 of

32% of the data will fall in that range, or 16%. The closest answer is 1/6.

19. **(C).** Percentiles define the proportion of a group that scores below a particular benchmark. Since John scored in the 32nd percentile, by definition, 32 percent of the class scored worse than John. Quantity A is equal to 32%.

Jane is in the 68th percentile, so 68% of the class scored worse than she did. Since 100 - 68 = 32, 32% of the class scored as high as or higher than Jane. Quantity B is also equal to 32%.

20. **(D).** To find the range of one standard deviation, add and subtract the standard deviation from the mean. For two standard deviations, take twice the value of the standard deviation ($7.1 \times 2 = 14.2$) and both add and subtract it from the mean.

$4.2 - 14.2 = -10$
$4.2 + 14.2 = 18.4$

21. **41.** The set of data from 25 to 48 has 48 - 25 + 1 = 24 terms, so each octile group would be made up of 24/8 = 3 terms. It's easier to find the 6th octile group by working backwards from the 8th one:

8th octile group: 48, 47, 46   7th octile group: 45, 44, 43   6th octile group: 42, 41, 40

The median of the 6th octile group is 41.

22. **(B).** Since the average is 7, you can use the average formula to find the sum of the scores in the class.

Average = Sum ÷ # of terms
7 = Sum ÷ 20
Sum = 140

Now you already know that at least one student got every possible score. There are eleven possible scores: 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10. This is an evenly spaced set, so you can calculate the sum by multiplying the average of the set by the number of terms in the set. The average is $(10 + 0)/2 = 5$ and the number of terms is 11, so the sum of the set is $5 \times 11 = 55$. If you subtract from the sum you found earlier, the remaining 9 students had to score 140 - 55 = 85 points.

Quantity B is asking you to find the lowest score that two students could have received. If 9 students scored a total of 85 points, and any one student could not score more than 10 points, then what is the lowest possible score that any one student could have received? In order to minimize that number, you need to maximize the numbers for the other students. If 8 students scored 10 points each, for a total of 80 points, then the 9th student must have scored at least 5. Quantity B must be larger than Quantity A. Notice that the average score of 7 forces you to make a lot of scores 10's to balance out the very low scores of 0, 1, 2, etc. that you must have in the class (at least one of each).

23. **(B).** Frequency refers to the number of times a particular result occurred. In other words, if the set represented by this table were written out, it would look like this: 4,4,4,4,4,4, 6,6,6,6,6, 8,8,8,8,8. The mode is the most commonly occurring result. The mode of this data is 4 because 4 appears more often than any other number. Thus, Quantity B is larger.

24. **(D).** Quintiles ("fifths" of the data) define relative scores, not absolute scores. Imagine two possible score distributions:

Example 1: The class's scores are 1, 2, 3, 4, 5 (20% of the class scored each of these). In this case, adding up two lowest quintile students would be 1 + 1 = 2, which is less than 5, the score of a top quintile student.

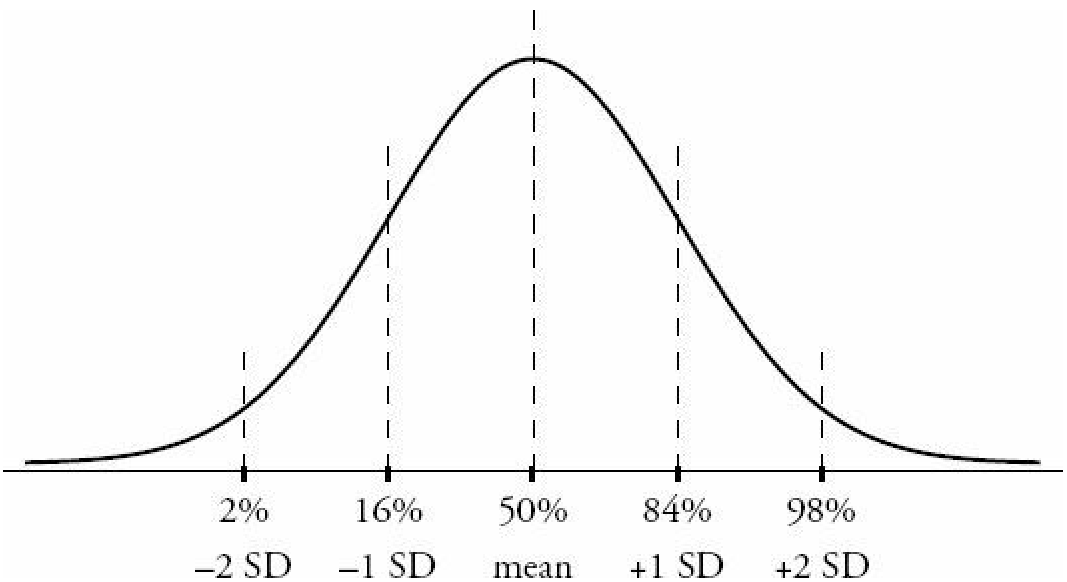
Example 2: The class's scores are 10, 11, 12, 13, 14 (20% of the class scored each of these — still not so sharp!). In this case, adding up two lowest quintile students would be 10 + 10 = 20, which is greater than 14, the score of a top quintile student.

Thus, you cannot determine which quantity is larger.

25. **(E).** A percentile ALWAYS represents one *percent* of a set of data. If the question had asked how many TERMS one percentile represented, that would be a different question (with a different answer).

26. **(D).** You don't know what x or y are, but since they are both positive integers, they can only be 1 and 18, 2 and 9, or 3 and 6 (because they have to multiply to 18). So the smallest number in the set is 1 and the largest is 20. Since 20 - 1 = 19, the range is 19.

27. **(D).** The diagram below shows the standard distribution curve for any normally distributed variable. The percent figures correspond roughly to the standard percentiles both 1 and 2 standard deviations (SD) away from the mean.



The 2nd percentile is 1720, roughly corresponding to 2 standard deviations below the mean. Therefore, the mean - 2 standard deviations = 1720.

Likewise, the 84th percentile is 1990. 84% of a normally distributed set of data falls below the mean + 1 standard deviation, so the mean + 1 standard deviation = 1990.

Call the mean M and the standard deviation S. You can now solve for these variables:

$$M - 2S = 1720$$
$$M + S = 1990$$

Subtract the first equation from the second equation:

$3S = 270$
$S = 90$

You are looking for the 16th percentile, which is the mean - 1 standard deviation or $M - S$. (It's a fact to memorize that approximately 2% of normally distributed data falls below $M - 2S$, and approximately 14% of normally distributed data falls between $M - 2S$ and $M - S$.)

You already know that $M - 2S = 1720$, so add another S to get $M - S$.

$(M - 2S) + S = 1720 + 90 = 1810$

Notice that the percentiles are *not* linearly spaced. The normal distribution is hump-shaped, so percentiles will be bunched up around the hump and spread out farther away.

28. **(D).** In most data sets, the range is larger than the interquartile range because the interquartile range ignores the smallest and largest data points. That's actually the purpose of interquartile range — to get a good picture of where *most* of the data is (think of the "big hump" on a bell curve). For instance:

Example Set A: 1, 2, 3, 4, 5, 6, 7, 100

Here, the range is 100 - 1 = 99.

The interquartile range is Q3 - Q1, or the median of the upper half of the data minus the median of the lower half of the data: 6.5 - 2.5 = 4.

In this example, the range is much larger. However, consider this set:

Example Set B: 4, 4, 4, 4, 5, 5, 5, 5

In this set, the range is 5 - 4 = 1. The interquartile range is also 5 - 4 = 1. While the interquartile range can never be *smaller* than the range, they can certainly be equal.

29. **67.** Since the standard deviation is 10 and 1 standard deviation above the mean is 77, simply subtract 77 - 10 = 67 to get the mean.

30. **(C).** Since one standard deviation below the mean is 250 and one standard deviation above the mean is 420, the mean/median must be halfway in between. Since 420 - 250 = 170 and half of 170 is 85, simply add 85 to 250 (or subtract it from 420) to get the mean/median of 335. (Note that in a normal distribution, the mean is equal to the median, so the two terms can be used interchangeably.)

31. **(A).** If the standard deviation were 3, then one standard deviation below the mean would be 9 and one standard deviation above the mean would be 15, so about 2/3 (more precisely 68%) of the data would be between 9 and 15 (in a normal distribution, it is always the case that about 2/3 of the data is within one standard deviation of the mean).

Since the *actual* standard deviation is *less than* 3, about 2/3 of the data is found within an *even smaller range* than 9 to 15. For instance, the standard deviation could be 1, and then about 2/3 of the data would be between 11 and 13. Or the standard deviation could be 2.5, and then about 2/3 of all the data would be found between 9.5 and 14.5.

Since 2/3 of the data is found within an *even smaller range* than 9 to 15, the range from 9 to 15 contains *more than* 2/3 of the data, so it definitely contains more than 60% of all the data points.

Don't be confused by the use of "number of data points." While you don't know the actual total number of data points, you can definitively conclude that Quantity A is equal to a larger percentage of that total than is Quantity B.

32. **(A).** Standard deviation is a measure of the data's spread from the mean. While the two sets have the same range (30 - 10 = 20), they do NOT have the same spread. The four extra terms in Quantity B are identical to the mean, meaning that, on average, the data in Quantity B is closer to the mean than the data in Quantity A. Thus, Quantity A is more spread out, on average, and has the larger standard deviation. You do not have to compute the actual standard deviations to find the answer here.

33. **(D).** Standard deviation depends on the difference between each number in a set and the average (arithmetic mean) of the set.

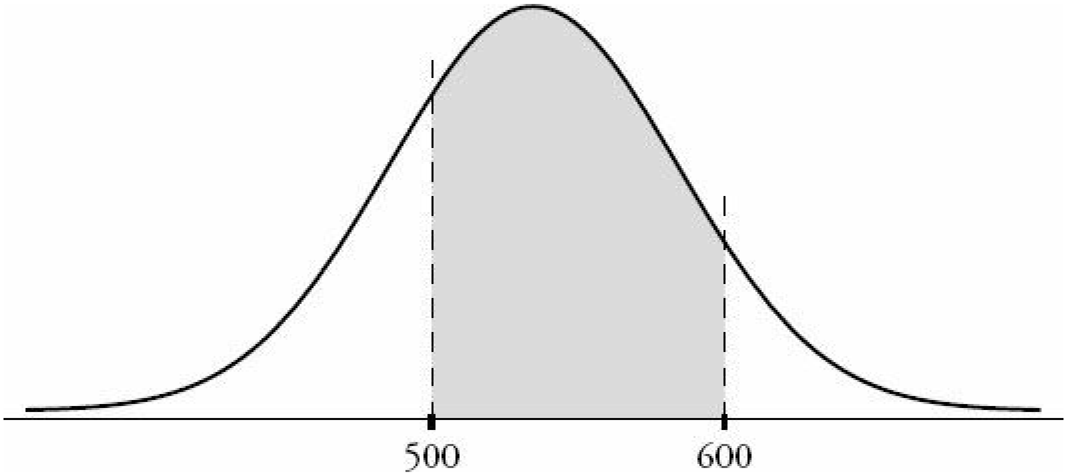
Quantity B is known, since the average of 3 consecutive multiples of 3 (such as 3, 6, 9) is the middle number (6), and the differences between each of the numbers in the set and that average are therefore known (-3, 0, and 3). While you don't need to calculate the actual standard deviation for the GRE, it certainly can be done: standard deviation is the square root of the average of the squares of these differences, or $\sqrt{\frac{(-3)^2+(0)^2+(3)^2}{3}}=\sqrt{\frac{18}{3}}=\sqrt{6}$, which is between 2 and 3.

Quantity A is *not* known, and in fact it can be greater or less than Quantity B. Again, you don't have to calculate standard deviation; you should just realize that the numbers in Quantity A could be more or less spread out than the numbers in Quantity B. Therefore, the answer is (D).

If you're interested in the calculations: the maximum value of Quantity A is 4, which you can get from a set of 2 numbers that differ by 8 (say, 1 and 9). The average of this set is 5; the difference of each number from this average is -4 and 4, and so the standard deviation works out to be $\sqrt{\frac{(-4)^2+(4)^2}{2}}=\sqrt{\frac{32}{2}}=\sqrt{16}=4$. Even without calculating, you should recognized that the set {1, 9} is more spread out than the set {3, 6, 9}.

On the other hand, the standard deviation of a set with range of 8 can be as close to zero as you want to make it. How? Start with 1 and 9 again in your set, but now add a whole bunch of 5's (right at the average). The differences of those 5's from the average of the set (which is still 5) are all 0. In essence, you're averaging in a lot of 0's to the standard deviation, swamping the effect of the two original deviations from 5 (the original numbers 1 and 9). You can make the whole set be super-close to the mean, 5, reducing the standard deviation to minuscule levels—well below that of the set {3, 6, 9}.

34. **(D).** While the shaded area may appear to be evenly located on either side of the mean, it isn't necessarily. For example, the 68% could be more lopsided, like so:



This area could still represent 68% of the scores, even if it's not one standard deviation to either side of the mean. For you to know that the median = 500, the problem would need to state explicitly that 500 and 600 each represent one standard deviation from the mean (or at least that 500 and 600 are equally far from the mean).

The fact that 68% of the data is located between 500 and 600 is definitely intended to trick you into assuming that 500 and 600 are -1 and +1 standard deviation from the mean, but you cannot assume this. While it is always true that, in a normal distribution, about 68% (some people memorize the approximation as two-thirds) of the data is within one standard deviation of the mean, the reverse is not true: you cannot assume that any chunk of data that is about 68% of the whole is therefore within one standard deviation of the mean.

35. **I and III only**.

I. True. Standard deviation describes how much a set of data diverges from the mean. Curve B is more widely spread than curve A, and thus *Y* has a larger standard deviation than *X*.

II. Not True. The probability that *any* normally distributed variable falls within 2 standard deviations of its mean is the same, approximately 0.14 + 0.34 + 0.34 + 0.14 = 0.96, or 96%. This is a value you should memorize for the GRE.

III. True. The mean of a normal curve is the point along the horizontal axis below the "peak" of the curve. The highest point of curve B is clearly to the right of the highest point of curve A, so the mean of Y is larger than the mean of X. Notice that the mean has nothing to do with the *height* of the normal curve, which only corresponds to how tightly the variable is gathered around the mean (i.e., how small the standard deviation is).

36. **(C).** Since the number of test results is divisible by 100, the percentiles cleanly divide the total into 100 percentile groups of 300 ÷ 100 = 3 results each. That is, there are 3 results in each percentile. So 2 other results are in the same percentile as Dominick's. Quantity A is 2.

Dominick's result is clearly in the 80th percentile, not the 79th or the 81st. So it must be possible to distinguish the 80th percentile (that group of 3) from the "next-door" percentiles. Say Dominick got a 58. How many *other* people could have gotten a 58? Maybe no one, maybe one, maybe two — but if *three* other people got a 58, then you'd have a total of four people with the same result. In that case, it would be impossible to assign Dominick's result *definitively* to the 80th percentile and not the neighboring percentiles. So the maximum number of other test-takers with the same result as Dominick is 2. Quantity B is also 2.

37. **(A).** 400 test scores are distributed among 50 possible outcomes (integers between 151 and 200, inclusive, which number 200 - 151 + 1 = 50 integers). There is an average of 400 ÷ 50 = 8 scores per integer outcome, and there are

$400 \div 100 = 4$ scores in each percentile. So, if all the scores were completely evenly distributed with exactly 8 scores per integer, there would be 2 percentile groups per integer outcome (0th and 1st percentiles at 151, 2nd and 3rd percentiles at 152, etc.). In that case, all 50 integers from 151 to 200 would correspond to more than one percentile group.

How can you reduce the number of integers corresponding to more than one percentile group? Bunch up the scores. Imagine that everyone gets a 157. Then that integer is the only one that corresponds to more than one percentile group (it corresponds to all 100 groups, in fact). However, you can't reduce further this way. You're left with 1 integer, so the minimum number of integers corresponding to more than one percentile group is 1, which is Quantity A.

As for Quantity B, though, a particular integer may have *no* percentile groups corresponding to it. In the previous example, if everyone gets a 157, then no one gets a 158, or a 200 for that matter. So the minimum number of percentile groups corresponding to a score of 200 (or to any other particular score) is 0, which is Quantity B.

38. **(C).** A two-humped shape could come from two overlapping normal distributions with different averages. Since the hump on the right is smaller, the distribution with a higher average should contain less data. Of the possible answer choices, only (C) describes such a scenario.

39. **(B).** The data appears essentially hump-shaped, but the left tail is cut off. This means that there is no data below a certain cutoff. Only (B) corresponds to this kind of situation.

40. **I, II, and III.**

I. Could Be True. Although biologists' salaries cluster around a lower number than physicists' salaries do, you cannot claim that *every* biologist's salary is lower than *every* physicist's salary. Some biologists' salaries can be high, and some physicists' salaries can be low.

II. Could Be True. Normal distributions are consistent with the hump shapes you see in the graph. You cannot *prove* that they're normal, but you cannot claim they're definitely not — they certainly *could* be normal.

III. Could Be True. From real-world normal distributions of an unknown amount of data, there's no way to tell the maximum or minimum values of the data. So the range certainly could be more than $150,000.

41. **II and III only.**

I. Not Necessarily True. Range is calculated this way: *largest value* - *smallest value*. From the graphs as shown (assuming that they do not continue "off screen" left and right), you would conclude that the two distributions have the *same* range, because the distributions are above zero on both the far left and the far right. (In the real world, you might assume that the graphs continue off screen, leading to even less confidence about the range of each distribution.)

II. True. The graph on the right (Town Y) has a smaller standard deviation (it is less spread out around its mean). So families in Town Y are more likely to be within 1 family member of the mean than families in Town X are.

III. True. The graph on the left is more spread out, so it has a larger standard deviation.

42. **(C).** The plot is not symmetrical, so you can eliminate (A), (D), and (E), which would all be symmetrically distributed around the median. You can also eliminate (B), which claims a range of 10 (the distance from whisker to

whisker) and an interquartile range of 6 (the width of the box). Since the distance between the whiskers is actually much more than twice the width of the box, (B) cannot be the answer. (C) fits: the median of the upper half of the data (the right edge of the box) is closer to the lowest value in the set (the left whisker) than to the highest value (the right whisker).

43. **(A).** The plot is symmetrical, so you can eliminate any non-symmetrical data sets (such as (C), (D), and (E)). In (B), all the data points are the same, so there would be no width to the box-and-whisker plot. (A) is the only remaining possibility: the data is evenly spaced, leading to equal widths for each segment of the plot, as shown.

44. **III only.**

I. Not True. The median of the whole set is the line in the middle of the box. As shown, it is closer to the *right* side of the box (the median of the upper half of the data) than to the *left* side of the box (the median of the lower half of the data) — the opposite of what this statement claims.

II. Not True. This non-symmetrical plot could never represent a symmetrical distribution such as the normal distribution. In fact, a *true* normal distribution cannot be represented by a box-and-whisker plot at all, because such a distribution stretches infinitely to the right and to the left, in theory.

III. True. Any set represented by a box-and-whisker plot has a standard deviation greater than zero, because the plot displays some spread in the data. The only set that has a zero standard deviation is a set containing identical data points with zero spread between them, such as $\{3, 3, 3, 3\}$.

45. **(D).** The line inside the box always represents the median. If the plot is symmetrical, the median equals the mean, but because this plot is not symmetrical, there's no way that the mean is 4. So you can eliminate (A) and (B).

The range is the distance between the whiskers. That distance is 24 - 1 = 23.

46. **I only.** Since the overall average length of all the earthworms is closer to the average length of earthworms in Sample A than to the average for Sample B, there are more earthworms in Sample A.

However, you cannot know anything about the median or the range of the data set without the individual values. For instance, the lengths of all the worms in Sample A could be exactly 2.4, or they could be spread out quite a bit from 2.4. Similarly, the worms in Sample B could measure exactly 3.0, or they could have a variety of different lengths that average to 3.0. Thus, the median and range could vary quite a bit.

Chapter 23
of

Probability, Combinatorics, and Overlapping Sets

In This Chapter…

Probability, Combinatorics, and Overlapping Sets

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A number is randomly chosen from a list of 10 consecutive positive integers. What is the probability that the number is greater than the mean?

(A) 3/10
(B) 2/5
(C) 1/2
(D) 7/10
(E) 4/5

2. A number is randomly chosen from the first 100 positive integers. What is the probability that it is a multiple of 3?

(A) 32/100
(B) 33/100
(C) 1/3
(D) 34/100
(E) 2/3

3. A restaurant menu has several options for tacos. There are 3 types of shells, 4 types of meat, 3 types of cheese, and 5 types of salsa. How many distinct tacos can be ordered assuming that any order contains exactly one of each of the above choices?

4. A history exam features 5 questions. 3 of the questions are multiple-choice with four options each. The other two questions are true or false. If Caroline selects one answer for every question, how many different ways can she answer the exam?

5. A certain company places a six-symbol code on each of their products. The first two symbols are one of the letters A–E and the last four symbols are digits. If repeats are allowed on both letters and numbers, how many such codes are possible?

6. The probability is 1/2 that a coin will turn up heads on any given toss and the probability is 1/6 that a number cube with faces numbered 1 to 6 will turn up any particular number. What is the probability of turning up a heads and a 6?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/4
(E) 2/3

7. An integer is randomly chosen from 2 to 20 inclusive. What is the probability that the number is prime?

8. Five students in a classroom are lining up one behind the other for recess. How many different lines are possible?

(A) 5
(B) 10
(C) 24
(D) 25
(E) 120

9. An Italian restaurant boasts 320 distinct pasta dishes. Each dish contains exactly one pasta, one meat, and one sauce. If there are 8 pastas and 4 meats available, how many sauces are there to choose from?

10. A 10-student class is to choose a president, vice president, and secretary from the group. Assuming that no person can occupy more than one post, in how many ways can this be accomplished?

11.

Quantity A	Quantity B
The number of 4-digit positive integers where all 4 digits are less than 5	625

12. BurgerTown offers many options for customizing a burger. There are 3 types of meats and 7 condiments: lettuce, tomatoes, pickles, onions, ketchup, mustard, and special sauce. A burger must include meat, but may include as many or as few condiments as the customer wants. How many different burgers are possible?

(A) 8!
(B) (3)(7!)
(C) (3)(8!)
(D) $(8)(2^7)$
(E) $(3)(2^7)$

13. The probability of rain is 1/6 for any given day next week. What is the chance it rains on both Monday and Tuesday?

(A) 1/36
(B) 1/12
(C) 1/6
(D) 1/3
(E) 2/3

14. How many five-digit numbers can be formed using the digits 5, 6, 7, 8, 9, 0 if no digits can be repeated?

(A) 64
(B) 120
(C) 240
(D) 600
(E) 720

15. A bag contains 3 red, 2 blue, and 7 white marbles. If a marble is randomly chosen from the bag, what is the probability that it is NOT blue?

16. A man has 3 different suits, 4 different shirts, 2 different pairs of socks, and 5 different pairs of shoes. In how

many ways can the man dress himself if he must wear 1 suit, 1 shirt, 1 pair of socks, and 1 pair of shoes?

17. A state issues automobile license plates using two letters selected from a 26-letter alphabet, as well as four numerals selected from the digits 0 through 9, inclusive. Repeats are permitted. For example, one license plate combination could be GF3352.

Quantity A	**Quantity B**
The number of possible unique license plate combinations	6,000,000

18. A small nation issues license plates that consist of just one number (selected from the digits 0 through 9, inclusive) and four letters, selected from a 20-letter alphabet. Repeats are permitted. However, there is one four-letter combination that is not allowed to appear on license plates. How many allowable license plate combinations exist?

(A) 1,599,990
(B) 1,599,999
(C) 1,600,000
(D) 4,569,759
(E) 4,569,760

19. A bag contains 6 black chips numbered 1–6 respectively and 6 white chips numbered 1–6 respectively. If Pavel reaches into the bag of 12 chips and removes 2 chips, one after the other, without replacing them, what is the probability that he will pick black chip #3 and then white chip #3?

20. Tarik has a pile of 6 green chips numbered 1–6 respectively and another pile of 6 blue chips numbered 1–6 respectively. Tarik will randomly pick 1 chip from the green pile and 1 chip from the blue pile.

Quantity A	**Quantity B**
The probability that both chips selected by Tarik will display a number less than 4	1/2

21. A bag contains 6 red chips numbered 1–6 respectively and 6 blue chips numbered 1–6 respectively. If 2 chips are to be picked sequentially from the bag of 12 chips, without replacement, what is the probability of picking a red chip and then a blue chip with the same number?

22. In a school of 150 students, 75 take Latin, 110 take Spanish, and 11 take neither.

Quantity A	Quantity B
The number of students who take only Latin	46

23. How many 10-digit numbers can be formed using only the digits 2 and 5?

(A) 2^{10}
(B) (22)(5!)
(C) (5!)(5!)
(D) 10!/2
(E) 10!

24. A 6-sided cube has sides numbered 1 through 6. If the cube is rolled twice, what is the probability that the sum of the two rolls is equal to 8?

(A) 1/9
(B) 1/8
(C) 5/36
(D) 1/6
(E) 7/36

25. A coin with heads on one side and tails on the other has a 1/2 probability of landing on heads. If the coin is flipped 5 times, how many distinct outcomes are possible if the last flip must be heads? Outcomes are distinct if they do not contain exactly the same results in exactly the same order.

26. In a class of 25 students, every student takes either Spanish, Latin, or French, or two of the three, but no students take all three languages. 9 take Spanish, 7 take Latin and 5 take exactly two languages.

Quantity A	Quantity B
The number of students who take French	14

27. Bob has a 24-sided die with an integer between 1 and 24 on each face. Every number is featured exactly once. When he rolls, what is the probability that the number showing is a factor of 24?

28. A baby has x total toys. If 9 of the toys are stuffed animals, 7 of the toys were given to the baby by its grandmother, 5 of the toys are stuffed animals given to the baby by its grandmother, and 6 of the toys are neither stuffed animals nor given to the baby by its grandmother, what is the value of x?

29. How many integers between 2,000 and 3,999 have a ones digit that is a prime number?

30. How many integers between 2,000 and 6,999 are even and have a digit that is a prime number in the tens place?

31. A group of 12 people who have never met are in a classroom. How many handshakes are exchanged if each pair shakes hands exactly once?

(A) 12
(B) 22
(C) 66
(D) 132
(E) 244

32. A classroom has 12 girls and 20 boys. One quarter of the girls in the class have blue eyes. If a child is selected at random from the class, what is the probability that he/she is a girl who does not have blue eyes?

(A) 3/32
(B) 9/32
(C) 3/8
(D) 23/32
(E) 29/32

33. A coin with heads on one side and tails on the other has a 1/2 probability of landing on heads. If the coin is flipped three times, what is the probability of flipping 2 tails and 1 head, in any order?

(A) 1/8
(B) 1/3
(C) 3/8
(D) 5/8

(E) 2/3

34. A 6-sided cube has sides numbered 1 through 6. If the cube is rolled twice, what is the probability that at least one of the rolls will result in a number higher than 4?

(A) 2/9
(B) 1/3
(C) 4/9
(D) 5/9
(E) 2/3

35. Tiles are labeled with the integers from 1 to 100 inclusive; no numbers are repeated. If Alma chooses one tile at random, replaces it in the group, and chooses another tile at random, what is the probability that the product of the two integer values on the tiles is odd?

(A) 1/8
(B) 1/4
(C) 1/3
(D) 1/2
(E) 3/4

36. If the word "WOW" can be rearranged in exactly 3 ways (WOW, OWW, WWO), in how many ways can the word "MISSISSIPPI" be rearranged?

$\boxed{}$

37. If a, b, and c are integers randomly chosen from the set of prime numbers greater than 2 and less than 30, what is the probability that $ab + c$ is equal to 23?

$\boxed{}$

38.

The probability of rain is 1/2 on any given day next week.

Quantity A	**Quantity B**
The probability that it rains on AT LEAST one out of the 7 days next week	127/128

39. Two fair dice with sides numbered 1 to 6 are tossed. What is the probability that the sum of the exposed faces on the dice is a prime number?

$$\frac{\boxed{}}{\boxed{}}$$

40. Jack has a cube with 6 sides numbered 1 through 6. He rolls the cube repeatedly until the first time that the sum of all of his rolls is even, at which time he stops. (Note: it is possible to roll the cube just once.) What is the probability that Jack will need to roll the cube more than 2 times in order to get an even sum?

(A) 1/8
(B) 1/4
(C) 3/8
(D) 1/2
(E) 3/4

41. Jan and 5 other children are in a classroom. The principal of the school walks in and chooses two children at random. What is the probability that Jan is chosen?

(A) 4/5
(B) 1/3
(C) 2/5
(D) 7/15
(E) 1/2

42. The probability that Gary will eat eggs for breakfast on any given day is 3/7. The probability that Gary will eat cereal for breakfast on any given day is 4/7. Gary never has both eggs and cereal for breakfast on the same day.

Quantity A	**Quantity B**
Probability that Gary eats eggs or cereal for breakfast on a particular day	1

43. The probability that Maria will eat breakfast on any given day is 0.5. The probability that Maria will wear a sweater on any given day is 0.3. The two probabilities are independent of each other.

Quantity A	**Quantity B**
The probability that Maria eats breakfast or wears a sweater	0.8

44. The probability of rain in Greg's town on Tuesday is 0.3. The probability that Greg's teacher will give him a pop quiz on Tuesday is 0.2. The events occur independently of each other.

Quantity A	**Quantity B**
The probability that either or both events occur	The probability that neither event occurs

45. The probability of event *X* occurring is the same as the probability of event *Y* occurring. The events occur independently of each other.

Quantity A	**Quantity B**
The probability that both events occur	The probability that neither event occurs.

46. A certain city has a 1/3 chance of rain occurring on any given day. In any given 3-day period, what is the probability that the city experiences rain?

(A) 1/3
(B) 8/27
(C) 2/3
(D) 19/27

(E) 1

47. Five students, Adnan, Beth, Carol, Dan, and Edmund are to be arranged in a line. How many such arrangements are possible if Beth is not allowed to stand next to Dan?

(A) 24
(B) 48
(C) 72
(D) 96
(E) 120

48. A polygon has 12 edges. How many different diagonals does it have? (A diagonal is a line drawn from one vertex to any other vertex inside the given shape. This line cannot touch or cross any of the edges of the shape. For example, a triangle has zero diagonals and a rectangle has two.)

(A) 54
(B) 66
(C) 108
(D) 132
(E) 144

49. A student council is to be chosen from a class of 12 students consisting of a president, a vice president, and 3 committee members. How many such councils are possible?

(A) $\frac{12!}{7!5!}$
(B) $\frac{12!}{7!3!}$
(C) $\frac{12!}{5!3!}$
(D) $\frac{12!}{7!}$
(E) 12!

50.

Quantity A	**Quantity B**
The number of possible pairings of 2 colors that can be selected from 5 possible options	The number of possible pairings of 8 colors that can be selected from 9 possible options

51.

Quantity A	**Quantity B**
The number of possible 4-person teams that can be selected from 6 people	The number of possible 2-person teams that can be selected from 6 people

52.

Quantity A	**Quantity B**

The number of ways 1st, 2nd, and 3rd place prizes could be awarded to 3 out of 6 contestants	The number of ways 1st, 2nd, 3rd, 4th, and 5th place prizes could be awarded to 5 contestants

53. An inventory of coins contains 100 different coins.

Quantity A	**Quantity B**
The number of possible collections of 56 coins that can be selected where the order of the coins does not matter	The number of possible collections of 44 coins that can be selected where the order of the coins does not matter

54. An office supply store carries an inventory of 1,345 different products, all of which it categorizes as "business use," "personal use," or both. 740 products are categorized as "business use" ONLY and 520 products are categorized as both "business use" and "personal use."

Quantity A	**Quantity B**
The number of products characterized as "personal use"	600

55. How many distinct 4-letter "words" can be made from the name "CHRISTYNA"? (A "word" is any arrangement of 4 letters regardless of whether it can be found in a dictionary.)

(A) 9
(B) 24
(C) 36
(D) 504
(E) 3,024

56. Seiko has a 6-sided number cube with sides labeled 1 through 6. If she rolls the cube twice, what is the probability that the product of the two rolls is less than 36?

(A) 1/6
(B) 1/3
(C) 2/3
(D) 5/6
(E) 35/36

57. There is an 80% chance David will eat a healthy breakfast and a 25% chance that it will rain. If these events are independent, what is the probability that David will eat a healthy breakfast OR that it will rain?

(A) 20%
(B) 80%
(C) 85%
(D) 95%
(E) 105%

58. The probability of rain is 1/2 for every day next week. What is the chance that it rains on at least one day during the workweek (Monday through Friday)?

(A) 1/2
(B) 31/32
(C) 63/64
(D) 127/128

(E) 5/2

59. Eight women and two men are available to serve on a committee. If three people are picked, what is the probability that the committee includes at least one man?

(A) 1/32
(B) 1/4
(C) 2/5
(D) 7/15
(E) 8/15

60. At Lexington High School, everyone takes at least one language — Spanish, French, or Latin — but no one takes all three languages. If 100 students take Spanish, 80 take French, 40 take Latin, and 22 take exactly two languages, how many students are there?

(A) 198
(B) 220
(C) 242
(D) 264
(E) 286

61.

Of 60 birds found in a certain location, 20 are songbirds and 23 are migratory. (It is possible for a songbird to be migratory, or not.)

Quantity A	**Quantity B**
The number of the 60 birds that are neither migratory nor songbirds	16

Probability, Combinatorics, and Overlapping Sets Answers

1. **(C).** In a list of 10 consecutive integers, the mean will be the average of the 5th and 6th numbers. Therefore, the 6th through 10th integers (five total integers) will be larger than the mean. Since probability is determined by the number of desired items divided by the total number of choices, the probability that the number chosen is higher than the mean is 5/10 = 1/2.

Another approach to this problem is to create a set of 10 consecutive integers; the easiest such list contains the numbers {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. The mean is one-half the sum of the first element plus the last element, or $\frac{1+10}{2}=5.5$. Therefore, there are 5 numbers higher than the mean in the list: 6, 7, 8, 9 and 10. Again, the probability of choosing a number higher than the mean is 5/10 = 1/2.

2. **(B).** The first 100 positive integers comprise the set of numbers containing the integers 1 to 100. Of these numbers, the only ones that are divisible by 3 are {3, 6, 9, …, 96, 99}, which adds up to exactly 33 numbers. This can be determined in several ways. You could simply count the multiples of 3, but that's a bit slow. Alternatively, you can compute 99/3 = 33 and realize that there are 33 multiples of 3 up to and including 99. The number 100 is not divisible by 3, so the correct answer is 33/100.

Alternatively, you can use the "add one and you're done trick," subtracting the first multiple of 3 from the last multiple of 3, dividing by 3 and then adding 1: $\frac{(99-3)}{3}+1=33$. Then, since probability is determined by the number of desired options divided by the total number of options, the probability that the number chosen is a multiple of 3 is 33/100.

3. **180.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Therefore, the total number of tacos is (3)(4)(3)(5) = 180 tacos.

4. **256.** This question tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The five separate test questions give you five independent choices. For the three multiple choice questions there are four options each, whereas for the two true/false questions there are two options each. Multiplying the independent choices yields (4)(4)(4)(2)(2) = 256 different ways to answer the exam.

5. **250,000.** This question tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. There are 5 choices for each of the letters at the beginning of the code (A, B, C, D, or E) and 10 choices for each of the 4 digits at the end of the code (0, 1, 2, 3, 4, 5, 6, 7, 8, or 9). Therefore, there are (5)(5)(10)(10)(10)(10) = 250,000 possible codes.

6. **(B).** The probability of independent events A AND B occurring is equal to the product of the probability of event A and the probability of event B. In this case, the probability of the coin turning up heads is 1/2 and the product of rolling a 6 is 1/6. Therefore, the probability of heads AND a 6 is equal to (1/2)(1/6) = 1/12. Alternatively, you could list all the possible outcomes: H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6. There are 12 total outcomes and only 1 with heads and a 6. Therefore, the desired outcome divided by the total number of outcomes is equal to 1/12.

7. **8/19.** Among the integers 2 through 20 inclusive there are 8 primes: 2, 3, 5, 7, 11, 13, 17, and 19. From 2 to 20 inclusive there are exactly 20 - 2 + 1 = 19 integers; remember to "add one before you're done" to include both endpoints. Alternatively, there are 20 integers from 1 to 20 inclusive, so there must be 19 integers from 2 to 20 inclusive. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the number chosen is prime is 8/19.

8. **(E).** This problem describes a specific arrangement of people, so this is an ordering problem. The total number of ways to arrange n items in order is $n!$ (the exclamation point means "factorial"). Therefore, since there are 5 students to be arranged in a line, the total number of possible orderings is $5! = (5)(4)(3)(2)(1) = 120$.

Alternatively, ask, "How many choices do I have for each place in the line?" Consider the first place in line. Since no students have been chosen yet, there are 5 total options for the first place in line. Similarly, for the second place there are 4 choices, because one student has already been chosen to occupy the first spot. Applying the same logic, there are 3 choices for the third place, 2 for the fourth, and 1 for the fifth. Using the fundamental counting principle, there are a total of $(5)(4)(3)(2)(1) = 120$ different lines.

9. **10.** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Let the number of sauces be represented by the variable S. The total number of possible pasta dishes can be represented by each separate choice multiplied together: $(8)(4)(S)$, or $32S$. The problem also indicates that the total number of pasta dishes must be equal to 320. Therefore, $32S = 320$ and $S = 10$.

10. **720**. One possible approach is to ask, "How many choices do I have for each of the class positions?" Begin by considering the president of the class. Since no one has been chosen yet, there are 10 students from whom to choose. Then, for the vice president there are 9 options because now one student has already been chosen as president. Similarly, there are 8 choices for the secretary. Using the fundamental counting principle, the total number of possible selections is $(10)(9)(8) = 720$.

Alternatively, you could use factorials. In this case order matters because you are choosing people for specific positions. This problem is synonymous to asking, "How many different ways can you line up 3 students as first, second, and third from a class of 10?" The number of ways to arrange the entire class in line is 10!. However, the problem is only concerned with the first 3 students in line, so exclude rearrangements of the last 7. The way in which these "non-chosen" 7 students can be ordered is 7!. Thus the total number of arrangements for 3 students from a class of 10 is $\frac{10!}{7!} = (10)(9)(8) = 720$.

11. **(B).** This is a combinatorics problem — to calculate Quantity A, make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

____ ____ ____ ____

Since all 4 digits must be less than 5, the possible options are limited to 0, 1, 2, 3 and 4. However, a 4-digit number cannot begin with 0 (the smallest four-digit number is 1,000). So, the first slot has only 4 possibilities, not 5:

$\underline{\ 4\ }\quad \underline{\ 5\ }\quad \underline{\ 5\ }\quad \underline{\ 5\ }$

$(4)(5)(5)(5) = 500$, which is less than 625. The answer is (B).

Note that the value in Quantity B comes from multiplying $5 \times 5 \times 5 \times 5$. Neglecting to note that 4-digit numbers cannot begin with 0 results in incorrect choice (C).

12. **(E).** This problem tests the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. The key to this problem is realizing how many choices there are for each option. For the meat, there are obviously 3 choices. For each of the condiments there are exactly 2 choices: yes or no. The only real choice regarding each condiment is whether to include it at all. As there are 7 condiments, the total number of choices is $(3)(2)(2)(2)(2)(2)(2)(2) = (3)(2^7)$.

Note: the condiment options cannot be counted as 8! (0 through 7 = 8 options) because, in this case, the order in which the options are chosen does not matter; a burger with lettuce and pickles is the same as a burger with pickles and lettuce.

13. **(A).** For probability questions, always begin by separating out the probabilities of each individual event. Then, if you need all the events to happen (an "AND question"), multiply the probabilities together. If you only need one of the multiple events to happen (an "OR question"), add the probabilities together.

In this case, there are two events: rain on Monday and rain on Tuesday. The question asks for the probability that it will rain on Monday AND on Tuesday, so multiply the individual probabilities together:

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

14. **(D).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. The problem asks how many five-digit numbers can be created from the digits 5, 6, 7, 8, 9, and 0. For the first digit, there are only five options (5, 6, 7, 8, and 9) because a five-digit number must start with a non-zero integer. For the second digit, there are 5 choices again, because now zero can be used but one of the other numbers has already been used, and numbers cannot be repeated. For the third number, there are 4 choices, for the fourth there are 3 choices, and for the fifth number there are 2 choices. Thus, the total number of choices is $(5)(5)(4)(3)(2) = 600$.

Alternatively, you can use the same logic and realize there are 5 choices for the first digit. (Separate out the first step because you have to remove the zero from consideration.) The remaining five digits all have an equal chance of being chosen, so choose four out of the remaining five digits to complete the number. The number of ways in which this second step can be accomplished is $(5!)/(1!) = (5)(4)(3)(2)$. Thus, the total number of choices is again equal to $(5)(5)(4)(3)(2) = 600$.

15. **5/6.** In the bag of marbles, there are 3 red marbles and 7 white marbles, for a total of 10 marbles that are NOT blue. There are a total of $3 + 7 + 2 = 12$ marbles in the bag. Since probability is defined as the number of desired items divided by the total number of choices, the probability that the marble chosen is not blue is $10/12 = 5/6$.

16. **120.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. Since the man must choose one suit, one shirt, one pair of socks, and one pair of shoes, the total number of outfits is the number of suits times the number of shirts times the number of socks times the number of shoes: $(3)(4)(2)(5) = 120$.

17. **(A).** This is a combinatorics problem. The license plates have 2 letters and 4 numbers, so make six "slots" and

determine how many possibilities there are for each slot. There are 26 letters in the alphabet and 10 digits to pick from, so:

$$\underline{\ 26\ }\quad \underline{\ 26\ }\quad \underline{\ 10\ }\quad \underline{\ 10\ }\quad \underline{\ 10\ }\quad \underline{\ 10\ }$$

Multiply 26 × 26 on your calculator to get 676. Add four zeroes for the four 10's to get 6,760,000. Quantity A is larger.

18. **(A).** First, calculate the total number of combinations, ignoring the one illegal "word." Since there are 10 possibilities for the first slot and 20 possibilities for the other 4 slots:

$$\underline{\ 10\ }\quad \underline{\ 20\ }\quad \underline{\ 20\ }\quad \underline{\ 20\ }\quad \underline{\ 20\ }$$

Multiply to get 1,600,000.

Now, consider all the license plates that contain the forbidden word. Say, for example, that the forbidden word is GURG. That means that the plates 0GURG, 1GURG, 2GURG, 3GURG, etc. are forbidden, for a total of 10 forbidden plates. You can also express this mathematically as:

$$\underline{\ 10\ }\quad \underline{\ 1\ }\quad \underline{\ 1\ }\quad \underline{\ 1\ }\quad \underline{\ 1\ }$$

Multiply to get 10 forbidden plates. Subtract: 1,600,000 - 10 = 1,599,990.

Alternatively, after finding the 1,600,000 figure, glance at the answer choices: the correct answer can't be (C), (D), or (E). Answer choice (B) subtracts just 1 plate, but more than 1 plate could have the forbidden "word." Therefore, (A) must be the correct answer.

19. $\mathbf{\frac{1}{121}}$ The probability of picking black chip #3 is $\frac{1}{12}$. Once Pavel has removed the first chip, only 11 chips remain, so the probability of picking white chip #3 is $\frac{1}{11}$. Multiply $\frac{1}{12}\times\frac{1}{11}=\frac{1}{121}$.

20. **(B).** In this problem, Tarik is NOT picking 1 chip out of all 12. Rather, he is picking 1 chip out of 6 green ones, and then picking another chip out of 6 blue ones. There are 3 green chips with numbers less than 4, so Tarik has a $\frac{3}{6}$ chance of selecting a green chip showing a number less than 4. Likewise, Tarik has a $\frac{3}{6}$ chance of selecting a blue chip showing a number less than 4. Therefore, Quantity A is equal to $\frac{3}{6}\times\frac{3}{6}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$. Quantity B is larger.

21. $\mathbf{\frac{1}{22}}$. The trap answer in this problem is $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$. This is NOT the answer to the question being asked—

rather, this is the answer to the question, "What is the probability of picking a red chip and then a blue chip that both have #3?" (or any other specific number). This is a more specific question than the one actually asked. In the question, asked, there are six possible ways to fulfill the requirements of the problem, not one, because the problem does not specify whether the number should be 1, 2, 3, 4, 5, or 6.

Thus, ANY of the 6 red chips is acceptable for the first pick. However, on the second pick, only the blue chip with the same number as the red one that was just picked is acceptable (you need whatever chip is the "match" for the first one picked). Thus:

$$\frac{6}{12} \times \frac{1}{11} = \frac{1}{2} \times \frac{1}{11} = \frac{1}{22}$$

22. **(B).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 - Both + Neither. (Adding the two groups—in this case Latin and Spanish—double-counts the students who take both classes, so the formula subtracts the "both" students.)

$150 = 75 + 110 - B + 11$
$150 = 196 - B$
$46 = B$

Careful! This is not the value of Quantity A. Since 46 students take both Latin and Spanish, subtract 46 from the total who take Latin to find those who take only Latin:

$75 - 46 = 29$

Thus, Quantity A is equal to 29 and Quantity B is larger.

23. **(A).** This problem relies on the fundamental counting principle, which says that the total number of ways for something to happen is the product of the number of options for each individual choice. For any digit of the 10-digit number there are exactly two options, a 2 or a 5. Thus, since there are two choices for each digit and it is a 10-digit number, there are $(2)(2)(2)(2)(2)(2)(2)(2)(2)(2) = 2^{10}$ total choices.

24. **(C).** The probability of any event equals the number of ways to get the desired outcome divided by the total number of ways for the event to happen. Starting with the denominator, use the fundamental counting principle to compute the total number of ways to roll a cube twice. There are 6 possibilities (1, 2, 3, 4, 5, or 6) for the first roll and 6 for the second, giving a total of (6)(6) = 36 possibilities for the two rolls. For the numerator, determine the number of possible combinations that will add to 8. For example, you might roll a 2 the first time and a 6 the second time. The full set of options is (2,6), (3,5), (4,4), (5,3), and (6,2). Thus there are 5 possible combinations that sum to 8, yielding a probability of 5/36.

25. **16.** This problem utilizes the fundamental counting principle, which states that the total number of choices is equal to the product of the independent choices. For the first flip there are 2 options: heads or tails. Similarly, for the second flip there 1 2 options, for the third there are 2 options, for the fourth there are 2 options, and for the fifth there is only one option because the problem restricts this final flip to heads. Therefore, the total number of outcomes is (2)(2)(2)(2)(1) = 16. A good rephrasing of this question is, "How many different outcomes are there if the coin is flipped 4 times?" The fifth flip, having been restricted to heads, is irrelevant. Therefore, the total number of ways to flip the coin five times with heads for the fifth flip is equal to the total number of ways to flip the coin four times; either way, the answer is 16.

26. **(C).** The problem specifies that no one takes all three languages. In addition, a total of 5 people take 2 languages. Thus, 5 people have been double-counted. Because you know the total number of people who have been double-counted (5) and triple-counted (0), you can use the standard overlapping sets formula:

Total = Spanish + French + Latin - (Two of the Three) - 2(All Three)

25 = 9 + French + 7 - 5 - 2(0)
25 = 11 + French
14 = French

The two quantities are equal.

27. $\mathbf{\frac{1}{3}}$ Probability equals the number of desired outcomes divided by the total number of possible outcomes. Among the integers 1 through 24 there are 4 factor pairs of 24: (1, 24), (2, 12), (3, 8), and (4, 6), for a total of 8 factors. The total number of possible outcomes when rolling the die once is 24. The probability that the number chosen is a factor of 24 is 8/24 = 1/3.

28. **17.** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 - Both + Neither. Here, the groups are "stuffed animal" and "given by the baby's grandmother." The problem indicates that the "both" category is equal to 5, and that the "neither" number is 6. The total is x.

Total = Group 1 + Group 2 - Both + Neither

$x = 9 + 7 - 5 + 6$
$x = 17$

29. **800.** This is a combinatorics problem. Make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

$\underline{\qquad}\quad\underline{\qquad}\quad\underline{\qquad}\quad\underline{\qquad}$

Since the number must begin with 2 or 3, there are 2 possibilities for the first slot. Because the ones digit must be prime and there are only 4 prime 1-digit numbers (2, 3, 5, and 7), there are 4 possibilities for the last slot.

$\underline{\quad 2 \quad}\quad\underline{\qquad}\quad\underline{\qquad}\quad\underline{\quad 4 \quad}$

The other slots have no restrictions, so put 10, since there are 10 digits from 0–9:

$\underline{\quad 2 \quad}\quad\underline{\quad 10 \quad}\quad\underline{\quad 10 \quad}\quad\underline{\quad 4 \quad}$

Multiply to get 800.

Alternatively, figure out the pattern and add up the number of qualifying 4-digit integers. In the first ten numbers, 2000–2009, there are exactly 4 numbers that have a prime units digit: 2002, 2003, 2005, and 2007. The pattern then repeats in the next group of ten numbers, 2010–2020, and so on. In any group of ten numbers, then, four qualify. In the first one hundred numbers, 2000–2099, there are ten groups of ten, or 10 × 4 = 40 numbers that have a prime units digit. In the first one thousand numbers, 2000–2999, there are ten groups of one hundred, or 100 × 4 = 400 numbers that have a prime units digit. There are a total of two groups of one thousand numbers (2000–2999 and 3000–3999), so there are a total of 400 × 2 = 800 numbers that have a prime units digit.

30. **1,000.** This is a combinatorics problem. Make four "slots" (since the numbers are all four-digit numbers), and determine how many possibilities there are for each slot:

____ ____ ____ ____

Since the number must begin with 2, 3, 4, 5, or 6, there are 5 possibilities for the first slot. Because the digit in the tens place must be prime and there are only 4 prime 1-digit numbers (2, 3, 5, and 7), there are 4 options for the tens place.

$\underline{\;5\;}$ ____ $\underline{\;4\;}$ ____

Since the number must be even, the final digit must be 0, 2, 4, 6, or 8—thus, there are 5 possibilities for the units digit. (Note that saying that a 4-digit number is even is simply saying that it ends in an even digit—for instance, 3,792 is even.) No restrictions were given for the hundreds slot, so there are 10 options, since there are 10 digits from 0–9.

$\underline{\;5\;}$ $\underline{\;10\;}$ $\underline{\;4\;}$ $\underline{\;5\;}$

Multiply to get 1,000.

Alternatively, figure out the pattern and add up the number of qualifying 4-digit integers. In the first one hundred numbers, 2,000–2,099, there are exactly four sets of ten numbers that have a prime tens digit: 202-, 203-, 205-, and 207-. Within each of those four sets, there are five possibilities for numbers that also end in an even digit. For example, the possibilities for the 202- set would be 2,020, 2,022, 2,024, 2,026, and 2,028. Out of the first one hundred numbers, then, there are 5 × 4 = 20 possibilities that have both a prime tens digit and end in an even digit.

From 2,000 to 6,999, there are 50 groups of 100 numbers (6,999 - 2,000 + 1 = 5,000, and 5,000 ÷ 100 = 50). Therefore, there are 20 × 50 = 1,000 numbers.

31. **(C).** Multiple approaches are possible here. One way is to imagine the scenario and count up the number of handshakes: you are the first person. How many hands do you need to shake? There are 11 other people in the room, so you need to shake hands 11 times. Now, move to the second person: how many hands must he shake? He has already shaken your hand, leaving him 10 others with whom to shake hands. The third person will need to shake hands with 9 others, and so on. Therefore, there are a total of 11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 handshakes. The fastest way to find the sum of a group of consecutive numbers is to take the average of the first and last terms and multiply it by the number of terms. The average is (11 + 1)/2 = 6 and there are 11 - 1 + 1 = 11 terms (find the difference between the terms and "add one before you're done"). The sum is 6 × 11 = 66.

Alternatively, rephrase the question as "How many different ways can any 2 people be chosen from a group of 12?" (This works because the problem ultimately asks you to "choose" each distinct pair of 2 people one time.) The key here is to realize that handshakes are independent of order, i.e., it doesn't matter if A shakes hands with B or if B shakes hands with A; it's the same outcome. Thus, you only care about how many pairs you can make. Any time you recognize a group of order-independent items being selected from a larger set, you can apply the formula $\frac{\text{total!}}{\text{in!out!}}$

to arrive at the total number of combinations. Thus: $\frac{12!}{2!10!}=\frac{12\cdot 11}{2}=66$.

32. **(B).** The probability of any outcome is equal to the number of desired outcomes divided by the total number of outcomes. There are 12 girls and 20 boys in the classroom. If one-quarter of the girls have blue eyes, then there are (12)(1/4) = 3 girls with blue eyes. Therefore, there are 12 - 3 = 9 girls who do NOT have blue eyes. The total number of ways in which you could choose a child is simply the total number of children in the class, namely 12 + 20 = 32. Therefore, the probability of choosing a girl who does not have blue eyes equals the number of girls without blue eyes divided by the total number of children, which is 9/32.

33. **(C).** There are only 2 possible outcomes for each flip and only 3 flips total. The most straightforward approach is to list all of the possible outcomes: {HHH, HHT, HTH, HTT, TTT, TTH THH, THT}. Of these 8 possibilities, 3 of the outcomes have one head and two tails, so the probability of this event is 3/8.

Alternatively, you can count the total number of ways of getting 1 head without listing all the possibilities. If the coin is flipped 3 times and you want only 1 head, then there are 3 possible positions for the single head: on the first flip alone, on the second flip alone, or on the third flip alone. Since there are 2 possible outcomes for each flip, heads or tails, there are (2)(2)(2) = 8 total outcomes. Again, the probability is 3/8.

Finally, you can compute the probability directly. The probability of flipping heads is 1/2 and the probability of flipping tails is also 1/2. The probability of getting heads in the first position alone, or HTT, is (1/2)(1/2)(1/2) = 1/8, where you multiply because you have heads AND tails AND tails. This represents the probability of heads in position 1, but heads could also be in position 2 alone or in position 3 alone. Since there are 3 possible positions for the heads, multiply by 3 to get the total probability (3)(1/8) = 3/8.

34. **(D).** Because this problem is asking for an "at least" solution, you can use the 1 - *x* shortcut. The probability that at least one roll results in a number higher than 4 is equal to 1 minus the probability that both of the rolls result in numbers 4 or lower. For one roll, there are 6 possible outcomes (1 through 6) and 4 ways in which the outcome can be 4 or lower, so the probability is 4/6 = 2/3. Thus, the probability that both rolls results in numbers 4 or lower is (2/3) (2/3) = 4/9. This is the result that you do NOT want; subtract this from 1 to get the probability that you do want. The probability that at least one of the rolls results in a number higher than 4 is 1 - (4/9) = 5/9.

Alternatively, write out the possibilities. The total number of possibilities for two rolls is (6)(6) = 36. Here are the ways in which at least one number higher than 4 can be rolled:

51, 52, 53, 54, 55, 56
61, 62, 63, 64, 65, 66
15, 25, 35, 45 (note: 55 and 65 have already been counted above)
16, 26, 36, 46 (note: 56 and 66 have already been counted above)

There are 20 elements (be careful not to double-count any options). The probability of at least one roll resulting in a number higher than 4 is 20/36 = 5/9.

35. **(B).** You need to use both probability and number properties concepts in order to answer this question. First, in order for two integers to produce an odd integer, the two starting integers must be odd. An odd times an odd equals an odd. An even times an odd, by contrast, produces an even, as does an even times an even.

Within the set of tiles, there are 50 even numbers (2, 4, 6, …, 100) and 50 odd numbers (1, 3, 5, …, 99). One randomly-chosen tile will have a 50/100 = 1/2 probability of being even, and a 1/2 probability of being odd. The probability of choosing an odd tile first is (1/2) and the probability of choosing an odd tile second is also (1/2), so the probability of "first odd AND second odd" is (1/2)(1/2) = 1/4.

Alternatively, you can recognize that there are only four options for odd/even pairs if two tiles are chosen: OO, OE, EO, EE. The only one of these combinations that yields an odd product is OO. Since all of these combinations are equally likely, and since OO is exactly one out of the four possibilities, the probability of choosing OO is 1/4.

36. **34,650.** This is a combinatorics problem, and the WOW example is intended to make it clear to you that any W is considered identical to any other W — switching one W with another would NOT result in a different combination, just as switching one S with another in MISSISSIPPI would not result in a different combination.

Therefore, solve this problem using the classic combinatorics formula for accounting for subgroups among which order does not matter:

<u>Total Number of Items!</u>
First Group! Second Group! Etc…

Because MISSISSIPPI has 11 letters, including one M, four S's, four I's, and two P's:

$$\frac{11!}{1!4!4!2!}$$

Now expand the factorials and cancel; use the calculator for the last step of the calculation:

$$\frac{11\times10\times9\times8\times7\times6\times5\times\cancel{4!}}{\cancel{1}\cancel{4!}(4\times3\times2\times1)(2\times1)}=\frac{11\times10\times9\times\cancel{8}\times7\times\cancel{6}\times5}{(\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1})(\cancel{2}\times\cancel{1})} = 11 \times 10 \times 9 \times 7 \times 5 = 34{,}650$$

37. **0.** Because *a, b,* and *c* are prime numbers greater than 2, they must all be odd. So, $ab + c$ = (ODD)(ODD) + ODD. ODD times ODD yields another odd number, so the calculation simplifies to ODD + ODD, which will always yield an even answer.

Thus, it is impossible for $ab + c$ to be odd. The probability is zero. (Note: if you were ever asked to type a nonzero probability into a box, you would need to express it as a decimal between 0 and 1.)

38. **(C).** Since Quantity A is an "at least" problem, you can use the 1 - x shortcut. Rather than calculate the probability of rain on exactly 1 day next week, and then the probability of rain on exactly 2 days next week, and so on (after which you would still have to add all of the probabilities together!), instead calculate the probability of no rain at all on any day, and then subtract that number from 1. That will give the combined probabilities for any scenarios that include rain on at least 1 day.

Probability of NO rain for any of the 7 days = $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{128}$

Subtract this probability from 1:

$$1-\frac{1}{128}=\frac{128}{128}-\frac{1}{128}=\frac{127}{128}$$

Quantities A and B are equal.

39. **5/12.** First think about the prime numbers less than 12, the maximum sum of the numbers on the dice. These primes are 2, 3, 5, 7, 11.

The probability of rolling 2, 3, 5, 7, or 11 = the number of ways to roll any of these sums, divided by the total number of possible rolls. The total number of possible dice rolls is 6 × 6 = 36.

Sum of 2 can happen 1 way: 1 + 1
Sum of 3 can happen 2 ways: 1 + 2 or 2 + 1
Sum of 5 can happen 4 ways: 1 + 4, 2 + 3, 3 + 2, 4 + 1
Sum of 7 can happen 6 ways: 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2, 6 + 1
Sum of 11 can happen 2 ways: 5 + 6, 6 + 5

That's a total of 1 + 2 + 4 + 6 + 2 = 15 ways to roll a prime sum.

Thus, the probability is 15/36 = 5/12.

40. **(B).** Jack will only continue to roll the cube if the sum of the individual rolls is odd. For the first roll, this will only occur if the number itself is odd; if Jack does not stop after the first roll, then, he must have rolled an odd number for the first roll. For the second roll, in order for the sum of the first and second to be odd, Jack must now roll an even (because odd + even = odd). You can rephrase the question: "What is the probability that Jack will roll an odd first and an even second?" The probability of event A AND event B equals the probability of A times the probability of B. Since the probability of odd = 1/2 and the probability of even = 1/2, the probability of the first number being odd AND the second number being even is (1/2)(1/2) = 1/4.

41. **(B).** The probability of any event equals the number of ways to get the desired outcome divided by the total number of outcomes.

Start with the denominator, which is the total number of ways that the principal can choose two children from the classroom. Use the fundamental counting principle. There are 6 possible options for the first choice and 5 for the second, giving (6)(5) = 30 possibilities. However, this double-counts some cases; for example, choosing Jan and then Robert is the same as choosing Robert and then Jan. Divide the total number of pairs by $\frac{6\cdot 5}{2}=15$. Alternatively, use the formula for a set where the order doesn't matter: $\frac{\text{total!}}{\text{in! out!}}$. In this case: $\frac{6!}{2!4!}=\frac{6\times 5\times 4!}{(2)(4!)}=\frac{6\times 5}{2}=15$.

Now compute the numerator, which is the number of pairs that include Jan. Since the pair only includes two children and one is already decided (Jan), there are exactly 5 options for the other child. Thus, there are 5 total pairs that include Jan: Jan with each of the other students.

The probability of choosing a pair with Jan is 5/15 = 1/3.

As a final alternative, you may simply list all the pairs of students and count how many of them include Jan. Label the students in the class as J, 1, 2, 3, 4, and 5, where J is Jan. Then all the pairs can be listed as (J1), (J2), (J3), (J4), (J5), (12), (13), (14), (15), (23), (24), (25), (34), (35), and (45). (Be careful not to include repeats.) There are 15 total elements in this list and 5 that include Jan, yielding a probability of $5/15 = 1/3$.

42. **(C).** The probability that Gary will eat eggs is 3/7. The probability that he will eat cereal is 4/7. In order to calculate the probability of eating one OR the other, add: $3/7 + 4/7 = 7/7 = 1$. Quantities A and B are the same.

43. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, the first step is to calculate the "or" situation, but you cannot stop there. When you add $0.5 + 0.3 = 0.8$, you double-count the occurrences when both events occur. Next, you have to subtract out the probability of both events occurring in order to get rid of the "double counted" occurrences.

Notice that this is a Quantitative Comparison. At this point, you could conclude that, because the 0.8 figure includes at least one "both" occurrence, the real figure for Quantity A must be smaller than 0.8. Therefore, Quantity B must be larger.

To do the actual math, find the probability of both events occurring (breakfast AND sweater): $(0.5)(0.3) = 0.15$. Subtract the "AND" occurrences from the total "or" probability: $0.8 - 0.15 = 0.65$

Quantity B is larger.

44. **(B).** The problem indicates that the events occur independently of each other. Therefore, in calculating Quantity A, do not simply add both events, even though it is an "or" situation. Adding $0.3 + 0.2 = 0.5$ is incorrect because the probability that both events occur is counted twice. (Only add probabilities in an "or" situation when the probabilities are mutually exclusive.)

While Quantity A's value should include the probability that both events occur, make sure to count this probability only once, not twice. Since the probability that both events occur is $0.3(0.2) = 0.06$, you must subtract this value from the "or" probability.

Quantity A: Add the two probabilities (rain OR pop quiz) and subtract BOTH scenarios (rain AND pop quiz):

$$0.3 + 0.2 - (0.3)(0.2) = 0.44$$

Quantity B: Multiply the probability that rain does NOT occur (0.7) and the probability that the pop quiz does NOT occur (0.8).

$$0.7(0.8) = 0.56$$

You could also note that, collectively, Quantities A and B include every possibility and are mutually exclusive of one another (Quantity A includes "rain and no quiz," "quiz and no rain," and "both rain and quiz," and Quantity B includes "no rain and no quiz"). Therefore, the values of Quantities A and B must add to 1. Calculating the value of either Quantity A or Quantity B would automatically tell you the value for the other quantity.

If you do this, calculate Quantity B first (because it's the easier of the two quantities to calculate) and then subtract

Quantity B from 1 in order to get Quantity A's value. That is, 1 - 0.56 = 0.44.

45. **(D).** The probabilities of both events are the same *as each other,* but that doesn't indicate anything about the value of those probabilities. For instance, if both probabilities are equal to 0.99, then the probability of both events occurring (Quantity A) is MUCH higher than the probability of neither occurring (Quantity B).

But what if the probability of each event is more like 0.000001 ("1 in a million")? Then the chance of neither occurring (Quantity B) would be much higher than the chance of both occurring (Quantity A).

In the case that the probabilities are each equal to 0.5, then — and only then — would the two quantities be equal.

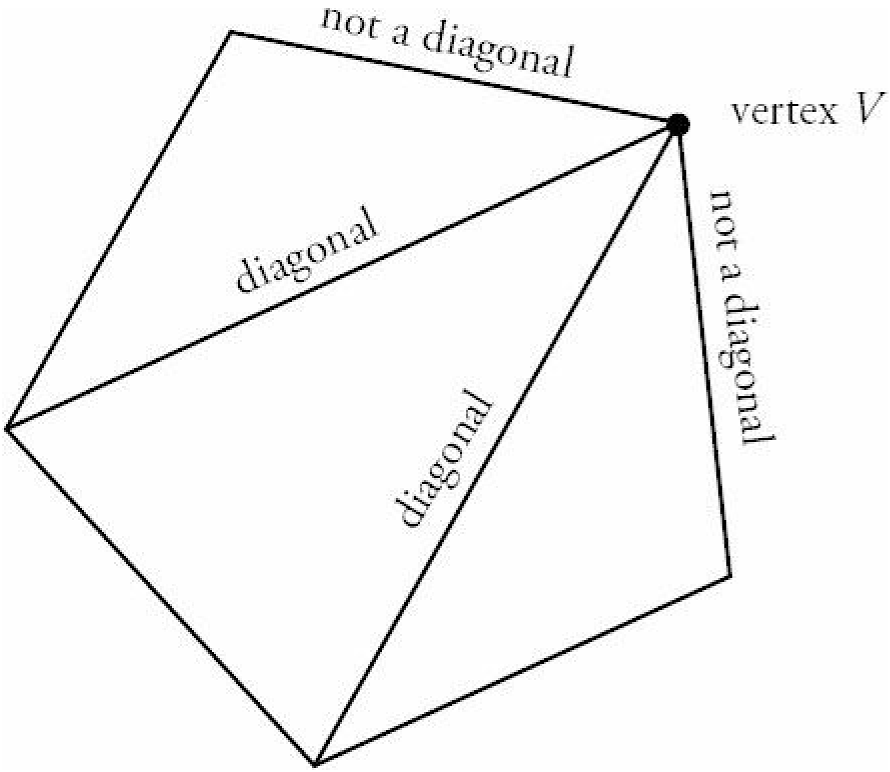
46. **(D).** In essence, the question is asking "What is the probability that one or more days are rainy days?" since any single rainy day would mean the city experiences rain. In this case, employ the 1 - x shortcut, where the probability of rain on one or more days is equal to 1 minus the probability of no rain on any day. Since the probability of rain is 1/3 on any given day, the probability of no rain on any given day is 1 - 1/3 = 2/3. Therefore, the probability of no rain on three consecutive days is (2/3)(2/3)(2/3) = 8/27. Finally, subtract from 1 to find the probability that it rains on one or more days: P(1 or more days) = 1 - P(no rain) = 1 - 8/27 = 19/27.

47. **(C).** The number of ways in which the students can be arranged with Beth and Dan separated is equal to the total number of ways in which the students can be arranged minus the number of ways they can be arranged with Beth and Dan together. The total number of ways to arrange 5 students in a line is 5! = 120. To compute the number of ways to arrange the 5 students such that Beth and Dan are together, group Beth and Dan as "one" person, since they must be lined up together. Then the problem becomes one of lining up 4 students, which gives 4! possibilities. However, remember that there are actually two options for the Beth and Dan arrangement: Beth first and then Dan or Dan first and then Beth. Therefore, there are (4!)(2) = (4)(3)(2)(1)(2) = 48 total ways in which the students can be lined up with Dan and Beth together. Finally, there are 120 - 48 = 72 arrangements where Beth will be separated from Dan.

Alternatively, compute the number of ways to arrange the students directly by considering individual cases. In this case, investigate how many ways there are to arrange the students if Beth occupies each spot in line and sum them to find the total. If Beth is standing in the first spot in line, then there are 3 options for the second spot (since Dan cannot occupy this position), 3 options for the next spot, 2 options for the next spot, and finally 1 option for the last spot. This yields (3)(3)(2)(1) = 18 total possibilities if Beth is first. If Beth is second, then there are 3 options for the first person (Dan cannot be this person), 2 options for the third person (Dan cannot be this person either), 2 options for the fourth person, and 1 for the fifth. This yields (3)(2)(2)(1) = 12 possibilities. In fact, if Beth is third or fourth in line, you arrive at the same situation as when Beth is second. Thus there will be 12 possible arrangements whether Beth is 2nd, 3rd, or 4th in line, yielding 36 total arrangements for these 3 cases. Using similar logic, the situation in which Beth is last in line is exactly equal to the situation where she is first in line. Thus, there are (18)(2) = 36 possibilities where Beth is first or last. In total, this yields 36 + 36 = 72 possible outcomes when considering all of the possible placements for Beth.

48. **(A).** A diagonal of a polygon is an internal line segment connecting any two unique vertices; this line segment does not lie along an edge of the given shape. Consider a polygon with 12 vertices. Construct a diagonal by choosing any two vertices and connecting them with a line. Remember that this is order independent; the line is the same regardless of which is the starting vertex. Therefore, this is analogous to choosing any 2 elements from a set of 12, and can be written as $\frac{12!}{10!2!}=\frac{12\times11\times10!}{10!(2)(1)}=\frac{12\times11}{2}=6\times11=66$. However, this method includes the vertices connected to their adjacent vertices, which form edges instead of diagonals. In order to account for this, subtract the number of edges on the polygon from the above number: 66 - 12 = 54.

Alternatively, if you choose a random vertex of the 12-sided shape, then there are 12 - 1 = 11 lines that can be drawn to other vertices since no line can be drawn from the vertex to itself. However, the lines from this vertex to the two adjacent vertices will lie along the edges of the polygon and therefore cannot be included as diagonals (see the figure of a pentagon below for an example).



Thus, there are 12 - 1 - 2 = 9 diagonals for any given vertex. Since there are 12 vertices, you might think that the total number of diagonals is equal to (12)(9) = 108. However, using this scheme you have counted each diagonal twice, using each side of the diagonal once as the starting point. Therefore, there are half this many different diagonals: 108/2 = 54.

49. **(B).** Consider each independent choice and then use the fundamental counting principle to calculate the total number of possibilities. Start by choosing the president, for which there are 12 total options. Next, for the vice president, there are 11 options. Finally, choose the committee of 3 from the remaining 10 students using the formula for a set where the order doesn't matter: $\frac{\text{total!}}{\text{in! out!}}$. The number of possible arrangements for the 3 committee members is $\frac{10!}{7!3!}$. Finally, using the fundamental counting principle, the total number of ways to choose a president, vice president, and committee of 3 is $\frac{12 \times 11 \times 10!}{7!3!}$ which can be rewritten as $\frac{12!}{7!3!}$.

50. **(A).** This is a classic combinatorics problem in which *order doesn't matter*—that is, the pairing "blue, green" is the same as "green, blue." A color is either "in" or "out." Use the standard "order doesn't matter" formula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Quantity A:

$$\frac{5!}{2!3!}=\frac{5\times4\times3!}{(2)(1)3!}=\frac{5\times4}{2}=10$$

For Quantity B:

$$\frac{9!}{8!1!}=\frac{9\times8!}{8!(1)}=9$$

Note that while the formula is necessary for Quantity A, you could reason your way to the value for Quantity B: every combination that selects 8 out of 9 colors will leave out exactly 1 color. Since there are 9 colors, there are 9 combinations.

51. **(C).** This is a classic combinatorics problem in which *order doesn't matter* — that is, picking Joe and Jane is the same as picking Jane and Joe. A person is either on the team or not. Use the standard "order doesn't matter" formula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Quantity A:

$$\frac{6!}{2!4!}=\frac{6\times5\times4!}{(2)(1)4!}=\frac{6\times5}{2}=15$$

For Quantity B:

$$\frac{6!}{4!2!}=\frac{6\times5\times4!}{4!(2)(1)}=\frac{6\times5}{2}=15$$

The quantities are equal. Note the first line of each Quantity: from that stage, you can already tell that the values will be the same.

This will always work—when order doesn't matter, the number of ways to pick 4 and leave out 2 is the same as the number of ways to pick 2 and leave out 4. Either way, it's one group of 4 and one group of 2. What actually happens to those groups (getting picked, not getting picked, getting a prize, losing a contest, etc.) is irrelevant to the ultimate solution.

52. **(C).** In this problem, order matters; if Jane comes in 1st place and Rohit comes in 2nd, there is a different outcome than when Rohit places 1st and Jane places 2nd. Use the fundamental counting principle to solve. To determine Quantity A make three slots (one for each prize). Six people are available to win 1st, and then five people could win 2nd, and four people could win 3rd:

$$\underline{\ 6\ }\quad\underline{\ 5\ }\quad\underline{\ 4\ }$$

Multiply: (6)(5)(4) = 120.

For Quantity B, make 5 slots, one for each prize. Five people can win 1st prize, then 4 people for 2nd prize, and so on:

$$\underline{\ 5\ }\quad\underline{\ 4\ }\quad\underline{\ 3\ }\quad\underline{\ 2\ }\quad\underline{\ 1\ }$$

Multiply (5)(4)(3)(2)(1) = 120. Quantities A and B are equal.

53. **(C).** This is a classic combinatorics problem in which *order doesn't matter* — in fact, the problem tells you that explicitly. Use the standard "order doesn't matter" formula:

$$\frac{\text{Everything!}}{\text{Picked! NotPicked!}}$$

For Quantity A:

$$\frac{100!}{56!44!}$$

Because the numbers are so large, there must be a way to solve the problem without actually simplifying (even with a calculator, this is unreasonable under GRE time limits). Try Quantity B and compare:

$$\frac{100!}{44!56!}$$

The quantities are equal. Note that this will always work — when order doesn't matter, the number of ways to pick 56 and leave out 44 is the same as the number of ways to pick 44 and leave out 56. Either way, it's one group of 56 and one group of 44. What actually happens to those groups (being part of a collection, being left out of the collection, etc.) is irrelevant to the ultimate solution.

54. **(A).** Use the overlapping sets formula for two groups: Total = Group 1 + Group 2 - Both + Neither. But first, add 740 ("business use ONLY") + 520 ("business use" and "personal use") to get 1260, the total number of products categorized as "business use."

Also note that the problem indicates that *all* of the products fall into one or both of the two categories, so "neither" in this formula is equal to zero.

Total = Business + Personal - Both + Neither
1,345 = 1,260 + P - 520 + 0
1,345 = 740 + P
605 = P

Quantity A is larger. Note that the question asked for the number of products characterized as "personal use" (which includes products in the "both" group). If the problem had asked for the number of products characterized as "personal use" ONLY, you would have had to subtract the "both" group to get 605 - 520 = 85. In this problem, however, Quantity A equals 605.

55. **(E).** Because all of the letters of the name "Christyna" are unique, there are 9 distinct choices of letters to form the 4-letter "word." In addition, order does matter: the "word" CHRI is different from the "word" CRHI. Use the fundamental counting principle to solve.

Begin by considering the first choice, for which there are 9 total options. Similarly, for the second choice there are 8 options, because one letter has already been chosen. Employing the same logic, there are 7 choices for the third letter and 6 choices for the fourth letter. Using the fundamental counting principle, (9)(8)(7)(6) = 3024 words.

Alternatively, try factorials. The total number of ways to arrange all 9 letters is 9! However, the problem is only concerned with "words" using four of these letters, meaning you must exclude rearrangements of the other 5. The number of ways in which you can order the 5 "non-chosen" letters is 5!. Thus the total number of "words" with 4 letters that can be made from the name "Christyna" is $\frac{9!}{5!}$ = (9)(8)(7)(6) = 3,024 "words."

56. **(E).** Use the 1 - *x* shortcut for this problem. How do you know to use this shortcut? In this case, you're being asked to solve for the probability that the product of the values from the two rolls will be less than 36. This will be time consuming to calculate directly because there are many different combinations that would produce a product less than 36 — (1)(6), (4)(5), (6)(3), and so on — the vast majority of combinations, actually! In fact, there's only one case when the product of the two rolls will *not* be less than 36: (6)(6). It is much easier to solve for this one value and then apply the 1 - *x* shortcut. In other words:

(The probability of rolling less than 36) + (The probability of rolling 36) = 1 $\longrightarrow$
(The probability of rolling less than 36) = 1 - (The probability of rolling 36)

This is an "AND question," because you need to get a 6 on the first roll *and* on the second roll. Multiply the two probabilities together:

$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Subtract from 1:

$$1 - \frac{1}{36} = \frac{35}{36}$$

57. **(C).** For probability questions, begin by separating out the probabilities of each individual event. Then, if all of the events must happen (an "AND question"), multiply the probabilities together. If only one of the multiple events needs to happen (an "OR question"), add. This question is an OR question, because it asks for the probability that David will eat a healthy breakfast *or* that it will rain.

At first glance, this may seem strange, because if you add the two probabilities together, you'll get something bigger than 100%, which is NEVER possible: $0.8 + 0.25 = 1.05$. This figure double-counts the cases where David eats a healthy breakfast AND it rains. Subtract out these cases in order to find the desired value.

In order to calculate the probability that David will eat a healthy breakfast AND that it will rain, multiply the individual probabilities together:

$$0.8 \times 0.25 = 0.2$$

Finally, subtract to find the probability that David will eat a healthy breakfast OR that it will rain:

$$1.05 - 0.2 = 0.85, \text{ or } 85\%$$

58. **(B).** Because this question uses "at least" language, use the $1 - x$ shortcut. In this case, the only outcome you do *not* want is rain on zero days next week. It will be much faster to solve for that probability and subtract from 1 in order to find the probability of all of the other outcomes (that there will be rain on one or more days next week).

(The probability of rain on at least one day) + (The probability of no rain) = 1

How do you find the probability of no rain? You want no rain on Monday AND Tuesday AND Wednesday AND Thursday AND Friday. Multiply together the individual probabilities of no rain on each of the five days.

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$$

Subtract this from 1 to get the desired answer:

$$1 - \frac{1}{32} = \frac{31}{32}$$

59. **(E).** Because this is an "at least" question, use the $1 - x$ shortcut.

(The probability of picking at least one man) + (The probability of picking no men) = 1

The probability of picking no men is an AND setup: woman AND woman AND woman.

For the first choice, there are 8 women out of 10 people: $8/10 = 4/5$
For the second choice, there are $7/9$ (because one woman has already been chosen)
For the third choice, there are $6/8 = 3/4$

Multiply the three probabilities together to find the probability that the committee will be comprised of woman AND woman AND woman:

$$\frac{4}{5} \times \frac{7}{9} \times \frac{3}{4} = \frac{1}{5} \times \frac{7}{3} \times \frac{1}{1} = \frac{7}{15}$$

To determine the probability of picking at least one man, subtract this result from 1:

1 - 7/15 = 8/15

60. **(A).** This overlapping sets question can be solved with the following equation:

Total # of People = Group 1 + Group 2 + Group 3 - (# of people in 2 groups) - (2)(# of people in all 3 groups) + (# of people in no groups)

The problem indicates that everyone takes at least one language, so the number of people in no groups is zero. The problem also indicates that nobody takes all three languages, so that value is also zero.

Total # of Students = 100 + 80 + 40 - 22 - (2)(0) + 0 = 198.

61. **(A).** It is not possible to solve for a single value for Quantity A, but it is possible to tell that Quantity A is greater than 17. Since 20 birds are songbirds and 23 are migratory, the total of these groups is 43, which is less than 60. It is possible for the overlap (the number of migratory songbirds) to be as little as 0, which would result in 20 songbirds, 23 non-songbird migratory birds, and 60 - 20 - 23 = 17 birds that are neither songbirds nor migratory.

It is also possible that there could be as many as 20 birds that overlap the two categories. (Find this figure by taking the number of birds in the smaller group; in this case, there are 20 songbirds). In the case that there are 20 migratory songbirds, there would also be 3 migratory birds that are not songbirds, in which case there would be 60 - 20 - 3 = 37 birds that are neither migratory nor songbirds.

Thus, the number of birds that are neither migratory nor songbirds is at least 17 and at most 37. No matter where in the range that number may be, it is greater than Quantity B, which is only 16.

Chapter 24
of

Data Interpretation

In This Chapter…

Data Interpretation

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\dfrac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

Problem Set A

9th Grade Students at Millbrook High School

	Boys	Girls
Enrolled in Spanish	12	13
Not Enrolled in Spanish	19	16

1. Approximately what percent of the 9th grade girls at Millbrook High School are enrolled in Spanish?

(A) 21%
(B) 37%
(C) 45%
(D) 50%
(E) 57%

2. What fraction of the students in 9th grade at Millbrook High School are boys who are enrolled in Spanish?

(A) 1/5
(B) 19/60
(C) 5/12
(D) 12/31
(E) 12/25

3. What is the ratio of 9th grade girls not enrolled in Spanish to all 9th grade students at Millbrook Middle School?

(A) 1 : 16
(B) 13 : 60
(C) 4 : 15
(D) 19 : 60
(E) 16 : 29

4. If *x* percent more 9th grade students at Millbrook High School are not enrolled in Spanish than are enrolled in Spanish, what is *x*?

(A) 20
(B) 25
(C) 30
(D) 40
(E) 50

5. If 2 of the 9th grade boys at Millbrook High school who are not enrolled in Spanish decided to enroll in Spanish, and then 8 new girls and 7 new boys enrolled in the 9th grade at Millbrook Middle School and also in Spanish, what percent of 9th grade students at Millbrook would then be taking Spanish?

(A) 52%
(B) 53%
(C) 54%
(D) 55%
(E) 56%

Problem Set B

Number of Hours Worked Per Week per Employee at Marshville Toy Company

# of employees	Hours worked per week
4	15
9	25
15	35
27	40
5	50

6. What is the median number of hours worked per week per employee at Marshville Toy Company?

(A) 25
(B) 30
(C) 35
(D) 37.5
(E) 40

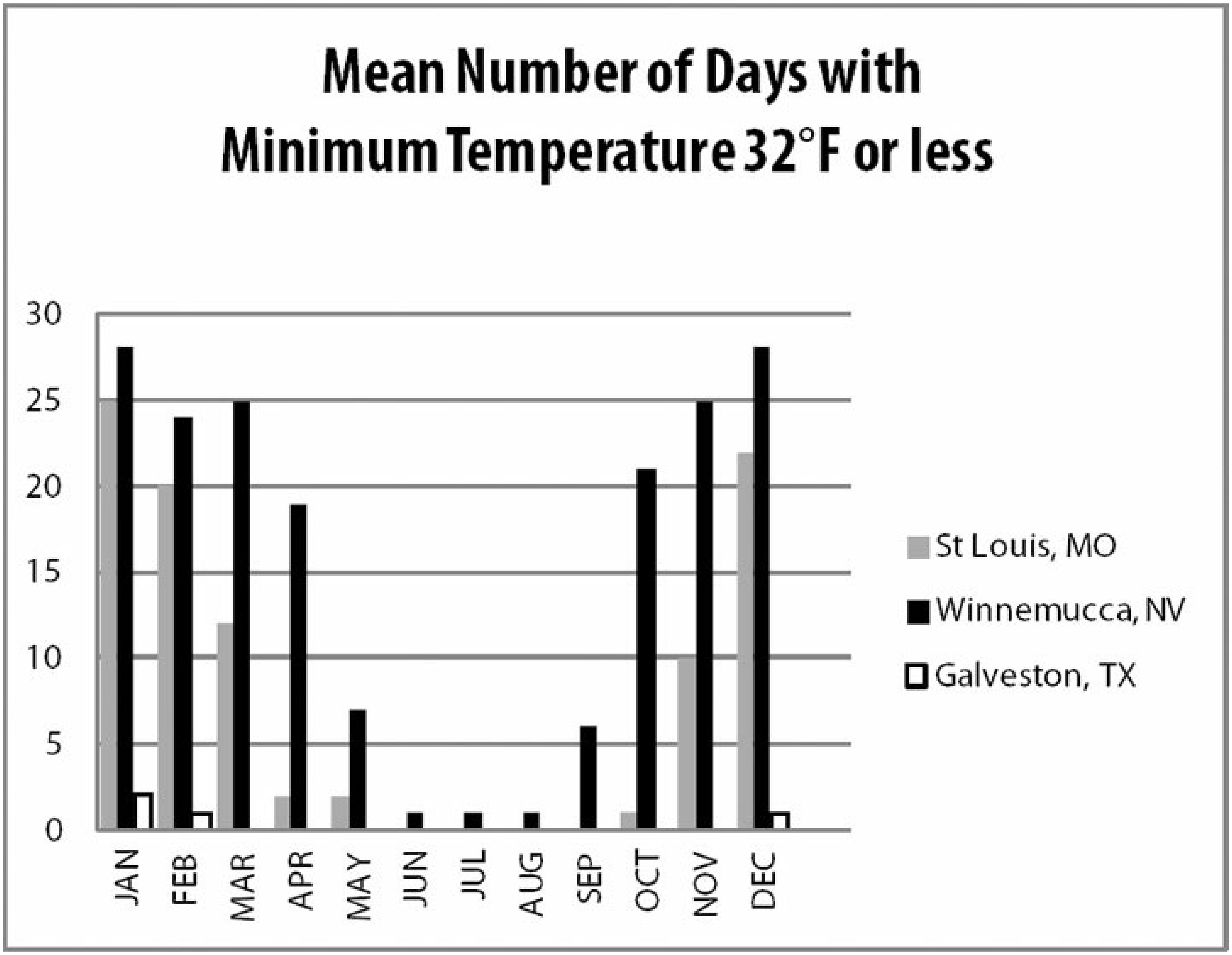
7. What is the average number of hours worked per week per employee at Marshville Toy Company?

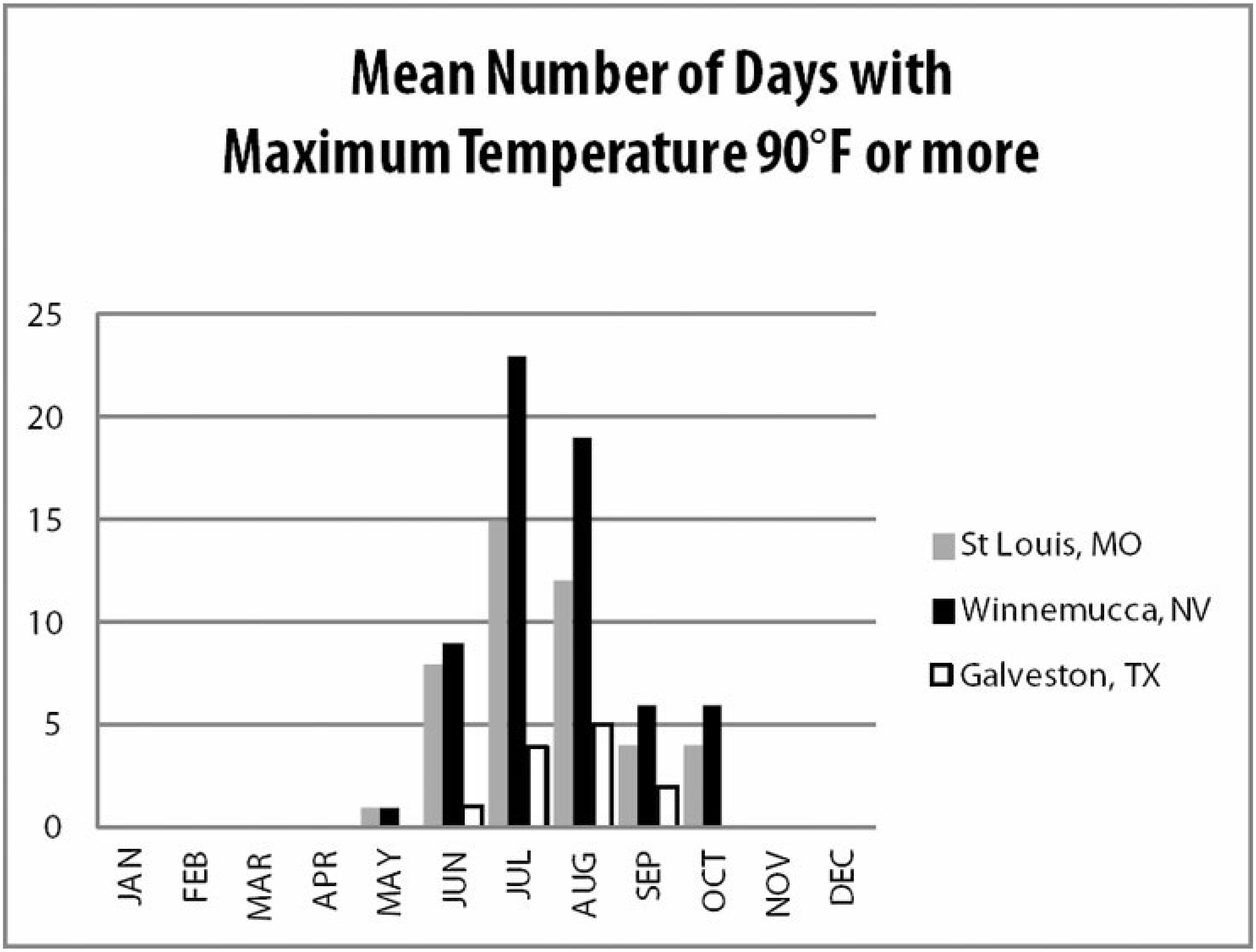
(A) 32
(B) 33
(C) 35
(D) $35\frac{2}{3}$
(E) $36\frac{1}{3}$

8. What is the positive difference between the mode and the range of the number of hours worked per week per employee at Marshville Toy Company?

(A) 0
(B) 4
(C) 5
(D) 8
(E) 26

Problem Set C





9. In how many months of the year were there more than 20 days with temperatures 32°F or less in Winnemucca?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 7

10. On how many days in the entire year did the temperature in Galveston rise to at least 90°F or fall to, or below, 32°F?

(A) 11
(B) 16
(C) 28
(D) 42
(E) 59

11. Approximately what percent of the days with maximum temperature of 90°F or more in St. Louis occurred in July?

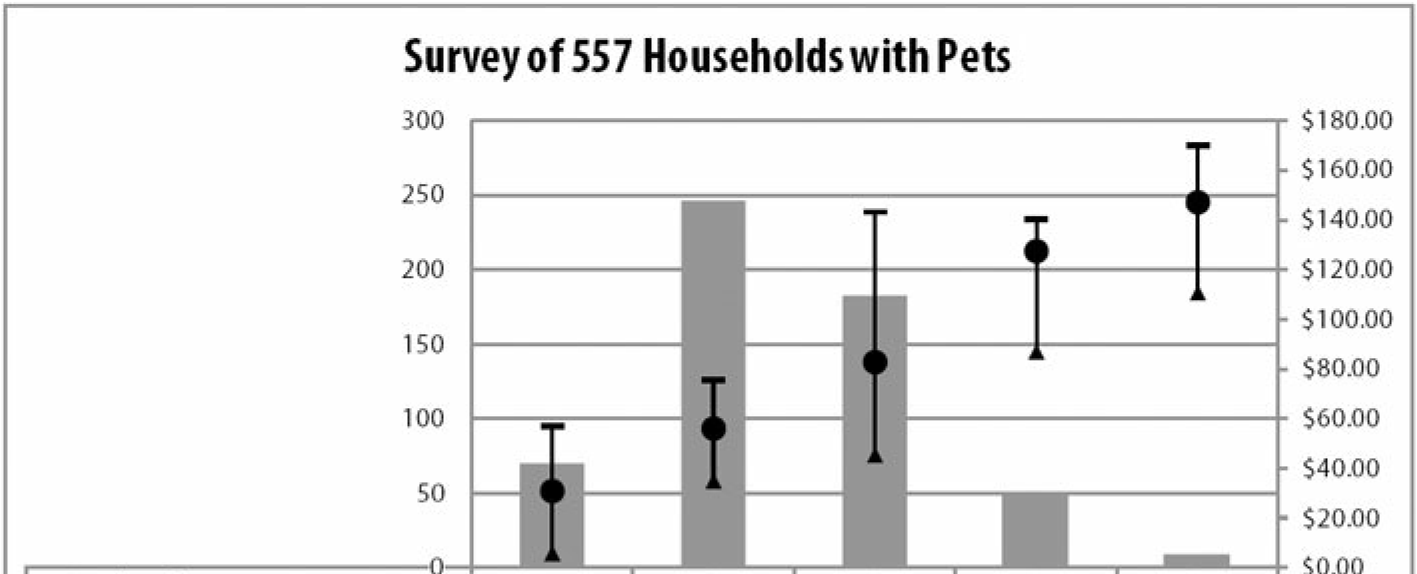
(A) 6%
(B) 15%
(C) 17%
(D) 34%

(E) 44%

12. The number of freezing January days in Winnemucca was approximately what percent more than the number of freezing January days in St. Louis? (A "freezing" day is one in which the minimum temperature is 32°F or less.)

(A) 3%
(B) 6%
(C) 12%
(D) 24%
(E) 28%

Problem Set D



Number of pets	1	2	3	4	5
■ Number of households with this number of pets	70	246	183	49	9
▬ Maximum Monthly Spending on Pet Supplies	$57.32	$75.98	$143.57	$140.48	$170.23
▲ Minimum Monthly Spending on Pet Supplies	$6.34	$35.47	$45.84	$87.46	$111.20
● Average Monthly Spending on Pet Supplies	$31.25	$56.42	$83.11	$127.74	$147.38

13. Approximately what percent of the surveyed households have more than three pets?

(A) 10%
(B) 20%
(C) 30%
(D) 40%
(E) 50%

14. What is the median number of pets owned by the households in the survey?

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

15. Grouping households by number of pets, what is the range of monthly spending on pet supplies for the group with the largest range?

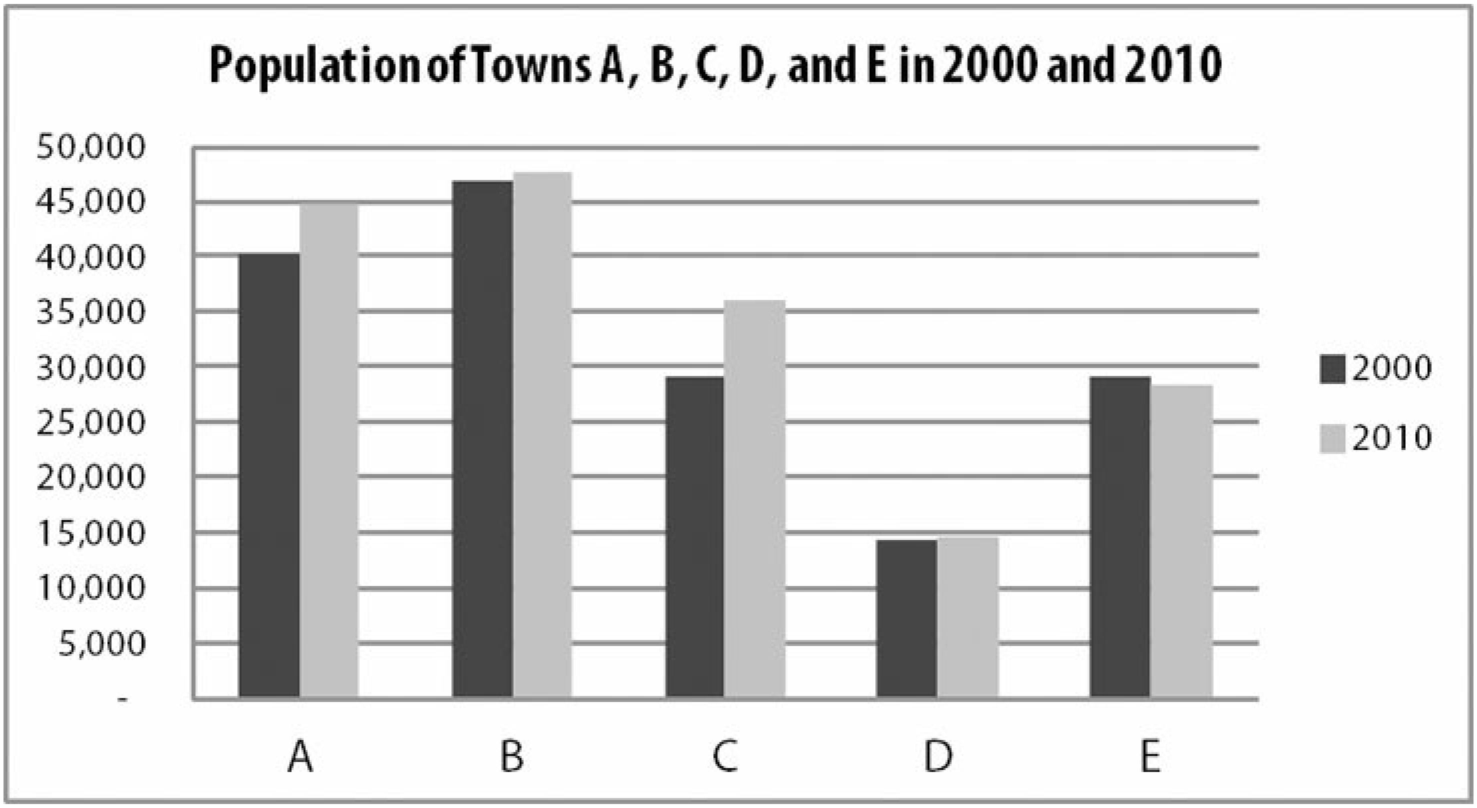
(A) $69.03
(B) $97.73
(C) $116.13
(D) $138.98

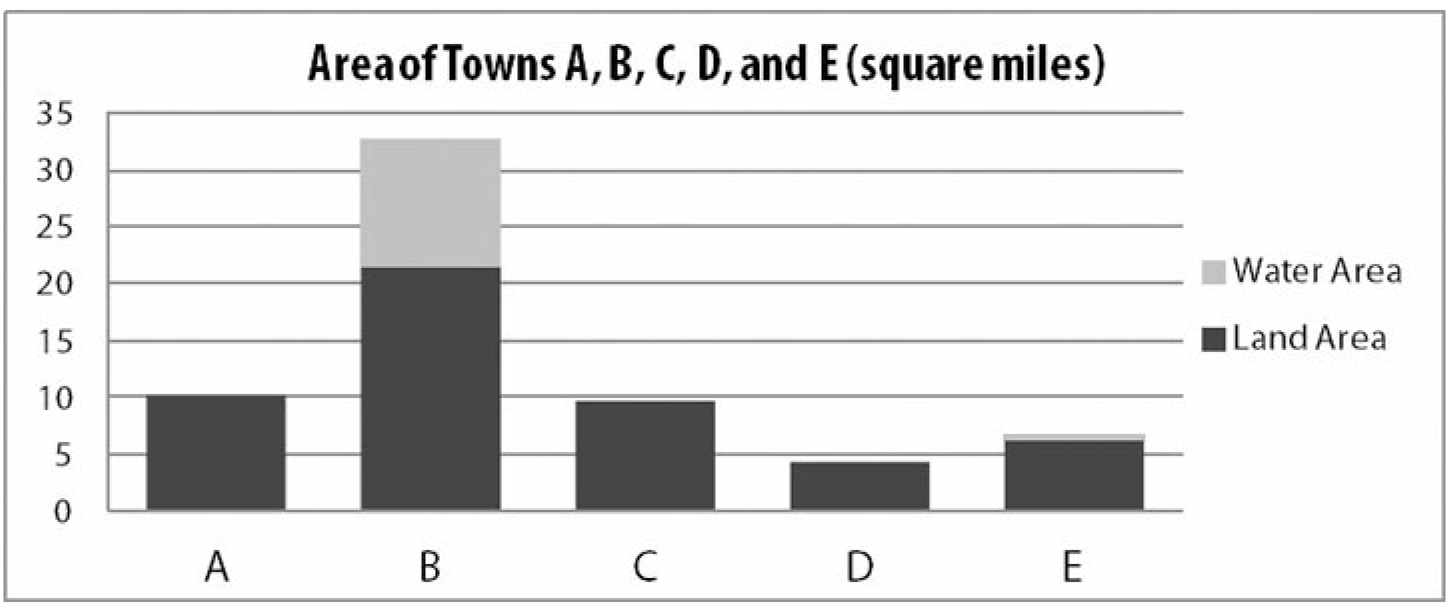
(E) $170.23

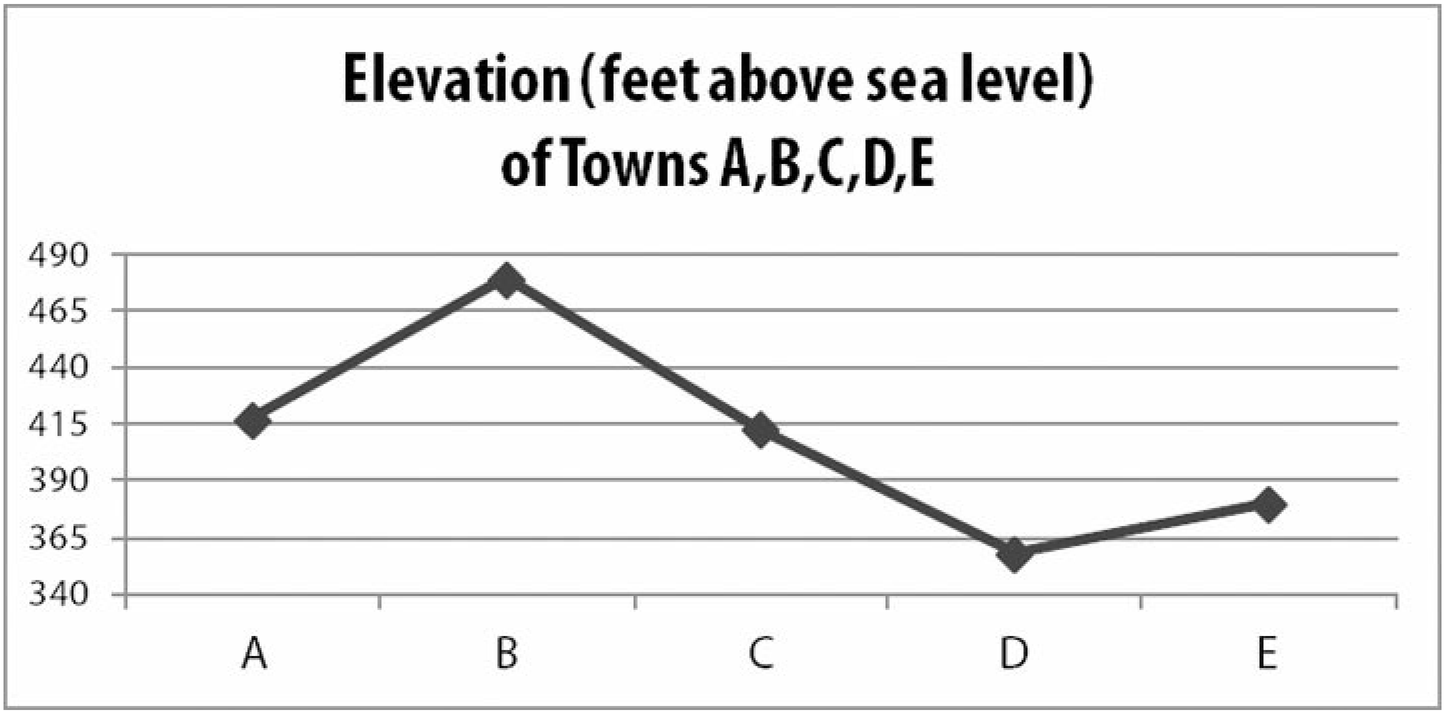
16. Households with how many pets have the greatest average monthly spending per pet?

(A) 1 pet
(B) 2 pets
(C) 3 pets
(D) 4 pets
(E) 5 pets

Problem Set E







17. In what town did the population increase by the greatest percent between 2000 and 2010?

(A) Town A
(B) Town B
(C) Town C
(D) Town D
(E) Town E

18. The ratio of the population of one town to the population of another remained most unchanged between 2000 and 2010 for which two towns?

Indicate two such towns.

☐ Town A
☐ Town B
☐ Town C
☐ Town D
☐ Town E

19. The water area of town B is most nearly equal the total land area of which two towns?

Indicate two such towns.

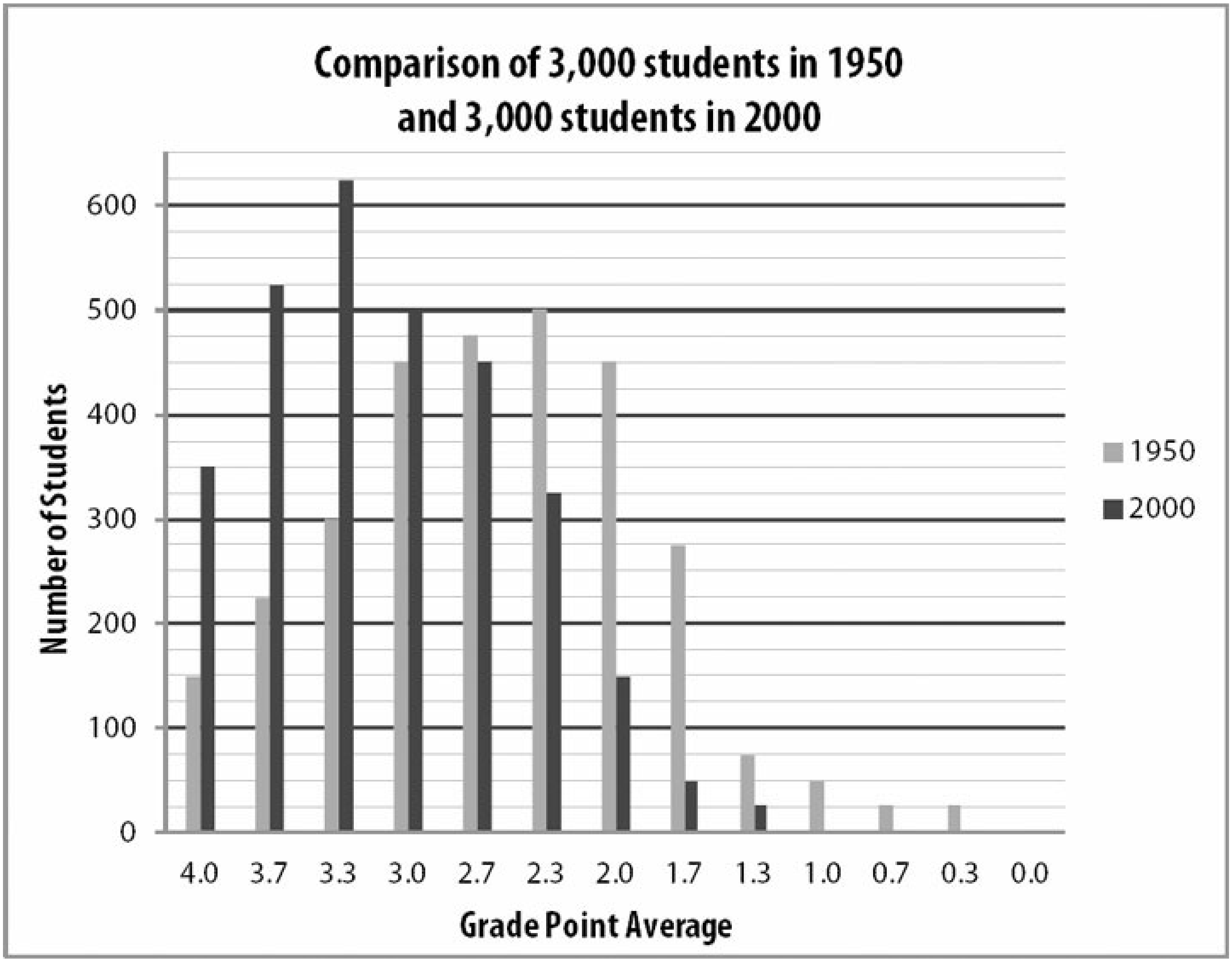
☐ Town A
☐ Town B
☐ Town C
☐ Town D
☐ Town E

20. Which two towns have the most nearly equal elevation in feet above sea level?

Indicate two such towns.

☐ Town A
☐ Town B
☐ Town C
☐ Town D
☐ Town E

Problem Set F



21. What was the mode for grade point average among the 3,000 students in 2000?

(A) 3.7
(B) 3.3
(C) 3.0
(D) 2.7
(E) 2.3

22. What was the median grade point average among the 3,000 students in 1950?

(A) 3.7
(B) 3.3
(C) 3.0
(D) 2.7
(E) 2.3

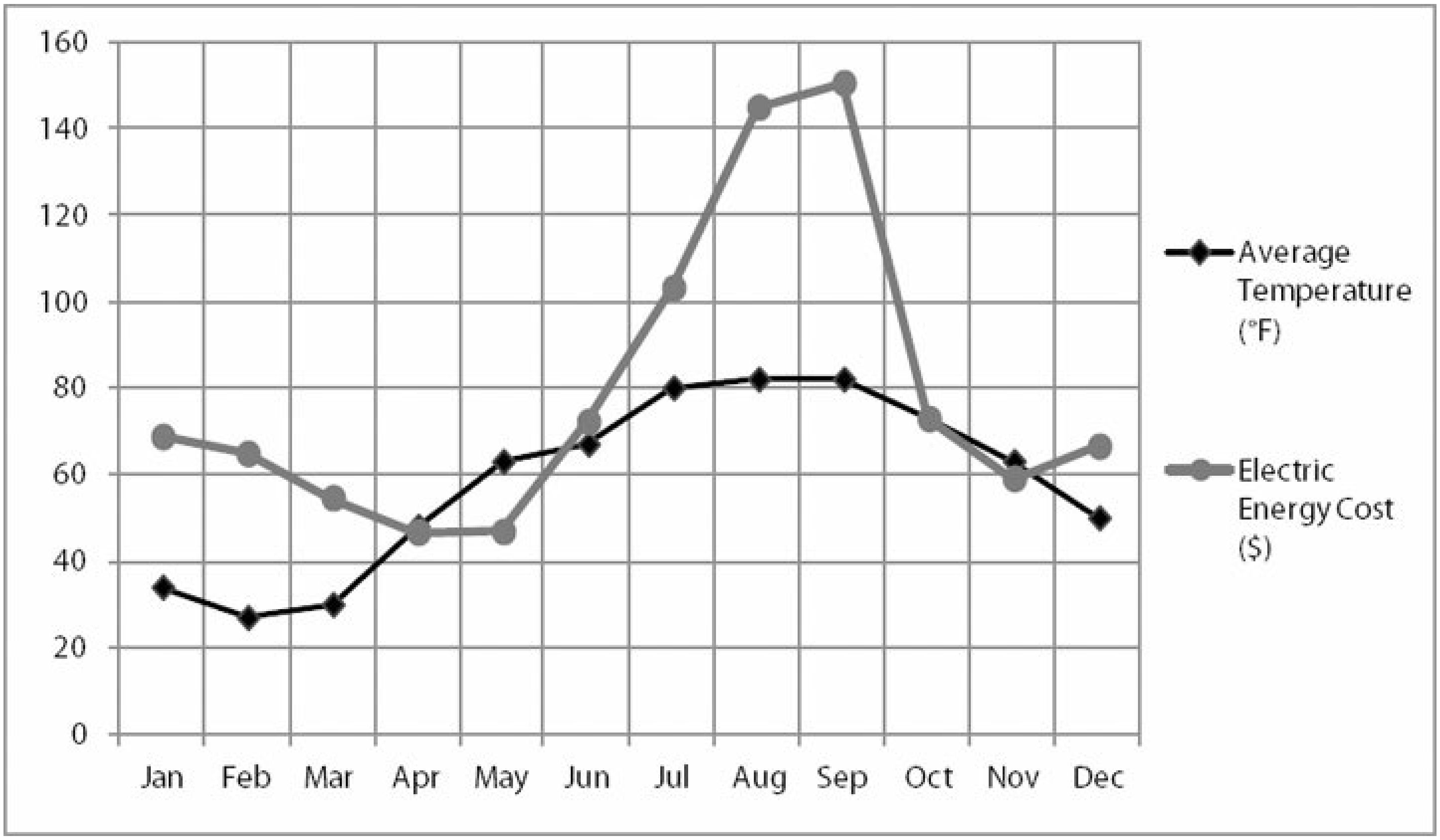
23. Approximately what percent of the students in 2000 earned at least a 3.0 grade point average?

(A) 25%
(B) 50%
(C) 67%
(D) 80%
(E) 97.5%

24. Approximately what percent of the students in 1950 earned a grade point average less than 3.0?

(A) 33%
(B) 37.5%
(C) 50%
(D) 62.5%
(E) 75%

Problem Set G



25. Electric energy cost increased most between which two consecutive months?

Indicate two such months.

- ☐ January
- ☐ February
- ☐ March
- ☐ April
- ☐ May
- ☐ June
- ☐ July
- ☐ August
- ☐ September
- ☐ October
- ☐ November
- ☐ December

26. Electric energy cost changed least between which two consecutive months?

Indicate two such months.

- ☐ January

☐ February
☐ March
☐ April
☐ May
☐ June
☐ July
☐ August
☐ September
☐ October
☐ November
☐ December

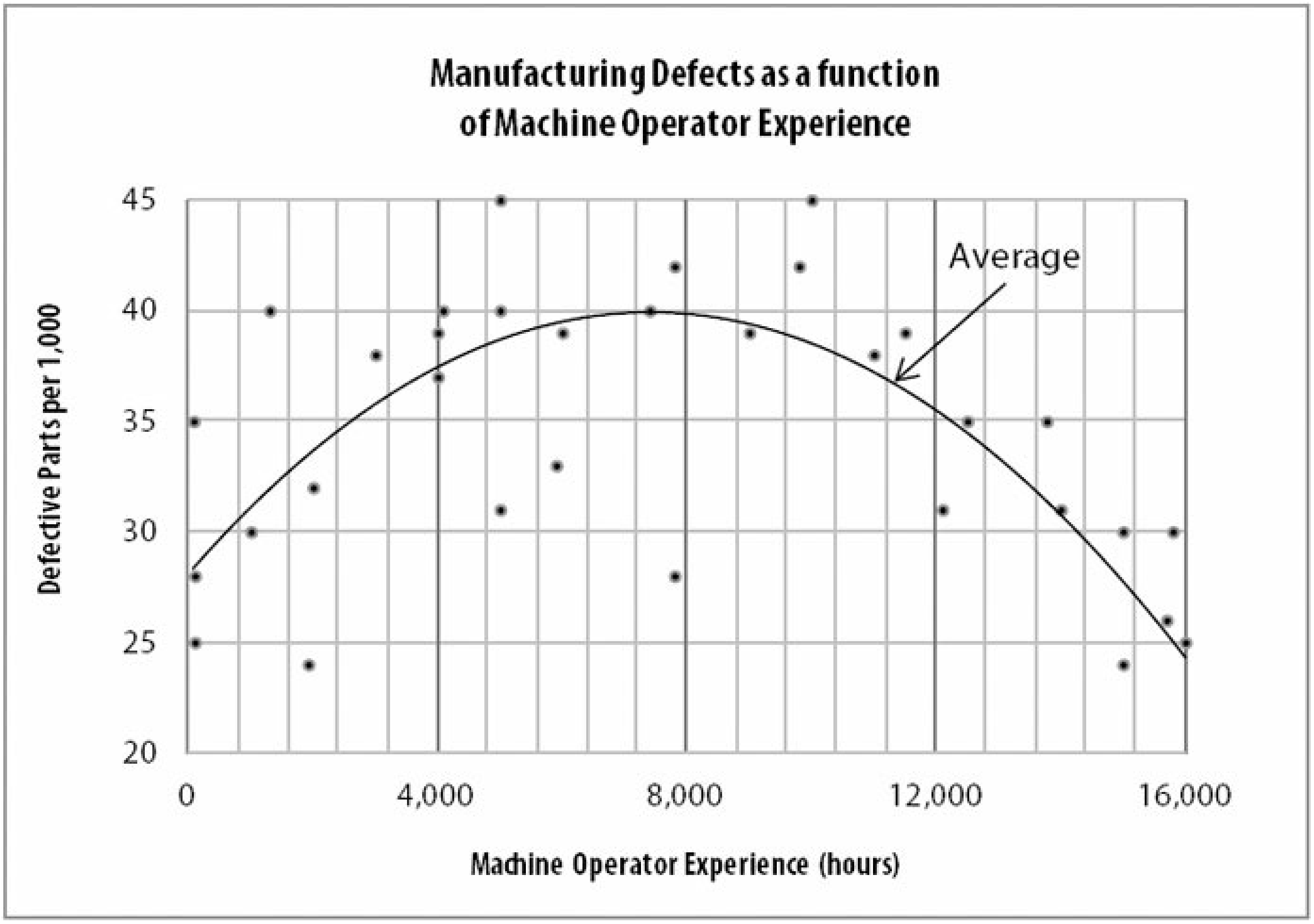
27. Approximately what was the average electric energy cost per month in the first half of the year?

(A) \$45
(B) \$50
(C) \$60
(D) \$70
(E) \$75

28. In what month was the electric energy cost per °F of average temperature least?

(A) April
(B) May
(C) October
(D) November
(E) December

Problem Set H



29. On average, the machine operators that produce the fewest defective parts per 1,000 have how many hours of experience?

    (A) 40
    (B) 4,000
    (C) 8,000
    (D) 12,000
    (E) 16,000

30. On average, the defective part rate is equal for machine operators with 12,000 hours and with approximately how many hours of experience?

    (A) 2,000
    (B) 2,700
    (C) 4,400
    (D) 8,400
    (E) 12,800

31. At approximately what experience level, in hours, do machine operators produce the most defective parts per 1,000, on average?

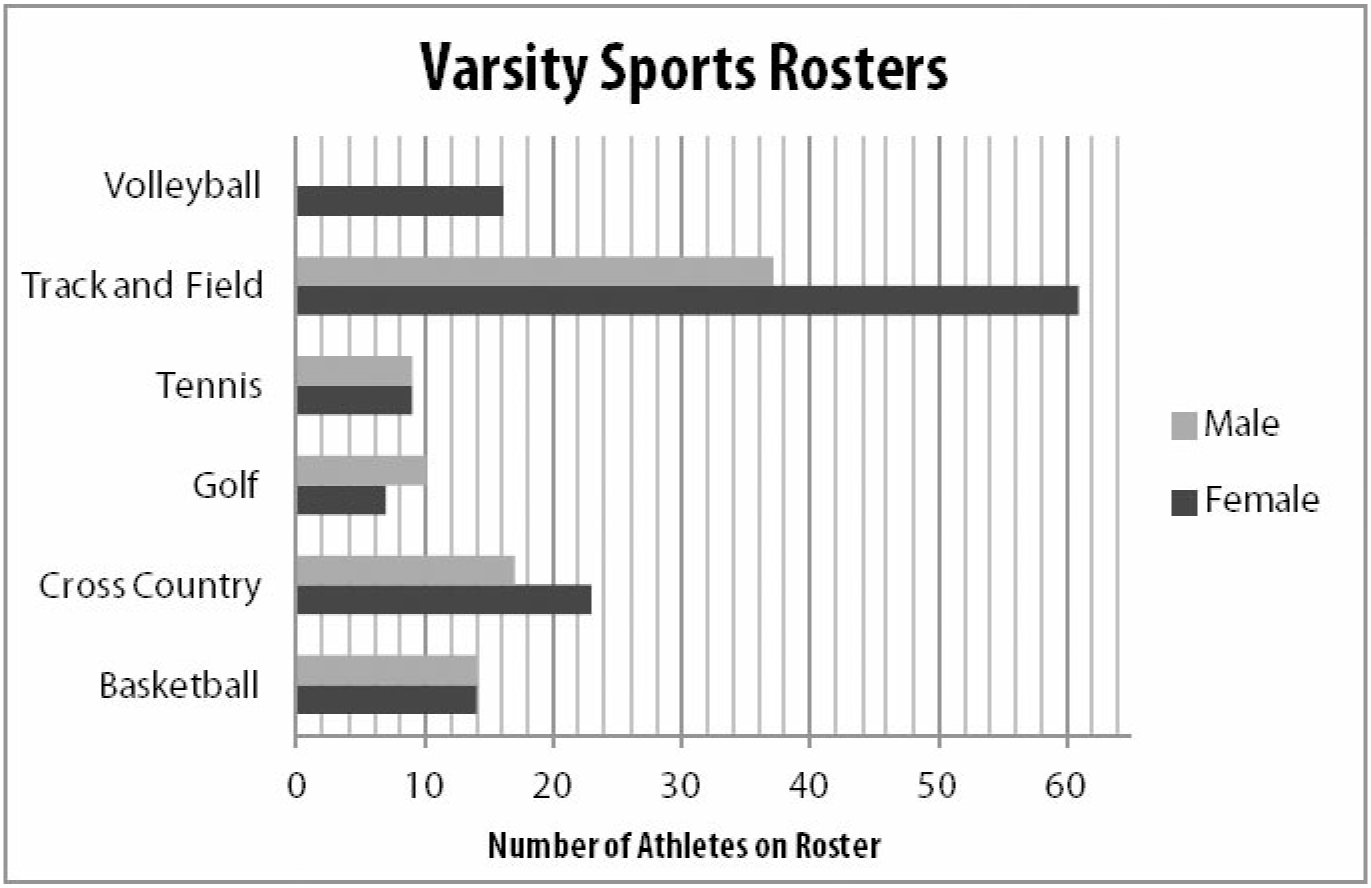
    (A) 40

(B) 4,000
(C) 8,000
(D) 12,000
(E) 16,000

32. Of the two individual machine operators who had a defective part rate of 4.2%, approximately how many hours of experience did the less experienced operator have?

(A) 2,300
(B) 5,000
(C) 7,700
(D) 9,800
(E) 15,100

Problem Set I



33. What is the ratio of male athletes to female athletes on the track and field roster?

(A) $\frac{37}{61}$
(B) $\frac{9}{14}$
(C) $\frac{17}{23}$
(D) $\frac{14}{9}$
(E) $\frac{61}{37}$

34. All athletes are on only one varsity sports roster EXCEPT those who run on both the Track and Field team and the Cross Country team. If there are 76 male athletes total on the varsity sports rosters, how many male athletes are on both the Track and Field team and the Cross Country team?

(A) 11
(B) 17
(C) 37
(D) 54
(E) 76

35. On what varsity sports rosters do male athletes outnumber female athletes?

Indicate all such rosters.

☐ Volleyball
☐ Track and Field
☐ Tennis
☐ Golf
☐ Cross Country
☐ Basketball

36. What is the ratio of female tennis players to male basketball players on the varsity sports rosters?

(A) $\frac{5}{12}$
(B) $\frac{9}{14}$
(C) $\frac{7}{8}$
(D) $\frac{14}{9}$
(E) $\frac{12}{5}$

Problem Set J

	Change in Total Revenue (2011 to 2012)	Percent Change in Number of Distinct Customers (2011 to 2012)	Percent Change in Total Costs (2011 to 2012)
Store W	-$400,000	+2%	+15%
Store X	+$520,000	+14%	+4%
Store Y	-$365,000	+5%	+12%
Store Z	+$125,000	-7%	-20%

37. For which store was the revenue per distinct customer greatest in 2012?

(A) Store W
(B) Store X
(C) Store Y
(D) Store Z
(E) It cannot be determined from the information given.

38. Between 2011 and 2012, total costs per distinct customer increased by the greatest percent at which store?

(A) Store W
(B) Store X
(C) Store Y
(D) Store Z
(E) It cannot be determined from the information given.

39. Store profit in 2012 could have been less than same store's profit in 2011 at which of the following store(s)?

Indicate <u>all</u> such stores.

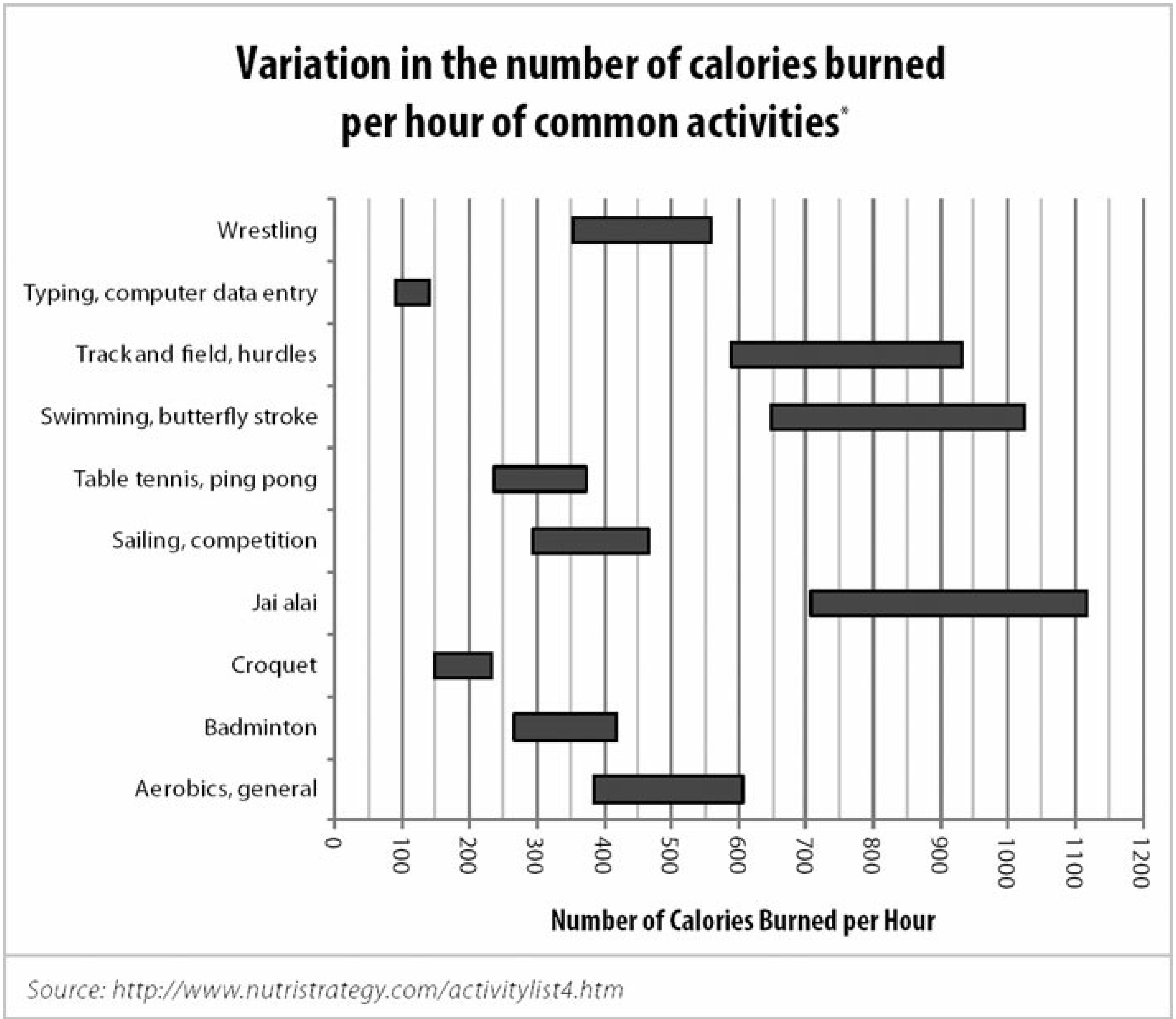
☐ Store W
☐ Store X
☐ Store Y
☐ Store Z
☐ None of the above.

40. Which of the following statements must be true?

(A) Of the four stores, Store X had the greatest percent increase in revenue from 2011 to 2012.
(B) Per customer revenue increased at Store Z from 2011 to 2012.
(C) Of the four stores, Store W had the greatest increase in total costs from 2011 to 2012.
(D) Of the four stores, Store Y had the highest percent of repeat customers.

(E) In 2012, Store W and Store Z combined had fewer distinct customers than did Store X.

Problem Set K



* Based on body weight of exercise subject. The lower limit represents the calories burned by a person weighing 130 pounds, while the upper limit represents the calories burned by a person weighing 205 pounds.

41. Which of the following statements could be true?

Indicate <u>all</u> such statements.

- ☐ A person weighing between 130 and 205 pounds performs one of the above activities for 10 hours yet burns fewer calories than another person in the same weight range performing another activity for only 1 hour.
- ☐ A 175 pound person playing jai alai for one hour burns fewer calories than a 180 pound person swimming the butterfly stroke for one hour.
- ☐ If all people in question weigh between 130 and 205 pounds, the average calories burned by two

people playing table tennis for 1 hour is more than the total calories burned by 2 people typing for 3 hours.

42. Which combination of activities burns the fewest calories total?

(A) A 130 pound person playing badminton for 1 hour and a 205 pound person playing table tennis for 1 hour
(B) A 130 pound person wrestling for 1 hour and a 205 pound person running track and field, hurdles for 1 hour
(C) A 130 pound person typing for 1 hour and a 205 pound person swimming the butterfly stroke for 1 hour
(D) A 130 pound person sailing in a competition for 1 hour and a 205 pound person doing aerobics for 1 hour
(E) A 130 pound person typing for 1 hour and a 205 pound person playing croquet for 1 hour

Problem Set L

Gross Domestic Product	Population and GDP for 50 African Countries						
		Population					
		more than 50 million	20 to 50 million	10 to 20 million	2 to 10 million	less than 2 million	Total
	more than $100 billion	3	2	0	0	0	5
	$20 – 100 billion	1	7	1	1	0	10
	$10 – 20 billion	1	3	3	3	3	13
	less than $10 billion	0	0	7	8	7	22
	Total	5	12	11	12	10	50

43. Among the 50 African countries represented in the chart above, how many countries have a population between 10 million and 50 million people and a GDP between $10 billion and $20 billion?

(A) 6
(B) 7
(C) 13
(D) 16
(E) 23

44. Among the 50 African countries represented in the chart above, what percent of the countries have a population less than 20 million people and a GDP of less than $20 billion?

(A) 38%
(B) 44%
(C) 62%
(D) 68%
(E) 90%

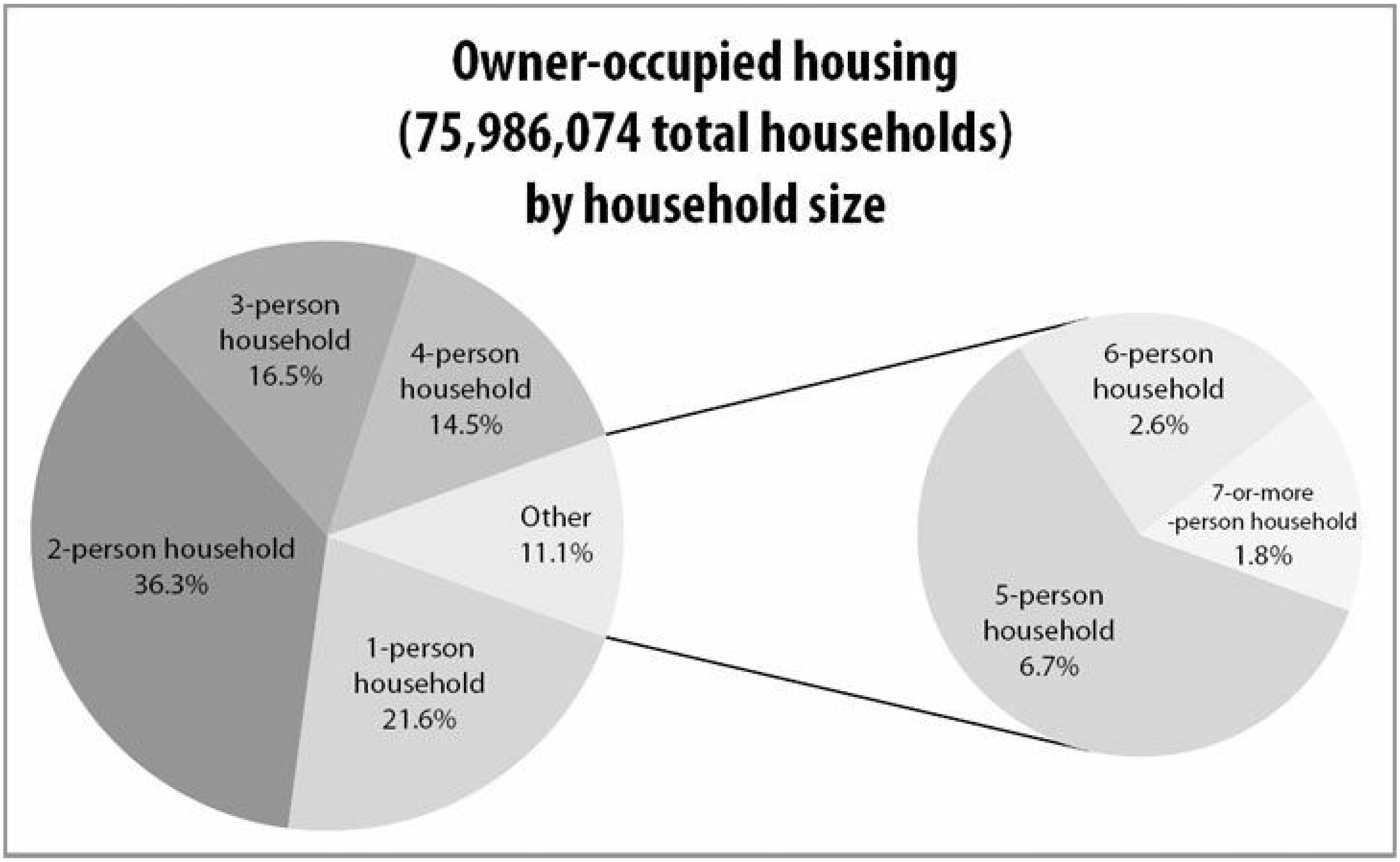
45. Approximately what percent of the African countries in the chart above with GDPs between $10 billion and $20 billion have populations between 10 million and 20 million?

(A) 6%
(B) 23%
(C) 26%
(D) 30%
(E) 51%

46. Referring to the 50 African countries represented in the chart above, which of the following is greatest?

(A) The number of countries with more than \$10 billion GDP and population less than 20 million
(B) The number of countries with less than \$20 billion GDP and population more than 10 million
(C) The number of countries with more than \$20 billion GDP
(D) The number of countries with less than \$100 billion GDP and population less than 10 million
(E) The number of countries with less than \$100 billion GDP and population between 10 million and 50 million

Problem Set M



47. What percent of owner-occupied housing units are households with fewer than four people?

(A) 11.1%
(B) 14.5%
(C) 25.6%
(D) 74.4%
(E) 88.9%

48. Among the owner-occupied housing units represented in the chart above, approximately how many households are 5-person households?

(A) 1 million
(B) 2 million
(C) 3 million
(D) 4 million
(E) 5 million

49. Which of the following is a correct ranking of 1-person households, 3-person households, and 5-person households based on the total number of people living in such households, from least to greatest number of people?

(A) 1-person households, 3-person households, 5-person households
(B) 1-person households, 5-person households, 3-person households
(C) 3-person households, 1-person households, 5-person households

(D) 3-person households, 5-person households, 1-person households
(E) 5-person households, 3-person households, 1-person households

50. Which range of household sizes, inclusive, accounts for more than 50% of all owner-occupied housing units?

(A) 2- to 3-person
(B) 3- to 4-person
(C) 4- to 5-person
(D) 5- to 6-person
(E) 6- to 7-person

Data Interpretation Answers

Problem Set A: First, read the title of the chart: everyone accounted for in the chart is a 9th grader at Millbrook Middle School. So, when problems mention "9th grade," you don't have to figure out how many people involved are 9th graders — everyone in the chart is.

When given a chart that depends on addition (boys plus girls = total students, and also those enrolled in Spanish plus those not enrolled in Spanish = total students), it can be helpful to sketch a quick version of the chart on your paper and to add a total column. (If the chart is large and this would be too time-consuming, just imagine that the "Total" row and "Total" column are present, and only calculate what you need.) For example:

	Boys	**Girls**	**TOTAL**
Enrolled in Spanish	12	13	
Not Enrolled in Spanish	19	16	
TOTAL			

Now add down and across:

	Boys	**Girls**	**TOTAL**
Enrolled in Spanish	12	13	25
Not Enrolled in Spanish	19	16	35
TOTAL	31	29	60

1. **(C).** There are 29 total girls and 13 are enrolled in Spanish. The fraction of girls enrolled in Spanish is $\frac{13}{29}$. Convert to a percent: $\left(\frac{13}{29}\times 100\right)\%$, or about 45%.

2. **(A).** There are 60 total students and 12 boys enrolled in Spanish. The answer is 12/60, which reduces to 1/5. (Read carefully! "What fraction of the students … are boys who are enrolled in Spanish?" is NOT the same as "What fraction of the boys are enrolled in Spanish?")

3. **(C).** There are 16 girls not enrolled in Spanish and 60 total students. The ratio is 16/60, which reduces to 4/15 or 4 : 15.

4. **(D).** 35 students are not enrolled in Spanish and 25 are. The question can be rephrased as, "35 is what percent greater than 25?" Using the percent change formula:

Percent Change = $\left(\frac{Difference}{Original} \times 100\right)\%$

Percent Change = $\left(\frac{10}{25} \times 100\right)\% = 40\%$

Thus, x is 40.

5. **(E)**. Sketch a new chart to reflect the changes. Switch 2 of the boys from "not enrolled" to "enrolled." Then, add 8 new girls and 7 new boys to the "enrolled" groups, like this:

	Boys	**Girls**	**TOTAL**
Enrolled in Spanish	12 + 2 + 7 = 21	13 + 8 = 21	42
Not Enrolled in Spanish	19 - 2 = 17	16	33
TOTAL	38	37	75

Update the Total rows and columns as well. You will see that both "Boys" and "Girls", as well as "Enrolled in Spanish" and "Not Enrolled in Spanish," now sum to a total of 75.

What percent of 9th grade students at Millbrook would then be taking Spanish? Since 42 out of 75 students would be enrolled in Spanish, enter 42/75 into your calculator, then multiply by 100 to convert to a percent. The answer is 56%.

Problem Set B: This chart shows the frequency with which certain values — numbers of hours worked per week — appear. That is, the chart is a short way of showing a list of values that would begin: 15, 15, 15, 15, 25, 25, 25, 25, 25, 25, 25, 25, 25… (then the number 35 fifteen times, then the number 40 twenty-seven times, then the number 50 five times). Add the numbers in the lefthand column to determine that 60 people's work hours are being accounted for.

6. **(E).** The median of any list is the middle number or the average of the two middle numbers. However, you CANNOT assume that the middle number in the list 15, 25, 35, 40, 50 is the median number of hours worked per week, because this does not take into account the *frequency* with which each of those numbers occurs in the list. The actual list includes the value 15 *four times* (once for each of the 4 employees who works 15 hours per week), the value 25 *nine times* (once for each of the 9 employees who works 25 hours per week), etc.

To find the median, add the numbers of employees: 4 + 9 + 15 + 27 + 5 = 60. Thus, the middle of this list will be the average of the 30th and 31st values. Since 4 + 9 + 15 = 28, the 29th, 30th, 31st, etc. values fall into the next highest group — the group of 27 people who work 40 hours per week. The median number of hours worked per week per employee is 40.

7. **(D).** To average the number of hours worked per week, you CANNOT simply average 15, 25, 35, 40, and 50. You must take into account *how many people* work each of those numbers of hours. That is, you must average the values for all 60 workers:

$$\frac{4(15)+9(25)+15(35)+27(40)+5(50)}{60} = 35.\overline{6} \text{ hours}$$

The answer is $35.\overline{6}$ or $35\frac{2}{3}$.

8. **(C).** The mode is the number that appears in the list with the greatest frequency. Since 27 people worked 40 hours a week and every other group has fewer than 27 people, the mode is 40. The range is the highest value in the list minus the lowest value in the list. 50 - 15 = 35. The positive difference between 40 and 35 is 40 - 35 = 5.

Problem Set C: The two charts show how often daily temperature extremes occurred in each month of the year for 3 cities. For the sake of simplicity, you can think of the top chart as "cold" and the bottom chart as "hot."

Also note that there is no information about exactly how hot or how cold the days tallied are: a day with a minimum temperature of 27°F counts as a "cold" day, just as a day with minimum temperature of -10°F would. Therefore, it is likely that questions will just reference one or both of the two temperature categories broadly (= 90 and = 32), which you can simply think of as "hot" or "cold" days.

9. **(D).** From the "cold" chart, the black bar referring to Winnemucca rises above 20 in Jan, Feb, Mar, Oct, Nov, and Dec, for a total of 6 months.

10. **(B).** This question asks about number of days with both temperatures extremes in Galveston. Galveston had 1 "hot" day in June, 4 in July, 5 in August, and 2 in September, for a total of 12. It had 2 "cold" days in January and 1 each in February and December, for a total of 4. The total number of days with either extreme temperature is 16 days.

11. **(D).** From the grey bars on the "hot" day chart, St. Louis had a total of 1 + 8 + 15 + 12 + 4 + 4 = 44 days when the temperature reached at least 90°F, and 15 of those were in July. These July days account for $\left(\frac{15}{44}\times 100\right)\% \approx 34\%$ of all the hot days in St. Louis (approximately).

12. **(C).** In January, Winnemucca had 28 freezing days, while St. Louis had 25. So the question is asking, "28 is what percent more than 25?" Answer this with the percent change formula: $\left(\frac{Difference}{Original}\times 100\right)\% = \left(\frac{3}{25}\times 100\right)\% = 12\%$. It is true that 28 is 12% more than 25% (check: 25 × 0.12 = 3).

Problem Set D: This table tallies the number of households, according to number of pets in the household, and each column captures information about these households. For example, the left-most column with numbers indicates that there are 70 households that have 1 pet, and these households spend an average of $31.25 per month on pet supplies. In that group, the household(s) that spent the least spent $6.34 on pet supplies, while the household(s) that spent the most spent $57.32. Notice that the bars and the max/min/average range lines duplicate the information in the table. For exact calculations, rely on the chart numbers. For broader questions, such as "which is greater," the more visual representation of the data can often provide a quick answer.

13. **(A).** There are 557 households, of which 49 have four pets and 9 have five pets. Thus a total of 58 households have more than three pets. To express this as a percent, take this number and divide by the total number of households. 58/557 = 10% (approximately).

14. **(B).** Since there are 557 households, the median household would be midway between the 1st and 557th households on the list when the households are ranked according to how many pets they own. The midpoint between the 1st and 557th households is the (1 + 557)/2 = 279th household. Check: There are 278 households below this one, and 278 households above (because 279 + 278 = 557). Ranked by number of pets, households 1 through 70 (the first 70 households) have one pet, which means households 71 through 316 (in other words, the next 246 households) have two pets. Since the 279th household falls in this interval, the median household owns two pets.

15. **(B).** The group with the largest range of monthly spending is the group of households that own three pets. This can be seen by looking at the length of the vertical line between the maximum spending bar and the minimum spending triangle. Within this group, the maximum amount spent is $143.57 and the minimum is $45.84, so the range is $143.57 - $45.84 = $97.73.

16. **(D).** The group with one pet spent an average of $31.25 per pet, as indicated in the chart. The group with two pets spent an average of $56.42 on two pets, which is $28.21 per pet ($56.42/2). The third group spent an average of $83.11 on three pets, or $83.11/3 = $27.70 per pet (approximately). The fourth group spent an average of $127.74 on four pets, or $127.74/4 = $31.94 per pet (approximately). The fifth group spent an average of $147.38 on five pets, or $147.38/5 = $29.47 per pet (approximately). The highest average is among the group that has four pets.

As an alternative to using the calculator for all five groups, you could benchmark to $30 per pet. If that were the average spending per pet, the bottom row of the table would read $30, $60, $90, $120, and $150 from left to right. Only the households with one or four pets exceed these numbers, so the correct answer is one of them. For just those households, calculate average monthly spending per pet as before.

Problem Set E:

17. **(C).** To compare population in 2000 and 2010, look at the difference in the heights of the dark and light gray bars in the population bar chart. Population decreased in Town E, and was barely changed in Towns B and D, so focus on the remaining Towns A and C. This difference is about 5,000 for town A (45,000 - 40,000), but is more than 5,000 for Town C (more than 35,000 - less than 30,000). The question asks for the "population increase by the greatest *percent*," which requires comparison to the original (2000) population: Percent Change = $\left(\frac{\text{Difference}}{\text{Original}} \times 100\right)\%$. Not only is the population increase greatest for Town C, the population of Town C is smaller than for Town A. The percent increase in population for Town A was $\left(\frac{5{,}000}{40{,}000} \times 100\right)\% = 12.5\%$, but the percent increase in population for Town C was about $\left(\frac{7{,}000}{29{,}000} \times 100\right)\% \approx 24\%$.

18. **Town B and Town D.** Translating "the ratio of the population of one town to the population of another remained most unchanged between 2000 and 2010 for which two towns" literally:

$$\frac{\text{FirstTown}_{2000}}{\text{SecondTown}_{2000}} \text{ is most equal to } \frac{\text{FirstTown}_{2010}}{\text{SecondTown}_{2010}} \text{ for which two towns?}$$

$$\text{Or } \frac{\text{FirstTown}_{2000}}{\text{SecondTown}_{2000}} \approx \frac{\text{FirstTown}_{2010}}{\text{SecondTown}_{2010}} \text{ for which two towns?}$$

Cross multiplying, this could be rephrased as $\frac{\text{SecondTown}_{2010}}{\text{SecondTown}_{2000}} \approx \frac{\text{FirstTown}_{2010}}{\text{FirstTown}_{2000}}$ for which two towns? Going back to words, this is like asking "Which two towns had the most similar population increase or decrease (as a percent or fraction)?

The percent change in population for each town was:

Town A: $\frac{5{,}000}{40{,}000} \times 100\% = 12.5\%$

Town B: about $\frac{1{,}000}{47{,}000} \times 100\% = \text{a bit over } 2\%$

Town C: about $\frac{7{,}000}{29{,}000} \times 100\% \approx 24\%$

Town D: about $\frac{\text{at most } 500}{\text{at least } 14{,}000} \times 100\% = \text{at most } 3.5\%$

Town E: negative % (population decreased)

While the calculations were rough and the numbers are not quite equal for towns B and D, these two percents are closest to each other, and all of the percents calculated for the other towns are quite unique.

19. **Town D, Town E.** In the bar chart for area, dark gray represents the land area and light gray (stacked on top) represents the water area. Thus, to find water area, subtract the height of the dark gray land area bar from the total height of the stacked bars.

Water area of Town B is about 32– 21 = 11. Since the vertical scale is not that precise, you may want to consider a range. Water area for Town B is a little more than 10, as the top of the dark gray bar is slightly *closer* to the horizontal line for 20 than the top of the light gray bar is to the horizontal line for 30. Similarly, water area for Town B must be less than 12.5, as the top of the light gray bar is halfway between 30 and 35, but the top of the dark gray bar is clearly higher than 20.

Towns C, D, and E all have land area less than 10 square miles (i.e. all are below horizontal grid line for 10). Adding the land area of Town C (a bit less than 10) to that of Town D (about 4), the result is too high. The sum of land area in Towns D and E is about 4 + about 6 (certainly less than 7.5), for a result closest to 11.

20. **Town A and Town C.** In the elevation chart, the towns are on the x-axis and the elevation (in feet above sea level) is on the y-axis. Two towns have the same elevation if marked at the same y value, i.e. if their data points are on the same horizontal line. Towns A and C are both close to the horizontal line for 415.

Problem Set F: The dark gray bars indicate number of students in 2000, and the light gray bars indicate number of students in 1950, having various grade point averages. Although the title makes it unnecessary to do so, if you totaled the number of students in each bar color, you would get 3,000 students.

Note the general contrast between students in the two years. Connecting the top of each light gray bar with a smooth line, the result would be a sort-of bell curve that peaks at grade point average of 2.3. Similarly, the dark gray bars form a similar bell curve, but its peak is at grade point average of 3.3, so the grades in general are clustered at the higher end of the scale in 2000.

21. **(B).** The mode of a list of numbers is the number that occurs most frequently in the list. In the bar graph for grade point average, dark gray bars represent the students in 2000, and the mode of that data set is indicated by the tallest dark gray bar. This is at grade point average of 3.3. There were 625 students with a grade point average of 3.3 in the year 2000, more students than had any other grade point average.

22. **(D).** The median is the "middle value" of an ordered list of numbers, dividing the list into roughly two equal parts. For the 3,000 students in 1950, the median grade point average is the average of the 1,500th highest grade point average and the 1,501st highest grade point average. The students in 1950 are represented by the light gray bars.

150 students had a 4.0 grade point average.
225 students had a 3.7 grade point average. (Total with this GPA and higher = 150 + 225 = 375)
300 students had a 3.3 grade point average. (Total with this GPA and higher = 375 + 300 = 675)
450 students had a 3.0 grade point average. (Total with this GPA and higher = 675 + 450 = 1,125)
475 students had a 2.7 grade point average. (Total with this GPA and higher = 1,125 + 475 = 1,600)

The 1,500th and 1,501st students fall between the 1,125th and 1,600th students. Thus, the 1,500th and 1,501st highest grade point averages are both 2.7.

23. **(C).** The students in 2000 are represented by the dark gray bars.

350 students had a 4.0 grade point average.
525 students had a 3.7 grade point average.
625 students had a 3.3 grade point average.
500 students had a 3.0 grade point average.

There were 350 + 525 + 625 + 500 = 2000 students who earned at least a 3.0 grade point average in the year 2000, out of a total of 3000 students. This is $\frac{2}{3}$ of the students, or about 67% of the students.

24. **(D).** The students in 1950 are represented by the light gray bars.

150 students had a 4.0 grade point average.
225 students had a 3.7 grade point average.
300 students had a 3.3 grade point average.
450 students had a 3.0 grade point average.

In 1950, 150 + 225 + 300 + 450 = 1,125 students had a grade point average of 3.0 or higher. Thus, 3,000 – 1,125 = 1,875 students earned a grade point average *less than* 3.0. As a percent of the class, this was

$$\left(\frac{1,875}{3,000}\times 100\right)\% = 62.5\%$$.

Problem Set G: The vertical number scale on the left side of the graph applies to both data sets, but for Average Temperature the units are °F and for Electric Energy Cost the units are dollars ($). For example, in January the average temperature was between 30°F and 40°F and the electric energy cost was about $70. Be careful to read data from the correct set; it would be easy to mix up which line is which. Consult the key frequently, and double-check your answers.

25. **July and August only.** Electric energy cost is represented by the light gray line and circular data points. A cost increase from one month to the next would mean a positive slope for the line segment between the two circular data points. The greater the slope of the light gray line segment, the greater the cost increase between those two months. There was an increase each month between May and September, and again between November and December. But the steepest positive slope is between July and August.

The cost increase from July to August was approximately \$145 - \$103 = \$42. For comparison, the cost increase from June to July was only about \$103 - \$70 = \$33. The correct answers are July and August.

26. **April and May only.** Electric energy cost is represented by the light gray line and circular data points. A cost increase from one month to the next would mean a positive slope for the line segment between the two data points, and a cost decrease would mean a negative slope. The steeper the slope of the line segment, the greater the cost change between two consecutive months. A cost change of \$0 would mean the line segment has a slope of 0 (i.e., it is horizontal).

To find the two consecutive months with the smallest electric energy cost change, look for the light gray line segment that is most horizontal. The line segment between April and May is nearly horizontal. The correct answers are April and May.

27. **(C).** There are two ways to approximate average electric energy cost per month in the first half of the year.

One way is to read the electric energy costs off of the chart and compute the average for the first 6 months, using the light gray circular data points:

Approximate average cost = $\frac{\$70+\$65+\$55+\$47+\$47+\$70}{6}=\frac{\$354}{6}=\$59$. Answer choice (C) \$60 is closest.

The other solution method is more visual. Consider choice (A) \$45, and imagine a horizontal line at \$45. All 6 cost data points for the first half of the year are above this horizontal line, so the average must be more than \$45. Similarly, imagine a horizontal line at \$75 for choice (E). All 6 cost data points for the first half of the year are below this horizontal line, so the average must be less than \$75. When a horizontal line at \$60 is considered, the 6 cost data points “balance”: 3 are above the line and 3 are below, by approximately the same amount.

28. **(B).** To minimize $\frac{\text{Electric Energy Cost (\$)}}{\text{Average Temperature (°F)}}$, minimize cost (light gray circular data points) while maximizing average temperature (black diamond data points). Only in April, May, October, and November is the black data point equal or greater than the gray data point (i.e., the $\frac{\text{Electric Energy Cost (\$)}}{\text{Average Temperature }(°F)}$ ratio is equal or less than 1). In April, October, and November, this ratio is close to 1. In May, the difference between the cost (\$) and the average temperature (°F) is greatest, so the electric energy cost per °F of average temperature is least. The correct answer is May.

Problem Set H: The chart shows defective parts per 1,000 (i.e., rate of making mistakes) as a function of machine operator experience. The dots indicate individual machine operators, and there is quite a bit of variance by individual. The line labeled "Average" shows the average performance of the group as a whole. A trend emerges: inexperienced machine operators and very experienced machine operators make fewer mistakes than those with medium level of experience. Also, certain individual machine operators are much "better" (i.e., they produce defective parts at a lower rate) than others, even with similar levels of experience.

29. **(E).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. The fewest defective parts per 1,000 on the chart is where this average curve is lowest: operators with 16,000 hours of experience produce a little less than 25 defective parts per 1,000. Another low point is for operators with minimal experience, but even they produce between 25 and 30 defective parts per 1,000. In contrast, the defective part rate is maximized at the top of the curve: operators with 8,000 hours of experience produce about 40 defective parts per 1,000.

30. **(B).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. Machine operators with 12,000 hours of experience produce an average of about 36 defective parts per 1,000.

The other group of machine operators that produces about 36 defective parts per 1,000 has a little less than 3,200 hours of experience. (Note that there are 5 grid lines for every 4,000 hours, so each vertical grid line is 800 hours apart. The grid mark to the left of the 4,000 mark represents 4,000 - 800 = 3,200 hours.) Choice (B) is close to and less than 3,200.

Alternatively, look up the average defective part rate for machine operators with the hours of experience listed in the choices:

(A) 2,000 hours (around 33 or 34 defective parts per 1,000).
(B) CORRECT. 2,700 hours (a bit over 35 defective parts per 1,000).
(C) 4,400 hours (around 38 defective parts per 1,000).
(D) 8,400 hours (a bit less than 40 defective parts per 1,000).
(E) 12,800 (around 34 defective parts per 1,000).

31. **(C).** Because the question specifies "on average," refer to the curve marked "Average" rather than the individual data points. The defective part rate is maximized at the top of the curve: operators with 8,000 hours of experience produce about 40 defective parts per 1,000.

32. **(C).** Because the question refers to "individual machine operators," refer to the individual data points rather than the curve marked "Average."

A defective part rate of 4.2% equates to $\frac{4.2}{100} \times 1{,}000 = 42$ defective parts per 1,000. On the chart, there are only 2 data points that fall between 40 and 45 defective parts per 1,000, and they do seem to be at 42 defective parts per 1,000. The less experienced of these two machine operators had just under 8,000 hours of experience.

Problem Set I:

33. **(A).** Note that there are 5 vertical grid lines for every 10 athletes, so each vertical grid line accounts for 2 people. On the track and field roster, there are between 36 and 38 men (so it must be 37) represented by the light gray bar. On the track and field roster, there are between 60 and 62 women (so it must be 61) represented by the dark gray bar. In fraction form, the "ratio of men to women" is $\frac{\text{men}}{\text{women}}$. The correct answer is $\frac{37}{61}$.

34. **(A).** Note that there are 5 vertical grid lines for every 10 athletes, so each vertical grid line accounts for 2 people. Male athletes are represented by the light gray bars for each sport. Sum the male athletes on each of the separate varsity sports rosters.

Males on Volleyball roster: 0
Males on Track and Field roster: between 36 and 38 (so it must be 37)
Males on Tennis roster: between 8 and 10 (so it must be 9)
Males on Golf roster: 10
Males on Cross Country roster: between 16 and 18 (so it must be 17)
Males on Basketball roster: 14

There are 0 + 37 + 9 + 10 + 17 + 14 = 87 male names on all of the rosters combined, but there are only 76 male athletes total. Since tennis, golf, and basketball players are all on one roster only, there must be 87 - 76 = 11 male athletes who are counted twice by being on both the Track and Field team and the Cross Country team. The correct answer is 11.

35. **Golf only.** Male athletes are represented by the light gray bars, female athletes by the dark gray bars. A sport in which male athletes outnumber female athletes will have a shorter dark gray bar than light gray bar.

This is only the case for golf, where there are 10 male athletes and 7 female athletes. Volleyball only has female athletes, so they outnumber the zero male athletes on the roster. In tennis and basketball, there are equal numbers of men and women. Female athletes outnumber male athletes on the Cross Country and Track and Field rosters.

36. **(B).** Note that there are 5 vertical grid lines for every 10 athletes, so each vertical grid line accounts for 2 people. There are between 8 and 10 female tennis players (so it must be 9) represented by the dark gray bar next to "Tennis." There are 14 male basketball players represented by the light gray bar next to "Basketball." In fraction form, the "ratio of female tennis players to male basketball players" is $\frac{\text{female tennis players}}{\text{male basketball players}}$. Thus, the answer is $\frac{9}{14}$.

Problem Set J: This chart compares four stores, providing information about change from 2011 to 2012 in three metrics: total revenue, number of distinct customers, and total costs. It is essential to note that change in total revenue is given in terms of dollars ($), whereas changes in number of distinct customers and in total costs are given only in percent terms. In general, percents provide less information than absolute numbers, as the total (i.e., percent of what?) is needed for context.

37. **(E).** It may be tempting to select Store Z, as revenue increased from 2011 to 2012 while number of distinct customers decreased, but be careful when mixing absolute numbers and percents. Without knowing the revenue in 2012 (only the change from the previous year is known) or the number of customers (only the percent change from the previous year is known) for any of the stores, you cannot determine the answer.

38. **(A).** Because the question is about costs per customer, both of which are given in percent change terms in the chart, and the question asks about percent change for this ratio, a comparison can be made among the stores. The percent change formula in general is $\left(\frac{\text{Difference}}{\text{Original}}\times 100\right)\%$. Thus, the percent change in total costs per distinct customer at a particular store is:

$$\left(\frac{\frac{\text{cost}_{2012}}{\text{customer}_{2012}}-\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}{\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}\times 100\right)\%$$

This looks like a mess, but remember that both cost_{2012} and customer_{2012} can be written in terms of cost_{2011} and customer_{2011}, respectively, based on the percent changes given in the table. Then cost_{2011} and customer_{2011} are in each term of the fraction and can be canceled. For example, for Store W, the percent change in total costs per distinct customer is:

$$\left(\frac{\frac{1.15\times\text{cost}_{2011}}{1.02\times\text{customer}_{2011}}-\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}{\frac{\text{cost}_{2011}}{\text{customer}_{2011}}}\times 100\right)\%=\left(\frac{\frac{1.15}{1.02}-1}{1}\times 100\right)\%=\left(\left(\frac{1.15}{1.02}-1\right)\times 100\right)\%$$

In other words, the magnitude of percent change in total costs per distinct customer depends only on the ratio of (1 + Percent Change in Total Costs) to (1 + Percent Change in Number of Distinct Customers). Perform this comparison for all of the stores.

(A) Store W: $\frac{1.15}{1.02}=1.12745$. GREATEST

(B) Store X: $\frac{1.04}{1.14}=0.91228$

(C) Store Y: $\frac{1.12}{1.05}=1.06667$

(D) Store Z: $\frac{0.80}{0.93}=0.86022$

39. **Store W, Store X, and Store Y.** Because Profit = Revenue - Cost, start by thinking about whether the changes to Revenue and Costs were positive or negative for each of the stores.

(A) CORRECT. Store W: Revenue decreased by \$400,000, and costs increased by 15%. Both changes negatively affect profit.

(B) CORRECT. Store X: Revenue increased by \$520,000, but costs also increased by 4%. Profit in 2012 could be greater than, less than, or equal to profit in 2011, depending on the store's cost structure. Try sample numbers to show that profit could have decreased. If in 2011, revenue was \$20,000,000 and costs were \$15,000,000, the profit was

$5,000,000. In 2012, revenue would be $20,520,000 and costs $15,600,000, making profit $4,920,000, less than in the previous year.

(C) CORRECT. Store Y: Revenue decreased by $365,000, and costs increased by 12%. Both changes negatively affect profit.

(D) Store Z: Revenue increased by $125,000, and costs decreased by 20%. Both changes positively affect profit.

40. **(B)**. Consider each statement individually:

(A) Not necessarily true. While Store X had the greatest increase in revenue, in dollars, it is impossible to calculate percent change in revenue $\left(\frac{\text{Difference}}{\text{Original}} \times 100\right)\%$ for any of the stores without information about the actual dollar amount of their revenue in either year.

(B) TRUE. Per customer revenue is $\frac{\text{Revenue}}{\text{Number of customers}}$. Store Z experienced an increase in revenue and a decrease in number of distinct customers, both of which increase per customer revenue.

(C) Not necessarily true. While Store W had the greatest *percent* increase in total costs, it is impossible to say whether this was the greatest increase *in dollars* without knowing the actual dollar amount of total costs for each of the stores that experienced a cost increase.

(D) Not necessarily true. The chart says nothing about repeat customers, only "distinct" customers.

(E) Not necessarily true. The chart says nothing about absolute numbers of distinct customers at any of the stores, only percent change from 2011 to 2012.

Problem Set K: Much of the detail in this chart is given in the title and other text. According to the title and the * note below, the chart shows range of calories burned per hour by people in the 130 to 205 pound weight range when doing various activities.

41. **I and II only.** Consider each statement individually:

I. Could Be True. A 130 pound person typing for 10 hours would burn less than 1,000 calories, which is less than the number of calories burned by a 205 pound person doing one of several activities on the chart for 1 hour (certainly jai alai and swimming the butterfly stroke burn more than 1,000 calories).

II. Could Be True. In general, the range of calories burned per hour is greater for jai alai than for swimming the butterfly stroke. The people in question are about the same weight, but don't make any assumptions about how number of calories burned might be a function of weight in this range (i.e., is the relationship linear?). All that matters is that the calorie burning ranges for the two activities overlap, and both people fall in the weight range, so it *could* be true that a 175 pound person playing jai alai for one hour burns fewer calories than a 180 pound person swimming the butterfly stroke for one hour.

III. Cannot Be True. The *average* calories burned by two people playing table tennis for 1 hour is a maximum of about 375. Two people typing for 3 hours burn as many calories *total* as one person typing for $2 \times 3 = 6$ hours, which is a minimum of about 550. "At most 375" cannot be greater than "at least 550."

42. **(E).** For each combination of activities, look at the minimum value on the chart for the 130 pound person and the maximum value on the chart for the 205 pound person.

(A) Badminton (minimum) + Table tennis (maximum) = 275 + 375 = 650
(B) Wrestling (minimum) + Track and field, hurdles (maximum) = 350 + 925 = 1,275
(C) Typing (minimum) + Swimming, butterfly stroke (maximum) = under 100 + 1025 = under 1,125
(D) Sailing, competition (minimum) + Aerobics (maximum) = 300 + 600 = 900
(E) CORRECT. Typing (minimum) + Croquet (maximum) = under 100 + over 225 = about 325

Alternatively, note that typing and croquet are the two activities that burn the fewest calories per hour overall. A 130 pound person and a 205 pound person doing 1 hour of activity from the chart each would only burn fewer calories total if both people were typing.

Problem Set L: The table categorizes 50 African countries according to GDP (rows) and population (columns). Notice that each row sums to a subtotal number of countries with that GDP range, and each column sums to a subtotal number of countries with that population range. Both the subtotal row and subtotal column sum to 50, the grand total. Moreover, notice that both population and GDP are shown in descending order: high population/high GDP countries are in the upper left corner of the table, while low population/low GDP countries are in the lower right corner of the table.

43. **(A).** GDP between \$10 billion and \$20 billion is a single row in the table. Population between 10 and 50 million people includes two columns in the table. Look at the intersections between this row and two columns. There are 3 countries at the intersection of 10 to 20 million population and \$10 billion to \$20 billion GDP, and there are 3 countries at the intersection of 20 to 50 million population and \$10 billion to \$20 billion GDP, for a total of 6 countries.

44. **(C).** Adding the entries that are in the intersection of the bottom two rows (less than \$20 billion GDP) and in the last three columns (population less than 20 million), the number of countries is 3 + 3 + 3 + 7 + 8 + 7 = 31. Out of 50 countries, 31 fit this description, so the percent is $\left(\frac{31}{50}\times 100\right)\%$ or 62%.

45. **(B).** There are 13 countries with GDPs between \$10 billion and \$20 billion, and of these, 3 have populations between 10 million and 20 million. Thus the percent is $\left(\frac{3}{13}\times 100\right)\%$ or approximately 23%.

46. **(D).** For each choice, carefully find the row(s)/column(s) that fit the description in, and sum all table entries that apply.

(A) More than \$10 billion GDP (top 3 rows) intersecting with population less than 20 million (3 columns on right, before the subtotal column): 0 + 0 + 0 + 1 + 1 + 0 + 3 + 3 + 3 = 11
(B) Less than \$20 billion GDP (bottom 2 rows, above the subtotal row) intersecting with population more than 10

million (3 columns on left): 1 + 3 + 3 + 0 + 0 + 7 = 14
(C) More than \$20 billion GDP (entire top 2 rows, so sum the subtotal column in those rows): 5 + 10 = 15
(D) Less than \$100 billion GDP (bottom 3 rows, above the subtotal row) intersecting with population less than 10 million (2 columns on right, before the subtotal column): 1 + 0 + 3 + 3 + 8 + 7 = 22
(E) Less than \$100 billion GDP (bottom 3 rows, above the subtotal row) intersecting with population between 10 million and 50 million (2nd and 3rd column): 7 + 1 + 3 + 3 + 0 + 7 = 21

Choice (D), 22, is the greatest.

Problem Set M: This pie chart represents about 76 million owner-occupied housing units, categorized by how many people live in the household. Don't let the smaller pie chart on the right throw you off. It is just a way to "zoom in" on the relatively small categories of households with at least 5 people. These categories could have been shown as small slivers in the pie chart on the left (notice that 6.7% + 2.6% + 1.8% = 11.1%, the "Other" category in the chart on the left).

47. **(D).** Sum the households with one, two, or three people (i.e., "fewer than four people"). Together these account for 21.6% + 36.3% + 16.5% = 74.4% of the total.

48. **(E).** According to the chart, 6.7% of the 75,986,074 households are 5-person households. In the calculator, multiply 0.067 by 76 (keep "million" in mind). The result is about 5, so the answer is 5 million households.

49. **(B).** Approximate the total number of households as 76 million (close enough to 75,986,074). There are 21.6% of the total, or approximately 16.4 million, 1-person households. Since each such household has only 1 person, this represents about 16.4 million people.

There are 16.5% of the total, or approximately 12.5 million, 3-person households. Since each of these households has 3 people, that is about 37.5 million people.

There are 6.7% of the total, or approximately 5.1 million, 5-person households. Since each of these households has 5 people, that is about 25.5 million people.

Since 16.4 million < 25.5 million < 37.5 million, the correct ranking is 1-person households, 5-person households, 3-person households.

50. **(A).** The 2- to 3- person range contains 36.3% + 16.5% = 52.8% of households, so this is the correct answer. Quickly rule out the other choices as a check.
(B) The 3- to 4- person range contains 16.5% + 14.5% = 31.0% of households.
(C) The 4- to 5- person range contains 14.5% + 6.7% = 21.2% of households.
(D) The 5- to 6-person range contains 6.7% + 2.6% = 9.3% of households.
(E) The 6- to 7-person range contains at most 2.6% + 1.8% = at most 4.4% of households (remember that some of the 1.8% could consist of households with more than 7 people).

All of the choices other than (A) are less than 50%.

Chapter 25

of

Polygons and Rectangular Solids

In This Chapter…

Polygons and Rectangular Solids

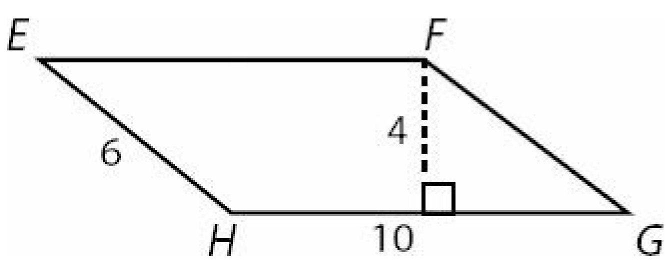
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

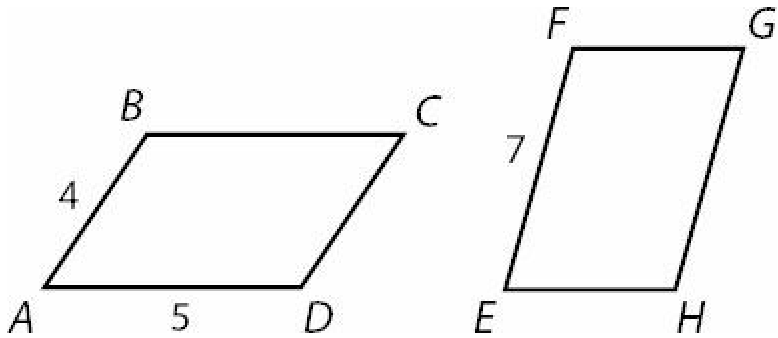
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.



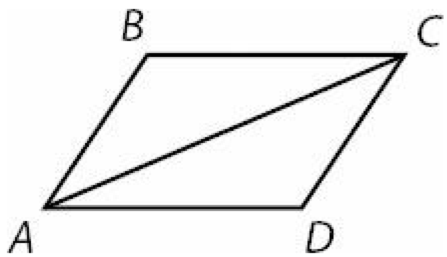
What is the area of parallelogram *EFGH*?

2.



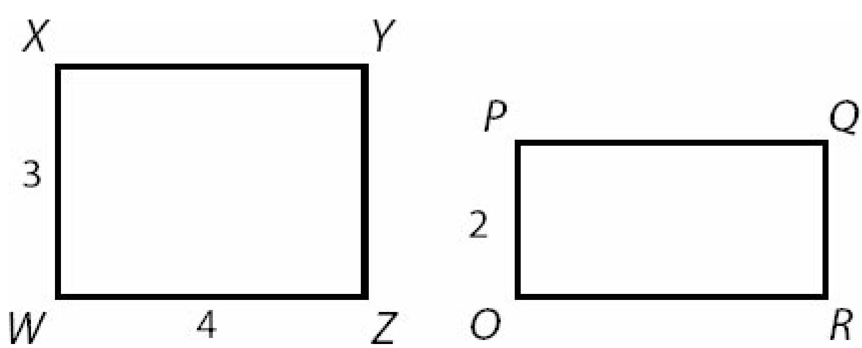
The two parallelograms pictured above have the same perimeter. What is the length of side *EH*?

3.



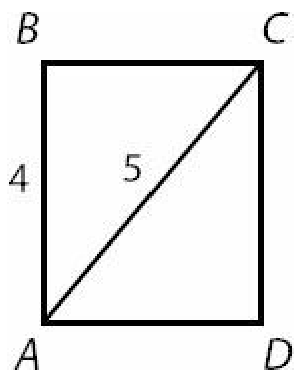
In Parallelogram *ABCD*, Triangle *ABC* has an area of 12. What is the area of Triangle *ACD*?

4.



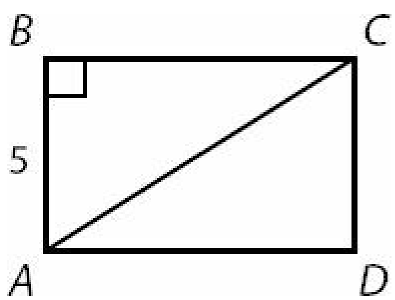
Rectangle *WXYZ* and Rectangle *OPQR* have equal areas. What is the length of side *PQ*?

5.



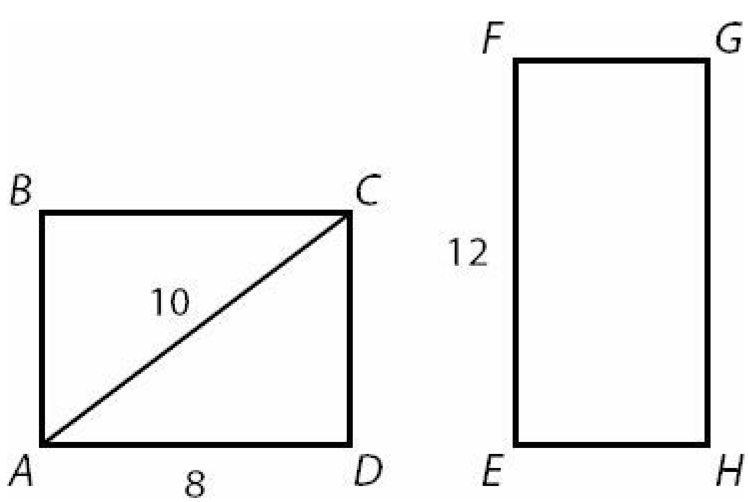
What is the area of Rectangle *ABCD*?

6.



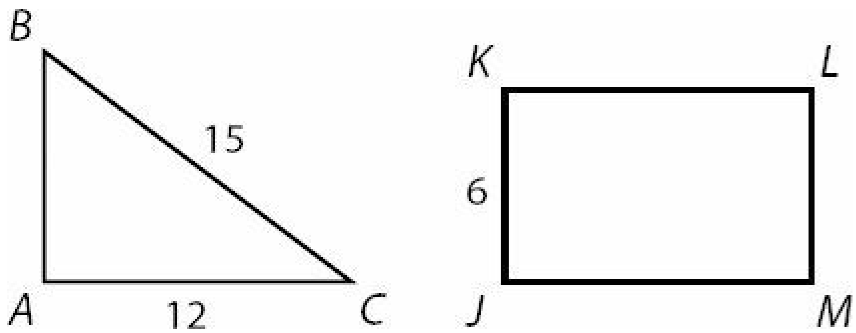
In Rectangle *ABCD*, the area of Triangle *ABC* is 30. What is the length of diagonal *AC*?

7.



Rectangles *ABCD* and *EFGH* have equal areas. What is the length of side *FG*?

8.



Triangle *ABC* and Rectangle *JKLM* have equal areas. What is the perimeter of Rectangle *JKLM*?

9.

Quantity A	**Quantity B**
The longest side of a rectangle with area 36	6

10.

Quantity A	**Quantity B**
The area of a rectangle with perimeter 40	110

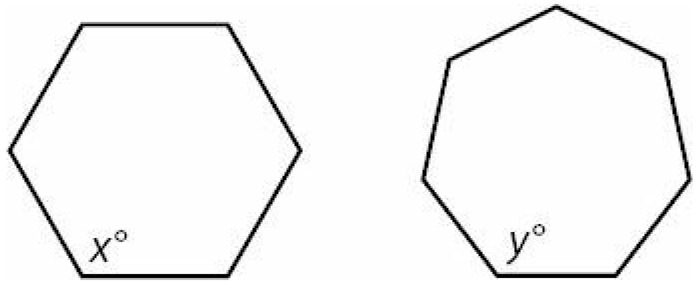
11. What is the area of a square with a diagonal measuring $4\sqrt{2}$ centimeters?

centimeters square

12.

Quantity A	**Quantity B**
The area of a parallelogram with a base of length 4 and height of 3.5	The area of a trapezoid with two parallel sides of lengths 5 and 9 and a height of 2

13.



Quantity A	Quantity B
x	y

14.

A trapezoid has an area of 42 and a height that is less than or equal to 6.

Quantity A	Quantity B
The height of the trapezoid	The length of the longer base of the trapezoid

15.

The perimeter of square W is 50% of the perimeter of square D.

Quantity A	Quantity B
The ratio of the area of square W to the area of square D	$\frac{1}{4}$

16. A 10 by 15 inch rectangular picture is displayed in a 16 by 24 inch rectangular frame. What is the area, in inches, of the part of the frame not covered by the picture?

(A) 150
(B) 234
(C) 244
(D) 264
(E) 384

17.

A rectangular box has edges of length 2, 3, and 4.

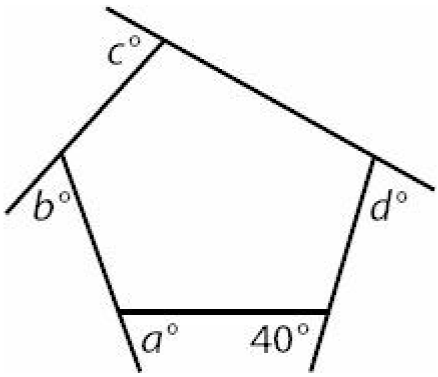
Quantity A	Quantity B
Twice the volume of the box	The surface area of the box

18. A perfect cube has surface area 96. What is its volume?

19. How many 2 inch by 2 inch by 2 inch solid cubes can be cut from six solid cubes that are 1 foot on each side? (12 inches = 1 foot)

(A) 8
(B) 64
(C) 216
(D) 1,296
(E) 1,728

20.



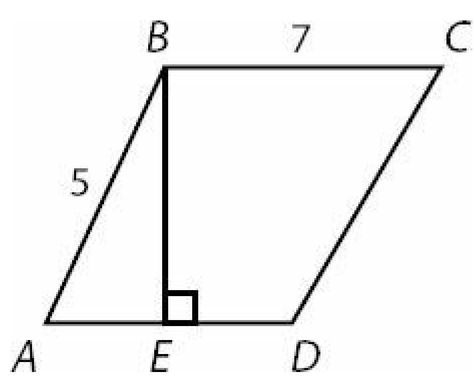
What is the value of $a + b + c + d$?

(A) 240
(B) 320
(C) 360
(D) 500
(E) 540

21. Garden A is a 225 meter by 180 meter rectangular vegetable garden, and Garden B is a rectangle with exactly half the length and width of Garden A. What is the ratio of the area of Garden A to the area of Garden B?

(A) 1 : 4
(B) 1 : 2
(C) 2 : 1
(D) 4 : 1
(E) 8 : 1

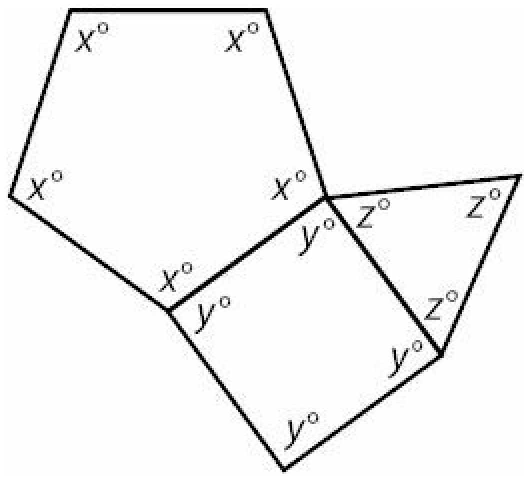
22.



In the trapezoid above, $AE = ED = 3$ and BC is parallel to AD.

Quantity A	**Quantity B**
The area of the trapezoid	35

23.



Quantity A	**Quantity B**
The value of $x + y + z$	270

24. A rectangle has an area of $54\sqrt{2}$ and a length of 6. What is the perimeter of the rectangle?

(A) $15\sqrt{2}$
(B) $30\sqrt{2}$
(C) $6 + 9\sqrt{2}$
(D) $12 + 18\sqrt{2}$
(E) $18 + 12\sqrt{2}$

25. A 1 meter by 1 meter by 1 meter sheet of paper is to be cut into 4 centimeter by 5 centimeter rectangles. How many such rectangles can be cut from the sheet of paper? (1 meter = 100 centimeters)

26.

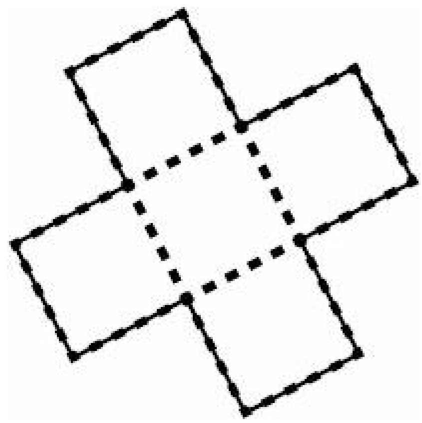
A parallelogram has two sides with length 10 and two sides with length 5.

Quantity A	**Quantity B**
The area of the parallelogram	30

27. What is the area of a regular hexagon with side length 2?

(A) $2\sqrt{3}$
(B) $2\sqrt{6}$
(C) $6\sqrt{2}$
(D) $6\sqrt{3}$
(E) $12\sqrt{3}$

28.



The figure above is composed of 5 squares of equal area, as indicated by the dotted lines. The total area of the figure is 45.

Quantity A	Quantity B
The perimeter of the figure	48

29.

Quantity A	Quantity B
The largest possible area of a rhombus with side 4.	The area of a square with side 4.

30. A 2 foot by 2 foot by 2 foot solid cube is cut into 2 inch by 2 inch by 4 inch rectangular solids. What is the ratio of the total surface area of all the resulting smaller rectangular solids to the surface area of the original cube? (1 foot = 12 inches)

(A) 2 : 1
(B) 4 : 1
(C) 5 : 1
(D) 8 : 1
(E) 10 : 1

31. If a cube has the same volume (in cubic units) as surface area (in square units), what is the length of one side?

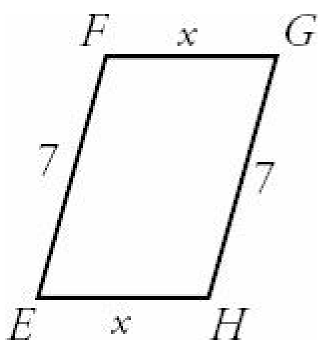
(A) 1
(B) 3
(C) $\frac{5}{3}$

(D) 6
(E) No such cube is possible.

Polygons and Rectangular Solids Answers

1. **40.** The area of a parallelogram is base × height. In this parallelogram, the base is 10 and the height is 4 (remember, base and height need to be perpendicular). So the area is 10 × 4 = 40.

2. **2.** First find the perimeter of Parallelogram *ABCD*. You know that 2 sides have a length of 4, and 2 sides have a length of 5. The perimeter is 2 × (4 + 5) = 18. That means Parallelogram *EFGH* also has a perimeter of 18. You know side *GH* also has a length of 7. You don't know the lengths of the other 2 sides, but you know they have the same length, so for now say the length of each side is *x*. Your parallelogram now looks like this:



So you know that $7 + x + 7 + x = 18 \rightarrow 2x + 14 = 18$ à $2x = 4 \rightarrow x = 2$

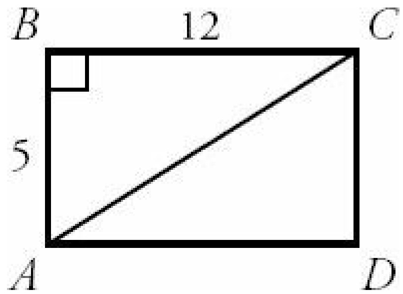
The length of side *EH* is 2.

3. **12.** One property that is true of any parallelogram is that the diagonal will split the parallelogram into two equal triangles. If Triangle *ABC* has an area of 12, then Triangle *ACD* must also have an area of 12.

4. **6.** You can start by finding the area of Rectangle *WXYZ*. Area of a rectangle is length × width, so the area of Rectangle *WXYZ* is 3 × 4 = 12. So Rectangle *OPQR* also has an area of 12. You know the length of side *OP*, so that is the width of Rectangle *OPQR*. So now you know the area, and you know the width, so you can solve for the length. $l \times 2 = 12 \rightarrow l = 6$. The length of side *PQ* is 6.

5. **12.** To find the area of Rectangle *ABCD*, you need to know the length of *AD* or *BC*. In a rectangle, every internal angle is 90 degrees, so Triangle *ABD* is actually a right triangle. That means you can use the Pythagorean Theorem to find the length of side *AD*. Actually, this right triangle is one of the Pythagorean Triplets—a 3-4-5 triangle. The length of side *AD* is 3. That means the area of Rectangle *ABCD* is 3 × 4 = 12.

6. **13.** You know the area of Triangle *ABC* and the length of side *AB*. Because side *BC* is perpendicular to side *AB*, you can use those as the base and height of Triangle *ABC*. So you know that 1/2(5) × (*BC*) = 30. That means the length of side *BC* is 12.



Now you can use the Pythagorean Theorem to find the length of diagonal *AC*, which is the hypotenuse of right triangle *ABC*. You can also recognize that this is a Pythagorean Triplet —a 5–12–13 triangle. The length of diagonal *AC* is 13.

7. **4.** The first thing to notice in this problem is that you can find the length of side *CD*. Triangle *ACD* is a right triangle, and you know the lengths of two of the sides. You can either use the Pythagorean Theorem or recognize that this is one of the Pythagorean Triplets—a 6–8–10 triangle. The length of side *CD* is 6. Now you can find the area of Rectangle *ABCD*. Side *AD* is the length and side *CD* is the width. $8 \times 6 = 48$.

That means that the area of Rectangle *EFGH* is also 48. You can use the area and the length of side *EF* to solve for the length of side *FG*. $12 \times (FG) = 48$. The length of side *FG* is 4.

8. **30.** If you can find the length of side *AB*, then you can find the area of Triangle *ABC*. You can use the Pythagorean Theorem to find the length of side *AB*. $(12)^2 + (AB)^2 = (15)^2 \rightarrow 144 + (AB)^2 = 225 \rightarrow (AB)^2 = 81 \rightarrow AB = 9$. (A 9–12–15 triangle is a 3–4–5 triangle, with all the measurements tripled.)

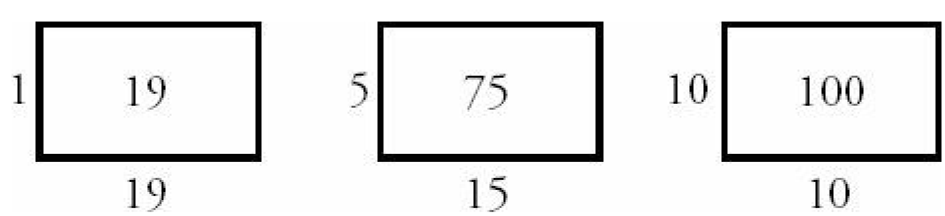
Now that you know *AB*, you can find the area of Triangle *ABC*. It's $1/2(12) \times 9 = 54$.

That means that Rectangle *JKLM* also has an area of 54. You have one side of the rectangle, so you can solve for the other. $6 \times (JM) = 54$. So the length of side *JM* is 9. That means that the perimeter is $2 \times (6 + 9) = 30$.

9. **(D).** A rectangle with area 36 could have length of 36 and width of 1, or length of 9 and width of 4, or an infinite number of other values, since the problem does not say that the side lengths must be integers. In every example except one, though, the longer sides are longer than 6 (and the shorter sides are less than 6), and Quantity A is greater. The only exception occurs if the rectangle is actually a square. If length is 6 and width is 6, the two quantities are equal. A square is definitely a type of rectangle! It is, in fact, an equilateral rectangle. This exception means the correct answer is (D).

Note also that Quantitative Comparisons are very often more interested in testing weird cases and exceptions to rules than they are in testing your knowledge of straightforward cases.

10. **(B).** While a rectangle with perimeter 40 could have many different areas, all of these areas are less than 110:



How can you be sure this will always be the case? It would be helpful to know the rule that the area of a rectangle with constant perimeter increases as length and width become more similar, and is maximized when the rectangle is a square. Thus, the 10 by 10 version of the rectangle represents the maximum possible area, which is still less than 110.

11. **16.** When a square is cut by a diagonal, two 45–45–90 triangles are created. Use the 45–45–90 formula (sides in the ratio $1 : 1 : \sqrt{2}$) to determine that the sides are equal to 4, and thus the area is $4 \times 4 = 16$. Alternatively, you could label each side of the square x (since they're the same) and use the Pythagorean Theorem:

$$x + x = (4\sqrt{2})^2$$
$$2x^2 = 32$$
$$x^2 = 16$$
$$x = 4$$

Thus, area = $4 \times 4 = 16$.

12. **(C).** The formula for area of a parallelogram is *base* × *height*, so Quantity A is $4 \times 3.5 = 14$.

The formula for area of a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$, where b_1 and b_2 are the lengths of the parallel sides, so Quantity B is $\frac{(5+9)}{2} \times 2 = 14$.

The two quantities are equal.

13. **(D).** Do not assume that any polygon is a regular figure unless you are told this. (For instance, if *every* angle in the hexagon were labeled with the same variable, you could be sure the hexagon was regular).

Using the formula $(n - 2)(180)$ where n is the number of sides, you can calculate that the sum of the angles in the 6-sided figure is 720 and the sum of the angles in the 7-sided figure is 900. However, you do not know how those totals are distributed among the interior angles, so either x or y could be greater.

14. **(B).** The area formula for a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$. For a fixed area, the average of the bases is minimized when the height is maximized, and vice versa. If the area is 42 and the maximum height is 6, then $\frac{(b_1 + b_2)}{2}$ is at least 7. Thus, the sum of the bases is at least 14. If two bases sum to 14, the longer base is greater than 7 (or else both bases are equal to 7).

Quantity A is less than or equal to 6.
Quantity B is greater than or equal to 7.

15. **(C).** If one square has twice the perimeter, it has twice the side length, it will have four times the area. Why is this? Doubling only the length doubles the area. Then, doubling the width doubles the area *again*.

You can also prove this with real numbers. Say square W has perimeter 8 and Square D has perimeter 16. Thus, square W has side 2 and Square D has side 4. The areas are 4 and 16, respectively. As a ratio, 4/16 reduces to 1/4.

16. **(B).** The area of the picture is $10 \times 15 = 150$. The area of the frame is $16 \times 24 = 384$. Subtract to get the answer: $384 - 150 = 234$.

17. **(B).** The volume of a rectangular box is $length \times width \times height = 2 \times 3 \times 4 = 24$. Quantity A is double this volume, or 48.

The surface area of a rectangular box is $2(length \times width) + 2(width \times height) + 2(length \times height) = 2(6) + 2(12) + 2(8) = 52$.

Quantity B is greater.

18. **64.** The surface area of a cube is given by the formula Surface Area = $6(side)^2$. Or just think about it logically: since all the faces are the same, the total surface area is 6 times the surface area of a single face. Since the Surface Area = 96:

$$96 = 6(side)^2$$
$$16 = (side)^2$$
$$4 = side$$

The volume of a cube is Volume = $(side)^3 = 4^3 = 64$.

19. **(D).** Each large solid cube is 12 inches × 12 inches × 12 inches. Each dimension (length, width, and height) is to be cut identically at 2 inch increments, creating 6 smaller cubes in each dimension. Thus, $6 \times 6 \times 6$ small cubes can be cut from each large cube. There are 6 large cubes to be cut this way, though, so the total number of small cubes that can be cut is $6(6 \times 6 \times 6) = 1{,}296$.

20. **(B).** The interior figure shown is a pentagon, although an irregular one. The sum of the interior angles of any polygon can be determined using the formula $(n - 2)(180)$, where n is the number of sides:

$$(5 - 2)(180) = (3)(180) = 540$$

Using the rule that angles forming a straight line sum to 180, the interior angles of the pentagon (starting at the top and going clockwise) are $180 - c$, $180 - d$, 140, $180 - a$, and $180 - b$. The sum of these angles can be set equal to 540.

$$540 = (180 - c) + (180 - d) + 140 + (180 - a) + (180 - b)$$
$$540 = 140 + 4(180) - a - b - c - d$$
$$540 - 140 - 720 = -(a + b + c + d)$$
$$-320 = -(a + b + c + d)$$

So, $a + b + c + d = 320$.

21. **(D).** Garden A has an area of $225 \times 180 = 40{,}500$. Garden B has an area of $112.5 \times 90 = 10{,}125$. The answer is 40,500/10,125, which reduces to 4/1, or a 4 : 1 ratio.

There is a more efficient solution, however. Halving only the length of a rectangle will divide the area by 2. Halving only the width will divide the area by 2. So halving both the length and width of the rectangle will divide the area by 4. The ratio is 4 : 1.

22. **(B).** First, note that while the figure may *look* like parallelogram, it is actually a trapezoid, as it has two parallel sides of unequal length ($AD = 6$ and $BC = 7$). A trapezoid has two parallel sides (the bases). The formula for the area of a trapezoid is $A = \frac{(b_1 + b_2)}{2} \times h$, where b_1 and b_2 are the lengths of the parallel sides and h is the height (BE in this figure).

Triangle ABE is a 3–4–5 special right triangle, so BE is 4. (You could also use the Pythagorean Theorem to determine this.)

Thus, the area is $\frac{(6+7)}{2} \times 4 = 26$. Quantity B is greater.

23. **(B).** Each angle in the pentagon is labeled with the same variable, so this is a regular pentagon. Using the formula $(n - 2)(180)$, where n is the number of sides, the sum of all the interior angles of the pentagon is $(3)(180) = 540$ degrees. Divide by 5 to get $x = 108$.

Now, the quadrilateral. All four-sided figures have interior angles that sum to 360. If you didn't have that memorized, you could also use $(n - 2)(180)$ to determine this. Divide 360 by 4 to get $y = 90$.

Now, the triangle. It is equilateral, so $z = 60$. (The sum of angle measures in a triangle is always 180; if the angles are equal, they will each equal 60.)

Thus, $x + y + z = 108 + 90 + 60 = 258$. Quantity B is greater.

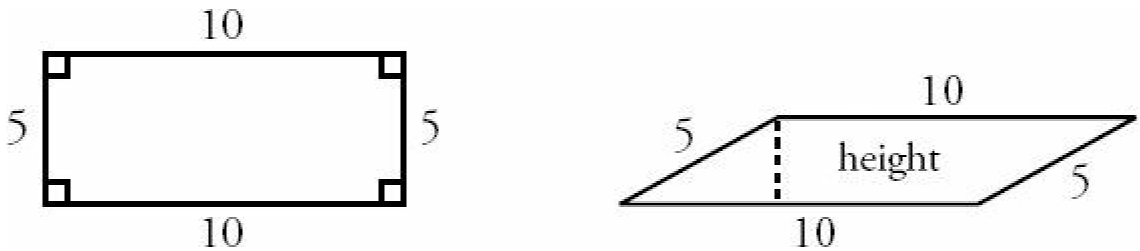
24. **(D).** Since the area of a rectangle is *length* × *width*:

$$54\sqrt{2} = 6 \times width$$
$$9\sqrt{2} = width$$

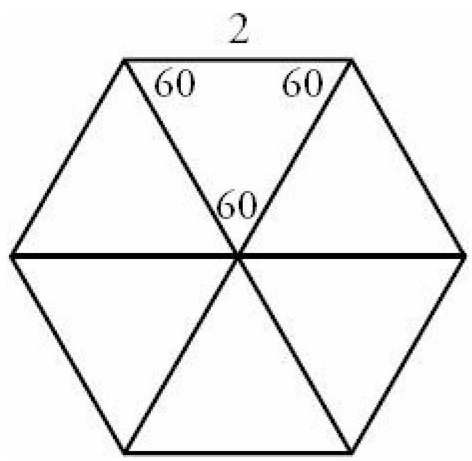
Since perimeter is 2*length* + 2*width*, the perimeter of the rectangle is $2(6) + 2(9\sqrt{2}) = 12 + 18\sqrt{2}$, or choice (D).

25. **500.** Since the sheet of paper is measured in meters and the small rectangles in centimeters, first convert the measures of the sheet of paper to centimeters. The large sheet of paper measures 100 cm by 100 cm. The most efficient way to cut 4 cm by 5 cm rectangles is to cut vertically every 4 cm and horizontally every 5 cm (or vice versa; the idea is that all the small rectangles should be oriented the same direction on the larger sheet). Doing so creates a grid of $\frac{100}{4} \times \frac{100}{5} = 25 \times 20 = 500$ small rectangles.

26. **(D).** The formula for the area of a parallelogram is *base* × *height*, where height is the perpendicular distance between the parallel bases, not necessarily the other side of the parallelogram. However, if the parallelogram is actually a rectangle, the height IS the other side of the parallelogram, and is thereby maximized. So, if the parallelogram is actually a rectangle, the area would be equal to 50, but if the parallelogram has more extreme angle measures, the height could be very, very small, making the area much less than 30.



27. **(D)**. Divide the hexagon with three diagonals (running through the center) to get six triangles. Since the sum of the angles in any polygon is (n - 2)(180), the sum for a hexagon is 720. Divide by 6 to get that each angle is 120. When you divide the hexagon into triangles, you split each 120 to make two 60 degree angles for each triangle. Any triangle that has two angles of 60 must have a third angle of 60 as well, since triangles always sum to 180. Thus, all six triangles are equilateral. Therefore, all three sides of each triangle are equal to 2.

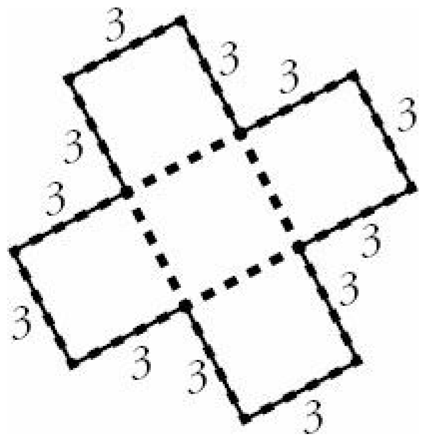


For any equilateral triangle, the height equals half the side times $\sqrt{3}$. Therefore, the height is simply $\sqrt{3}$. Since $A = \frac{bh}{2}$, the area of each equilateral is $\frac{2\sqrt{3}}{2} = \sqrt{3}$.

Since there are six such triangles, the answer is $6\sqrt{3}$.

28. **(B).** If a figure with area of 45 is composed of 5 equal squares, simply divide to get that the area of each square is 9 and thus the side of each square is 3.

Don't make the mistake of adding up EVERY side of every square to get the perimeter—make sure you only count lengths that are actually part of the perimeter of the overall figure. (Note that the central square does not have any lengths that are part of the perimeter).

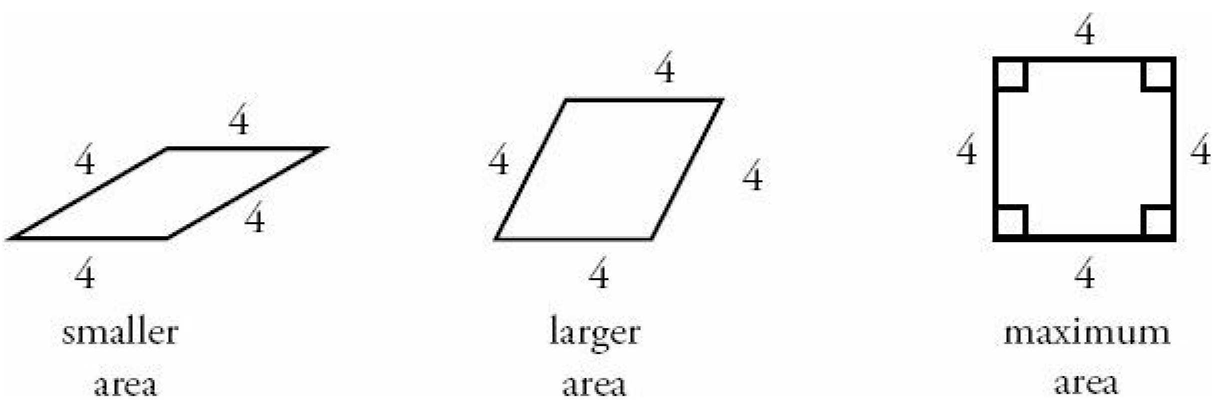


The perimeter is made of 12 segments, each with length 3. The perimeter is 36.

Incorrect choice (A) comes from reasoning that 5 squares have 20 total sides, each of length 3, and thus the combined length would be 60. You also cannot just subtract the four dotted line lengths, as each of these was actually counted twice, as part of the central square and one of the others. This mistake would incorrectly yield choice (C). The best approach here is to make a quick sketch of the figure, label the sketch with what you know, and count up the perimeter.

29. **(C).** First, note that Quantity B is simply 16.

For Quantity A, a rhombus is a parallelogram with 4 sides of equal length. A square is a type of rhombus (specifically, it is a rhombus that has four equal angles). The rhombus with the largest possible area would be a square—a square identical to the one described in Quantity B.



30. **(E).** To find the surface area of the original cube, first convert the side lengths to inches (it is NOT okay to find surface area or volume and then convert using 1 foot = 12 inches; this is only true for straight-line distances). The equation for surface area is $6s^2$. So, the surface area of the large original cube is 6(24 inches)^2 = 3,456 square inches.

Each large solid cube is 24 inches × 24 inches × 24 inches. To cut the large cube into 2 inch by 2 inch by 4 inch rectangular solids, two dimensions (length and width, say) will be sliced every 2 inches, while one dimension (height, say) will be sliced every 4 inches. Thus, $\frac{24}{2}\times\frac{24}{2}\times\frac{24}{4}=12\times12\times6=864$ small rectangular solids can be cut from the large cube.

The equation for the surface area of a rectangular solid is: $2lw + 2wh + 2lh$. In this case, that is 2(2 × 2) + 2(2 × 4) + 2(2 × 4) = 8 + 16 + 16 = 40 square inches per small rectangular solid. There are 864 small rectangular solids, so the total surface area is:

$40 \times 864 = 34{,}560$ square inches.

Finally, the ratio of the total surface area of all the resulting smaller rectangular solids to the surface area of the original cube is the ratio of 34,560 to 3,456. This ratio reduces to 10 to 1.

31. **(D)**. To solve this question, you need the equations for the volume and the surface area of a cube:

Volume = s^3 Surface area = $6s^2$

If a cube has the same volume as surface area, set these equal:

$$s^3 = 6s^2$$
$$s = 6$$

Chapter 26
of

Circles and Cylinders

In This Chapter…

Circles and Cylinders

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. A circle has an area of 16π. What is its circumference?

(A) 4π
(B) 8π
(C) 16π
(D) 32π
(E) It cannot be determined from the information given.

2. A circle has a circumference of 20π. What is its area?

(A) 10π
(B) 20π
(C) 40π
(D) 100π
(E) 400π

3. A circle has a circumference of 8. What is its area?

(A) $\frac{4}{\pi}$

(B) $\frac{4}{\pi^2}$

(C) $\frac{16}{\pi}$

(D) $\frac{16}{\pi^2}$

(E) 16π

4. A circle has a diameter of 5. What is its area?

(A) $\frac{25\pi}{4}$

(B) $\frac{25\pi}{2}$

(C) $\frac{25\pi^2}{2}$

(D) 10π

(E) 25π

5. A circle's area equals its circumference. What is its radius?

(A) 1
(B) 2
(C) 4
(D) 8
(E) 16

6.

Circle C has a radius r such that $1 < r < 5$

Quantity A	**Quantity B**
The area of Circle C	The circumference of Circle C

7.

Quantity A	**Quantity B**
The radius of a circle with area 36π	The radius of a circle with circumference 12π

8.

Quantity A	Quantity B
The area of a circle with radius 4	The circumference of a circle with radius 6

9. A circle has radius $\frac{5}{3}$. What is its area?

(A) $\frac{\sqrt{5}\pi}{3}$

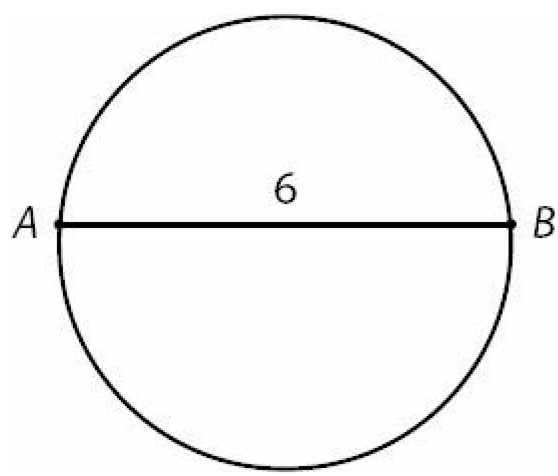
(B) $\frac{5\pi}{3}$

(C) $\frac{25\pi}{9}$

(D) $\frac{10\pi}{3}$

(E) $\frac{100\pi}{9}$

10.



AB is <u>not</u> a diameter of the circle

Quantity A	Quantity B
The area of the circle	9π

11. A circle has radius 0.01. What is its area?

(A) $\frac{\pi}{10}$

(B) $\frac{\pi}{100}$

(C) $\frac{\pi}{1,000}$

(D) $\frac{\pi}{10,000}$

(E) $\frac{\pi}{100,000}$

12. A circle has radius $\sqrt{x}$. What is its circumference?

(A) πx
(B) $2\pi x$
(C) $2\pi\sqrt{x}$
(D) $2\pi x^2$
(E) πx^2

13.

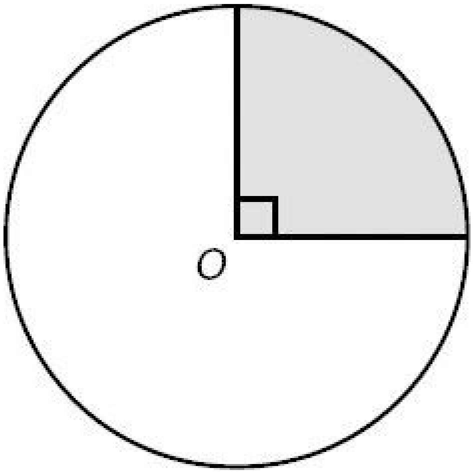
The circumference of a circle is greater than 7π.

Quantity A	**Quantity B**
The area of the circle	15π

14. A circle has an area of 4π. If the radius were doubled, the new area of the circle would be how many times the original area?

(A) 2
(B) 3
(C) 4
(D) 5
(E) It cannot be determined from the information given.

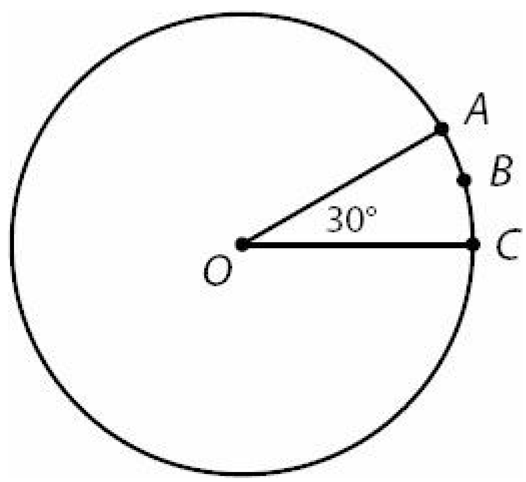
15.



In the figure above, point O is the center of the circle. If the radius of the circle is 8, what is the area of the shaded sector?

(A) 2π
(B) 4π
(C) 8π
(D) 16π
(E) 32π

16.



The radius of the circle with center O is 6.

Quantity A	**Quantity B**
The length of arc ABC	3

17. A sector of a circle has an arc length of 7π. If the diameter of the circle is 14, what is the measure of the central angle of the sector, in degrees?

(A) 45
(B) 60
(C) 90
(D) 120
(E) 180

18. A sector of a circle has a central angle of 270°. If the circle has a radius of 4, what is the area of the sector?

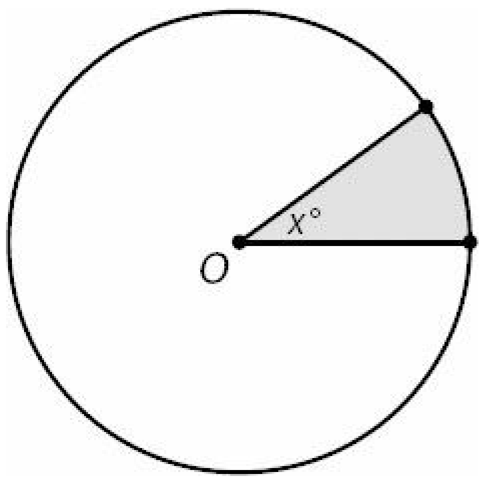
(A) 4π
(B) 8π
(C) 12π
(D) 16π
(E) 20π

19.

Within a circle with radius 12, a sector has an area of 24π.

Quantity A	**Quantity B**
The measure of the central angle of the sector, in degrees	90

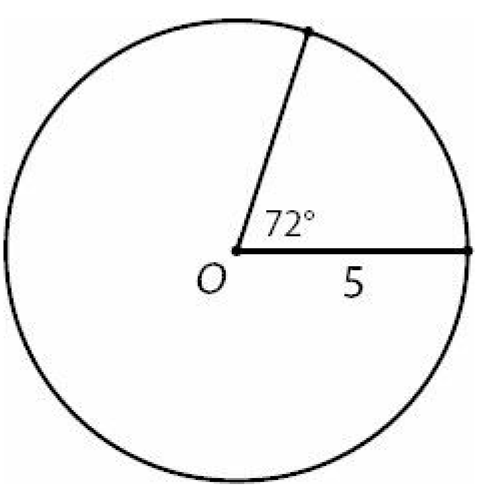
20.



The area of the shaded sector is $\frac{1}{10}$ of the area of the full circle.

Quantity A	**Quantity B**
$2x$	75

21.



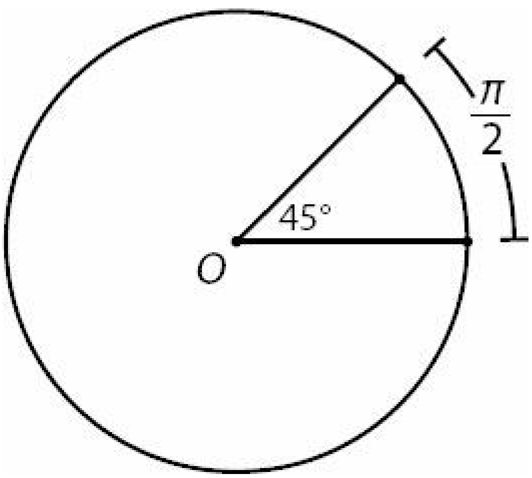
If O is the center of the circle, what is the perimeter of the sector with central angle 72°?

(A) $5 + 2\pi$
(B) $10 + 2\pi$
(C) $10 + 4\pi$
(D) $10 + 5\pi$
(E) $20 + 2\pi$

22. A sector of a circle has a radius of 8 and an area of 8π. What is the arc length of the sector?

(A) π
(B) 2π
(C) 4π
(D) 6π
(E) 8π

23.



If point O is the center of the circle in the figure above, what is the radius of the circle?

24.

Sector A and Sector B are sectors of two different circles.
Sector A has a radius of 4 and a central angle of 90°.
Sector B has a radius of 6 and a central angle of 45°.

Quantity A	**Quantity B**
The area of Sector A	The area of Sector B

25. What is the volume of a right circular cylinder with a radius of 2 and a height of 4?

(A) 8π
(B) 12π
(C) 16π
(D) 32π
(E) 72π

26. What is the height of a right circular cylinder with radius 1 and volume 16π?

27.

A right circular cylinder has volume 24π.

Quantity A	**Quantity B**
The height of the cylinder	The radius of the cylinder

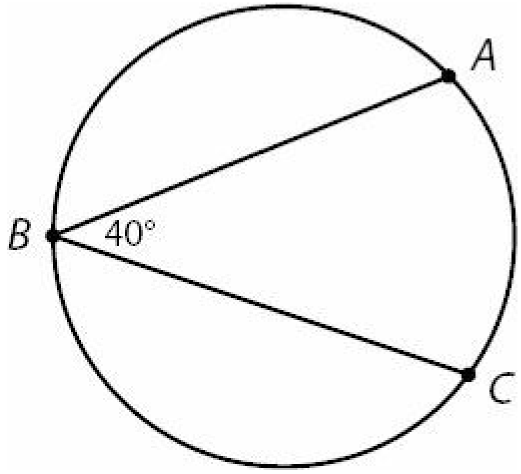
28. If a half-full 4-inch by 2-inch by 8-inch box of soymilk is poured into a right circular cylindrical glass with radius 2 inches, how many inches high will the soymilk reach? (Assume that the capacity of the glass is greater than the volume of the soymilk.)

(A) 8
(B) 16
(C) $\frac{4}{\pi}$
(D) $\frac{8}{\pi}$
(E) $\frac{16}{\pi}$

29. If a right circular cylinder's radius is halved and its height doubled, by what percent will the volume increase or decrease?

(A) 50% decrease
(B) no change
(C) 25% increase
(D) 50% increase
(E) 100% increase

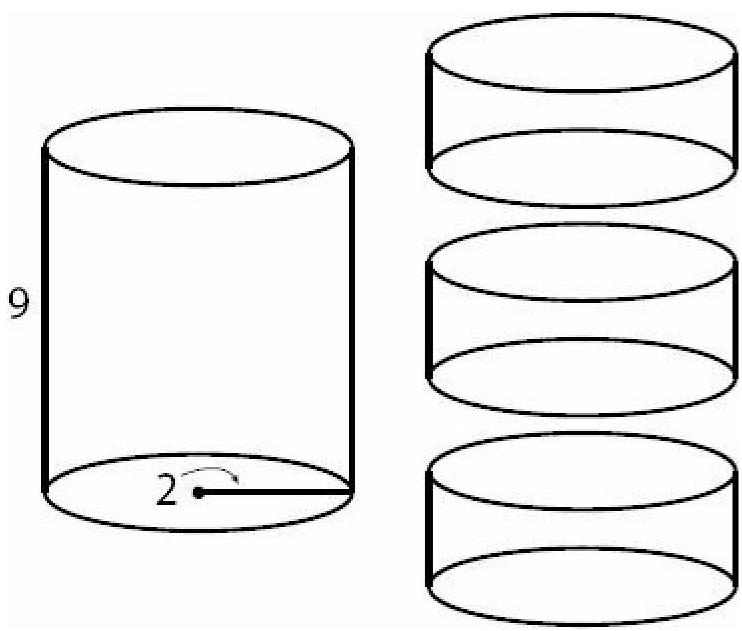
30.



If the diameter of the circle is 36, what is the length of arc *ABC*?

(A) 8
(B) 8π
(C) 28π
(D) 32π
(E) 56π

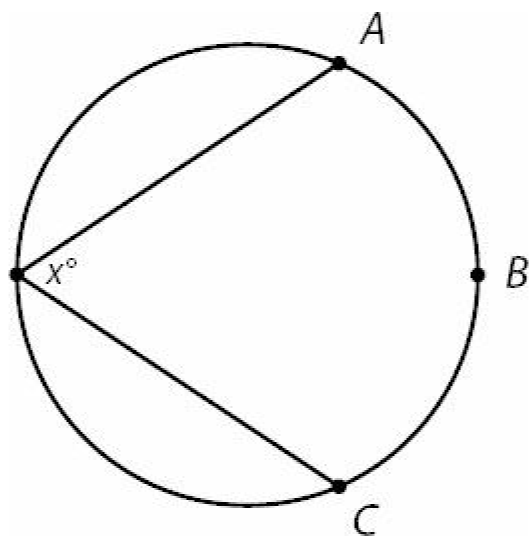
31.



If a solid right circular cylinder with height 9 and radius 2 is cut as shown into three new cylinders, each of equal and uniform height, how much new surface area is created?

(A) 4π
(B) 12π
(C) 16π
(D) 24π
(E) 36π

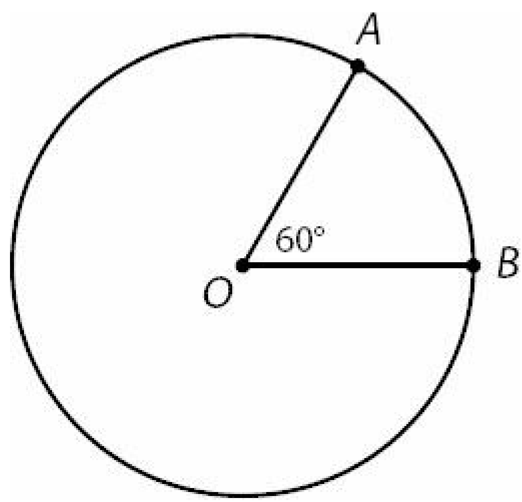
32.



$x > 60°$

Quantity A	**Quantity B**
The ratio of the length of arc *ABC* to the circumference of the circle	1/3

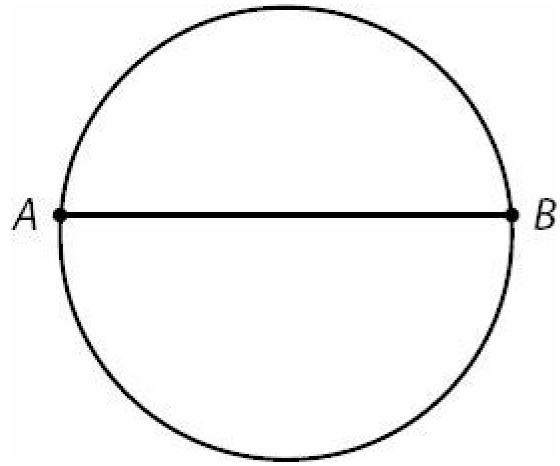
33.



Point *O* is the center of the circle above.

Quantity A	**Quantity B**
The ratio of the length of minor arc *AB* to major arc *AB*	1/6

34.



The circle above has area 25.

Quantity A	**Quantity B**
The length of chord *AB*	10

Circles and Cylinders Answers

1. **(B).** Since the area formula for a circle is $A = \pi r^2$:

$$16\pi = \pi r^2$$
$$16 = r^2$$
$$4 = r$$

Since the circumference formula is $C = 2\pi r$ and $r = 4$:

$$C = 2\pi(4)$$
$$C = 8\pi$$

2. **(D).** Since the circumference formula is $C = 2\pi r$:

$$20\pi = 2\pi r$$
$$20 = 2r$$
$$10 = r$$

Since the area formula for a circle is $A = \pi r^2$ and $r = 10$:

$$A = \pi(10)^2$$
$$A = 100\pi$$

3. **(C).** Since the circumference formula is $C = 2\pi r$:

$$8 = 2\pi r$$

Note that the circumference is just 8, not 8π, so the radius is going to look a bit unusual. First, divide both sides by 2:

$$4 = \pi r$$
$$\frac{4}{\pi} = r$$

Now, plug the radius $\frac{4}{\pi}$ into the area formula for a circle:

$$A = \pi\left(\frac{4}{\pi}\right)^2$$

$$A = \pi \times \frac{16}{\pi^2}$$

$$A = \frac{16}{\pi}$$

4. **(A).** If a circle's diameter is 5, its radius is $\frac{5}{2}$. Plug this into the area formula:

$$A = \pi\left(\frac{5}{2}\right)^2$$

$$A = \pi \times \frac{25}{4}$$

$$A = \frac{25\pi}{4}$$

5. **(B).** To find the radius that would make the area and the circumference of a circle equal, simply set the area and circumference formulas equal to one another:

$\pi r^2 = 2\pi r$

Since both sides have both r and π, divide both sides by πr:

$r = 2$

6. **(D).** Picking numbers is the easiest way to prove (D). If you begin with a radius of 3, the area is 9π and the circumference is 6π, so Quantity A is greater. If you try a radius of 4, the area is 16π and the circumference is 8π, so once again Quantity A is greater. But if you try a radius of 2, both the area and the circumference equal 4π. Therefore, Quantity A is not always greater, so the answer is (D). Note also that r is not required to be an integer. If you try a value close to the minimum, such as 1.1, Quantity B would be greater.

7. **(C).** Since the area formula for a circle is $A = \pi r^2$, calculate Quantity A by plugging 36π into the formula as the area:

$36\pi = \pi r^2$
$36 = r^2$
$6 = r$

Since the circumference formula for a circle is $C = 2\pi r$, calculate Quantity B by plugging 12π into the formula as the circumference:

$$12\pi = 2\pi r$$
$$12 = 2r$$
$$6 = r$$

The two quantities are equal. In other words, a circle with area 36π will also have circumference 12π.

8. **(A).** Since the area formula for a circle is $A = \pi r^2$, calculate Quantity A by plugging radius 4 into the formula:

$$A = \pi(4)^2$$
$$A = 16\pi$$

Since the circumference formula for a circle is $C = 2\pi r$, calculate Quantity B by plugging radius 6 into the formula:

$$C = \pi 2(6)$$
$$C = 12\pi$$

Quantity A is greater.

9. **(C).** Since the area formula for a circle is $A = \pi r^2$, plug radius $\frac{5}{3}$ into the formula:

$$A = \pi\left(\frac{5}{3}\right)^2$$

$$A = \frac{25\pi}{9}$$

10. **(A).** Since a diameter is the longest straight line you can draw from one point on a circle to another (that is, a diameter is the longest chord in a circle), the actual diameter must be *greater* than 6.

IF the diameter were 6, the radius would be 3, and the area would be:

$$A = \pi(3)^2$$
$$A = 9\pi$$

However, since the diameter must be greater than 6, the area must be greater than 9π. Do NOT make the mistake of picking (D) for Quantitative Comparison geometry questions in which you cannot "solve." There is often still a way to determine which quantity is greater.

11. **(D).** Since the formula for the area of a circle is $A = \pi r^2$, plug radius 0.01 into the formula. However, since the answers are in fraction format, it is probably easier to convert to fraction form now rather than at the end. Since

$0.01 = \frac{1}{100}$ (you can verify this in your calculator):

$$A = \pi\left(\frac{1}{100}\right)^2$$

$$A = \pi\left(\frac{1}{10,000}\right)$$

$$A = \frac{\pi}{10,000}$$

12. **(C).** Since the circumference formula for a circle is $C = 2\pi r$, plug $\sqrt{x}$ in for the radius:

$$C = 2\pi\sqrt{x}$$

This expression does not simplify — no more work is required. Note that incorrect answer choice (A) is the result of accidentally using the area formula rather than the circumference formula.

13. **(D).** The circumference is "greater than 7π." Do not make the mistake of thinking that it has to be at least 8π! There is NO rule that the number before the π must be an integer. If the circumference of the circle were 8π, the radius would be 4 and the area therefore 16π, making Quantity A greater.

However, the circumference of the circle could be 7.5π, in which case the radius would be 3.75 and the area would be 14.0625π, making Quantity B greater. Thus, the answer is (D).

14. **(C).** To begin, find the original radius of the circle: Area = $\pi r^2 = 4\pi$, so $r = 2$. Once doubled, the new radius is 4. A circle with a radius of 4 has an area of 16π. The new area of 16π is 4 times the old area of 4π.

15. **(D).** If the sector has a central angle of 90°, then the sector is 1/4 of the circle, because $\frac{90}{360} = \frac{1}{4}$. To find the area of the sector, first find the area of the whole circle. The radius is 8, which means the full circle area is $\pi(8)^2 = 64\pi$. If the circle area is 64π, then the sector's area is $1/4 \times 64\pi = 16\pi$.

16. **(A).** If the sector has a central angle of 30°, then it is 1/12th of the circle, because $\frac{30}{360} = \frac{1}{12}$. To find the arc length of the sector, first find the circumference of the entire circle. The radius of the circle is 6, so the circumference is $2\pi(6) = 12\pi$. That means that the arc length of the sector is $(1/12)(12\pi) = \pi$. Since π is about 3.14, Quantity A is greater.

17. **(E).** To find the central angle of the sector, first determine what fraction of the full circle the sector represents. The diameter of the circle is 14, so the circumference is $\pi(14) = 14\pi$. Since the arc length is 7π, the sector is 1/2 the full circle. That means that the central angle of the sector is 1/2 of 360°, or 180°.

18. **(C).** The sector is 3/4 of the circle, because $\frac{270}{360} = \frac{3}{4}$. To find the area of the sector, first find the area of the whole circle. The radius of the circle is 4, so the area is $\pi(4)^2 = 16\pi$. That means the area of the sector is $(3/4)(16\pi) = 12\pi$.

19. **(B).** First find the area of the whole circle. The radius is 12, which means the area is $\pi(12)^2 = 144\pi$. Since the sector has an area of 24π and $\frac{24\pi}{144\pi} = \frac{1}{6}$, the sector is 1/6th of the entire circle. That means that the central angle is 1/6th of 360, or 60°.

20. **(B).** If the area of the sector is $\frac{1}{10}$ of the area of the full circle, then the central angle is $\frac{1}{10}$ of the degree measure of the full circle, or $\frac{1}{10}$ of 360 = 36 = x. Thus, Quantity A = 2(36) = 72.

21. **(B).** To find the perimeter of a sector, you need the radius of the circle and the arc length of the sector. Begin by determining what fraction of the circle the sector is. The central angle of the sector is 72°, so the sector is $\frac{72}{360} = \frac{1}{5}$ of the circle. The radius is 5, so the circumference of the circle is $2\pi(5) = 10\pi$. The arc length of the sector is 1/5 of the circumference: $(1/5)(10\pi) = 2\pi$. The perimeter of the sector is simply this 2π, plus the two radii that make up the straight parts of the sector: $10 + 2\pi$.

22. **(B).** Compare the given area of the sector to the calculated area of the circle. The radius of the circle is 8, so the area of the circle is $\pi(8)^2 = 64\pi$. The area of the sector is 8π, or $\frac{8\pi}{64\pi} = \frac{1}{8}$ of the circle. The radius is 8, so the circumference of the whole circle is $2\pi(8) = 16\pi$. Since the sector is 1/8 of the circle, the arc length is $(1/8)(16\pi) = 2\pi$.

23. **2.** If the sector has a central angle of 45°, then the sector is $\frac{45}{360} = \frac{1}{8}$ of the circle. Thus, the arc length of the sector is 1/8 of the circumference of the circle, or stated differently, the circumference is 8 times the arc length of the sector. The circumference is $8(\pi/2) = 4\pi$. From the circumference formula, $4\pi = 2\pi r$, so $r = 2$. The radius of the circle is 2.

24. **(B).** Sector A is $\frac{90}{360} = \frac{1}{4}$ of the circle with radius 4. The area of this circle is $\pi(4)^2 = 16\pi$, so the area of Sector A is 1/4 of 16π, or 4π.

Sector B is $\frac{45}{360} = \frac{1}{8}$ of the circle with radius 6. The area of this circle is $\pi(6)^2 = 36\pi$, so the area of Sector B is 1/8 of 36π or 4.5π.

4.5π is greater than 4π, so the area of Sector *B* is greater than the area of Sector *A*.

25. **(C).** Use the formula for volume of a right circular cylinder, $V = \pi r^2 h$. (This formula is easy to memorize as it is simply the area of a circle, multiplied by height). $V = \pi(2)^2(4) = 16\pi$.

26. **16.** From the formula for volume of a right circular cylinder, $V = \pi r^2 h$:

$$16\pi = \pi(1)^2 h$$
$$16 = h$$

27. **(D).** Plugging into the formula for volume of a right circular cylinder, $V = 24\pi = \pi r^2 h$. However, there are many combinations of r and h that would make the volume 24π. For instance, $r = 1$ and $h = 24$, or $r = 4$ and $h = 1.5$. Keep in mind that the radius and height don't even have to be integers, so there truly are an infinite number of possibilities, some for which h is greater and some for which r is greater.

28. **(D).** A box is a rectangular solid whose volume formula is simply V = length × width × height. So, the volume of the box is 4 inches × 2 inches × 8 inches = 64 inches^3. Since the box is half full, there are 32 inches^3 of soymilk. This volume will not change when the soymilk is poured from the box into the cylinder. The formula for the volume of a cylinder is $V = \pi r^2 h$, so:

$32 = \pi(2)^2 h$, where r and h are in units of inches.

$$\frac{32}{4\pi} = h$$

$$\frac{8}{\pi} = h$$

The height is $8/\pi$ inches. Note that the height is "weird" (divided by π) because the volume of the cylinder did *not* have a π.

29. **(A).** According to the formula for the volume of a right circular cylinder, the original volume is $V = \pi r^2 h$. To halve the radius, simply replace r with $r/2$. To double the height, simply replace h with $2h$. The only caveat: be sure to use parentheses!

$$V = \pi\left(\frac{r}{2}\right)^2(2h) = \frac{2\pi r^2 h}{2^2} = \frac{\pi r^2 h}{2}$$

Thus, the volume, which was once $\pi r^2 h$, is now $\frac{\pi r^2 h}{2}$. In other words, it has been cut in half, or reduced by 50%.

Alternatively, plug in numbers. If the cylinder originally had radius 2 and height 1, the volume would be $V = \pi(2)^2(1) = 4\pi$. If the radius were halved to become 1 and the height were doubled to become 2, the volume would be $V = \pi(1)^2(2) = 2\pi$. Again, the volume is cut in half, or reduced by 50%.

30. **(C).** Note that a MINOR arc is the "short way around" the circle from one point to another, and a MAJOR arc is the "long way around." Arc *ABC* is thus the same as major arc *AC*.

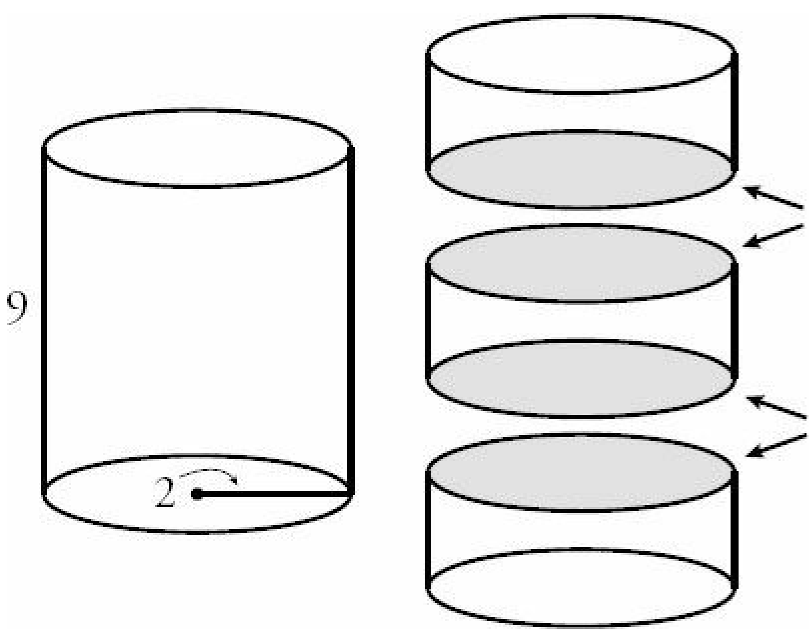
For a given arc, an inscribed angle is always half the central angle, which would be 80° in this case. The minor arc *AC* is thus $\frac{80}{360} = \frac{2}{9}$ of the circle. Since the circumference is 36π:

$$\text{minor arc } AC = \frac{2}{9}(36\pi) = 8\pi$$

Arc *ABC*, or major arc *AC*, is the entire circumference minus the minor arc:

$$36\pi - 8\pi = 28\pi$$

31. **(C).** You *could* find the surface area of the large cylinder, then the surface areas of the three new cylinders, then subtract the surface area of the large cylinder from the combined surface areas of the three new cylinders. However, there is a much faster way. When the large cylinder is cut into three smaller ones, only a few *new* surfaces are created — the bottom base of the top cylinder, the top and bottom bases of the middle cylinder, and the top surface of the bottom cylinder.



Thus, these four circular bases represent the new surface area created. Since the radius of each base is 2, use the area formula for a circle, $A = \pi r^2$:

$$A = \pi(2)^2$$
$$A = 4\pi$$

Since there are 4 such bases, multiply by 4 to get 16π.

32. **(A).** If x were equal to 60°, arc *ABC* would have a central angle of 120°. (Inscribed angles, with the vertex at the far side of the circle, are always half the central angle.) A 120° arc is 120/360 = 1/3 of the circumference of the circle. Since x is actually greater than 60°, the arc is actually greater than 1/3 of the circumference. Thus, the ratio of the arc length to the circumference is greater than 1/3.

33. **(A).** Since the angle that determines the arc is equal to 60 and 60/360 = 1/6, minor arc *AB* is 1/6 of the circumference of the circle. (There are always 360 degrees in a circle. Minor arc *AB* is the "short way around" from *A* to *B*, while major arc *AB* is the "long way around.")

Since minor arc *AB* is 1/6 of the circumference, major arc *AB* must be the other 5/6. Therefore, the ratio of the minor arc to the major arc is 1 to 5 (NOT 1 to 6!) You could calculate this as $\frac{1/6}{5/6} = \frac{1}{6} \times \frac{6}{5} = \frac{1}{5}$, or you could just reason the ratio of 1 of *anything* (such as sixths) to 5 of the same thing (again, sixths) is a 1 to 5 ratio.

Of course, the trap answer here is (C). This is a common mistake. 1/6 of the total is not the same as a 1 to 6 ratio of two parts.

34. **(B).** The equation for the area of a circle is $A = \pi r^2$. Note that the given area is just 25, *not* 25π! So:

$$\pi r^2 = 25$$
$$r^2 = \frac{25}{\pi} \approx 8$$
r = a bit less than 3.

So the diameter of the circle is a bit less than 6. The diameter is the chord with maximum length, so wherever *AB* is on this circle, it's significantly shorter than 10.

Chapter 27
of

5 lb. Book of GRE® Practice Problems

Triangles

In This Chapter…

Triangles

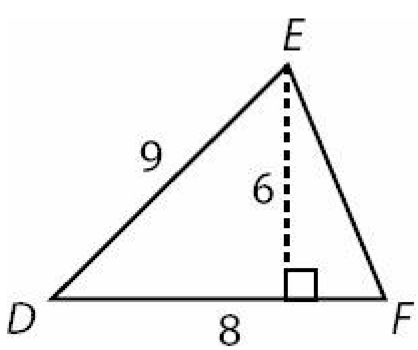
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

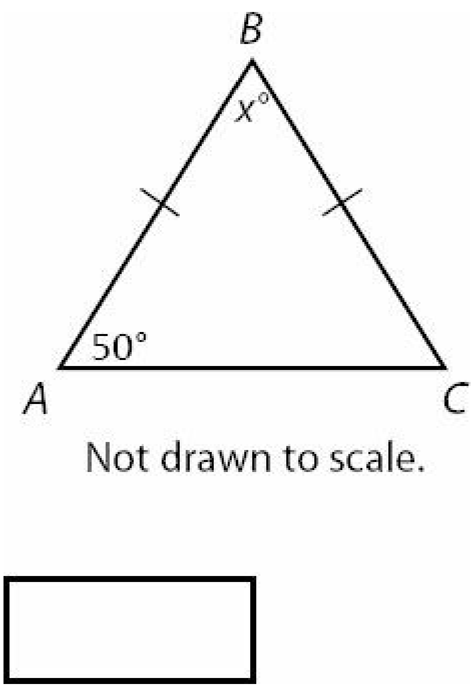
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1. What is the area of Triangle *DEF*?

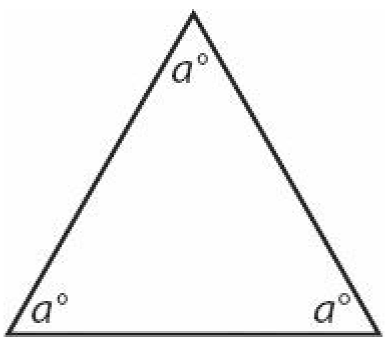


(A) 23
(B) 24
(C) 48
(D) 56
(E) 81

2. What is the value of *x*?

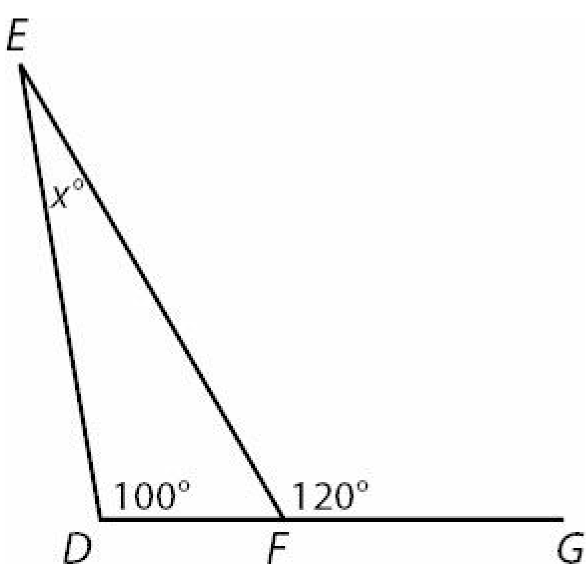


3.



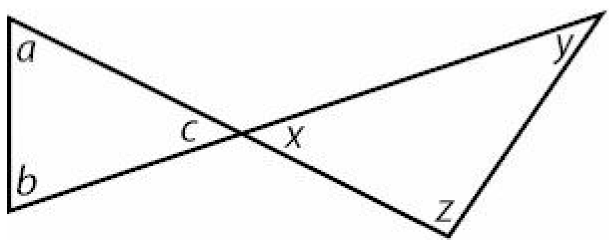
<u>**Quantity A**</u>	<u>**Quantity B**</u>
$2a + b$	$3a + \frac{b}{3}$

4.



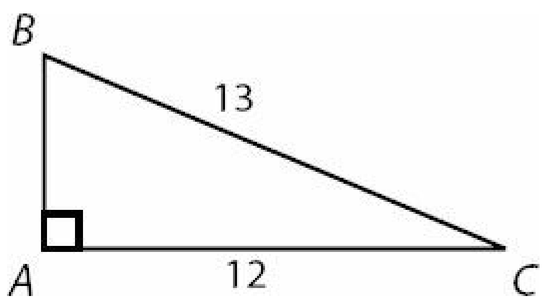
DFG is a straight line. What is the value of *x*?

5



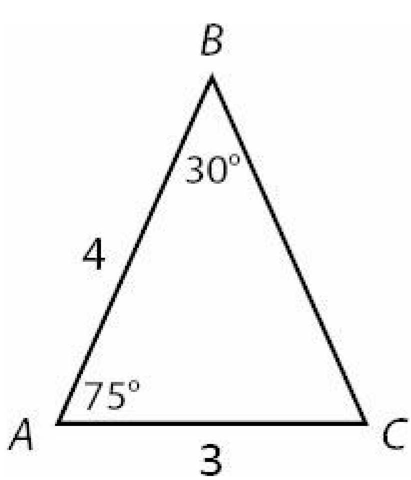
Quantity A	**Quantity B**
$a + b + x$	$c + y + z$

6.



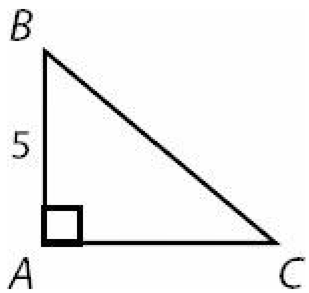
What is the area of right triangle *ABC*?

7.



What is the perimeter of triangle *ABC*?

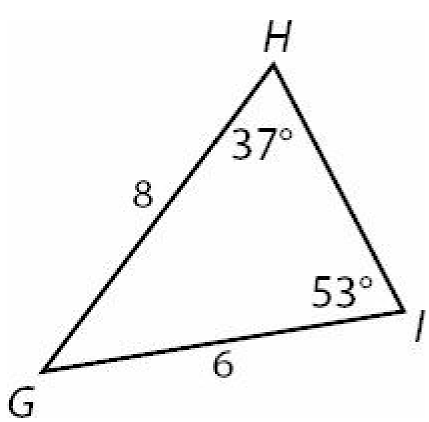
8.



The area of right triangle *ABC* is 15. What is the length of hypotenuse *BC*?

(A) $\sqrt{34}$
(B) 6
(C) $\sqrt{51}$
(D) $\sqrt{61}$
(E) $\sqrt{71}$

9.

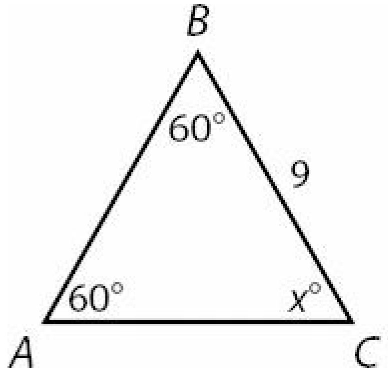


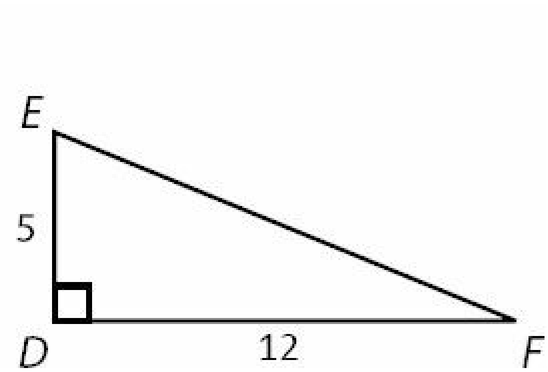
What is the length of side *HI*?

10. If the hypotenuse of an isosceles right triangle is $7\sqrt{2}$, what is the area of the triangle?

(A) 14
(B) 18
(C) 24.5
(D) 28
(E) 49

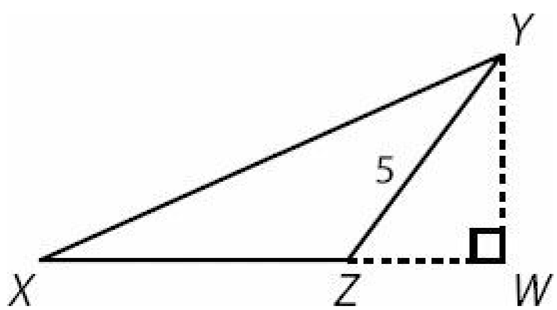
11.





Quantity A	**Quantity B**
Perimeter of triangle *ABC*	Perimeter of triangle *DEF*

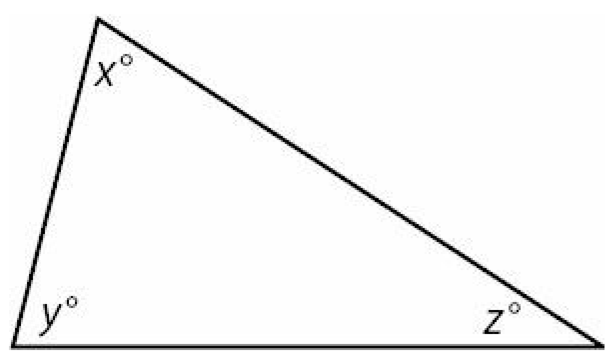
12.



WZ has a length of 3 and *ZX* has a length of 6. What is the area of Triangle *XYZ*?

$\boxed{}$

13.



In the figure shown, $z + x$ is 110 degrees.

Quantity A	**Quantity B**
x	y

14.

Isosceles triangle *ABC* has two sides with lengths 8 and 5.

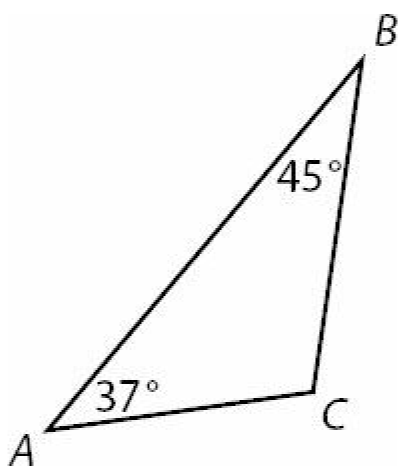
Quantity A	Quantity B
The length of the third side	8

15.

Isosceles triangle *ABC* has two sides with lengths 2 and 11.

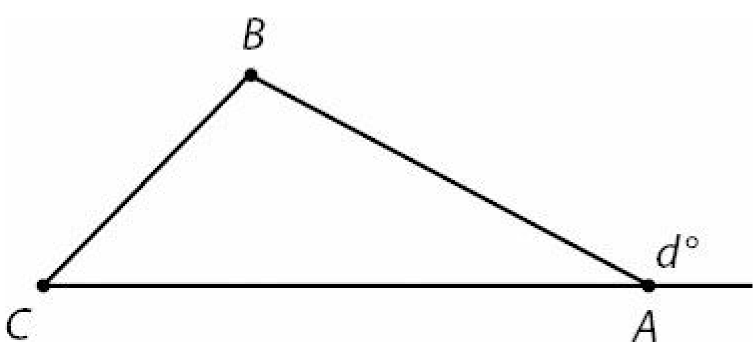
Quantity A	Quantity B
The length of the third side	11

16.



Quantity A	Quantity B
Side length *AC*	Side length *BC*

17.



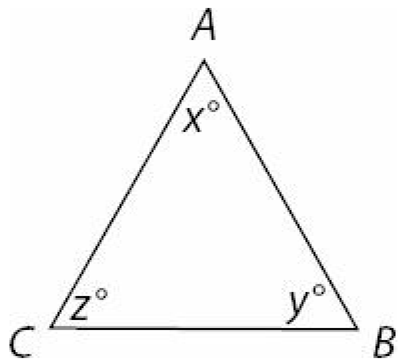
Quantity A	Quantity B
The sum of the measures of angles *B* and *C*	*d*

18.

The sides of a right triangle are 3, 4, and z.

Quantity A	**Quantity B**
z	5

19.

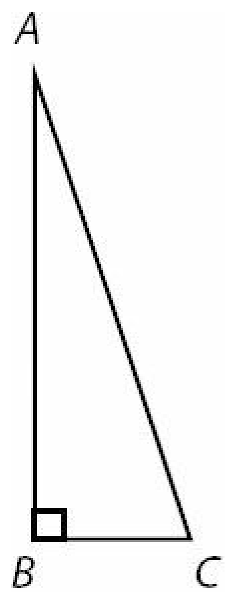


Note: Figure NOT drawn to scale

$x > z$
$z > 60$

Quantity A	**Quantity B**
The length of side AC	The length of side AB

20.



$AC = 4\sqrt{10}$

BC is 1/3 the length of AB

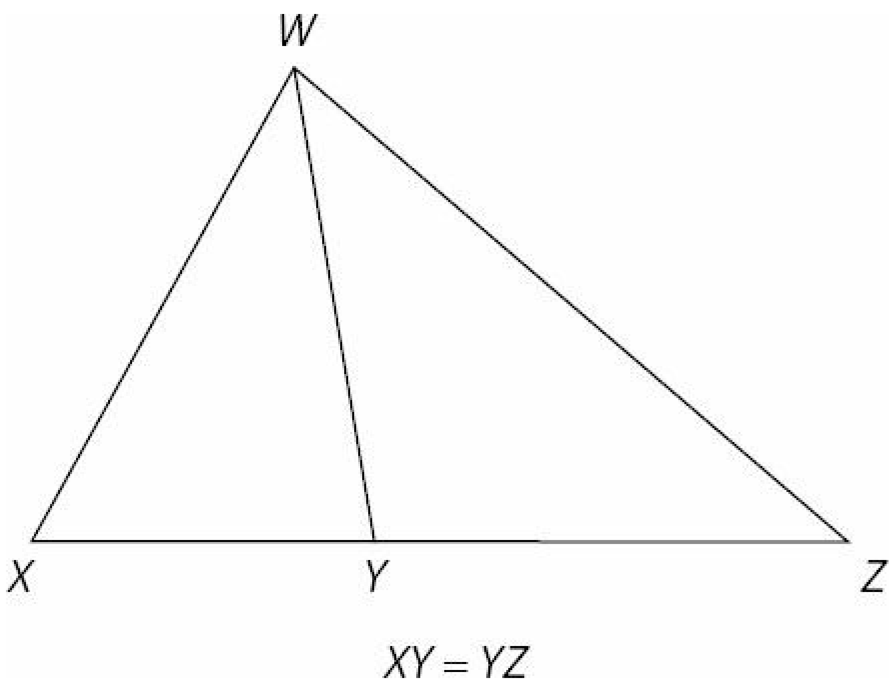
Quantity A	Quantity B
The length of *AB*	10

21.

A triangle has perimeter 24.

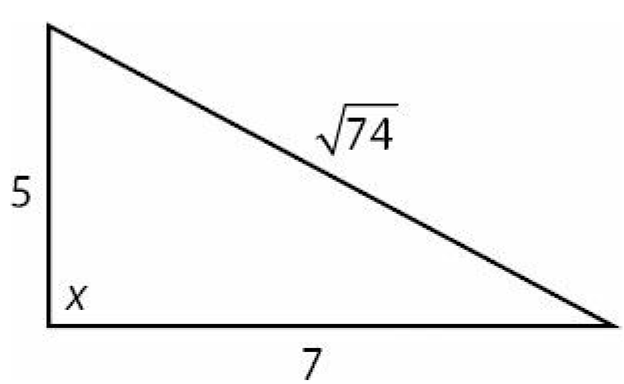
Quantity A	Quantity B
The area of the triangle	20

22.



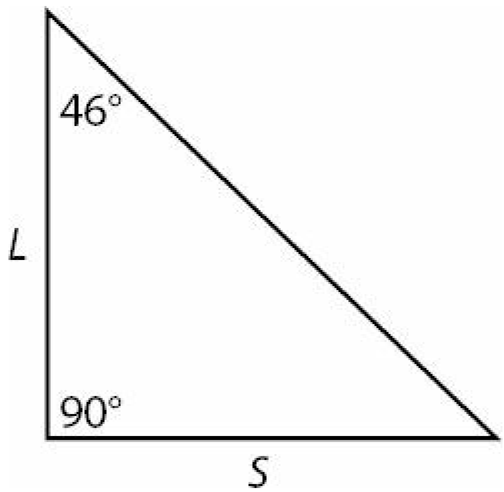
Quantity A	Quantity B
The area of *WYX*	The area of *ZYW*

23.



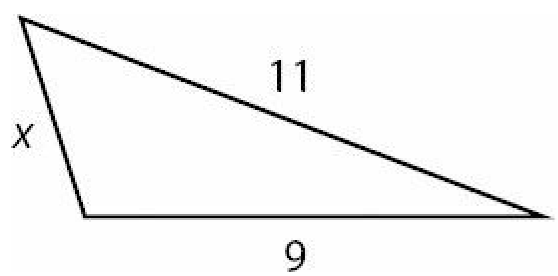
Quantity A	Quantity B
x	90

24.



Quantity A	Quantity B
$\frac{L}{S}$	1

25.



x is an integer

Quantity A	Quantity B
The number of possible values of x	17

26. If p is the perimeter of a triangle with one side of 6 and another side of 9, what is the range of possible values for p?

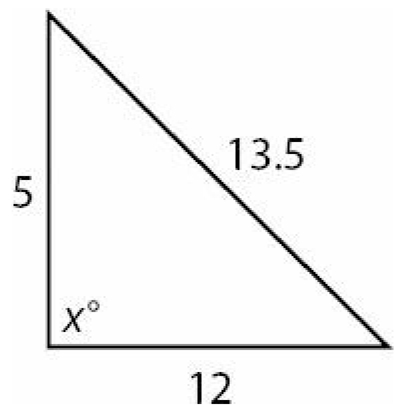
(A) $3 < p < 15$
(B) $15 < p < 24$
(C) $18 < p < 30$
(D) $18 < p < 42$
(E) $21 < p < 42$

27.

A right triangle has hypotenuse 8 and legs of 6 and *x*.

Quantity A	**Quantity B**
x	10

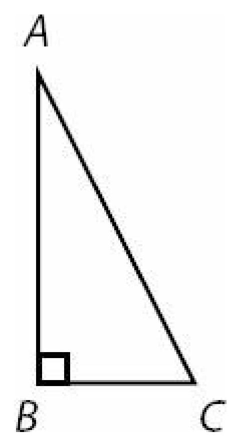
28.



Note: Figure NOT drawn to scale

Quantity A	**Quantity B**
x	90

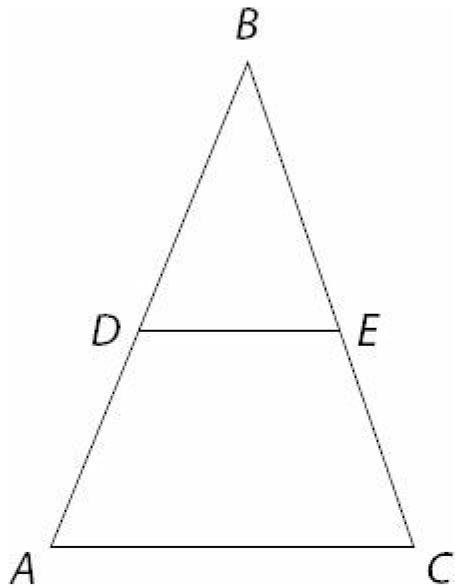
29.



The length of *BC* is equal to *x* *AB* is twice as long as *BC*

Quantity A	**Quantity B**
The length of *AC*	$x\sqrt{3}$

30.



DE is parallel to *AC*
$BE = EC$
$AC = 14$

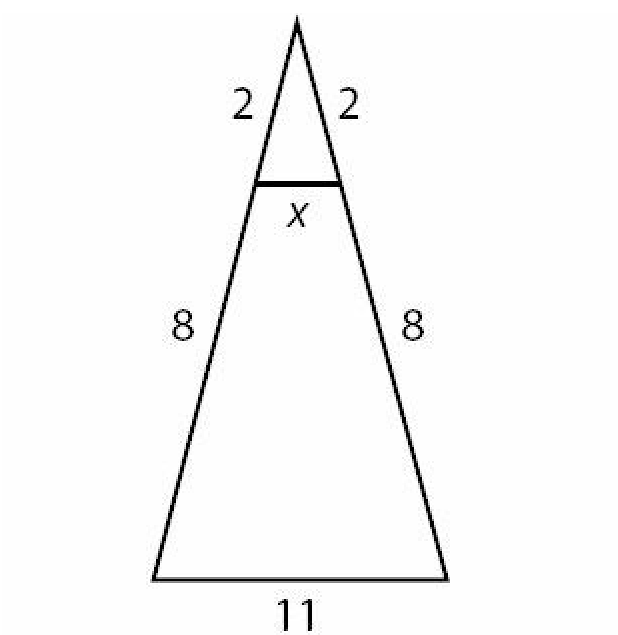
Quantity A	**Quantity B**
DE	7

31.

A triangle has sides of 8, *m,* and *n*.

Quantity A	**Quantity B**
$\|m - n\|$	8

32.



Note: Figure NOT drawn to scale

If the line segment with length x is parallel to the line segment with length 11, what is the value of x?

(A) 1

(B) $\sqrt{2}$

(C) $\frac{11}{5}$

(D) $\frac{11}{4}$

(E) 5.5

33.

Two sides of a triangle have measures 13 and 9.

Quantity A	**Quantity B**
The measure of the third side of the triangle.	$\sqrt{226}$

34. What is the area of an equilateral triangle with side length 4?

(A) $2\sqrt{3}$

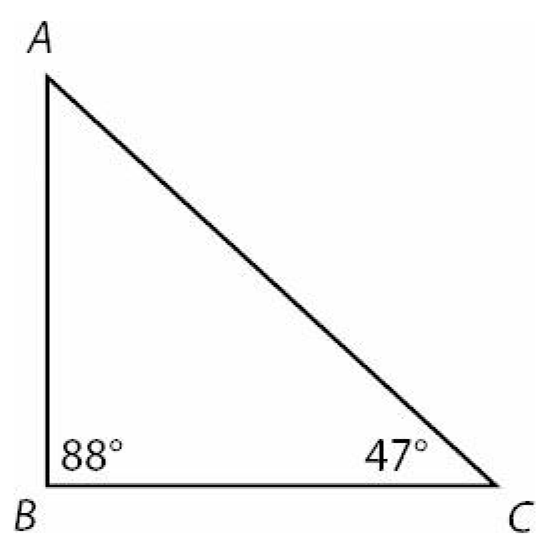
(B) 4.5

(C) $4\sqrt{2}$

(D) $4\sqrt{3}$

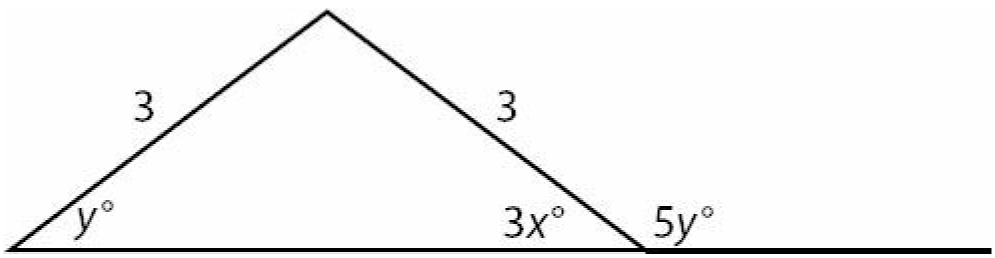
(E) 8

35. Triangle *ABC* is given below with angle measures *B* and *C* given.



Quantity A	Quantity B
The length of side *AB*	The length of side *BC*

36.



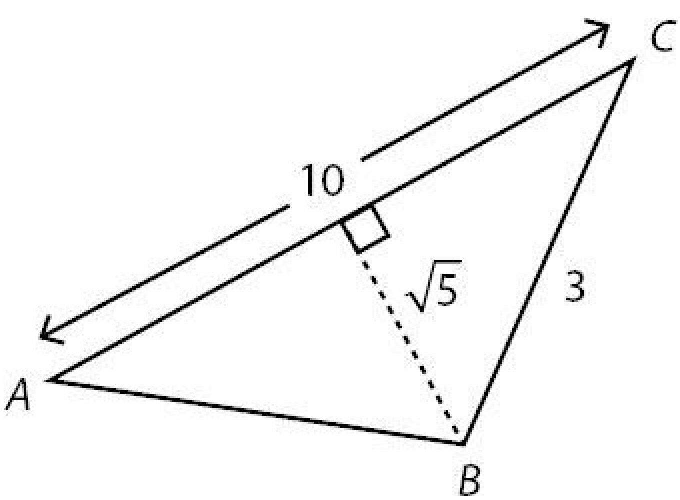
What is the value of x in the figure above?

(A) 5
(B) 10
(C) 18
(D) 30
(E) 54

37. An isosceles right triangle has an area of 50. What is the length of the hypotenuse?

(A) 5
(B) $5\sqrt{2}$
(C) $5\sqrt{3}$
(D) 10
(E) $10\sqrt{2}$

38.

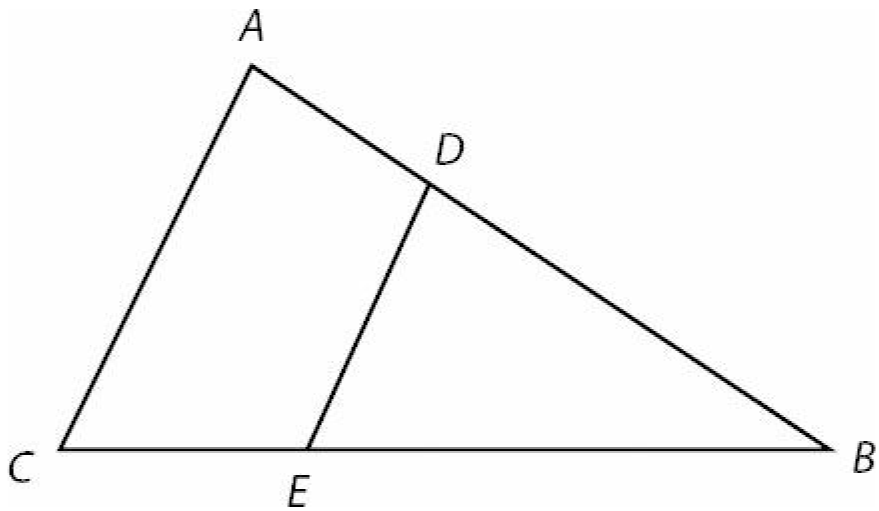


Note: Figure NOT drawn to scale

In the figure above, what is the length of side *AB*?

(A) 5
(B) $\sqrt{30}$
(C) $5\sqrt{2}$
(D) 8
(E) $\sqrt{69}$

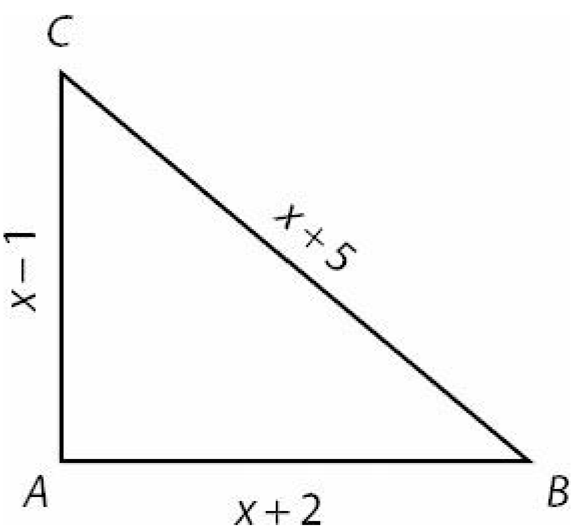
39. In the figure below, *AC* is parallel to *DE* and the length of *DE* is equal to the length of *EB*.



Note: Figure NOT drawn to scale

Quantity A	**Quantity B**
The length of side *AC*	The length of side *CB*

40.

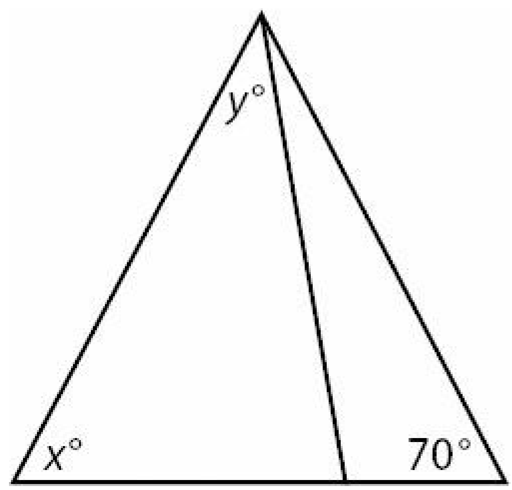


In the right triangle above, what is the length of *AB*?

(A) 9
(B) 10
(C) 12
(D) 13

(E) 15

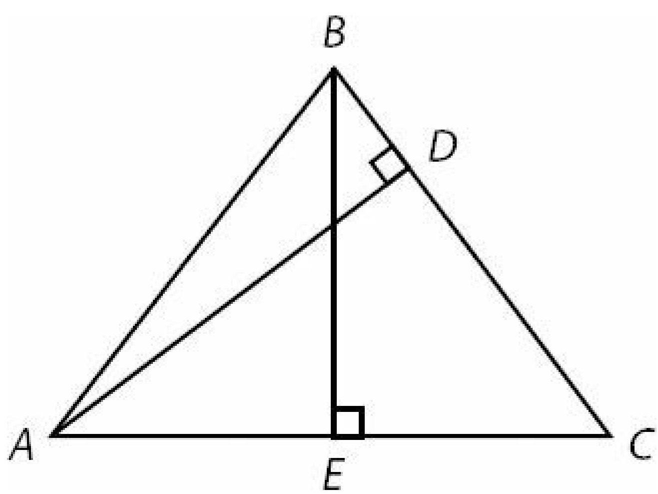
41.



Note: Figure NOT drawn to scale

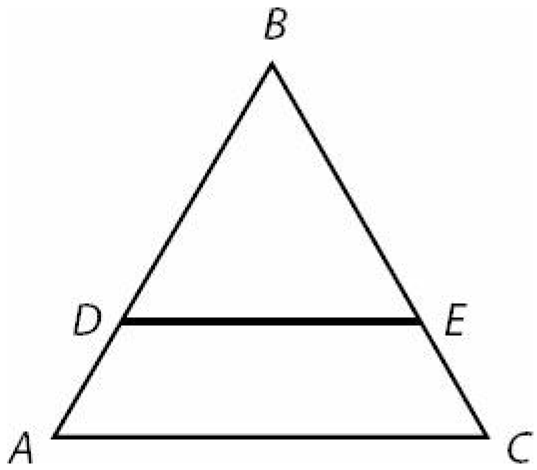
Quantity A	**Quantity B**
$x + y$	110

42.



Quantity A	**Quantity B**
The product of *BE* and *AC*	The product of *BC* and *AD*

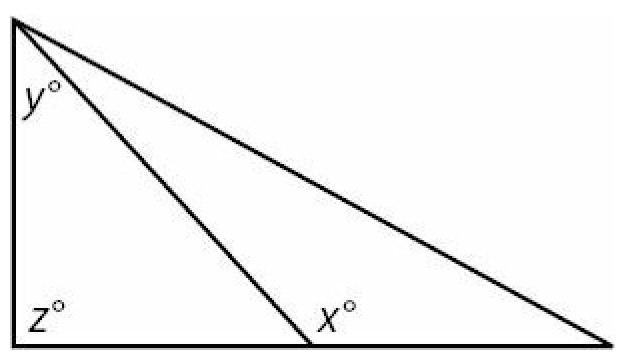
43.



In the figure above, *DE* and *AC* are parallel lines. If *AC* = 10, *DE* = 6, and *CE* = 2, what is the length of side *BC*?

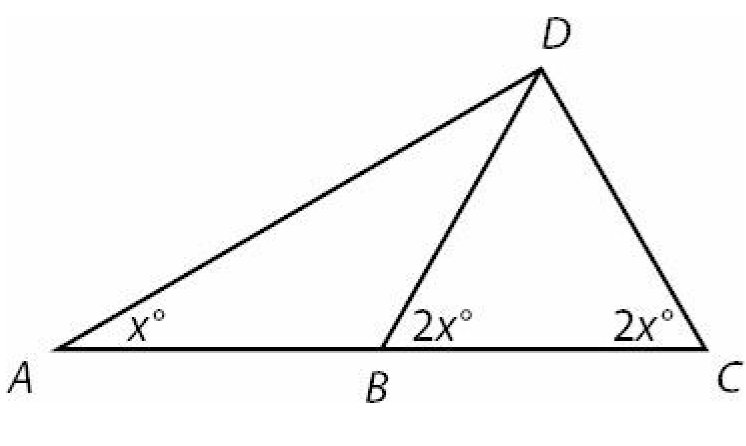
(A) 2
(B) 3
(C) 5
(D) 6
(E) 8

44.



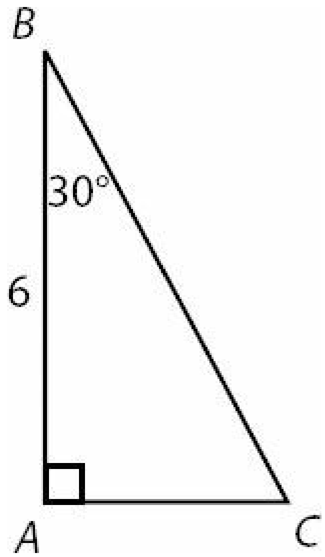
Quantity A	**Quantity B**
x	$y + z$

45.



Quantity A	Quantity B
The length of side *DC*	The length of side *AB*

46.



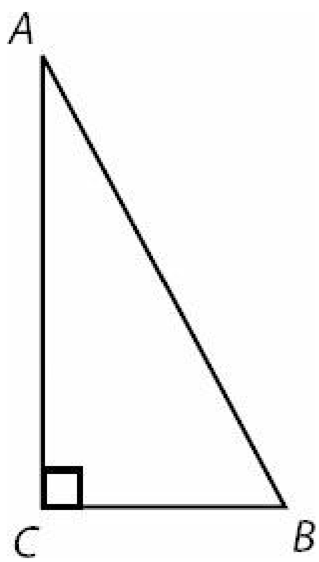
What is the perimeter of right triangle *ABC* above?

(A) $6 + 4\sqrt{3}$
(B) $6 + 6\sqrt{3}$
(C) $6 + 8\sqrt{3}$
(D) $9 + 6\sqrt{3}$
(E) $18 + 6\sqrt{3}$

47. A 10 foot ladder leans against a vertical wall and forms a 60 degree angle with the floor. Assuming the ground below the ladder is perfectly horizontal, how far above the ground is the top of the ladder?

(A) 5 feet
(B) $5\sqrt{3}$ feet
(C) 7.5 feet
(D) 10 feet
(E) $10\sqrt{3}$ feet

48.

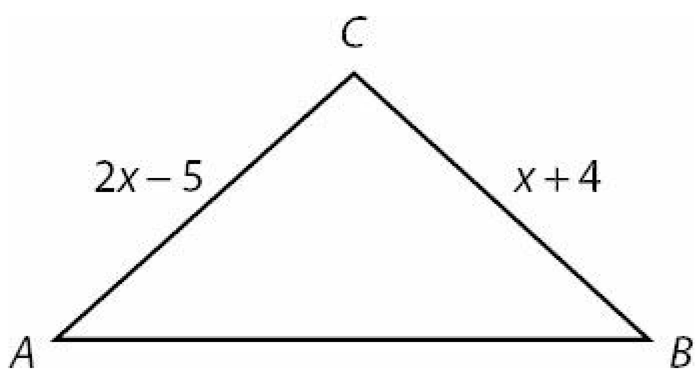


Note: Figure NOT drawn to scale

Triangle *ABC* has area 36. If side *AC* is twice as long as side *CB*, what is the length of side *AB*?

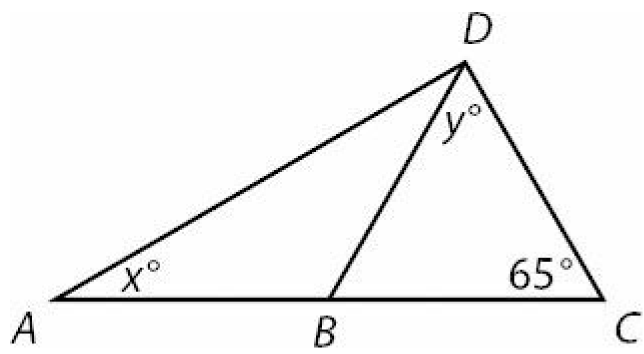
(A) 6
(B) 12
(C) $6\sqrt{5}$
(D) 18
(E) $12\sqrt{5}$

49. In the figure shown, the measure of angle *A* is equal to the measure of angle *B*.



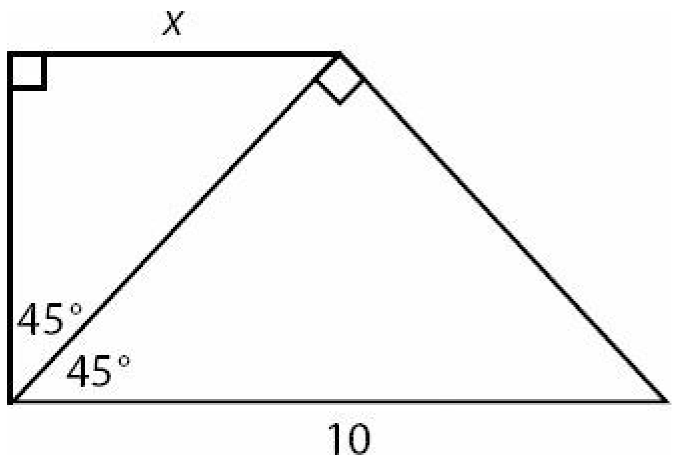
Quantity A	**Quantity B**
Side length *CB*	7

50. In the figure shown, side lengths *AB*, *BD*, and *DC* are all equal.



Quantity A	**Quantity B**
x	y

51.



In the figure shown, what is the value of x?

(A) 2.5

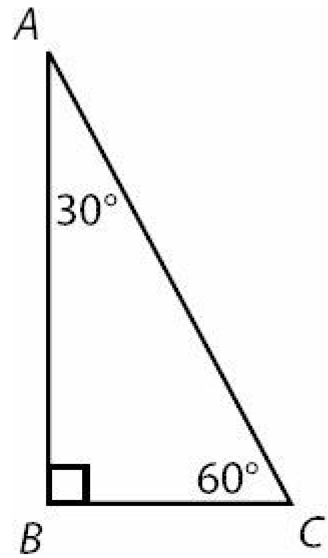
(B) $\frac{5}{\sqrt{2}}$

(C) 5

(D) $5\sqrt{2}$

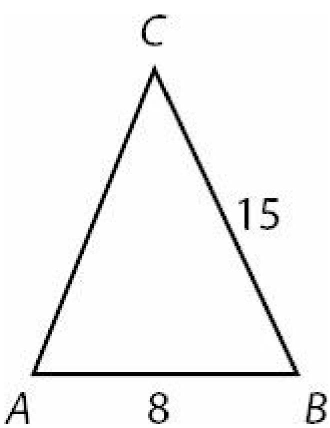
(E) $\frac{10}{\sqrt{2}}$

52.



Quantity A	**Quantity B**
The ratio of the length of side *BC* to the length of side *AB*	$\frac{10}{17}$

53.

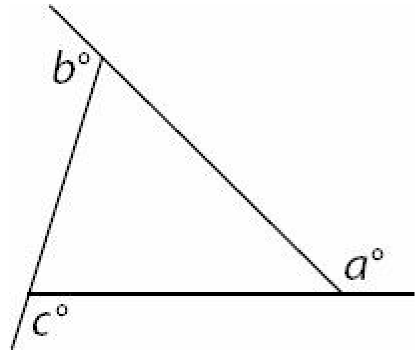


Note: Figure NOT drawn to scale

Which of the following statements individually provide sufficient information to calculate the area of triangle *ABC*?

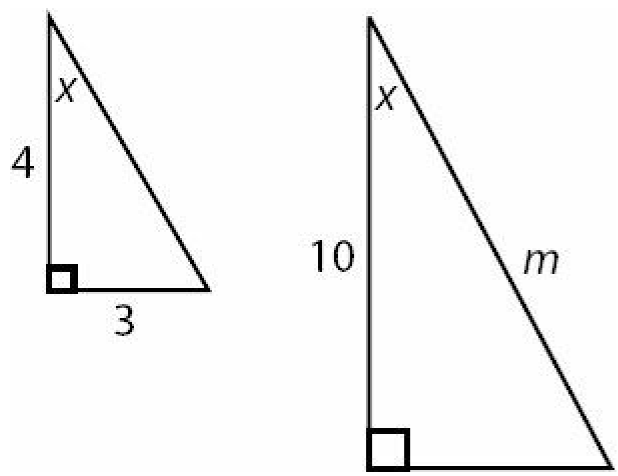
- ☐ Angle *B* equals 90.
- ☐ Side *AC* equals 17.
- ☐ *ABC* is a right triangle

54.



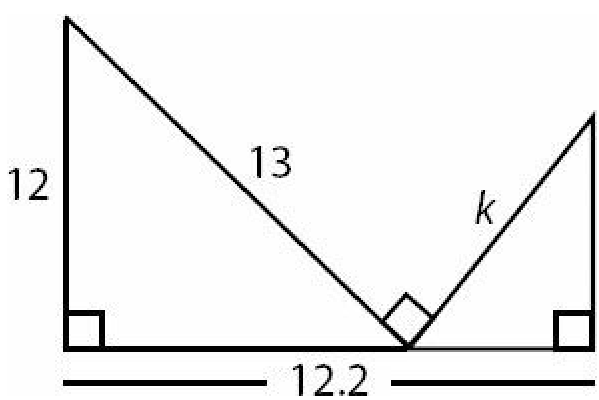
Quantity A	Quantity B
$a + b + c$	180

55.



Quantity A	Quantity B
m	15

56.

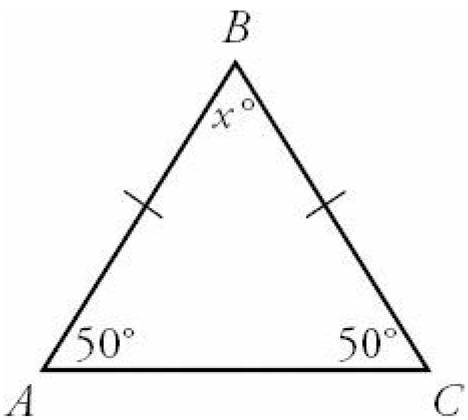


What is the length of hypotenuse k?

Triangles Answers

1. **(B).** The area of a triangle is equal to $\frac{bh}{2}$. Base and height must always be perpendicular. Use 8 as the base and 6 as the height. $A = \frac{(8)(6)}{2} = 24$.

2. **80.** If you know the other 2 angles in a triangle, then you can find the third, because all 3 angles must add up to 180. In Triangle *ABC*, sides *AB* and *BC* are equal. That means their opposite angles are also equal. That means that angle *ACB* is also 50°.

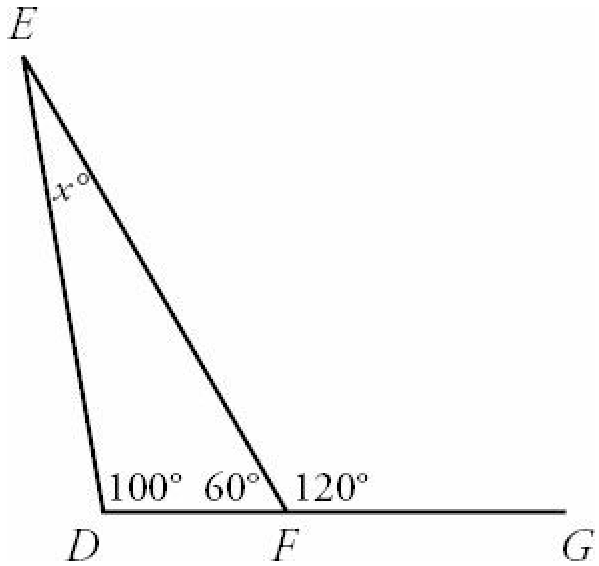


Now that you know the other 2 angles, you can find angle x. You know that $50 + 50 + x = 180$, so $x = 80$.

3. **(C).** The three angles in a triangle must add up to 180°, so $3a = 180$ and $a = 60$ (the triangle is equilateral). The four angles in a quadrilateral must add up to 360°, so $4b = 360$ and $b = 90$ (the angles are right angles, so the figure is a rectangle).

Substitute the values of a and b into Quantity A to get $2(60) + 90 = 210$. Likewise, substitute into Quantity B to get $3(60) + \frac{90}{3} = 210$. The quantities are equal.

4. **20.** To find the value of x, you need to find the degree measures of the other two angles in Triangle *DEF*. You can make use of the fact that *DFG* is a straight line. Straight lines have a degree measure of 180, so angle $DFE + 120 = 180$, which means angle $DFE = 60$.



Now you can solve for x, because $100 + 60 + x = 180$. Solving for x, you get $x = 20$.

5. **(C).** Since c and x are vertical angles, they are equal. So you can swap their positions in the quantities, to put all the angles in the same triangle together.

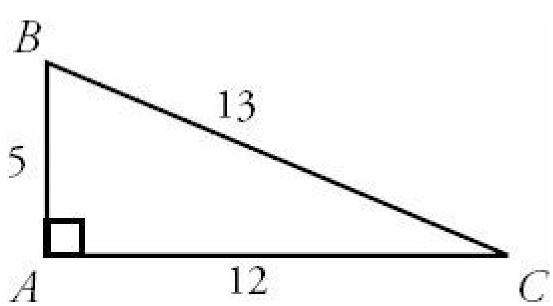
Quantity A	**Quantity B**
$a + b + c$	$x + y + z$

The three angles inside a triangle add up to 180°, so both sides are equal to 180. The quantities are equal.

6. **30.** To find the area, you need a base and a height. If you can find the length of side *AB*, then *AB* can be the height and *AC* can be the base, because the two sides are perpendicular to each other.

You can use the Pythagorean Theorem to find the length of side *AB*. $(a)^2 + (12)^2 = (13)^2$. $a^2 + 144 = 169$. $a^2 = 25$. $a = 5$. Alternatively, you could recognize that the triangle is a Pythagorean triplet 5–12–13.

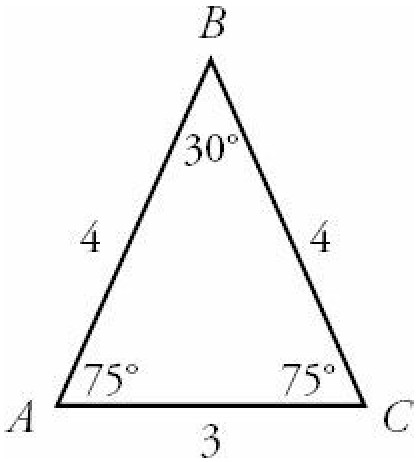
Now that you know the length of side *AB* you can find the area. $\text{Area} = \frac{(12)(5)}{2} = 30.$



7. **11.** To find the perimeter of Triangle *ABC*, you need the lengths of all 3 sides. There is no immediately obvious way to find the length of side *BC*, so let's see what inferences you can make from the information the question gave you.

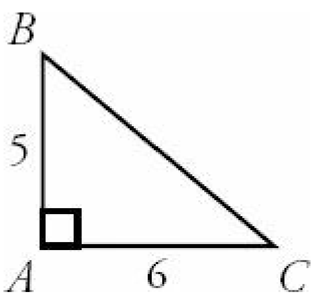
You know the degree measures of two of the angles in Triangle *ABC,* so you can find the degree measure of the third. You'll label the third angle x. You know that $30 + 75 + x = 180$. Solving for x you find that $x = 75$.

Angle *BAC* and angle *BCA* are both 75, which means Triangle *ABC* is an isosceles triangle. If those two angles are equal, you know that their opposite sides are also equal. Side *AB* has a length of 4, so you know that *BC* also has a length of 4.



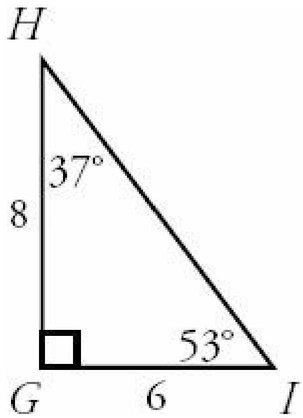
To find the perimeter, you add up the lengths of the three sides. 4 + 4 + 3 = 11.

8. **(D).** To find the length of the hypotenuse, you need the lengths of the other two sides. Then you can use the Pythagorean Theorem to find the length of the hypotenuse. You can use the area formula to find the length of *AC*. $\text{Area} = \frac{bh}{2}$, and you know the area and the height. So $15 = \frac{(\text{base})(5)}{2}$. When you solve this equation, you find that the base = 6.



Now you can use the Pythagorean Theorem. $(5)^2 + (6)^2 = c^2$. $25 + 36 = c^2$. $61 = c^2$. $\sqrt{61} = c$. Since 61 is not a perfect square, you know that c will be a decimal. 61 is also prime, so you cannot simplify $\sqrt{61}$ any further. (It will be a little less than $\sqrt{64} = 8$.)

9. **10.** There is no immediately obvious way to find the length of side *HI*, so let's see what you can infer from the picture. You know two of the angles of Triangle *GHI*, so you can find the third. You'll label the third angle x. $37 + 53 + x = 180$. That means $x = 90$. So really your triangle looks like this:



You should definitely redraw once you discover the triangle is a right triangle!

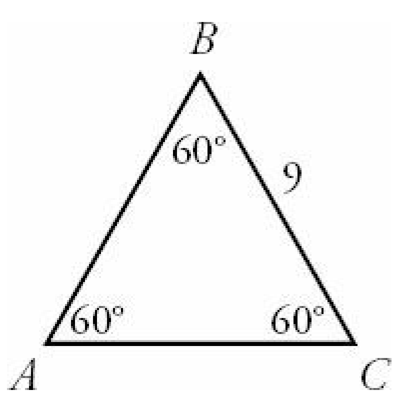
Now that you know Triangle *GHI* is a right triangle, you can use the Pythagorean Theorem to find the length of *HI*. *HI* is the hypotenuse, so $(6)^2 + (8)^2 = c^2$. $36 + 64 = c^2$. $100 = c^2$. $10 = c$. The length of *HI* is 10.

Alternatively, you could have recognized the Pythagorean triplet. Triangle *GHI* is a 6–8–10 triangle.

10. **(C) 24.5.** All isosceles right triangles (or 45–45-90 triangles) have sides in the ratio $1 : 1 : \sqrt{2}$. Thus, an isosceles right triangle with hypotenuse $7\sqrt{2}$ has all its sides in the ratio $7 : 7 : 7\sqrt{2}$. The base and height are each 7. From $A = \frac{bh}{2}$, you get $A = \frac{(7)(7)}{2} = 24.5$.

11. **(B).** To determine which triangle has the greater perimeter, you need to know the side lengths of all three sides of both triangles. Begin with Triangle *ABC*.

There's no immediate way to find the lengths of the missing sides, so let's start by seeing what you can infer from the picture. You know two of the angles, so you can find the third. You'll label the unknown angle x. $60 + 60 + x = 180$. $x = 60$.



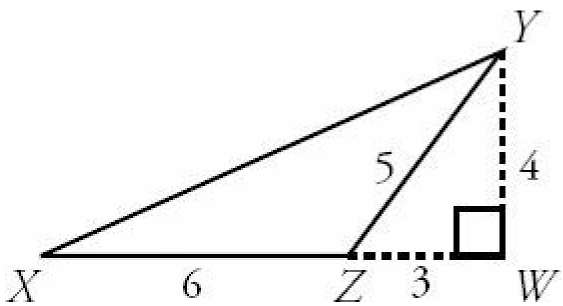
All three angles in Triangle *ABC* are 60°. If all three angles are equal, that means all three sides are equal in this equilateral triangle. So every side of Triangle *ABC* has a length of 9. That means the perimeter $= 9 + 9 + 9 = 27$. Now look at Triangle *DEF*. Triangle *DEF* is a right triangle, so you can use the Pythagorean Theorem to find the length of side *EF*. *EF* is the hypotenuse, so $(5)^2 + (12)^2 = c^2$. $25 + 144 = c^2$. $169 = c^2$. $13 = c$. That means the perimeter is $5 + 12 + 13 = 30$. Alternatively, 5–12–13 is a Pythagorean triplet.

30 > 27, so Triangle *DEF* has a greater perimeter than Triangle *ABC*.

12. **12.** Start by filling in everything you know about Triangle *XYZ*.

To find the area of Triangle *XYZ*, you need a base and a height. If Side *XZ* is a base, then *YW* can act as a height. You can find the length of *YW* because Triangle *ZYW* is a right triangle, and you know the lengths of two of the sides. *YZ* is the hypotenuse, so $(a)^2 + (3)^2 = (5)^2$. $a^2 + 9 = 25$. $a^2 = 16$. $a = 4$.

Alternatively, you could recognize the Pythagorean triplet: *ZYW* is a 3–4–5 triangle.



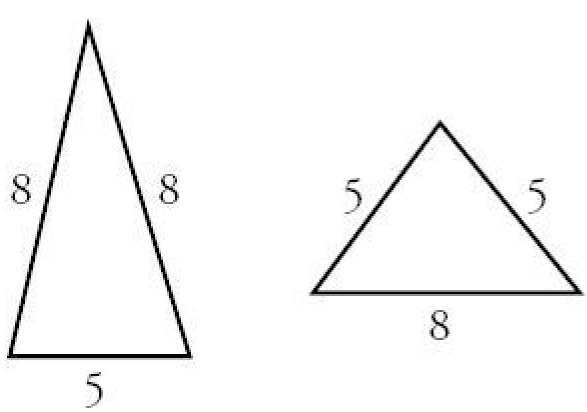
Now you know that the area of Triangle *XYZ* is $\frac{bh}{2} = \frac{(6)(4)}{2} = 12$.

13. **(D).** The problem tells you that $z + x = 110$ degrees. Given that angles of a triangle must sum to 180 degrees, you also know that $x + y + z = 180$. Substitute 110 for $x + z$ on the left side:

$y + 110 = 180$
$y = 70$ degrees

The problem asks you to compare angles *x* and *y*. Although you have solved for *y*, you are still uncertain about the measure of angle *x* (all you know is that it must be greater than 0 and less than 110 degrees). Therefore, you cannot determine which quantity is greater.

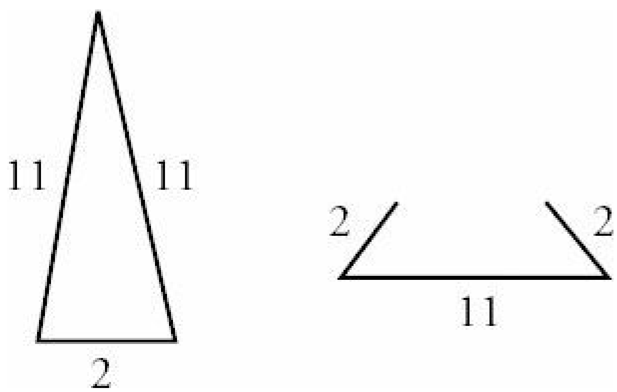
14. **(D).** An isosceles triangle has 2 equal sides, so this triangle must have a third side of either 8 or 5. Use the Third Side Rule (any side of a triangle must be greater than the difference of the other two sides and less than their sum) to check whether both options are actually possible.



Since 8 - 5 = 3 and 8 + 5 = 13, the third side has to be greater than 3 and less than 13. Therefore, that side could indeed be either 5 or 8. You don’t know which it is, though, so you cannot determine which quantity is greater.

15. **(C).** An isosceles triangle has 2 equal sides, so this triangle must have a third side of either 2 or 11. Because one side is so long and the other so short, it is worth testing via the Third Side Rule (any side of a triangle must be greater than the difference of the other two sides and less than their sum) to see whether both possibilities are really possible.

From the Third Side Rule, a triangle with sides of 2 and 11 must have a third side greater than 11 - 2 = 9 and less than 11 + 2 = 13. Since 2 is not between 9 and 13, you simply cannot have a triangle with sides of length 2, 2, and 11. However, you *can* have a third side of length 11: a 2–11–11 triangle is possible. So the third side must be 11.



Thus, the two quantities are equal.

16. **(A).** In this triangles problem, you are asked to compare the relative lengths of two sides of a triangle. In any triangle, the following rule is true: the larger the angle, the longer the side opposite that angle.

Therefore, since angle B is larger than angle A, the side opposite angle B must be longer than the side opposite angle A. Side length AC is longer than side length BC. Quantity A is greater.

17. **(C).** By definition, the exterior angle d is supplementary to the adjacent interior angle and equal to the sum of the two non-adjacent angles. Thus, d = angle B + angle C.

Alternatively, try plugging in numbers. If $d = 120$, angle A equals 180 - 120 = 60. How many degrees are left for angles B and C to share? 180 - 60 = 120, so angles B and C must add up to 120—the same as d. As long as your example obeys the rules of triangles (the 3 angles in the triangle add to 180) and straight lines (the 2 angles on the line also add to 180), your example will show that d = angle B + angle C. The two quantities are equal.

18. **(D).** If the question had said the two LEGS of the triangle were 3 and 4, then the hypotenuse would be 5 (as you know from the 3–4–5 special right triangle). However, you can't assume that 3 and 4 are the legs. In fact, 4 could be the *hypotenuse*. In that case, 3 would still be a leg, and the length of the other leg would be $\sqrt{16-9}=\sqrt{7}$, as you could show from the Pythagorean theorem. You don't need to calculate this value—just recognize that it must be less than 4 (because 4 is the hypotenuse in this case). Since z could be either equal to or less than 5, you cannot determine which quantity is greater.

19. **(B).** Since $z > 60$ and $x > z$, x must also be greater than 60. Thus, $x + z$ must be greater than 120, which leaves less than 60 degrees for the third angle y. You can now order the angles by size: $y < z < x$.

The smallest side is across from the smallest angle, which is y, so the smallest side must be AC. The middle side is across from the middle angle, which is z, so the middle side must be AB. At this point, you know that the length of AB

is greater than the length of AC. Quantity B is greater.

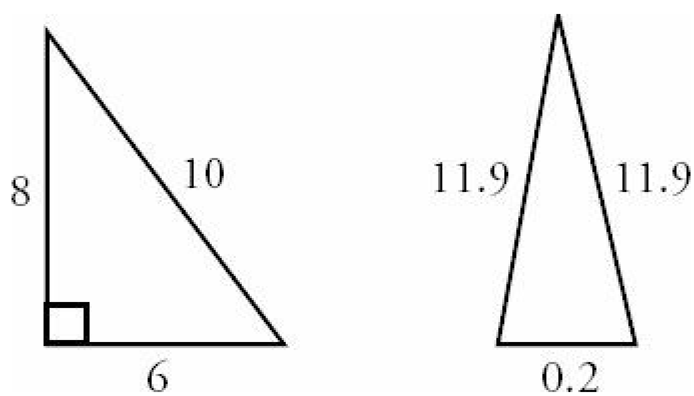
20. **(A).** Since BC is 1/3 the length of AB, relate the two sides using a variable. The easiest way to do this is to label AB "$3x$" and label BC "x." (Calling the smaller side x lets you avoid using a fraction.)

Now apply the Pythagorean Theorem:

$$x^2 + (3x)^2 = (4\sqrt{10})^2$$
$$x^2 + 9x^2 = 160$$
$$10x^2 = 160$$
$$10x^2 = 16$$
$$x = 4$$

You are looking for AB, which equals $3x = 3(4) = 12$. Quantity A is greater.

21. **(D).** The perimeter of a triangle is not enough to find its area, and vice-versa. A triangle with perimeter 24 could be a right triangle with sides of 6, 8, and 10, in which case the area would be $A = \frac{bh}{2} = \frac{6(8)}{2} = 24$, which is larger than 20. Or the triangle could have sides of 11.9, 11.9, and 0.2, in which case the area would be incredibly small.



You should note that you don't have *complete* freedom with the area. There is a maximum area that a triangle with perimeter 24 can have—namely, the area of an equilateral triangle with side lengths of 8. Such a triangle would have an area of $\frac{s^2\sqrt{3}}{4} = \frac{8^2\sqrt{3}}{4} = 16\sqrt{3} \approx 16(1.7)$, which is greater than 20. (You can derive that formula for the area of an equilateral triangle by dropping a height, making two 30–60–90 triangles.) But any positive area less than this maximum is possible. You can drastically shrink the area of a triangle with perimeter 24, making that area as close to zero as you wish. Thus, you cannot determine which quantity is greater.

22. **(C).** The area of a triangle is equal to $\frac{bh}{2}$. You know that the two triangles have equal bases, since $XY = YZ$. They also have the same height, since they both have the same height as the larger triangle WXZ. The two quantities are equal.

23. **(C).** You certainly cannot *assume* that $x = 90$. But since you have three values for the sides of the triangle, you can test whether the triangle is a right triangle by applying the Pythagorean Theorem to the three values and seeing whether

you get a true statement.

$$5^2 + 7^2 = (\sqrt{74})^2$$
$$25 + 49 = 74$$
$$74 = 74$$

Since 74 obviously equals 74, the Pythagorean theorem does apply to this triangle. So the triangle is a right triangle. Notice also that you used the side across from x as the hypotenuse. Thus, you can be sure that $x = 90$. The two quantities are equal.

24. **(B).** Within a single triangle, there is a direct relationship between the side length and the opposite angle. That is, the biggest side is opposite the biggest angle; the smallest side is opposite the smallest angle; and the middle side is opposite the middle angle.

Since the angles in a triangle sum to 180, the angle opposite L is 44. So L is smaller than S, and $\frac{L}{S}$ is less than 1. Quantity B is greater.

25. **(C).** From the Third Side Rule, any side of a triangle must be greater than the difference of the other two sides and less than their sum. Since $11 - 9 = 2$ and $11 + 9 = 20$, x must be between 2 and 20, *not* inclusive. Note that x is an integer, so x must be between 3 and 19, *inclusive*.

Now you can list the possibilities and count: x can be 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, or 19. Or you can subtract the *inclusive* endpoints and "add 1 before you're done:" $19 - 3 + 1 = 17$. Either way, there are 17 total possibilities. The two quantities are equal.

26. **(C) 18 < p < 30.** From the Third Side Rule, any side of a triangle must be greater than the difference of the other two sides and less than their sum. Since $9 - 6 = 3$ and $9 + 6 = 15$, the unknown third side must be between 3 and 15, *not* inclusive. To get the lower boundary for the perimeter, add the lower boundary of the third side to the other two sides: $3 + 6 + 9 = 18$. To get the upper boundary for the perimeter, add the upper boundary for the third side to the other two sides: $15 + 6 + 9 = 30$. Thus, p must be between 18 and 30, not inclusive—in other words, $18 < p < 30$.

27. **(B).** You may have memorized the 6–8–10 Pythagorean triple, a fact that this problem is trying to exploit—don't be tricked into thinking that x equals 10! In a 6–8–10 triangle, 10 would have to be the *hypotenuse*. In any right triangle, the hypotenuse must be the longest side.

Since the given triangle has 8 as the hypotenuse, the leg of length x must be less than 8. So x must also be less than 10. At this point, you can safely choose (B). If you really want the actual value of x, simply apply the Pythagorean theorem:

$$6^2 + x^2 = 8^2$$
$$36 + x^2 = 64$$
$$x^2 = 28$$

$x = \sqrt{28}$, which is between 5 and 6 (so it is less than 10).

Quantity B is greater.

28. **(A).** One good approach here is to test the value in Quantity B. If angle x equals 90°, then you have a right triangle. Use the legs of 5 and 12 to find the hypotenuse. $5^2 + 12^2 = c^2$ will tell you that c equals 13 in this case. (Or, memorize the 5–12–13 Pythagorean triple, since it appears often on the GRE.)

Since the hypotenuse is slightly *longer* than 13, the angle across from the 13 must actually be slightly *larger* than 90°. Therefore, x is larger than 90. Quantity A is greater.

29. **(A).** Since the figure is a right triangle, set up a Pythagorean theorem to solve for the hypotenuse in terms of x:

$$x^2 + (2x)^2 = c^2$$
$$x^2 + 4x^2 = c^2$$
$$5x^2 = c^2$$
$$\sqrt{5x^2} = c$$
$$x\sqrt{5} = c$$

The hypotenuse is equal to $x\sqrt{5}$, which is greater than $x\sqrt{3}$ (as long as x is positive, which of course it is since x is a distance).

The "trick" in this problem is that if you accidentally simplify $(2x)^2$ as $2x^2$ rather than as $4x^2$, you end up with incorrect choice (C).

Quantity A is greater.

30. **(C).** If $AC = 14$ and is parallel to DE, then triangles DBE and ABC are similar.

Since $BE = EC$, $BC = 2BE$ (that is, the side of the big triangle is twice the side of the small triangle). Since the two triangles are similar, *all* the sides of the small triangle will equal half of the corresponding sides of the big triangle. Thus, $DE = 7$.

Alternatively, you can write out a proportion:

$$\frac{BE}{BC} = \frac{DE}{AC}$$

You don't know the exact lengths of BE and BC, but you do know that they are in a 1 to 2 ratio, which is all you need (even if you did know the exact lengths, you'd be able to reduce that fraction to 1/2 anyway).

$$\frac{1}{2} = \frac{DE}{14}$$

$$DE = 7$$

The two quantities are equal.

31. **(B).** From the Third Side Rule, any side of a triangle must be less than the sum of the other two sides and greater than their difference.

Since one side equals 8, the other two sides must differ by less than 8. Thus:

$|m - n| < 8$

The absolute value signs are not totally necessary here, in fact—all they do is guarantee that you're taking the positive difference of *m* and *n* (it doesn't matter which one is longer).

Since $|m - n|$ must be less than 8, Quantity B is greater.

32. **(C).** The two triangles—the small triangle and the large triangle (which encompasses the small triangle)—share one angle, the angle at the top of both triangles.

Furthermore, because the parallel lines create equal angles when cut by transversals (in this case, the sides of the triangle), the other two angles of both of the triangles are also in equal measure. Therefore, the two triangles are similar.

Because the two triangles are similar, they are in proportion to one another. The small triangle has two sides of 2, while the large triangle has corresponding sides of 10 (NOT 8! The lengths labeled 8 are NOT the sides of an actual triangle—the large triangle has two sides of 8 + 2 = 10). So the two triangles are in a 2 : 10 proportion to one another. Reduce 2 : 10 to 1 : 5 and write a proportion:

$$\frac{1}{5} = \frac{x}{11}$$

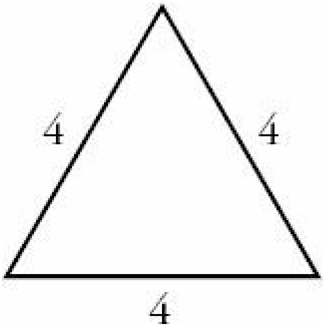
$$5x = 11$$

$$x = \frac{11}{5}$$

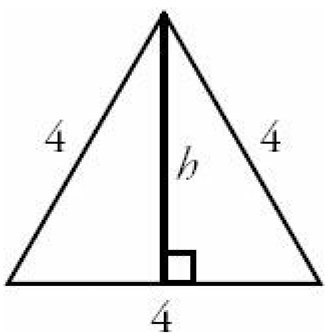
33. **(D).** From the Third Side Rule, a triangle with sides of 13 and 9 must have a third side greater than 13 - 9 = 4 and less than 13 + 9 = 22.

Use the square root button on the calculator to see that $\sqrt{226} \approx 15.033$, or note that since $\sqrt{225} = 15$, $\sqrt{226}$ must be just a little more than 15 (but certainly less than 16). Since the third side's length is between 4 and 22, it could be more or less than $\sqrt{226}$. You cannot determine which quantity is greater.

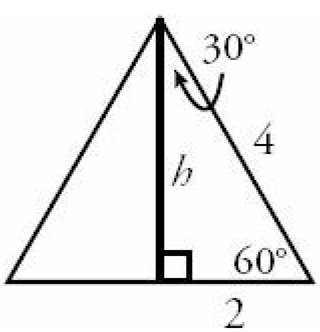
34. **(D).** An equilateral triangle with side length 4 can be drawn as:



In order to find the area, recall that the area of a triangle is given by the formula $A = \frac{bh}{2}$. The base of the triangle is already known to be 4, so you must find the height in order to solve for area. The height is the straight line from the highest point on the triangle dropped down perpendicular to the base:



The angle opposite *h* must be 60°, since it is one of the three angles of the original equilateral triangle. Thus, the triangle formed by *h* is a 30–60–90 triangle as shown below in red.



Using the properties of 30–60–90 triangles, you know that *h* is equal to the shortest side multiplied by $\sqrt{3}$. Thus, $h = 2\sqrt{3}$ and the area is given by

$$A = \frac{bh}{2} = \frac{4 \times 2\sqrt{3}}{2} = 4\sqrt{3}$$

The shortcut formula for the area of an equilateral triangle is $A = \frac{s^2\sqrt{3}}{4}$, which can be derived by the same logic as shown above.

35. **(A).** Label angle *A* with the variable *x*. Since, the three angles of a triangle must add up to 180 degrees, you know

that

$x + 88 + 47 = 180$
$x + 135 = 180$
$x = 45$

Therefore, angle A has a measure of 45 degrees. Now, although you do not know the lengths of the sides, the largest side is opposite the largest angle, and the smallest side is opposite the smallest angle. Because angle C is larger than angle A, you know that the side opposite angle C is longer than the side opposite angle A. In other words, side AB is longer than side BC. Quantity A is greater.

36. **(B).** You are asked for the value of x. Since there are two unknowns, look for two equations to help you solve. The first equation comes from the fact that $3x$ and $5y$ make a straight line, so they must add to 180:

$3x + 5y = 180$

The second equation comes from the isosceles triangle theorem, which states that angles across from sides of a triangle with equal length are equal. In this case, the two sides with length 3 are equal, so the angles across from them (y and $3x$) must also be equal:

$y = 3x$

Substitute for y in the first equation:

$3x + 5(3x) = 180$
$3x + 15x = 180$
$18x = 180$
$x = 10$

37. **(E).** You are told that the area is 50, so $\frac{bh}{2} = 50$. In an isosceles right triangle, base = height, so you can substitute another b in for h:

$\frac{b^2}{2} = 50$
$b^2 = 100$
$b = 10$

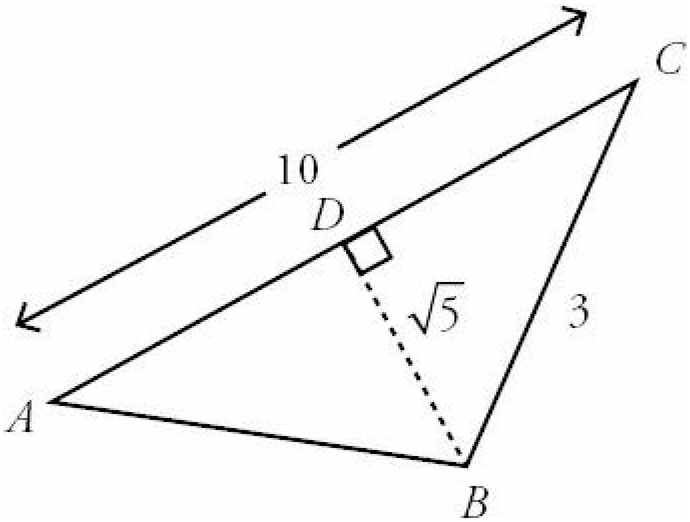
An isosceles right triangle follows the 45–45–90 triangle formula, so the hypotenuse is $10\sqrt{2}$.

Alternatively, use the Pythagorean Theorem to find the hypotenuse:

$10^2 + 10^2 = c^2$
$200 = c^2$

Thus, $c = \sqrt{200} = \sqrt{100 \times 2} = 10\sqrt{2}$.

38. **(E).** You are asked for the length of segment *AB*. For convenience, put the letter *D* on the point at the right angle between *A* and *C*, as shown:



Solve this multi-step problem by working backwards from your goal. To find the length of *AB*, you can use the Pythagorean theorem on triangle *ADB*, since angle *ADB* must be a right angle. In order to use the Pythagorean theorem, you need the lengths of the two legs. *BD* is known, so you just need *AD*. Since *AD* and *DC* add up to a line segment of length 10, you know that *AD* = 10 - *DC*.

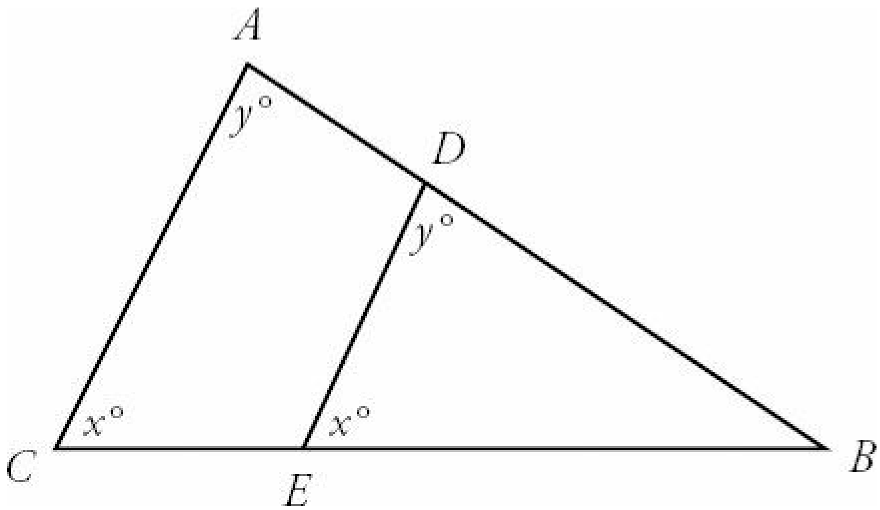
Finally, to find *DC*, apply the Pythagorean theorem to triangle *BDC*:

$$(\sqrt{5})^2 + (DC)^2 = 3^2$$
$$5 + (DC)^2 = 9$$
$$(DC)^2 = 4$$
$$DC = 2$$

So *AD* = 10 - *DC* = 10 - 2 = 8. Apply the Pythagorean theorem to *ADB*:

$$(\sqrt{5})^2 + 8^2 = (AB)^2$$
$$5 + 64 = (AB)^2$$
$$69 = (AB)^2$$
$$AB = \sqrt{69}$$

39. **(C).** You are asked to compare the lengths of *AC* and *CB*. Call angle *DEB* $x°$. *DE* and *AC* are parallel, and they are both cut by transversal *CB*. So angles *DEB* and *ACB* are corresponding angles—they have the same measure, and angle *ACB* will also be $x°$. Similarly, if angle *EDB* is labeled as $y°$, then angle *A* will also be $y°$. At this point, the diagram looks like this:



Now you have two triangles with all three angles the same, *ACB* and *DEB* (angle *B* will obviously be the same in both triangles). So these triangles are similar, and the sides of *ACB* will be in the same proportions as the corresponding sides of *DEB*.

You are told that the length of *DE* is equal to the length of *EB*. (When the figure is not to scale, don't trust your eyes—trust what the problem tells you!) To maintain similarity, the corresponding sides on the larger triangle (*AC* and *CB*) must also be equal. Thus, although you do not know the lengths of *AC* and *CB*, you know they must be the same. The two quantities are equal.

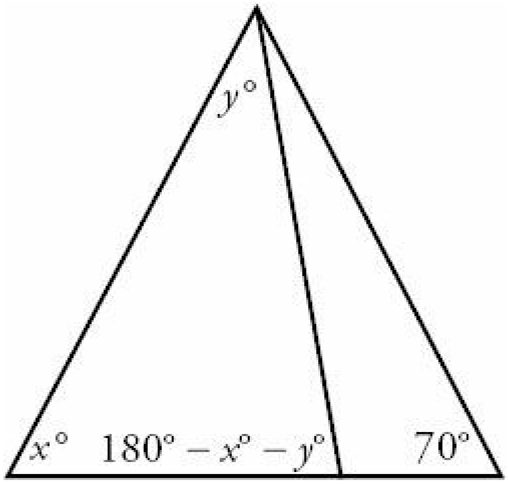
40. **(C).** Because you are told that this is a right triangle, you can use the Pythagorean theorem to solve for the lengths of the sides. The Pythagorean theorem states that $a^2 + b^2 = c^2$ where c is the hypotenuse and a and b are the legs of a right triangle. In this case, $x + 5$ must be the length of the hypotenuse (the longest side), because $x + 5$ is definitely greater than both $x + 2$ and $x - 1$. Plug the expressions into the theorem and simplify:

$$(x - 1)^2 + (x + 2)^2 = (x + 5)^2$$
$$(x^2 - 2x + 1) + (x^2 + 4x + 4) = x^2 + 10x + 25$$
$$2x^2 + 2x + 5 = x^2 + 10x + 25$$
$$x^2 - 8x - 20 = 0$$
$$(x - 10)(x + 2) = 0$$
$$x = 10 \text{ or } x = -2$$

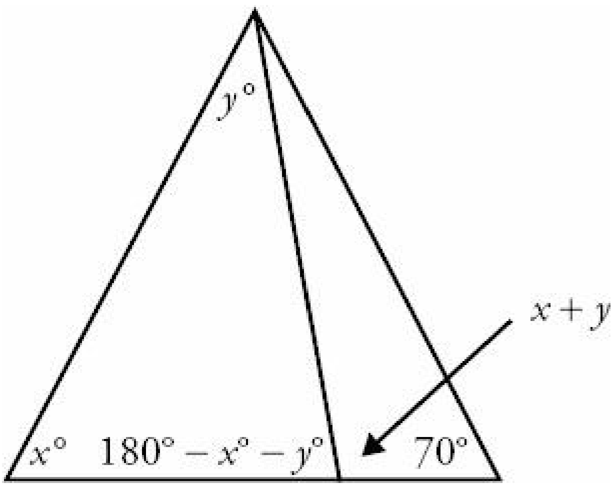
However, $x = -2$ is not an option; side lengths can't be negative. So x must equal 10. This is NOT the final answer, however. You are ultimately asked for side length *AB*:

$$AB = x + 2 = 10 + 2 = 12.$$

41. **(B).** You are asked to compare $x + y$ with 110. To do so, fill in the missing angles on the triangles. In the triangle on the left, all three angles must add up to 180 degrees. Therefore, the missing angle must be $(180 - x - y)$, as shown here:



Now consider the angle next to the one you just solved for. These two angles add up to 180, forming a straight line. So the adjacent angle must be $x + y$.



Alternatively, you could notice also that $x + y$ is the exterior angle to the triangle on the left, so it must be the sum of the two non-adjacent angles (namely, x and y).

Now, the three angles of a triangle must add up to 180, and no angle can equal 0. So any *two* angles in a triangle must add up to *less* than 180. Consider the triangle on the right side, which contains angles of $x + y$ and 70. Then their sum is less than 180.

$$(x + y) + 70 < 180$$

Subtract 70 from both sides:

$$x + y < 110$$

Quantity B is greater.

42. **(C).** First determine how Quantities A and B relate to the triangle. For instance, examine Quantity A, the product of BE and AC. Notice that BE is the height of the triangle, while AC is the base. This product should remind you of the formula for area: $A = \frac{bh}{2}$.

With $b = AC$ and $h = BD$, and moving the 2, you get

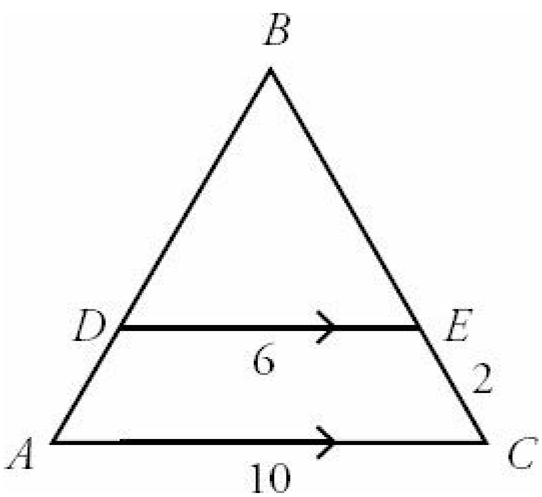
$$2 \times \text{Area} = (AC)(BD)$$

What about *BC* and *AD*? Well, you can consider *BC* the base—and if you do so, then *AD* is the height to that base. So you can put $b = AC$ and $h = BD$ into the area formula, yielding

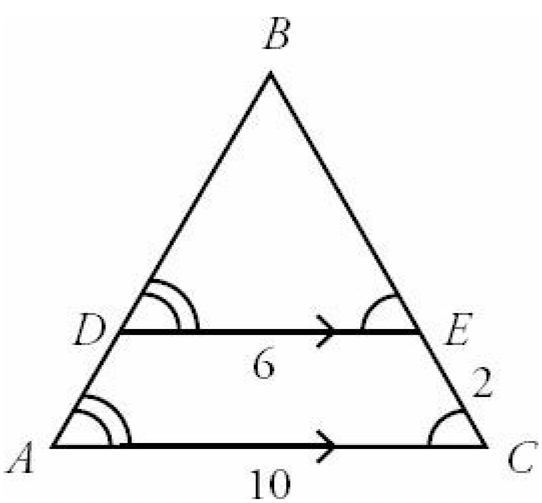
$$2 \times \text{Area} = (BC)(AD)$$

Both Quantity A and Quantity B are twice the area of the triangle. The two quantities are equal.

43. **(C)** First, label the diagram with the information given:



where the arrows represent parallel lines. Because *DE* and *AC* are parallel, angle *DEB* must be the same as angle *ACB*, as they are formed by the same transversal (line segment *BC*). Similarly, angle *BDE* must be the same as angle *BAC*.



Triangle *BDE* has all the same angles as triangle *BAC* (angle *B* is shared), so the two triangles are similar. The ratio of any two corresponding sides on similar triangles is the same, whichever pair of sides you pick. You can find this constant ratio from *DE* and AC: $\frac{DE}{AC} = \frac{6}{10} = \frac{3}{5}$. So all other corresponding sides must also be in the ratio of 3 to 5. Assign *x* to the unknown length of *BE*. Then *BC* will be $x + 2$. Apply the ratio above:

$$\frac{BE}{BC} = \frac{x}{x+2} = \frac{3}{5}$$

Cross multiply and solve for x:

$5x = 3(x + 2)$
$5x = 3x + 6$
$2x = 6$
$x = 3$

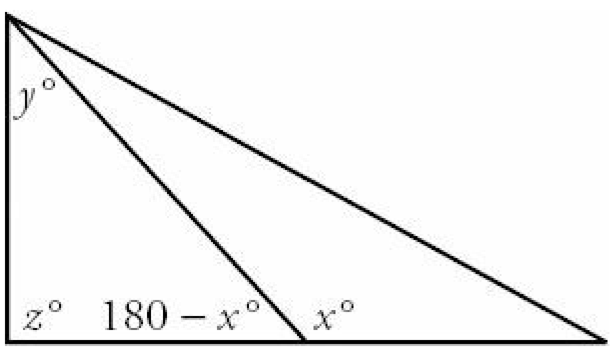
Remember that x is the length of side BE, but you want the length of BC.

$BC = x + 2 = 3 + 2 = 5$

44. **(C).** You can solve this problem in two ways. The quick way is to notice that x is the exterior angle to the smaller triangle on the left (which contains y and z). Since y and z are the non-adjacent interior angles, you can immediately apply the rule that the exterior angle (x) is equal to the sum of the two non-adjacent interior angles (y and z).

The longer way is to derive that relationship, essentially, from two rules: (1) the three angles in a triangle add up to 180, and (2) the two angles formed by a segment running into a straight line (such as x and its unlabeled neighbor) also add up to 180.

First, label that missing angle as $180 - x$ as shown:

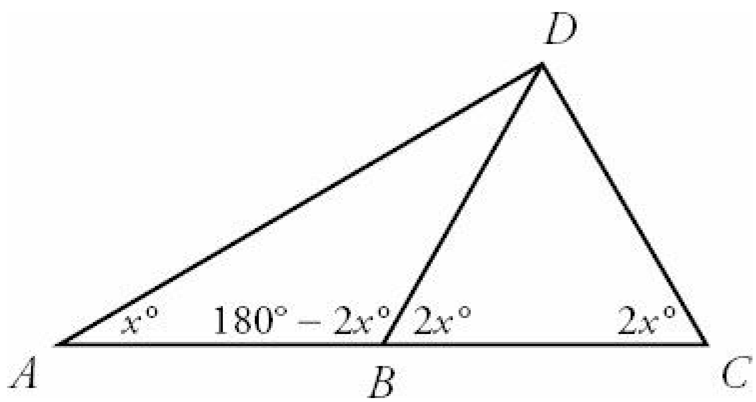


Now apply the rule that the three angles in a triangle add up to 180:

$y + z + (180 - x) = 180$
$y + z + 180 - x = 180$
$y + z - x = 0$
$y + z = x$

The two quantities are equal.

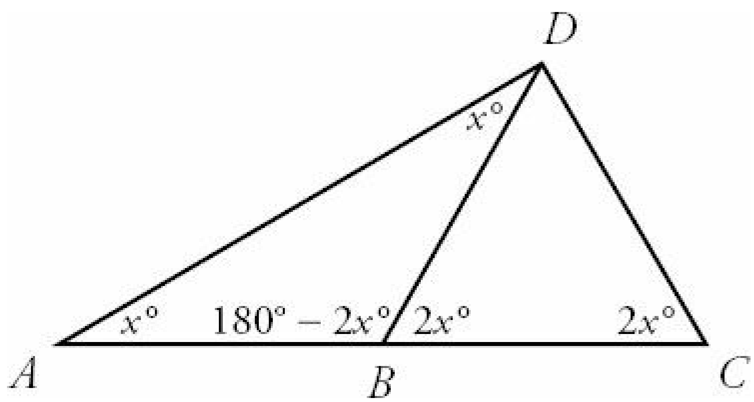
45. **(C).** To compare DC and AB, first solve for the unlabeled angles in the diagram. The two angles at point B make a straight line, so they add up to 180, and the unlabeled angle is $180 - 2x$, as shown:



Now make the angles of triangle *ABD* add up to 180:

> (Angle *A*) + (Angle *B*) + (Angle *D*) = 180
> x + (180 - 2x) + (Angle *D*) = 180
> x + 180 - 2x + (Angle *D*) = 180
> -x + (Angle *D*) = 0
> Angle *D* = x

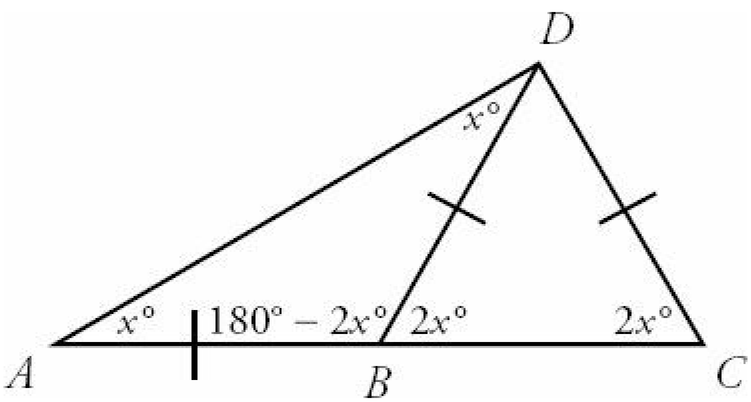
Therefore, the figure becomes



(You could have also gotten here if you noticed that angle *DBC* (equal to 2x) is the exterior angle to the triangle on the left, and so it equals the sum of the two non-adjacent angles in that triangle. One of those angles, namely angle *A*, is x, so the other one must be x as well.)

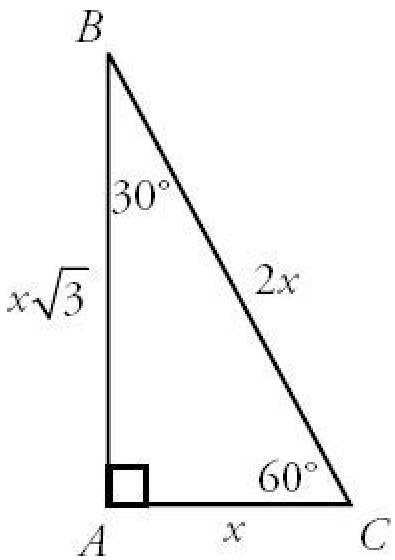
Now apply the properties of isosceles triangles. The two angles labeled x are equal, so the triangle that contains them (triangle *ABD*) is isosceles, and the sides opposite those equal angles are also equal. Put a slash through those sides (*AB* and *DB*) to mark them as the same length.

Likewise, the two angles labeled 2x are equal, so the triangle that contains them (triangle *DBC*) is isosceles, and the sides opposite those angles (*DB* and *DC*) are equal. Adding one more slash through *DC*, you get



Thus, sides *AB* and *DC* have the same length. The two quantities are equal.

46. **(B).** To compute the perimeter of this triangle, you need the lengths of all three sides. Because *A* is a right angle and angle *B* is 30°, right triangle *ABC* is a 30–60–90 triangle. For any 30–60–90 triangle, the sides are in these proportions:



Now you must match up this universal 30–60–90 triangle to your given triangle, so that you can find *x* in this particular case. The only labeled side in the given triangle (6) matches the $x\sqrt{3}$ side in the universal triangle (they're both opposite the 60° angle), so set them equal to each other:

$$6 = x\sqrt{3}$$
$$x = \frac{6}{\sqrt{3}}$$

Rationalize the denominator by multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$ (which does not change the value of *x*, as $\frac{\sqrt{3}}{\sqrt{3}}$ is just a form of 1):

$$x = \frac{6}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$x = \frac{6\sqrt{3}}{3}$$

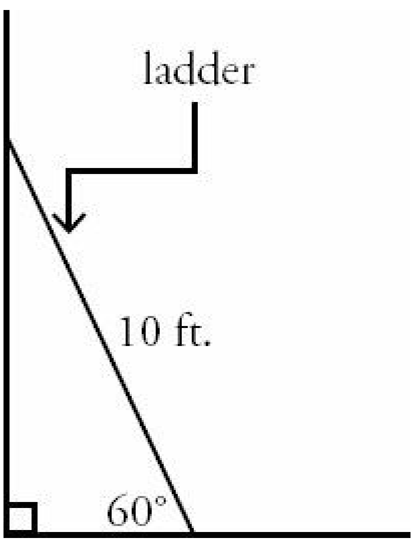
$$x = 2\sqrt{3}$$

Now figure out all the sides in your given triangle. The length of side *AC* (x) is $2\sqrt{3}$, the length of side *AB* is given by 6, and the length of side *BC* is $2x = 2(2\sqrt{3}) = 4\sqrt{3}$.

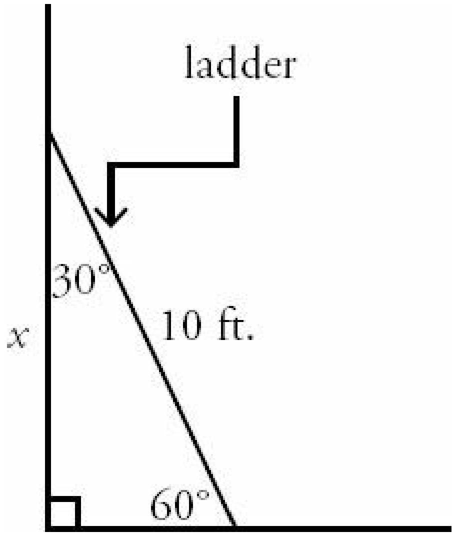
Finally, add up all the sides to get the perimeter:

$$\text{Perimeter} = 6 + 2\sqrt{3} + 4\sqrt{3}$$
$$\text{Perimeter} = 6 + 6\sqrt{3}$$

47. **(B).** First, draw a diagram and label all the givens:

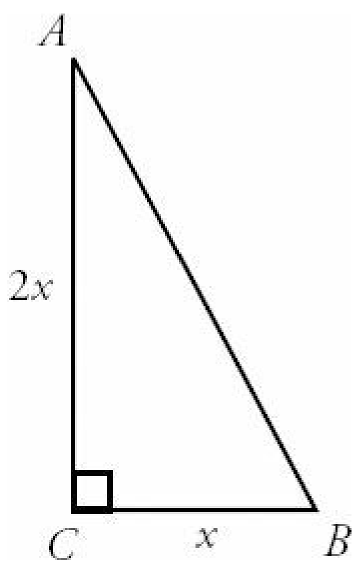


Since the wall is vertical and the floor is perfectly horizontal, the angle where they meet is 90°. So the triangle is 30–60–90. You are asked to find the vertical distance from the top of the ladder to the floor, so represent this length as x.



In any 30–60–90 triangle, the short leg (opposite the 30° angle) is the hypotenuse divided by 2, making the floor-side $10 \div 2 = 5$ feet. The longer leg (opposite the 60° angle) is $\sqrt{3}$ times the short leg. So $x = 5\sqrt{3}$ feet.

48. **(C).** Draw a diagram and label the sides of the triangle with the information given. Since *AC* is twice as long as *CB*, label *CB* as *x* and *AC* as 2*x*, as shown



You are given the area of the triangle, and you can use *x* as the base and 2*x* as the height in the formula for area (which equals $\frac{bh}{2}$). Plug in and solve for *x*:

$$36 = \frac{x(2x)}{2} = \frac{2x^2}{2}$$
$$36 = x^2$$
$$x = 6$$

So *CB* = 6 and *AC* = (2)(6) = 12. Use the Pythagorean Theorem to find *AB*:

$$(AB)^2 = 6^2 + 12^2$$

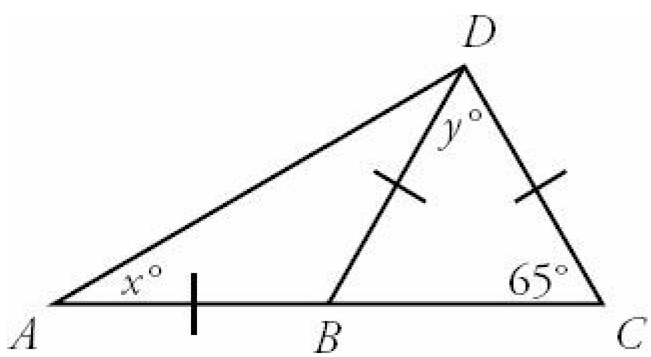
$(AB)^2 = 36 + 144$
$(AB)^2 = 180$
$AB = 6\sqrt{5}$

49. **(A).** In order to make the comparison, you need the length of *CB*. Since angles *A* and *B* are equal, the triangle is isosceles, and the sides opposite those angles (*AC* and *CB*) must also be equal. Write the equation:

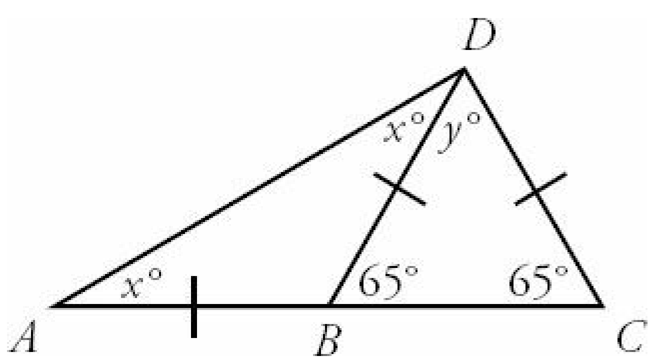
$2x - 5 = x + 4$
$x - 5 = 4$
$x = 9$

CB is therefore equal to 9 + 4 = 13. Quantity A is greater.

50. **(B).** Redraw the figure and label the equal sides:



The two small triangles (on the left and on the right) are each isosceles, because they each contain two equal sides. From the isosceles triangle theorem, the angles opposite equal sides are also equal. Fill in more angles on the figure:



The three angles in the triangle on the right must add up to 180 degrees:

$65 + 65 + y = 180$
$130 + y = 180$
$y = 50$

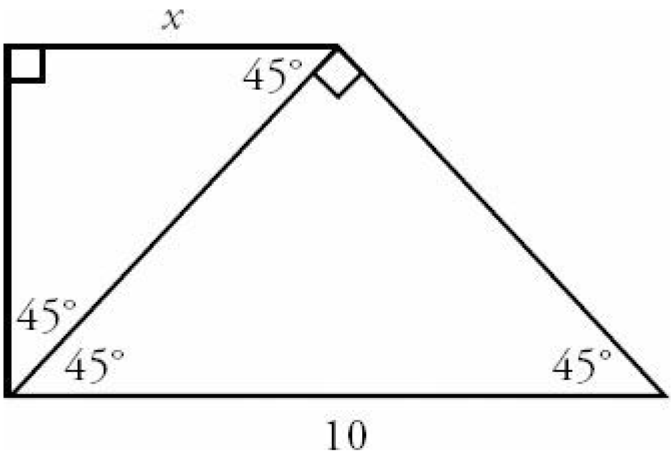
The two angles at point *B* make a straight line, so they add up to 180 degrees. So the unlabeled angle must be 180 - 65 = 115 degrees.

Finally, the three angles in the triangle on the left must sum to 180 degrees:

$x + x + 115 = 180$
$2x = 65$
$x = 32.5$

So y is greater than x. Quantity B is greater.

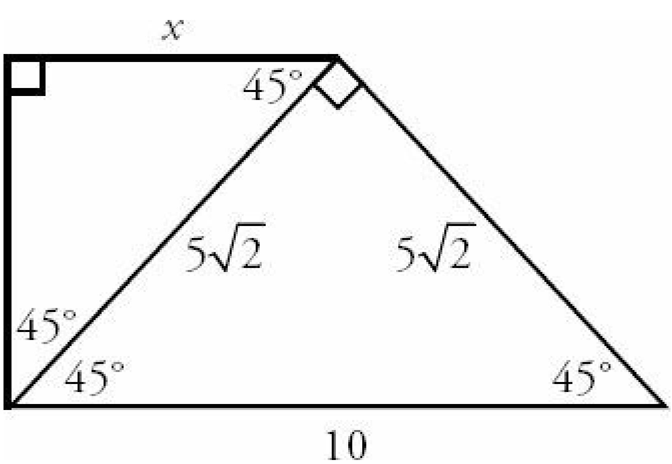
51. **(C).** Redraw the figure and label all angles, applying the rule that the angles in a triangle add up to 180:



You can now see that you have two separate 45–45–90 triangles. In a 45–45–90 triangle, the sides are in a $1 : 1 : \sqrt{2}$ ratio. Thus, the length of each leg equals the length of the hypotenuse divided by $\sqrt{2}$. The hypotenuse of the larger triangle is 10, so each leg of that triangle is $\frac{10}{\sqrt{2}}$. Rationalize the denominator by multiplying by $\frac{\sqrt{2}}{\sqrt{2}}$:

$$\frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

Add more labels to your figure:



Now you know that the hypotenuse of the smaller triangle is $5\sqrt{2}$. Apply the 45–45–90 triangle ratio once more ($1 : 1 : \sqrt{2}$) to see that $x = 5$.

52. **(B).** The sides of a 30–60–90 triangle are always in a $1:\sqrt{3}:2$ ratio. Since *BC* is across from the 30° angle and *AB* is across from the 60° angle, Quantity A is equal to $\frac{1}{\sqrt{3}}$.

In the calculator, this is 0.578…. Use the calculator to see that Quantity B = 10/17 = 0.588… and is slightly larger than Quantity A.

Alternatively, set the two fractions next to each other and cross multiply to compare them:

$$\frac{1}{\sqrt{3}} \ ? \ \frac{10}{17}$$
$$17 \ ? \ 10\sqrt{3}$$
$$1.7 \ ? \ \sqrt{3}$$
$$1.7^2 \ ? \ 3$$

$1.7^2 = 2.89$. Note that 1.7 is a good approximation of $\sqrt{3}$, but $\sqrt{3}$ is actually a bit bigger.

Quantity B is larger.

53. **I and II only.** If Angle $B = 90°$, then 8 and 15 are the base and height, and you can calculate the area. Statement I is sufficient.

If side $AC = 17$, you can plug 8, 15, and 17 into the Pythagorean theorem to see whether you get a true statement. Use 17 as the hypotenuse in the Pythagorean theorem because 17 is the longest side:

$$8^2 + 15^2 = 17^2$$
$$64 + 225 = 289$$
$$289 = 289$$

Since this is obviously true, the triangle is a right triangle with the right angle at *B*. If Angle $B = 90°$, then 8 and 15 are the base and height, and you can calculate the area. (8–15–17 is a Pythagorean triplet, so if you had that fact memorized, you could skip the step above.) Statement II is sufficient.

Knowing that *ABC* is a right triangle (Statement III) is *not* sufficient to calculate the area because you don't know which angle is the right angle. A triangle with sides of 8 and 15 could have hypotenuse 17, as you've already seen, but another scenario is possible: perhaps 15 is the hypotenuse. In this case, the third side is smaller than 15, and the area is smaller than in the 8–15–17 scenario.

54. **(A).** The three interior angles of the triangle add up to 180. Try an example: say each interior angle is 60°. In that case, *a*, *b*, and *c* would each equal 120° (since two angles that make up a straight line add to 180), and Quantity A would equal 360.

You can prove this result in general by expressing each interior angle in terms of a, b, and c, and then setting their sum equal to 180:

$$(180 - a) + (180 - b) + (180 - c) = 180$$
$$540 - a - b - c = 180$$
$$360 = a + b + c$$

Quantity A is greater.

55. **(B).** Since both triangles have a 90° angle and an angle x, the third angle of each is the same as well (because the three angles in each triangle add up to 180). All the corresponding angles are equal, so the triangles are similar, and the ratio of corresponding sides is constant.

The smaller triangle is a 3–4–5 Pythagorean triple (the missing hypotenuse is 5). Set up a proportion that includes two pairs of corresponding sides. The words "4 is to 10 as 5 is to m" become this equation:

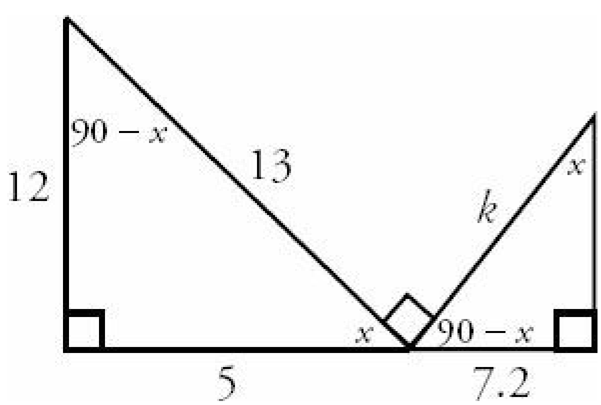
$$\frac{4}{10} = \frac{5}{m}$$
$$4m = 50$$
$$m = 12.5$$

Quantity B is larger.

56. **7.8.** Begin by noting that the triangle on the left is a 5–12–13 Pythagorean triple, so the bottom side is 5. Subtract 12.2 - 5 = 7.2 to get the bottom side of the triangle on the right.

Next, the two unmarked angles that "touch" at the middle must add up to 90, because they form a straight line together with the right angle of 90° between them, and all three angles must add up to 180. Mark the angle on the left x. The angle on the right must then be $90 - x$.

Now the other angles that are still unmarked can be labeled in terms of x. Using the rule that the angles in a triangle add up to 180, the angle between 12 and 13 must be $90 - x$, while the last angle on the right must be x, as shown:



Since each triangle has angles of 90, x, and $90 - x$, the triangles are similar. This observation is the key to the problem. Now you can make a proportion, carefully tracking which side corresponds to which. 7.2 corresponds to 12, since each side is across from angle x. Likewise, k corresponds to 13, since each side is the hypotenuse. Write the equation and solve for k:

$$\frac{7.2}{12} = \frac{k}{13}$$

$$\frac{(13)7.2}{12} = k$$

$$k = 7.8$$

Chapter 28
of

Coordinate Geometry

In This Chapter…

Coordinate Geometry

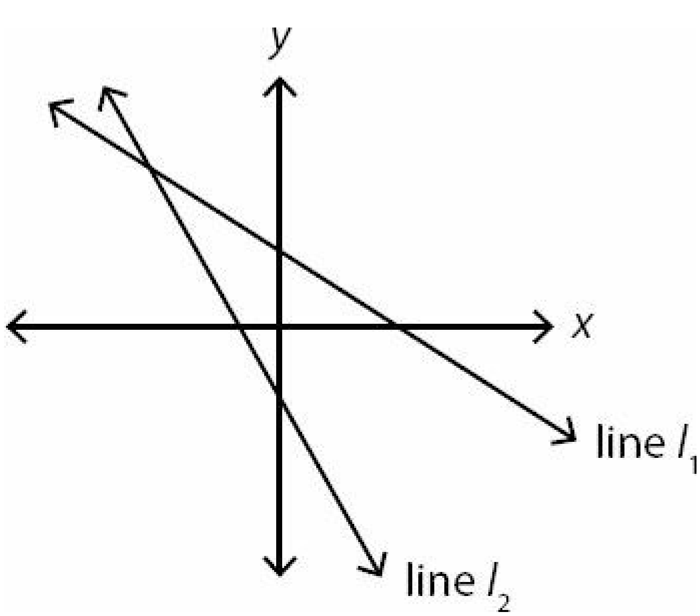
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

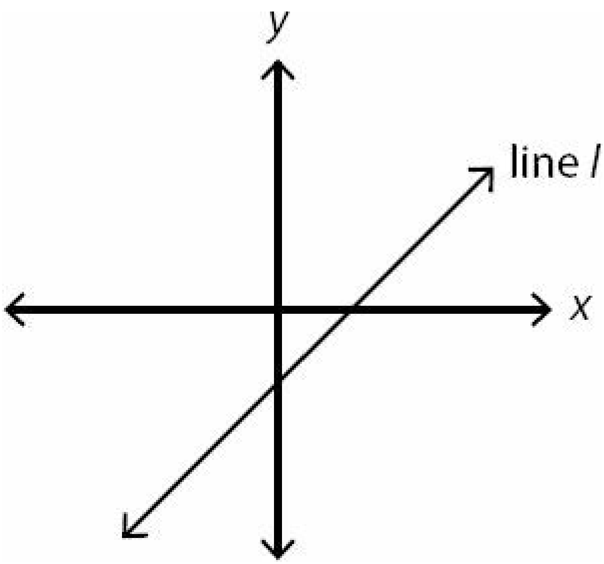
1.



Quantity A | **Quantity B**

The slope of line l_1 The slope of line l_2

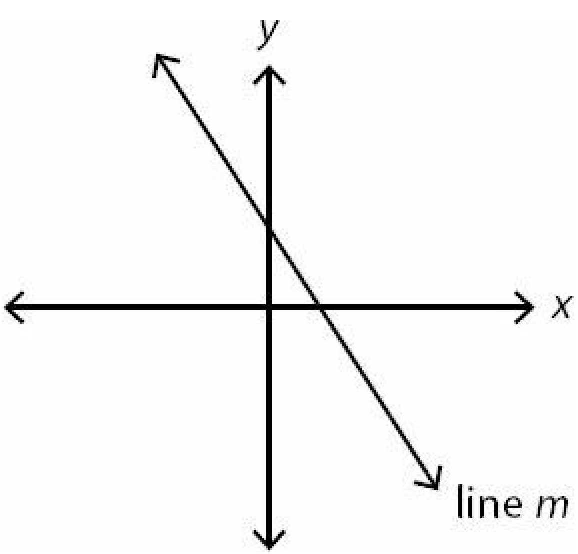
2.



Which of the following is most likely to be the equation of line *l*?

(A) $y = 4x + 4$
(B) $y = 4x - 4$
(C) $y = x - 6$
(D) $y = x + 1/2$
(E) $y = -x - 3$

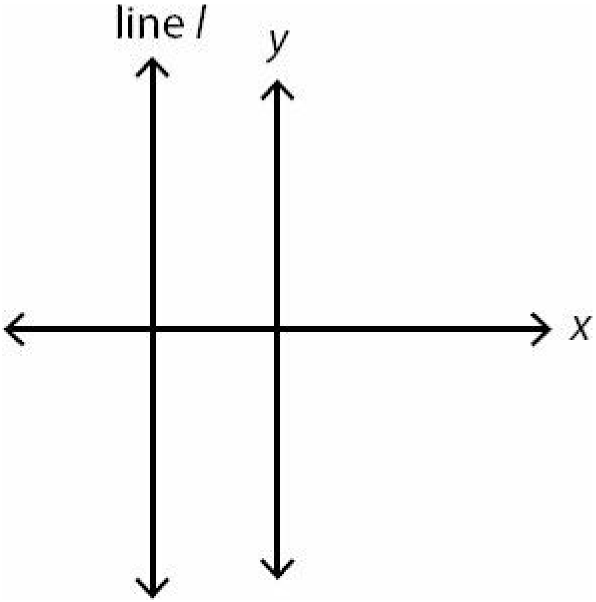
3.



Which of the following could be the equation of line *m*?

(A) $6y + 6x = 7$
(B) $3y = -4x - 3$
(C) $5y + 10 = -4x$
(D) $y = 2$
(E) $x = -2$

4.



If line *l* is parallel to the *y*-axis, what could be the equation of line *l*?

(A) $x = 2$
(B) $x = -2$
(C) $y = 2$
(D) $y = -2$
(E) $y = -2x$

5. What is the equation of the line that passes through (-1, -3) and has a slope of -2?

(A) $y = -2x - 1$
(B) $y = -2x - 2$
(C) $y = -2x - 5$
(D) $y = -4x - 2$
(E) $y = -5x + 2$

6. What is the slope of a line that passes through the points (-4, 5) and (1, 2)?

(A) $-\frac{3}{5}$

(B) -1

(C) $-\frac{5}{3}$

(D) $-\frac{7}{3}$

(E) -3

7. Which of the following could be the slope of a line that passes through the point (-2, -3) and crosses the *y*-axis above the origin?

Indicate all such values.

☐ $-\frac{2}{3}$

☐ $\frac{3}{7}$

☐ $\frac{3}{2}$

☐ $\frac{5}{3}$

☐ $\frac{9}{4}$

☐ 4

8. If a line has slope -2 and passes through the points (4, 9) and (6, y), what is the value of y?

[ ]

9. What is the distance between the points (-1, -1) and (5, 6)?

(A) 6
(B) 7
(C) $\sqrt{79}$
(D) $\sqrt{85}$
(E) 11

10. If the longest distance between any two of the points (-1, -2), (6, -2), and (7, 10) is $p\sqrt{13}$, what is the value of p?

[ ]

11.

A line has the equation $2y - 4x - 8 = 0$.

Quantity A	**Quantity B**
The slope of the line	4

12. Which of the following points lies on the line $y = 2x - 8$?

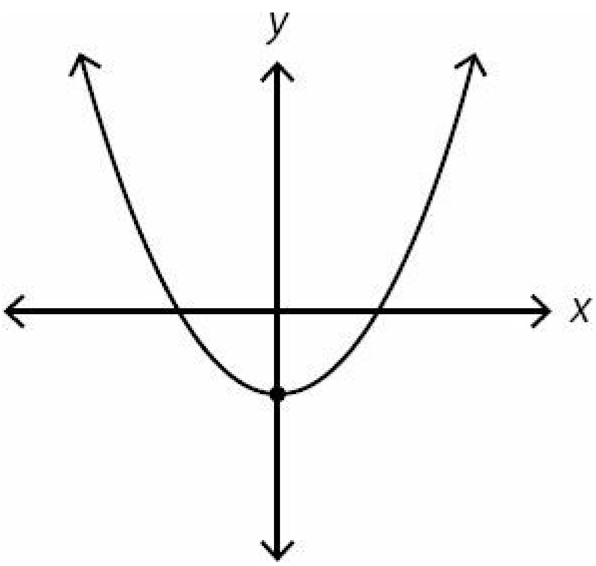
Indicate all such values.

- ☐ (3, -2)
- ☐ (-8, 0)
- ☐ (1/2, -7)

13. Which of the following points does NOT lie on the curve $y = x^2 - 3$?

(A) (3, 6)
(B) (-3, 6)
(C) (0, -3)
(D) (-3, 0)
(E) (0.5, -2.75)

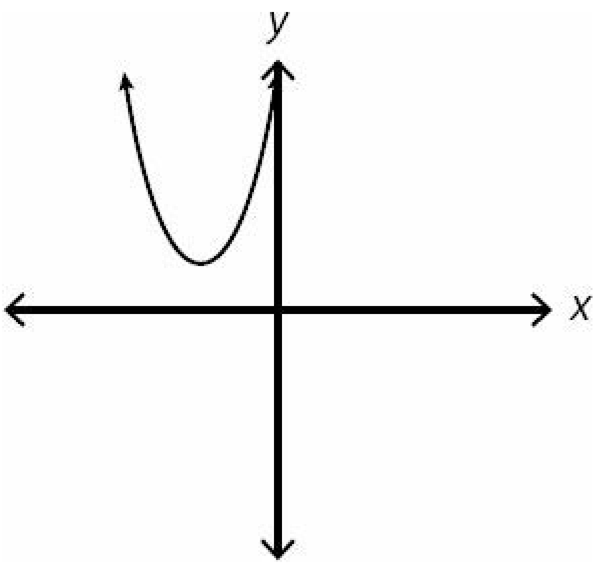
14.



Which of the following could be the equation of the figure above?

(A) $y = x - 2$
(B) $y = x^2 - \text{x}$
(C) $y = x^2 - 2$
(D) $y^2 = x^2$
(E) $y = x^3 - 2$

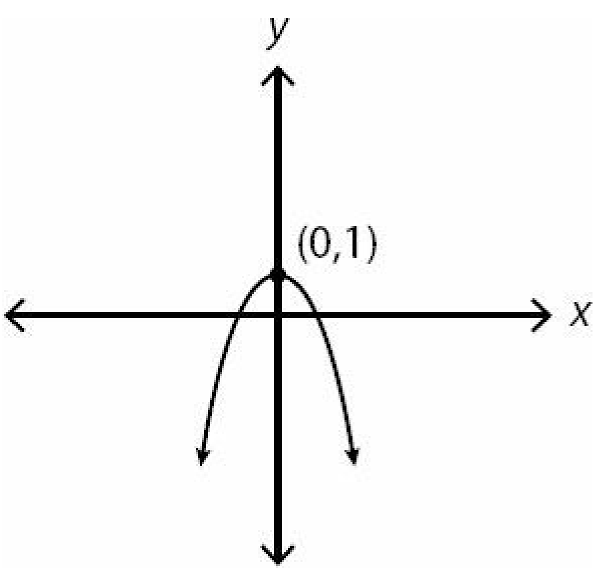
15.



Which of the following could be the equation of the parabola pictured above?

(A) $y = x^2 + 3$
(B) $y = (x - 3)^2 + 3$
(C) $y = (x + 3)^2 - 3$
(D) $y = (x - 3)^2 - 3$
(E) $y = (x + 3)^2 + 3$

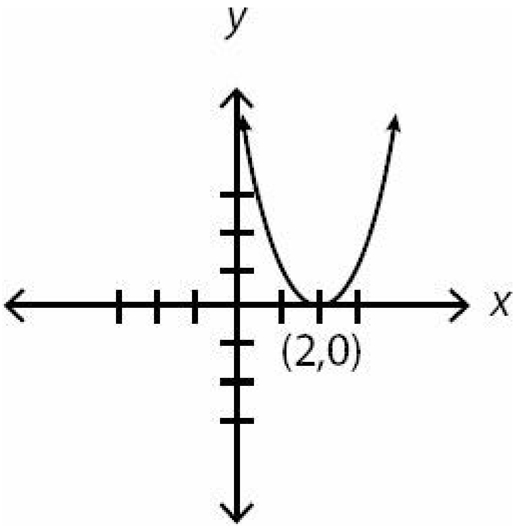
16.



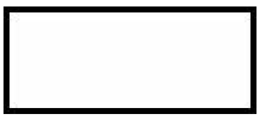
Which of the following could be the equation of the parabola pictured above?

(A) $y = -x - 1$
(B) $y = x^2 + 1$
(C) $y = -x^2 - 1$
(D) $y = -x^2 + 1$
(E) $y = -(x - 1)^2$

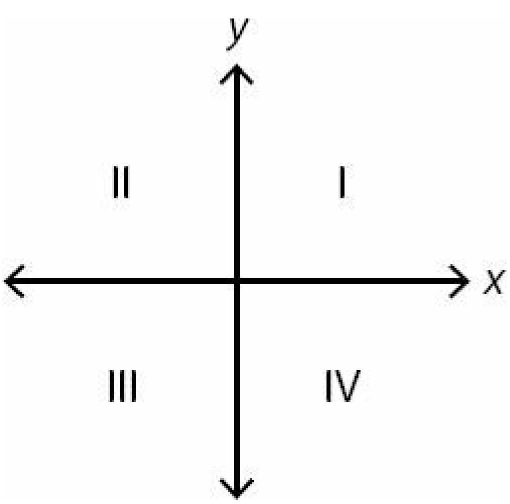
17.



If the equation of the parabola pictured above is $y = (x - h)^2 + k$ and $(-3, n)$ is a point on the parabola, what is the value of n?



18.



Which quadrant, if any, contains no point (x, y) that satisfies the inequality $y \geq (x - 3)^2 - 1$?

(A) I
(B) II
(C) III
(D) IV
(E) All quadrants contain at least one point that satisfies the given inequality.

19.

In the coordinate plane, line p has an equation of $3y - 9x = 9$.

Quantity A	**Quantity B**

The slope of line p	The x-intercept of line p

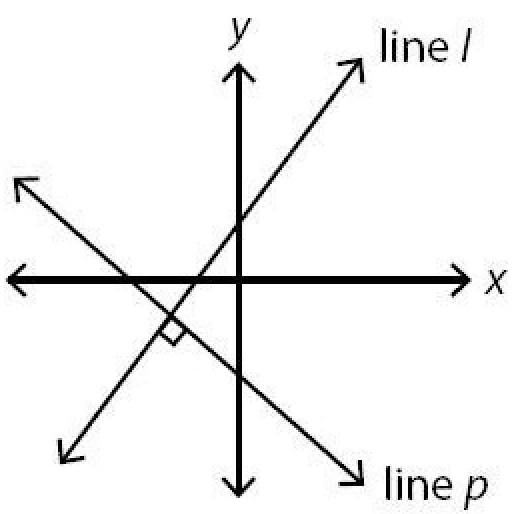
20. In the xy coordinate plane, lines l_1 and l_2 intersect at $(2, 4)$. If the equation of l_1 is $y = px + 16$ and the equation of l_2 is $y = mx + p$, where m and p are constants, what is the value of m?

21. If $(3, 5)$ and $(4, 9)$ are points on line L, which of the following is also a point on that line?

Indicate <u>all</u> such values.

☐ (2, 1)
☐ (5, 12)
☐ (6, 17)

22.



Line l has slope > 1.

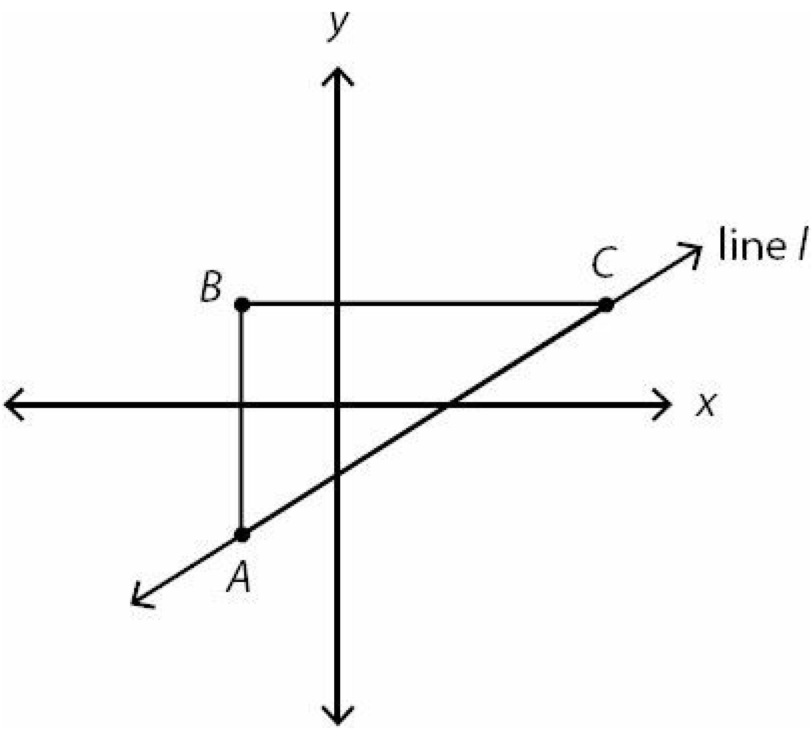
<u>Quantity A</u>	**<u>Quantity B</u>**
Slope of line p	-1

23.

Lines l_1 and l_2 are parallel and have slopes that sum to less than 1.

<u>Quantity A</u>	**<u>Quantity B</u>**
The slope of a line perpendicular to lines l_1 and l_2	$-\frac{1}{2}$

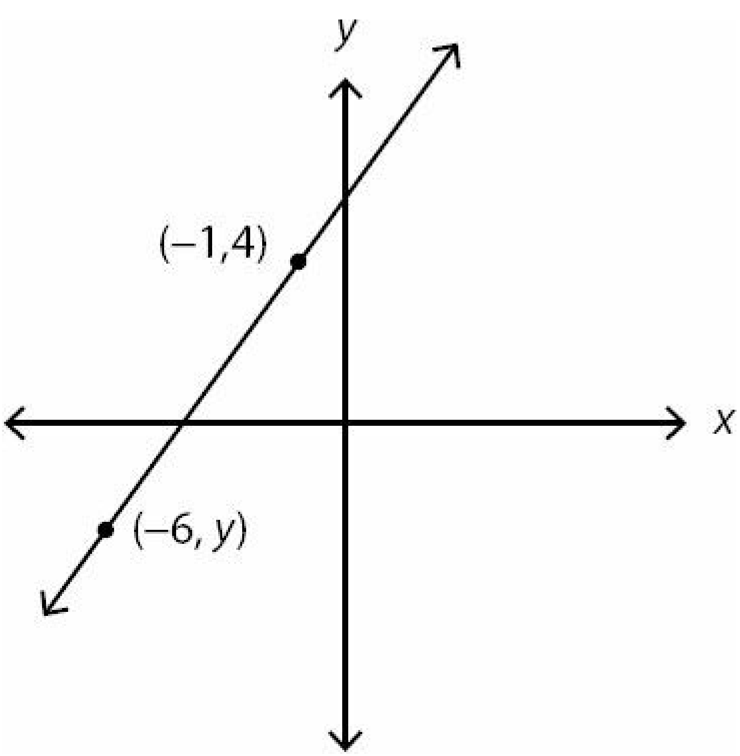
24.



If the slope of line *l* is 1/3 and the length of line segment *BC* is 4, how long is line segment *AB*?

(A) 3/4
(B) 4/3
(C) 3
(D) 4
(E) 12

25.



If the slope of the line is $\frac{15}{14}$, what is the value of *y*?

(A) $\frac{2}{7}$

(B) $\frac{7}{2}$

(C) $-\frac{7}{2}$

(D) $-\frac{14}{19}$

(E) $-\frac{19}{14}$

26. What is the area of a triangle with vertices (-2, 4), (2, 4) and (-6, 6)?

$\boxed{}$

27.

Lines k and p are perpendicular, neither is vertical, and p passes through the origin.

Quantity A	**Quantity B**
The product of the slopes of lines k and p	The product of the y-intercepts of lines k and p

28.

In the coordinate plane, points (a, b) and (c, d) are equidistant from the origin.
$|a| > |c|$

Quantity A	**Quantity B**
$|b|$	$|d|$

29.

In the coordinate plane, lines j and k are parallel. The x-intercept of line j is greater than that of line k and the product of their slopes is positive.

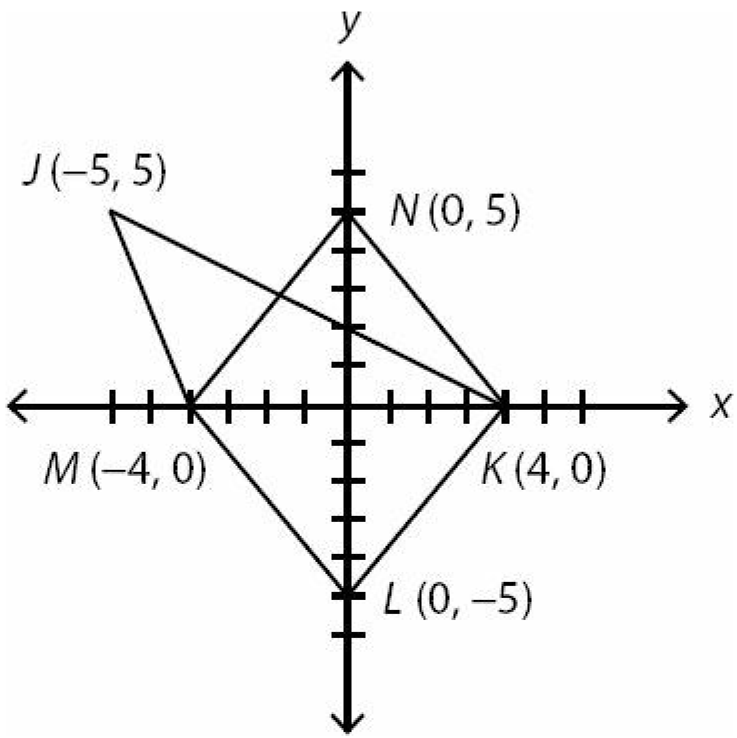
Quantity A	**Quantity B**
The y-intercept of line j	The y-intercept of line k

30. In the xy plane, which of the statements below individually provide enough information to determine whether line z passes through the origin?

Indicate all such statements.

☐ The equation of line z is $y = mx + b$ and $b = 0$.
☐ The sum of the slope and the y-intercept of line z is 0.
☐ For any point (a, b) on line z, $|a| = |b|$.

31.



Quantity A	Quantity B
The area of parallelogram *KLMN*	The area of quadrilateral *JKLM*

32. Which of the following could be the equation of a line parallel to the line $5x - 6y = 9$?

(A) $y = -\frac{5}{6}x + 1$

(B) $y = \frac{6}{5}x + 1$

(C) $y = \frac{5}{6}x + 1$

(D) $y = \frac{3}{2}x - 1$

(E) $y = \frac{2}{3}x - 1$

33. Which of the following could be the equation of a line perpendicular to the line $y = -6x + 4$?

(A) $6y - x = 12$

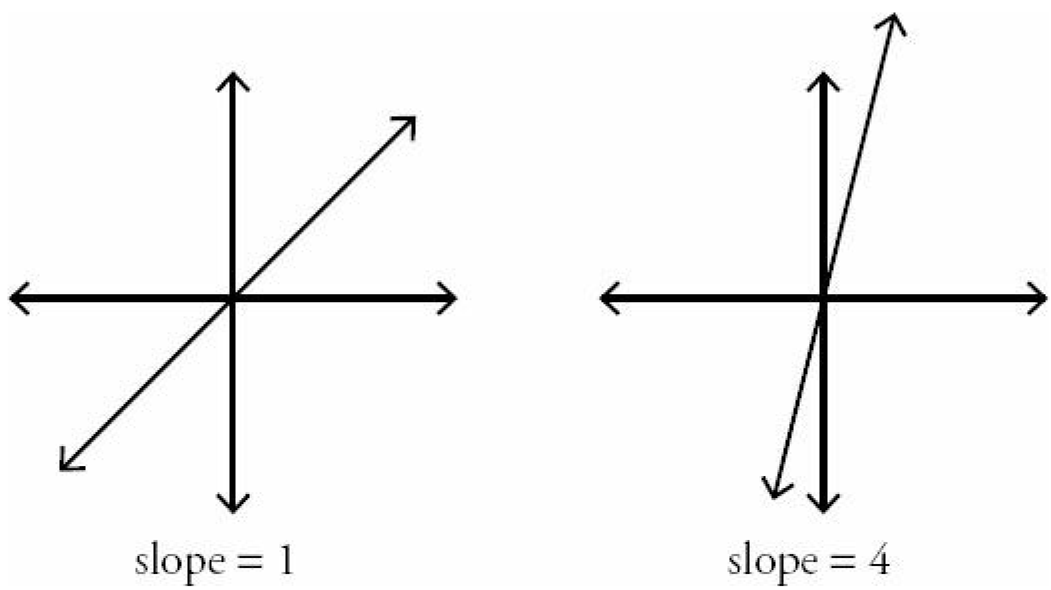
(B) $x = -6y - 12$
(C) $y + 4x = 2$
(D) $\frac{y}{2} = -3x + 5$
(E) $y + 1 = 6x$

Coordinate Geometry Answers

1. **(A).** Both slopes are negative (pointing down when reading from left to right), and line l_2 is clearly steeper than line l_1. Thus, the slope of l_2 has a larger *absolute value*. But since the values are both negative, the slope of l_1 is a larger number. For instance, the slope of l_1 could be -1 and the slope of l_2 could be -2. Whatever the actual numbers are, the slope of l_1 is closer to 0 and therefore larger.

2. **(C).** Since there are no numbers on the graph, you can't determine the actual equation of the line, but the line clearly has a positive slope (it points upward when reading from left to right) and a negative y-intercept (it crosses the y-axis below the origin). All of the answers are already in slope-intercept form ($y = mx + b$ where m = slope and b = y-intercept). Choices (A), (B), (C), and (D) have positive slope. Of those, only choices (B) and (C) have a negative y-intercept.

Now, is the slope closer to positive 4 or positive 1? A slope of 1 makes 45° angles when it cuts through the x and y axes, and this figure looks very much like it represents a slope of 1. A slope of 4 would look much steeper than this picture. Note that xy-planes are drawn to scale on the GRE, and units on the x-axis and on the y-axis are the same, unless otherwise noted.



The correct answer is (C). Note that the GRE would only give questions like this where the answers are far enough apart that you can clearly determine the intended answer.

3. **(A).** Since there are no numbers on the graph, you can't determine the actual equation of the line, but the line clearly has a negative slope (it points down when reading from left to right) and a positive y-intercept (it crosses the y-axis above the origin).

Change the answers to slope-intercept form ($y = mx + b$ where m = slope and b = y-intercept). First note that (D) and (E) cannot be the answers—choice (D) represents a horizontal line crossing through (0, 2), and choice (E) represents a vertical line passing through (-2, 0). Now, look at choice (A):

$$6y + 6x = 7$$

$6y = -6x + 7$

$y = -x + \frac{7}{6}$

This line (choice A) has slope -1 and y-intercept $\frac{7}{6}$. Next, test choice (B):

$3y = -4x - 3$

$y = -\frac{4}{3}x - 1$

This line (choice B) has slope $-\frac{4}{3}$ and y-intercept. Finally, look at (C):

$5y + 10 = -4x$
$5y = -4x - 10$

$y = -\frac{4}{5}x - 2$

This line (choice C) has slope $-\frac{4}{5}$ and y-intercept -2.

The only choice with a negative slope and a positive y-intercept is choice (A).

4. **(B).** Since the line is vertical, it always has the same x-coordinate. That is what the correct answer, $x = -2$, expresses. No matter what the y-coordinate is, x is always some negative value. All equations of vertical lines have the form x = [number]. Similarly, all equations of horizontal lines have the form y = [number].

5. **(C).** In $y = mx + b$ form, m is the slope and b is the y-intercept. Since the slope is -2:

$y = -2x + b$

Now, plug in the point (-1, -3) to determine b:

$-3 = -2(-1) + b$
$-3 = 2 + b$
$-5 = b$

Since $b = -5$, the equation of the line is:

$y = -2x - 5$

Note that the coordinates (-1, -3) do not belong in the final answer. The point (-1, -3) was merely an example of one of the infinite number of points along the line.

6. **(A).** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. It doesn't matter which point is first; just be consistent. Using (-4, 5) as x_1 and y_1 and (1, 2) as x_2 and y_2:

$$m = \frac{2-5}{1-(-4)} = -\frac{3}{5}$$

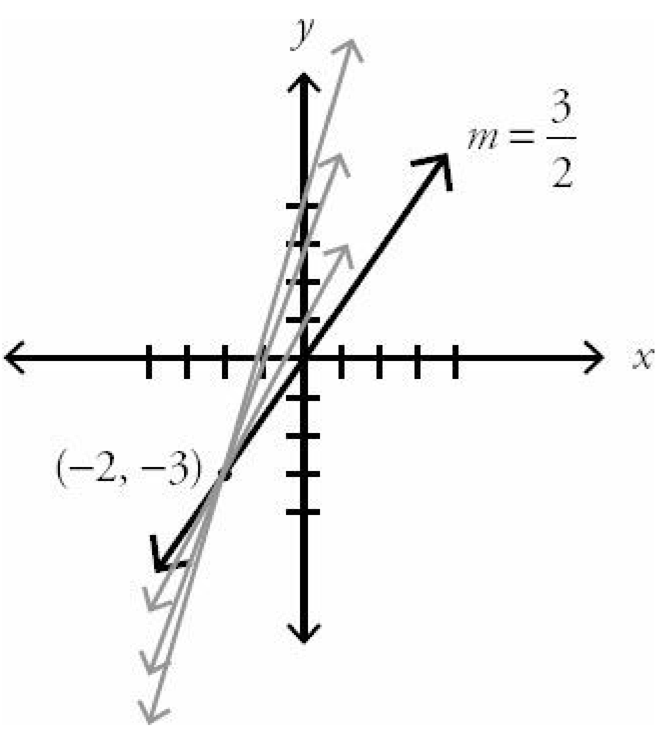
The slope is $\frac{-3}{5}$, which appears in the choices as $-\frac{3}{5}$ (these are identical).

7. **IV, V, and VI only.** The line must hit a point on the y-axis above (0, 0). That means the line could include (0, 0.1), (0, 25), or even (0, 0.00000001). Since the x-intercept could get very, very close to (0, 0), use the point (0, 0) to calculate the slope—and then reason that since the line can't *actually* go through (0, 0), the slope will actually have to be steeper than that.

The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using (0, 0) as x_1 and y_1 and (-2, -3) as x_2 and y_2 (you can make either pair of points x_1 and y_1, so make whatever choice is most convenient):

$$m = \frac{-3-0}{-2-0} = \frac{-3}{-2} = \frac{3}{2}$$

Since the slope is positive and the line referenced in the problem needs to hit the x-axis above (0, 0), the slope of that line will have to be greater than $\frac{3}{2}$, as in the gray lines below:



Select all answers with a slope greater than $\frac{3}{2}$. Thus, $\frac{5}{3}$, $\frac{9}{4}$, and 4 are correct.

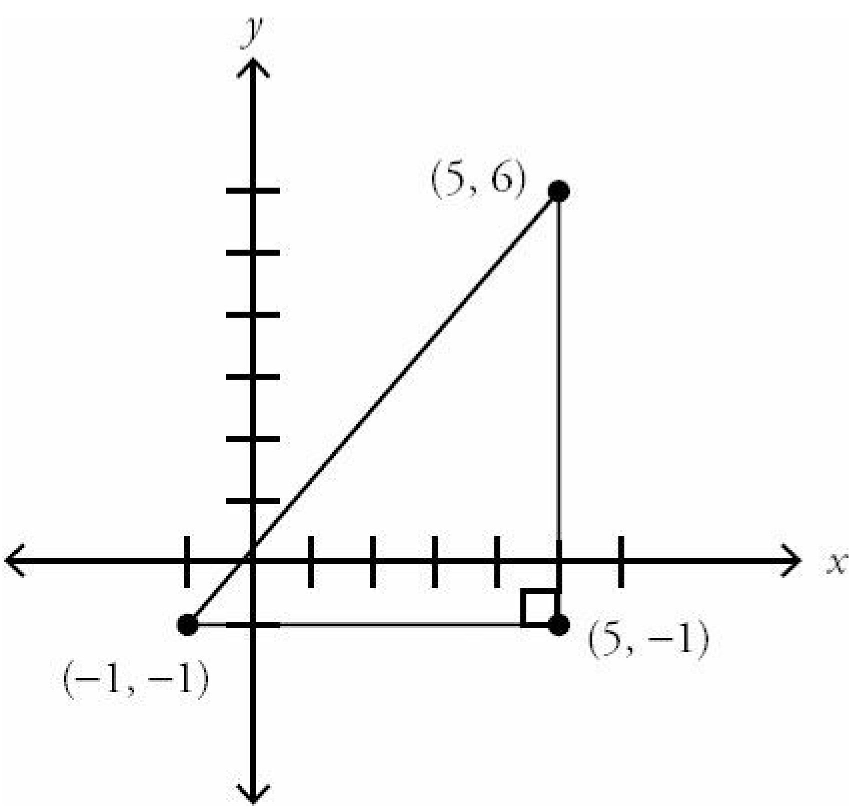
8. **5.** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using (4, 9) as x_1 and y_1 and (6, y) as x_2 and y_2, and plugging in -2 for the slope:

$$-2 = \frac{y-9}{6-4}$$
$$-2 = \frac{y-9}{2}$$
$$-4 = y = -9$$
$$5 = y$$

9. **(D).** You could use the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$:

$$d = \sqrt{(6-(-1))^2 + (5-(-1))^2}$$
$$d = \sqrt{(7)^2 + (6)^2}$$
$$d = \sqrt{85}$$

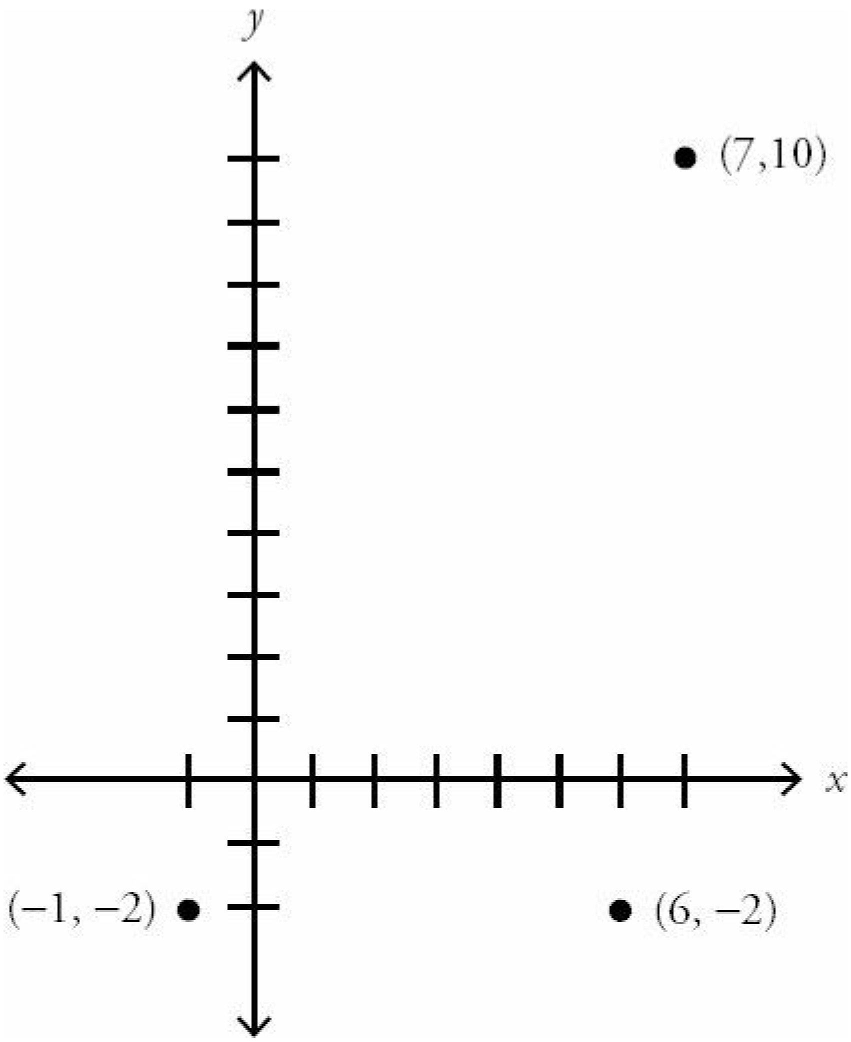
Or you could construct a right triangle and use the Pythagorean theorem:



The third point, (5, -1) lets you construct a right triangle with the distance *d* as the hypotenuse. The distance between (-1, -1) and (5, -1) is 6. The distance between (5, -1) and (5, 6) is 7. From the Pythagorean theorem:

$$6^2 + 7^2 = d^2$$
$$36 + 49 = d^2$$
$$85 = d^2$$
$$d = \sqrt{85}$$

10. **4.** Make a quick sketch:



It should be obvious that the two furthest points are (-1, -2) and (7, 10). Also keep in mind that since (-1, -2) and (6, -2) share a *y*-coordinate, the distance between the two points is just the distance between their *x*-coordinates: 6 - (-1) = 7. The correct answer should definitely be longer than 7.

To find the distance between (-1, -2) and (7, 10), you could use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(10 - (-2))^2 + (7 - (-1))^2}$$
$$d = \sqrt{(12)^2 + (8)^2}$$

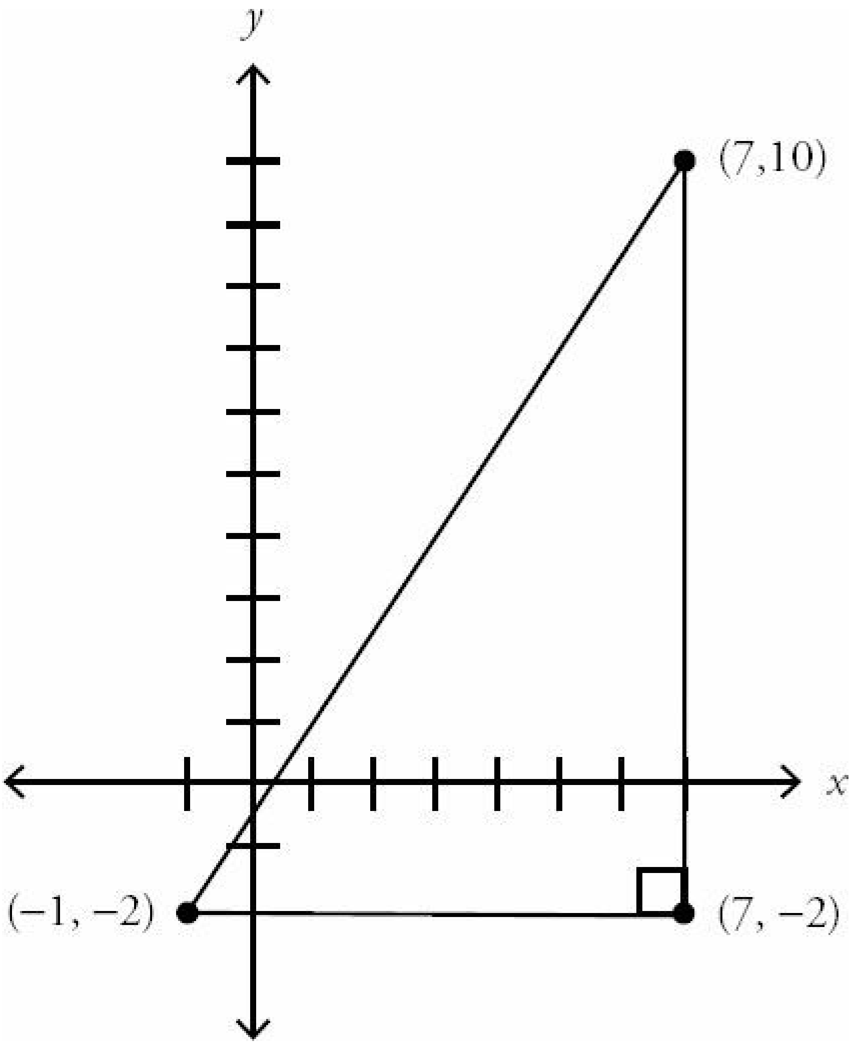
$d = \sqrt{208}$

Use your calculator, if needed, to find the biggest perfect square that goes into 208. It is 16:

$$d = \sqrt{208} = \sqrt{16 \times 13} = 4\sqrt{13}$$

Since the distance $p\sqrt{13}$ is equal to $4\sqrt{13}$, $p = 4$.

Alternatively, you could construct a right triangle and use the Pythagorean theorem.



The point (7, -2) lets you construct a right triangle with the distance *d* as the hypotenuse. (Don't get confused—this point has nothing to do with the (6, -2) from the problem; that point was irrelevant to the question being asked.) You create the point (7, -2) by dropping a line straight down from (7, 10) and stopping at (7, -2), which has the same *y*-coordinate as the point (-1, -2).

The distance between (-1, -2) and (7, -2) is 8. The distance between (7, 10) and (7, -2) is 12. From the Pythagorean Theorem:

$$8^2 + 12^2 = d^2$$
$$64 + 144 = d^2$$

$208 = d^2$
$d = \sqrt{208} = \sqrt{16 \times 13} = 4\sqrt{13}$

Again, since the distance $p\sqrt{13}$ is equal to $4\sqrt{13}$, $p = 4$.

11. **(B).** Manipulate $2y - 4x - 8 = 0$ into slope intercept form ($y = mx + b$, where m is the slope and b is the y-intercept):

$2y - 4x = 8$
$2y = 4x + 8$
$y = 2x + 4$

The slope of the line is 2. (The y-intercept is 4, which may be the "trick" intended in Quantity B). Quantity B is larger.

12. **I and III only.** For the point (3, -2) to lie on the line $y = 2x - 8$, y needs to equal -2 when you plug in 3 for x.

$y = 2(3) - 8$
$y = 6 - 8 = -2$

y does equal -2 when x equals 3, so the point does lie on the line and statement I is correct.

However, when you plug in -8 for x, y does not equal 0, so statement II is not correct. When you plug in 1/2 for x, y does equal -7, so statement III is correct.

13. **(D).** The problem asks for the point that does NOT lie on the curve. $y = x^2 - 3$ is the equation of a parabola, but you don't need to know that fact in order to answer this question. For each choice, just plug in the coordinates for x and y. For instance, try choice (A):

$6 = (3)^2 - 3$
$6 = 6$

Since this is a true statement, choice (A) lies on the curve. The only choice that yields a false statement when plugged in is choice (D), the correct answer.

For the point (-3, 0) to lie on the curve $y = x^2 - 3$, y needs to equal 0 when you plug in -3 for x.

$y = (-3)^2 - 3$
$y = 9 - 3 = 6$

y does not equal 0 when x equals -3, so the point does not lie on the curve.

14. **(C).** The figure is a parabola. Choices (A), (D), and (E) do not represent parabolas. An equation for a parabola should look something like this: $y = 5x^2 + 4x + 3$, where the 5, 4, and 3 can be other numbers. When the left side is just y, the right side has to have an x^2 term (and no higher power). There can be an x term and/or a constant term.

Both (B) and (C) represent parabolas, but the parabola in (B) is not centered on the y-axis (there must be no x term). The equation in (C), $y = x^2 - 2$, represents a parabola that opens upward, that is centered on the y-axis, and that has a y-intercept of -2. These features are consistent with the diagram.

15. **(E).** The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Here is the vertex of the parabola described by each answer choice:

(A) (0, 3) On the axis
(B) (3, 3) Incorrect Quadrant
(C) (-3, -3) Incorrect Quadrant
(D) (3, -3) Incorrect Quadrant
(E) (-3, 3) Correct

Only choice (E) places the vertex in the correct quadrant.

16. **(D).** The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Eliminate choice (A), as it is not the equation of a parabola. Here is the vertex of the parabola described by each remaining answer choice:

(B) (0, 1) Correct
(C) (0, -1) Incorrect
(D) (0, 1) Correct
(E) (1, 0) Incorrect

Both (B) and (D) have the correct vertex. However, choice (B) describes a parabola pointing upward from that vertex, because the x^2 term is positive. The negative in front of choice (D) indicates a parabola pointing downward from that vertex.

17. **25.** The equation of the parabola is $y = (x - h)^2 + k$. The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). (Since the equation of this particular parabola does not have constant a, a must be equal to 1.)

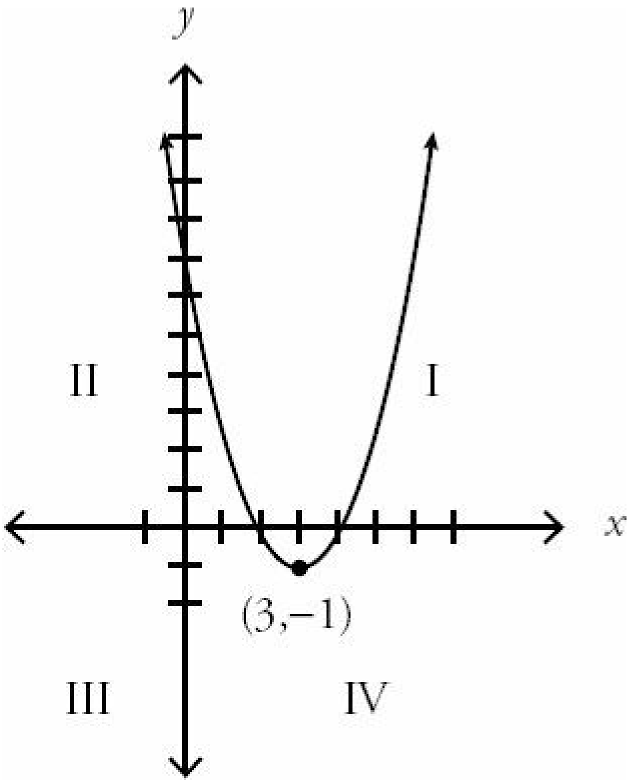
Using $y = (x - h)^2 + k$ and the vertex (2, 0) shown in the graph:

$$y = (x - 2)^2 + 0$$
$$y = (x - 2)^2$$

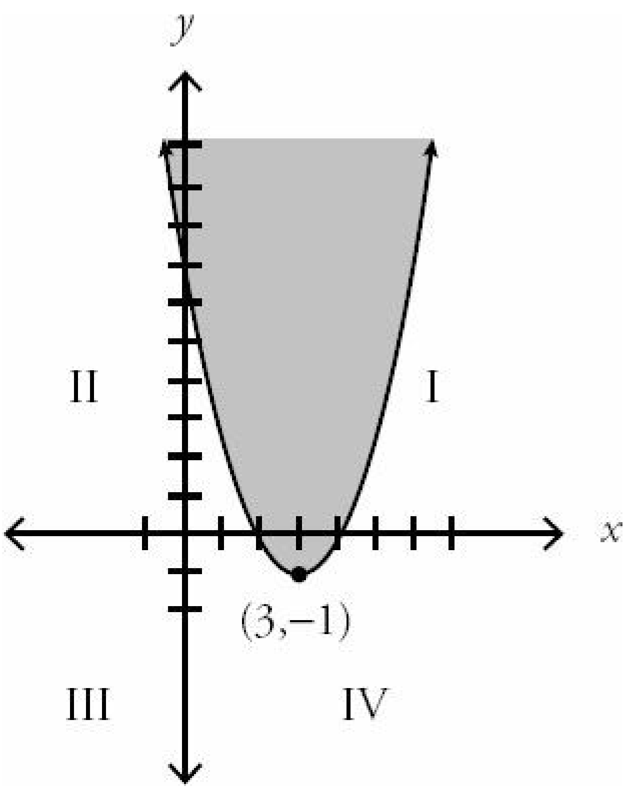
Since $(-3, n)$ is a point on the parabola, plug in -3 and n for x and y:

$$n = (-3 - 2)^2$$
$$n = (-5)^2$$
$$n = 25$$

18. **(C).** One method for solving this problem is to graph the curve. To do this, first graph the parabola as though the $\geq$ sign were an equals sign. The standard equation of a parabola in vertex form is $y = a(x - h)^2 + k$, where the vertex is (h, k). Thus, $y = (x - 3)^2 - 1$ is the graph of a parabola with its vertex at (3, -1). (3, 1)



Since $y \geq (x - 3)^2 - 1$ is actually an inequality in which y (i.e., all the y-coordinates) are greater than the graph, shade above the curve.



The inequality has no points in Quadrant III.

Alternatively, you could use algebra to prove this. Coordinates in Quadrant III have negative x-coordinates and negative y-coordinates. There is no such pair of coordinates that will satisfy $y \geq (x - 3)^2 - 1$.

Specifically, the only way the y-coordinate could be negative is if $(x - 3)^2$ were less than 1 (so that subtracting 1 from it yielded a negative). The only way for $(x - 3)^2$ to be less than 1 is for x to be less than 1 away from 3. That is, y is only negative when $2 < x < 4$. You can also demonstrate this by setting $(x - 3)^2 - 1$ in an inequality with 0:

$(x - 3)^2 - 1 < 0 \longleftarrow$ Note: this is NOT necessarily a true statement; you are investigating what would have to be true about x in order for this to be true.

$$(x - 3)^2 < 1$$
$$x - 3 < 1 \text{ or } x - 3 > -1$$
$$x < 4 \text{ or } x > 2$$
$$2 < x < 4$$

Thus, y is only negative when $2 < x < 4$. Therefore, no coordinate pair in Quadrant III will satisfy the inequality.

19. **(A).** In slope intercept form ($y = mx + b$, where m is the slope and b is the y-intercept):

$$3y - 9x = 9$$
$$3y = 9x + 9$$
$$y = 3x + 3$$

The slope is 3. The y-intercept is also 3, but the problem asks for the x-intercept. To get an x-intercept, substitute 0 for y:

$$0 = 3x + 3$$
$$-3 = 3x$$
$$-1 = x$$

Thus, the slope is 3 and the x-intercept is -1. Quantity A is greater.

20. **5.** If l_1 and l_2 intersect at (2, 4), then 2 can be plugged in for x and 4 plugged in for y in either equation. Equation l_1:

$$y = px + 16$$
$$4 = p(2) + 16$$
$$-12 = 2p$$
$$-6 = p$$

Now, plug (2, 4) as well as $p = -6$ into Equation l_2 to get m, the final answer:

$$y = mx + p$$
$$4 = m(2) - 6$$
$$10 = 2m$$
$$5 = m$$

21. **I and III only.** To be on the same line as (3, 5) and (4, 9), the slope between any given point and either (3, 5) or (4, 9) must be the same as the slope between (3, 5) and (4, 9).

The (very) long way to do this problem would be to find the slope of (3, 5) and (4, 9). Using "change in y" divided by "change in x," you get a slope of 4/1, or 4. Test the choice (2,1) with either (3, 5) or (4, 9) to see if the slope is the same —for instance, the slope of the line segment between (2,1) and (3,5) is clearly 4/1, since the difference between the y-coordinates is 4 and the difference between the x-coordinates is 1. Since the slopes of these connecting line segments are the same, they are in fact parts of the same line.

It is possible to do this procedure for each choice. However, like most GRE problems, this problem has a "trick": Using the original two points, notice that to get from (3, 5) to (4, 9), the x-coordinate goes up 1, while the y-coordinate goes up 4. Now just continue that pattern upward from (4, 9), adding 1 to the x and 4 to the y. You get (4 + **1**, 9 + **4**), or (5, 13). This point is not in the choices, and in fact you can now eliminate (5, 12) since the line passes above that point.

Keep going up on the line. (5 + **1**, 13 + **4**) is (6, 17), so this point is on the line.

You can do the same trick going *down*. Start from (3, 5) and instead of adding 1 and 4, *subtract* 1 and 4. (3 - **1**, 5 - **4**) is (2, 1), so this point is on the line.

Thus, (2, 1) and (6, 17) are on the line, and (5, 12) is not.

22. **(A).** If the slope of line l is > 1 and line p is perpendicular (you know this because of the right angle symbol on the figure), then line p will have a slope greater than -1 because perpendicular lines have negative reciprocal slopes—that is, the product of the two slopes is -1.

Try a few examples to better illustrate this: line l could have slope 2, in which case line p would have slope -1/2. Line l could have slope 3/2, in which case line p would have slope -2/3. Or, line l could have slope 100, in which case line p would have slope -1/100.

All of these values (-1/2, -2/3, and -1/100) are greater than -1. This will work with any example you try; since line l has a slope greater than 1, line p will have a slope with an absolute value less than 1. Since that value will also be negative, it will always be the case that -1 < (slope of line p) < 0.

23. **(D).** Since lines l_1 and l_2 are parallel, they have the same slope. Call that slope m. Since the slopes add to less than 1:

$$m + m < 1$$
$$2m < 1$$
$$m < 1/2$$

Thus, lines l_1 and l_2 each have slopes less than 1/2. A line perpendicular to those lines would have a negative reciprocal slope. However, there isn't much more you can do here. Lines l_1 and l_2 could have slopes of 1/4 (in which case a perpendicular line would have slope = -4), or slopes of -100 (in which case a perpendicular line would have slope = 1/100). Thus, the slope of the perpendicular line could be less than or greater than $-\frac{1}{2}$.

24. **(B).** You are told that the slope is 1/3. Since slope = rise/run (or "change in y" divided by "change in x"), for every 1 unit the line moves up, it will move 3 units to the right.

Since the *actual* move to the right is equal to 4, you can now create a proportion:

$$\frac{1}{3} = \frac{n}{4}$$

Here, n is the distance from A to B (which is also the change in the y-coordinates).

Cross multiply to get $3n = 4$ or $n = 4/3$.

25. **(E).** The slope formula is $m = \frac{y_2 - y_1}{x_2 - x_1}$. Using $\frac{15}{14}$ as the slope, (-6, y) as x_1 and y_1, and (-1, 4) as x_2 and y_2:

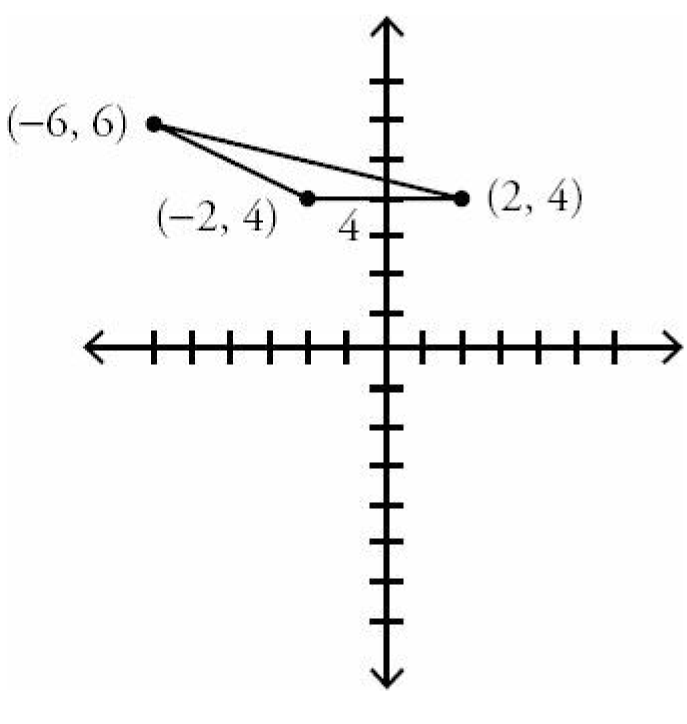
$$\frac{15}{14} = \frac{4 - y}{-1-(-6)}$$

$$\frac{15}{14} = \frac{4 - y}{5}$$

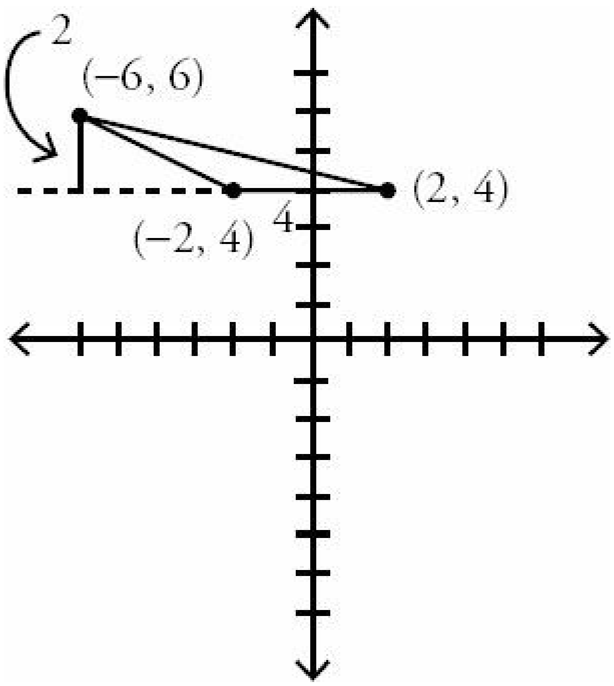
Cross multiply and solve for y:

$15(5) = 14(4 - y)$
$75 = 56 - 14y$
$19 = -14y$
$-\frac{19}{14} = y$

26. **4.** Make a quick sketch of the three points, joining them to make a triangle. Since (-2, 4) and (2, 4) make a horizontal line, use this line as the base. Since these two points share a y-coordinate, the distance between them is simply the distance between their x-coordinates: 2 - (-2) = 4.



The height of a triangle is always perpendicular to the base. Drop a height vertically from (-6, 6). Subtract the y-coordinates to get the distance: 6 - 4 = 2.

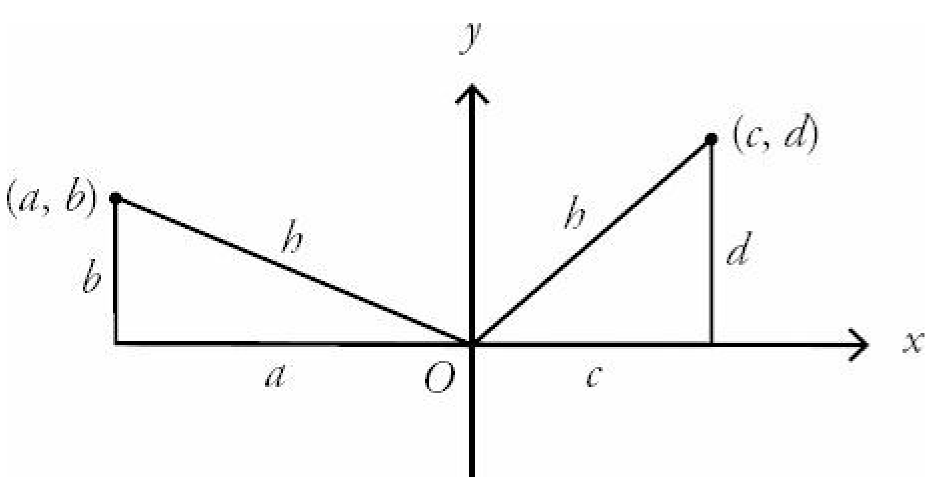


The formula for area of a triangle is $\frac{bh}{2}$. Thus, the area is $\frac{(4)(2)}{2}$, or 4.

27. **(B).** The slopes of perpendicular lines are the negative inverse of each other, so their product is -1. For example, perpendicular lines could have slopes of 2 and -1/2, or -5/7 and 7/5. In all of these cases, Quantity A is equal to -1. (The only exception is when one of the lines has an undefined slope because it's vertical, but that case has been specifically excluded.)

If line p passes through the origin, its y-intercept is 0, so regardless of the y-intercept of line k, Quantity B is equal to zero.

28. **(B).** A point's distance from the origin can be calculated by constructing a right triangle in which the legs are the vertical and horizontal distances. Sketch a diagram in which you place (a, b) and (c, d) anywhere in the coordinate plane that you wish; then construct two right triangles using (0, 0) as a vertex.



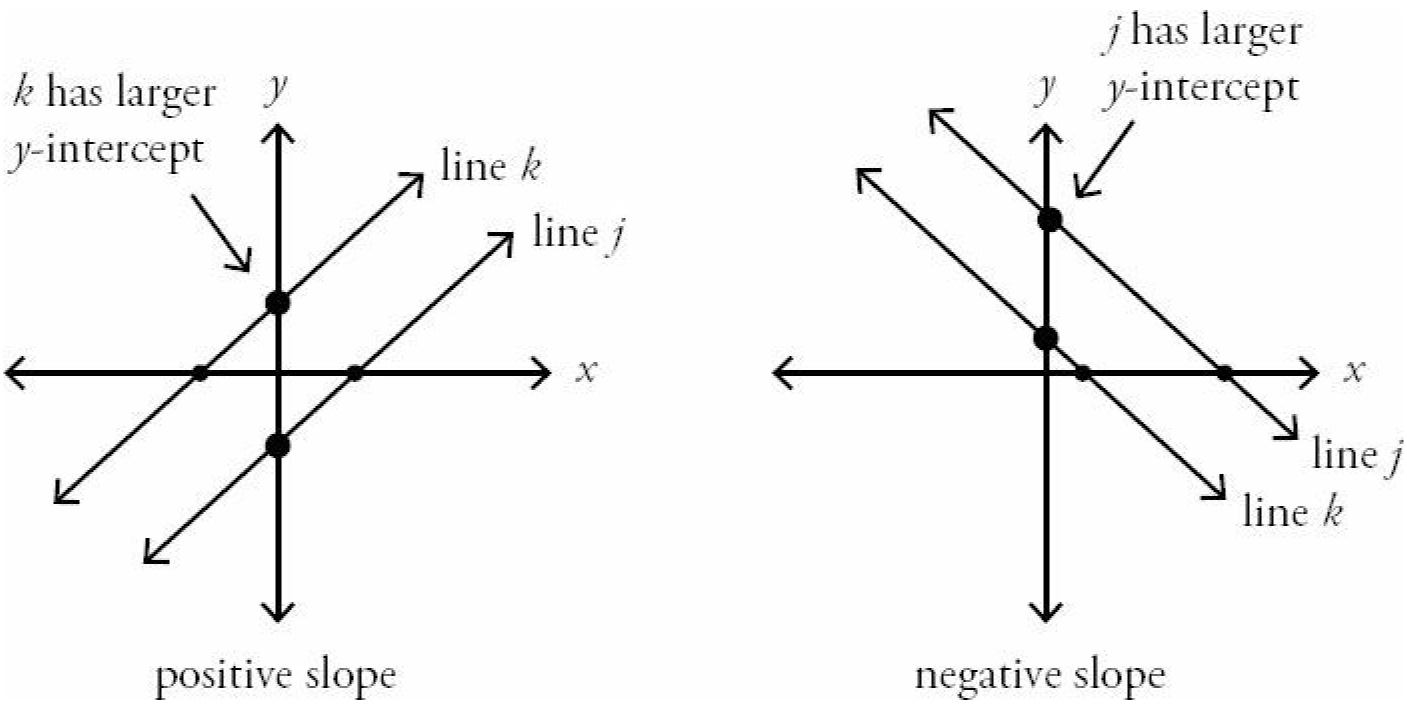
Both hypotenuses are labeled h, since the points are equidistant from the origin. Set up two Pythagorean theorems:

$a^2 + b^2 = h^2$
$c^2 + d^2 = h^2$

So $a^2 + b^2 = c^2 + d^2$.

Since $|a| > |c|$, you know that $a^2 > c^2$. (Try it with test numbers.) To make the equation $a^2 + b^2 = c^2 + d^2$ true, you must have $b^2 < d^2$. This means that $|b| < |d|$, and Quantity B is larger.

29. **(D).** Parallel lines have the same slope. Since the product of the two slopes is positive, either both slopes are positive or both slopes are negative. Here are two examples in which line j has a larger x-intercept, as specified by the problem:



If the slopes are positive, k will have the greater y-intercept, but if the slopes are negative, l will have the greater y-intercept.

30. **I and III only.** Statement I tells you directly that b, the y-intercept, is equal to 0. Thus, the line passes through the origin.

For statement II, both the slope and the y-intercept could be 0, in which case line z is a horizontal line lying on the x-axis and therefore passes through the origin. Or, the slope and y-intercept could simply be opposites, such as 2 and -2. A line with a y-intercept of -2 and a slope of 2 would not pass through the origin. Therefore, this statement is not sufficient to determine whether line z passes through the origin.

As for statement III, since $|a| = |b|$ must hold for every point on the line, then (0, 0) is a point on the line, since $|0| = |0|$.

31. **(C).** Both figures share triangle MLK, so you don't need to calculate anything for this part of the figure. Parallelogram $KLMN$ and quadrilateral $JKLM$ each have a "top" (the part above the x-axis) that is a triangle with base MK (= 8) and height 5. If two triangles have the same base and equal heights, their areas are equal. No calculation is needed to pick (C).

32. **(C).** Rearrange the equation to get it into $y = mx + b$ format where m is the slope:

$$5x - 6y = 9$$
$$-6y = -5x + 9$$
$$y = \frac{5}{6}x - \frac{3}{2}$$

The slope is $\frac{5}{6}$. Parallel lines have the same slope, so only choice (C) is parallel to the given line.

33. **(A)**. The line $y = -6x + 4$ is already in $y = mx + b$ format, so the slope is -6. Perpendicular lines have negative reciprocal slopes, so you are looking for a line with slope $\frac{1}{6}$. Rearrange each choice into $y = mx + b$ format, if needed, to find a match.

Choice (A):

$$6y - x = 12$$
$$6y = x + 12$$
$$y = \frac{1}{6}x + 2$$

The slope of this line is $\frac{1}{6}$. Choice (A) works, so it is not necessary to try the other choices.

Chapter 29
of

Mixed Geometry

In This Chapter…

Mixed Geometry

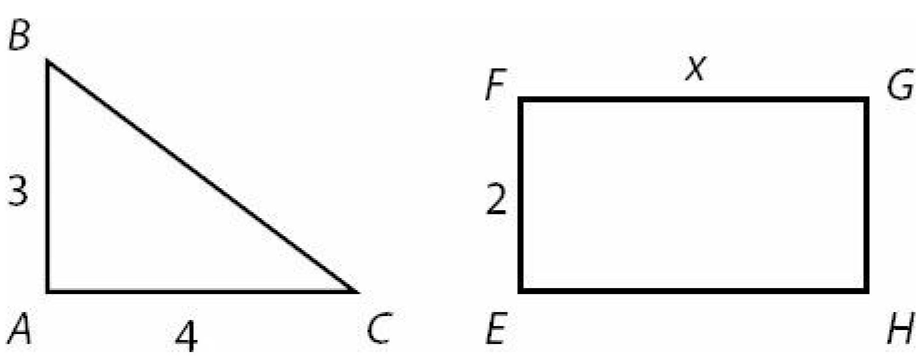
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

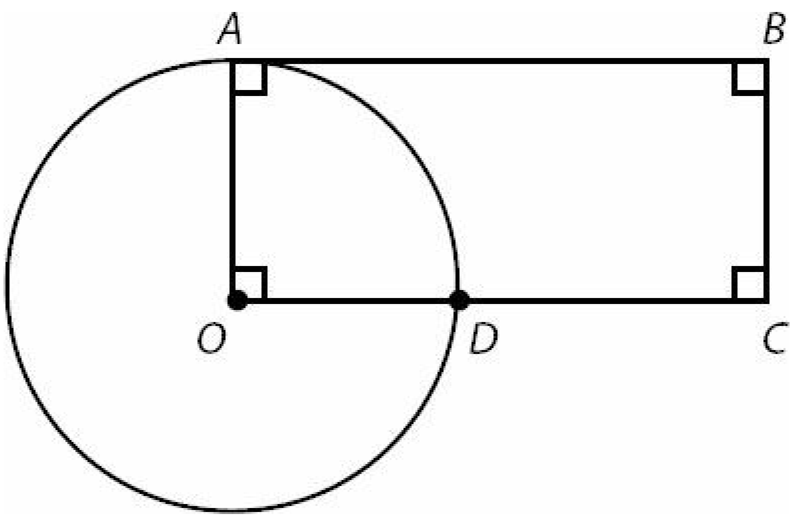
All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.



Right Triangle *ABC* and Rectangle *EFGH* have the same perimeter. What is the value of *x*?

2.



Point *O* is the center of the circle.

If the area of the circle is $36p$ and the area of the rectangle is 72, what is the length of *DC*?

3. The center of a circle is (5, -2). (5, 7) is outside the circle, and (1, -2) is inside the circle. If the radius, *r*, is an integer, how many possible values are there for *r*?

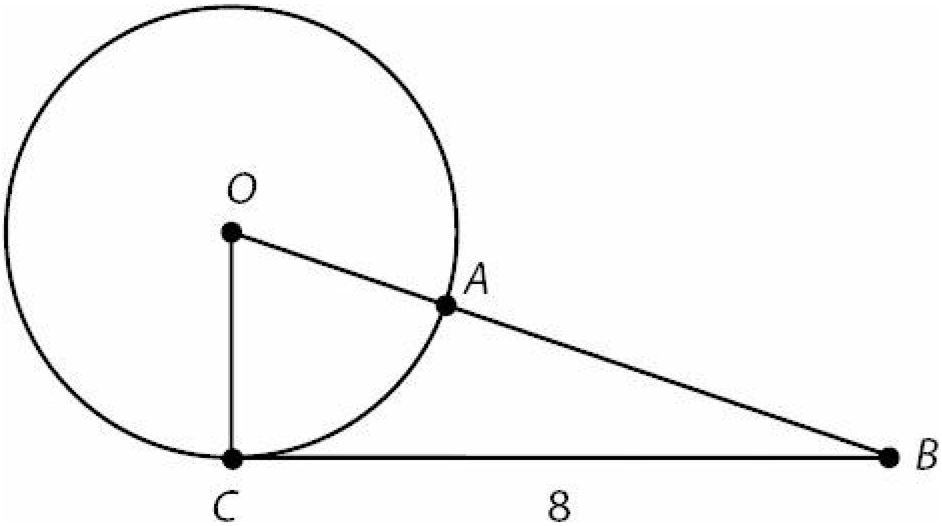
(A) 4
(B) 5
(C) 6
(D) 7
(E) 8

4.

A square's perimeter in inches is equal to its area in square inches.
A circle's circumference in inches is equal to its area in square inches.

Quantity A	**Quantity B**
The side length of the square.	The diameter of the circle.

5.



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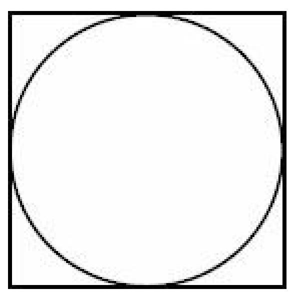
In the figure above, point *O* is the center of the circle, points *A* and *C* are located on the circle, and line segment *BC* is tangent to the circle. If the area of triangle *OBC* is 24, what is the length of *AB*?

(A) 2
(B) 4
(C) 6
(D) 8
(E) 10

6.

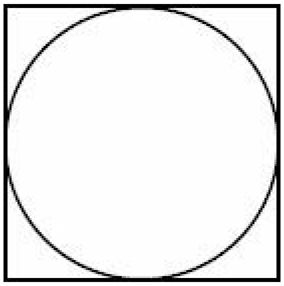


The circle is inscribed in the square.
The area of the circle is 25π.

Quantity A	**Quantity B**
The area of the square	50

7. If a circle is inscribed in a square with area 16, the area of the circle is equal to how many π?

8.



If the circle is inscribed in the square above, and the area of the square is 50, what is the area of the circle?

(A) $\frac{25\pi}{4}$

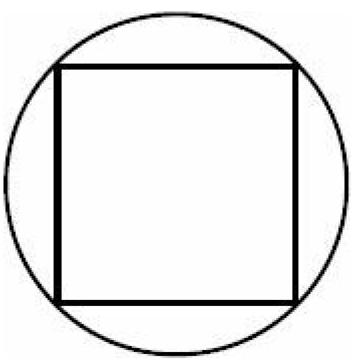
(B) $\frac{25\pi}{2}$

(C) 25π

(D) 50π

(E) $\frac{625\pi}{16}$

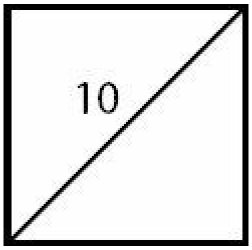
9.



In the figure above, a square is inscribed in a circle. If the area of the square is 4, what is the area of the circle?

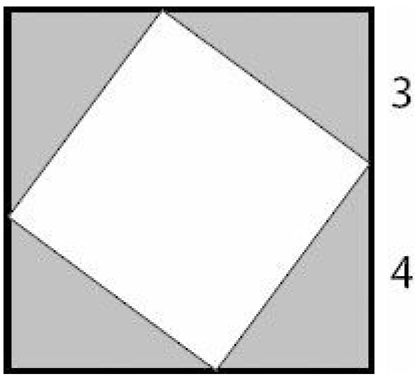
(A) π
(B) 2π
(C) 4π
(D) 6π
(E) 8π

10.



What is the area of the square in the figure above?

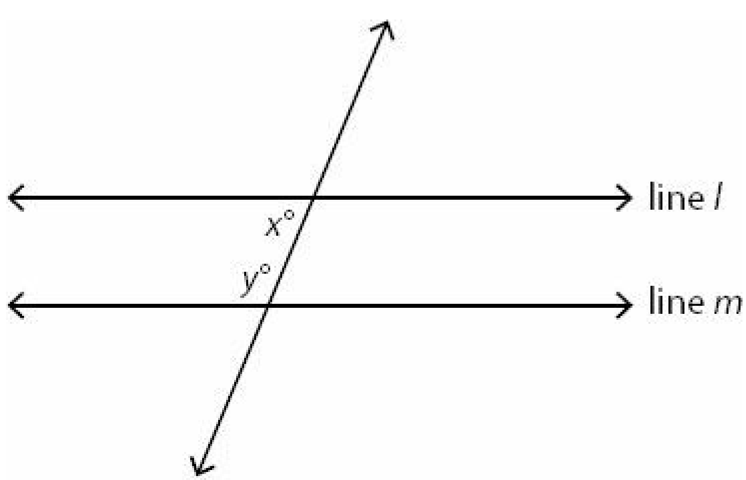
11.



In the 7-inch square above, another square is inscribed. What fraction of the larger square is shaded?

(A) 3/12
(B) 24/49
(C) 1/2
(D) 25/49
(E) 7/12

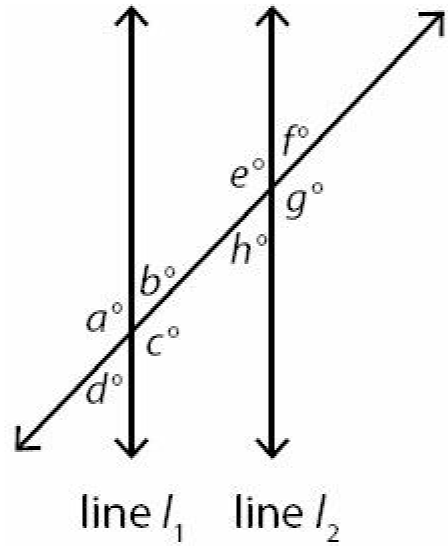
12.



Lines *l* and *m* are parallel.

Quantity A	Quantity B
$x°$	$180 - y°$

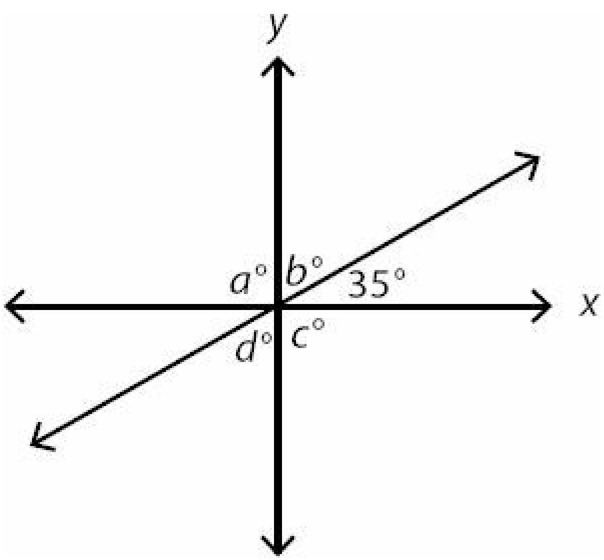
13.



Lines l_1 and l_2 are parallel.
$a > 90$

Quantity A	Quantity B
$a + g + f$	$e + b + h$

14.



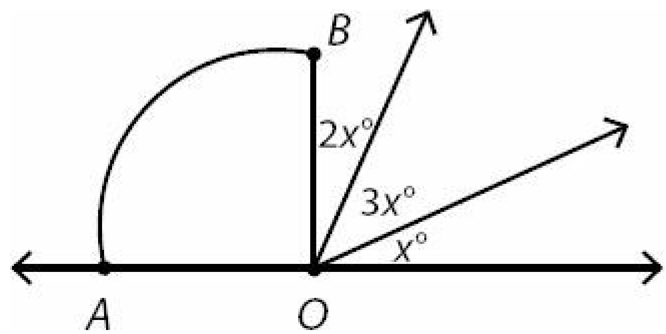
What is the value of $a + b + c + d$?

15.

A right isosceles triangle with a leg of length *f* has the same area as a square with a side of length 5.

Quantity A	Quantity B
f	s

16.



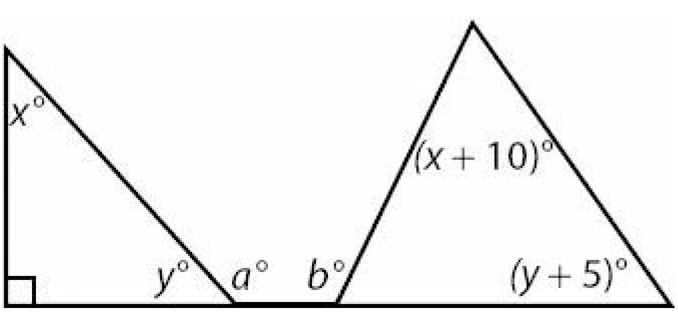
Sector *OAB* is a quarter-circle.

Quantity A	Quantity B
x	15

17. In the *xy*-plane, an equilateral triangle has vertices at (0, 0) and (9, 0). What could be the coordinates of the third vertex?

(A) (0, 4.5)
(B) (4.5, 4.5)
(C) $\left(\frac{9\sqrt{3}}{2}, \frac{9\sqrt{3}}{2}\right)$
(D) $(4.5,\ 9\sqrt{3})$
(E) $(4.5,\ \frac{9\sqrt{3}}{2})$

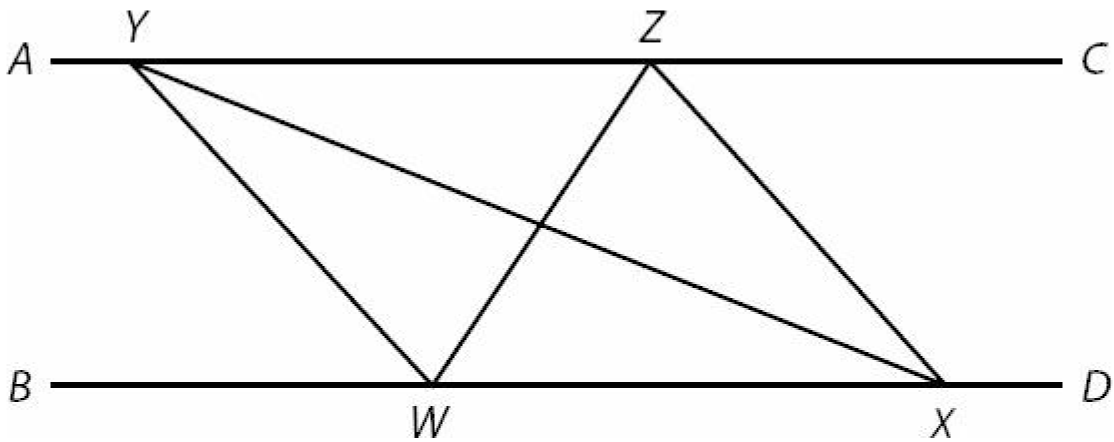
18.



What is *a* in terms of *b* and *y*?

(A) $b + y + 65$
(B) $b - y + 65$
(C) $b + y + 75$
(D) $b - 2y + 45$
(E) $b - y + 75$

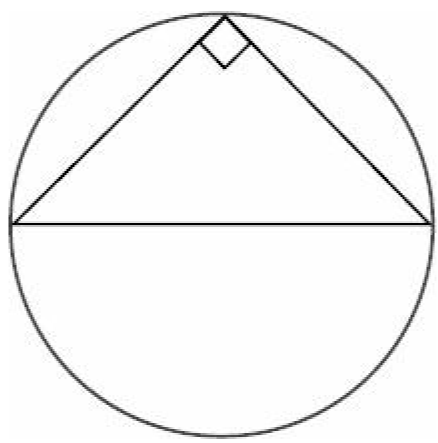
19.



In the figure above, line segments *AC* and *BD* are parallel.

Quantity A	**Quantity B**
The area of triangle *WYX*	The area of triangle *WZX*

20.



A right triangle is inscribed in a circle with an area of 16π centimeters^2 as shown above.

Quantity A	**Quantity B**
The hypotenuse of the triangle, in centimeters	8

21. A rectangular box has a length of 6 cm, a width of 8 cm, and a height of 10 cm. What is the length of the diagonal of the box, in cm?

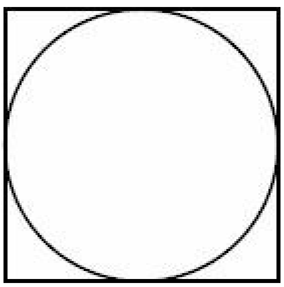
(A) 10
(B) 12
(C) $10\sqrt{2}$

(D) $10\sqrt{3}$
(E) 24

22. If the diagonal of a square garden is 20 feet long, what is the perimeter of the garden?

(A) $10\sqrt{2}$ feet
(B) $20\sqrt{2}$ feet
(C) 40 feet
(D) $40\sqrt{2}$ feet
(E) 80 feet

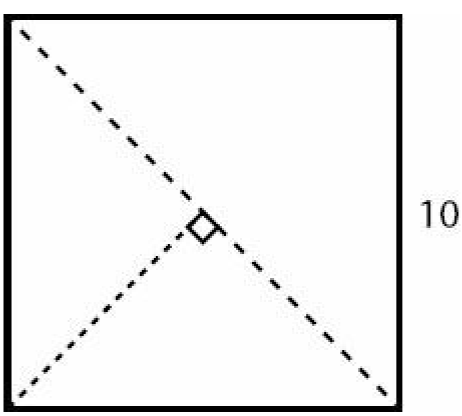
23.



In the figure above, if the diagonal of the square is 12, what is the radius of the circle?

(A) $3\sqrt{2}$
(B) 6
(C) $6\sqrt{2}$
(D) 9
(E) 18

24.



Julian takes a 10- by 10-inch square piece of paper and cuts it in half along the diagonal. He then takes one of the halves and cuts it in half again from the corner to the midpoint of the opposite side. All cuts are represented in the figure with dotted lines. What is the perimeter of one of the smallest triangles, in inches?

(A) 10
(B) $10\sqrt{2}$

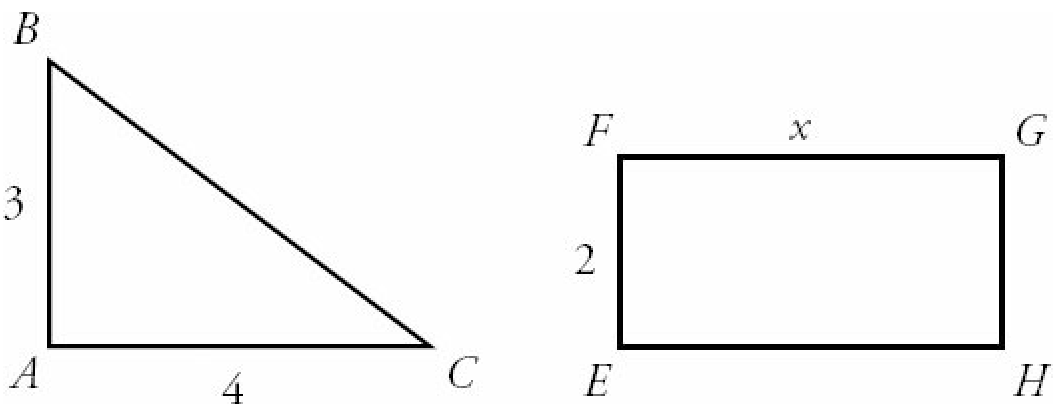
(C) 20

(D) 10 + $10\sqrt{2}$

(E) 10 + $20\sqrt{2}$

Mixed Geometry Answers

1. **4.**



Triangle *ABC* is a right triangle, so you can find the length of hypotenuse *BC*. This is a 3–4–5 triangle, so the length of side *BC* is 5. That means the perimeter of Triangle *ABC* is 3 + 4 + 5 = 12.

That means the perimeter of Rectangle *EFGH* is also 12. That means that $2 \times (2 + x) = 12$.
So $4 + 2x = 12 \rightarrow 2x = 8 \rightarrow x = 4$.

2. **6.** The area of this circle is 36π and the area of any circle is πr^2, so the radius of this circle is 6. Label both radii (*OA* and *OD*) as 6. Because *ABCO* is a rectangle, its area is base times height, where radius *OA* is the height.

Area of a rectangle = bh
$72 = b(6)$
$b = 12$

Since *OC* is a base of the rectangle, it is equal to 12. Subtract radius *OD* from base *OC* to get the length of segment *DC*: 12 - 6 = 6.

3. **(A).** This problem does not actually require any special formulas regarding circles. Calculate the distance between (5, -2) and (5, 7). Since the x-coordinates are the same and 7 - (-2) = 9, the two points are 9 apart. Because (5, 7) is outside the circle, the radius must be less than 9.

Similarly, (1, -2) is inside the circle. Calculate the distance between (5, -2) and (1, -2). Since the y-coordinates are the same, the distance is 5 - 1 = 4. Because (1, -2) is inside the circle, the radius must be more than 4.

The radius must be an integer that is greater than 4 and less than 9, so it can only be 5, 6, 7, or 8. Thus, there are 4 possible values for r.

4. **(C).** The perimeter of a square is $4s$ and the area of a square is s^2 (where s is a side length). If the square's perimeter equals its area, set the two expressions equal to each other and solve:

$4s = s^2$
$0 = s^2 - 4s$
$0 = s(s - 4)$
$s = 4$ or 0

Only $s = 4$ would result in an actual square, so $s = 0$ is not a valid solution.

The circumference of a circle is $2\pi r$ and the area of a circle is πr^2 (where r is the radius). If the circle's circumference equals its area, set the two expressions equal to each other and solve:

$2\pi r = \pi r^2$
$2r = r^2$
$0 = r^2 - 2r$
$0 = r(r - 2)$
$r = 2$ or 0

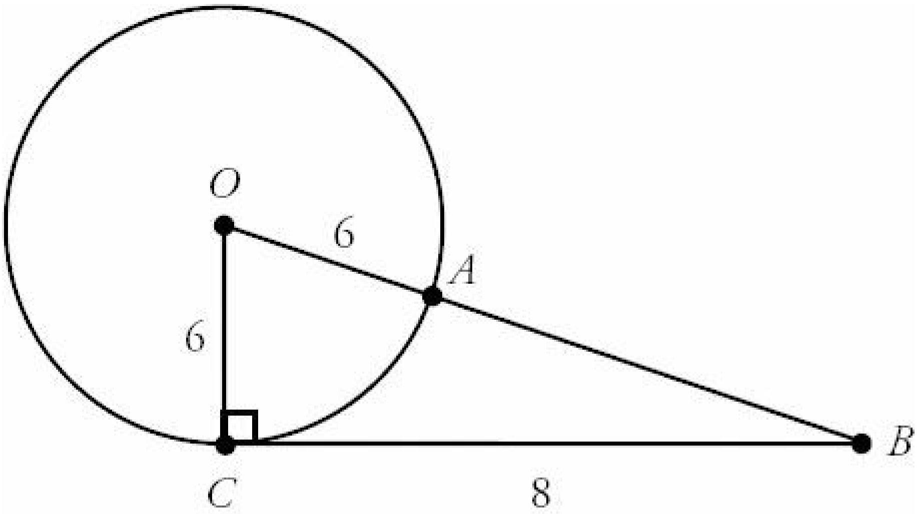
Only $r = 2$ would result in an actual circle, so $r = 0$ is not a valid solution.

If the radius of the circle is 2, then the diameter is 4. Thus, the two quantities are each equal to 4.

5. **(B).** Because BC is tangent to the circle, angle OCB is a right angle. Thus, radius OC is the height of the triangle. If the area of the triangle is 24, use the area formula for a triangle (and 8 as the base, from the figure) to determine the height:

$$A = \frac{bh}{2}$$
$$24 = \frac{8 \times OC}{2}$$
$$48 = 8 \times OC$$
$$6 = OC$$

Thus, the radius of the circle is 6 (note that you have TWO radii on the diagram, OC and OA). Since you now have two sides of a right triangle, use the Pythagorean theorem to find the third:



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$6^2 + 8^2 = (OB)^2$
$36 + 64 = (OB)^2$
$100 = (OB)^2$
$10 = OB$

(Of course, the 6–8–10 triangle is one of the special right triangles you should memorize for the GRE!)

Since the hypotenuse *OB* is equal to 10 and the radius *OA* is equal to 6, subtract to get the length of *AB*. The answer is 10 - 6 = 4.

6. **(A).** The area of the circle = $25\pi = \pi r^2$, so the radius is 5 and therefore the diameter of the circle is 10. The diameter of the circle is equal to the side of the square (the circle and square are "equally tall"), so the area of the square is $10 \times 10 = 100$.

Alternatively, the area of the circle is 25π, which is approximately 25(3.14), or greater than 75. The square is clearly larger than the circle, so the area of the square is greater than 75, which is greater than 50.

7. **4.** If the square has area 16, its sides equal 4. If the square is 4 "tall," so is the circle. That is, the side of the square is equal to the diameter of the circle. Since the diameter of the circle is 4, the radius is 2. For the circle, area $A = \pi r^2 = \pi(2)^2$ or 4π. Since the question asks "how many π?" and π is already written next to the box, type only 4 in the box.

8. **(B).** If the area of the square is 50, the sides of the square are $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$.

If the square is $5\sqrt{2}$ "tall", so is the circle. That is, the side of the square is equal to the diameter of the circle. Since the circle diameter is $5\sqrt{2}$, the radius is $\frac{5\sqrt{2}}{2}$. Using the formula for the area of a circle, $A = \pi r^2$:

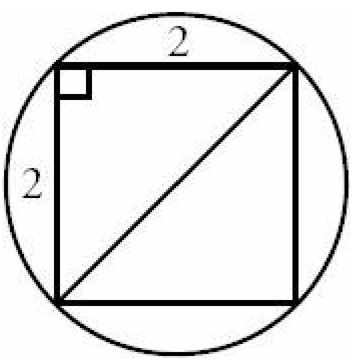
$$A = \pi\left(\frac{5\sqrt{2}}{2}\right)^2$$

$$A = \pi\left(\frac{25 \times 2}{4}\right)$$

$$A = \frac{25\pi}{2}$$

Note that even if you got a bit lost in the math, you could estimate quite reliably! The square is clearly a bit larger than the circle, so the circle area should be a bit less than 50. Put all the answers in your calculator, using 3.14 as an approximate value for π, and you will quickly see that choice (A) = 19.625, which is too small, and choice (B) = 39.25, while the other three choices are much too large (larger than the square!)

9. **(B).** If the area of the square is 4, then the side length is 2. To find the area of the circle, you need the circle's radius, which is not obvious yet. Draw a diagonal in the square—this line segment is also a diameter of the circle. Then use the Pythagorean theorem (or the 45–45–90 angle formula) to find the diagonal length:



$2^2 + 2^2 = d^2$ (where d is the diagonal of the square and the diameter of the circle)
$8 = d^2$
$\sqrt{8} = d$

Because $\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$ is the diameter of the circle, the radius is $1\sqrt{2}$ or just $\sqrt{2}$. The area of the circle is:

$A = \pi r^2$
$A = \pi(\sqrt{2})^2$
$A = 2\pi$

Note that even if you got a bit lost in this problem, you could just use common sense to estimate. The circle is obviously larger than the square, so the answer should be somewhat larger than 4. Plug in $\pi = 3.14$ using your calculator to see which choices are reasonable. Choice (A) is too small. Choice (B) is about 6.28. Choice (C) is *twice* as big, and (D) and (E) are even larger. Only (B) is reasonable.

10. **50.** One way to solve this problem is by using the Pythagorean theorem. All sides of a square are equal to s, so:

$s^2 + s^2 = 10^2$
$2s^2 = 100$
$s^2 = 50$

Note that you *could* solve for s ($s = \sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$), but the area of the square is s^2, which is already calculated above. The area of the square is 50.

11. **(B).** Each of the shaded triangles is a 3–4–5 Pythagorean triple. (Or, just note that each shaded triangle has legs of 3 and 4; the Pythagorean theorem will tell you that each hypotenuse = 5).

Since each hypotenuse is also a side of the square, the square has area $5 \times 5 = 25$.

The larger square (the overall figure) has area $7 \times 7 = 49$.

Subtract to find the area of the shaded region: 49 - 25 = 24.

The fraction of the larger square that is shaded is therefore 24/49.

12. **(C).** When two parallel lines are cut by a transversal, same-side interior angles are supplementary. Thus, $x + y = 180$, and $x = 180 - y$.

13. **(A).** While the exact measures of any of the angles are not given, when parallel lines are cut by a transversal, only two angle measures are created: all the "big" angles are the same, and all the "small" angles are the same. Since $a > 90$ (i.e., the picture is, indeed, the way it looks) and $a = c = e = g$, infer that a, c, e, and g are all the same "big" angle measure, which is greater than 90.

Similarly, $b = d = f = h$, so these are the same "small" angle measure, which is less than 90.

Quantity A is the sum of two "big" angles and one "small."
Quantity B is the sum of one "big" angle and two "small."
Quantity A is greater.

If you wish to try this with real numbers, plug in $a = 100$ (for example), and you will see that a, c, e, and g are all equal to 100, and b, d, f, and h are all equal to 80, so Quantity A would equal 280 and Quantity B would equal 260. For any example with $a > 90$, Quantity A will be larger.

14. **290.** Angles that "go around in a circle" sum to 360 degrees. It may be tempting to simply subtract 35 from 360 and answer 325, but don't overlook the unlabeled angle, which is opposite and therefore equal to 35°. So, subtract 35 + 35 = 70 from 360 to get the answer, 290.

15. **(A).** In the right isoceles triangle, the base and height are the perpendicular sides, which are each of length f. Thus, Area = $\frac{f^2}{2}$. The square, of course, has area s^2. Thus, $\frac{f^2}{2} = s^2$, or $f^2 = 2s^2$. Since f and s are definitely positive, f is larger than s.

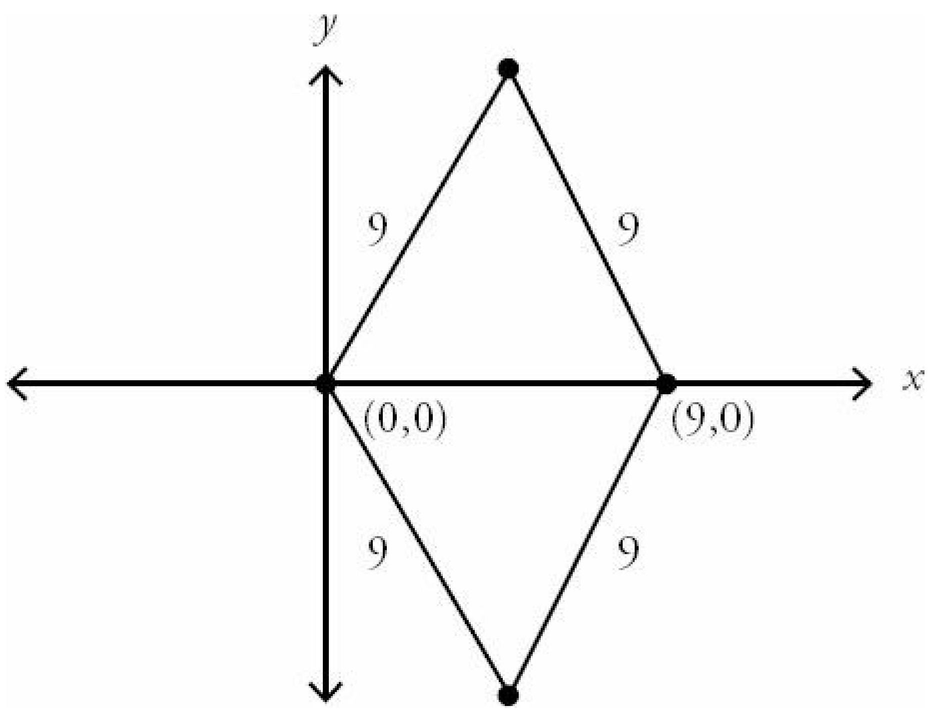
Alternatively, you could envision that if the areas were equal, the triangle sides would have to be much longer. After all, an isosceles right triangle is only half of a square.

16. **(C).** If sector *OAB* is a quarter-circle, then the angle at *O* measures 90°. Thus, since angles that make up a straight line must sum to 180, $2x + 3x + x$ must sum to 90:

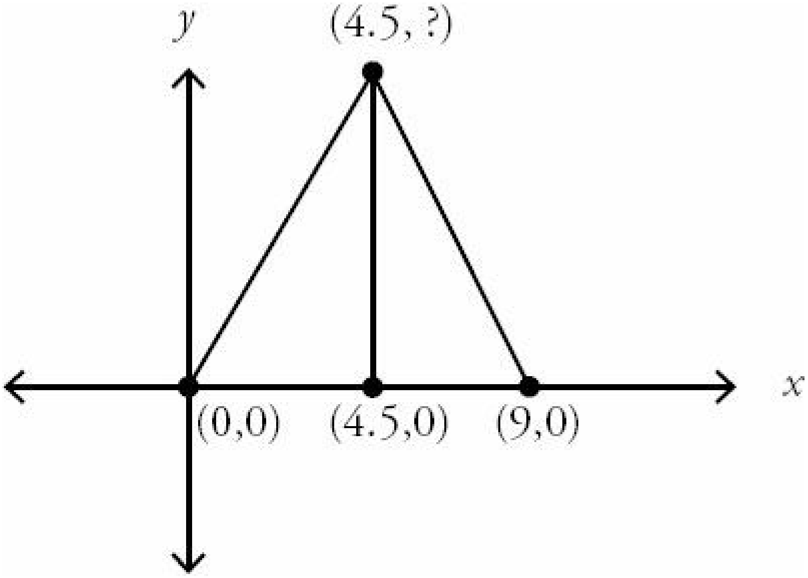
$$2x + 3x + x = 90$$
$$6x = 90$$
$$x = 15$$

The two quantities are equal.

17. **(E).** Given the two vertices at (0, 0) and (9, 0), there are only two possible locations for the third coordinate: (9,0)



All of the answer choices are positive, so focus on the "upper" option. Since equilateral triangles are symmetrical, the *x*-coordinate of the third vertex will be halfway between 0 and 9, or 4.5. Thus, the answer must be (B), (D), or (E). Draw the height of the triangle:



The height of an equilateral triangle is given by $h = \frac{s\sqrt{3}}{2}$. Alternatively, note that the height cuts the 60–60–60 triangle into two 30–60–90 triangles. Use the side length ratios for a 30–60–90 triangle (1 : $\sqrt{3}$: 2) to determine that, since the side across from the 30-degree angle is equal to 4.5, the side across from the 60-degree angle will be equal to 4.5 $\sqrt{3}$ or $\frac{9\sqrt{3}}{2}$.

This height is the *y*-coordinate of the third vertex. The answer is (4.5, $\frac{9\sqrt{3}}{2}$).

18. **(E).** An exterior angle of a triangle is equal to the sum of the two opposite interior angles. From the left triangle, $a = x + 90$. From the right triangle, $b = (x + 10) + (y + 5) = x + y + 15$.

Alternatively, you could use the facts that the interior angles of a triangle sum to 180, as do angles that form a straight line. From the left triangle, $x + y + 90$ and $y + a$ both equal 180, so $x + y + 90 = y + a$, or $x + 90 = a$. From the right triangle, $180 = (x + 10) + (y + 5) + (180 - b)$, or $b = x + y + 15$.

The question asks for *a* in terms of *b* and *y*, so *x* is the variable that needs to be eliminated. Do so by solving one equation for *x*, and substituting this expression for *x* in the other equation.

From the right triangle: $b = x + y + \longrightarrow x = b - y - 15$

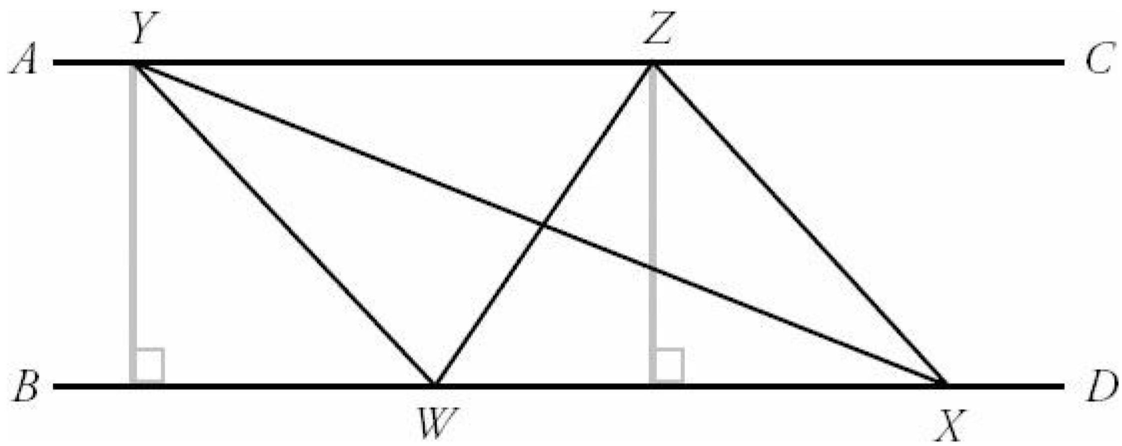
From the left triangle:

$a = x + 90$
$a = (b - y - 15) + 90$
$a = b - y + 75$

19. **(C).** Both triangles, *WYX* and *WZX*, share a common base of segment *WX*. As the area of a triangle is given by

Area = (1/2)(base)(height)

and both triangles have equal bases, you can determine which has a greater area by determining which has a greater height. The height is a perpendicular line drawn from the highest point on the triangle to the base. In this case, the heights would be given in gray below:



By the definition of parallel lines, *AC* and *BD* are uniform distance apart. Therefore, the heights shown are the same. Because these triangles have equal bases and heights, they must have equal area.

20. **(C).** To solve this problem, recall that a triangle inscribed in a semi-circle will be a right triangle *if and only if* one side of the triangle is the diameter (i.e., the center of the circle must lie on one side of the triangle). Because this is a right triangle, the hypotenuse must be the diameter of the circle.

To find the diameter of the circle, recall the formula for area: Area = πr^2.

$16\pi \text{ cm}^2 = \pi r^2$
$16 \text{ cm}^2 = r^2$
$r = 4$ cm

Given that diameter is twice the radius, the diameter (i.e., the hypotenuse of the triangle) is 8 cm. Quantity A is 8, making the two quantities equal.

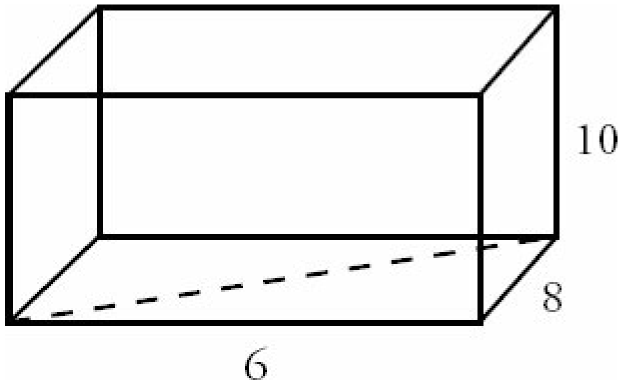
21. **(C).** The fastest approach to solving this problem is to use the "Super Pythagorean Theorem," which states that the diagonal of any rectangular box is given by:

$d^2 = l^2 + w^2 + h^2$

Where *l*, *w*, and *h* are the length, width, and height of the box, respectively. Plugging in yields

$d^2 = 6^2 + 8^2 + 10^2$
$d^2 = 36 + 64 + 100$
$d^2 = 200$
$d = 10\sqrt{2}$

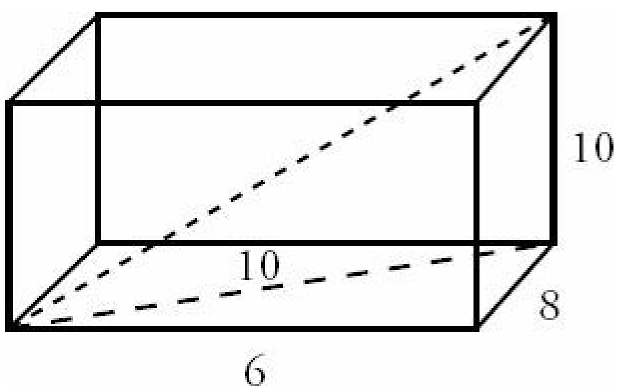
Alternatively, one could avoid the Super Pythagorean Theorem by applying the normal Pythagorean theorem twice. To find the diagonal of the box, you must first find the diagonal of one of the sides. Choosing this side as the base,



where the dashed line represents the diagonal of the base. Applying the Pythagorean theorem:

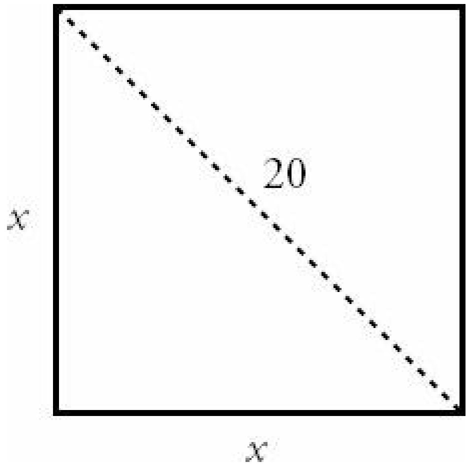
$c^2 = a^2 + b^2$
$c^2 = 6^2 + 8^2$
$c^2 = 36 + 64$
$c^2 = 100$
$c = 10$

From here, draw the diagonal of the box and apply the Pythagorean theorem again as shown.



$d^2 = 10^2 + 10^2$
$d^2 = 100 + 100$
$d^2 = 200$
$d = 10\sqrt{2}$

22. **(D).** Draw the following figure to represent the square garden.



Label the diagonal with the given length of 20 and the sides with the variable x. The diagonal forms the hypotenuse of a right triangle with legs of length x. Using Pythagorean theorem, solve for the length of the legs as

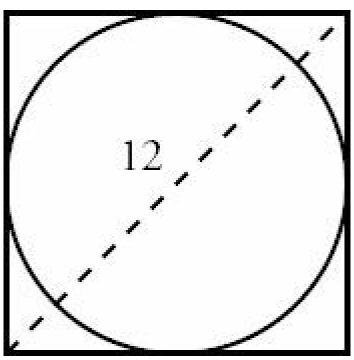
$$x^2 + x^2 = 20^2$$
$$2x^2 = 400$$
$$x^2 = 200$$
$$x = 10\sqrt{2}$$

The perimeter of a square is 4 × (side length).

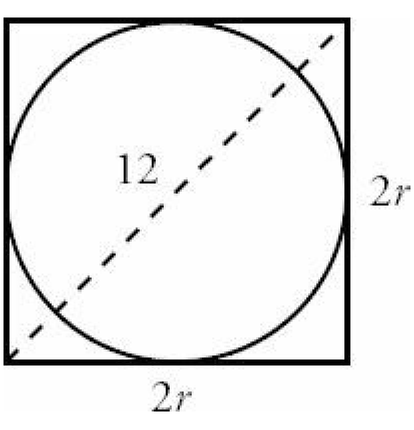
Perimeter = $4 \times (10\sqrt{2})$

Perimeter = $40\sqrt{2}$

23. **(A).** Begin by diagramming the figure, labeling the diagonal of the circle as 12.



From here, recognize that the square is as "tall" as the circle, or the side length of the square equals the diameter of the circle, which is $2r$.



By the Pythagorean theorem:

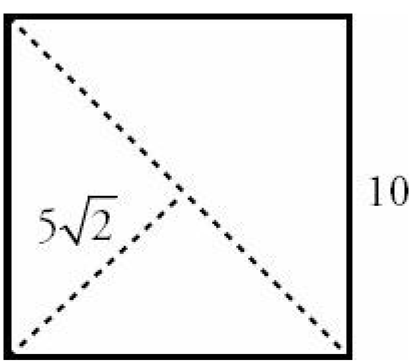
$$(2r)^2 + (2r)^2 = 12^2$$
$$4r^2 + 4r^2 = 144$$
$$8r^2 = 144$$
$$r^2 = 18$$
$$r = 3\sqrt{2}$$

24. **(D).** In order to compute the perimeter of one of the smaller triangles, first compute the length of the diagonal. For a square with side length 10 inches, the length of the diagonal can be computed by the Pythagorean theorem:

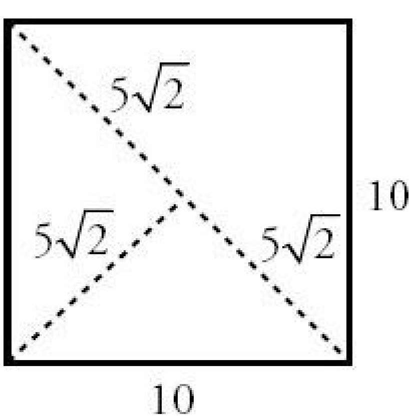
$$(\text{diagonal})^2 = (\text{side})^2 + (\text{side})^2$$
$$(\text{diagonal})^2 = 10^2 + 10^2$$
$$(\text{diagonal})^2 = 200$$
$$\text{diagonal} = 10\sqrt{2}$$

Alternatively, recognize that the diagonal of a square is always $\sqrt{2}$ times the side length.

The second cut goes from the corner to the midpoint of the diagonal, so that slice is half as long as the diagonal of the square: $\frac{10\sqrt{2}}{2} = 5\sqrt{2}$. This can be seen as



Similarly, because the remaining line in each of the smaller triangles is half of a diagonal, each is of length $5\sqrt{2}$ inches.



Adding up the lengths of the sides, the perimeter of the smallest triangle is

Perimeter = $10 + 5\sqrt{2} + 5\sqrt{2}$

Perimeter = $10 + 10\sqrt{2}$

Chapter 30

of

Advanced Quant

In This Chapter…

Advanced Quant

The following questions are *extremely* advanced for the GRE. We have included them by popular demand — students who are aiming for perfect math GRE scores often wish to practice on problems that may even be harder than any they see on the real GRE. We estimate that a GRE test taker who does well on the e first math section and therefore is given a difficult second section might see one or two problems, at most, of this level of difficulty.

If you are NOT aiming for a perfect math score, we absolutely recommend that you skip these problems!

If you are taking the GRE for business school or another quantitative program, you may wish to attempt some of these problems. For instance, you might do one or two of these problems — think of them as "brain teasers" — to cap off a study session from elsewhere in the book. (For reference, getting 50% of these problems correct would be a pretty incredible performance!)

Even if you *are* aiming for a perfect math score, though, please make sure you are *flawless* at the types of math problems in the *rest* of this book before you work on these. You will gain far more points by reducing silly mistakes (through practice, steady pacing, and good organization) on easy and medium questions than by focusing on ultra-hard questions.

For more such problems, visit the Manhattan Prep GRE blog for our weekly Challenge Problem. (Access to the archive of over 100 Challenge Problems is available to our course and Guided Self-Study students for free and to the public for a small fee.)

That said, attempt these Advanced Quant problems — if you dare!

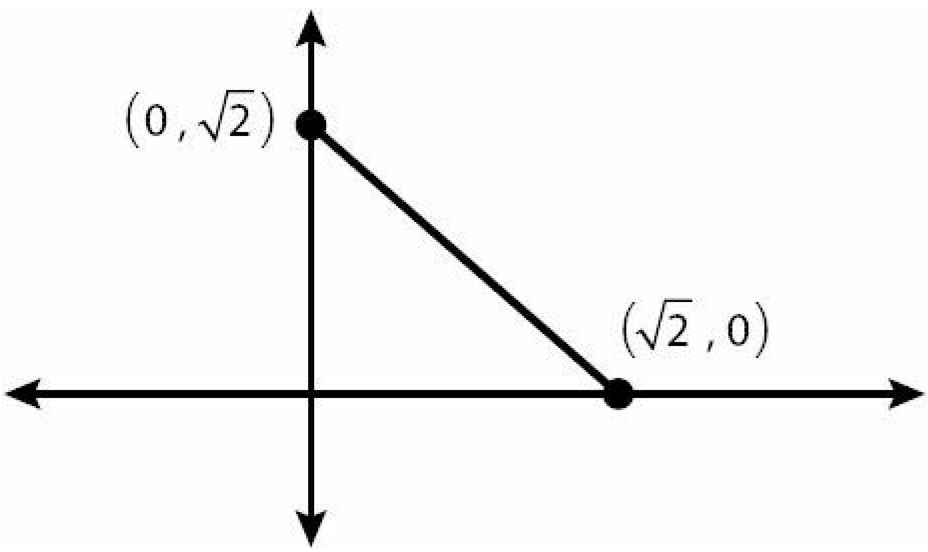
1. The probability of rolling any number on a weighted 6-sided die, with faces numbered 1 through 6, is directly proportional to the number rolled. What is the expected value of a single roll of the die?

   (A) $4\frac{1}{6}$
   (B) $4\frac{1}{3}$
   (C) $4\frac{1}{2}$
   (D) $4\frac{2}{3}$
   (E) $4\frac{5}{6}$

2. 21 people per minute enter a previously empty train station beginning at 7:00:00 p.m. (7 o'clock and zero seconds). Every 9 minutes beginning at 7:04:00 p.m., a train comes and everyone who has entered the station in the last 9 minutes gets on the train. If the last train comes at 8:25:00, what is the average number of people who get on each of the trains leaving from 7:00:00 to 8:25:00?

(A) 84
(B) 136.5
(C) 178.5
(D) 189
(E) 198.5

3. The random variable X has the following continuous probability distribution in the range $0 \leq X \leq \sqrt{2}$, as shown in the coordinate plane with X on the horizontal axis:



The probability that $X < 0$ = the probability that $X > \sqrt{2} = 0$.

What is the median of X?

(A) $\frac{\sqrt{2}-1}{2}$
(B) $\frac{\sqrt{2}}{4}$
(C) $\sqrt{2}-1$
(D) $\frac{\sqrt{2}+1}{4}$
(E) $\frac{\sqrt{2}}{2}$

4.

x < 0

Quantity A	**Quantity B**
$x^2 - 5x + 6$	$x^2 - 9x + 20$

5. If x is a positive integer, what is the units digit of $(24)^{5+2x}(36)^6(17)^3$?

(A) 2
(B) 3
(C) 4
(D) 6
(E) 8

6. A rectangular solid is changed such that the width and length are increased by 1 inch apiece and the height is decreased by 9 inches. Despite these changes, the new rectangular solid has the same volume as the original rectangular solid. If the width and length of the original rectangular solid are equal and the height of the new rectangular solid is 4 times the width of the original rectangular solid, what is the volume of the rectangular solid?

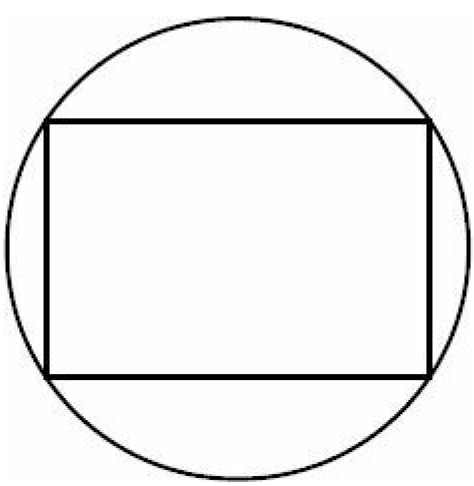
(A) 18
(B) 50
(C) 100
(D) 200
(E) 400

7. The sum of all solutions for x in the equation $x^2 - 8x + 21 = |x - 4| + 5$ is equal to:

(A) -7
(B) 7
(C) 10
(D) 12
(E) 14

8. In the figure shown, the circumference of the circle is 10π. Which of the following is NOT a possible value for the area of the rectangle?

(A) 30
(B) 40
(C) $20\sqrt{2}$
(D) $30\sqrt{2}$
(E) $40\sqrt{2}$



9. The length of one edge of a cube equals 4. What is the distance between the center of the cube and one of its

vertices?

(A) 2
(B) $2\sqrt{2}$
(C) $2\sqrt{3}$
(D) $4\sqrt{2}$
(E) $4\sqrt{3}$

10. If c is randomly chosen from the integers 20 to 99, inclusive, what is the probability that $c^3 - c$ is divisible by 12?

11. If x and y are positive integers greater than 1 such that $x - y$ and x/y are both even integers, which of the following numbers must be non-prime integers?

Indicate <u>all</u> such statements.

☐ x
☐ $x + y$
☐ y/x

12. The remainder when 120 is divided by single-digit integer m is positive, as is the remainder when 120 is divided by single-digit integer n. If $m > n$, what is the remainder when 120 is divided by $m - n$?

13. A circular microchip with a radius of 2.5 centimeters is manufactured following a blueprint scaled such that a measurement of 1 centimeter on the blueprint corresponds to a measurement of 0.05 millimeters on the microchip. What is the area of the blueprint, in square centimeters? (1 centimeter = 10 millimeters)

π

14.

For a certain quantity of a gas, pressure P, volume V, and temperature T are related according to the formula $PV = kT$, where k is a constant.

Quantity A	**Quantity B**
The value of P if $V = 20$ and $T = 32$	The value of T if $V = 10$ and $P = 78$

15.

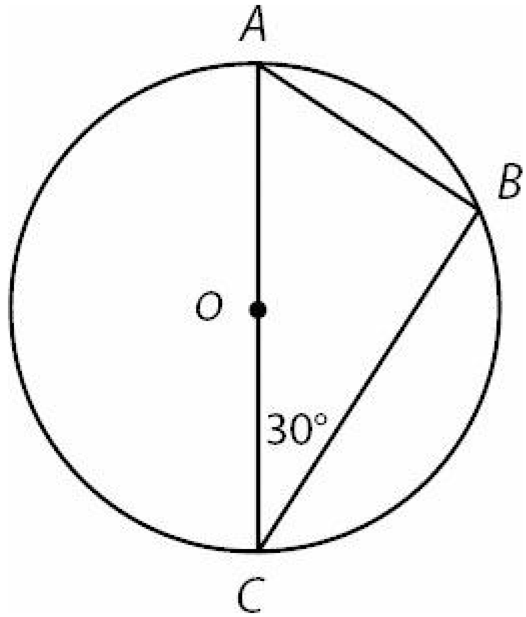


Figure not drawn to scale.

The circle with center O has a circumference of $6\pi\sqrt{3}$. If AC is a diameter of the circle, what is the length of line segment BC?

(A) $\frac{3}{\sqrt{2}}$
(B) 3
(C) $3\sqrt{3}$
(D) 9
(E) $9\sqrt{3}$

16. A batch of widgets costs $p + 15$ dollars for a company to produce and each batch sells for $p(9 - p)$ dollars. For which of the following values of p does the company make a profit?

(A) 3
(B) 4
(C) 5
(D) 6
(E) 7

17. If K is the sum of the reciprocals of the consecutive integers from 41 to 60 inclusive, which of the following is less than K?

Indicate <u>all</u> such statements.

- ☐ 1/4
- ☐ 1/3
- ☐ 1/2

18. Triplets Adam, Bruce, and Charlie enter a triathlon. There are nine competitors in the triathlon. If every competitor has an equal chance of winning, and three medals will be awarded, what is the probability that at least two of the triplets will win a medal?

(A) 3/14
(B) 19/84
(C) 11/42
(D) 15/28
(E) 3/4

19. The expression $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}}$ extends to an infinite number of roots. Which of the following choices most closely approximates the value of this expression?

(A) $\sqrt{3}$
(B) 2
(C) $1+\sqrt{2}$
(D) $1+\sqrt{3}$
(E) $2\sqrt{3}$

20. Half an hour after Car A started traveling from Newtown to Oldtown, a distance of 62 miles, Car B started traveling along the same road from Oldtown to Newtown. The cars met each other on the road 15 minutes after Car B started its trip. If Car A traveled at a constant rate that was 8 miles per hour greater than Car B's constant rate, how many miles had Car B driven when they met?

(A) 14
(B) 12
(C) 10
(D) 9
(E) 8

21.

x and y are positive integers such that $x^2 5^y = 10{,}125$

Quantity A	**Quantity B**
x^2	5^y

22. If $x = 2^b - (8^8 + 8^6)$, for which of the following b values is x closest to zero?

(A) 20
(B) 24
(C) 25
(D) 30
(E) 42

23. If $k > 1$, which of the following must be equal to $\dfrac{2}{\sqrt{k+1}+\sqrt{k-1}}$?

(A) 2
(B) $2\sqrt{2k}$
(C) $2\sqrt{k+1}+\sqrt{k-1}$
(D) $\dfrac{\sqrt{k+1}}{\sqrt{k-1}}$
(E) $\sqrt{k+1}-\sqrt{k-1}$

24. Bank account A contains exactly x dollars, an amount that will decrease by 10% each month for the next two months. Bank account B contains exactly y dollars, an amount that will increase by 20% each month for the next two months. If A and B contain the same amount at the end of two months, what is the ratio of $\sqrt{x}$ to $\sqrt{y}$?

(A) 4 : 3
(B) 3 : 2
(C) 16 : 9
(D) 2 : 1
(E) 9 : 4

25. Let a be the sum of x consecutive positive integers. Let b be the sum of y consecutive positive integers. For which of the following values of x and y is it NOT possible that $a = b$?

(A) $x = 2; y = 6$
(B) $x = 3; y = 6$
(C) $x = 6; y = 4$
(D) $x = 6; y = 7$
(E) $x = 7; y = 5$

26.

Body Mass Index (BMI) is calculated by the formula $\dfrac{703w}{h^2}$, where w is weight in pounds and h is height in inches. (12 inches = 1 foot.)

Quantity A	**Quantity B**
The number of pounds gained by a 6 foot, 2 inch tall person whose BMI increased by 1.0.	The number of pounds lost by a 5 foot, 5 inch tall person whose BMI decreased by 1.2.

27. Bag A contains 3 white and 3 red marbles. Bag B contains 6 white and 3 red marbles. One of the two bags will be chosen at random, and then two marbles will be drawn from that bag at random without replacement. What is the probability that the two marbles drawn will be the same color?

(A) 7/20
(B) 9/10
(C) 9/20
(D) 11/20
(E) 13/20

28. How many positive four-digit integers contain the digit grouping “62” (in that order) at least once? For instance,

2628 and 6244 are two such integers to include, but 2268 and 5602 do not meet the restrictions.

(A) 180
(B) 190
(C) 279
(D) 280
(E) 360

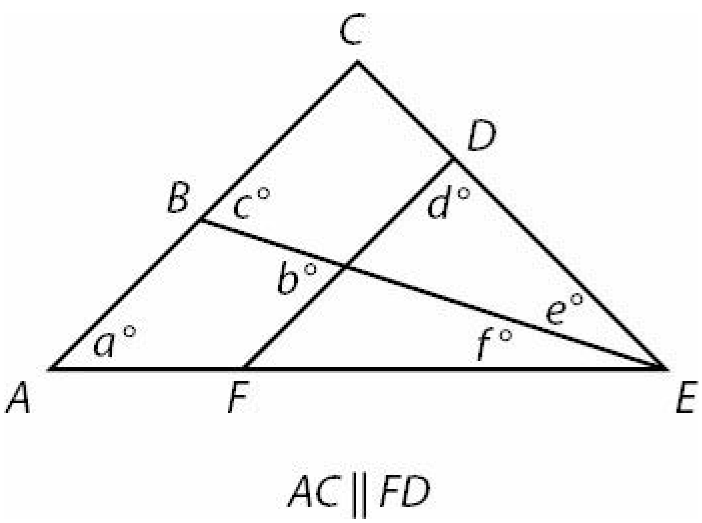
29. How many 5 digit numbers that are divisible by 9 can be formed using the digits 0, 1, 2, 4, 5, 6 if repeats are not allowed?

(A) 66
(B) 120
(C) 360
(D) 488
(E) 720

30.

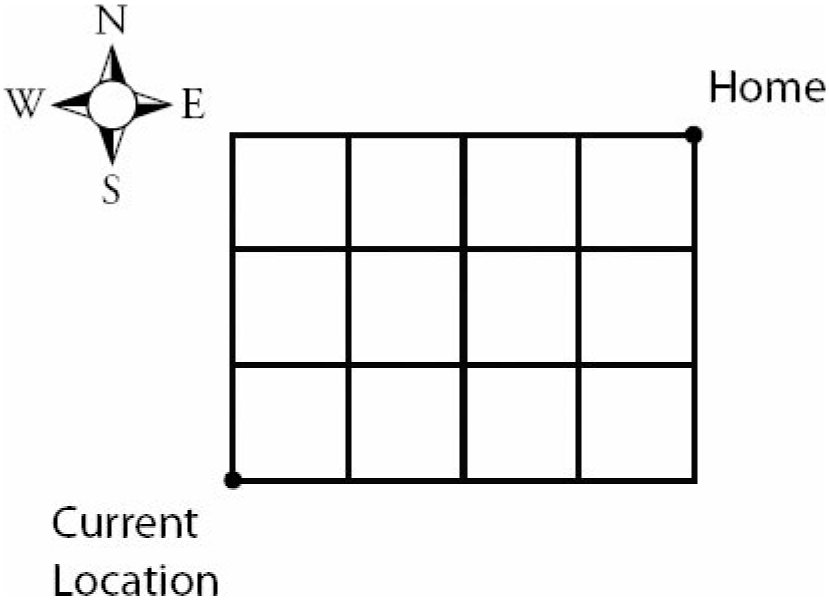
Quantity A	**Quantity B**
The average of all the multiples of 3 between 101 and 598	The average of all the multiples of 4 between 101 and 598

31.



Quantity A	**Quantity B**
$a + d - c - 90$	$90 - e - b - f$

32.



A man walks to his home from his current location on the rectangular grid shown. If he may choose to walk north or east at any corner, but may never move south or west, how many different paths can the man take to get home?

(A) 12
(B) 24
(C) 32
(D) 35
(E) 64

33. A bag contains 3 white, 4 black, and 2 red marbles. Two marbles are drawn from the bag. What is the probability that the second ball drawn will be red if replacement is NOT allowed?

(A) 1/36
(B) 1/12
(C) 7/36
(D) 2/9
(E) 7/9

34.

$x < 0$

Quantity A	**Quantity B**
$\left(\left(25^{x}\right)^{-2}\right)^{3}$	$\left(\left(5^{-3}\right)^{2}\right)^{-x}$

35.

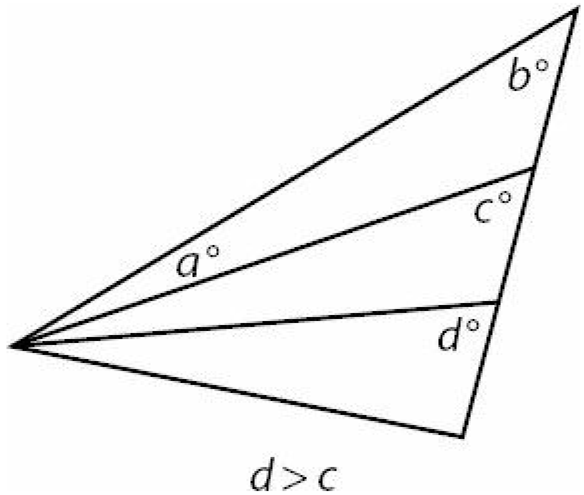
Quantity A	**Quantity B**
The sum of the multiples of 3 between -93 and 252, inclusive	9,162

36.

x is an integer

Quantity A	Quantity B
$(-1)^{x^2} + (-1)^{x^3} + (-1)^{x^4}$	$(-1)^{x} + (-1)^{2x} + (-1)^{3x} + (-1)^{4x}$

37.



Quantity A	Quantity B
a	$d - b$

38.

Sequence S is such that $S_n = S_{n-1} + \frac{5}{2}$ and $S_1 = 1$
Sequence A is such that $A_n = A_{n-1} - 2.5$ and $A_1 = 36$

Quantity A	Quantity B
The sum of the terms in S from S_1 to S_{14}, inclusive	The sum of the terms in A from A_1 to A_{14}, inclusive

39.

Quantity A	Quantity B
$\dfrac{a^{64} - 1}{(a^{32}+1)(a^{16}+1)(a^{8}+1)^{2}}$	1

40.

The circumference of a circle is 7/8 the perimeter of a square.

Quantity A	Quantity B
The area of the square	The area of the circle

41.

Per Serving of:	Calories	Cost
Snack A	320	$1.50
Snack B	110	$0.45

Choosing from the snacks in the table above, a group of people consumes 2,370 calories of snacks that cost a total of $10.65.

Quantity A	Quantity B
The number of servings of Snack A the group consumed	5

42.

$a_1, a_2, a_3, \ldots, a_n,$

… In the sequence above, each term after the first is equal to the average of the preceding term and the following term.

Quantity A	Quantity B
$a_{51} - a_{48}$	$a_{37} - a_{34}$

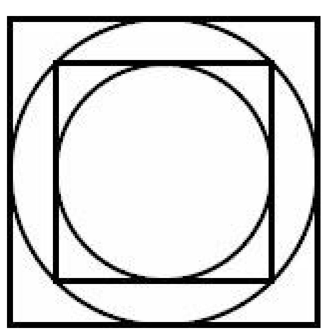
43.

The greatest common factor of $12x$ and $35y$ is $5y$

x and y are positive integers

Quantity A	Quantity B
The remainder when $12x$ is divided by 10	The greatest common factor of x and y

44.



Quantity A	Quantity B
The ratio of the area of the larger square to the area of the smaller square	Twice the ratio of the area of the smaller circle to the area of the larger circle

45.

$$m = 2^{16}3^{17}4^{18}5^{19}$$
$$n = 2^{19}3^{18}4^{17}5^{16}$$

Quantity A	**Quantity B**
When integer m is multiplied out, the number of zeroes at the end of m	When integer n is multiplied out, the number of zeroes at the end of n

46.

The sequence of numbers $a_1, a_2, a_3, \ldots, a_n, \ldots$ is defined by $a_n = 2^n - \frac{1}{2^{n-33}}$ for each integer $n \geq 1$.

Quantity A	**Quantity B**
The sum of the first 32 terms of this sequence	The sum of the first 31 terms of this sequence

47. Set $S = \{-1, 4, 30, -21\}$. If the mean = 4, then the standard deviation, rounded to the nearest tenth, is equal to

$\boxed{}$

48. Each of 100 balls has an integer value from 1 to 8, inclusive, painted on the side. The number n_x of balls representing integer x is given by the formula $n_x = 18 - (x - 4)^2$. The interquartile range of the 100 integers is

(A) 1.5
(B) 2.0
(C) 2.5
(D) 3.0
(E) 3.5

49. When x is divided by 13 the answer is y with a remainder of 3. When x is divided by 7 the answer is z with a remainder of 3. If x, y, and z are all positive integers, what is the remainder of $\frac{yz}{13}$?

(A) 0
(B) 3
(C) 4
(D) 7
(E) 10

50. In a certain sequence, the term a_n is defined as the value of x that satisfies the equation $2 = (x/2) - a_{n-1}$. If $a_6 = 156$, what is the value of a_2?

(A) 1
(B) 6
(C) 16

(D) 26
(E) 106

51. The operator ! is defined such that $a!b = a^b \times b^{-a}$

Quantity A	Quantity B
$(x!4) \div (4!x)$	$\frac{x^8}{16^x}$

52. What is the ratio of the sum of the even positive integers between 1 and 100 (inclusive) and the sum of the odd positive integers between 100 and 150?

(A) 102 to 125
(B) 50 to 51
(C) 51 to 56
(D) 202 to 251
(E) 2 to 3

53. For integer $n \geq 3$, a sequence is defined as $a_n = a_{n-1}^2 - a_{n-2}^2$ and $a_n > 0$ for all positive integer n. The first term, a_1, is 2, and the fourth term is equal to the first term multiplied by the sum of the second and third terms. What is the third term, a_3?

(A) 0
(B) 3
(C) 5
(D) 10
(E) 16

54. In a certain sequence, each term beyond the second term is equal to the average of the previous two terms. If a_1 and a_3 are positive integers, which of the following is NOT a possible value of a_5

(A) -9/4
(B) 0
(C) 9/4
(D) 75/8
(E) 41/2

55. The operator ? is defined by the following expression: $a?b = \left|\frac{a+1}{a}\right| - \frac{b+1}{b}$ where $ab \neq 0$. What is the sum of the solutions to the equation $x?2 = \frac{x?(-1)}{2}$?

(A) -1
(B) -0.75
(C) -0.25
(D) 0.25
(E) 0.75

56. X is a non-negative number and the square root of $(10 - 3X)$ is greater than X.

Quantity A	Quantity B
$\|X\|$	2

57. The area of an equilateral triangle is greater than $25\sqrt{3}$ but less than $36\sqrt{3}$

Quantity A	Quantity B
The length of one of the sides of the triangle	9

58. The inequality $|8 - 2x| < 3y - 9$ is equivalent to which of the following?

(A) $2x < (17 - 3y)/2$
(B) $3y + 2x > 1$
(C) $6y - 2 < 2x$
(D) $1 - y < 2x < 17 + y$
(E) $3y - 1 > 2x > 17 - 3y$

59. In the sport of mixed martial arts (MMA), more than 30% of all fighters are skilled in both the Muy Thai and Brazilian Jiu Jitsu styles of fighting. 20% of the fighters who are not skilled in Brazilian Jiu Jitsu are skilled in Muy Thai. 60% of all fighters are skilled in Brazilian Jiu Jitsu.

Quantity A	Quantity B
The percent of fighters who are skilled in Muy Thai	37%

60. The rate of data transfer, r, over a particular network is directly proportional to the bandwidth, b, and inversely proportional to the square of the number of networked computers, n.

Quantity A	Quantity B
The resulting rate of data transfer if the bandwidth is quadrupled and the number of networked computers is more than tripled	$\frac{4}{9}r$

Advanced Quant Answers

1. **(B).** First, figure out the probability of each outcome. The die has six faces, numbered 1 through 6. Since the probability of rolling any particular number is directly proportional to that number, you can write each probability with an unknown multiplier x like so:

Probability of rolling a 1 = $1x = x$
Probability of rolling a 2 = $2x$
Probability of rolling a 3 = $3x$
Probability of rolling a 4 = $4x$
Probability of rolling a 5 = $5x$
Probability of rolling a 6 = $6x$

These are the only possible outcomes, so the probabilities must sum to 1:

$x + 2x + 3x + 4x + 5x + 6x = 1$
$21x = 1$
$x = \frac{1}{21}$

Now you can find all the probabilities, since they are just multiples of x.

The expected value, or mean, of a roll of the die is found this way:

1. Multiply each outcome (1, 2, 3, 4, 5, and 6) by its corresponding probability.
2. Sum up all those products.

So the mean equals the following sum:

$$(1)\left(\frac{1}{21}\right)+(2)\left(\frac{2}{21}\right)+(3)\left(\frac{3}{21}\right)+(4)\left(\frac{4}{21}\right)+(5)\left(\frac{5}{21}\right)+(6)\left(\frac{6}{21}\right)$$
$$= (1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2)\left(\frac{1}{21}\right)$$
$$= (1 + 4 + 9 + 16 + 25 + 36)\left(\frac{1}{21}\right)$$
$$= \frac{91}{21} = \frac{13}{3} = 4\frac{1}{3}$$

2. **(C).** From 7pm to 7:04, 84 people enter the station (21 per minute). These 84 people will get on the 7:04 train.

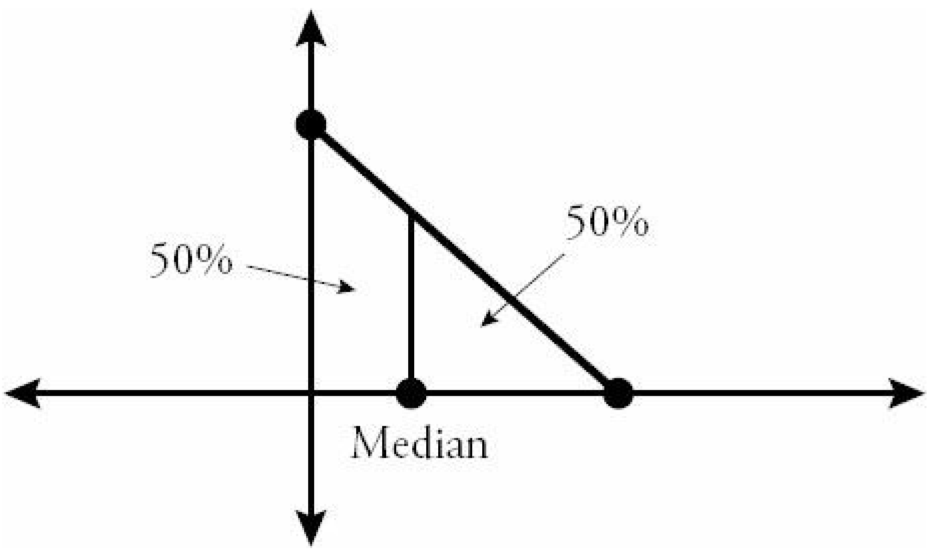
After that, for each 9 minute period, 9(21) = 189 people will enter the station and then get on a train. These trains will leave at 7:13, 7:22, 7:31, 7:40, 7:49, 7:58, 8:07, 8:16, and 8:25.

Since 9 trains each have 189 people and the first train has 84 people, the average is:

$$\frac{9(189)+1(84)}{10}=178.5$$

Note that the strange time format (minutes and seconds) doesn't make the problem any harder — the problem is actually more clear if you know that the train comes at 7:04 and zero seconds, rather than 7:04 and 30 seconds, at which point more people would have entered the station.

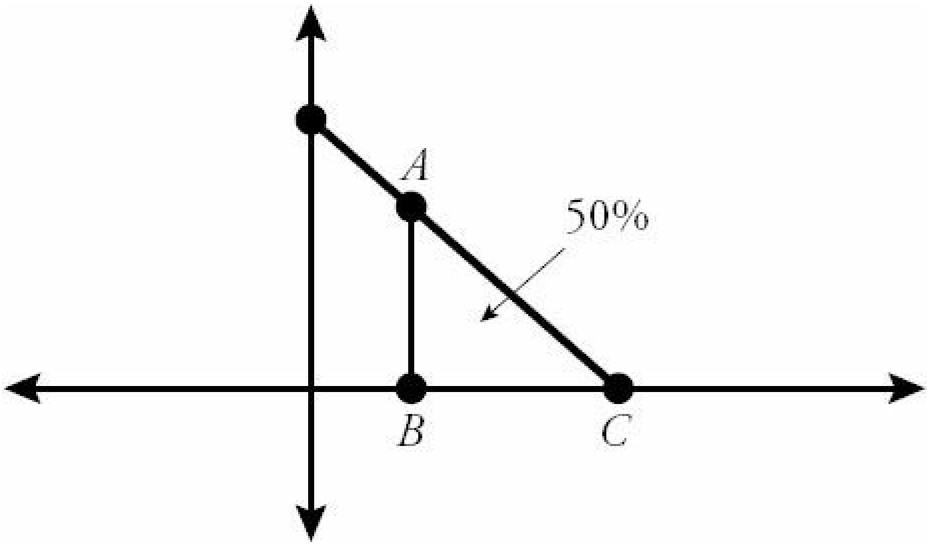
3. **(C).** A continuous probability distribution has a total area of 100%, or 1, underneath the entire curve. The median of such a distribution splits the area into two equal halves, with 50% of the area to the left of the median and the other 50% to the right of the median:



In simpler terms, the random variable X has a 50% chance of being above the median and a 50% chance of being below the median. You can ignore the regions to the right or the left of this triangle, since the probability that X could fall in either of those regions is zero. So the question becomes this: what point on the X-axis will divide the large right triangle into two equal areas?

One shortcut is to note that the area of the large isosceles right triangle must be 1, which equals the total area under any probability distribution curve. You can easily confirm this fact, though, by finding the area of this right triangle: $\frac{1}{2}bh=\frac{1}{2}\left(\sqrt{2}\right)\left(\sqrt{2}\right)=\frac{2}{2}=1$.

The quickest way to find the median is to consider the *small* isosceles right triangle, ΔABC as shown:



Triangle ABC must have an area of $\frac{1}{2}$. So what must be the length of each of its legs, AB and BC? From the formula $\frac{1}{2}bh = \frac{1}{2}$, and noting that the base BC equals the height AB, you can see that the base BC must be 1 (the same as the height). Since the coordinates of point C are ($\sqrt{2}$, 0), the coordinates of point B must be ($\sqrt{2}$, 0). That is, the median is $\sqrt{2} - 1$.

4. **(B).** One way to solve is to set up an implied equation or inequality, then make the same changes to both quantities, and finally compare after simplifying.

Quantity A		**Quantity B**	
$x^2 - 5x + 6$	?	$x^2 - 9x + 20$	
$\underline{-(x^2 - 5x + 6)}$		$\underline{-(x^2 - 5x + 6)}$	
0	?	$-9x - (-5x) + 20 - 6$	Notice that x^2 is common to both quantities, so it can be ignored (i.e. it cancels).
0	?	$-9x + 5x + 14$	
0	$<$	$-4x + 14$	

Because x is negative, $-4x + 14 = -4(neg) + 14 = pos + 14$, which is greater than 0.

Another way to solve is to factor and then compare based on number properties. Quantity A factors to$(x - 2)(x - 3)$. Quantity B factors to $(x - 4)(x - 5)$. Because x is negative, "x minus a positive number" is also negative. Each quantity is the product of two negative numbers, which is positive.

Quantity A: $(x - 2)(x - 3) = (neg)(neg) = pos$

Quantity B: $(x - 4)(x - 5) = (\text{more } neg)(\text{more } neg) = \text{more } pos$

Thus, Quantity B is larger.

5. **(A).** 24 to any power will end in the same units digit as 4 to the same power (it is always true that, if you only need the last digit of the product, you only need the last digits of the numbers being multiplied).

4 to any power ends in either 4 or 6 ($4^1 = 4$, $4^2 = 16$, $4^3 = 64$, etc.) If the power is odd, the answer will end in 4; if the power is even, the answer will end in 6.

Since the exponent "$5 + 2x$" will be odd for any integer power of *x,* 24^{5+2x} will end in 4.

36 to any power will end in the same units digit as 6 to the same power. Interestingly, powers of 6 always end in 6, so 36^6 will end in 6.

17 to any power will end in the same units digit as 7 to the same power. While the units digits of the powers of 7 do indeed create a pattern, 7^3 is just 343, which ends in 3.

Thus:

24^{5+2x} ends in 4
36^6 end in 6
7^3 ends in 3

Multiplying three numbers that end in 4, 6, and 3 will yield answer that ends in 2, because $(4)(6)(3) = 72$, which ends in 2.

6. **(E).** The old solid has:

Width = w
Length = l
Height = h

But then you are told that width and length are equal, so substitute right away to reduce the number of variables:

The old solid has:

Width = w
Length = w
Height = h

After the changes detailed in the problem, the new solid has:

Width = $w + 1$
Length = $w + 1$
Height = $h - 9$

You are then told that the old and new solids have equal volume. Since volume = length × width × height:

$w^2h = (w + 1)^2(h - 9)$

Before you get too far into simplifying this, there is one more fact yet to be considered: the height of the new solid is four times the width of the original solid. Thus:

$h - 9 = 4w$

or

$h = 4w + 9$

Substitute into both spots in $w^2h = (w + 1)^2(h - 9)$ where h appears:

$w^2(4w + 9) = (w + 1)^2(4w)$

Distribute $w^2(4w + 9)$ and FOIL $(w + 1)^2$:

$4w^3 + 9w^2 = (w + 1)^2(4w)$
$4w^3 + 9w^2 = (w^2 + 2w + 1)(4w)$
$4w^3 + 9w^2 = 4w^3 + 8w^2 + 4w$

Fortunately, you can now subtract $4w^3$ from both sides and simplify from there:

$4w^3 + 9w^2 = 4w^3 + 8w^2 + 4w$
$9w^2 = 8w^2 + 4w$
$w^2 = 4w$
$w = 4$ (since w cannot be 0)

You now need the *volume* of the original solid. The old solid has:

Width = w
Length = w
Height = h

You also know that $h = 4w + 9$.

Thus, width = 4, length = 4, and height = 4(4) + 9 = 25, and the volume of the original solid is (4)(4)(25) = 400.

7. **(D).** Since the part of the equation inside the absolute value could have a positive value (in which case the absolute value is irrelevant) or a negative one, solve the equation twice, once for each scenario:

Scenario 1:	**Scenario 2:**
$x - 4 \geq 0$	$x - 4 \leq 0$
$x^2 - 8x + 21 = \lvert x - 4\rvert + 5$	$x^2 - 8x + 21 = \lvert x - 4\rvert + 5$
$x^2 - 8x + 21 = x - 4 + 5$	$x^2 - 8x + 21 = -(x - 4) + 5$

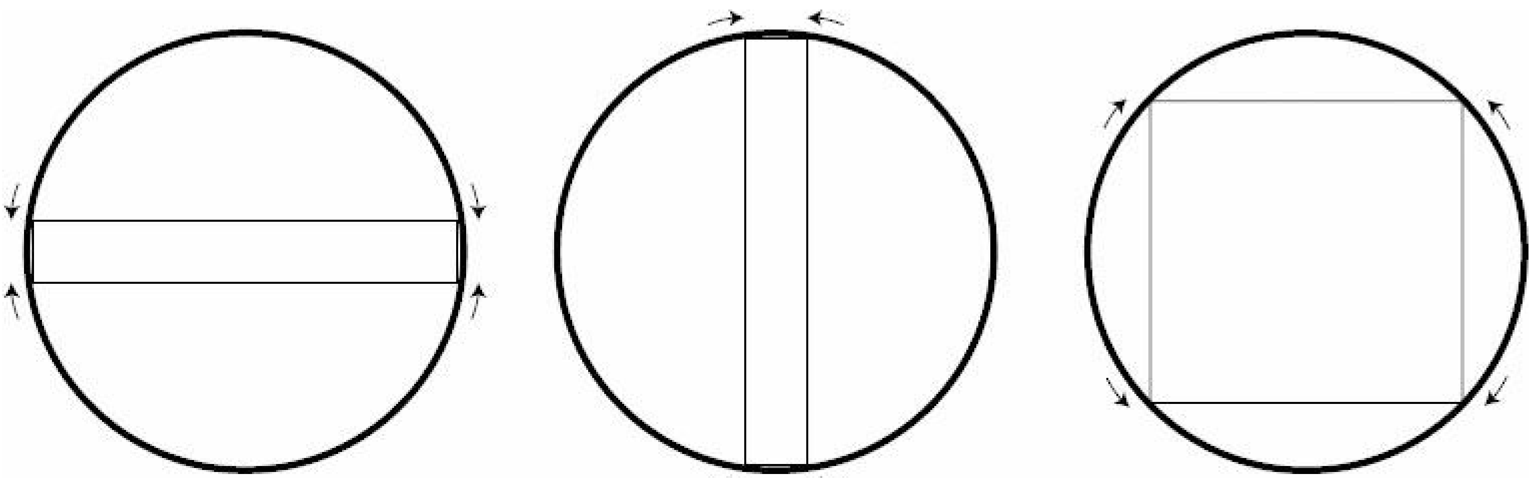
$$x^2 - 9x + 20 = 0 \qquad\qquad x^2 - 8x + 21 = -x + 4 + 5$$
$$(x - 5)(x - 4) = 0 \qquad\qquad x^2 - 7x + 21 = 0$$
$$x = 5 \text{ or } 4 \qquad\qquad (x - 4)(x - 3) = 0$$
$$x = 4 \text{ or } 3$$

Sum of the different solutions: $5 + 4 + 3 = 12$.

The repeated solution (4 occurs twice) may simply be ignored — the work shows two different ways of achieving the solution 4, but that is still just one solution to the equation. There are three total solutions that sum to 12.

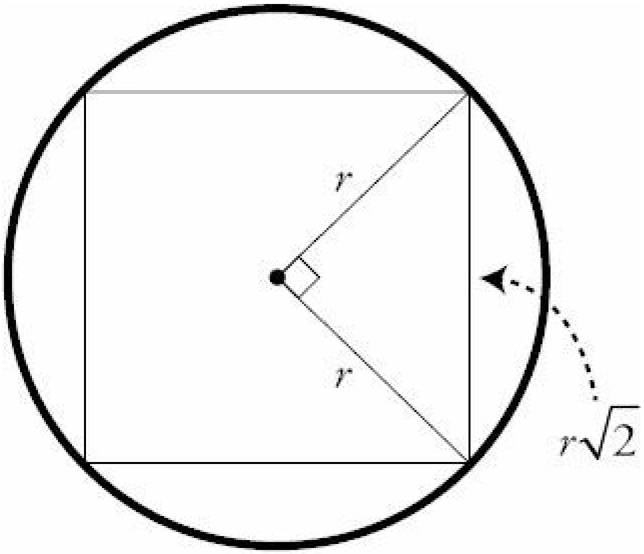
8. **(E).** This question asks you to determine which answer choice lists an area for the inscribed rectangle that is not feasible. What makes one possible area feasible and another one infeasible?

The inscribed rectangle can be stretched and pulled to extremes: extremely long and thin, extremely tall and narrow, and somewhere in between:



Long and thin **Tall and narrow** **In between**

The "long and thin" and "tall and narrow" rectangles will have a very small area, and the "in between" rectangle will have the largest possible area. In fact, the largest possible rectangle inscribed inside a circle will be a square:

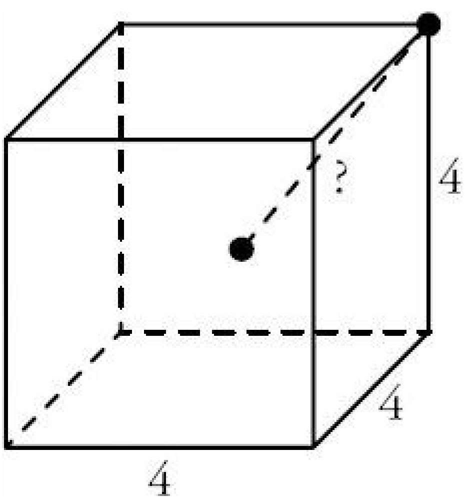


Square: maximal area

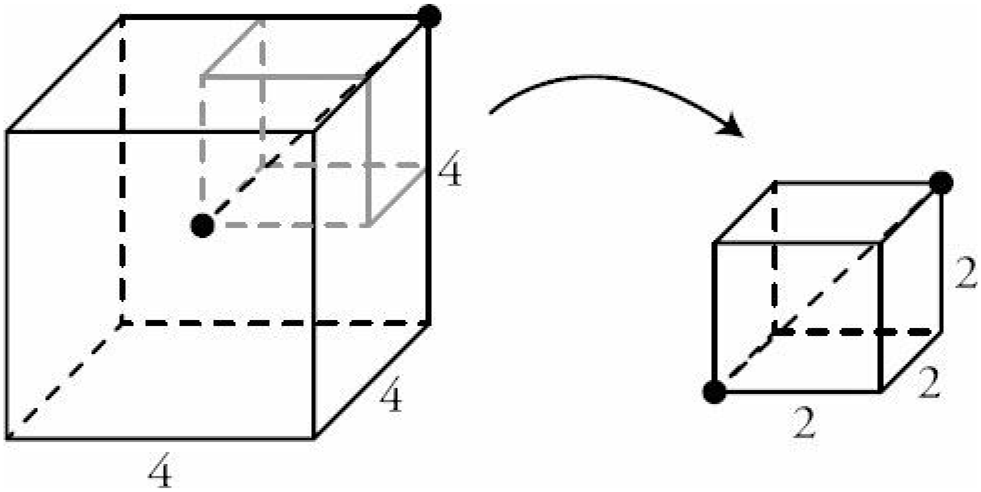
In this problem, the circumference = $10\pi = 2\pi r$. Thus $r = 5$, and the diagonal of the square is $2r = 10$. The square then has a side length of $5\sqrt{2}$ and an area of $\left(5\sqrt{2}\right)^2 = 50$

Only answer choice (E) is larger than 50. The correct answer is (E).

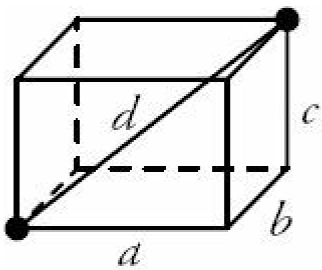
9. **(C).** First, you should represent the object with pictures, as is good practice with any 3-dimensional situation.



The length of any side of the cube is 4, and you are asked for the distance between the center of the cube and any of its vertices (corners). If you chop up the cube into 8 smaller cubes, you can see that the distance from the center of the $4 \times 4 \times 4$ cube to any corner is the diagonal of a $2 \times 2 \times 2$ cube.



You can find the diagonal of a cube in a variety of ways. Probably the fastest (besides applying a memorized formula) is to use the "super-Pythagorean" Theorem, which extends to three dimensions:



$a^2 + b^2 + c^2 + = d^2$

In the special case when the three sides of the box are equal, as they are in a cube, then you have this equation, letting s represent any side of the cube:

$s^2 + s^2 + s^2 = d^2$
$3s^2 = d^2$
$s\sqrt{3} = d$

Since $s = 2$, you know that $d = 2\sqrt{3}$.

10. **3/4.** Probability is (favorable outcomes)/(total # of possibilities). There are 99 – 20 + 1 = 80 possible values for c, so the unknown is how many of these c values yield a $c^3 - c$ that is divisible by 12.

The prime factorization of 12 is 2 × 2 × 3. There are several ways of thinking about this: numbers are divisible by 12 if they are divisible by 3 and by 2 twice, or if they are multiples of both 4 and 3, or if half of the number is an even multiple of 3, etc.

The expression involving c can be factored.

$c^3 - c = c(c^2 - 1) = c(c - 1)(c + 1)$

These are consecutive integers. It may help to put them in increasing order: $(c - 1)c(c + 1)$. Thus, this question has a lot to do with *Consecutive Integers*, and not only because the integers 20 to 99 themselves are consecutive.

In any set of three consecutive integers, a multiple of 3 will be included. Thus, $(c - 1)c(c + 1)$ is always divisible by 3 for any integer c. This takes care of part of the 12. So the question simply becomes "How many of the possible $(c - 1)c(c + 1)$ values are divisible by 4?" Since the prime factors of 4 are 2's, it makes sense to think in terms of odds and evens.

$(c - 1)c(c + 1)$ could be (E)(O)(E), which is definitely divisible by 4, because the two evens would each provide at least one separate factor of 2. Thus, $c^3 - c$ is divisible by 12 whenever c is odd, which are the cases $c = 21, 23, 25, \ldots, 95, 97, 99$. That's $((99 – 21)/2) + 1 = (78/2) + 1 = 40$ possibilities.

Alternatively, $(c - 1)c(c + 1)$ could be (O)(E)(O), which will only be divisible by 4 when the even term itself is a multiple of 4. Thus, $c^3 - c$ is also divisible by 12 whenever c is a multiple of 4, which are the cases $c = 20, 24, 28, \ldots, 92, 96$. That's $((96 - 20)/4) + 1 = (76/4) + 1 = 20$ possibilities.

The probability is thus $(40 + 20)/80 = 60/80 = 3/4$.

11. **I and II only.** x cannot equal y, as that would make $x/y = 1 \neq$ even. So either $x > y$ or $y > x$.

x and y are both positive, and x/y is an integer, so $x > y$.

If $x - y$ is even, either x and y are both even, or they are both odd.

Since x/y = an even integer, $x = y \times$ an even integer.

Odd $\neq$ Odd × an even integer, so x and y can't be odd.

Even = Even × an even integer, so x and y must be even.

I. TRUE. x and y are both even, and x/y is an even integer. The smallest value of x is 4, when y is 2, and $x/y = 4/2 = 2$. No even number greater than 2 is prime, so x can't be prime.

II. TRUE. x and y are each positive even numbers and $x \neq y$. Thus, $x + y$ is even, and the smallest possible value of $x + y = 4 + 2 = 6$. All even numbers greater than or equal to 6 are non-prime.

III. FALSE. It could be that $x = 4$ and $y = 2$, so $y/x = 1/2$, which is technically non-prime, but is not an integer. In fact, if x/y = an even integer, y/x = 1/an even integer = positive fraction.

12. **0.** Since the remainder is defined as what is left over after one number is divided by another, it makes sense that the leftover amount would be positive. So why is this information provided, if the remainder is "automatically" positive? Because there is a third possibility: that the remainder is 0! So when you are told here that the remainder when 120 is divided by m is positive, you are really being told that $120/m$ does not have a remainder of 0. In other words, 120 is not divisible by m, or m is not a factor of 120. Similarly, n is not a factor of 120.

Another constraint on both m and n is that they are single-digit positive integers. So m and n are integers between 1 and 9, inclusive, that are not factors of 120. Only two such possibilities exist: 7 and 9.

Since $m > n$, $m = 9$ and $n = 7$. Thus, $m - n = 2$, and the remainder when 120 is divided by 2 is 0.

13. **225,000.** Microchip radius = (2.5 cm)(10 mm/cm) = 25 mm

Blueprint radius = 1 cm per every 0.05 mm on the microchip
= 10 mm per every 0.05 mm on the microchip
= (10 mm/0.05 mm on microchip)(25mm on microchip)
= (10 mm/0.05)(25)
= (1,000 mm/5)(25)
= (1,000 mm)(5)
= 5,000 mm
= (5,000 mm)(cm/10 mm)
= 500 cm

Blueprint area = $\pi \times r^2$
$= \pi \times (500 \text{ cm})^2$
$= 250{,}000\pi \text{ cm}^2$

14. **(D).** If $PV = kT$, then $P = \frac{kT}{V}$. Quantity A is $P = \frac{k(32)}{(20)} = \frac{8}{5}k$.

If $PV = kT$, then $T = \frac{PV}{k}$. Quantity B is $T = \frac{(78)(10)}{k} = \frac{780}{k}$.

Don't rush to judgment, thinking that $780 > \frac{8}{5}$ means that Quantity B is greater. Notice that the k term is in the numerator of one quantity (so Quantity A increases with k) and the denominator of the other (so the larger k is, the smaller Quantity B is).

If $k = 1$, then Quantity B is greater $\left(780 > \frac{8}{5}\right)$. But if $k = 100$, Quantity A is greater ($160 > 7.8$). Thus, (D) is the answer.

15. **(D).** Some intuitive recollection of geometry rules and a picture drawn to scale can help you determine reasonable answer choices. If AC is a diameter of the circle, then Triangle ABC is a right triangle, with angle $ABC = 90$ degrees. The shortest side of a triangle is across from its smallest angle, and the longest side of a triangle is across from its largest angle. Therefore, $AC > BC > AB$.

The circumference of the circle $= \pi d = 6\pi\sqrt{3}$, so $d = 6\sqrt{3} \approx 6(1.7) = 10.2$. Thus, $AC \approx 10.2$ and $BC < 10.2$. But you can clearly see from the picture drawn to scale that BC is longer than half the diameter, so you can conservatively determine that $BC > 5.1$.

(A) $\frac{3}{\sqrt{2}} \approx \frac{3}{1.4} = \frac{30}{14} = 2\frac{1}{7}$ TOO LOW

(B) 3 TOO LOW

(C) $3\sqrt{3} \approx 3(1.7) = 5.1$ TOO LOW

(D) 9 OK

(E) $9\sqrt{3} \approx 9(1.7) = 15.3$ TOO HIGH

Alternatively, you could use rules of geometry to solve directly for the answer. Line *AC* passes through the center of the circle, so the inscribed Triangle *ABC* is a right triangle with angle *ABC* = 90°. Since angle *ACB* is 30°, angle *CAB* is 60°.

The sides in a 30–60–90 triangle have the ratio 1 : $\sqrt{3}$: 2, so given any side, you can compute the other two sides.

First, use the circumference to solve for *AC* (the diameter):

$$6\pi\sqrt{3} = \pi d = \text{Circumference}$$
$$\frac{6\pi\sqrt{3}}{\pi} = d$$
$$6\sqrt{3} = d$$

Now you can use ratios (specifically, the unknown multiplier) to find *BC*.

	AB	***BC***	***AC***
Basic Ratio	$1x$	$\sqrt{3}x$	$2x$
Known Side			$6\sqrt{3}$
Unknown Multiplier			$x = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$
Computer Sides	$3\sqrt{3}$	$(\sqrt{3})(3\sqrt{3}) = 9$	$2(3\sqrt{3}) = 6\sqrt{3}$

Line segment *BC* has length 9.

16. **(B).** Profit equals revenue minus cost. The company's profit is:

$$p(9 - p) - (p + 15) = 9p - p^2 - p - 15$$
$$= -p^2 + 8p - 15$$

$= -(p^2 - 8p + 15)$
$= -(p - 5)(p - 3)$

Profit will be zero if $p = 5$ or $p = 3$, which eliminates answers (A) and (C). For $p > 5$, both $(p - 5)$ and $(p - 3)$ are positive. In that case, the profit is negative (i.e., the company loses money). The profit is only positive if $(p - 5)$ and $(p - 3)$ have opposite signs, which occurs when $3 < p < 5$.

The correct answer is (B).

17. **I and II only.** The sum (1/41 + 1/42 + 1/43 + 1/44 + … + 1/57 + 1/58 + 1/59 + 1/60) has 20 fractional terms. It would be nearly impossible to compute if you had to find a common denominator and solve without a calculator and a lot of time. Instead, look at the maximum and minimum possible values for the sum.

Maximum: The largest fraction in the sum is 1/41. K is definitely smaller than $20 \times 1/41$, which is itself smaller than $20 \times 1/40 = 1/2$.

Minimum: The smallest fraction in the sum is 1/60. K is definitely larger than $20 \times 1/60 = 1/3$.

Therefore, $1/3 < K < 1/2$.

I. YES: $1/4 < 1/3 < K$
II. YES: $1/3 < K$
III. NO: $1/2 > K$

18. **(B).** First, make some observations. With 9 competitors and only 3 medals awarded, only 1/3 of the competitors will win overall. Although a simplification, it is reasonable for each competitor to see his or her chance of winning a medal as 1/3, or to expect to win 1/3 of a medal (pretending for a moment that medals can be "shared").

You are asked for the probability *at least* 2 of the triplets will win a medal. In other words, you want 2/3 to 3/3 of the triplets to win medals, or for each triplet to win 2/3 to 3/3 of a medal. Since 2/3 and 3/3 are both greater than 1/3, you are looking for the probability that the triplets will win medals at a rate greater than that expected for competitors overall. This would certainly be an unusual outcome. Thus, the probability should be less than 1/2. Eliminate (D) and (E). You could then at least make an educated guess from among the remaining choices with at least a 1 in 3 shot at success.

To solve, use the probability formula and combinatorics:

$$\text{Probability} = \frac{\text{specified outcome}}{\text{all possible outcomes}} = \frac{\#\text{ of ways at least 2 triplets win medal}}{\#\text{ of ways 3 medals can be awarded}}$$

First, find the total number of outcomes for the triathlon. There are nine competitors; three will win medals and six will not. Set up an anagram grid where Y represents a medal, N no medal:

competitor:	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
medal:	Y	Y	Y	N	N	N	N	N	N

# of ways 3 medals can be awarded= $\frac{9!}{3!6!} = \frac{(9)(8)(7)}{(3)(2)(1)} = (3)(4)(7) = 84$

Now, you need to determine the number of instances when *at least two* brothers win a medal. Practically speaking, this could happen when (1) exactly three brothers win or (2) exactly two brothers win.

Start with *all three* triplets winning medals, where Y represents a medal:

triplet:	A	B	C		non-triplet:	C_1	C_2	C_3	C_4	C_5	C_6
medal:	Y	Y	Y		medal:	N	N	N	N	N	N

The number of ways this could happen is $\frac{3!}{3!} \times \frac{6!}{6!} = 1$. This makes sense, as there is only one instance in which all three triplets would win medals and all of the other competitors would not.

Next, calculate the instances when *exactly two* of the triplets win medals:

triplet:	A	B	C		non-triplet:	C_1	C_2	C_3	C_4	C_5	C_6
medal:	Y	Y	N		medal:	Y	N	N	N	N	N

Since both triplets and non-triplets win medals in this scenario, we need to consider possibilities for both sides of the grid. For the triplets, the number of ways that two could win medals is $\frac{3!}{2!1!} = 3$.

For the non-triplet competitors, the number of ways that one could win the remaining medal is $\frac{6!}{1!5!} = 6$.

Multiply these two numbers to get the total number of instances: 3 × 6 = 18.

The brothers win *at least two* medals in 18 + 1 = 19 cases. The total number of cases is 84, so the probability is 19/84.

The correct answer is (B).

19. **(B).** Since the answer asks for an approximation, you should use decimal approximations for all square roots in the question and answer choices.

(A) $\sqrt{3} \approx 1.7$
(B) 2
(C) $1 + \sqrt{2} \approx 1 + 1.4 = 2.4$

(D) $1 + \sqrt{3} \approx 1 + 1.7 = 2.7$

(E) $2\sqrt{3} \approx 2(1.7) = 3.4$

Note that there is a minimum difference of 0.3 between answer choices. This implies that you must be *reasonably* careful when approximating, but will have no trouble choosing an answer if you approximate every square root to the nearest tenth.

$$\sqrt{2} \approx 1.4$$
$$\sqrt{2+\sqrt{2}} \approx \sqrt{2+1.4} \approx \sqrt{3.4} \approx 1.8$$
$$\sqrt{2+\sqrt{2+\sqrt{2}}} \approx \sqrt{2+1.8} \approx \sqrt{3.8} \approx 1.9$$
$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}} \approx \sqrt{2+1.9} \approx \sqrt{3.9} \approx 2$$
$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}} \approx \sqrt{2+2} \approx \sqrt{4} = 2$$

At this point, you can see that the expression is converging on 2.

Alternatively, an algebraic solution is possible if you recognize that the infinite expression is nested within itself:

$$x = \sqrt{2+\left(\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}}\right)} = \sqrt{2+x}$$

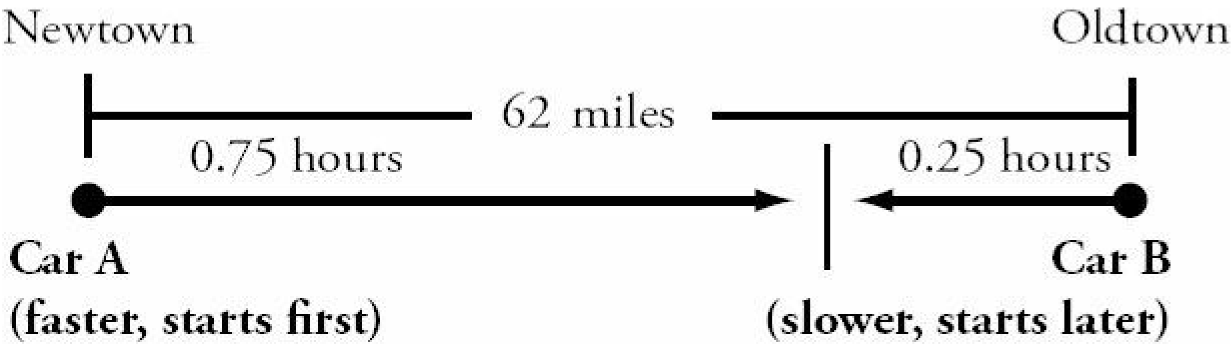
You can solve for x as follows:

$x = \sqrt{2+x}$
$x^2 = 2 + x$
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$

This implies that $x = 2$ or $x = -1$. Since x is the square root of a real, positive number, it must be positive, and you can conclude that $x = 2$.

The correct answer is (B).

20. **(A).** Draw a diagram to illustrate the moment at which A and B pass each other moving in opposite directions:



You could test the answer choices:

	B's distance (miles)	B's rate (mph) = D/T = $D/0.25$	A's rate (mph) = B's rate + 8	A's distance (miles) = $R \times T$ = $R \times 0.75$	Total distance
(A)	**14**	**56**	**64**	**48**	**62**
(B)	12	48	56	42	54
(C)	10	40	48	36	46
(D)	9	36	44	33	42
(E)	8	32	40	30	38

Or you could solve algebraically, using an *RTD* chart. Note that you must convert 15 minutes to 1/4 (or 0.25) hours:

	Rate	**Time**	**Distance**
Car A	$(r + 8)$ mph	0.75 hours	$(0.75)(r + 8)$ miles
Car B	r mph	0.25 hours	$0.25r$ miles
Total			62 miles

Set up and solve an equation for the total distance:

$$(0.75)(r + 8) + (0.25r) = 62$$
$$0.75r + 6 + 0.25r = 62$$
$$r = 56$$

Therefore, Car B traveled a distance of $0.25r = (0.25)(56) = 14$ miles.

The correct answer is (A).

21. **(D).** Factor 10,125 to its prime factors: $10{,}125 = 3^4 5^3$.

So, $x^2 5^y = 3^4 5^3$.

In order to have 5^3 on the right side, there have to be three factors of 5 on the left side. All three could be in the 5^y term (i.e., y could equal 3). Or, one of the 5's could be in the 5^y term, and two of the 5's in the x^2 term; i.e. y could equal 1 and x could have a single factor of 5.

In order to have 3^4 on the right side, x^2 must have $3^4 = (3^2)^2$ as a factor. In other words, x must have 3^2 as a factor, because 3^2 is certainly not a factor of 5. Thus, x is a multiple of 9.
The possibilities:

Quantity A: x^2		Quantity B: 5^y	Check: The product must be 10,125	Check: Quantity A must be a perfect square	Check: Quantity B must be a power of 5
$x^2 = 9^2 = 81$	<	$5^y = 5^3$ = 125	(81)(125) = 10,125	yes	yes
$x^2 = (9 \times 5)^2$ = 2,025	>	$5^y = 5^1$ = 5	(2,025)(5) = 10,125	yes	yes

In one case, Quantity A is greater. In the other, Quantity B is greater. The correct answer is (D).

22. **(B).** Testing the choices would be a natural way to solve this problem, since the question doesn't ask you to solve for b in general, but rather "for which of the following is x closest to zero?" However, numbers between 2^{20} and 2^{42} are too large to plug and compute. You must manipulate the terms with base 8 to see how they might balance with 2^b:

$$x = 2^b - (8^8 + 8^6)$$
$$0 \approx 2^b - (8^8 + 8^6)$$
$$2^b \approx (8^8 + 8^6)$$
$$2^b \approx (8^6)(8^2 + 1)$$
$$2^b \approx ((2^3)^6)\,((2^3)^2 + 1)$$
$$2^b \approx (2^{18})(2^6 + 1)$$

Since 1 is very small in comparison to 2^6, we can approximate $(2^6 + 1) \approx (2^6)$. Therefore,

$$2^b \approx (2^{18})(2^6)$$
$$2^b \approx 2^{24}$$
$$b \approx 24$$

The correct answer is (B).

23. **(E).** Since there are variables in the answer choices, we should pick a number and test the choices. If $k = 2$, then $\frac{2}{\sqrt{k+1}+\sqrt{k-1}} = \frac{2}{\sqrt{3}+\sqrt{1}} \approx \frac{2}{1.7+1} = \frac{2}{2.7}$, which is less than 1. Now test the answer choices and try to match the target:

(A) 2 TOO HIGH

(B) $2\sqrt{2k} = 2\sqrt{4} = 4$ TOO HIGH

(C) $2\sqrt{k+1} + \sqrt{k-1} = 2\sqrt{3} + \sqrt{1} \approx 2(1.7) + 1 = 4.4$ TOO HIGH

(D) $\dfrac{\sqrt{k+1}}{\sqrt{k-1}} = \dfrac{\sqrt{3}}{\sqrt{1}} \approx 1.7$ TOO HIGH

(E) $\sqrt{k+1} - \sqrt{k-1} = \sqrt{3} - \sqrt{1} \approx 1.7 - 1 = 0.7$ OK

Alternatively, you could solve this problem algebraically. The expression given is of the form $\dfrac{2}{a+b}$, where $a = \sqrt{k+1}$ and $b = \sqrt{k-1}$.

You need to either simplify or cancel the denominator, as none of the answer choices have the denominator you start with, and most of the choices have no denominator at all. To be able to manipulate a denominator with radical signs, you must first try to eliminate the radical signs entirely, leaving only a^2 and b^2 in the denominator. To do so, multiply by a fraction that is a convenient form of 1:

$$\frac{2}{(a+b)} = \frac{2}{(a+b)} \times \frac{(a-b)}{(a-b)} = \frac{2(a-b)}{a^2 - b^2}$$

Notice the "difference of two squares" special product created in the denominator with your choice of (a - b).

Substituting for a and b,

$$\frac{2}{(\sqrt{k+1}+\sqrt{k-1})} \times \frac{(\sqrt{k+1}-\sqrt{k-1})}{(\sqrt{k+1}-\sqrt{k-1})} = \frac{2(\sqrt{k+1}-\sqrt{k-1})}{(k+1)-(k-1)} = \frac{2(\sqrt{k+1}-\sqrt{k-1})}{2} = \sqrt{k+1}-\sqrt{k-1}$$

The correct answer is (E).

24. **(A).** First, note the answer pairs (A)&(C) and (B)&(E), in which one ratio is the square of the other. This represents a likely trap in a problem that asks for the ratio of $\sqrt{x}$ to $\sqrt{y}$ rather than the more typical ratio of x to y. You can eliminate (D), as it is not paired with a trap answer and therefore probably not the correct answer. You should also suspect that the correct answer is (A) or (B), the "square root" answer choice in their respective pairs.

For problems involving successive changes in amounts — such as population-growth problems, or compound interest problems — it is helpful to make a table:

	Account A	**Account B**
Now	x	y
After 1 month	$\left(\frac{9}{10}\right)x$	$\left(\frac{12}{10}\right)y$

After 2 month	$\left(\frac{9}{10}\right)\left(\frac{9}{10}\right)x=\left(\frac{81}{100}\right)x$	$\left(\frac{12}{10}\right)\left(\frac{12}{10}\right)y=\left(\frac{144}{100}\right)y$

If the accounts have the same amount of money after two months, then:

$$\left(\frac{81}{100}\right)x=\left(\frac{144}{100}\right)y$$
$$81x=144y$$

This can be solved for $\frac{\sqrt{x}}{\sqrt{y}}$:

$$\frac{x}{y}=\frac{144}{81}$$
$$\frac{\sqrt{x}}{\sqrt{y}}=\frac{\sqrt{144}}{\sqrt{81}}=\frac{12}{9}=\frac{4}{3}$$

The correct answer is (A).

25. **(C).** Consecutive integers have two key characteristics: they differ by a known, constant value (i.e., 1), and they alternate odd, even, odd, even, etc. You should use the odd/even property to evaluate these choices. This general approach is usually faster than considering specific values. It works particularly well for very general questions about whether something CANNOT or MUST be true.

(A)	a = E + O = O, or a = O + E = O	b = 3 pairs (E + O) = O, or b = 3 pairs (O + E) = O	a could equal b (both Odd)
(B)	a = E + O + E = O, or a = O + E + O = E	b = 3 pairs (E + O) = O, or b = 3 pairs (O + E) = O	a could equal b (both Odd)
(C)	**a = 3 pairs (E + O) = O, or a = 3 pairs (O + E) = O**	**b = 2 pairs (E + O) = E, or b = 2 pairs (O + E) = E**	$a \neq b$ **(Odd ≠ Even)**
(D)	a = 3 pairs (E + O) = O, or a = 3 pairs (O + E) = O	b = O + 3 pairs (E + O) = E, or b = E + 3 pairs (O + E) = O	a could equal b (both Odd)
(E)	a = O + 3 pairs (E + O) = E, or a = E + 3 pairs (O + E) = O	b = O + 2 pairs (E + O) = O, or b = E + 2 pairs (O + E) = E	a could equal b (both Odd or both Even)

The correct answer is (C).

26. **(A).** First of all, note that the height of each person in question is fixed (no one grew taller or shorter); only

weights changed. Second, note that BMI is always positive, and is proportional to w; as weight increases, BMI increases, and vice versa. So the language of the quantities — pounds gained … BMI increased" and "pounds lost … BMI decreased" — is aligned with this proportionality. Both quantities are a positive number of pounds.

Since $\text{BMI} = \frac{703w}{h^2}$, change in $\text{BMI} = \frac{703w_{before}}{h^2} - \frac{703w_{after}}{h^2} = \frac{703}{h^2}\left(w_{before} - w_{after}\right)$.

To simplify things, you can write this in terms of ΔBMI and Δw, the positive change in BMI and weight, respectively:

$$\Delta BMI = \frac{703}{h^2}\Delta w$$

(The triangle symbol indicating positive change in a quantity does not appear on the GRE — it is used here for convenience in notating an explanation.)

Since the quantities both refer to Δw, rewrite the relationship as $\Delta w = \frac{h^2}{703}\Delta BMI$. Both ΔBMI and h are given in each quantity, so Δw can be calculated and the relationship between the two quantities determined. (The answer is definitely not (D).)

Quantity A:
A 6' 2" tall person is 6(12) + 2 = 74 inches tall.

$$\Delta w = \frac{h^2}{703}\Delta BMI = \frac{74^2}{703}(1.0) = \frac{74^2}{703}$$

Quantity B:
A 5' 5" tall person is 5(12) + 5 = 65 inches tall.

$$\Delta w = \frac{h^2}{703}\Delta BMI = \frac{65^2}{703}(1.2)$$

Since the 703 in the denominator is common to both quantities, the comparison is really between $74^2 = 5{,}476$ and $65^2(1.2) = 4{,}225(1.2) = 5{,}070$. Quantity A is greater.

27. **(C).** There are four different outcomes that can yield two balls of the same color: Bag A with white, Bag A with red, Bag B with white, or Bag B with red. The first decision that must be made is to choose a bag. Because the problem states that one of the two bags will be chosen at random, you are no more likely to choose one bag than the other. Therefore, the probability of choosing Bag A, P(A), and the probability of choosing Bag B, P(B), must be the same, i.e. P(A) = P(B) = 1/2.

If Bag A is chosen, what is the probability of a matched pair? First, compute the probability of two whites. The probability of the first white is 3/6 and the probability of the second white is 2/5, so the probability of a first AND second white is (3/6)(2/5) = 1/5. Similarly, the probability of two reds is (3/6)(2/5) = 1/5. If Bag A is chosen, you can obtain a match by either grabbing a pair of white OR a pair of red, so you must add their probabilities to get the total chance of a pair. This gives P(Bag A Pair) = 1/5 + 1/5 = 2/5.

Similarly, if Bag B is chosen the probability of a pair of white marbles is (6/9)(5/8) = 5/12 and the probability of a pair of red marbles is (3/9)(2/8) = 1/12. Therefore, the probability of a pair is P(Bag B pair) = 5/12 + 1/12 = 6/12 = 1/2. The probability of choosing Bag A AND a pair from Bag A is the product of the two events, (1/2)(2/5) = 1/5. Similarly, the probability of choosing Bag B AND a pair from Bag B is (1/2)(1/2) = 1/4. The total probability of choosing a pair will be the probability of choosing Bag A and a pair from Bag A OR choosing Bag B and a pair from bag B, meaning you must sum these two events. This gives: P(pair) = 1/5 + 1/4 = 4/20 + 5/20 = 9/20.

28. **(C).** There are three different cases in which you must count: 62_ _, _62_, and _ _62. In the case of 62_ _, any digits from 00 to 99 will work, which gives you 100 numbers. In the case of _62_, you have 9 choices for the first digit as you are allowed to use any number from 1–9 inclusive, but not zero because you must meet the requirement of using a four digit positive integer. For the last digit you still allow any number from 0–9, which is 10 choices. Thus, by the fundamental counting principle, for _62_ you have (9)(10) = 90 choices. For the case of _ _62 you again have 1–9 inclusive for the first digit and 0–9 inclusive for the second digit for a total of 90 choices. However, in this case you are double counting one number, since 6262 already appeared in the 62_ _ case. Therefore there are only 89 new numbers that meet the criteria. Since you could create the case 62_ _ OR _62_ OR _ _62, you must add the number of possibilities together for each case to achieve the total. This gives you 100 + 90 + 89 = 279.

29. **(B).** In order for a number to be divisible by 9 the sum of the digits must be a multiple of 9. The lowest number that can be made by summing 5 of the digits is given by 0 + 1 + 2 + 4 + 5 = 12 and the highest number that can be made is 1 + 2 + 4 + 5 + 6 = 18. The only number in this range that sums to a multiple of 9 is 18, and thus the only possible combination of numbers you can use is {1, 2, 4, 5, 6}. In other words, no combination of numbers using the number 0 will ever yield a multiple of 9. The question can now be rephrased as, "How many different 5 digit numbers can be made using the digits {1, 2, 4, 5, 6} without repetition?" as these numbers will always sum to 18 and thus will always be divisible by 9. In this case, the answer is simply 5!, as there are 5 choices for the first number, 4 for the second, 3 for the third, and so on. Thus, there are 5! = 120 possible 5 digit numbers that are divisible by 9.

30. **(B).** This problem is greatly simplified if you realize that you do not need to sum all the multiples of 4 (or of 3) in the given range and divide by the number of such multiples, using the typical average formula. The average for a set of evenly spaced integers is equal to the average of the first and last term.

Quantity A: The multiples of 3 between 101 and 598 are 102, 105, 108, ..., 591, 594, 597. The average of the whole set is $\frac{102+597}{2} = \frac{699}{2} = 349.5$

Quantity B: The multiples of 4 between 101 and 598 are 104, 108, 112, ..., 588, 592, 596. The average of the whole set is $\frac{104+596}{2} = \frac{700}{2} = 350$

31. **(C).** Set up an implied inequality and perform identical operations on each quantity, grouping variables.

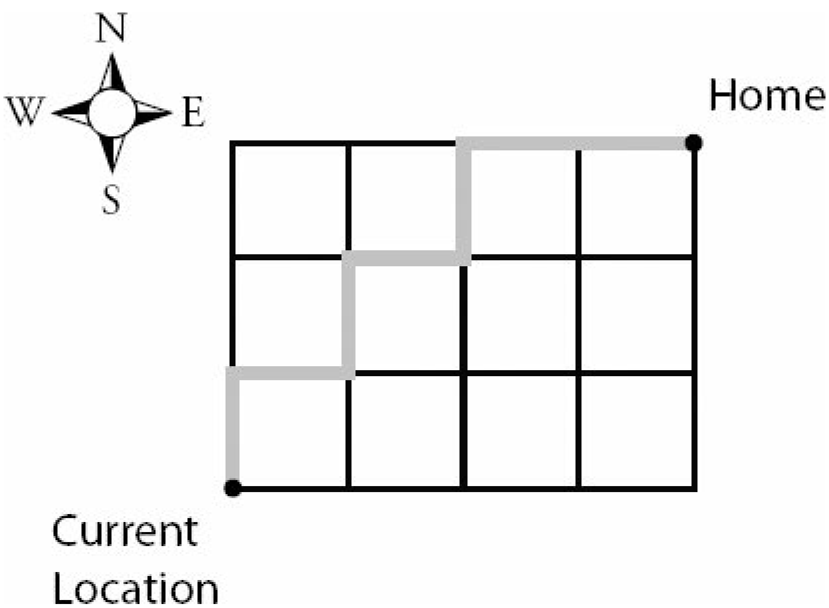
Quantity A		**Quantity B**
$a + d - c - 90$	?	$90 - e - b - f$
$a + d - c$	?	$180 - e - b - f$
$a + d - c + e + b + f$	?	180
$(a + d + e + f) + (b - c)$	?	180

In the last step above, only the order of the variables was changed, and parentheses added to group certain terms. Notice that the angle at point C and point D is the same, as AC and FD are parallel lines intersected by the transversal CE. So, the first set of parentheses holds the sum of the interior angles of the biggest triangle ACE, which is 180. Also because AC and FD are parallel lines intersected by transversal BE, $b = c$, so $b - c = 0$ in the second set of parentheses.

Quantity A		**Quantity B**
$(a + d + e + f) + (b - c)$	?	180
$(180) + (0)$	=	180

Thus, the quantities each equal 180 and the correct answer is (C).

32. **(D).** Given that the man can only move north and east, he must advance exactly 7 blocks from his current location to get home regardless of which path he takes. Of these 7 blocks, 4 must be moving east and 3 must be moving north. An example path is given below:



The problem can then be rephrased as follows: "Of the 7 steps, when does the man choose to go east and when does he choose to go north?" Labeling each step as N for north and E for east, you can see the problem as the number of unique rearrangements of NNNEEEE (for example, this arrangement corresponds to going north 3 times and then east 4 times straight to home). This is given by $\frac{total!}{repeats!} = \frac{7!}{3!4!} = 35$.

33. **(D).** Denote red as R, white as W, and black as B. There are exactly three ways in which the selections may occur where you get red as the second marble, either RR, WR, or BR. Since you may have any of these options the problem is an OR, so calculate the probability of each event and then add them together. First, for RR, you have the probability of the first red as (2/9) and the second red as (1/8), yielding a probability of red AND red as (2/9)(1/8) = 1/36. Similarly, the probability of first white AND second red is (3/9)(2/8) = 1/12. Finally, the probability of first black and second red is (4/9)(2/8) = 1/9. Thus, the total probability of the second marble being red is P(RR) + P(WR) + P(BR) = 1/36 + 1/12 + 1/9 = 1/36 + 3/36 + 4/36 = 8/36 = 2/9. Of course, an easier way to solve this problem is to consider that the first draw is completely irrelevant, so you may consider the second draw alone. For the second draw, there are 2 red marbles out of a total of 9 marbles, giving you 2/9. Keep in mind that even though there are often difficult solution methods, sometimes a clever insight can greatly simplify the problem.

34. **(A).** Simplify both quantities, remembering that a power to a power means you multiply the exponents. Also, 25 is 5 squared, so you can substitute, putting both quantities in terms of a base of 5.

Quantity A: $\left((25^x)^{-2}\right)^3 = 25^{-6x} = (5^2)^{-6x} = 5^{-12x}$

Quantity B: $\left((5^{-3})^2\right)^{-x} = 5^{6x}$

Typically, when you are comparing exponents with the same base, the one with the larger exponent is greater. It might be tempting to conclude that $6x > -12x$, but be careful with negative variables.

If $x = -1$, Quantity A = 5^{12} and Quantity B = 5^{-6}, or $\frac{1}{5^6}$. In this case, Quantity A is much larger.

If $x = -\frac{1}{2}$, Quantity A = 5^6 and Quantity B = 5^{-3}, or $\frac{1}{5^3} = \frac{1}{125}$. Again, Quantity A is much larger.

If $x = -10$, Quantity A = 5^{120} and Quantity B = 5^{-60}. You can see that, the more negative x gets, the larger the difference between Quantity A and Quantity B becomes. Quantity A will always be larger.

Another way to look at it:

Quantity A: $5^{-12x} = 5^{-12 \times negative} = 5^{negative}$

Quantity B: $5^{6x} = 5^{6 \times negative} = 5^{negative}$

Even if $|x|$ is a tiny fraction, i.e. you are taking some high order root of 5 such as $\sqrt[8]{5}$ or $\sqrt[100]{5}$, these quantities would approach 1 such that

Quantity A: $51^{\text{positive}} > 1$

Quantity B: $51^{\text{negative}} < 1$

Since Quantity A is greater than 1 and Quantity B is less than 1, Quantity A is larger.

35. **(A).** First, notice that -93 and 252 are both multiples of 3 and "inclusive" means they should be included in the sum with all the multiples of 3 in between them. Listing and adding the numbers would be time consuming and error prone, so some strategies are useful.

Quantity A: (-93) + (-90) + (-87) + … + (-6) + (-3) + 0 + 3 + 6 + … + 87 + 90 + 93 + 96 + 99 + … 246 + 249 + 252

Since 252/3 = 84, 252 is the 84th positive multiple of 3. By the same logic, minus 93 is the 31st negative multiple of 3 (because -93/3 = -31). So the sum in question is 3 times the sum of the integers from -31 to +84, inclusive.

Quantity A: $3 \times [(-31) + (-30) + (-29) + \ldots + (-2) + (-1) + 0 + 1 + 2 + \ldots + 29 + 30 + 31 + 32 + 33 + \ldots 82 + 83 + 84]$

Notice that all of the negative integers have an additive inverse elsewhere in the sum that cancels them out. For example, (-31) + 31 = 0, and (-30) + 30 = 0, etc. So the sum in question is really 3 times the sum of just the integers from 32 to 84, inclusive.

Quantity A: $3 \times [32 + 33 + \ldots 82 + 83 + 84]$

Now, apply the formula for summing consecutive integers: $\frac{First + Last}{2} \times Number\ of\ terms$

The number of terms is Last - First + 1 = 84 - 32 + 1 = 53.

Quantity A: $3 \times [32 + 33 + \ldots 82 + 83 + 84] = 3\times\left[\frac{32+84}{2}\times 53\right] = 3\times\left[\frac{116}{2}\times 53\right] = 3 \times [58 \times 53]$
$= 3 \times 3{,}074 = 9{,}222$

Thus, Quantity A is larger.

36. **(B).** When you see a negative base raised to an integer power, the question is about positives and negatives: $(-1)^{odd} = -1$ and $(-1)^{even} = +1$.

If *x* is even, all of the exponents in this question are even.

Quantity A: $(-1)^{even^2} + (-1)^{even^3} + (-1)^{even^4} = (-1)^{even} + (-1)^{even} + (-1)^{even} = 1 + 1 + 1 = 3$

Quantity B: $(-1)^{even} + (-1)^{2\times even} + (-1)^{3\times even} + (-1)^{4\times even} = 1 + 1 + 1 + 1 = 4$

If *x* is odd, *some* of the exponents in this question are odd.

Quantity A: $(-1)^{odd^2} + (-1)^{odd^3} + (-1)^{odd^4} = (-1)^{odd} + (-1)^{odd} + (-1)^{odd} = (-1) + (-1) + (-1) = -3$

Quantity B: $(-1)^{odd} + (-1)^{2\times odd} + (-1)^{3\times odd} + (-1)^{4\times odd} = (-1)^{odd} + (-1)^{even} + (-1)^{odd} + (-1)^{even}$
$= (-1) + 1 + (-1) + 1$
$= 0$

In both cases, Quantity B is greater than Quantity A.

37. **(B).** Because an exterior angle of a triangle is equal to the sum of the two opposite interior angles of the triangle (in this case, the top small triangle), $c = a + b$.

Since $d > c$ and you can substitute $a + b$ for c, you know:

$$d > a + b$$

Subtract b from both sides:

$$d - b > a$$

Thus, Quantity B is larger.

38. **(B).** Many students find sequence notation intimidating, but it doesn't have to be. Let's rephrase each sequence in normal language.

$S_n = S_{n-1} + \frac{5}{2}$ is just saying that *every term in S is equal to the term before it, plus 5/2.*

$A_n = A_{n-1} - 2.5$ is just saying that *every term in A is equal to the term before it, minus 2.5.*

Of course, 5/2 and 2.5 are equal, which is a good clue that there's probably some simple way to solve this problem without actually summing up a sequence.

Since $S_1 = 1$ and every term in S is just 2.5 greater than the term before it, Sequence S begins like this: 1, 3.5, 6, 8.5, 11, 13.5, 16, 18.5, 21, 23.5, …

Since $A_1 = 36$ and every term in A is just 2.5 less than the term before it, Sequence A begins like this: 36, 33.5, 31, 28.5, 26, 23.5, 21, 18.5…

At this point, it looks as though the two sequences are going to have a lot of terms in common! Remember, any common elements appearing in both Quantity A and Quantity B can just be canceled out.

You could just write out all 14 terms for each column, or you could "skip up" to S_{14} by noting that S_{14} is just going to be S_1 plus 2.5, thirteen times (since it takes thirteen "jumps" to get from 1 to 14). Take the first term, 1, plus 13(2.5) = 32.5 to get $S_{14} = 33.5$.

You can also "skip up" to the final term in A. To get to A_{14}, take A_1 and subtract 2.5 thirteen times (since it takes thirteen "jumps" to get from 1 to 14). Take the first term, 36, minus 13(2.5) = 32.5 to get $A_{14} = 3.5$.

So, Quantity A looks like this: The sum of 1, 3.5, 6, …, 28.5, 31, 33.5

And Quantity B looks like this: The sum of 36, 33.5, 31, …, 8.5, 6, 3.5

That is, all the terms from 3.5 to 33.5, inclusive, are held in common by both sets, so you can safely subtract them out. Here's what's left.

Quantity A: 1

Quantity B: 36

Quantity B is greater.

39. **(B).** This problem depends on knowing how to factor the "difference of squares." The basic formula that you learned in the Manhattan Prep's *Algebra GRE® Strategy Guide* is this:

$x^2 - y^2 = (x - y)(x + y)$

The important part about learning this formula is that *anything* can be "subbed in" for x and y. Another way to think about it is that two perfect squares can be "subbed in" for x^2 and y^2. For instance, a^{64} and 1 can serve as x^2 and y^2 (remember, 1 is a perfect square — if it helps, think of it as 1^2).

So, you can factor a^{64} - 1 in the numerator according to the pattern above:

$$\text{Quantity A:} \quad \frac{\left(a^{32}+1\right)\left(a^{32}-1\right)}{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{8}+1\right)^{2}}$$

In order to do this, you also need to know how to take the square root of a^{64}. To take the square root of any number with an even exponent, just cut the exponent in half. Do not take the square root of the exponent itself! So, $\sqrt{a^{64}}$ = a^{32}, not a^8.

Now, notice that a^{32} - 1 also matches the pattern (a perfect square minus a perfect square). a^{32} + 1, however, cannot be factored. Let's factor a^{32} - 1 only:

$$\text{Quantity A:} \quad \frac{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{16}-1\right)}{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{8}+1\right)^{2}}$$

Looking good. But wait! a^{16} - 1 ALSO matches the pattern! You can factor again:

$$\text{Quantity A:} \quad \frac{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{8}+1\right)\left(a^{8}-1\right)}{\left(a^{32}+1\right)\left(a^{16}+1\right)\left(a^{8}+1\right)^{2}}$$

You actually could factor a^8 - 1, but nothing on the bottom is going to cancel with terms broken down further on top. Instead, cancel common terms on top and bottom.

Quantity A: $\dfrac{\cancel{(a^{32}+1)}\cancel{(a^{16}+1)}(a^8+1)(a^8-1)}{\cancel{(a^{32}+1)}\cancel{(a^{16}+1)}(a^8+1)^2} = \dfrac{(a^8+1)(a^8-1)}{(a^8+1)^2} = \dfrac{\cancel{(a^8+1)}(a^8-1)}{\cancel{(a^8+1)}(a^8+1)} = \dfrac{(a^8-1)}{(a^8+1)}$

Whatever a^8 is, the numerator of Quantity A is less than the denominator of Quantity A (which is also definitely positive, since $a^8 \geq 0$). Thus, Quantity A < 1. The correct answer is (B).

40. **(A).** This problem introduces a square and a circle, and stating that the circumference of the circle is 7/8 the perimeter of the square.

This is license to plug in. Since both a square and a circle are regular figures — that is, all squares are in the same proportion as all other squares, and all circles are in the same proportion as all other circles — you can be certain that plugging only *one* set of values will give the same result you'd get from plugging *any* set of values. Because the figures are regular and related in a known way (circumference = 7/8 × square perimeter), there is no need to repeatedly try different values as is often necessary on Quantitative Comparisons.

You could say the radius of the circle is 2, so the circumference is 4π. Then, the perimeter of the square is $(8/7)(4\pi) = 32\pi/7$. This isn't ideal, because then you are stuck with π in the calculations for the square, where it is unnecessarily awkward.

It is best to pick values for the square. If the side of the square is 2, the perimeter is 4(2) = 8 and the area is (2)(2) = 4. Then, circumference of the circle is (7/8)(8) = 7. Since circumference is $2\pi r = 7$, the radius of the circle is

$$r = \frac{7}{2\pi}$$

Using these numbers:

Quantity A: The area of the square = 4

Quantity B: The area of the circle =

$$\pi r^2 = \pi\left(\frac{7}{2\pi}\right)^2 = \pi\left(\frac{49}{4\pi^2}\right) = \frac{49}{4\pi} \approx 3.9$$

(Use the calculator and the approximation 3.14 for π to determine that Quantity A is larger.)

41. **(C).** You can set up and simplify two equations, one for the calories consumed and the other for cost, using variables A and B for the number of servings of Snacks A and B, respectively.

Calories:
$320A + 110B = 2{,}370$
$32A + 11B = 237$ {divided by 10}

Cost:
$\$1.50A + \$0.45B = \$10.65$
$150A + 45B = 1{,}065$ {multiply by 100 to eliminate decimals}

$30A + 9B = 213$ {divided by 5}

So you now have a system of two equations and two variables, which could be solved for A and B. But even in their simplified form, these two equations have awkward coefficients that will make solving messy.

Since this is a Quantitative Comparison question, it would be smarter to "cheat off of the easy statement." That means to plug in the 5 from Quantity B as a possible number of servings of Snack A, and see what happens.

Plug $A = 5$ in to each equation.

Calories: $32(5) + 11B = 237$, so $160 + 11B = 237$, and $11B = 77$. Therefore $B = 7$.

Cost: $30(5) + 9B = 213$, so $150 + 9B = 213$, and $9B = 63$. Therefore $B = 7$.

This shows that $A = 5$ and $B = 7$ is the solution you would get by solving the system of equations yourself.

Thus, Quantity A and Quantity B are both 5.

42. **(C).** In this recursive function, each term is dependent on two others:

$$a_2 = (a_1 + a_3)/2$$
$$a_3 = (a_2 + a_4)/2$$
$$a_4 = (a_3 + a_5)/2$$

… and so on. Without actual numbers to plug in, it will be difficult to compare Quantity A and Quantity B.

You could try to put all a_n in terms of a_1 algebraically, and hope to find a pattern. If you haven't already, go ahead, try it! It's a mess.

The best way to make sense of the sequence definition is to list some (randomly made-up) actual numbers that follow the sequence rules.

If $a_1 = 1$ and $a_2 = 3$, you can extrapolate that the sequence is: 1, 3, 5, 7, 9, 11, etc.
If $a_1 = -100$ and $a_2 = 50$, you can continue the sequence: -100, 50, 200, 350, 500, 650, etc.
If $a_1 = 0$ and $a_2 = -4$, the sequence is: 0, -4, -8, -12, -16, -20, etc.

No matter the value of a_1 and a_2, the pattern is the same. After the first term, each term in the sequence is equal to the preceding term plus some constant. The constant in the test sequences was equal to $a_2 - a_1$, according to the numbers you started with.

Now you can more easily put all a_n in terms of a_1:

$$a_1 = a_1$$
$$a_2 = a_1 + c$$

$a_3 = a2 + c = a_1 + 2c$
$a_4 = a_1 + 3c$
$a_5 = a_1 + 4c$
…
$a_n = a_1 + (n - 1)c$

Thus:

Quantity A	**Quantity B**
$a_{51} - a_{48} = (a_1 + 50c) - (a_1 + 47c) = 3c$	$a_{37} - a_{34} = (a_1 + 36c) - (a_1 + 33c) = 3c$

The two quantities are the same.

43. **(B).** The term $12x$ has prime factors 2, 2, 3, and x (actually, you don't know whether x is prime, but since you don't know anything else about it right now, leave it as x).

The term $35y$ has prime factors 5, 7, and y. (Again, you don't know whether y is prime, but you can't do anything more with it right now.)

You are told that the greatest factor held in common between $12x$ and $35y$ is $5y$. Therefore, $12x$ and $35y$ each contain both 5 and y. Of course, you already knew that $35y$ contained both 5 and y, but you have definitely just learned something new about $12x$ - it also contains $5y$. You also now know that y CANNOT contain 2 and/or 3, since, if it did, the greatest common factor would also contain the 2 and/or 3 (since $12x$ and $35y$ would then *both* contain the 2 and/or 3). Similarly, x cannot contain a 7 — if it did, the 7 would appear in the greatest common factor (which it does not).

Thus, so far, you know:

$12x$ contains 2, 2, 3, 5, y, and possibly other factors but NOT a 7
$35y$ contains 5, 7, y and y does NOT contain 2 or 3

Take a look at some examples. If $x = 55$ and $y = 11$, then x correctly contains both 5 and y, and the GCF of 12(55) and 35(11) would indeed be $5y$, or 55. Alternatively, if $x = 5$ and $y = 1$, then the GCF of 12(5) and 35(1) would again be $5y$, which in this case would be 5.

In both examples, the remainder when $12x$ is divided by 10 is 0 and thus Quantity A is equal to 0. You can be certain that this will always be true because $12x$ definitely contains both 2 and 5. Any integer with 2 and 5 in its prime factors will always be a multiple of 10.

In the first example, $x = 55$ and $y = 11$, the GCF of x and y is 11. In the second example, $x = 5$ and $y = 1$, the GCF of x and y is 1. Since x and y will always be integers, their greatest common factor will always be 1 or more. Thus, Quantity B is larger.

44. **(A).** One good way to work through this problem is to pick a number, ideally starting with the innermost shape, the small circle. Let's say this circle has radius 1 and diameter 2, which would also make the side of the smaller square equal to 2.

If the small square has side 2, its diagonal would be $2\sqrt{2}$ (based on the 45-45-90 triangle ratios, or you could do the Pythagorean Theorem using the legs of 2 and 2). If the diagonal is $2\sqrt{2}$, then the diameter of the larger circle is also $2\sqrt{2}$ (and the radius of the larger circle is one-half of that, or $\sqrt{2}$), making the side of the larger square also equal to $2\sqrt{2}$. Therefore:

Small circle: radius = 1, area = π

Large circle: radius = $\sqrt{2}$, area = 2π

Small square: side = 2, area = 4

Large square: side = $2\sqrt{2}$, area = 8

Thus, the large circle has twice the area of the small circle, and the large square has twice the area of the small square. This will work for any numbers you choose. In fact, you may wish to memorize this as a shortcut: if a circle is inscribed in a square that is inscribed in a circle, the large circle has twice the area of the small circle; similarly, if a square is inscribed in a circle that is inscribed in a square, the large square has twice the area of the small square.

In Quantity A, the ratio of the area of the larger square to the smaller square is 2/1 = 2.

In Quantity B, twice the ratio of the area of the smaller circle to the area of the larger circle = 2(1/2) = 1.

45. **(A).** This problem is much easier than it looks! Of course, the integers are much too large to fit in your calculator. However, all you need to know is that a pair consisting of one 2 and one 5 multiplies to 10 and therefore adds a zero to the end of a number. For instance, a number with two 2's and two 5's in its prime factors will end with two zeroes, because the number is a multiple of 100.

Quantity A has 19 5's and many more 2's (since 2^{16} and 4^{18} together is obviously more than 19 2's — if you really want to know, it's 2^{16} and $(2^2)^{18}$, or 2^{16} and 2^{36}, or 2^{52}, or 52 2's). Since you need *pairs* made up of one 2 and one 5, you can make exactly 19 pairs (the leftover 2's don't matter), and the number ends in 19 zeroes.
Quantity B has 16 5's and many more 2's (specifically, there are 53 2's, but you should be able to tell at a glance that there are obviously more than 16 2's, so you don't need to calculate this). Since you need *pairs* made up of one 2 and one 5, you can make exactly 16 pairs (the leftover 2's don't matter), and the number ends in 16 zeroes. Thus, Quantity A is larger.

46. **(A).** Calculate several terms of the sequence defined by $a_n = 2^n - \frac{1}{2^{n-33}}$ and look for a pattern.

$$a_1 = 2^1 - \frac{1}{2^{-32}} = 2^1 - 2^{32}$$

$$a_2 = 2^2 - \frac{1}{2^{-31}} = 2^2 - 2^{31}$$

...

$$a_{16} = 2^{16} - \frac{1}{2^{-17}} = 2^{16} - 2^{17}$$

$$a_{17} = 2^{17} - \frac{1}{2^{-16}} = 2^{17} - 2^{16}$$

...

$$a_{31} = 2^{31} - \frac{1}{2^{-2}} = 2^{31} - 2^2$$

$$a_{32} = 2^{32} - \frac{1}{2^{-1}} = 2^{32} - 2^1$$

Notice that the 16th and 17th terms (the two middle terms in a set of 32 terms) are arithmetic inverses, that is, their sum is zero. Likewise, the 1st and 32nd terms sum to zero, as do the 2nd and 31st terms. In the first 32 terms of the sequence, there are 16 pairs that each sum to zero. Thus, Quantity A is zero.

For the sum of the first 31 terms, you could either

1. Subtract a^{32} from the sum of the first 32 terms: $0 - (2^{32} - 2^1) = 2^1 - 2^{32} = 2 -$ (a very large number) = negative, or
2. Realize that in the first 31 terms, all terms except a_1 can be paired such that the pair sums to zero, so the sum of the first 19 terms = $a_1 = 2^1 - 2^{32} = 2 -$ (a very large number) = negative.

Thus, Quantity B is negative, which is less than zero. Quantity A is larger.

47. **36.6.** To calculate first find the squared differences between the average and the terms. For example, the difference between the average, 4, and the first term, -1, is 5. $5^2 = 25$. Do this for all four terms:

-1: $(4 - (-1))^2 = 5^2 = 25$
4: $(4 - (0))^2 = 4^2 = 16$
30: $(4 - (30))^2 = (-26)^2 = 676$
-21: $(4 - (-21))^2 = (25)^2 = 625$

Add all of these terms together: 25 + 16 + 625 + 676 = 1,342.

Take the square root (use the calculator!): Square root of 1342 = 36.633.

The question asked you to round to the nearest tenth, so the answer is 36.6.

Note: Thus far, all of the questions we've seen on the real GRE dance around the issue of calculating standard deviation — no question has actually asked you to calculate it. However, we've included this example here, just in case the GRE ups the ante on us in future.

48. **(C).** Before figuring out how many balls you have of each integer value, consider what the question is asking: the "interquartile range" of a group of 100 integers. To find this range, split the 100 integers into two groups, a lower 50 and an upper 50. Then find the median of each of those groups. The median of the lower group is the first quartile (Q_1), while the median of the upper group is the third quartile (Q_3). Finally, Q_3-Q_1 is the interquartile range.

The median of a group of 50 integers is the average (arithmetic mean) of the 25th and the 26th integers when ordered from smallest to largest. Out of the ordered list of 100 integers from smallest to largest, then, find #25 and #26 and average them to get the first quartile. Likewise, find #75 and #76 and average them to get the third quartile. Then perform the subtraction.

Each ball has an integer value painted on the side — either 1, 2, 3, 4, 5, 6, 7, or 8. Figure out how many balls there are for each integer by applying the given formula, starting with the lowest integer in the list (1) and going up from there.

Number of balls labeled number 1 = 18 - $(1 - 4)^2$ = 18 - (-3)2 = 18 - 9 = 9 balls. These represent balls #1 through #9.

Number of balls labeled number 2 = 18 - $(2 - 4)^2$ = 18 - $(-2)^2$ = 18 - 4 = 14 balls, representing balls #10 through #23. Be careful when counting; the 14th ball is #23, not #24, because #10 is the first, #11 is the second, and so on.

Number of balls labeled number 3 = 18 - $(3 - 4)^2$ = 18 - $(-1)^2$ = 18 - 1 = 17 balls, representing #24 through #40.

At this point, you can tell that balls #25 and #26 both have a 3 on them. So the first quartile Q_1 is the average of 3 and 3, namely 3. Now keep going!

Number of balls labeled number 4 = 18 - $(4 - 4)^2$ = 18 - $(0)^2$ = 18 balls, representing balls #41 through #58.

Number of balls labeled number 5 = 18 - $(5 - 4)^2$ = 18 - $(1)^2$ = 18 - 1 = 17 balls, representing #59 through #75.

You can stop here. Ball #75 has a 5 on it (in fact, the last 5), while ball #76 must have a 6 on it (since 6 is the next integer in the list). Thus, the third quartile Q_1 is the average of 5 and 6, or 5.5. Notice that you have to count carefully — if you are off by even just one either way, you'll get a different number for the third quartile.

Finally, Q_3-Q_1 = 5.5 - 3 = 2.5, the interquartile range of this list of integers.

49. **(A).** Setting up the information in the question in the form of an equation, you see that:

$x/13 = y + 3/13$

$x = 13y + 3$

and

$x/7 = z + 3/7$
$x = 7z + 3$

Setting the two values for x equal to one another you see that

$13y + 3 = 7z + 3$
$13y = 7z$

Because y and z must be whole numbers, y must have 7 as a factor and z must have 13 as a factor. y and z can share an unlimited number of factors, but y must have a 7 in its prime box and z must have a 13 in its prime box.

The question now asks what is the remainder of $\frac{yz}{13}$. Since 13 is in the numerator, it can be canceled out of the fraction, leaving a 1 in the denominator and resulting in a whole number which has a remainder of 0.

50. **(B).** First solve for x in the equation to reveal the recursive formula for calculating a_n: $2 = (x/2) - a_{n-1}$

$4 = x - 2(a_{(n-1)})$
$x = 4 + 2(a_{(n-1)})$

Since "the term a_n is defined as the value of x that satisfies the equation," substitute a_n for x to get the real formula you are being asked to use:

$a_n = 4 + 2(a_{(n-1)})$

Since you have a_6 and want to calculate a previous term, a_4, it may be useful to rewrite the equation in a form that allows you to solve for the previous term. That is, solve for $(a_{(n-1)})$:

$a_{(n-1)} = (a_n - 4)/2$

Now let a_b be a_n and a_5 be $a_{(n-1)}$:

$a_5 = (156 - 4)/2 = 76$
$a_4 = (76 - 4)/2 = 36$
$a_3 = (36 - 4)/2 = 16$
And $a_2 = (16 - 4)/2 = 6$

51. **(C).** Compute the expressions for each of the terms:

$x!4 = x^4 \times 4^{-x}$ and $4!x = 4^x \times x^{-4}$

Dividing the first by the second yields

$$\frac{x^4 4^{-x}}{4^x x^{-4}} = \frac{x^4}{x^{-4}} \times \frac{4^{-x}}{4^x} = x^8 4^{-2x}$$

There are a number of ways we could write $x^8 4^{-2x}$:

$$x^8 4^{-2x} = \frac{x^8}{4^{2x}} = \frac{x^8}{16^x}$$

The two quantities are equal.

52. **(A).** First, calculate the sum of the even integers between 1 and 100 (2, 4, 6… 98, 100). You can think of this list as 50 even integers to be summed or, more usefully, you can think of it as 25 integer pairs each of which sum to 102 (2 + 100, 4 + 98, …, 50 + 52). The sum of these 25 pairs is simply 25 × 102. This is an alternate approach to the usual formula that tells you to compute the average value times the number of terms. Next, calculate the sum of the odd integers between 100 and 150 (101, 103, 105, …, 147, 149). Like before, you can turn this list into pairs, though you need to be careful because there are an odd number of integers in this list and the middle number (125) will not get a pair: (101 + 149, 103 + 147, …, 123 + 127). There are 12 pairs that each add to 250 and one leftover integer, 125. The sum is 12 × 250 + 125 or 12.5 × 250. The ratio be asked for in the question is (25 × 102) to (12.5 × 250). You can divide both sides by 25 to yield the ratio 102 to 125.

53. **(C).** The problem gives two ways to calculate the fourth term: (1) the definition of the sequence tells you that $a_4 = a_3^2 - a_2^2$ and (2) you are told that $a_4 = a_1(a_2 + a_3) = 2(a_2 + a_3)$. Setting these two equal gives $a_3^2 - a_2^2 = 2(a_2 + a_3)$. Factor the left side: $(a_3 + a_2)(a_3 - a_2) = 2(a_2 + a_3)$. Since $a_n > 0$ for all possible n's, you know that $(a_3 + a_2)$ does not equal 0 and you can divide both sides by it: $a_3 - a_2 = 2$ and $a_3 = a_2 + 2$. Using the definition of a_3, you know $a_3 = a_2^2 - a_1^2 = a_2^2 - 4$. Substituting for a_3 yields: $a_2 + 2 = a_2^2 - 4$ and $a_2^2 - a_2 - 6 = 0$. Factor and solve: $(a_2 - 3)(a_2 + 2) = 0$; $a_2 = 3$ or -2. a_n must be positive, so $a_2 = 3$ and $a_3 = a_2 + 2 = 3 + 2 = 5$.

54. **(D).** Since a_1 and a_3 are integers, a_2 must also be an integer: $a_3 = (a_1 + a_2)/2$ or INT = (INT + a_2)/2 so 2(INT) = INT + a_2 and a_2 = 2(INT) - INT which is itself an integer. a_4 will be the average of two integers. If $a_2 + a_3$ is even, a_4 will be an integer. If $a_2 + a_3$ is odd, a_4 will be a decimal ending in 0.5. If a_4 is an integer, a_5 can be an integer or can be a decimal ending in 0.5. If a_4 is a decimal ending in 0.5, a_5 must be a decimal ending in 0.25 or 0.75. a_5 cannot be a decimal ending in 0.375 such as 75/8 = 9.375. Note that a_5 can be negative: Even if a_1 and a_3 are positive, that does not rule out the possibility that a_2 (and subsequent terms) could be negative.

55. **(D).** Use the definition of ? to rewrite the equation: $\left|\frac{x+1}{x}\right| - \frac{2+1}{2} = \frac{1}{2}\left(\left|\frac{x+1}{x}\right| - \frac{-1+1}{-1}\right)$. Simplifying yields: $\left|\frac{x+1}{x}\right| - \frac{3}{2} = \frac{1}{2}\left|\frac{x+1}{x}\right|$. Let $z = \left|\frac{x+1}{x}\right|$. Substitute z into the equation: $z - 3/2 = (1/2)z$ or $z = 3$. To solve $\left|\frac{x+1}{x}\right| = 3$, take two cases

(1) $\frac{x+1}{x} > 0$. $\frac{x+1}{x} = 3$ or $x = 0.5$.

(2) $\frac{x+1}{x} < 0$. $\frac{x+1}{x} = -3$ or $x = -0.25$. The sum of the solutions is $0.5 + (-0.25) = 0.25$.

56. **(B).** Expressed algebraically, $\sqrt{10-3X} > X$. Because both sides of this inequality are non-negative, you can square both sides to result in the following:

$10 - 3X > X^2$
$0 > X^2 + 3X - 10$
$0 > (X + 5)(X - 2)$

Now, because the product of $(X + 5)$ and $(X - 2)$ is negative, you can deduce that the larger of the two expressions, $(X + 5)$, must be positive and the smaller expression, $(X - 2)$, must be negative. Therefore, $X > -5$ and $X < 2$. Combining these yields $-5 < X < 2$.

However, because the question indicates that X is non-negative, X must be 0 or greater. Therefore, $0 \leq X < 2$. The absolute value sign in Quantity A doesn't change anything — X is still greater than or equal to zero and less than 2, and Quantity B is larger.

Alternatively, plug the value from Quantity B into $\sqrt{10-3X} > X$:

$$\sqrt{10-3(2)} > 2$$
$$\sqrt{4} > 2$$
$$2 > 2$$

This is FALSE, so X cannot be 2.

Now, plug in a smaller or larger value to determine whether X needs to be greater than or less than 2. If $x = 1$:

$$\sqrt{10-3(1)} > 1$$
$$\sqrt{7} > 1$$

$\sqrt{7}$ is between 2 and 3, so this is true.

Trying values will show that only values greater than or equal to zero and less than 2 make the statement true, so Quantity A must be smaller than 2.

57. **(A).** The area of an equilateral triangle is $\frac{b^2\sqrt{3}}{4}$ where b is the length of one side. Since this area is between 25

$\sqrt{3}$ and $36\sqrt{3}$, you can substitute to get $25\sqrt{3} < \frac{b^2\sqrt{3}}{4} < 36\sqrt{3}$. Dividing all sides by $\sqrt{3}$ yields $25 < b^2/4 < 36$. Multiplying all sides by 4 yields $100 < b^2 < 144$, and taking the square root of all sides, one gets $10 < b < 12$. Since every possibility for b is greater than 9, Quantity A is larger.

58. **(E).** When dealing with absolute values, you must typically consider two outcomes. First determine the outcome if the expression within the absolute value sign is positive. So, if $8 - 2x > 0$, then $|8 - 2x| = 8 - 2x$, and therefore $8 - 2x < 3y - 9$ or $2x > 17 - 3y$.

You also must determine the outcome if the expression within the absolute value sign is negative. So, if $8 - 2x < 0$, then $|8 - 2x| = 2x - 8$, and therefore $2x - 8 < 3y - 9$ or $2x < 3y - 1$. Combining these two inequalities, one arrives at $3y - 1 > 2x > 17 - 3y$.

Now a quick sanity check to make sure the inequality makes sense: $3y - 9$ must be greater than 0 or the absolute value could not be less than $3y - 9$. So $y > 3$. This means $17 - 3y < 8$, and $3y - 1 > 8$, so there is definitely room for $2x$ to fit between those values. If the potential values of $17 - 3y$ and $3y - 1$ had overlapped, this would be an indication either that a mistake had been made or that the problem required further investigation to refine the result. As it is, (E) will work as an answer for this problem.

59. **(A).** This is an overlapping set problem. Matrix 1 shows an initial setup for a double-set matrix. The columns are headed “Skilled in BJJ” and “Not Skilled in BJJ.” The rows are headed “Skilled in Muy Thai” and “Not Skilled in Muy Thai.” There is also a total row and a total column.

When dealing with overlapping sets, consider whether the question is giving information regarding the population as a whole or regarding a subset of the population. While the first statement (“30% of all fighters”) refers to the whole population, the second statement (“20% of the fighters who are not skilled in Brazilian Jiu Jitsu”) refers to a subset of the population, in this case the 40% who are not skilled in Brazilian Jiu Jitsu. Thus, 8% are skilled in Muy Thai but not in Brazilian Jiu Jitsu, as seen in Matrix 1.

Matrix 1

	Skilled in BJJ	**Not Skilled in BJJ**	**Total**
Skilled in Muy Thai	> 30	8	
Not Skilled in Muy Thai			
Total	60	40	100

Matrix 2 shows how to fill out additional cells. Notably, there are some ranges of values that are possible for the cells in the first column. These ranges are limited by 0 on the low end and 60 on the high end.

Matrix 2

	Skilled in BJJ	**Not Skilled in BJJ**	**Total**
Skilled in Muy Thai	> 30 but $\leq$ 60	8	
Not Skilled in Muy Thai	$\leq$ 0 but < 30	32	
Total	60	40	100

Matrix 3 shows the ranges of values that are possible for the percentage of people skilled in Muy Thai and the percentage of people not skilled in Muy Thai. Particularly, the percent of fighters who are skilled in Muy Thai is greater than 38 but less than or equal to 68. Thus, Quantity A is larger.

Matrix 3

	Skilled in BJJ	**Not Skilled in BJJ**	**Total**
Skilled in Muy Thai	> 30 but ≤ 60	8	> 38 but ≤ 68
Not Skilled in Muy Thai	≤ 0 but < 30	32	≤ 32 but < 62
Total	60	40	100

60. **(B).** This is a good example of a problem where one can use the idea of extreme values. You can express this situation with the equation $r = \frac{kb}{n^2}$, where k is a constant. Quadrupling b and more than tripling n yields the following equation: $r_1 = \frac{k \times 4b}{(\text{"greater than } 3n\text{"})^2}$, where r_1 represents the new rate of data transfer.
Squaring a value that is greater than 3 produces a value that is greater than 9, allowing one to rewrite the equation as $r_1 = \frac{k \times 4b}{\text{"greater than } 9n^2\text{"}}$. Rearranging this equation yields $r_1 = \frac{4}{\text{"greater than } 9\text{"}} \times \frac{k \times b}{n^2}$ divided by a value greater than 9 is a value that is less than $\frac{4}{9}$. Thus, Quantity B is greater.

Chapter 31
of

Essays

In This Chapter…

How to Study for the AWA Using This Book

Included in the AWA (Analytical Writing Assessment) section of the GRE are two essays — an Issue Essay and an Argument Essay — written in 30 minutes each.

If you feel extremely confident about your writing and just want to get a feel for GRE-specific essay writing, feel free to simply read over the topics and some of the sample essays and comments that appear in the answer section.

However, if you need to seriously improve your AWA performance, do NOT read all of the essay topics at once.

Instead, set yourself up with a word processing program on a computer (any text program is fine — the simpler the program, the more like the GRE the experience will be) and a timer. Turn off any spellchecking feature in your word processing program.

Bookmark the pages in this book that contain the Issue and Argument essay topics. Set the timer for 30 minutes. When the clock begins, read Issue Topic #1 and plan and write an essay on this topic. STOP when the 30 minutes are up.

Then, without pausing or taking a break, turn to the Argument essays, reset the timer for 30 minutes, read Argument Topic #1 and plan and write an essay on that topic. STOP when the 30 minutes are up.

To review your work, read the commentary and sample essays on these essay topics (found in *Issue Essay Guidelines and Sample Responses* and *Argument Essay Guidelines and Sample Responses*). Compare your essays to the samples. While obviously there are many positions one can take on a particular topic, ask whether your own essays are as well-structured, persuasive, and on-topic as the examples. (We find that Issue Essays on the same topic tend to differ quite a bit from each other, but good Argument Essays on the same topic are often quite similar — each Argument topic has several flaws planted by the test writers for test takers to write about.)

Practice with the other topics in the same way. You may decide after a few rounds of practice that you only want to practice Issue topics or only want to practice Argument topics. This is fine. But please do observe good practice habits — always time yourself, and keep in mind that reading and thinking about the topic is included in the 30 minutes. Ideally, you should spend about 2–3 minutes reading and thinking about the topic, which will give you 27–28 minutes to write and revise.

Finally, should you exhaust the resources in this book, you may wish to know that ETS, the makers of the GRE, have actually released all of their actual Issue and Essay topics! You can read and practice with hundreds of such topics on ETS.org.

Issue Essay Topics

The GRE's AWA (Analytical Writing Assessment) includes two essays — an Issue Essay and an Argument Essay — to be written in 30 minutes each. There is no break in between the essays or after the AWA.

On the computer-based GRE, the Issue Essay prompt will appear on the screen above a text box in which you may write as much as you wish, or as much as you are able in the 30-minute time limit. Most top-scoring essays tend to be 4–6 full paragraphs long (although this is not a requirement). You should write an introductory paragraph, 2 or more body paragraphs with one main point per paragraph, and a concluding paragraph.

You will have scratch paper on which to plan your essay (or you may make notes in the text box and then delete them). The system does not have any type of spell check, but does allow you to cut and paste.

According to ETS:

> Trained GRE readers will evaluate your response for its overall quality based on how well you:
>
> - respond to the specific task instructions
> - consider the complexities of the issue
> - organize, develop, and express your ideas
> - support your ideas with relevant reasons and/or examples
> - control the elements of standard written English

Here are the official instructions from the test screen:

> You have 30 minutes to plan and compose a response to the issue below. A response to any other issue will receive a score of zero. Make sure that you respond according to the specific instructions and support your position on the issue with reasons and examples drawn from such areas as your reading, experience, observations, and/or academic studies.

To practice your essay writing, we suggest typing your work into any word processing program on your computer. Start a timer for 30 minutes BEFORE READING THE TOPIC. The time you spend reading the topic and planning your work is part of the 30 minutes. Before you take the real GRE, you need to be able to read a GRE essay topic and plan your work in less than 5 minutes, so you have enough time to actually write the essay.

Discussion of the topics and sample essays are located in the answer key to this section of the book.

<u>Issue Topic #1</u>

> Claim: In order to help small businesses thrive, government should play a minimal role in private business matters.
>
> Write a response in which you discuss the extent to which you agree or disagree with the claim. In

developing and supporting your position, be sure to address the most compelling reasons and/or examples that could be used to challenge your position.

Issue Topic #2

Schools should do more to prepare students for the non-academic aspects of adulthood.

Write a response in which you discuss the extent to which you agree or disagree with the recommendation and explain your reasoning for the position you take. In developing and sup porting your position, be sure to address the most compelling reasons and/or examples that could be used to challenge your position.

Issue Topic #3

People should question the rules of authority as opposed to accepting them passively.

Write a response in which you discuss the extent to which you agree or disagree with the statement and explain your reasoning for the position you take. In developing and supporting your position, you should consider the reasons for which the statement may or may not be true and explain how these considerations shape your position.

Issue Topic #4

If two applicants for a job are otherwise equally qualified, the job should go to the applicant with more experience.

Write a response in which you discuss the extent to which you agree or disagree with the statement and explain your reasoning for the position you take. In developing and supporting your position, you should consider the specific circumstances in which adopting the position would or would not be advantageous and explain how these examples shape your position.

Issue Topic #5

Schools should cut funding for extracurricular activities such as sports and the arts when school buildings are in need of repair.

Write a response in which you discuss the extent to which you agree or disagree with this recommendation and explain your reasoning for the position you take. In developing and sup porting your position, describe specific circumstances in which adopting the recommendation would or would not be advantageous and explain how these examples shape your position.

Issue Topic #6

Creativity should be used as the only true measure of intelligence.

Write a response in which you discuss the extent to which you agree or disagree with the claim. In developing and supporting your position, be sure to address the most compelling reasons and/or examples that could be used to challenge your position.

Issue Topic #7

Government funding for pure science endeavors, such as space exploration, should be reduced in order to direct more funding towards humanitarian science projects.

Write a response in which you discuss the extent to which you agree or disagree with the claim. In developing and supporting your position, be sure to consider specific instances for which this statement may or may not be true.

Issue Topic #8

The fact that technology is outpacing the needs of those in cultures that can afford the technology creates cultures of excess consumerism.

Write a response in which you discuss the extent to which you agree or disagree with the claim. In developing and supporting your position, be sure to address the most compelling reasons and/or examples that could be used to challenge your position.

Issue Topic #9

Luck plays more of a role in determining success than work ethic does.

Write a response in which you discuss the extent to which you agree or disagree with the claim. In developing and supporting your position, be sure to address the most compelling reasons and/or examples that could be used to both support and challenge your position.

Issue Topic #10

Some people believe that competition drives young athletes to perform at their best, while others believe that competition discourages those who are not athletically talented from participating in organized sports.

Write a response in which you discuss which view more closely aligns with your own and explain your reasoning for your position. In developing and supporting your position, be sure to address both views presented.

Argument Essay Topics

The GRE's AWA (Analytical Writing Assessment) includes two essays — an Issue Essay and an Argument Essay — to be written in 30 minutes each. There is no break in between the essays or after the AWA.

On the computer-based GRE, the Argument Essay prompt will appear on the screen above a text box in which you may write as much as you wish, or as much as you are able in the 30-minute time limit. Most top-scoring essays tend to be 4–6 full paragraphs long (although this is not a requirement). You should write an introductory paragraph, 2 or more body paragraphs with one main point per paragraph, and a concluding paragraph.

You will have scratch paper on which to plan your essay (or you may make notes in the text box and then delete them). The system does not have any type of spellcheck, but does allow you to cut and paste.

According to ETS:

> Trained GRE readers will evaluate your response for its overall quality based on how well you:
>
> - Respond to the specific task instructions
> - Consider the complexities of the issue
> - Organize, develop, and express your ideas
> - Support your ideas with relevant reasons and/or examples
> - Control the elements of standard written English

Here are the official instructions from the test screen:

> You have 30 minutes to plan and compose a response to the issue below. A response to any other issue will receive a score of zero. Make sure that you respond according to the specific instructions and support your position on the issue with reasons and examples drawn from such areas as your reading, experience, observations, and/or academic studies.

To practice your essay writing, we suggest typing your work into any word processing program on your computer. Start a timer for 30 minutes BEFORE READING THE TOPIC. The time you spend reading the topic and planning your work is part of the 30 minutes. Before you take the real GRE, you need to be able to read a GRE essay topic and plan your work in less than 5 minutes, so you have enough time to actually write the essay.

Discussion of the topics, as well as sample essays, are in the answer key to this section of the book.

Argument Topic #1

> A recent study by the Center for Disease Control and Prevention found that employees with paid sick leave are 28 percent less likely to be involved in a work-related accident than employees who do not receive payment for sick leave. Researchers hypothesize that employees with unpaid sick leave feel pressured to work during time of illness for fear of lack of pay. On-the-job accidents are then spurred by impaired

judgment or motor skills due to illness or illness-related medications. The highest-risk occupations, such as construction, showed the highest discrepancy between paid and unpaid leave.

Write a response in which you discuss what questions would need to be answered in order to determine whether the researchers' hypothesis is reasonable. Be sure to explain what effects the answers to these questions would have on the validity of the hypothesis.

Argument Topic #2

The city council of Town X has proposed reducing the city's electric expenses by switching all the lights in public buildings from incandescent bulbs to light-emitting diodes (LEDs). The switch would be made gradually as the old incandescent bulbs burn out, and the city council reasons that since LED lights burn brighter and cost no more to purchase, the switch would help Town X save money on electrical costs in the future.

Write a response in which you discuss what questions would need to be answered to help evaluate the efficacy of the city council's proposal to save money on electrical costs. Be sure to explain how the answers to these questions would help to evaluate the council's prediction.

Argument Topic #3

Company X has just switched to a 4-day workweek, mandating that employees work 10 hours per day from Monday to Thursday instead of 8 hours per day from Monday to Friday. Although the policy is new, Company X claims that the policy will help to increase profits by shutting down offices on Fridays and to boost employee morale by reducing commuting time to and from work each week.

Write a response in which you examine the stated and/or unstated assumptions of the argument. Be sure to explain how the argument depends on these assumptions and what the implications are for the argument if the assumptions prove unwarranted.

Argument Topic #4

Five years ago, the local university built two new dormitories through different contractors: Aleph Construction and Gimmel Builders. The buildings were nearly identical, though it cost Gimmel Builders approximately 20 percent more to construct their dormitory. Aleph's dormitory, however, has required approximately 10 percent more in maintenance costs per year over the past five years. Therefore, to construct their new dormitory with the lowest overall cost, the local university should hire Aleph Construction.

Write a response in which you examine the stated and/or unstated assumptions in the expert's claim. Be sure to explain how the argument depends on these assumptions and what the implications are for the argument if the assumptions prove unwarranted.

Argument Topic #5

Airline industry representatives have recently argued that flying is safer than driving, citing two separate studies. First, US statistics show that each year there are approximately 40,000 deaths in automobile accidents versus only approximately 200 in flight accidents. Second, studies indicate that pilots are four times less likely than average to have accidents on the road.

Write a response in which you discuss what questions would need to be answered in order to determine whether the argument is reasonable. Be sure to explain what effects the answers to these questions would have on the validity of the argument.

Argument Topic #6

In a laboratory study of two different industrial cleansers, CleanAll was found to remove 40% more dirt and kill 30% more bacteria than the next best cleanser. Furthermore, a study showed that employees working at buildings cleaned with CleanAll used far fewer sick days than employees working in buildings cleaned with other cleansers. Therefore, to prevent employee illness, all companies should use CleanAll as their industrial cleanser.

Write a response in which you examine the stated and/or unstated assumptions of the argument. Be sure to explain how the argument depends on these assumptions and what the implications are for the argument if the assumptions prove unwarranted.

Argument Topic #7

Downtown Zurzi is becoming increasingly congested with traffic, increasing commuting time for those who work downtown or near downtown. The nearby city of Loft was faced with the same problem several years ago and implemented a small weekly tax for driving one’s car downtown. Downtown traffic almost immediately subsided in Loft and the local government also raised much needed money for fixing roads elsewhere. Obviously, this plan should be implemented in Zurzi in order to solve the brewing traffic congestion problem.

Write a response in which you discuss what specific evidence is needed to evaluate the argument and explain how the evidence would weaken or strengthen the argument.

Argument Topic #8

In the last year’s mayoral election in Town T, candidate Miller led candidate Keating by a substantial margin in the polls leading up to the election. At the last minute, candidate Keating launched a widely viewed series of television advertisements that focused on preserving the natural environment of Town T, a topic neglected by candidate Miller. Subsequently, candidate Keating won the election by a narrow margin. This year, if candidate Miller hopes to win the upcoming mayoral election, he must increase his coverage of the topic of preserving the natural environment of Town T.

Write a response in which you examine the stated and/or unstated assumptions of the argument. Be sure to explain how the argument depends on these assumptions and what the implications are for the argument if the assumptions prove unwarranted.

Argument Topic #9

Last year PrepUp had record enrollment in their test prep courses, but yearly profits fell by nearly 30 percent. To the contrary, TopPreparation had comparable enrollment to the year be fore, with profits rising by approximately 20 percent. This discrepancy most likely results from the fact that PrepUp teaches only live, in-person classes, which require expensive rental spaces and teaching equipment. Although TopPreparation’s total enrollment remained unchanged, its online enrollment increased by 50 percent, and online classes have a much lower overhead. In order to address these issues of profitability in the coming year, PrepUp will begin offering online test prep courses.

Write a response in which you discuss what questions would need to be answered in order to decide whether PrepUp's plan for the coming year and the argument on which it is based are reasonable. Be sure to explain how the answers to these questions would help to evaluate the plan.

Argument Topic #10

Cot-Ten, a cotton production company, has recently faced profitability issues based on the use of Chemical X in its manufacturing process. The main by-product produced when using Chemical X is covered under stringent environmental regulations, making it very difficult and expensive to dispose of. A similar processing product, Chemical Y, has recently been discovered, and can be used by Cot-Ten at a minimal cost of switching. The CEO of Cot-Ten has declared that the company will increase profits by switching to Chemical Y by the end of the month.

Write a response in which you discuss what specific evidence is needed to evaluate the argument and explain how the evidence would weaken or strengthen the argument.

Issue Essay Guidelines and Sample Responses

<u>Issue Topic #1</u>

> Claim: In order to help small businesses thrive, government should play a minimal role in private business matters.
>
> Write a response in which you discuss the extent to which you agree or disagree with the claim. In developing and supporting your position, be sure to address the most compelling reasons and/or examples that could be used to challenge your position.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Jennifer Dziura:

Unlike some Issue Essay topics, this one isn't too extreme. If a topic says "never" or "always," you almost have to disagree with it. But this topic says "a minimal role," so you really could agree or disagree without taking too extreme a position. Since you are being asked for your opinion — inasmuch as you can back it up with reasons and examples! — there's no reason you *can't* take an extreme position, but extreme positions are simply much harder to defend with solid reasons and examples. There are usually exceptions to any extreme position, and nuanced positions both look a little more sophisticated and are easier to defend.

I can see that government meddling too much in the affairs of small businesses could make it really hard to start a small business (big businesses have people whose entire job it is to do paperwork, but smaller operations can be seriously burdened by complicated systems of permits and permissions). But obviously (to me, anyway!) the government needs to keep businesses from discriminating against employees, polluting the environment, and selling dangerous products (for companies that sell food, vitamins, beauty products, baby products, cars, etc.).

But wait! Before I write about how it's important for government to regulate product safety, pollution, etc. — *actually, I don't think that's on-topic!* I was asked to write about how the government should act "in order to help small business thrive," not "in order to make the world a better place" (I guess I care more about the second part!)

Okay, in order to make sure I stay on-topic, I'm going to argue that the government should regulate small businesses *so the public will trust small businesses, which will help small businesses thrive.*

It's important to make sure that every reason or example you use ADDRESSES THE TOPIC, and that all of your argumentation is unified under a coherent and on-topic main point.

Note: I also had some other ideas about how maybe the government should protect small business from big business. I was thinking about Wal-Mart putting local shops out of business, or Chick-fil-A (a US fast food joint that uses the slogan "Eat More Chicken") suing a man in Vermont who makes t-shirts that say "Eat More Kale." I think this could also make a good point for this essay topic (or a good focus for a whole essay on this topic!), but it doesn't really fit into my other idea (that the government should leave small businesses alone except to protect the public), so I'm not going to use this.

Finally, the topic says "be sure to address the most compelling reasons and/or examples that could be used to challenge your position," so I'll make that my third body paragraph. What would my opponents say? I'm thinking here about people who argue that corporations have the legal status of people, and that people have the right to voluntarily transact as they wish. Honestly, I was just arguing about this on Facebook ("We'd be paying private companies for the privilege of driving over bridges that don't have to meet any government safety standards! There'd be mice in our sausages! Children working 12-hour days in factories! Lunch counters in the South could still be segregated!"), so I feel very prepared. (As well as extremely opinionated!)

Okay, I'm ready to start writing. Here are the notes I'd type into the text box (and then delete as I was writing the real essay):

> thesis: gov should stay out of small business affairs but should protect human rights and safety
> I. staying out: too many regulations can hamper entrepreneurship
> II. protecting the public is important so public trusts small business
> III. challenges to my position: if people want to freely trade money for services, no one should get in their way (my response: governments always provide services that are impractical for people to provide for themselves, like safety testing)

Note: One good essay time-saver is to copy and paste your introduction into the conclusion spot, and then re-word it as your conclusion. It's easier than writing from scratch!

Essay:

Small businesses need many conditions in place in order to thrive: startup capital, a suitable location and physical infrastructure, shrewd founders who can adapt to changing circumstances — and also, of course, customers. While, in general, a government's hands-off policy towards small business can help small businesses by freeing owners from time-consuming and cumbersome regulations, it is also the case that some government regulations are needed to protect the public from unscrupulous business owners. This type of regulation not only helps the public, but helps small businesses thrive by increasing consumer confidence in small businesses, as well as protecting ethical small businesses from unfair competition from businesses that break the rules.

The United States has one of the highest rates of entrepreneurship in the developed world — much higher than the rates in Western Europe. Why? In the U.S., virtually anyone can start a business. For about $35, one can obtain a business license, and then you're officially in businesses. Hiring the first employee or two is also relatively simple, and some workers can be paid as contractors, which eliminates the need to administer payroll taxes. In contrast, in Western European nations where you need employment papers (and sometimes references) to even open a simple checking account, naturally, the regulations and paperwork needed to start an enterprise can be daunting. Furthermore, laws intended to protect workers mean that it is virtually impossible to fire anyone, which can be a real burden for a small business that simply cannot survive with employees who are not up to the task. The lack of regulation in the U.S. is a strong contributor to the burgeoning of small business in that country.

On the other hand, a pure laissez-faire policy would be disastrous for small business, as well as for society. Should we allow small businesses to sell "health cures" that actually make people sick, or car seats for babies that haven't been through safety testing? Of course not — even if requiring safety testing puts a burden on small business. The safety of the public is important, to be sure, but public confidence is also important if small businesses are to have any customers. Furthermore, some small business owners (just like large business owners) practice racial discrimination, sexual harassment, and other abuses of workers, such as paying undocumented workers less than minimum wage. It is important for human rights to curtail these abuses; it also helps small businesses for the government to put a check on such practices, because then legitimate, ethically-run businesses can compete on a fair playing field.

While this essay propounds a system of minimal to moderate regulation of small businesses, some might argue that government has no right to intrude on the private dealings of businesses and their customers. According to this view, if someone wants to offer unlicensed cosmetic injections, and budget-minded consumers are willing to have their wrinkles filled with who-knows-what, then it is the right of both parties to transact as they wish. However, in developed nations, governments take on, at very least, the provision of services that individuals cannot provide for themselves — military defense, the creation of highways and other infrastructure, and the testing of food and medicine on a wide scale, among others. Individuals do not have the ability to test every can of baby food for botulism or to evaluate the credentials of a "doctor." Thus, in industries in which human health and safety are at issue, some regulation is justified.

How should governments best help small businesses thrive? In some sense, the answer is that governments should back off and let people do as they will. In a free market, many small businesses will be launched, many will fail, and the best will survive. However, some government intervention is required to protect the public from dangerous products and harmful business practices. These regulations ultimately help small business as a whole by encouraging consumer confidence in these businesses.

Here is a second opinion from Manhattan Prep instructor Emily Sledge:

Concerning government involvement in private business matters, we might imagine a scale from "total" on the left to "none" on the right. Jen has successfully argued for slightly right of center on this scale, making her position clear and taking a more easily defensible position than either extreme would be. Notice some of her techniques: the second paragraph explains her most "extreme" position, giving the United States as an example of how thriving small businesses are correlated with less regulation. So if less regulation is good, wouldn't no regulation be even better? Not so fast! The remaining body paragraphs swing the essay back toward the center, offering examples of why various and diverse stakeholders (the public, small business owners, employees) benefit from some regulation, and why certain situations require some government involvement. The reiteration of her thesis at the beginning of the fourth paragraph is a nice touch; Jen gives many detailed examples in the third and fourth paragraphs, so it is wise to remind the reader of the big picture periodically.

Issue Topic #2

School should do more to prepare students for the non-academic aspects of adulthood.

Write a response in which you discuss the extent to which you agree or disagree with the recommendation and explain your reasoning for the position you take. In developing and sup porting your position, be sure to address the most compelling reasons and/or examples that could be used to challenge your position

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Tommy Wallach:

This is a pretty juicy prompt, as they go, because it's about education. Prompts about business experience or scientific endeavors require a lot more hypothesizing on my part, as I've never been in business or conducted scientific research. But for this prompt, I can draw on personal experience. Everyone's been to school!

While you can rest an entire essay on your own experience, I like to mix it in with at least one fact/hypothetical example, so it doesn't feel like I'm writing some kind of personal essay. However, don't be afraid to use the first person in your essays; it will not cause you to lose points. I would certainly draw on my own opinion of how my academic and non-academic education served me in order to write this essay.

Maybe because I'm the artsy type, I'm going to choose to agree with the prompt, though this one is definitely open enough that you could go either way. My thesis will be that most of post-scholastic life is non-academic, so actually the non-academic should be the *focus* of school. This is slightly more extreme than necessary, but I feel confident that I can support it.

Finally, the topic says "be sure to address the most compelling reasons and/or examples that could be used to challenge your position," so I'll make that my third body paragraph. Providing a straw man argument (defined as a sham argument you set up just to knock it down) is never a bad idea in the Issue Essay, but it is of course required with a prompt like this.

Alright. Next, I'd type up a brief outline to make sure I had enough meat for a good essay. Don't forget to delete it from the box before you submit the essay. It probably wouldn't kill you either way, but why confuse your reader?

> Thesis: life after school is non-academic, so non-academic > academic
> P1: Economic crisis because of financial illiteracy
> P2: My experience: private school with math/science focus
> P3: Challenges to my position: specialized tech jobs require serious academics: response = Microsoft hiring riddle

Essay:

Education is certainly the hot-button issue of the moment, with national policies like Race to the Top and No Child Left Behind changing the way we teach, measure academic success, and even learn. At the heart of the debate over the future of education is the question broached in this prompt: are academic subjects, such as mathematics and history, more worthy of students' time than life skills, such as critical thinking, ethics, and self-expression? Many refuse to let go of the system of rote memorization and strict quantitative evaluation that has long characterized education in America, yet in a fast-paced world characterized by huge systemic problems and constant seismic shifts in science, business, and technology, this system is no longer functional. Adulthood is primarily non-academic, and school is meant to prepare students for adulthood. Thus, schools should indeed do more to prepare students for the non-academic aspects of life.

At the heart of the recent financial crisis was the proliferation of sub-prime mortgages, in which people without any capital were able to procure a mortgage the payments of which they would never be able to afford. How were so many people hoodwinked by predatory loans? It is because their education, in spite of all of its math classes, did not prepare them in any way for the mathematics of the real world. What use is calculus, which only a tiny subset of students will ever use in their lives or jobs, in comparison to lessons in reading contracts or investing intelligently, something that every person will need to do at one point or another? It is not too extreme to suggest that if our schools spent more of their time focused on finance as it actually affects people's lives, the crisis could have been avoided, or at least significantly mitigated.

And non-academic skills are as important on the micro level as they are on the macro level. I attended a private school in Seattle, Washington with a firm focus on mathematics and science; we were required to take both a math and a science class every year, and most of us were encouraged to take AP and honors classes in these subjects. While there is something to be said for working outside of your comfort zone, the intense attention paid to academics came at the expense of those subjects that most interested me. During high school, I competed as a concert pianist and performed in professional theater productions in Seattle. Like me, the vast majority of my classmates did not go into fields involving either math or science, and those years we spent studying academic subjects to advanced levels caused us to lose out on time we could have spent developing passions from which we might have derived joy for the rest of our lives. Studies show that playing an instrument provides numerous benefits relevant to the non-academic aspects of adulthood, such as managing stress, persevering in the face of failure, and maintaining focus.

Of course, many educational theorists argue that the workforce is increasingly being channeled into either the service economy, comprised primarily of jobs requiring no education at all, and those jobs that require a high level of technical proficiency and thus a specialized academic education. These theorists believe that our only hope of remaining competitive in the world economy is to ensure that our schools focus on academics such as math and science, so that America's children will be prepared to step into those highly specialized jobs. However, it turns out that even the most advanced tech companies in the world claim that mental flexibility is far more important to their hiring decisions than pure knowledge (much of which they would rather pass on through training on-site, in order to promote their own methodologies). At interviews for Microsoft, potential candidates are said to have been asked the following riddle: "Why is a sewer grate round, rather than square?" The answer is that a round grate can never fall down the hole (a square can if you tilt it up and rotate it into the diagonal). Surely no amount of purely academic education would prepare one to answer a question such as this; only an educational regimen that propounded critical and flexible thinking about real world problems would be of any use.

An academic education is inarguably important. No student should be allowed to leave school without a solid understanding of basic mathematics and the ability to read critically and write coherently. But these skills are merely one part of what makes a well-rounded individual. For too long, American schools have been overly focused on quantifiable metrics of achievement; if we are to compete in the global economy, as well as fix some of the terrible problems afflicting our world, it will be necessary to design a new education premised on preparing students for the lives they are most likely to lead, rather than some idealistic concept.

Here is a second opinion from Manhattan Prep instructor Emily Sledge:

I thought this essay was strongest when Tommy used even-handed language, as he did in his summation, writing that an academic education is "inarguably important," and that certain basic skills are a minimum educational requirement, but that "these skills are *merely one part* of what makes a well-rounded individual." That's much tougher to dispute than the more extreme language Tommy used in the preceding paragraph: "Surely *no* amount of purely academic education would prepare one to answer a question such as this; *only* an educational regimen that propounded … *thinking about real world problems* would be of *any* use." Fortunately, Tommy's thesis — agreeing with the prompt — is fairly moderate, arguing that we should "do more" to prepare students for the non-academic aspects of life.

While Tommy did an admirable job in a 30-minute essay, an ideal essay might have countered certain objections. Tommy asks "What use is calculus … in comparison to lessons in reading contracts or investing intelligently?" But aren't academic topics such as reading comprehension, grammar, vocabulary, and algebra fundamental to these real life skills? One could make the case that calculus requires and teaches the very "critical and flexible thinking" skills Tommy later cites as important. The sewer grate question is a good one — intriguing if you've never given it any thought. Yet he cites geometry concepts in the answer to the puzzle, suggesting that academic topics are at the core of this "mental flexibility" exercise.

In summary, I liked the mix of personal, business, and education examples, and thought they were well-suited to an argument that academic and non-academic skills are both important to learn.

In general, I would recommend toning down strong assertions (such as those about the relative value of academic and non-academic study) to make your GRE essays more credible and more difficult to dispute.

<u>**Issue Topic #3**</u>

People should question the rules of authority as opposed to accepting them passively.

Write a response in which you discuss the extent to which you agree or disagree with the statement and explain your reasoning for the position you take. In developing and supporting your position, you should consider the reasons for which the statement may or may not be true and explain how these considerations shape your position.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Stephanie Moyerman:

First, I must establish my "take" on the statement. Paying careful attention to the exact wording, I notice that the statement suggests that people should only *question* the rules of authority, but does not imply or suggest any necessary actions. As I can see no harm in the merely questioning the rules of authority, I initially decide that I agree with the prompt. After all, great milestones in modern history, such as the Enlightenment and the American Revolution, have resulted from the questioning of authority.

I must remember, however, that I am not to explain whether I agree or disagree with the prompt, but the *extent* to which I agree or disagree. Any essay in which I express 100% agreement with the statement will be very difficult to defend. Therefore, I decide that my position will be to mainly agree with the statement, but to consider at least one situation in which the statement may not be true. I decide that in the case of elected officials and a government with a system of checks and balances, such as the United States, it would become very difficult to pass any law if all constituents questioned each and every rule. Further, the idea of elected leaders is that they best represent their constituents as a whole. Countries with a varied body of constituents that follow this statement could experience people questioning the laws that obviously benefit the society as a whole. This could lead to disorder, a more segregated society, or, in the very least, the hindrance of governmental progress.

With these points in mind, I type the following notes into the text box:

Main Idea: In most instances, people should question the rules of authority as opposed to accepting them passively. However, there are specific cases in which adhering strictly to this statement could have negative effects.

I. Questioning the rules of authority, in and of itself, does not suggest necessary actions, just thought.
II. The French and American Revolutions both resulted from questioning the rules of authority.
III. Questioning the rules of authority can lead to disorder, segregation, and/or hindrance of progress in a varied nation with elected officials.

Essay:

Glancing back through modern history, one will undoubtedly notice that many remarkable changes have been sparked by people questioning the rules of authority. One might argue that the outcomes of these changes have weighed on both the good and bad side of human history and that questioning authority can be wrong or dangerous. But it is not the questioning of authority that caused these end results; it is the action of the people involved. Questioning the rules of authority only reveals the necessity for change within an established institution. Thus, I strongly agree with the statement that people should question the rules of authority as opposed to accepting them passively. The questioning of authority incites necessary social and political change without implying any necessary action.

Take, for example, the formation of the United States of America. For years under British rule citizens in the colonies were unfairly taxed and treated in accordance with British law. Not until some very powerful men questioned British rule and stood up to defend their beliefs did the colonies come to think of themselves as an independent entity. Years later, when released from British rule, their original questioning of the rules of authority led the leaders of the United States to take a different authoritative stance than the British. The United States became a democracy ruled by the people. The questioning of authority not only brought about social change, but also had a strong influence on the shaping of future events. In fact, one of our famous forefathers, Benjamin Franklin, famously quoted that, "It is the first responsibility of every citizen to question authority."

Similarly, but more extreme, the citizens of France in the 18th century lived under the delusion that their monarchy was ordained by God for many years. These years were filled with hardship, starvation, and greed, where the monarchy and those around them lived untainted palatial lives while their citizens lay hungry in the streets. It was not until the masses came to question the monarchy's rules that social change was brought about and the government was overthrown. In this instance, however, many radical groups came to power after the monarchy collapsed, each reacting to the harsh rules of the previous authority. Eventually, by continuing to question these rules, the French were able to remove their tyrannical rulers and form a republic.

Some might claim that questioning the rules of authority necessarily implies violent action in the case of civil change; that one cannot exist without the other. This is certainly not the case. Take, for example, Mahatma Gandhi and his fight for India's independence. His questioning of authority not only inspired others to do so, but also inspired them to affect social change via non-violent protest. The questioning of authority, coupled with moral actions, led to the end of much tyranny and injustice throughout the world all through non-violent means.

This is not to say that I think that this statement is the best course of action in every situation and for every person. There are certain situations in which questioning authority could lead to consequences such as segregation, disorder, and/or a hindrance of progress. Take, for instance, the government of the United States, one with elected officials and a system of established checks and balances. When citizens independently elect their leaders, one must bear in mind that these officials are elected to service the needs of the constituents as a whole. In a system where every person questions each new rule, as the statement implies, political leaders must cater to many different viewpoints, hindering progress. Further, natural divisions arise from people with differing views on the laws, such as Democrats and Republicans. These divisions segregate populations and hinder progress as well.

In essence, I agree with this statement as long as it is not taken to the extreme. One cannot achieve social or political change without questioning the authority, and certainly changes have been necessary in our history and will be necessary in our future. However, certain situations do exist where an entire population adhering strictly to this statement could find itself hindering progress and drawing somewhat arbitrary lines among people.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

What an interesting take on this topic! As an American, my first thought when I read this topic was, "Of *course* people should question the rules of authority!" After all, questioning the rules doesn't mean you have to break them — sometimes questioning the rules leads a person to start a political movement or write an article or just sign a petition, and sometimes it causes a person grow up, major in political science, and then run for office. Questioning the rules is the basis of critical thinking and citizenship!

So, I was pretty surprised to see Stephanie argue that too much questioning of the rules could lead to disorder. While Stephanie made the decision to write a more nuanced essay by taking a middle-ground point of view, I would have written a more hard-hitting essay in favor of questioning the rules all the time. I might have used examples of the civil rights movement in the US (in which Martin Luther King, Jr. and many others went to jail by breaking the rules in a fight for justice), versus countries in which no dissent is tolerated (such as North Korea). I also tend to use John Stuart Mill's concept of the marketplace of ideas in many essays, and that would have fit very nicely here.

Overall, though, Stephanie has written a well-structured and well-developed essay. Bonus points for the Ben Franklin quote!

Issue Topic #4

> If two applicants for a job are otherwise equally qualified, the job should go to the applicant with more experience.
>
> Write a response in which you discuss the extent to which you agree or disagree with the statement and explain your reasoning for the position you take. In developing and supporting your position, you should consider the specific circumstances in which adopting the position would or would not be advantageous and explain how these examples shape your position.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Stephanie Moyerman:

As a relatively young person in America's work force, I read this statement and my gut disagrees. However, I must be careful not to write a personal reaction essay, but instead to qualify the reasons for my disagreement and use them to shape my writing. In addition, the prompt states that I should consider the specific circumstances in which adopting the position would or would not be advantageous and discuss how these shaped my position.

To brainstorm my essay, I first try to pinpoint exactly why being young gives me such a strong personal reaction to this statement. Adhering to this prompt would certainly put younger applicants at a disadvantage, as they, by virtue of their fewer years on the planet, tend to have less work experience. In fact, if this were taken to the extreme and every company adhered strictly to this statement, it would become difficult for any young person to be hired into any competitive job due to the sheer number of applicants in today's job market. This would lead to a self-perpetuating cycle: young people lack experience, they miss job opportunities based on their lack of experience, and then are dismissed from future job opportunities because they cannot gain valuable job experience. I am not, however, advocating that jobs should be given to younger applicants, simply stating that judging by experience alone presents a bias against a certain group.

To claim that two people are "otherwise equally qualified" is also extremely vague. In every situation, a less arbitrary test could be constructed. Surely, if experience is that important, it should play a role in the test that decides the outcome. For instance, a coding challenge for otherwise equally qualified candidates for a computer programming job should favor the candidate with more experience, as their experience should equate to being better at such a task. However, if it does not, they should not be chosen for the job simply because their experience is greater.

After careful consideration, I realize that I strongly disagree with the statement. The job should not be given to the candidate with more experience, but should be decided on a case-by-case basis. Perhaps in the extreme case of budget or time constraints a concrete test such as experience would be helpful.

> Thesis: The job should not go to the applicant with more experience, but should be evaluated on a case-by-case basis.

(A) "Otherwise equally qualified" is a vague term. Can a distinction still be made? Situation dependent — the job itself, colleagues, overall goals, etc. play a rather large role.
(B) The number of applicants for competitive jobs would limit a young person's ability to achieve said job, creating a perpetual cycle.
(C) Extreme cases to favor the other side.

Essay:

In 2006, more than 1200 applicants applied for a single professorship opening at Harvey Mudd College, a small, liberal arts college in Southern California. The sheer number of applicants, all with PhDs in physics, made the application process extremely competitive. Judging only by resumes, transcripts, statements of intent, and recommendations, the teaching staff of the college narrowed the pool down to three finalists. With so many applicants for such a competitive position, it became all but impossible to distinguish between those that were most highly rated. In essence, these people were all equally qualified for the job. However, marked distinctions among the finalists remained. They were comprised of a late twenties recent PhD graduate from CalTech, a late-thirties professor from the University of North Carolina, and an early-thirties post-doctoral fellow from Stanford. The school did not defer to their differences in age or experience to decide. Instead, each candidate was flown to the college and presented a prepared lecture to the freshman class of approximately 200 students. The students evaluated each candidate in categories such as clarity and likability. One might imagine that lecturing experience might favor the professor, but in the end, the recent CalTech graduate achieved the highest ratings and was hired by the college. All three finalists had a fair and equal chance at the job upon final decision, though experience was not disregarded by the test. Had the college deferred to experience alone, our CalTech graduate would have been eliminated.

Harvey Mudd College is not unique in its hiring process. Almost all schools defer to another means of judgment for incoming candidates. The sheer number of applicants meeting the benchmark requirements for every open professorship guarantees two or more equally qualified applicants at the top. And, if every professorship nationwide were decided from this point based on experience, it would be all but impossible for young people to break into college and university teaching positions. Allowing experience to be the deciding factor would create a perpetual cycle — young applicants would be turned down due to their lack of experience, limiting their ability to gain experience and denying them access to future jobs if decided in the same fashion. There is, of course, nothing special about a university teaching position. Many lusted-after job openings experience a similar number of applicants. Judging identically equal applicants by experience appears to be an unnecessary bias against the young, especially when impartial tests remain that could aid in the decision making process.

There are circumstances, however, in which it would be advantageous to simply choose the job applicant with more experience. For instance, had the hiring budget for Harvey Mudd College been much lower, it would have been impossible to fly the candidates out and give them the impartial test that was mentioned. Or, if an applicant had to be chosen on a very short, strict time schedule, such tests of merit would not be feasible. In said cases, it seems acceptable to use experience as the deciding criteria between otherwise identically equal candidates.

Ultimately, unbiased hiring tests appear to be the best option for deciding between otherwise identically equal candidates. Experience factors into candidate performance, but does not become the only deciding factor. However, extreme circumstances, such as short time-scales and limited financial resources, may limit the ability to perform unbiased hiring tests. In these cases, hiring based on experience alone does appear to have its merits.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

I love the unusual introduction of this essay! While many Issue Essays tend to begin with a restatement of the topic, a developing sentence or two, and then a thesis, this essay begins with a story and actually builds a bit of suspense! Who will get the job?! As it turns out, not the most experienced candidate. A fantastic and appropriate story.

While Stephanie disagrees with the prompt, she offers a bit (just a bit!) of ground to the other side — she acknowledges that experience should be the determining factor *if* there are budgetary or time limitations. This is ceding very little ground indeed! She's basically saying, "I guess you could do it *your* way — if you don't have time to do it the *right* way."

I love that the essay doesn't just go against the prompt — it also offers a suitable alternative ("unbiased hiring tests"). As such, this essay was very persuasive.

Issue Topic #5

> Schools should cut funding for extracurricular activities such as sports and the arts when school buildings are in need of repair.
>
> Write a response in which you discuss the extent to which you agree or disagree with this recommendation and explain your reasoning for the position you take. In developing and sup porting your position, describe specific circumstances in which adopting the recommendation would or would not be advantageous and explain how these examples shape your position.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Stacey Koprince:

This is a tricky little topic because we can interpret the circumstances in so many different ways. If we take the phrase "in need of repair" literally, then we might think, "Well, if you *need* to repair it, you have to! What else can we do?"

Real life is never quite this black and white, though. "In need of repair" can mean anything from the imminent collapse of the walls to, hey, there's a dent in this wall over here.

As I thought this through, I realized that, first, I needed to articulate when you would actually need to make a certain type of repair and when you would have more leeway to let something go. In this case, I came up with the idea that a school, first and foremost, delivers academic services. So, if neglecting to make a certain repair would prevent a school from delivering those academic services (or otherwise significantly impair the school's ability to do so), then that repair probably does need to be made. On the other, hand, something of a more aesthetic nature that would not interfere with academics, such as that dent in the wall, may or may not be more important than funding the tennis team.

Ultimately, then, a blanket policy is too heavy-handed. There needs to be a more nuanced ability to decide on a case-by-case basis. At this point, I'd typed up some notes for paragraphs 1 through 3, and I thought, "Hmm. What am I going to do for paragraph 4?" I realized that there was something else bothering me about the statement: why is it a choice between repairs and extracurriculars? Are there no other possible sources of funding? Maybe not, but let's actually establish that before we cut off Friday night football and Monday afternoon debate club.

And voila! I had my paragraph 4 and was able to start writing.

> P1: Disagree that this should be a blanket policy. Depends on other considerations… General principle: academics come first
> P2: Do repairs and cut extracurric's: if interferes with academic (heat doesn't work. roof is caving in)
> P3: Don't repair if doesn't interfere (paint is fading but not in any way problematic; cosmetic)
> P4: Is it really either/or? Alternate sources of funding? Special bonds when major repairs / renovations are needed?
> P5: Conclusion

Essay:

The author claims that, when school buildings are in need of repair, the schools should do two things:

prioritize these repairs over extracurricular activities and use the funds from those extracurricular activities to pay for the repairs. It is entirely possible that school districts will have to make a difficult choice such as this one, but it would be hasty to put into place a blanket policy without first examining the goals and priorities of the schools. Do schools serve solely to deliver academic services? I would argue no — athletics and art programs are very important as well, though academics are clearly the first priority. A school faced with such a choice might instead base such a decision upon whether the needed repair is one that would interfere with academics if it were not made.

For example, if a school's heating system breaks down in the winter or the roof weakens and is in danger of caving in, then clearly the school will need to make the necessary repairs immediately. If the necessary funding can be obtained only from curtailing extracurricular activities, then those activities must go by the wayside. If the school cannot deliver on its fundamentals (academics), then it cannot (or should not!) offer additional programs that are designed to enrich the core offering but are not a part of the core offering themselves. Note that there are two levels of decision to be made here: whether the problem, if not repaired, will interfere with academics, and whether the funds must come specifically from the after-school activities budget.

Alternatively, suppose that the school could use re-painting but there is nothing dangerous or otherwise problematic about the current paint job; the issue is, in other words, aesthetic. Suppose, again, that the funds for re-painting would have to come from the extracurricular activities fund. Is it more important for the school to be aesthetically pleasing or to offer after-school programs? Reasonable people may disagree as to the answer, and this is the whole point: all repairs are not equal, and it is not necessarily the case that any repair trumps all extracurricular activities.

The examination of these two hypotheticals allows us to uncover a problematic assumption with the original assertion. Is it the case that the school must make an either/or choice: either school repair or extracurricular funding? If significant, necessary repairs are needed, there are perhaps other avenues for funding: government grants, bonds voted for by the local community, even a fundraising drive. At the same time, for ongoing maintenance issues of a more aesthetic nature, schools might reasonably decide to make some minor compromises — repaint every 6 years instead of every 4, say — in order to stretch their money to cover extracurricular programs. After all, everyone has to stick to a budget, and we all make trade-offs every day in deciding how to spread our limited funds.

In conclusion, the author's assertion is too narrowly drawn. It is entirely possible that a school may have to cut funding to some extra programs when it finds itself faced with a costly maintenance problem that would interfere with academics if not addressed. After all, the school must first deliver on its mission to educate its students. There are many shades of grey, however, in distinguishing between necessary and "nice to have" maintenance, and it is by no means a foregone conclusion that any kind of maintenance trumps any kind of after-school program. In fact, there are many circumstances in which a school might reasonably defer or minimize long-term maintenance in order to maintain adequate funding for additional activities that can immeasurably enrich a student's life and learning.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

I think that this essay would score a 6 because it questions the definition of terms within the argument ("in need of repair"), and then *actually addresses the entire topic* — both the idea about repairs and the idea about extracurriculars. I've read enough mediocre essays to tell you what most test takers would do with this topic — they would write a whole essay about how sports and the arts are really important. That type of essay is unsophisticated, not entirely on topic, and quite frankly, childish. Adult life is full of hard decisions: sure, extracurriculars are important, but so are functional school buildings. So, now what? How do we decide between two important things? That's where a good essay should start.

By the way, Stacey's writes "grey" in the final paragraph. This is a Canadian/UK spelling (in the US, "gray" is more commonly used). I mention that because the official graders for the GRE see essays from students around the globe — they see American English, British English, Indian English, and other varieties, and they're used to seeing things phrased in slightly different ways. While excellent grammar would be a nice feature in an essay, the AWA section of the GRE really is more about the ideas.

Issue Topic #6

Creativity should be used as the only true measure of intelligence.

Write a response in which you discuss the extent to which you agree or disagree with the claim. In developing and supporting your position, be sure to address the most compelling reasons and/or examples that could be used to challenge your position.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Stacey Koprince:

I really struggled to think of examples for this topic. I knew immediately that the position was too extreme and I couldn't possibly agree with it — but specific arguments or examples to support my position? Ugh. So I did something extremely unusual (for me) here. I wrote notes for my first paragraph but, after banging my head against the wall for a minute or so, I then wrote my entire first paragraph without any notes written down for the other paragraphs, hoping that the act of writing would help the brainstorming side of my brain to kick into gear.

And it worked! I was able to keep writing. About halfway through the third paragraph, I went back to my notes to type up a very brief outline of what I was in the midst of doing, just to make sure that it made sense and that I wouldn't forget what I wanted to do in paragraph 4 when I got there.

You'll also see that I have three capitalized words in my notes for paragraph 1. I wrote this in response to the second sentence of the instructions: we have to address the ways in which others could "challenge [our] position." I always make it a point to acknowledge the other main point of view in my Issue Essays, but because they specifically requested it, I'm going to spend more time on this than usual — likely a paragraph rather than just a sentence or two.

P1: Examples. ACKNOWLEDGE OTHER SIDE!!! "Only" is extreme — one measure, not "only."
P2: ?
P3: ?
P4: protest by author: this is just creativity in disguise
P5: concl

Essay:

Intelligence can be measured in many forms — so many, in fact, that while the concept of intelligence has been around for millennia, we're still debating how to define, let alone measure, such a nebulous concept. Is it a measure of how much someone learns in school? How well someone performs on standardized tests, such as the SAT, or IQ tests, such as the Stanford-Binet? Surely the author of the essay prompt would protest that these are not valid measures of intelligence because an artistic genius — Picasso or Mozart — might not perform very well on such "standard" measures of intelligence. And I agree with the author to this extent: a standardized test should not be the measure by which we seek to quantify or classify intelligence in general — not the only measure, that is. To the extent that we do need to quantify intelligence, we must first answer the question: what particular type of intelligence are we trying to measure?

A standardized test designed for admission to a certain educational program can be a valid measure for someone seeking to quantify how well someone has learned the material tested on that exam, as well as how flexibly someone can apply that knowledge to the particular types of questions seen on the exam. Such an exam, if well-constructed, could help the makers to ascertain how well the person is able to reason through various ideas or solution paths for a narrowly-defined set of material, and we might call that a form of intelligence. But is this the only measure of intelligence? Certainly not, as this is not the only type of intelligence.

Let's examine another concept entirely: the EQ, or emotional quotient. In recent decades, researchers have developed this concept to describe how well an individual assesses, expresses, and manages her own emotions as well as those of others. We've all known or heard of at least one "genius" who has an off-the-charts IQ but cannot interact productively with others. We've also likely all known someone who is extremely good with people, though he or she may not have graduated near the top of the class. Which is more intelligent? We haven't even begun to address other types of genius — the comedic genius of Lucille Ball, the gymnastic perfection of Nadia Comaneci, the marketing acumen of Steve Jobs, the "street smarts" of someone with little formal education and none of the "typical" signs of success who has nevertheless developed a strong, stable, and satisfying life despite hardship.

The author of the prompt might argue that all of these examples are ultimately a manifestation of creativity — that creative thinking allows someone to perform well on a standardized test, or to "read" the emotions or expressions of others and react accordingly, or to dream up the perfect marketing campaign that goes viral in a matter of minutes. If we define creativity that broadly, then I will agree with the author that creativity should always be considered when assessing intelligence. But I maintain that it is only one factor among many, including knowledge, expertise, flexibility, and observation.

If I had to propose one concept to cover all, I would propose that what we're really measuring is the ability to solve problems (within the context and parameters of whatever the particular "problem" is) — how to capture those water lilies on canvas to best effect, how to time a certain joke to draw the biggest laughs, how to recognize and best respond to an expression of dismay, or disgust, or grief on another's face. I would prefer, however, not to limit the discussion at all. I am content to let intelligence rest as a complicated, difficult to define, and nearly impossible to measure concept that nonetheless exists in everyone to some degree or another.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

My first thought when I read this topic was, "Wow, a lot of people might be tempted to write a whole essay about how the GRE is dumb and they should be able to submit an art portfolio or something instead." (I can't cite any official rule against it, but you probably shouldn't submit a GRE essay about how you don't like the GRE.)

In any case, I like several things about Stacey's essay — she takes a very fair and balanced position (Lucille Ball, Nadia Comaneci, Steve Jobs), but also follows a rather unique essay structure, leading us through several types of intelligence before synthesizing her ideas in the final paragraph, where she concludes that the best definition of intelligence is the ability to solve problems. I think all of my own GRE essays have given a clear thesis in the first paragraph; this essay flips that structure very effectively, making the conclusion an actual *conclusion,* rather than just a summarization of what came before.

I also liked that Stacey shared her reasoning process with us — who among us doesn't know the feeling of *freaking out* because you can't think of what to write? Sometimes you just have to start writing and work it out as you go.

Issue Topic #7

Government funding for pure science endeavors, such as space exploration, should be reduced in order to direct more funding towards humanitarian science projects.

Write a response in which you discuss the extent to which you agree or disagree with the claim. In developing and supporting your position, be sure to consider specific instances for which this statement may or may not be true.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Stacey Koprince:

I found it easy to come up with my "take" on this one but difficult to come up with specific examples. I knew in general that technologies originally developed for space exploration are now being used in a whole host of things here on Earth, but I wasn't 100% sure about the details. In particular, as I wrote my essay, I realized that healthcare examples would relate best to my other examples, but I was also the most shaky on the healthcare details. So, honestly, I sort of guessed — I wrote in some details as though I were confident, but I could be wrong. (And, no, I did not go back and correct them before publishing this! I want you to see what I really did.)

We're actually allowed to do this; the essays are not fact-checked. I'm not going to say that man first landed on the moon in 1842, because I know that's completely not true, but if I do have some details that aren't quite right, I'm not going to be penalized for that. If you think about it, it makes perfect sense: they can't possibly take the time to fact check every last thing, and so they check nothing.

I want to point out one more thing: my final essay is a bit shorter than usual (although still longer than many high-scoring Issue Essays), and that reflects that fact that it took me longer than usual to work out what I wanted to write. At some point, though, I had to cut off my planning and start writing or I wouldn't have had enough time to finish — and that's when I realized that I needed to "fudge" on my details a little.

Here are my original notes. After I wrote my opening paragraph, I realized that I should switch paragraphs 2 and 3 in order to provide a better transition into paragraph 4.

P1: Specific examples. Prioritize humanitarian science over pure science. Why? Benefits? Goals?
P2: Techs developed via space expl. projects are used extensively on earth (communications: cell phone; healthcare: robotic components that help in surgery; energy: solar cell panels)
P3: "humanitarian" = broad, but let's say health and education projects? Sometimes we do want to prioritize these
P4: But find a balance between the two, b/c "pure science" can result in "humanitarian" help.
P5: conclusion

Essay:

The author claims that humanitarian science projects should be prioritized over pure science projects, to the point of redirecting funding from pure science to humanitarian science. There are perhaps some instances in which this would be judicious, but in the absence of any discussion of the goals for such funding and the possible benefits to be obtained, such a black-and-white policy is likely too simplistic. Further, the author appears to assume that this is a zero-sum game: that such funding must go to one or the other and that the same amount of funding will be "worth more" by some (unspecified) measure if spent on humanitarian projects.

It would be helpful to examine specific cases in order to make this abstract argument a bit more concrete. "Humanitarian" is a broad term, but let's assume that the author is referring to projects that will provide healthcare or education to communities currently lacking such services in some significant way. (The term humanitarian could, of course, refer to many other types of research, community outreach programs, and so on — but we're trying to discuss one tangible example.) Vaccinating children against smallpox and measles is an obviously high-priority task: the technology exists, it is not prohibitively expensive, and it will both save lives and improve quality of life immeasurably. In any prioritization of funds, this particular usage must be near the top.

It is also the case, though, that many technologies originally developed during space exploration projects are now used in mission-critical "humanitarian" arenas. For instance, remote monitoring of vital statistics (such as heart rate and blood pressure) via miniature devices that can be affixed to or implanted in the body — or even swallowed! — allows us to monitor someone's health when that person is not physically in a healthcare setting. Other miniature remote-controlled devices (robots, if you will) are used in surgical settings to minimize the invasiveness of the procedure, or even to perform a maneuver that is impossible with traditional surgery. Satellite networks allows us to monitor weather patterns and broadcast information about the approaching hurricane immediately, potentially saving tens of thousands of lives.

Further, it isn't necessarily sufficient to look at the goals and expected benefits in the short-term. When NASA scientists were developing ways to monitor the health and manage the treatment of astronauts from afar, they likely weren't anticipating the myriad ways in which the technology would ultimately be used "on Earth." Early photovoltaic technology was developed almost exclusively in space because access to the sun's rays was free and easy. Decades later, solar cells are a leading source of terrestrial "clean" energy and one of the leading candidates to replace fossil fuel energy should we one day find ourselves in the position of running out of Earth-based energy sources.

Any approach, then, that paints all "humanitarian" projects with one broad stroke ("worthwhile") and "pure science" projects with another ("less important") is too crude an approach to make such very nuanced decisions. Given limited funds, of course priorities will have to be determined and a tough balance struck. History demonstrates convincingly, however, that what begins as a "pure science" project might one day prove to be a "humanitarian" effort that can affect the lives of every person on the planet.

Here is a second opinion from Manhattan Prep instructor Tommy Wallach:

In spite of Stacey's claim that she struggled to come up with ideas for the prompt, you certainly wouldn't know it from reading this essay! In truth, you should never look like you're at a loss for facts, because any details you're unsure about can be fictionalized, as long as you'd be willing to bet good money that your *overall concept* is correct. In other words, you may not know exactly why sugar is bad for the body, but you can feel free to say, "Sugar is bad for the body because the calories are burned quickly—causing the notorious sugar rush—leaving the body starved for nutrients in the long term." Are you sure that's exactly how it goes down? Nope. But it's more than good enough for your purposes. That being said, if you *can* write an essay entirely with stuff you know is true, you're likely to be more confident, and thus sound more confident.

If you wanted to take the other side of this essay, you could start by discussing the enormous amount of federal money that goes to weapons development, and argue that this investment has paid very little dividends in terms of raising our standard of living. In the next paragraph, you could discuss a small investment that has paid huge dividends in this regard, such as vaccination research in the 19th century, research into antibiotics, or agricultural improvements that have allowed for the production of greater and greater amounts of food to be grown on the same amount of land. Finally, because the prompt asks you to consider the other side, you could use Stacey's example, discussing how research conducted by NASA often results in practical products and techniques used in a more humanitarian way down the line. You could then argue that this is all well and good, but surely it would be more efficient to simply invest that money in those humanitarian projects in the first place.

Issue Topic #8

The fact that technology is outpacing the needs of those in cultures that can afford the technology creates cultures of excess consumerism.

Write a response in which you discuss the extent to which you agree or disagree with the claim. In developing and supporting your position, be sure to address the most compelling reasons and/or examples that could be used to challenge your position.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Roman Altshuler:

This is a pretty broad statement. Supposedly, it applies to *all* cultures that are affluent enough to afford technology. The claim looks really obvious, so my first thought is this: GRE essays are supposed to show complexity. If you agree with a prompt that says something obvious, it's going to be pretty difficult to say anything complex about it. On the other hand, if you disagree with a prompt that says something that seems obvious, you have an easy way to make your essay complex: you just explain that what the prompt claims is *too* obvious; it's based on a superficial understanding of the facts.

Notice, also, that the prompt asks you to "address the most compelling reasons and/or examples that could be used to challenge your position." The strategy of explaining why the claim is too obvious does that automatically: I just describe why the appearances are misleading, and by describing them I am already listing reasons that could be used to challenge my position.

So now I am thinking about content. What am I going to write about? I don't want to write about technology in general, because it seems like that will be too broad. So instead I am going to pick a particular instance of technology that I'll focus on for most of the essay as an especially salient example (and I'll call it an "especially salient example" in the essay, because that sounds fancy and uses a GRE word). How about cell phones? I just got a new one, so I have a bunch of thoughts about cell phones that I can draw on. The example seems a little too banal, but that's okay, as long as I can say things about it that aren't.

So now what about my argument? I think about whether people would buy crap they don't need if it didn't exist. Well, obviously not. But all that crap wouldn't exist if people didn't want to buy stuff. So maybe the prompt gets things backwards: it's not that the existence of lots of unnecessary technology makes people buy it. It's that people in a consumer culture want to have more stuff to buy, and that's why there is so much unnecessary stuff.

The prompt's mention of "cultures that can afford the technology" makes me think of affluent cultures and the poor societies their consumerism exploits. Maybe I can say something about that. I put that thought on the back burner; I'll try to get something out of it if I need an extra paragraph.

The prompt also mentions that the problem is with technology that is "outpacing the needs" of people in certain consumer cultures. How do we know when technology "outpaces needs"? I see a few problems right off the bat. First of all, needs change. It's easy to say that something is unnecessary or frivolous when looking from the outside, but from the inside the growth of technology creates its own needs. For example: I *need* a tablet, because I read a lot and want to carry large amounts of reading material with me everywhere. Also, we don't always know that a culture "needs": technology often *seems* unnecessary, but turns out to satisfy needs in hindsight. I remember how cell phones, with their access to social networks, were crucial in mobilizing protesters in the revolution in Egypt. I'll use that as an example.

And anyway, is it really clear that there *is* lots of technology that outpaces needs driving consumer culture? Some people will buy anything that's new, but maybe most people don't buy stuff that they really don't need. I could be wrong about that, but it's worth making an argument for it.

Now I clearly have enough to write about, so I won't need to talk about affluent vs. disadvantaged cultures after all.

Instead, I'll focus on the idea that technology outpaces needs. By questioning the premise of the prompt, I can give a nuanced, somewhat unusual take on the issue. And then I'll wrap up that approach by insisting that *even if the premise is right* — that there is lots of technology on the market that nobody needs — it's *still* not true that this causes people to buy stuff they don't need.

I've been jotting notes as I go, so at this point I have the following written:

> Intro: cell phones. Thesis: it looks like tech nobody needs drives consumerism, but this gets things backwards and mistakenly assumes that people are buying stuff they don't need
>
> 1. people don't buy things that nobody needs
> 2. needs change. sometimes tech satisfies needs nobody knows about yet. Arab Spring example.
> 3. consumer culture leads to new technology, not the reverse

As I get to each point, I delete it. And it's important to always keep the broader picture in mind, so when I develop the points, I have to explain what they have to do with the overall theme.

<u>Essay:</u>

As a result of constant competition and innovation, technology evolves at a staggering rate. Fifteen years ago mobile phones were a novelty for the privileged few; now they are the possession of roughly 70 percent of the world's population. And, of course, it is not enough simply to release a new kind of product and sell it: the product must be constantly updated, developed, and marketed in a new form. Better battery life and reception, global positioning, access to the internet, the ability to download music from the "cloud", or to take photographs and immediately share them across one's social network; all these are developments that have sprung sequentially from manufacturers in relatively short order, each one used to market a new generation of phone to eager consumers. And, of course, cell phones are only one example among many, though an especially salient one. The constant release of updated forms of the same technology, combined with the existence of a ready market to adopt these upgrades, conveys the appearance that the updates drive purchasing for the sake of purchasing; that if not for the oncoming tides of new products, the waves of spending would abate. But the appearance gets the relation backwards, and it relies on too simplistic a view of what the relation between technology and the needs of a culture might be.

First of all, it is far from obvious that technology that genuinely outpaces the needs of its consumers retains a consumer base. Items that fail to find a public often vanish without a trace. Consider the push on the part of a number of computer manufacturers at the dawning of the last decade to market expensive tablet computers. The attempt went nowhere, and tablets vanished from the market to return only ten years later when a need for them appeared. The history of technology is replete with such market failures which, precisely because they were failures, rarely make it into public memory. This tendency to forget where the market has failed is perhaps the best explanation for the tendency to believe that unnecessary technological advances drive consumers' tendency to purchase every new bit of technology, no matter how little needed. But in fact, it seems, technology that outpaces the needs of a culture simply vanishes from that culture without taking hold. Of course every affluent culture has a wave of so-called "early adopters," who will buy the newest gadget, no matter how useless. But segments of the population who will spend on items they cannot even fathom needing are, unsurprisingly, small.

Granting that technology that outpaces the needs of its market base sometimes fails, it may still seem as if this is often not the case; that, in fact, frequently technological advances are purchased and adapted on a wide scale by a culture, even when ostensibly no need for such a technology is present. The judgment that occurrences of this sort are common, however, faces serious objections. What, first of all, can we mean by the idea of technology "outpacing the needs" of its users? If we think of needs in the most basic terms, as universally shared by all human beings, then the list of genuine needs will be quite small: food, shelter, clothing, and companionship. But even this small list of basics requires supplementation.

Medicine, for example, seems like a basic need; but the need for medical technology is in principle limitless as long as human beings remain mortal. It is not so clear, then, that—at least in some areas—technology really can outpace need. It is also worth considering that technological advances in some seemingly frivolous fields may transfer to advancements of vital necessity. Finally, just as basic needs appear in some respects infinite, so the advancement of technology can create new needs, perhaps ones unforeseen prior to its development. A technologically advanced culture, for example, may well have needs far beyond those of its less affluent analogues; such needs may well appear trivial to outsiders. And, of course, the fact that some bit of technology does not satisfy a clear need at present does not show that it satisfies no need of the culture: one need only consider the role played by mobile technology — which may have appeared excessive when it first appeared — in the recent events of the Arab Spring.

Just as we find it difficult to definitely declare that any particular technological upgrade has outpaced the needs of its culture, so we should be suspicious that such outpacing, where it occurs, genuinely drives the consumerism. Affluent cultures constantly demand new outlets for their affluence; technology is merely one of the many industries that responds to that demand. It is not hard to imagine that those with money to spend will want to spend it. Thus, it seems reasonable to assume that excess consumerism drives the demand for ever new products, not the reverse. Moreover, the consumerism is fanned by any number of marketing techniques, themselves produced by companies eager to increase their profit margin. Often, new developments in technology, as in other industries, come about in response to the need to market new products. But again, it is the marketing and the cultural response to it that drives innovation here. The innovation exists for the sake of the marketing and consumerism, not the other way around. That new, seemingly frivolous technology is constantly produced, marketed, and purchased may give the appearance that it is new technology that causes the consumerism, but the reverse may well be the case.

We should be skeptical, then, of the view that unnecessary technology creates cultures of excess consumerism. While often this is how things may look on the surface, in reality the complex relations between technological innovation, need, and consumerism are far more complex. New technology creates new needs, and the consumers' demand for new products creates those products. To blame technology, especially technology that seems to have no necessary application, for the increase in consumerism is to misdiagnose the situation; it is to mistake the symptom for the underlying cause.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

When I read this essay, I almost laughed — "This incredibly lengthy, well-reasoned, well-argued essay is going to intimidate the heck out of our students!" It's worth mentioning that the friendly GRE instructor we know as Roman is known in some circles as "Dr. Altshuler, philosophy professor." So, consider this essay an ideal — not a standard most test takers would be able to meet in a mere 30 minutes!

One thing that's especially excellent about this essay is the complex argumentative structure. Roman didn't just argue yes or no. Rather, he broke the topic into two claims: "technology is outpacing needs" and "this creates cultures of excess consumerism." He then questions the definition of "needs," argues against the first claim, and then argues that *even if the first claim is right,* the second claim still doesn't work.

This is a thorough way to attack an argument! (You say A will lead to B? A isn't true, but EVEN IF IT WERE, it wouldn't lead to B.) It's like you're winning the argument *twice.*

I also really enjoyed the example about the Arab Spring. This is a perfect example of a seeming "luxury" serving a noble and important purpose. (If I were writing this example, though, I'd include at least a sentence explaining what the Arab Spring is, just in case the grader has been living under a rock!) This is also a good reason to keep up with current events — students who complain they have "nothing to write about" would generally benefit from a daily dose of CNN.com.

Finally, keep in mind that there's no requirement that you take such an argumentative approach — this is the Issue Essay, not the Argument Essay. So, you could certainly choose instead to agree with the prompt, and give examples of technology leading to excess consumerism.

Issue Topic #9

> Luck plays more of a role in determining success than work ethic does.
>
> Write a response in which you discuss the extent to which you agree or disagree with the claim. In developing and supporting your position, be sure to address the most compelling reasons and/or examples that could be used to both support and challenge your position.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Chris Berman:

The stimulus asks that the response address the evidence for both positions — this must be accomplished while establishing a clear and defensible position. To receive a top score, the essay must consider defining the terms ("success") used in such broad statements. Precisely understanding the statement presented is also very important; this one revolves around luck playing *more* of a role, and an essay that argued that success was due entirely to one or the other would not be completely responsive.

Finally, note any assumptions underlying either position that could be utilized. For instance, if one were to argue in favor of the preeminence of work ethic, one could discuss the assumption that luck does or doesn't even out in the long run. However, when all is said and done, one has to argue the position for which points and examples leap to mind. I will posit the greater importance of luck.

Main point — the importance of luck is supported by the vast majority of the historical evidence, and the opposing position rests on questionable assumptions and less compelling evidence.

1. Examples that support this point.
2. Opposing points and their flaws
3. Opposing assumptions and their flaws
4. Brief summation

Essay:

While no one should deny the utility of a strong work ethic in pursuit of material success and professional recognition, luck has a greater power and can deliver or destroy the dreams of both the earnest and the slothful. The truth of this perhaps ironic or even tragic aspect of life is borne out by the historical record. Military history is rife with examples of how fortune changes "the affairs of men." One such example would be Frederick the Great, the 18th century Prussian king. Famous both for his personal work ethic and for his imposition of the same on his soldiers, he is virtually universally acknowledged as the foremost military leader of his time. Yet, he was only saved from utter defeat by the unexpected death of the Empress of Russia. Sports history is also rife with examples of luck trumping effort. "Hollywood" also provides a plethora of examples of how good or bad luck determines the fate of hard working actors — Lana Turner, a star of the 1940's, was "discovered" at Schwab's drugstore. More quotidian lives follow the same pattern — inventors who lose the fruits of their labors through chance, and countless industrious people whose lives were undone by chance accidents or disease.

This aspect of life is also frequently mirrored in literature, plays, and songs. The deus ex machina of Greek tragedy is the beginning of a long lineage of art describing the superiority of fate to human will. Some would counter by arguing that fiction embraces the triumph of hard work and point to the Horatio

Alger stories and similar works. However, not only are such examples smaller in number, but many of them were crafted expressively to tout that virtue in the service of building a society rather than to accurately depict the realities of life. Furthermore, it could also be argued that those who strived and succeeded against obstacles merely had enough good luck to dodge a fatal blow in one form or another. While Puritan and Lutheran philosophies laud the worth of work ethic, these essays were based on theological rather than pragmatic grounds. In fact, many were expressly composed to explain and counter the reality of the preeminence of random fortune.

Additionally, dubious assumptions underlie some of the main arguments in favor of the primacy of work ethic. Implicitly, the opposing view equates work ethic with the quality of the work. While there is certainly a correlation, there is no causation between effort and excellence. No matter how hard I were to work, I will never be a professional athlete of any sort. Also, the counterargument assumes that there is sufficient demand for the supposed excellence derived from effort. To return to the "Hollywood" example, there it is glaringly obvious that there is an oversupply of capable and objectively similarly skilled actors; thus, logically enough, random factors and good fortune become the primary determinants. I have personally seen actors cast because they remind the directors of their brothers. I have also seen actors not cast for exactly the same reason. Such oversupply is a characteristic of many, albeit not all, fields. Finally, the assumption that an individual's luck will return to the mean, as it were, is dubious. Again, history, the arts, and most people's anecdotal evidence point to the conclusion that some are luckier than others, even if no one lives an absolutely charmed life. Perhaps one lifetime is too small a sample size.

To reiterate, while a strong work ethic is sometimes necessary for and usually contributes to success, luck is the more powerful factor. The record of human activity leads to no other conclusion. To that point, there is an old saying that remains only too relevant: "If you want to make God laugh, tell him your plans."

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

Great essay! Chris writes in a literary and somewhat formal style, which I think makes a powerful first impression ("luck … can deliver or destroy the dreams of both the earnest and the slothful"). Because of the writing style, I can say I wasn't surprised at all to see the essay introduce an example about literature, from Greek tragedy to Horatio Alger.

I also very much enjoyed the essay's critical takedown of Alger as an example of work ethic being more important than luck and of Protestant denominations' propounding of work ethic (which some people still call "Protestant work ethic.") In both cases, Chris argues that these do *not* make good examples of the primacy of work ethic over luck, because the sources had ulterior motives. The argument about an oversupply of talent was also persuasive (if there are more hard workers than there are jobs for those hard workers, luck will end up being the deciding factor).

If I had been writing an essay on this topic, I would have also argued in favor of luck, although I'm sure it would have been a very different essay. I probably would've written a lot about how what country you're born in has a lot more to do with your success than the efforts you put in. In a developed nation, you benefit from health care, roads, infrastructure, public education, and some kind of safety net for hard times, whereas none of these are reliably available in many of the nations of the world. Some people are born in war zones or refugee camps; some people think they're signing up for a job opportunity and end up becoming victims of human trafficking. Hard work is irrelevant in these cases.

If you wanted to go the other way on this topic and argue that work ethic is more important than luck, you can't *just* write about the importance of hard work. You also need to address the obvious objections — what about all the people who obviously got lucky (or unlucky)? I think that would be a hard essay to write. However, you could pursue some kind of middle ground approach, perhaps something like, "Good luck is an essential ingredient of success, but most strokes of luck are useless without the work ethic to pursue those opportunities."

Issue Topic #10

Some people believe that competition drives young athletes to perform at their best, while others believe that competition discourages those who are not athletically talented from participating in organized sports.

Write a response in which you discuss which view more closely aligns with your own and explain your reasoning for your position. In developing and supporting your position, be sure to address both views presented.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Jesse Cotari:

When I look at this prompt, my first thought is: *How do I take a side when both sides are pretty accurately describing facts, rather than offering than opinions?* But instead of dwelling on that issue, I kick the question to my subconscious and get to work thinking about each side of the issue. I do some brainstorming, write the first sentence of the introduction, and then start in on the first two paragraphs. My goal at this point is to establish both sides' validity with examples of how they could both be correct, which will set up for whatever twist is coming that will drive my later paragraphs.

(I'm a big believer in giving my unconscious mind something to work on while I'm focusing attention forward. Typing takes mostly time and attention, whereas figuring out a creative "take" on an idea sometimes just takes right-brained inspiration. Instead of sitting around waiting for inspiration to strike, I get working on less-creative processes like typing out an already-outlined paragraph.)

By the time I finish the first body paragraph, I have a pretty good idea of where the rest of the essay is going, so I take a moment to do a little more outlining, focusing on the deeper implications of either viewpoint.

If it were necessary to take an unqualified view in support of one side, I would have had a hard time. However, the possibility of including nuance makes this much easier. First, I decide to define the "unspoken assumption" behind each argument. Why would someone say one or the other of those statements? Probably to push a particular type of sports program. Okay, so let's talk about that.

At this point, despite the fact that I see the validity of both sides, I take a qualified side with "everybody wins!" This means positing that collaboration is more important than perseverance. Do I believe that that's 100% true all of the time? No, but I can make a case for other sources of perseverance in addition to sports, so I'll go with it. I finish up the introductory paragraph, write the second body paragraph, and then, the conclusion.

OF NOTE: I know that my process in this case is risky. Since I'm writing the essay in a non-linear way, I leave a little time to go back and be sure that everything flows.

Essay:

The statement above presents two conflicting presentations of the same true facts: It is unquestionable that athletic competition both drives and discourages young athletes. However, unspoken assumptions as to the purpose of organized sports underlie significant differences in the implications of either statement. By understanding these underlying assumptions, it is possible to decide which aspect of organized sports is most beneficial to potential young athletes. While being driven to one's best through competition certainly has its benefits, the values of teamwork and collaboration that can be derived from any athletic participation are more important to success in the rest of life.

The first view, that competition drives young athletes to perform at their best, can be clearly examined through the lens of track and field. The immediate feedback received from watching a competitor pull ahead makes a direct psychological impact on any runner. In head-to-head competition, the direct ability to compare one's performance with that of one's rivals can be intensely motivational. Even after the race, the knowledge that with a little more practice victory could have been achieved can impel a young competitor to run those extra miles in preparation. This direct comparison with the abilities of others is one factor that can drive young athletes to be their best, but also makes the opposing viewpoint valid as well. Those who are not "athletically talented" make the comparison and instead of seeing reasons to put in more hours of practice, see reasons not to bother. Constantly failing to win can indeed be highly discouraging. Thus it is unquestionably true that both discouragement and personal drive are potential side effects of organized sports.

At a more fundamental level, however, it is not the viewpoints themselves, but the unspoken assumptions as to the purpose of organized sports that are incompatible and make siding against competition possible. The first view, that competition drives athletes to be their best, is a valid aim if being good at sports is a useful end goal. Does being able to run 400 meters faster than anyone else in the country serve any practical purpose? In a world generally free from the necessity to escape large predators, probably not. Nonetheless, it is true that the perseverance and dedication necessary to achieve that goal are worthy skills. The ends of development of dedication should, however, be compared with the opposing assumption: that playing a team sport is valuable, regardless of the level of success. Through participating in a team sport, young athletes can develop the ability to work with others to a common purpose. In the modern workplace — focused on teams, interactivity and collaboration — this life skill offers a clear advantage. While perseverance can be developed in other areas (particularly academic ones, where the benefits are more tangible), few pursuits offer the ability to develop the skills required to participate in a team. Deemphasizing the competitive aspect of team sports (or at least providing the option to play team sports in a less competitive environment) allows the development of collaboration to reach a wider youth population and spreads the benefits of organized sports to more young athletes.

In the end, the statements present two true facts that have different implications. Both perseverance and teamwork are vital to the success of any adult, and are worthy goals of an athletic program. However, by reducing the emphasis on competition, the benefits of working with a team can accrue to a larger cohort of young athletes.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

Essays on very specific topics are challenging! Personally, I would be very annoyed to get this topic — as a reasonably educated person, I feel like I have a lot of fields to draw on for relevant and persuasive examples, but here I cannot use any of my knowledge because I am pretty much forced to write about children's sports.

Jesse does a great job with this topic, correctly identifying that both statements about youth sports have some truth to them, and so we must decide what we most value if we are to decide which truth is most important.

I was also *fascinated* by Jesse's writing process, in which he uses his subconscious mind to do some processing while he does the more tedious work of typing. If you wonder how some people think through an essay topic and write such long and well-developed essays all in 30 minutes, well … this is one way to do it!

Personally, I find that I'm able to think of and write so much in half an hour because I have a lot of practice. But maybe more importantly, I have a structure in mind before I begin writing (intro, 3 body paragraphs, conclusion), so I feel almost like I'm just dropping some details into a form or something. Finally, successful essay writers are often people who read a *lot,* on a variety of topics — and people who look things up when they don't know something! It can be frustrating when you want to write about the electric car because it perfectly fits the topic, but you realize you don't actually know anything about electric cars. Prepared test takers are people who do a lot of reading (world news, science news, business news, the *New York Times Review of Books,* etc.), and whose natural curiosity leads them to Google or Wikipedia to fill in the gaps in their knowledge.

Of course, when you get a topic about children's sports, it's hard to make a lot of that knowledge base fit, but it can be done — off the top of my head, Rousseau's *Emile* and the theories of Piaget on child development could be used in an essay on this topic, even though they have nothing specifically to do with sports. Similarly, an example about the history of the Olympics could apply — as long as you explain how an example about adults applies to children. Also keep in mind that there isn't actually a requirement to bring in outside examples or information, as long as you can find enough to write about just making general/hypothetical arguments about the prompt itself. Jesse's essay is an excellent example of this approach.

Argument Essay Guidelines and Sample Responses

Argument Topic #1

> A recent study by the Center for Disease Control and Prevention found that employees with paid sick leave are 28 percent less likely to be involved in a work-related accident than employees who do not receive payment for sick leave. Researchers hypothesize that employees with unpaid sick leave feel pressured to work during time of illness for fear of lack of pay. On-the-job accidents are then spurred by impaired judgment or motor skills due to illness or illness-related medications. The highest-risk occupations, such as construction, showed the highest discrepancy between paid and unpaid leave.
>
> Write a response in which you discuss what questions would need to be answered in order to determine whether the researchers' hypothesis is reasonable. Be sure to explain what effects the answers to these questions would have on the validity of the hypothesis.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Stacey Koprince:

I read the instructions first, then the argument. The instructions focus on the "questions" needed to evaluate the argument; because I knew this when I started making notes, I was able to put as many as possible into question form. Later, when I began to write, I already had many questions that I could fold right into the argument.

The argument has two significant flaws. First, the study results (in sentences 1 and 4) do not address any part of the hypothesis (that is, whether people are working while ill and whether that then causes accidents). Second, the argument fails to address (and dismiss) any alternate explanations for the data.

My raw notes are fairly long, but notice how I've got them organized paragraph by paragraph. When I start writing, I actually type right underneath the notes for each paragraph, so that I can re-use exactly what I've already typed and then simply delete any extraneous words. This is similar to writing a detailed outline first and then fleshing out the first draft. The only thing I don't brainstorm is the conclusion — my conclusion will just be a restatement of everything I've already said.

> P1: Hypothesis = pressured to work. But evidence doesn't show WHY there are more accidents. Conjecture.
> P2: Can the existing data be made to show actual causation? Study could poll people as to why they had the accident. Did they actually come to work sick?
> P3: Other possible explanations? Differences based upon job title or level? Desk-job workers probably less likely to have accidents; are they also more likely to have paid sick leave? Blue-collar jobs more likely to cause accidents and also less likely to offer paid sick leave? Salary vs. hourly wage work?
> P4: Other reasons why people feel pressured to work? Perhaps unpaid sick leave correlates with certain kinds of jobs where it's more problematic for a team if someone doesn't show up for work that day. *[Note: as I wrote the essay, I realized that this was already too much like paragraph 3 — luckily, by then, I'd thought of something else to say in paragraph 4.]*
> P5: conclusion

The instructions specifically ask us to "explain what effects the answers" would have on the argument. It's not enough to ask the questions — I also have to state how various responses might help or hurt the argument. Note that I did not necessarily say "If the answer is yes, then this will happen; if the answer is no, then that will happen." It's enough to say something like "if X happens, this would support the hypothesis." I did, though, make sure that some questions were presented in a way that allowed me to say "If so, this strengthens the hypothesis" and others were presented in a way that allowed me to say "If so, this weakens the hypothesis."

Essay:

The author provides two pieces of evidence in support of the researchers' contention that workers without paid sick leave are more likely to work when ill, and that such workers are more likely to experience a job-related accident as a result. The results of the study cannot be disputed — we must accept as true, for example, that those with paid sick leave are significantly less likely to have work-related accidents than those without paid sick leave. We might, however, question whether the study was large enough, or representative enough, to draw broad conclusions. Further, the study does not demonstrate causation: it does not tie the incidence of work-related accidents to illness. While the researchers' hypothesis is certainly one possibility, more research is needed to eliminate other possibilities and to bolster the strength of this argument.

The largest leap in the argument is the assumption that those without paid sick leave feel pressured to work when ill. No evidence is presented to establish this supposition. In order to strengthen this part of the argument, the researchers might ask study participants whether they have actually come to work ill during the same timeframe covered by the original study and, if so, why they chose to come to work when ill. If the study participants who did experience a work-related accident were also more likely to come to work ill for fear of lack of pay, then the hypothesis would be much more strongly supported, particularly if this occurred with a correspondingly large proportion of workers (to match the 28% greater incidence of accidents in the original study).

The researchers would also strengthen their case by addressing alternate explanations for the data in the original study. For instance, are there differences between the two groups based upon industry or job performed that might explain the data? For example, are hourly workers more likely to lack paid sick leave, while salaried workers are more likely to receive it? Are hourly workers more likely to work in blue-collar or more manual occupations, where on-the-job accidents are more frequent? If so, then we would expect a correlation between unpaid sick leave and a higher incidence of workplace accidents because the work itself is inherently more dangerous, not because people are choosing to work when ill. Further, if it is the case that higher-risk occupations in general are more likely to lack paid leave, then the second piece of evidence also loses its significance. In such a case, the researchers' hypothesis would be significantly weakened.

The data presented also lacks a depth of detail that would help us to evaluate the significance of the study results. How many people were surveyed? What is the margin of error and how was the study conducted? Is the 28% figure statistically significant? If the study represented a large enough survey group to extrapolate to the general population, across regions, industries, and job responsibilities, then the study results may be conveying something significant. If, alternatively, few people were surveyed or the incidence of job-related accidents were very low, then perhaps the 28% difference represented a small number of people, well within the statistical variance expected.

While the argument presents an interesting hypothesis, the data presented is not strong enough to establish the validity of the conclusion to even a small degree. First, at a basic level, we need to know whether the existing data is statistically significant and sufficiently representative. There are also several gaps in the logic chain, assertions made without supporting evidence. Finally, the researchers could strengthen their case by examining, and dismissing, alternate explanations for the data presented thus far. These steps might not be enough to establish the validity of the hypothesis beyond a doubt, but they would allow the researchers to determine whether the pursuit of the hypothesis is a good use of time, funds, and attention.

Here is a second opinion from Manhattan Prep instructor Tommy Wallach:

Phew! Stacey has done a phenomenal job with this brutal Argument Essay. What makes it so hard? Well, the argument itself doesn't actually make a huge number of outright logical errors (e.g., "This burrito has guacamole, therefore it is the best burrito in Los Angeles!"). Instead, the majority of errors here are errors of *omission*. Stacey needs to use all the rules she knows about making a good argument in order to describe everything the author left out.

Every Argument Essay presents you with a terrible argument, and it's up to you to explain why it's terrible. But in this case, the argument just isn't that bad as written. Thankfully, the prompt gives some hint of this. By asking what questions would need to be answered to justify the conclusion, it's more or less telling you that you'll need to consider things the prompt ignored. Make sure you take note of the special instructions on every essay, because they often provide helpful hints such as this one.

Honestly, I can't think of much I would do differently in this essay, though I do think Stacey missed one juicy low-hanging point. The Center for Disease Control and Prevention is likely to have as its prime focus the desire to stop the spread of disease (it's not particularly interested in making sure random construction workers get good benefits). The best way to stop the spread of disease is to make sure sick people stay home, and the best way to get sick people to stay home is to ensure that they all get paid sick leave. Since the CDC's conclusion happens to coincide with that ulterior motive (getting sick people to stay home), we might have reason to doubt the CDC's conclusion.

<u>Argument Topic #2</u>

> The city council of Town X has proposed reducing the city's electric expenses by switching all the lights in public buildings from incandescent bulbs to light-emitting diodes (LEDs). The switch would be made gradually as the old incandescent bulbs burn out, and the city council reasons that since LED lights burn brighter and cost no more to purchase, the switch would help Town X save money on electrical costs in the future.
>
> Write a response in which you discuss what questions would need to be answered to help evaluate the efficacy of the city council's proposal to save money on electrical costs. Be sure to explain how the answers to these questions would help to evaluate the council's prediction.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Stacey Koprince:

As always, I read the question first, not the argument, so that I know what angles I need to take right from the start. This one's asking me to suggest "questions to help evaluate" the plan presented in the argument. I need to explain how the answers would help to evaluate the plan, not simply present the questions.

Great! My angle is "attack" — that is, I'm going to pick apart the plan, asking "Are you sure? Have you thought about XYZ? What about ABC? Is it really going to work that way?"

I also note that the question tells me what the goal is: ""to save money on electrical costs." Okay, now I dive into the argument. The first paragraph of my essay is going to summarize the argument, so my argument notes will go there (I type these notes in as I'm reading; they'll become my outline). Then I start attacking, labeling all my questions P2, P3, or P4:

> P1: Goal: reduce electric cost. Plan: gradual switch from IB to LED. LED brighter, cost same to purchase.

p2: Do they cost the same to use? Brighter, maybe use more energy? Do they last as long?
p3: Can they just be switched out directly? Will the new bulbs fit the existing fixtures? Etc.
p4: Other drawbacks? Unintended consequences?
p5: Conclusion

My P5 note is standard — I always write "Conclusion" (actually, "concl.", but I spelled out my notes above for you more than I would for myself) and worry about it later.

In this case, I notice that the argument offers to "supporting" reasons to switch — but, hey, the new bulbs cost the same as the old ones. That doesn't help me *save* money. Plus, the initial cost to purchase is only part of the total cost — what about the electricity bill? And then I'm off and running.

Notice that I did actually think to myself: okay, what's the conclusion, and what evidence is the author trying to present in support of that conclusion? I can then ask myself "does each premise really support the conclusion?" and "is the plan really going to work in the way that the author thinks?" In both cases, the answer is going to be no — I just have to figure out how each piece of the argument falls short.

Finally, I reminded myself that I need to have a neutral tone throughout. That is, I'm not trying to argue that the town council is wrong or its plan will not work; I'm simply pointing out flaws and indicating that the council has some more research to do in order to demonstrate that its plan is sound. I had to remind myself of that on this one because I think the town council's plan actually does sound pretty bad!

Essay:

The council proposes reducing electric expenses by changing from one type of light bulb to another over time. While the council provides two reasons why this could help to save money, only one is a potential support (the fact that the new bulbs burn more brightly). The other reason, that the LED bulbs cost no more to purchase, does not help to save money; rather, it only shows that the town will not spend more money to purchase the bulbs. Further, the one supporting fact may or may not result in a cost savings — we don't have enough information to tell for sure.

The initial cost to purchase the new bulbs is the same, but this is only one of the costs associated with lighting. The bulk of the cost is incurred over time, as the bulbs are used. Do the new bulbs use the same amount of electricity to run? The argument mentions that they're brighter; perhaps they use more energy? If so, then the electrical costs could actually increase over time, not decrease — the opposite of what the council expects. Further, do the new bulbs last as long as the old ones? If they last longer, then costs might decrease; if they burn out more quickly, however, then costs could increase.

The council's plan is to replace the incandescent bulbs with the new LEDs over time, as the old ones burn out. Is it possible to make a direct replacement without incurring any extra costs? Will the new bulbs fit into the old fixtures, and do they use the same type of electrical connection? If not, there could be a significant cost associated with retrofitting the fixtures in order to accommodate the new type of bulb — and that would be made more complicated by the plan to replace the bulbs gradually as they burn out. If one fixture has three incandescent bulbs, and one burns out, and if that fixture would need to be retrofitted in order to be able to take an LED, can you retrofit only one portion of the fixture? Likely not, so now, in addition to the cost of retrofitting, we will also be tossing out two perfectly good incandescent bulbs in order to switch to LEDs.

Finally, are there any other drawbacks associated with switching from the old bulbs to the new that might add to the costs or reduce the efficacy of the new bulbs? For instance, perhaps the new lights, which burn more brightly, also emit a larger amount of heat, which might cause the building to have to increase the usage of air conditioning, thereby increasing electricity costs (albeit from a different source). Possible unintended consequences need to be addressed before implementing such a plan, or the council might find itself with a surprise on its next electric bill.

The plan presented by the council includes just one piece of evidence intended to support the idea that electric costs will decrease, and even that piece of evidence is suspect (as we saw, the fact that the LEDs burn more brightly might mean that they actually use more energy!). While it may ultimately be the case that the plan is sound and will save Town X money, we cannot conclude this from the argument as it stands now. There are simply too many unexplored variables, including total cost, not just replacement cost, and possible drawbacks, including retrofitting or other unexpected consequences, that could actually result in increased costs.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

This is definitely NOT the only argument prompt I've read that says "B is better than A, so let's switch!" In fact, Topic #10 in this book also matches this format. With any topic of this type, switching time and cost are issues, which Stacey hit on in great detail in paragraph 3.

There is also always a danger in switching from a known thing to an unknown thing, which Stacey addresses in paragraph 4.

If I had gotten this topic, I would have written a pretty similar essay — the topic writers build a small set of flaws into each argument for you to find, so good essays on the same Argument topic tend to be pretty similar. I don't think I would have thought of the thing about fixtures having multiple bulbs, though!

It does occur to me that certain types of lighting can trigger migraines in some people, so that makes me think that there might be more health consequences related to a new type of lighting. If sitting under sun lamps is a treatment for depression, maybe really unnatural types of light could have the opposite effect? I'm not saying that that's true — just that more study is needed.

<u>Argument Topic #3</u>

Company X has just switched to a 4-day workweek, mandating that employees work 10 hours per day from Monday to Thursday instead of 8 hours per day from Monday to Friday. Although the policy is new, Company X claims that the policy will help to increase profits by shutting down offices on Fridays and to boost employee morale by reducing commuting time to and from work each week.

Write a response in which you examine the stated and/or unstated assumptions of the argument. Be sure to explain how the argument depends on these assumptions and what the implications are for the argument if the assumptions prove unwarranted.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Stacey Koprince:

The instructions specifically ask me to address assumptions made in the argument, so I first typed "P1: assumptions" and I also made a mental note to concentrate on that while I read the argument. Once I found the conclusion ("Company X claims that…"), I put it in my Paragraph 1 notes, and then I started brainstorming what assumptions would lead to that conclusion.

The best way to brainstorm assumptions is to figure out what premise leads to a particular claim. In this case, the argument says that "shutting down offices" will lead to an "increase [in] profits." Hmm, well, I know that profits are calculated by subtracting costs from revenues. This premise says the costs are going to go down, so the author must be assuming that revenues are going to stay the same. Now, I just need to brainstorm a couple of possibilities for why revenues might not stay the same if the company is closed down every Friday.

Next, the argument says that "reducing commuting time" will "boost employee morale." Here, I immediately thought, "What if someone's commute time is 10 minutes? He or she really isn't going to care much about commute time." That led me to the basic assumption here: that commute time is a significant source of concern for the vast majority of employees — and that told me what to think about next! It's possible that working an 8-hour day is just fine but 10 hours crosses a line in some way. If someone needs to pick a child up from daycare, or get a child onto the school bus in the morning, that person is not going to be thrilled with extended work hours. I have a Tae Kwon Do class after work one night a week; they're not going to change the time or day of the class just to accommodate my new work schedule!

See what I did at the end there? I put myself into the argument. What if my company were proposing this? Because the instructions specifically ask us to talk about what would happen "if the assumptions prove unwarranted," I put my critical thinking cap on. Why would this be annoying or problematic for me? (If you think it'd be great for you, then imagine a friend who would have legitimate problems with the proposal.)

Here are my notes for the argument (note: I type my notes in and then I write paragraph by paragraph, erasing each line of notes as I go).

> p1 Assumptions. Claim: increase profits, boost morale.
> p2 Assumes commute time = major pain point. Maybe 10h per day = bad for morale (kids, after-work activities?)
> p3 Assumes can make same revenues in 4 days as 5.
> p4 Assumes they can just shut down on Fri. Customers? Deliveries? Etc.
> p5 Conclusion

Essay:

Company X claims that switching to a 4-day workweek will help the company to increase both profits and employee morale. While it is possible that the plan will work as stated, the company's argument depends upon several assumptions that may or may not be valid. It is possible that a 4-day workweek will have no effect on profits; however, under certain circumstances, a 4-day workweek could actually decrease profits. Further, while some employees will no doubt be thrilled to have a 3-day week-end every week, others may not appreciate a 10-hour work day.

The company assumes that commute time is a significant "point of pain" for most of its employees, and that those employees would be happy to work 10 hours rather than 8 in order to avoid one day of commute time. The company fails to address, however, how a 10-hour workday might alter the schedules and lives of the employees outside of work. Let's assume that employees work a standard 8am to 5pm schedule. Now, employees will have to start earlier or work later (or both). Those with partners and / or children may find that this new schedule disrupts the family dinner or the before-school morning routine. Those who participate in regularly scheduled activities — an exercise routine, a sports match, a book club — may find their personal and social activities curtailed. If so, the new plan is not likely to improve employee morale.

The company believes that profits will increase because the company will reduce expenses associated with keeping the office open on Fridays. The company assumes, then, that it will make the same amount of revenue in 4 days that it typically makes in 5. Perhaps the nature of the company's business allows this but it would be wise to validate this assumption before proceeding. For instance, the business model might require salespeople to call on customers. What hours do those customers work? If it is very difficult to call on customers before 8am or after 5pm, then extending the hours for salespeople while cutting off one workday a week could actually result in decreased revenues. Alternatively, employees may be more mentally fatigued by the end of a 10-hour day and lose productivity; it's entirely possible that four 10-hour days will produce less work than five 8-hour days, though both add up to 40 hours.

This brings us to our final assumption: the company appears to believe that there are no negative consequences relative to shutting down completely on Fridays. Does the company ever field customer inquiries or have to provide support for its products and services? If so, customers will be unhappy if they cannot reach the company on Fridays. Does the company receive deliveries of any kind or have to interact with vendors or suppliers in any way? In such a case, Friday closures could impact the company's efficiency, which could in turn impact revenues, expenses, or morale.

Company X has developed an interesting idea that may be worth further exploration, but committing to such a plan immediately would be rash. There are a number of variables that need to be examined before concluding that the plan is likely to achieve the stated goals without resulting in (possibly quite negative) unintended consequences. It would be prudent for the company to determine, via direct dialogue with workers, what measures in general are likely to improve employee morale. Further, the company must carefully examine the possible financial consequences associated with spreading 40 hours of work over 4 days rather than 5 and with closing its offices entirely on a day when businesses are typically expected to be open.

Here is a second opinion from Manhattan Prep instructor Tommy Wallach:

Great essay from Stacey here. You can see from her explanation of how she went about writing it that Stacey cares a lot about organization. The three major assumptions are listed in the introduction, and then you hit them one at a time in three nice long paragraphs. This is *always* the goal, as long as you can come up with enough material while you're outlining.

Notice how Stacey uses the real world without resorting to specific examples; this is a hallmark of a good Argument Essay. In the second paragraph, she brings up "an exercise routine, a sports match, a book club." These are simple, solid, real-world examples. There's no need to talk about specific companies, and you should try to avoid it, or you risk writing more about the issue than the argument itself. You should be able to hypothesize plenty of realistic situations ("employees may be more mentally fatigued by the end of a 10-hour day," "the business model might require salespeople to call on customers," etc.) without resorting to *actual* situations from history/current events. (Specific examples are great in the Issue Essay, but not so much in the Argument Essay.)

Stacey hits a million great points here, but you'd be surprised how much stuff is hidden inside each prompt. For example, Stacey did not mention the possibility that Friday is the most lucrative day of the week, and so losing it would be worse than losing another day. She also never discusses the fact that when people are forced onto a different schedule than most of their friends and family, that free time might not be worth as much. These aren't mistakes, by the way, as Stacey found *plenty* of great stuff to talk about here. It's just a reminder that essay prompts always provide more than enough material for you to write a solid three paragraph essay.

Argument Topic #4

Five years ago, the local university built two new dormitories through different contractors: Aleph Construction and Gimmel Builders. The buildings were nearly identical, though it cost Gimmel Builders approximately 20 percent more to construct their dormitory. Aleph's dormitory, however, has required approximately 10 percent more in maintenance costs per year over the past five years. Therefore, to construct their new dormitory with the lowest overall cost, the local university should hire Aleph Construction.

Write a response in which you examine the stated and/or unstated assumptions in the expert's claim. Be sure to explain how the argument depends on these assumptions and what the implications are for the argument if the assumptions prove unwarranted.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Chris Berman:

No matter how the stimulus is phrased, a successful essay must correctly identify the conclusion and evidence topics. Usually, one must also note any central assumptions. Here, the stimulus expressly asks one to do so. Also, the specific question asks that the response discuss the implications if the assumptions are flawed — so, one must essentially note how to weaken the argument. The structure here is classic of the GRE. The premises involve percentages. Almost always with such a structure, there are flawed assumptions concerning the relationship between the percentages and actual numbers. This argument contains some other assumptions to question — for example, the supposed lifespan of the structures and the meaning of "nearly identical," to name two.

1. Identify the structural features of the argument.
2. Identify the questionable assumptions
3. Note how those assumptions, if unwarranted, would weaken the conclusion

Essay:

The argument concludes that Aleph is the more cost efficient contractor because its building cost was a substantial percent less than the competition's, even though its maintenance costs were higher than Gimmel's, albeit by a smaller percentage. Besides the assumption that the facts are accurate, this argument hinges on several other assumptions of dubious currency. Most strikingly, the argument assumes that a lower percentage automatically indicates a lower numerical cost. This is mathematically untenable. The author merely assumes that the maintenance base price is low enough that a smaller percentage indicates a smaller cost. If this is incorrect, the conclusion becomes erroneous. Furthermore, the argument apparently assumes that the lifespan of these buildings is not long enough for the additional maintenance costs to outweigh the construction savings. A miscalculation here would also completely undermine the conclusion.

While the aforementioned assumptions are certainly enough to question the validity of the conclusion, this argument contains several other quite possibly unwarranted assumptions. First, the author assumes that the contractors' prices have not changed in the interim. Additionally, the argument states that the buildings are "nearly identical," but the author only assumes that those implied small differences do not account for the difference in price. Furthermore, the argument takes for granted that the costs of all other features of the buildings — the land, zoning costs, transportation, etc. — were identical. Another unsupported assumption is that the maintenance costs will not change. Similarly, the author blithely relates the higher maintenance costs to the contractor without providing evidence that the costs do not result from unrelated factors. Finally, the argument supposes that there is no other contractor that would prove the least costly of all.

This litany of unsupported assumptions seriously jeopardizes the certainty of the conclusion. If any one of the suppositions contained in the argument proved false, the conclusion would be factually wrong, as Aleph would then not be the most cost efficient option.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

I like that we've included in this book a shorter essay that still covers all the bases! Our instructors can tend to be a bit long-winded.

While my own Argument essays tend to follow a set structure (short intro, 3 body paragraphs, short conclusion), and I tend to shoehorn whatever I have to say into that structure (if I have 8 problems with the argument, I will logically group them into about 3 body paragraphs), Chris here sticks *exactly* to the instructions, which ask about assumptions. So he gives us a "litany" of them, and ends with a strong conclusion: "If any one of the suppositions contained in the argument proved false, the conclusion would be factually wrong."

I think that the structure Chris used here, in which he introduces many of his arguments in the introduction, might be a bit risky. In less adept hands, this could look unplanned or disorganized, although Chris makes it work nicely.

When in doubt in your own writing, I'd stick to "one main point per paragraph."

Argument Topic #5

> Airline industry representatives have recently argued that flying is safer than driving, citing two separate studies. First, US statistics show that each year there are approximately 40,000 deaths in automobile accidents versus only approximately 200 in flight accidents. Second, studies indicate that pilots are four times less likely than average to have accidents on the road.
>
> Write a response in which you discuss what questions would need to be answered in order to determine whether the argument is reasonable. Be sure to explain what effects the answers to these questions would have on the validity of the argument.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Daniel Yudkin:

Crucial to writing a strong Argument essay is to refer consistently to the key words mentioned in the given prompt. The prompt in this case tells us to discuss which "questions need to be answered" to make the argument more "reasonable," and what their answers would have on the argument's "validity." Keeping these key points in mind, we can turn to specifics.

What is wrong with this argument? At first blush, it may seem convincing to learn that so few people die in flight compared to in automobile accidents. This is indeed a strong start to an argument, but it is missing a key ingredient, a "question" that needs to be "answered" in order for this statistic to make any sense. We need to know more than objective quantities; we need to know *proportions*. To know that only 200 people die in planes isn't enough—we need to know what *fraction* this is of the entire amount. Suppose only 200 people took planes every year, and every single one of them died. Flying doesn't sound quite as safe anymore, does it? Therefore, an important question to be answered involves the total number of people who drive and fly every year.

Turning to the second supporting sentence, we learn that pilots are safer-than-average drivers. Of course, what this doesn't tell us is anything about how safe they are in the air. Therefore, in order for this sentence to add to the argument's validity, we would need to know the extent to which being a safe driver makes you a safe flier. Now, even with this information, we still wouldn't know what impact this would have for the average airplane passenger—but at least it would be a start. We'll explore this topic further in the essay itself.

One final question that may not be worth mentioning in our essay but is important to keep in mind is where the statistics came from—in other words, were the studies cited from reliable, reputable sources? Or were they conducted by organizations that had a vested interest in proving a particular point? When all else fails—when the argument seems sound and you're having trouble pinpointing its flaws—this is always a good idea to fall back on.

We can construct our essay now using our first two issues as the skeleton, expanding and using examples to illustrate our points, and taking care to refer repeatedly to the specific issues raised in the prompt.

Questions:

What is the relative proportion of deaths per year between driving and flying?
To what extent does being a good driver make a good pilot?

Essay:

The purpose of the airline industries' argument, elucidated in the passage above, is to convince the reader that flying is safer than driving. The industry representatives cite two pieces of evidence to support their conclusion—first, that fewer people die per year in flight; second, that airline pilots get into fewer driving accidents. To show that these arguments are reasonable, several questions need to be answered. The first question has to do with the proportion of deaths represented by the figures of annual deaths; the second has to do with the degree to which being a safe driver translates into being a safe pilot.

Suppose the CEO of a car company tried to convince shareholders that the company was succeeding in a marketing campaign to sell a certain kind of car by saying that over 10,000 cars had sold since the campaign began. The obvious question on the shareholders' minds should be the fraction of the total number of cars sold that this figure represents. If 10,000 cars represents less than 1% of the entire market on cars for this period, presumably they would not be very pleased with the campaign. Similarly, knowing that only 200 people, versus 40,000 people, died in flight versus on the road is not informative until we know the proportion of the total number of fliers and drivers that these numbers represent. If far fewer people fly than drive, 200 could represent a large fraction of all fliers. And if the proportion of flight-deaths is higher than the proportion of car-deaths, then regardless of the total number, it would be fair to say that flying is not safer than driving. The question that needs to be answered, in other words, is how many people fly and drive each year. This would then allow us to calculate the portion of the total that these figures represent.

Turning to the second piece of evidence used to support the argument that flying is safer than driving, we learn that airline pilots are safer-than-average drivers, having four times fewer accidents than the norm. In order for this fact to be convincing, several questions would need to be answered. First of all, we would need to know whether being a safe driver translates into being a safe pilot. But more than this, we would need to know what being a safe pilot *means*. Keep in mind that we are solely evaluating the argument that flying is safe *relative to driving*. So knowing that pilots are "safe fliers" is nonsensical unless we can evaluate that claim in the context of driving. Therefore, a question that needs to be answered to make this argument valid is what having a "safe pilot" means for airline passengers. An example of an answer to this question that would make the argument more valid would be that for any given hour of flight, pilots are vastly less likely to crash than for a given hour of driving. This is quite clearly a long distance away from the statement as initially phrased—evidence that the argument as-is is in need of serious revision.

In sum, we see in this passage the beginnings of a coherent argument, but more information is needed in order to fully evaluate the strength of the claims. Namely, we would need to know what fractions 200 and 40,000 represent of the entire population of fliers and drivers, respectively, and also what implications being a safe driver has for being a safe airline pilot.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

Some Argument prompts present you with frankly *ridiculous* arguments. Flying is safer because pilots are safer *drivers*? On what planet does that make any sense?! Don't toss aside your initial feeling of "What the heck?!" A ridiculous premise is like a gift from the GRE — you can easily write a whole paragraph (as Daniel did) about how pilots being safe drivers is not the same as flying being safer. Daniel adeptly points out that *even if* being a safe driver were highly correlated with being a safe pilot, it's not clear that having a safe pilot makes air travel all that safe (what if there’s a tropical storm, or the plane’s wings fall off?)

Note also that Daniel adopts a very natural structure here — the prompt gave two reasons why flying is supposedly safer. Daniel wrote an intro, a large body paragraph about each of those two reasons, and a short conclusion.

Finally, always check the instructions that come with your topic. The instructions here ask us to “discuss what questions would need to be answered in order to determine whether the argument is reasonable.” Accordingly, Daniel’s tone is more like “Here is exactly what information we would need,” rather than “This argument is stupid.” The graders like you to sound balanced, thoughtful, and reasonable.

Argument Topic #6

> In a laboratory study of two different industrial cleansers, CleanAll was found to remove 40% more dirt and kill 30% more bacteria than the next best cleanser. Furthermore, a study showed that employees working at buildings cleaned with CleanAll used far fewer sick days than employees working in buildings cleaned with other cleansers. Therefore, to prevent employee illness, all companies should use CleanAll as their industrial cleanser.
>
> Write a response in which you examine the stated and/or unstated assumptions of the argument. Be sure to explain how the argument depends on these assumptions and what the implications are for the argument if the assumptions prove unwarranted.

Here’s a take on this topic — followed by an actual essay — from Manhattan Prep instructor Daniel Yudkin:

Typically, when we are asked to examine the assumptions that drive an argument, we are presented with certain pieces of evidence which do not totally add up to the final conclusion. The job, then, is to point out which pieces of evidence are missing that would be necessary to generate the conclusion. In this argument, we read that all companies should adopt CleanAll. The reasons for this are that it’s more effective than the “next best cleanser” and that when it’s adopted employees seem to take fewer sick days.

The phrase “next best cleanser” should be a dead giveaway of a great discussion target. Since no objective standards are cited, we don’t know if this cleanser is the second worst, or what. Next, the fact that employees take fewer “sick days” is insufficient evidence of the efficacy of the cleanser; there could be a variety of other causes for this. Finally, there may be other considerations that companies have in which cleanser they use, so even if the above two assumptions were satisfied, it wouldn’t necessarily warrant the conclusion.

We can make our outline:

Assumptions

- “Next best cleanser” is something important
- “Sick days” are a significant indicator of cleanliness
- Effectiveness warrants adoption

We’ll systematically describe these assumptions in our essay, taking care to indicate, as the prompt requests, the implications for the argument if the assumptions prove unwarranted.

Essay:

In this argument, the author suggests that all companies should adopt CleanAll as their industrial cleanser. This is based on several facts that the author cites. The argument depends, however, on certain additional assumptions which are not explicitly stated and which, should they prove unfounded, may critically weaken the final conclusion.

For example, the author references the fact that CleanAll performed better than the next best cleanser in a laboratory study of two industrial cleansers. Before this piece of evidence can be used to support the argument that CleanAll should be adopted by all companies, several things need to be established. First of all, it is important to know the origin of this study. Is the organization that performed this study reliable? The argument assumes that it is, but if it is not this would greatly weaken its thrust. Additionally, and importantly, we are never told what this "next best cleanser" is. Suppose that CleanAll was the second-worst industrial cleanser. If it were compared with the worst cleanser, it may very well boast the same facts that it removes more dirt and bacteria. In order for this claim to have weight, then, it is crucial to know the scope of the comparison. As is, the assumption that this comparison cleanser is high-quality is unsupported.

Another assumption on which the argument depends is the notion that sick days are a valid measure of the effectiveness of an industrial cleaner. The study demonstrates that the employees of companies using CleanAll take far fewer sick days. But we must not jump immediately to the conclusion that this differences was *caused* by the effectiveness of CleanAll. Were the two companies randomly selected? Were other possible variables controlled for? And finally, does fewer sick days mean a better cleaner? As is, the argument assumes that each of these is the case—for if any of these *weren't* the case, it would be unwarranted to jump to the final conclusion that *all* companies should adopt the cleaner.

The final assumption the argument makes is that a sweeping claim can be made from limited evidence. Suppose that all the assumptions stated above were indeed correct—that CleanAll performed better than leading cleansers, not just low-quality ones, and that its use in a company does indeed cause employees to take fewer sick days. Even if these were all the case, it wouldn't necessarily indicate that all companies should adopt the product. Perhaps there are other considerations that influence companies' choices. Price is one example. Perhaps CleanAll is much more expensive than competitors. Even such seemingly inane factors as scent could influence companies' choice as to whether they should use CleanAll. In short, the argument assumes that sick days and dirt removal account for the entirety of the factors influencing companies' decision in which cleanser to adopt; if these assumptions prove unwarranted the argument will have far fewer grounds to stand on in its conclusions.

In sum, while the evidence cited in this argument does provide initial reasons to believe that CleanAll is a superior cleanser, the argument's conclusion rests on other assumptions—namely, that the cleaning studies it cites are valid and reliable, that the use of "sick days" is an appropriate measure of cleanser effectiveness, and that companies care about only these two things. Without concrete evidence that these assumptions can be satisfied, we must remain skeptical.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

This well-written, well-argued, and well-organized essay really hits all the marks!

Personally, it kind of occurred to me that killing *all* the bacteria isn't necessarily a plus for a cleaning product. Apparently (according to what I read in the news, anyway) excessive use of antibacterial products is causing resistant superbugs to evolve! These bugs could kill us all! And couldn't harsh cleaning products also aggravate people's allergies, chemical sensitivities, etc.? (Some offices actually ban perfume for this reason.)

Anyway, that's just my personal brainstorming — Daniel's essay certainly has more than enough to go on. I loved the point that performing better than the "next best" cleanser could just mean your product is second-worst. Daniel also astutely points out that cleaning power isn't the only determinant of which product is best for a particular buyer. After all, it's an office building, not an operating room.

<u>Argument Topic #7</u>

> Downtown Zurzi is becoming increasingly congested with traffic, increasing commuting time for those who work downtown or near downtown. The nearby city of Loft was faced with the same problem several years ago and implemented a small weekly tax for driving one's car downtown. Downtown traffic almost immediately subsided in Loft and the local government also raised much needed money for fixing roads elsewhere. Obviously, this plan should be implemented in Zurzi in order to solve the brewing traffic congestion problem.
>
> Write a response in which you discuss what specific evidence is needed to evaluate the argument and explain how the evidence would weaken or strengthen the argument.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Daniel Yudkin:

When you read an argument topic that makes a comparison, the first thing you should be thinking about is the degree to which that comparison is warranted. Sure enough, with this argument topic, we will be spending much of our time discussing the ways in which Zurzi and Loft may differ in their respective traffic situations. Because many assumptions are built into the notion that what works well for Loft will work well for Zurzi, this is a rich issue for discussion.

A second common assumption made in argument topics is that something is a "good thing." In this case, we see that the argument assumes that it is a good thing that "traffic immediately subsided in Loft" and the town "raised much needed money for fixing roads elsewhere." Of course, there may be downsides to these incidents which we'll have to address in the essay.

The prompt asks you to discuss the "specific evidence" needed to decide whether the argument is good or bad, and how that evidence might strengthen or weaken the argument. With the above two thoughts in mind, you can now make a clear sketch of which issues your essay is going to address:

Comparison (Loft & Zurzi)

- Is the driving population the same?
- Is the downtown traffic situation the same?

Good Thing?

- Did the lack of traffic deprive local business owners of customers?
- Could those who paid the tax afford to pay it?
- Did the traffic simply move elsewhere, congesting other areas?

We now have five decent nuggets from which to craft an Argument Essay. We can start with a general introduction and then move on to discuss the specific examples in the subsequent paragraphs, making sure to point out how each example is directly relevant to the prompt. Then we round it off with a conclusion that summarizes the points.

<u>Essay:</u>

We learn in the above argument that a town, Zurzi, is confronted with a traffic problem, and we learn how another town, Loft, dealt with this problem. We are asked to subscribe to the notion that since Loft's efforts to curtail traffic were effective, Zurzi should enact similar measures. In order to better evaluate this argument, additional evidence is needed, falling generally into the categories of whether Loft's efforts were indeed effective, and, even if this is the case, whether there is reason to believe that Zurzi will enjoy similar success were it to follow suit.

Were Loft's efforts effective? To answer this question we would need to know more about the side effects of its actions. There are several possible negative side effects that could have occurred as a results of Loft's implementing a tax on cars that enter the downtown area. For example, if traffic is suddenly avoiding that area because of the tax, where is it going? Are people simply driving less, or have they merely moved to another part of town? Evidence showing that traffic is simply bypassing that area and congesting another part of town would weaken the argument. Additionally, we may want to know *who* this tax is most affecting. In many cities, people who live on the outskirts of town are less able to afford the more expensive real estate in the city center. They then drive to work downtown. If this is the case in Loft, then the tax may be putting financial strain on those who cannot afford it, but have no other choice. If we were to find evidence that this was the case, this would weaken the argument that Zurzi should enact a similar plan. Finally, another important side effect of the tax may be the impact on local business owners. If suddenly traffic downtown is drastically curtailed, local business owners may find themselves with a precipitous decline in business. Evidence to this effect would serve to undermine the original statement.

We turn next to the second big assumption of the prompt, which is that Zurzi should do as Loft does. Suppose we find evidence that Loft's measures were indeed successful—that traffic patterns aren't harmfully disrupted, that the tax isn't too great a financial burden, and that business owners are enjoying, say, an increasing influx of pedestrian clientele. Does this imply that Zurzi should necessarily do in kind? Not until we have evidence that Zurzi's situation is comparable to Loft's. For example, we would need to know whether Zurzi's driving population is the same, both geographically and financially, as Loft's. Just because Loft's drivers can afford to pay that tax doesn't mean that Zurzi's can—we need more information to confirm that this the case. Additionally, as with Loft, we would need evidence indicating that businesses are subject to similar conditions. Evidence to this effect—namely, evidence indicating that Zurzi faces conditions similar to Loft—would strengthen, and indeed are necessary to the logical thrust of, the argument.

In summary, we see two main pillars of evidence that would need to be established in order to better evaluate the argument. First, we'd need evidence that Loft's efforts were successful. Second, we'd need evidence that Zurzi is like Loft. With such confirming evidence, the argument begins to cohere. Without it, the argument is bereft of critical foundations.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

Great essay, Daniel! Any time you get a GRE essay topic that makes an analogy, you can pretty much write a whole essay (if you have to) about what a bad analogy it is. For Argument Essay purposes, all analogies are terrible analogies!

Daniel's third paragraph covers in detail all the things we'd need to know about Loft and Zurzi to make sure that what worked in Loft would be likely to work in Zurzi. Daniel's substantial second paragraph about the possible side effects of the plan makes for a very thorough essay. I especially liked the closing about the argument's being "bereft of critical foundations."

Argument Topic #8

In the last year's mayoral election in Town T, candidate Miller led candidate Keating by a substantial margin in the polls leading up to the election. At the last minute, candidate Keating launched a widely viewed series of television advertisements that focused on preserving the natural environment of Town T, a topic neglected by candidate Miller. Subsequently, candidate Keating won the election by a narrow margin. This year, if candidate Miller hopes to win the upcoming mayoral election, he must increase his coverage of the topic of preserving the natural environment of Town T.

Write a response in which you examine the stated and/or unstated assumptions of the argument. Be sure to explain how the argument depends on these assumptions and what the implications are for the argument if the assumptions prove unwarranted.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Tommy Wallach:

This is a nice, long argument topic. I generally find that the more words there are in the prompt, the more problems there are in the argument, which is good news if you want to pull together three solid body paragraphs. Remember that your thesis on any Argument Essay is more or less the same: "While the author comes to a plausible conclusion, the argument made to reach that conclusion is flawed."

Our goal is to find three problems with this argument. Remember that they have been purposely placed in the prompt, so all we have to do is locate them. The first one I can see is that an error with time is being made here. Just because an issue is problematic one year doesn't mean it will be problematic another year. The natural environment issue may already have been resolved.

The next problem has to do with the comparison between the two elections. In the previous election, Miller went up against Keating and lost. But the prompt hasn't told us that Miller is going up against Keating again. Miller's new opponent may have no plans to focus on that issue, or his opponent may have already staked out that territory early in the campaign.

The final problem I see is that a causal relationship has been implied between the TV ads and Miller's loss, yet this could just be a correlation. Miller could have lost for some other reason. I don't love this particular flaw, so I'm going to leave it for last. You should always do your best to find three flaws, but if one of them feels weak, leave it for last. Your reader will often begin to skim at the end if they think you're going strong.

Let's take a look at the outline.

Thesis: Author comes to decent conclusion, argument no good
I. Issues are not the same every election
II. Candidates are not the same/polling situation not the same
III. No definitive evidence given that TV ads were deciding factor

<u>Essay:</u>

The prompt above describes a mayoral election between Miller and Keating that Miller lost soon after a particular TV ad was run. From this, the author derives a conclusion about what Miller should do to win the election this year. In coming to this conclusion, the author makes a number of unfounded assumptions that undermine the validity of the argument. These assumptions are based on faulty comparisons— between candidates, topical issues, and other contextual elements—and a mistaking of correlation for causation. If the author wants to support his conclusion, he would need to provide evidence proving that these two elections are comparable, and that Miller's loss can factually be attributed to its implied cause.

The first error made by the author is in implying that just because an issue was topically relevant during one election, it will necessarily be relevant during the next election. For all we know, the natural environment could have been a major issue in the previous election because of some climatic issue, such as saving a particular waterway or passing a specific law to protect local wetlands, that has since been resolved (during Keating's mayoralty, for example, which would make logical sense given that he ran on a preservationist platform). If the author wants to make this connection, he would need to provide evidence that the natural environment remains a hot-button issue that could decide the election one way or another.

The next faulty assumption in the argument also involves a comparison, namely between Miller and whoever his opponent is. The prompt seems to imply that Miller will be going up against Keating again, but there's no evidence to support this. This matters, because if Miller is running against a different candidate, then there's no reason to believe a strategy that would have served him well against Keating would still be useful. For example, he could be running against a park ranger, who will always be able to beat him on issues relevant to the natural environment, in which case increasing his coverage of those issues would be a waste of time. To fix this assumption, the author would need to tell us definitively that Miller is running against Keating again (or a very similar candidate).

Another major assumption the argument makes is in deciding that the television ads Keating ran were the sole or even primary reason that Miller lost the first election. This is a causation/correlation error, in that we have no reason to believe the two things didn't simply happen separately, rather than one causing the other. It's just as likely that Miller committed some terrible faux pas near the end of the election, and that turned out to be more of a deciding factor than the television ads. The author needs to provide evidence that the primary impetus behind so many people changing their vote between the time of the polling showing Miller in the lead and Keating's win was the television ads.

It's certainly plausible that Miller might fare better in the next election if he pays attention to a subject he is said to have neglected. However, we cannot know that for certain unless the author provides definitive evidence that the issue of environmental preservation is still in the forefront of constituents' minds, that Miller and Keating are the two candidates in the next election, and that the television ads and the issue they underscored were the primary reason Miller lost his lead in the previous election. With this information in place, the author's conclusion becomes not just plausible, but convincing.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

I was especially interested in Tommy's comment that his thesis is always pretty much, "While the author comes to a plausible conclusion, the argument made to reach that conclusion is flawed." Mine is always something like, "This argument has numerous logical flaws, rests on many unwarranted assumptions, and would be improved with the addition of more information." In other words, Tommy's approach is more middle-ground, and mine is more, "This argument is awful."

Tommy's approach works very well in this essay. We're certainly not called on to argue that Miller should *not* campaign on environmental issues — just that there are gaps in the particular reasoning presented in the argument for this conclusion.

Argument Topic #9

Last year PrepUp had record enrollment in their test prep courses, but yearly profits fell by nearly 30 percent. To the contrary, TopPreparation had comparable enrollment to the year be fore, with profits rising by approximately 20 percent. This discrepancy most likely results from the fact that PrepUp teaches only live, in-person classes, which require expensive rental spaces and teaching equipment. Although TopPreparation's total enrollment remained unchanged, its online enrollment increased by 50 percent, and online classes have a much lower overhead. In order to address these issues of profitability in the coming year, PrepUp will begin offering online test prep courses.

Write a response in which you discuss what questions would need to be answered in order to decide whether PrepUp's plan for the coming year and the argument on which it is based are reasonable. Be sure to explain how the answers to these questions would help to evaluate the plan.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Jesse Cotari:

The topic of this Argument Essay provides both advantages and risks for me. On the upside, I've recently been doing loads of business case studies that focus on profits, revenues, and competitive advantage. This means that I have a lot of familiarity with the types of analytical frameworks that go into tackling this type of situation, and hence loads of ideas! But this presents a number of risks. A bevy of ideas can be a burden when writing a timed essay, so it's going to be essential to brainstorm quickly, group common ideas together, prioritize the best and discard the second-tier ideas. Furthermore, I felt the distinct draw to attack this problem as a business case, and not as an AWA essay. However, doing this the AWA way means identifying and focusing on the logical flaws in the argument, not just examining the issues involved in making a proper business decision. The last risk was jargon. Dropping a term like "allocated fixed cost" into the argument without definition is going to irritate a grader (with good reason). On the other hand, defining terms can eat up vital time and make an essay's logic drag. To attack this essay effectively therefore means keeping the *ideas* behind the jargon in mind while staying clear of any particularly economic or financial terms.

My first bit of processing is to outline the argument, to get a sense for the flow and identify common logical errors.

PrepUp had "record enrollment" whereas TopPrep had unchanged numbers.

PU:
High enrollment, dropping profits
Live in-person classes
Expensive rental spaces
expensive teaching equipment

TP:
same enrollment ⟶ 30% rise in profits
50% rise in online classes

The key flaws I see here are a lack of quantification (we're given percents, not numbers), vague terms ("record enrollment"?), "correlation isn't causation," and issues of "skill and will." I also have an idea that relates the sizes of the companies, the initial investment required for an online teaching system, and the comparative per-student cost of that initial investment. However, despite the fact that it's probably a solid idea, I dump it because I realize that the verbiage and chains of logic necessary to make the point would slow me down and potentially reduce the overall flow of the essay. I decide to lump lack of quantification and vague terms into one issue, and come up with the following outline:

Intro
Lack of numbers (% vs $)
Correlation is not causation: Profitability and in-person classes
Even if: Skill and will: Investment, training, ⟶ profit
Wrap up.

(I always like having an "even if" in the chain of my argument. It's usually my third body paragraph. At that point in my essay, I've pointed out a number of flaws in the logic. Then I say, *even if you fixed all of these problems, you* ***still*** *wouldn't be able to make your argument work.*)

Outline in hand (actually, typed on the screen, to be deleted at the end), the last thing I do is always to reread the prompt. Noting that it says "questions … to be answered," I make a mental note to phrase the essay in those terms, and begin writing.

With two minutes left, I go back and made a few last checks. In my fervor of ripping up the argument, I notice that I neglected to phrase "logical omissions" as "questions to be answered," so I fix that. I make my intro a little more pointed. The last thing I do is be sure to copy-paste to fix the places where I abbreviated TopPreparation as "TP."

Essay:

At first glance, the recommendation appears obvious: If TopPreparation's business model is yielding large increases in profit, PrepUp should emulate this business model to improve profitability. However, the logic of the argument as it stands is utterly incomplete, and there are major questions that need to be answered before deciding whether to undertaking a plan of offering online courses makes sense for PrepUp.

The first area of deficit in the argument lies in the numbers, or rather, in their absence. All of the profits are stated in percent increases or decreases, with no initial values to start from. If PrepUp had been insanely profitable in previous years, whereas TopPreparation barely made ends meet, emulating TopPreparation's business model would be unwise. It would be essential to determine the starting profitability of both businesses to make any comparison.

A second major question stems from the assumed connection between increased enrollment and decreased profits at PrepUp. To make this connection, we would need to answer the question of whether per-student costs have actually increased. Without knowing the underlying causes, increased enrollment could positively or negatively affect profits, or could have absolutely no relationship to their present decline. If new classes have been held in new, more expensive classrooms, increasing enrollment could have indeed led to increased costs and the decline of profits. On the other hand, if increased enrollment has led to increases in average class sizes, the per-student cost would have actually gone down, while per-student income would have remained the same, leading to an increase in per-student profitability. It is equally possible that profits have declined because costs unrelated to instruction have increased. Perhaps the CEO decided to increase her pay by a large percentage. Perhaps PrepUp invested in new IT infrastructure. Without knowing the breakdown of the current income and expenses, it is impossible to impute the profitability decline solely to the expense of in-person teaching.

Even if PrepUp and TopPreparation were initially producing comparable profits, and increasing enrollment in classes has pushed up per-student costs, there is still a major question to be answered: How much is the new system going to cost compared to how much it will save? Development, implementation and personnel training are required for the roll-out of any new system, and these things are not free; there will likely be a large initial investment required to implement such a system. While a "much lower overhead" sounds like an attractive goal, a savings $10,000 per year for a system that costs $100,000 to implement would imply a minimum 10-year timeline for recovery of the initial investment. Without knowing the initial investment required to implement a new online system, it is impossible to assess the overall contribution to cost savings.

All told, while the online teaching system does appear attractive, exact quantitative data is lacking in the argument. Without a basis to compare the two companies, and without understanding the current drivers of profitability and the costs of the new system, it is impossible to properly evaluate the argument in favor of online classes.

Here is a second opinion from Manhattan Prep instructor Jennifer Dziura:

I enjoyed Jesse's strategy of always having an "even if" in the chain of the argument — in Jesse's case, in the third body paragraph.

A sophisticated Argument Essay doesn't just say, "Everything in the prompt is wrong." Rather, it addresses *relationships* among the components of the argument. Jesse writes two full paragraphs about problems in comparing TopPreparation and PrepUp, and then says that *even if* the comparisons are valid and the assumptions true, the cost of switching might *still* eliminate any advantage to the plan to switch.

Note that Jesse doesn't come out against PrepUp's plan — he concludes that "the online teaching system does appear attractive," but that much more quantitative data is needed. Because Jesse has gone into great detail about *exactly* what data we would need, this conclusion is persuasive.

Argument Topic #10

> Cot-Ten, a cotton production company, has recently faced profitability issues based on the use of Chemical X in its manufacturing process. The main by-product produced when using Chemical X is covered under stringent environmental regulations, making it very difficult and expensive to dispose of. A similar processing product, Chemical Y, has recently been discovered, and can be used by Cot-Ten at a minimal cost of switching. The CEO of Cot-Ten has declared that the company will increase profits by switching to Chemical Y by the end of the month.
>
> Write a response in which you discuss what specific evidence is needed to evaluate the argument and explain how the evidence would weaken or strengthen the argument.

Here's a take on this topic — followed by an actual essay — from Manhattan Prep instructor Jennifer Dziura:

Let's run through this argument: Disposing of Chemical X (well, the by-product of Chemical X) is expensive and is hurting profits.

But Chemical Y has just been discovered! Cot-Ten can use Chemical Y instead. (Hmmn, sketchy. Brand-new chemicals kind of freak me out. Maybe our clothes will start giving us cancer!)

We are then told that the cost of switching will be minimal. Um, okay, but that's hardly the point! Tell me something — anything! — about this mysterious Chemical Y!

OMG, so many unanswered questions! Such as:

- Are there also "stringent environmental regulations" about Chemical Y? (Or is it likely that there *will* be, since Chemical Y is new?) Or regulations on its by-products?
- Does Chemical Y *have* by-products? If so, are there *more* by-products? For instance, maybe it costs less per pound to dispose of Chemical Y's byproducts, but maybe Chemical Y makes *way more overall byproducts,* so that the overall cost is higher.
- Does it cost more to *buy* Chemical Y in the first place???
- Is the cotton made with Chemical Y just as good? Might REVENUES go down even as expenses are slashed? THERE ARE TWO SIDES TO PROFIT: revenue and expenses, and revenue is kind of the more important one!
- Are there other reasons profits might not increase? Are consumers opposed to this relatively unknown new chemical? (I am! And this isn't even real!)

Okay, this is plenty for me to get started writing, especially since the instructions ask me to talk about "what specific evidence is needed." Easy! I know I need to shoehorn these points into three body paragraphs, but since my bullet points above have a lot of overlap… I'm just going to start writing and see how it goes. After I write the body paragraphs, I'll go back and revise my introduction to make sure my thesis reflects what I actually talked about.

Also — very important — I want to make sure that I stay ON TOPIC. Keep in mind that writing a paragraph about how Chemical Y might destroy the environment and kill all the wildlife and give everyone diseases would NOT help you score well on this essay. In fact, this point would be IRRELEVANT, because the argument's conclusion is that Cot-Ten will be able to *increase its profits* (not just "do good stuff") by switching. So, if I wanted to argue that the new chemical might have all kinds of bad effects, I need to tie those bad effects back into "profitability," such as by pointing out that poisoning people is bad for the bottom line.

Essay:

In response to its profitability problems, cotton production company Cot-Ten intends to switch from Chemical X, which requires expensive disposal of its by-products, to Chemical Y. However, Chemical Y is a truly unknown actor in this drama. To evaluate the argument at hand, we need far more information about Chemical Y, its byproducts and disposal processes, its cost, and its efficacy in producing saleable cotton. Otherwise, Cot-Ten will be moving from a profitability deficit to a total unknown that could very well prove worse.

The reason provided for Cot-Ten's switch from Chemical X is that the byproduct of that chemical falls under strict environmental regulations, and is thus expensive to dispose, thus impacting Cot-Ten's profitability. Then we are told that the newly discovered Chemical Y is "similar." Similar in what ways? Presumably, the chemical serves the same function in the manufacturing process, but the argument has done nothing to assure us that Chemical Y is not, in fact, similar in terms of environmental regulations and disposal costs. What if Chemical Y is actually more expensive to dispose of? To properly evaluate Cot-Ten's plans, we need to know how much byproduct, if any, Chemical Y produces, and the cost and difficulty of that byproduct's disposal. In order to know that, it would be helpful to know whether Chemical Y falls under any environmental regulations or whether — since Chemical Y is new — such environmental regulations are likely to be passed in the near future. In order to calculate any cost savings, we would also need to know whether the manufacturing process would use the same amount of Chemical Y as it did of Chemical X (even if Chemical Y's byproducts are cheaper to dispose of, pound-for-pound, if much more Chemical Y is needed or if Chemical Y yields far more byproduct, this cost savings could be easily canceled out).

Keep in mind, however, that the argument's conclusion relates not just to cutting expenses, but to profitability overall. What if the switch to Chemical Y causes revenue to fall? We are told that Chemical Y is a "similar processing product," but will the cotton produced with this chemical be identical (or better)? Even if the product produced is technically the same, will consumers be aware of the switch? If so, would they have any reason to doubt the new product's quality or otherwise be opposed to the switch? The same factors that might cause environmental regulations to be levied against Chemical Y, as they were against Chemical X, could also cause environmentally-conscious consumers to oppose to the new product, thus lowering revenues.

If it turned out that Chemical Y were truly an innocuous alternative to Chemical X, that it cost less to dispose of Chemical Y's byproducts, and that consumers would purchase the new product for at least the same price and frequency as the old product, then the argument would be immeasurably strengthened. However, if Chemical Y were to have the same problems — high disposal costs due to environmental regulations — or introduce other costs, or if it were to reduce revenues, then the argument's conclusion would be unjustified. While Cot-Ten executives may have scored a coup by locating a cost-saving alternative, much research must be done before these executives can be congratulated. Whether Cot-Ten's profitability will rise, and whether switching from Chemical X to Chemical Y is a wise move, remains to be seen.

Here is a second opinion from Manhattan Prep instructor Tommy Wallach:

This is a really great essay from GRE superstar Jen Dziura. Notice how her introduction manages to be playful and professional at once ("Chemical Y is a truly unknown actor in this drama."). Don't be afraid to make your essays entertaining; those poor readers have to score dozens and dozens of these things every day, and they'll be grateful for a little bit of fun.

Jen focuses her big second paragraph on the primary assumption made in the passage, that Chemical Y is going to improve the whole by-product disposal issue. If I were writing this essay, I might break this paragraph into two smaller ones, just to give the reader a bit of a break. I might have the first paragraph focus on the issue of Chemical Y's own by-products and disposal costs, and the second paragraph on the issue of whether manufacturing/purchasing Chemical Y might cost more. Jen hits both of these points already, so the change would merely be an organizational one. (My OCD brain always strives for exactly five paragraphs, even though there's no rule about it!).

Notice also how many ideas Jen had before she started writing. There were so many, some of them had to get left by the wayside! If you know you have plenty of good ideas, don't hurt yourself trying to stretch the essay to fit them all in. Organization is key, so keeping your paragraphs focused is better, even if you leave out an idea or two (I might have grabbed on to the "minimal cost of switching" myself, but there's so much here, there's really no need). In this essay, it's easy to see that paragraph two is about the profit side of things, where paragraph three is about the revenue side of things. Jen knows that essay readers need to be able to see exactly what each paragraph is about, preferably within the first couple sentences of that paragraph.

Chapter 32
of

Verbal Practice Sections

In This Chapter…

Verbal Practice Section 1: Easy Difficult

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20 Questions
Time: 30 Minutes

For questions 1–6, select one entry for each blank from the corresponding column of choices. Fill in the blank in the way that best completes the text.

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1. A dry-farmed tomato raised in low-nitrogen soil will often have the nutrient value of a much larger conventionally grown tomato, and its flavor may be similarly __________.

delicious
healthful
scanty
concentrated
shrunken

2. Most viewers of today's reality television underestimate the degree of __________ with which the seemingly artless narrative of each episode is assembled: oftentimes, actors interviewed on camera are asked to repeat their story half a dozen times before producers are satisfied.

dramaturgy
opportunism
fallacy
contrivance
histrionics

3. Many (i) __________ have commentated that football reflects an industrial perspective because the game time is inflexibly determined by the clock, whereas baseball stems from an agrarian one, in which, the passage of time is more (ii) __________ and determined by events.

Blank (i)	Blank (ii)
pundits	amorphous
amateurs	pacific
parvenus	asymmetrical

4. Ernest Hemingway, the novelist and proponent of traditional masculine virtues that were already considered (i) __________ by his more progressive peers, eventually lost currency with the general public; although his earlier literary works remained popular, his continuing adherence to that code drove him to disdain society, which, in turn, (ii) __________ him.

Blank (i)	Blank (ii)
effeminate	shunned
anachronistic	eulogized
sardonic	murdered

5. While a (i) __________, the smallest amount that can exist independently, of water could be said to be a single molecule, a (ii) __________ of dust could be made up of pollen, hair, human skin cells, minerals from soil, or even burnt meteor particles. Thus, one definition of dust is "solid particles with a diameter of less than 500 micrometers"— a (iii) __________ definition that is based only on size and state of matter rather than on structure.

Blank (i)	Blank (ii)	Blank (iii)
plethora	trove	precise
parameter	covey	loose
quantum	mote	deleterious

6. From the battle's opening (i) __________ to its (ii) __________ conclusion, the forces of destruction razed a path through the city, ultimately leaving behind a (iii) __________ stillness where there once had been streets and squares bustling with life.

Blank (i)	Blank (ii)	Blank (iii)
denouement	sanguinary	brusque
salvo	celebrated	capitulatory
philippic	blithe	disquieting

For each of questions 7–10, select one answer choice unless otherwise instructed.

It is no revelation that people prefer immediate rewards. What is less well known is that people are willing to renounce a significant portion of a given reward in order to expedite delivery. This phenomenon is known as "discounting," because the value of a delayed reward is discounted, or reduced, in the mind of the receiver. Discounting helps explain the straightforward "time value of money" (a dollar now is worth more than a dollar later), but its manifestations can be far more dramatic.

Behavioral economists have identified extreme discounting in experiments in which subjects were offered either a dollar immediately or three dollars the next day. Individuals who consistently choose significantly smaller rewards for their immediacy are described as "present-biased." Present-bias may seem innocuous, but it has serious ramifications. In another experiment, young children were given a marshmallow, then told that if they could wait a few minutes to eat it, they would receive a second one. Those unable to endure the delay suffered from more behavioral problems in adolescence and scored markedly lower on standardized tests than the children who were able to wait and thereby earn another treat. Traits such as indolence and apathy may indeed be manifestations of present-bias; material success is predicated on one's ability to recognize hedonistic impulses, understand their consequences, and delay or suppress gratification.

What was once known as "exponential discounting" (because the length of the delay before a reward was given seemed to correlate directly with the size of the perceived discount) has been renamed "hyperbolic discounting," because the effects of time delay do not seem strictly linear. A study showed that people offered \$50 now or \$100 in a year were likely to choose the former. But when people were offered either \$50 in 5 years or \$100 in 6 years (the same choice 5 years in the future), the vast majority chose the latter. This experiment reveals the difficulty of making effective financial decisions about one's future priorities, just as the choice to procrastinate requires the unlikely supposition that one's future self will have a greater set of resources to accomplish the postponed task than one's present self.

7. The function of the second paragraph within the passage may be most appropriately characterized as

(A) analyzing the psychological sources of the phenomenon introduced in the first paragraph
(B) elaborating upon and qualifying the main proposition put forth in the first paragraph
(C) providing an alternative point of view on the positions already established by the author
(D) illustrating one aspect of the phenomenon named in the first paragraph and offering possible social implications
(E) drawing general conclusions about specific examples presented earlier

8. The passage suggests which of the following about the traits of indolence and apathy?

(A) They are predicated on one's material success.
(B) They may be regarded as the effects of a tendency toward extreme discounting.
(C) They may be manifested in present-bias.
(D) They may seem innocuous, but they have serious ramifications.
(E) They are the cause of one's ability to identify and suppress hedonistic impulses.

9. It can be inferred that the word *hyperbolic* in the phrase "hyperbolic discounting" signifies, in this context,

(A) the nonlinearity in the consequences of various delays
(B) the exaggeration of the effects of time delay
(C) the direct correlation between a delay's duration and a perceived discount's magnitude
(D) the exponential growth of the size of the discounting
(E) the lack of certainty in one's perception of longer and shorter delays

10. According to the passage, which of the following is true of the children described in the passage who were able to wait to eat their first marshmallow?

(A) They scored lower on standardized tests than children unable to wait.
(B) They suffered more frequently from behavior problems as adolescents.
(C) They each received a second marshmallow as part of the experiment.
(D) They could be characterized as indolent or apathetic.
(E) They can accurately be described as present-biased.

Question 11 is based on the following reading passage.

The female arkbird will lay eggs only when a suitable quantity of nesting material is available, and the climate is suitably moderate. This winter is the coldest on record, but somewhat counterintuitively, the temperature change has actually increased the amount of nesting material as trees and plants die, shedding twigs and leaves. However, although nesting material is abundant, ___________________________.

11. Which of the following options for the blank above is best supported by the passage?

(A) The female arkbird will likely migrate to avoid the cold.
(B) Arkbird mortality rates increase as the weather becomes less moderate.
(C) Female arkbirds prefer the type of nesting material produced in warmer weather.
(D) An abundance of nesting material provides increased protection for arkbird eggs.
(E) The female arkbird will not lay eggs this winter.

For questions 12–15, select the two answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole and produce completed sentences that are alike in meaning.

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12. Floodwaters had already breached the library's walls, but hopeful volunteers in hip boots worked tirelessly to __________ the damage.

☐ exacerbate
☐ ameliorate
☐ recant
☐ forfeit
☐ recount
☐ extenuate

13. Eleanor of Aquitaine, who married Henry II of England, was dead and forgotten for hundreds of years, until Katherine Hepburn __________ her in "The Lion in Winter," injecting her own vitality into the depiction of that queen.

☐ mummified
☐ mocked
☐ resurrected
☐ glamorized
☐ immortalized
☐ parodied

14. However beneficent the intentions, if the civilian death toll continues to climb, ongoing contact with the local populace may well prove __________ to the aim of normalizing relations.

☐ asinine
☐ bedazzling
☐ unpropitious
☐ inimical

☐ calibrated
☐ incongruous

15. The theme song from the film *Team America: World Police* is meant as a parody, sending up the __________ slogans and anthems that have recently become a disconcertingly ubiquitous aspect of American life.

☐ aggressive
☐ punctilious
☐ boosterish
☐ jingoistic
☐ pedantic
☐ bloodthirsty

Question 16 is based on the following reading passage.

The chemicals division at Company M spent 4% of its 2008 budget on marketing. The consumer products division spent 35% of its 2008 budget on marketing, while the machinery division spent only 2% of its 2008 budget on marketing.

16. Which of the following conclusions is best justified by the data above?

(A) The consumer products division spent more on marketing in 2008 than the chemicals and machinery divisions combined.
(B) Consumers are more swayed by marketing than are the mostly corporate buyers of chemicals and machinery.
(C) On average, all three divisions combined spent less than 35% of their 2008 budgets on marketing.
(D) The company's overall spending on marketing is between 4% and 35%.
(E) The chemicals division spent 100% more on marketing in 2008 than did the machinery division.

Questions 17–19 are based on the following reading passage.

Maps are essential décor for any social studies class, and though they are helpful tools in beginning to understand geography, maps are merely 2-D representations of a 3-D world and will always carry certain inherent inaccuracies. Because of their flatness and size restrictions, maps require manipulation, rendering them incapable of showing the actual shape of the earth and the continents, nations, and other features upon it. Though these might seem like necessary concessions, the implications of such manipulations move beyond the blackboard and can have damaging effects; forcing students to see the world in 2-D each day has the attendant effect of teaching them to understand the world in two-dimensional terms.

Furthermore, maps present borders as fixed, unchanging entities, which is a misleading implication to present in a history course. Borders have been changing throughout the history of civilization, and the United States is a perfect example of a country with borders that have blurred and bled into one another for decades as states continued to form and join as recently as the 20th century.

17. The author's main idea is that

(A) maps are a necessary evil
(B) maps present borders as static
(C) 3-D representation of the world is impossible
(D) outdated information makes education less effective
(E) graphic representation can encourage cognitive misconceptions.

18. Which of the following, if true, would most undermine part of the author's evidence?

(A) Some students exposed to maps grasp 3-D and dynamic concepts about the world.
(B) Most teachers rely very little on the maps displayed in their classrooms.
(C) Computer generated map displays increasingly in use in classrooms show changes in boundaries almost instantaneously.
(D) Maps from hundreds of years ago contain errors.
(E) 2-D maps do not indicate topographical features effectively.

19. The author does which of the following in the passage

(A) employs circular logic
(B) cites a historical case
(C) uses physical description to support an accusation
(D) discusses a hierarchy of problems with maps
(E) rebuts a commonly held view

Question 20 is based on the following reading passage.

In 1928, Sir Alexander Fleming, working at St. Mary's Hospital in London, observed that a bluish-green mold had contaminated a culture of Staphylococcus, and that the areas of the Staphylococcus bacteria nearest to the mold were being destroyed. Upon testing a pure culture of this mold, Fleming discovered that the mold killed many types of bacteria. He named the substance *penicillin* and published his results in 1929.

It was not, however, until over a decade later that a team of researchers from Oxford, aided by an American laboratory, were able to increase the growth rate of penicillin — by then recognized to be the strongest antibacterial agent known at that time — such that enough of it could be produced to treat Allied soldiers wounded on D-Day, in 1944.

Consider each of the answer choices separately and indicate all that apply.

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20. Which of the following can be inferred from the passage?

☐ Fleming did not fully appreciate the therapeutic value of penicillin.
☐ At some point prior to 1944, penicillin could not be grown fast enough.
☐ Since ancient times, antibacterial agents have been used to treat wounded soldiers.

Answers to Verbal Practice Section 1

1. **Concentrated.** "Similarly" tells you that the relatively small dry-farmed tomato has all the flavor of a conventionally grown large tomato, just as it has all the nutrients. You want a word that means something like "condensed." Only *concentrated* works. *Shrunken* and even *scanty* might be attractive, but neither captures the idea that the tomato has a great deal packed into a small package.

2. **Contrivance.** This blank takes its clue from the phrase "seemingly artless." As is often the case in GRE questions, the word *seemingly* should set you up to be thinking of an opposite to "artless," which means "genuine, not artificial." In this case, then, you want a word that denotes something deliberately constructed or fabricated: something *contrived*.

3. **Pundits, amorphous.** The clue for the first blank is "commentated" (*commentators* are professionals, not *amateurs*). The pivot word *whereas* means that the second blank opposes "inflexibly determined." *Amorphous* fits this meaning.

4. **Anachronistic, shunned.** Following the clues, the first blank must reflect the opinion of "progressive peers" regarding that which is "traditional" (*progressive* indicates wanting to move forward — away from traditions). *Anachronistic* means "in the wrong time period." The second blank must match *disdain* because of the expression *in turn,* which indicates reciprocating an action or "getting someone back" for something.

5. **Quantum, mote, loose.** A *quantum* is "the smallest amount that can exist independently." A *mote* is a speck or small amount; the word is specifically associated with dust. In the third blank, since the definition is *based only on size and state of matter rather than on structure*, it is *loose,* rather than strict.

6. **Salvo, sanguinary, disquieting.** A *salvo* is a "simultaneous release of bombs," and is often used metaphorically to mean the start of some kind of fight. Certainly, every clue in the sentence is negative, so you want to describe the battle's conclusion in a negative way — only *sanguinary* matches. Finally, a "stillness where there once had been streets and squares bustling with life" is a bit disturbing or *disquieting*.

7. **(D).** The first paragraph introduces the phenomenon of *discounting*: the reduction in the perceived value of a delayed reward. You are told that discounting can account for "straightforward" effects such as the time value of money, but "more dramatic" manifestations are also hinted at. Some of these dramatic manifestations of "extreme discounting" are described in the second paragraph. You learn about "present-bias" in two experiments (one in which \$1 now is preferred to \$3 tomorrow, and another in which children struggle to resist marshmallows). The consequences of the marshmallow experiment are stated and ruminated upon in a larger social context (how present-bias may prevent material success). Thus, the second paragraph elaborates upon a particular, extreme version of the phenomenon introduced in the first paragraph. It also draws out a few larger implications. Regarding (A), the second paragraph provides nothing to explain where discounting comes from psychologically. You cannot assume that children are the "psychological sources" or that discounting originates in childhood. Regarding (B), the first paragraph does not really introduce a "proposition" per se, which would be a claim of some sort. Rather, the first paragraph simply defines a phenomenon. Even if you said that this definition is a proposition, the second paragraph does not "qualify" (limit) it in any way. As for choice (C), the author has staked out no position in the first paragraph; he or she has only described a phenomenon. The second paragraph does not take an alternative point of view, either. As for (E), there are no true "specific examples" introduced in the first paragraph for the second paragraph to draw conclusions

from.

8. **(B).** Since you learn from the passage, "Traits such as indolence and apathy may indeed be manifestations of present-bias," you are looking for a choice that says more or less the same thing. Choice (A) mixes up words from the text (*predicated, material success*), but definitely does not match the meaning of the passage. The passage says that material success is predicated on the "ability to recognize hedonistic impulses," not that indolence and apathy are predicated on material success. In fact, the passage suggests that indolence and apathy have something to do with a *lack* of material success. Choice (B) is correct but is in disguise. You are looking for *manifestations of present-bias*, but you have here "regarded as the effects of a tendency toward extreme discounting." The key is that at the beginning of the second paragraph, you have essentially defined "present-bias" in terms of "extreme discounting." That is, present-bias is really nothing more than a tendency toward extreme discounting. This disguise makes choice (B) tricky. Regarding choice (C), be careful with language! Saying that these traits "are manifested IN present-bias" means exactly the reverse of "are manifestations OF present-bias." In the former, the traits are somehow hidden, but they show up IN or THROUGH something on the surface called present-bias. What you are looking for, however, is this: present-bias is the hidden thing. The traits of indolence and apathy are manifestations (demonstrations) of that hidden thing. Choice (D), like choice (A), mixes up words from the text. Present-bias — not indolence and apathy — is what "may seem innocuous," but have "serious ramifications." Choice (E) also grabs language from the text but uses it in a mixed-up way. The language "traits such as indolence and apathy" shows up in the same sentence as the *ability to recognize hedonistic impulses… and delay or suppress gratification*, but those two phrases are not connected in an "X causes Y" way. If anything, you could infer that these traits and this ability run counter to each other: if you have the latter, you probably won't have the former.

9. **(A).** In this Inference question, you must deduce something about the meaning of a particular jargon word, *hyperbolic*, as used in the phrase "hyperbolic discounting." Do not try to figure out its meaning in your head; wrong answer choices have been devised to play off of your possible knowledge of the word *hyperbolic.* Rather, go to the text. You are told that *"exponential discounting"… has been renamed "hyperbolic discounting," because the effects of time delay do not seem strictly linear*. Focus on this last part. The meaning of "hyperbolic" must have to do with this idea, that *the effects of time delay do not seem strictly linear*. Only (A) works. *Nonlinearity* means the quality of not being linear. *Consequences* is a synonym for effects. Translated slightly, this is what you are looking for. Be careful of choice (B) — the use of the word *hyperbole* in everyday speech to mean "exaggeration" could mislead you here

10. **(C).** This Specific Detail question requires you to find something absolutely true according to the passage. In this case, you want to know what you can restate about the children who were able to wait. Careful: since there are two groups of children, an easy way for the test to construct a wrong answer is to state something true about the *other* group of children. That is, a comparison may be reversed. Incorrect choices (A), (B), (D), and (E) describe the children who *didn't* wait.

11. **(E).** When the question stem asks for the option for the blank *best supported by the passage*, it is asking for a *conclusion* that is the logical synthesis of the premises in the argument. The argument gives two requirements for egg laying: there must be nesting material, and the climate must be moderate. While *moderate* hasn't been specifically defined, it's safe to say that *the coldest winter on record* is the opposite of *moderate.* Therefore, the bird will not lay eggs. The fact that the nesting material requirement has been filled — even above and beyond what is required — doesn't fix the problem. Choices (A), (B), (C), and (D) offer statements for which you do not have enough information to evaluate. Note that in choice (B), "mortality rates" are not the same thing as laying eggs or not laying eggs.

12. **Ameliorate, extenuate.** You know that floodwaters had already entered the library, but the "hopeful" volunteers are working anyway — they hope to *limit* the damage. *Ameliorate* and *extenuate* both have the sense of making something better without completely solving it. Note that *exacerbate* means the opposite (to make worse), and *recant*

and *forfeit* are negative words that just don't fit here (you could *recant* your former statements and *forfeit* an athletic competition, for instance).

13. **Resurrected, immortalized.** The answers must oppose "dead and forgotten." While *glamorized* might be tempting, you cannot justify it with clues from the sentence (*vitality* is certainly not enough to be *glamorous*). *Mocked* and *parodied* is an incorrect pair — the sentence gives no indication that Eleanor is being made fun of. Do not weave stories or insert your own ideas. Note that *resurrected* and *immortalized* are not synonyms, but both certainly oppose the clue "dead and forgotten."

14. **Unpropitious, inimical.** The sentence tells you that if the death toll continues to climb, contact with the local populace—despite being intended for helpful (*beneficent*) purposes—may prove the opposite. You are looking for words that mean something like harmful. *Unpropitious* and *inimical* both mean adverse or harmful, and are just the words you are looking for. *Incongruous* means not fitting. While this *could* be a match for the blank, meaning-wise — since contact with the locals may not fit with the aim of normalizing relations — *incongruous* is used with "with" rather than "to," and it lacks a match among the answer choices.

15. **Boosterish, jingoistic.** You are asked to describe "slogans and anthems" that are then described as "disconcertingly ubiquitous." While *aggressive* and *bloodthirsty* are a near-pair, they are far too strong to describe slogans and anthems. *Punctilious* and *pedantic* are also a near-pair, but refer to arrogance related to rule-following or learning, which aren't relevant here. *Boosterish* and *jingoistic*, both implying an uncritical support of something (specifically one's country, in the case of *jingoistic*) are much better.

16. **(C).** When a logic problem gives you a bunch of numbers, it is very likely that the problem will then try to trick you into assigning *reasons* for those numbers. It is important to stick only to the facts you were given so that you do not select an unsupported conclusion. Regarding (A), the consumer products division spent a higher *percentage* on marketing, but you don't know that it spent more actual dollars (maybe the consumer products division is much smaller than chemicals and machinery). (B) should trip a red alert! You are absolutely not authorized to guess *why* the numbers worked out the way they did. You have absolutely no information that would justify this conclusion, or any conclusion about consumer behavior. (C) is correct — on average, all three divisions spent less than 35% of their budgets on marketing. While you don't know how big the budgets were in actual dollars or anything about the sizes of the three budgets relative to one another, 35% of any number, averaged in with less than 35% of some other numbers, will certainly generate an average under 35%. Note that, since a conclusion you draw in a Logic Reading Comprehension question cannot introduce new assumptions, correct conclusions are often fairly obvious or are near-paraphrases of information you've already read in the passage. Regarding (D), you don't know if the company has other divisions besides chemicals, machinery, and consumer products, so you cannot draw this conclusion. (E) is wrong because the chemicals division may have spent a higher *percentage,* but you have no information about the actual number of dollars spent or the relative sizes of the two departments' marketing budgets.

17. **(E).** The author's thrust is that 2-D representation warps students' perception; it pushes them to think of the world as flat and static rather than 3-D and dynamic. Choice (A) is not the point and the author does not say maps are necessary or evil, which is too extreme. (B) is a true detail from the passage but is less than the author's overall point. Again, (C) might be implied but the purpose of the passage is to discuss the effect of such tools. (D) might be inferred from the second paragraph but ignores the main issue of the essay.

18. **(C).** In the second paragraph, the static quality of maps is mentioned as a negative and choice (C) eliminates that problem. As for choice (A), "some" means at least one, not a majority — the example of what may just be a few exceptional students does not do much damage to the evidence or the point that, in general, maps cause students to think about the world in 2-D. The author's evidence involves the presence of maps in the classroom, so the amount of use (B) is irrelevant. (D) is also irrelevant, as the passage premises involve modern maps. (E) is backwards; it

strengthens the evidence that maps impede 3-D comprehension.

19. **(B).** In the second paragraph, the author uses the example of the 20th century United States to buttress his or her argument. (A) and (E) are outright falsehoods — there is no circular logic nor a rebuttal … for that matter, there is no commonly held view discussed. As for (C), the author does use physical description but not in support of an "accusation." (D) is incorrect because the author listed problems but did not create a hierarchy.

20. **II only.** The passage tells you that Fleming discovered penicillin and that others developed it; you don't know anything about his feelings or predictions about his discovery. You do know that *it was not until a decade later that a team of researchers from Oxford, with the help of an American laboratory, were able to increase the growth rate of penicillin ... such that it could be produced in sufficient quantity to treat Allied soldiers wounded on D-Day, in 1944*. Thus, at some point prior to 1944, penicillin was being grown too slowly. Finally, while penicillin was used to treat Allied soldiers in 1944, you don't know that antibacterial agents were used "since ancient times." A little bit of common sense should indicate that this was almost certainly *not* the case in the very earliest battles (ancient people didn't know about bacteria), but of course outside information is not needed to answer GRE Reading Comprehension questions — we can simply note that the passage covers only a very short period of time.

Verbal Practice Section 2: Medium Difficult

20 Questions
Time: 30 Minutes

For questions 1–6, select one entry for each blank from the corresponding column of choices. Fill in the blank in the way that best completes the text.

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1. Cormack McCarthy writes in an idiom both spare and flowery, with paragraphs of short, declarative sentences interspersed with long, __________ passages of description and philosophizing.

boring
floral
baroque
classical
fictional

2. Simony is one of the three primary (i) ____________ in Joyce's *Dubliners*, recurring in almost every story in the collection. In some stories, the simony is more or less literal, with characters attempting to purchase salvation with money. In other stories, it arrives in a more (ii) _____________ form.

Blank (i)	Blank (ii)
theses	spiritual
leitmotifs	pecuniary
characters	figurative

3. Mixed-media artist Mae Chevrette begins each painting by affixing to canvas a photo from her travels, then embedding _____________ such as old maps and sheet music before applying paint. The final works, emblazoned with quotes as well as evidence of Chevrette's extensive and wide-ranging travels, convey a sense of joyful ___________.

Blank (i)	Blank (ii)
devices	philosophy
apocrypha	wanderlust
ephemera	anomie

4. Taxation of legal substances known to be of a (i) __________ nature necessarily threads a thin line; it threatens to (ii) __________ government support for consumption of a product the use of which it is rightly interested in

curtailing.

Blank (i)	Blank (ii)
noxious	legitimize
salubrious	incentivize
hortatory	signify

5. The newspaper's essay contest soliciting defenses of anti-vegetarianism yielded only a handful of entries that did not allow authors' (i) __________ to dictate their arguments: focusing on the seemingly universal human (ii) __________ for consuming animal products, these authors successfully navigated the gray area between simple self-justification and genuine apologia. The successful submissions argued not so much that desire or tradition could justify the current palate, but that the eradication of factory farming, the (iii) __________, and the return to natural feed can blunt the evils caused by its excesses.

Blank (i)	Blank (ii)	Blank (iii)
erudition	penchant	effacement of resources
proclivities	salutation	imputation of ecology
tenacity	earmark	mitigation of suffering

6. The long-term Senator began his career as an unrepentant (i) __________ for his party's excesses, defending policies which posterity has since judged to be reactionary, even (ii) __________; more recently he has taken (iii) __________ line, denying that those very policies ever reflected the party's values.

Blank (i)	Blank (ii)	Blank (iii)
critic	surreptitious	an analeptic
apologist	acerbic	an unconscionable
appellant	retrograde	a revisionist

Questions 7–9 are based on the following reading passage.

The increasing number of published scientific studies ultimately shown to have been based on erroneous data threatens not only reputations of individual scholars but also perceptions of the field as a whole. Since the general public often interprets such debunkings as evidence of malicious or conspiratorial intentions on the part of researchers, these incidents risk being construed as evidence that fraudulent practices pervade the discipline. Such conclusions are rendered all the more potent by the **rousing** prospect of exposing hypocrisy in a field that prides itself on its rigor. It would therefore behoove interested parties to go to lengths to demonstrate that such episodes, while incidentally regrettable, are not necessarily signs of malfeasance, and are in fact fully consistent with a healthy science. Indeed, the very practices of hypothesis-testing and scientific replication are in place precisely to redress such concerns. Spurious results may linger briefly in the communal ethos, but the more attention they garner for their ingenuity and impact, the more likely they are to be subjected to the crucible of attempted replication. Just as in a thriving garden, small weeds may crop up from time to time only to get pulled out at signs of trouble, so too in science do specious findings occasionally attempt to infiltrate the canon only to get uprooted and tossed aside in the end by the inexorable process of scientific natural selection.

7. In the context of the passage, the word *rousing* is used to indicate that:

(A) scientists often take deep satisfaction in adhering to their own rules.
(B) people may find the chance to catch others in their own web to be galvanizing.
(C) scientific non-experts are aware that their criticisms of more specialized areas of study could be construed as exposing hypocrisy.
(D) uncertainty itself can be something that the general public finds exciting.
(E) scientific revolutions often happen when most experienced academics least expect them.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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8. The author of the passage would most likely defend which of the following scenarios as instances of "healthy science"?

☐ A young researcher discovers an important error in an established text and makes careful note of it in her personal logbook.
☐ A highly influential scholar publishes a controversial finding in a well-regarded journal only to be shown by follow-up studies to have inadvertently relied on an invalid statistical method.
☐ Unbeknownst to his collaborators, a scientist tweaks his data to be more consistent with a theory that has already won much empirical support.

9. Which of the following best describes the overall purpose of the passage?

(A) To develop a scientific hypothesis and then describe evidence refuting it.
(B) To argue that a problem which many people believe to be endemic to a specific domain is in fact much more widespread.
(C) To encourage more robust dialogue between scientific experts and laypeople.
(D) To highlight a possible interpretation of a phenomenon and then point out how that interpretation is mistaken.
(E) To build support for a position, then contend that that position is fundamentally flawed.

Question 10 is based on the following reading passage.

Mayor of Middletown: Two years ago, in order to improve the safety of our town's youth, I led the charge for a law requiring all bicycle riders to wear helmets when riding within city limits. My opponents claim the law is a failure because, last year, we had a higher incidence of bicycle accident victims with severe head injuries than in the previous year. The more important statistic, however, is the bicycle accident fatality rate, which has dropped nearly 30 percent since the law passed. Clearly, the helmet law has been a success.

10. Which of the following, if true, would best support the mayor's claim that the helmet law has been a success?

(A) If accident victims do not die as a result of a head injury, they often suffer from permanent brain damage.
(B) While only 15 percent of all bicycle accidents resulting in injury occur as a result of a collision with a motor vehicle, those accidents represent 90 percent of fatal accidents.
(C) In bicycle accidents, injuries to hands, knees, and elbows are far more common than head injuries.
(D) Typically, fatality rates for bicycle accident victims who sustain serious head injuries are twice as high when the victims are not wearing helmets as when they are.
(E) The effect of the helmet law in Middletown is typical; other towns also experienced a higher incidence of head injuries but a lower fatality rate.

Questions 11–12 are based on the following reading passage.

Ultraviolet radiation (with a wavelength in the range of 290 to 400 nanometers), visible light (400 to 760 nm), and infrared radiation (760 to 3,000 nm) are the three forms of energy that the earth receives from the sun. Within the ultraviolet spectrum, the three sub-categories are UVC (200–290 nm), UVB (290–320 nm), and UVA (320–400 nm). Most UVC rays do not reach the earth because ozone and other gases in the upper atmosphere absorb them, but exposure to this type of radiation from germicidal lamps and mercury lamps may still be hazardous. Excessive exposure to visible light is also thought to be harmful, but it is not of extreme importance with respect to prevention of skin damage. Most skin damage is caused by UVA and UVB radiation.

The amount of UVA that reaches the earth is ten times greater than UVB, but the amount of UVA need to produce sunburn in human skin is 800 to 1,000 times higher than UVB. **Nevertheless, UVA intensifies the sunburn effects of UVB through a delayed erythema and aids in cancer formation.** Thus, effective sunscreens and sun blocks must protect throughout both the UVB and UVA ranges.

Consider each of the answer choices separately and indicate all that apply.

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11. Which of the following can be inferred from the passage?

☐ A form of energy with a wavelength in the range of 2,000 to 2,500 nanometers would not be categorized as ultraviolet radiation.
☐ UVB rays cannot cause cancer formation without the intensifying effects of UVA.
☐ Ultraviolet radiation is not a form of infrared radiation.

Consider each of the answer choices separately and indicate all that apply.

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12. The highlighted sentence serves which of the following roles?

☐ provides a factual basis for creating sunscreens and sun blocks with certain properties.
☐ undermines an argument presented earlier in the passage.
☐ suggests that, within the wavelength range of ultraviolet radiation, radiation with wavelength lower than 320 nm can exacerbate the carcinogenic properties of radiation with a wavelength higher than 320 nm.

For questions 13–16, select the two answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole and produce completed sentences that are alike in meaning.

13. Even the most accomplished performers at times have difficulty with some of Rachmaninoff's more perilous passages—the composer seems often to have written his music with the deliberate intent to __________.

- ☐ flummox
- ☐ unnerve
- ☐ retract
- ☐ transmogrify
- ☐ distribute
- ☐ malign

14. Americans can scarcely suppose that all 100,000,000 speakers of Indian English are united in error; it is past time that we recognized that distinctively Indian constructions are not __________.

- ☐ fads
- ☐ solecisms
- ☐ idioms
- ☐ dialects
- ☐ pidgins
- ☐ lapses

15. The ____________ adventurer Rickard Brawnson spends the majority of his autobiography trying to make his life sound like something out of an Indiana Jones movie, but in the end, according to some reviewers, it all comes off rather forced and unconvincing.

- ☐ hapless
- ☐ traitorous
- ☐ self-styled
- ☐ unlucky
- ☐ soi-disant
- ☐ intrepid

16. One of the more bizarre powers of the US presidency is the more or less ___________ authority to grant pardons, negating months or even years of criminal litigation in an instant.

- ☐ impartial
- ☐ plenary
- ☐ unbiased
- ☐ executive
- ☐ sweeping
- ☐ tyrannical

Questions 17–20 are based on the following reading passage.

Long regarded as a necessary evil, the royal mistress is a classic staple of the French court. It was hardly a new trick for a monarch to use mistresses and political advisors as scapegoats, but the Bourbons did it with their own particular flare and brand of ceremony. Much of life in the French court was dictated by tradition, ritual, and custom, and the role of the mistress was no exception to this. Mistresses were there to please the king and be the target for unwanted criticism, but they were also expected to stay out of political affairs.

This, of course, was hardly ever the case. The mistresses of Louis XIV, however, were rather well behaved in comparison to those of the future kings. Louis XIV fathered 13 illegitimate children with his mistresses over the course of his life. Thus, his many mistresses were often more concerned with securing rights for their illegitimate offspring than with meddling in affairs of the state. This lack of political meddling made them somewhat less prone to the tremendously harsh scrutiny faced later by those of Louis XV. Additionally, Louis XIV's absolutist rule certainly had much to do with his mistresses remaining in their "proper places." Furthermore, Louis' strict control of the presses kept much of the harshest criticism at bay.

Nevertheless, there was still a steady stream of underground literature and cartoons that demonstrated abhorrence for many of Louis' paramours. What was important about the criticism that did proliferate against his mistresses, however, was that it was used to great advantage by Louis XIV. Indeed, he used it to deflect criticism off of himself. By having an easily disposable female to shoulder the blame for various monarchical mishaps, Louis was able to retain his appearance of absolute control and otherworldly perfection.

There would, however, be consequences for such skillful puppet-mastery in the coming century. Louis XIV was the singular architect of a vast veil of fictive space inlaid between him and his people, creating a dangerous precedent of masterful manipulation that could not be maintained to the same degree by later monarchs. It was clear that Louis XIV crafted this fictive space cleverly and with great skill, peppering it with self-promoting propaganda to control his image in the collective imagination of his people. His progeny, however, were simply not as adept at doing so. Even more problematic, although future monarchs were not able to dexterously manipulate this fictive space themselves, it did not go away. Instead, it was the satirists, pamphleteers, and playwrights who took over its construction in the years leading up to the Revolution. In short, though it was Louis XIV who wrote his own mythology, Louis XVI would have his written for him.

17. The primary purpose of the passage is to

(A) critique the morals of the court of Louis XIV
(B) discuss the popular opinion of French royal mistresses
(C) contrast the mistresses of Louis XIV and Louis XV
(D) suggest the main cause of the French Revolution
(E) describe the utility and flaws of a political tradition

18. According to the passage, all of the following were reasons that the mistresses of Louis XIV were less problematic than those of Louis XV EXCEPT:

(A) They were more concerned with securing the futures of their offspring.
(B) There was little freedom for the press under Louis XIV.
(C) They produced more offspring than did those of Louis XV.
(D) Louis XIV was a skillful politician.
(E) They were relatively uninterested in affairs of state.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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19. The passage suggests which of the following?

- ☐ Necessary evils are part of monarchies.
- ☐ Writing one's own mythology can be good statesmanship.
- ☐ Louis XIV viewed Louis XV as an unsatisfactory heir

20. The passage implies that

(A) Louis XIV made mistakes that led to the Revolution
(B) Louis XIV was a member of the Bourbon family
(C) Louis XV wrote his own mythology
(D) the most troublesome mistresses were those of Louis XVI
(E) Louis XIV had more mistresses than Louis XVI

Answers to Verbal Practice Section 2

1. **Baroque.** Because the description of "short, declarative sentences" matches up with the word *spare* in the first part of the sentence, you need something in the blank that matches up with "flowery," meaning "full of elaborate literary words or phrases." *Boring* is judgmental, and *floral* just means "of flowers." *Baroque*, meaning "highly ornate and extravagant in style," fits your sentence.

2. **Leitmotifs, figurative.** A theme that reappears throughout a work of art is called a *leitmotif* (simony — attempting to "to purchase salvation with money" — is not a *thesis* in *Dubliners*, nor is it a *character*). You are told that simony is literal in some stories, and then the final sentence begins with "in other stories," signaling a change in direction. The opposite of literal is *figurative*.

3. **Ephemera, wanderlust.** "Old maps and sheet music" are not *devices,* nor are they fake (*apocrypha*). Rather, they are *ephemera,* printed matter not intended to be saved. If you knew the more common *ephemeral* (fleeting), you could work this out. In the second blank, *wanderlust* describes a love of travel.

4. **Noxious, incentivize.** If the government is rightly interested in curtailing the use of a product, this tells you that the product is somehow harmful, or of a *noxious* nature, so that's the first blank. (The other words are both positive, and the government can't have a rightful interest in limiting the use of such products.) By taxing such products, the government gains a profit from their consumption. That the government is *threading a thin line* suggests that there is a conflict: on the one hand, the government then has an interest in increasing profits, on the other hand it has an interest in curtailing the use of the taxed substance. So the problem is that the profit gives the government an *incentive* to encourage use of the product, meaning that the taxes *incentivize* the government to support the product.

5. **Proclivities, penchant, mitigation of suffering.** The clue for the first two blanks is given in the claim that *authors successfully navigated the gray area between simple self-justification and genuine apologia*. *Apologia* means "defense," so you know that the authors were defending eating meat. But they did not allow their *desires* or *tendency* to eat meat to derail their arguments. The first two blanks require something like "tendency," and both *proclivity* and *penchant* mean that. The third blank requires something can "blunt the evils" of eating meat, and that goes along with the eradication of factory farming and the return to natural feed, so you are looking for something that contributes to the well-being of or detracts from the harms done to animals raised for food. *Mitigation of suffering* fits this bill.

6. **Apologist, retrograde, revisionist.** For the first blank, you want a word that means "a defender." Surprisingly, that's precisely what *apologist* means (the word *apology* once meant a speech offered to defend or justify). For the second blank, we'd like something similar to "reactionary," but even stronger and more negative. Since *reactionary* means "opposed to change," you want a word that means "very opposed to change." *Retrograde* will do, since it suggests a retreat to some earlier state (the word also has technical meanings in biology, music, and astronomy). The third word describes an approach that recasts history, or challenges a conventional history. That sort of "recasting" is called *revisionist*.

7. **(C).** To properly answer this question, you must first determine who is being "roused." The beginning of the sentence states that *conclusions are rendered all the more potent*. The group drawing the conclusions is the general public. The implication of the sentence is thus that the general public is eager to leap to conclusions involving malicious intent, and also eager to catch scientists in their own hypocrisy. The correct answer is therefore (C).

8. **II only.** The author of the passage argues that erroneous findings will be eventually corrected via the process of scientific natural selection through being subjected to scrutiny *the very practices of hypothesis-testing and scientific replication are in place precisely to redress such concerns*. The first statement is incorrect, since there is no such scrutiny — no one else will read the researcher's logbook. The third statement is incorrect because the scientist tweaks his data deliberately — this is more like the "fraudulent practices" described earlier in the passage than the "healthy science" described later — and because no scrutiny takes place.

9. **(D).** The passage primarily points out that the discovery of erroneous findings *could* be seen as malfeasance by researchers, then goes on to show how these same findings are actually a healthy and natural part of the scientific process. (A) isn't correct because the author does not develop a scientific hypothesis; (C) isn't correct because generating dialogue is not the author's primary concern — it is, rather, to dispel misguided notions.

10. **(D).** The mayor claims the law is a success because the accident fatality rate has dropped nearly 30 percent since the law passed. Opponents claim the law is a failure because more people have suffered from severe head injuries since the law passed. To strengthen the mayor's conclusion, you need to rebut the opponents' claim, or somehow weaken its effect. While (A) may be true, it does not address the mayor's claim that the helmet law in particular has been successful in its goal to protect bicycle riders. If anything, (A) could weaken the mayor's claim, because the mayor bases the law's success on the lower fatality rate. If those people who don't die instead suffer from permanent brain damage, it is difficult to claim success in "protecting" the population. (B) is tempting because it is a compelling reason why the town might want to enact a helmet law in the first place. The question does not ask you to address this, however; the helmet law has already been enacted. Rather, you need information that will point to the law as a success, and this choice does not provide any information allowing you to assess the success (or failure) of the helmet law. Furthermore, (B) introduces "motor vehicles," which are not mentioned in the argument. (C) may be tempting because the goal of the law is to protect the town's youth, and, presumably, it is desirable to limit all kinds of injuries. There are two problems, however. First, because the focus of the argument is on a helmet law, the focus is also on injuries associated with the area where a helmet is worn: the head. As such, other injuries are out of scope. Second, even if other types of injuries were within the scope of the argument, this choice does not provide any information allowing you to assess the *success* of the helmet law.

Choice (D) is correct — when bicyclists suffer serious head injuries, the chances of dying are twice as high for those not wearing helmets. The converse is that people with head injuries are less likely to die from those injuries if they were wearing a helmet. In other words, the decreased fatality rate is actually responsible, at least partially, for the increase in injuries: some of the injured would have been on the fatality list instead had they not been wearing helmets. This supports the mayor's case by showing that the premise used by the mayor's opponents does not actually indicate a failure in the law. Regarding choice (E), the fact that the results are similar in other towns means merely that the outcome is predictable in some way; it does not necessarily indicate success. In fact, if all towns experienced a sharp increase in fatalities after enacting a particular law, the outcome could be described as typical, but hardly a success.

11. **I and III only.** You are told that ultraviolet radiation is in the range of 290–400 nm, so energy with a wavelength of 2,000–2,500 nm would not fall in that range (in fact, it would fall into the range of infrared radiation. You are told that UVA rays intensify UVB damage and can aid in cancer formation, but you cannot infer that UVA rays are *necessary* for cancer to form. Finally, since ultraviolet radiation has a wavelength in the range of 290 to 400 nanometers and infrared radiation is in the range of 760 to 3,000 nm, ultraviolet radiation is not a form of infrared radiation.

12. **I only.** The fact that UVA rays can intensify sunburn and aid in cancer formation "provides a factual basis" for the recommendation in the next sentence, that sunscreens block both UVB and UVA rays. The highlighted sentence does

not “undermine an argument” — in fact, no “argument” exists in the passage, which is informative and factual. Finally, the third statement is the opposite of what you were told: UVA (above 320 nm) “exacerbates the carcinogenic properties” of UVB (below 320 nm), not the other way around. Keep in mind that while it may seem logical that if UVA could make the cancer-causing properties of UVB worsen, it would work the other way around — both things cause cancer, after all, right? You should not make this kind of assumption. The question did NOT ask, “What’s true in real life?” It asked what role the highlighted sentence was playing. Make sure you answer exactly the question being asked.

13. **Flummox, unnerve.** You rely largely on the context of the sentence for a hint as to this blank’s meaning. Very experienced performers have so much difficulty with some of Rachmaninoff’s music that it sometimes seems as though the music was *designed* to “confuse, baffle, bewilder,” or *flummox*, or *unnerve*.

14. **Solecisms, lapses.** That which *we can scarcely suppose* is false, so *distinctively Indian constructions* are not errors. The words you want must mean “errors.” *Lapses* mean errors, and *solecisms* means specifically “errors in grammar.”

15. **Self-styled, soi-disant.** The most important words here are *forced* and *unconvincing*, used to describe Brawnson’s account of his adventures. That means that Brawnson is decidedly NOT Indiana Jones, so you need a word for the blank that will in some way negate “adventurer.” *Soi-disant* and *self-styled* both imply that Brawnson may call *himself* an adventurer, but other people might not. *Traitorous* is far too extreme.

16. **Plenary, sweeping.** The key here is that the power to grant pardons can negate months or years of criminal litigation “in an instant.” That implies the power is quick and absolute. Both *plenary* and *sweeping* correctly capture the absolute nature of this power. *Tyrannical* is too judgmental, and *executive* wouldn’t mean much of anything (all presidential powers are, technically, *executive*).

17. **(E).** In the first paragraph, the author outlines the political role of royal mistresses and goes on to discuss the virtues and pitfalls of this system. (A) is incorrect and out of scope, as the author avoids moral judgments. (B) is too narrow; popular opinion is a detail, one of the pitfalls. (C) is wrong because the passage does not contrast the mistresses — about whom there is very little information — but rather their effect on French politics. (D) is incorrect (and too extreme) because the author does not say it was the *main* cause.

18. **(C).** This issue is largely discussed in the second paragraph, which provides support for the other four choices. (C) is correct because, while the author does mention the number of Louis XIV’s illegitimate children, the passage does not mention the number for Louis XV or compare those numbers.

19. **II only.** In the last paragraph, the passage describes the success of Louis XIV by writing his own mythology and compares that to the less satisfactory attempts by his successors. The first statement is out of scope as the author only states that it was a staple of the French court, not all courts. Similarly, no indication is given of Louis’ opinion of his successor.

20. **(B).** In the first paragraph the author states that the Bourbons brought a unique flair to the French custom of royal mistresses. Regarding (A), the passage discusses the success and skill of Louis XIV; mistakes are only attributed to his successors. Choice (C) is unsupported — in the last paragraph, the passage notes that Louis XVI did not write his own mythology but Louis XV is not mentioned. (D) and (E) are wrong for similar reasons—the passage compares the number of and problems caused by the mistresses of Louis XIV and Louis XV. But nothing is mentioned in that regard for Louis XVI.

Verbal Practice Section 3: Hard Difficult

20 Questions
Time: 30 Minutes

For questions 1–6, select one entry for each blank from the corresponding column of choices. Fill in the blank in the way that best completes the text.

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1. While it is tempting to think that artists like Picasso literally see the world in a markedly different way, examination of the artist's creative process reveals __________ movement from roughly realist sketches toward his famous Cubist style through a series of ever more abstract steps.

an obsessive
an iterative
a random
a historical
a dicey

2. Researchers from the University of Southampton concluded that ethnic differences are likely not the cause of mutual mistrust, citing government surveys which show that cooperation and trust are no higher in racially ___________ neighborhoods than in mixed communities.

militant
parochial
provincial
homogeneous
sectarian

3. Though in her home country of Denmark the singer always enjoyed an outpouring of support verging on the (i) __________, she continues overseas to confront certain impediments to success — evidence, perhaps, that not all art is (ii) __________.

Blank (i)	Blank (ii)
evanescent	fully decipherable
adulatory	universally translatable
totalitarian	entirely tractable

4. That (i) __________ rhetoric is so easily (ii) __________ the language of patriotism is perhaps one of the greatest

dangers of relying too heavily on the latter to bolster public morale.

Blank (i)	Blank (ii)
pacifist	phased in
obsequious	couched in
bellicose	implied by

5. The very title of Evelyn Waugh's Sword of Honour trilogy suggests a robust and (i) __________ tale, and the bare outline seems to fit, as the protagonist Guy Crouchback serves as a commando, trains as a paratrooper, and is dispatched to Yugoslavia to aid the partisans. In fact, however, Crouchback is an extraordinarily (ii) __________ man, ill-at-ease with his younger and more (iii) __________ fellow officers, and almost never motivated by appetite or impulse.

Blank (i)	Blank (ii)	Blank (iii)
pell-mell	effete	vigorous
red-blooded	venerable	puerile
avant-garde	literary	timorous

6. In the contemporary climate of academic specialization, the typical university lecturer of only two centuries ago, who was expected to (i) __________ views on subjects as diverse as geography, physics, and the fine arts, seems a veritable (ii) __________, and we forget at our peril that it was precisely such breadth of learning that led to some of the great discoveries and even (iii) __________ shifts in the sciences, as when Darwin drew upon his knowledge of philosophy and economics to articulate his famous theory of evolution.

Blank (i)	Blank (ii)	Blank (iii)
propound	astrophysicist	hegemonic
gainsay	polymath	paradigmatic
delineate	autodidact	minuscule

Questions 7–10 are based on the following reading passage.

Without a doubt, one of the pinnacle achievements of modern physics is the development of Maxwell's equations. Their beauty lies in their elegant simplicity, while the breadth and depth of Maxwell's equations speak for themselves. These four simple equations, coupled with the Lorenz Force Equation, form a full basis for modeling the behavior of an entire branch of physics: classical electrodynamics and optics. Further, despite their deceptive simplicity, Maxwell's equations have withstood the test of time. While equations modeling most other fields of physics have been modified to accommodate new experimental results and theories, Maxwell's equations have not been altered since their original conception in 1861. Take, for instance, Einstein's theory of general relativity, first published in 1916. Although the equation governing general relativity was also elegant and powerful, and laid the framework for most modern astrophysics, Einstein himself did not realize and correct an error within his equation until nearly fifteen years later. Newtonian mechanics has given way to more powerful theoretical frameworks and analytical mechanics has bent under the weight of quantum theory, but Maxwell's equations stand as originally written, tried and true.

Maxwell's four equations, the majority of which are less than twenty characters, are the mathematical formulation of four very simple ideas. First, any free electric charge will result in an electric field. Second, magnets do not have free charges, but are always paired together with a positive and negative end, yielding a magnetic field that has a looped structure. Third, a magnetic field that changes in time will result in an electric field and, fourth, an electric current or changing electric field will produce a magnetic field. It is truly amazing that these four simple rules, unmodified, have been used to model all electric, magnetic, and optics studies for more than 150 years.

7. Which of the following best expresses the author's intent in writing the passage?

(A) To argue that Maxwell's equations are the most important equations in all of physics
(B) To explain the significance and meaning of Maxwell's equations
(C) To argue that Maxwell is a more important name in physics than Einstein
(D) To describe the implications of each of Maxwell's four equations
(E) To advocate further studies in the field of electromagnetism and optics

8. The author references Einstein's theory of general relativity for which of the following reasons?

(A) To argue that the equations of electricity and magnetism are more important than the equations of relativity
(B) To provide an example of an equation that has been unwavering in time
(C) To advocate that Maxwell was a more important historical figure than Einstein
(D) To show that the implications of Maxwell's equations are far more powerful than General Relativity
(E) To provide an example of an important equation that has been modified over time

9. Which of the following is not mentioned as a rule in any of Maxwell's four equations?

(A) The looped structure of a magnetic field results from coupled charges.
(B) A magnetic field that changes in time results in an electric field.
(C) Any free electric charge results in an electric field.
(D) Magnetic fields are generated by unpaired magnetic charges.
(E) An electric current will produce a magnetic field.

Consider each of the answer choices separately and indicate <u>all</u> that apply.

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10. Which of the following can be correctly inferred from the passage?

☐ Maxwell's four equations form a full basis for modeling the behavior of classical electrodynamics and optics.
☐ Einstein's original equations of general relativity were incorrect as originally written in 1916.
☐ Newtonian mechanics has been modified by quantum theory.

Question 11 is based on the following reading passage.

When people are told that some behavior is common, they are more likely to indulge in that behavior even when society disapproves of it. For example, if many people are shown littering in an anti-litter advertisement, observers may subconsciously feel that littering is a normal, accepted activity. Thus, in order to influence behavior effectively, it is critical not to show or discuss anyone engaging in an activity that the advertisement seeks to discourage.

11. Which of the following, if true, most undermines the argument's conclusion?

(A) In a study, the most effective anti-smoking advertisement featured a person smoking amidst a disapproving crowd.
(B) The most effective way to influence behavior is for parents to teach their children not to litter.
(C) People who watch public service advertisements are typically aware that actors are merely pretending to engage in the disapproved behavior.
(D) Teenagers are more likely to litter than the general population and less likely to be influenced by anti-litter advertisements.
(E) In a study, the most effective anti-littering advertisement featured a pristine public park with children playing in the background.

For questions 12–15, select the two answer choices that, when used to complete the sentence, fit the meaning of the sentence as a whole and produce completed sentences that are alike in meaning.

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12. Although bonobos are a good deal more gregarious than chimpanzees, they do not hesitate to ___________ those whose continued presence would otherwise undermine the safety or even equanimity of the group.

☐ patronize
☐ imperil
☐ oust
☐ jeopardize
☐ safeguard
☐ ostracize

13. Perhaps because his military training discouraged indirection, the National Incident Commander sought a ___________ and open conversation with the Governor.

☐ plain
☐ profane
☐ frank
☐ brusque
☐ pert
☐ boisterous

14. Though croquet is proverbially a genteel game, it is not enough to play your own ball well — you must ___________ your opponent's play as well, even when impeding his or her progress costs you strokes.

☐ scotch
☐ anticipate
☐ obviate

☐ underscore
☐ eliminate
☐ stymie

15. A report in General Hospital Psychiatry finds that panic attacks ___________ the effects of diabetes, probably by interfering with patients' self-care, leading to a 75-percent increase in the frequency of symptoms.

☐ exacerbate
☐ hinder
☐ impede
☐ aggravate
☐ indemnify
☐ degrade

Question 16 is based on the following reading passage.

A certain medication used to treat migraine headaches acts by blocking pain receptors in the brain. When a person takes the medication within one hour after ingesting grapefruit or grapefruit juice, however, the effectiveness of the medication is significantly diminished. Researchers have determined that the grapefruit contains a compound that alters the shape of the pain receptors, with the result that the medication can no longer bind with them completely.

16. Which of the following conclusions could be most properly drawn from the information given above?

(A) If one takes the medication more than an hour after ingesting grapefruit, its effectiveness is not diminished.
(B) Ingesting grapefruit after taking the medication does not diminish the effectiveness of the medication.
(C) There is only one type of pain receptor in the brain.
(D) The medication is fully effective only when it properly binds with its target pain receptors.
(E) It is not possible to design a medication for migraine headaches that can bond with the altered receptors.

Questions 17–19 are based on the following reading passage.

Jeffrey C. Goldfarb suggests public-spirited dialog need not happen after a traditional theater show, as it is most successful when it happens through a show. He believes that the live component of the theater distinguishes it from other media objects, and allows meaning to arise from the interaction between performers and audience as the performance is happening.

Whereas television or film, for instance, has no room for active dialogue, theater does because the performers and audience are present in the space together. The theatrical text becomes the medium, and the performers speak through the way in which they perform the text, while the audience does so through a number of culturally sanctioned actions: applause, laughter (both laughing with and laughing at), sighing, gasping, cheering, and booing. Goldfarb recounts a particular occurrence surrounding a production of *Dziady* (Forefather's Eve) in Poland in 1968. The show had been ordered to close and, on its last night, the theater was overcrowded with supporters. They were an enthusiastic, vocal audience who entered into "dialogue" with the actors and read into the play's anti-czarist language a critique of Soviet government. When the performance ended, the crowd went into the streets to protest. The play's content became political through dialogue and, in a way, the theater building held a public sphere where an anti-Soviet public gathered to affirm their political sentiment before taking it to the street in open, public protest.

What Goldfarb does not write about is how uncommon such an event is, especially for today's American theatergoers. Augusto Boal was probably closer to the reality of current Western theater when he complained about how still everyone is expected to keep during any performance, constantly policed by other audience members. The high prices on professional theater tickets and an elitist value on cultural tradition (versus popular, technology-based mass media) combine to produce an aristocratic culture surrounding theater. In this manner, a "high class" code of etiquette is imposed upon the performance space, dictating that audience members are to remain quiet: the actors speak, the audience listens. As Boal criticizes in *Legislative Theatre*, traditional form sets up a relationship where "everything travels from stage to auditorium, everything is transported, transferred in that direction — emotions, ideas, morality! — and nothing goes the other way." He argues that this relationship encourages passivity and thus cancels theater's political potential.

17. The primary purpose of the passage is

(A) To lay out a viewpoint and present a perceived omission
(B) To articulate an original thesis
(C) To deride an established tradition
(D) To contrast two opposing ideas
(E) To reconcile two opposing ideas

Consider each of the answer choices separately and indicate all that apply.

---

18. The author implies which of the following about American theater?

☐ In some social settings, passivity is considered a virtue.
☐ Augusto Boal would approve of the events of the closing performance of *Dziady*.
☐ Physical presence has a bearing on the creation of active dialogue.

Consider each of the answer choices separately and indicate all that apply.

---

19. Which of the following must be true according to the passage?

☐ By American social mores, the Polish audience described would be exhibiting other than "high class" behavior.
☐ *Dziady* criticized the Soviet regime.
☐ Theater is more educational than film.

Question 20 is based on the following reading passage.

In the 1930's, Pablum, the first pre-cooked, dried baby food was sold in America. Pablum took its name from the Latin word *pabulum*, which meant "foodstuff" and was also used in medicine to refer to a passively absorbed source of nutrition. While Pablum contained vitamin D and thus helped to prevent rickets in an era in which child malnutrition was still widespread, ironically, the word *pablum* — undoubtedly influenced by the negative connotation of the word *pabulum* as well as the physical reality of a mushy, bland, rehydrated cereal — today means "trite, naive, or simplistic ideas or writings; intellectual pap."

20. Which of the following best describes the irony of the shifting meanings of the word *pablum*?

(A) A word for a passively absorbed source of nutrition is used for a substance actively fed to babies.
(B) Many babies would have died of malnutrition without Pablum.
(C) A word derived from Latin is still in use in modern English, although the meaning has changed somewhat.
(D) A cereal designed to be nourishing now lends its name to a word for something lacking in substance.
(E) Just as babies are fed bland food, *pablum* today means "bland writing or ideas."

Answers to Verbal Practice Section 3

1. **An iterative.** The first part of the sentence suggests that the way Picasso sees the world is different from how it looks to you, while the second part tells you that his artistic process moves in stages *from* the way the world normally looks *toward* something called Cubism, which is in contrast to his “roughly realist” art. This means that the movement involves different, more and more abstract, *iterations* or versions of the same image, meaning that it is an iterative movement. *Iterative* means “involving repetition or reiteration.” Also, none of the other choices fit the idea of a movement that both takes place in steps and has a definite direction—it cannot be *random* or *dicey*, since it has a clear aim and structure. If you picked *obsessive,* watch out for inserting your own assumptions or opinions!

2. **Homogeneous.** You want a word that is opposed to “mixed,” and that describes areas without differences. If you borrow those very words, you might anticipate something like “unmixed, the same throughout.” *Homogeneous* has just this meaning.

3. **Adulatory, universally translatable.** If the singer enjoys an outpouring of support, we’d expect it to “verge” on extremely positive support—in this case, *adulation*, which means extreme adoration. But the fact that her art is not well received in other places means that it might not carry over, or *translate*, to those places. The two wrong answers for the second blank, which contain two words related to being “understandable” or “usable,” *decipherable* and *tractable*, are close, but do not capture the sense of artistic adaptation implied in the second clause.

4. **Bellicose, couched in.** Since the rhetoric presents a danger, you are looking for something dangerous: *bellicose*, meaning “aggressive or warlike,” is a perfect fit. *Obsequious* means servile, and *pacifist* means “favoring or supporting peace,” so neither of those seems particularly dangerous or likely to follow from patriotism. The bellicose rhetoric is *expressed in, phrased in, or laid out in* the language of patriotism; *couched in* means the same thing as *laid out in*. This is a better fit than *implied by*, since to say that patriotism *implies* bellicose rhetoric is to make the assumption that patriotism is aggressive in itself, rather than simply yielding easily to aggression.

5. **Red-blooded, effete, vigorous.** The first word should echo “robust,” and fit the military derring-do described in the bare outline. *Red-blooded* suggests virility and heartiness. “In fact, however” suggests that Crouchback is not actually robust, and he is further described as “ill-at-ease,” and *never motivated by appetite or impulse*. These suggest something like “feeble” or “impotent.” *Effete* is even better, as it describes someone lacking vigor and energy. For your third word, the phrasing “and more” suggests a contrast with Crouchback; we’re not looking, then, for something that is a pure synonym for “younger” but for something that goes against the description of Crouchback as “effete.” Maybe “energetic.” The best answer is *vigorous*. *Puerile* is a trap — it seems to echo “younger,” but the sentence does not suggest that these younger officers are immature.

6. **Propound, polymath, paradigmatic.** The university lecturer was expected to have, or to set out, views on diverse subjects. To *propound* a view is to set it forward or lay it out. *Delineate* (lay out the boundaries of) is not a good fit; *gainsay* means to contradict, and while the lecturer may have the ability to contradict views on diverse subjects, this doesn’t make as much sense as laying them out. You know that these lecturers were widely learned; a person with a wide, practically encyclopedic breadth of knowledge is a *polymath*. *Astrophysicist* doesn’t fit, since it is only the name of one specialized branch of study; an *autodidact* is someone who is self-taught. In the final blank, you are looking for something stronger than discoveries: something like a change (or shift) in the way science is done, or the way scientific theories are understood—the theory of evolution is an example of such a change. A *paradigm* is a model or template for doing something, and is what you are looking for.

7. **(B).** The author uses the first paragraph of the passage to explain why Maxwell's equations are so significant. The second paragraph discusses the meaning of each of Maxwell's equations. Thus, as answer choice (B) presents both of these ideas, it is the correct answer. Further, (A) incorrectly states that this passage is used to argue that Maxwell's equations are the most important in physics, which is never stated in the passage. (C) incorrectly compares Maxwell to Einstein, which is never done in the passage. (D) is too narrow; it fails to address the author's intent in writing the first paragraph. Finally, the author does not advocate for further studies as described in answer choice (E).

8. **(E).** Before introducing Einstein's theory of general relativity, the author states that Maxwell's equations have withstood the test of time. He/She goes on to say that, in contrast, although Einstein's equations are elegant and powerful, they had to be modified not long after their postulation. Therefore, the author uses Einstein's theory of general relativity to provide an example of an important equation that has been modified over time, as in answer (E). (A) is incorrect as the author does not directly compare the fields of electricity and magnetism and relativity. (B) is incorrect as it states the opposite of what is mentioned in the paragraph. (C) incorrectly compares Maxwell to Einstein as opposed to their equations. Finally, (D) is incorrect because the implications of Maxwell's equations are never compared to the power of Einstein's general relativity.

9. **(D).** The passage states that magnets do not have free charges but are always paired together with a positive and negative end. Therefore, answer choice (D) is not mentioned, as it claims that magnetic fields are generated by unpaired magnetic charges. Answer choice (A) is presented in Maxwell's first equation: paired, or coupled, charges yield a looped magnetic field. (B) is presented in Maxwell's third equation, (C) is presented in Maxwell's first equation, and (E) is presented in Maxwell's fourth equation.

10. **II only.** The passage states that Einstein did not realize and correct an error in his theory of general relativity until 15 years after the theory was postulated in 1916. Therefore, Einstein's original equations of general relativity must have been incorrect as written in 1916. The first statement cannot be inferred as the passage says that Maxwell's equations, coupled with the Lorenz Force Equation, form the full basis for electrodynamics and optics. From this, one cannot infer that Maxwell's equations alone form a full basis for electrodynamics and optics. The third statement incorrectly infers that Newtonian mechanics has been modified by quantum theory, whereas the last sentence in the first paragraph states that it is analytical mechanics, and not Newtonian mechanics, that has been modified by quantum theory.

11. **(A).** According to the argument, if *many* people are shown littering in an anti-litter ad, then those watching the ad *may* feel that littering is normal or accepted. The author concludes from this that the "bad" behavior should not be shown or discussed *at all* in an ad that seeks to discourage this "bad" behavior. We're asked to find a piece of information that weakens the author's conclusion. (A) is correct. The author assumes that *any* mention of the negative behavior is enough to make people feel that the behavior is normal or accepted. The premise indicates only that this occurs when *many* people are shown to actively engage in the negative behavior. This choice illustrates that the author's assumption is not valid: a very effective anti-smoking ad featured one person smoking. That is, an effective ad did feature someone engaging in the negative behavior. While (B) may be true, the conclusion is not concerned with the *most* effective way to influence a particular type of behavior. Rather, the conclusion focuses on what should *not* be done if one wants to discourage a negative behavior. Regarding (C), the conclusion focuses on what not to do (show or discuss the discouraged behavior) in order to influence behavior in an effective manner. The fact that observers are aware that the ads may feature actors does not influence that particular conclusion. Regarding (D), the conclusion is not concerned with whether advertisements are more or less effective than other means for influencing teenagers, nor is it concerned with what *should* be done to influence this group (or others). Rather, the conclusion focuses on what should *not* be done if one wants to discourage a certain behavior. Finally, (E) strengthens the author's position: the best advertisement does not show the behavior that the ad seeks to discourage. You were asked to weaken the argument, not strengthen it.

12. **Oust, ostracize.** "Although" implies that you want an action that you wouldn't ordinarily expect from "gregarious,"

or friendly, animals, so you want something like "be unfriendly to." The "otherwise" in *continued presence would otherwise undermine* implies that bonobos sometimes do something to "exclude" others. *Oust* and *ostracize* fit. *Imperil* and *jeopardize* both mean something like "endanger," but this pair does not oppose "continued presence" as directly as do *oust* and *ostracize.*

13. **Plain, frank.** Since his training "discouraged indirection," you can expect a word like "direct." The word *open* suggests that the Commander wanted to be "candid." *Plain* and *frank* both mean something like "direct and candid." They both suggest unornamented but not necessarily unfriendly speech. *Brusque*, on the other hand (like *curt* or *terse*), suggests a discourteous bluntness.

14. **Scotch, stymie.** "Though" suggests that the word is opposed to gentility, and so you might expect something like "behave coarsely toward." The word *impeding* gives you a more precise idea: you want something like "aggressively block." *Scotch*, which means "abruptly end," and *stymie*, which means "thwart," are both good.

15. **Exacerbate, aggravate.** Since panic attacks eventually lead to *a 75 percent increase in the frequency of symptoms*, they make the effects of diabetes much worse. You should anticipate a word that means "make worse." *Exacerbate* and *aggravate* have precisely this meaning. In some contexts *hinder* and *impede*, which mean "block," would mean "make worse," but blocking the symptoms of a disease would actually make those symptoms better. Similarly, *degrade* is a bad word, meaning variously "demote, debase, impair," but none of its meanings work here.

16. **(D).** The passage says that the medication has been shown to be less effective when taken after grapefruit consumption because grapefruit consumption has been shown to affect the binding of the medication to pain receptors. It can be concluded that effective binding is needed to enable the full effectiveness of the medication, which is what (D) states. Regarding (A), the passage says that grapefruit or grapefruit juice ingested within an hour will "significantly" diminish the effectiveness of the medication. This does not mean that grapefruit or grapefruit juice ingested more than an hour before the medication is ingested will have no effect on the medication; it may have a mild effect. Regarding (B), the passage speaks only of taking the medication after ingesting grapefruit; it says nothing about what might happen when eating grapefruit after taking the medication. As for (C), the passage implies that the pain receptors that the specific medication works on are the pain receptors that are affected by grapefruit. This does not mean that these are the only pain receptors in the brain. (E) is out of scope. You are given no information about possible research or design of new migraine medications.

17. **(A).** The author presents Goldfarb's view in the first paragraph and then highlights an omission in Goldfarb's work in the second paragraph. (B) is incorrect because "original" contradicts the text — the author recounts the opinions of Goldfarb and Boal. (C) is wrong, as the author does not "deride" anything, and the word *established* is not supported by the text. (D) is eliminated because there is no contrast nor opposing ideas — Goldfarb neglected to mention that something he discussed was quite uncommon, but Goldfarb's ideas do not oppose Boal's. Similarly, in (E), no reconciliation is attempted and there are still no opposing ideas.

18. **I, II, and III.** The first statement must be true, as the second paragraph indicates that American audiences consider passive behavior polite. The second statement must be true since, according to the last sentence of the second paragraph, Boal criticized passivity because it "cancels theater's political potential." The third statement is a nice match for this sentence from the first paragraph: "Whereas television or film, for instance, have no room for active dialogue, theater does because the performers and audience are present in the space together."

19. **I only.** In the first paragraph, the passage describes the interaction between the Polish audience and the cast; in the second paragraph, the author tells you that American "high class" behavior involves remaining quiet in the theater. Together, these two pieces of information support the first statement. The second statement is incorrect — the passage states that the audience "read into" or *interpreted* the anti-czarist language as a critique of the Soviet regime;

while the play may very well have been intended that way, you can't know this from the passage. The comparison concerning education in the third statement is unjustified because the passage only states that the live element is unique to theater.

20. **(D).** The word *irony* is defined as "an outcome of events contrary to what was, or might have been, expected." (A) is not ironic — the medical term *pablum* means a passively absorbed nutrition, and babies receive the baby food Pablum in a passive way. (B), (C), and (E) may be true, but also do not fit the definition of ironic. (D) is indeed ironic — the word now means nearly the opposite of what was originally intended. Only (D) presents this kind of "twist."

Chapter 33
of

Math Practice Sections

In This Chapter…

Math Practice Section 1: Easy Difficult

Math Practice Section: Easy
20 Questions
35 Minutes

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

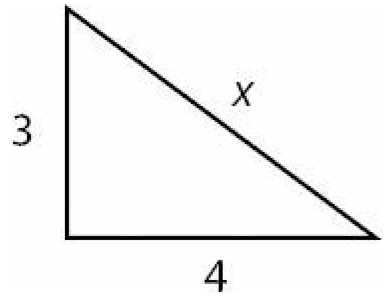
x, y, and z are consecutive integers such that $x < y < z$

Quantity A	**Quantity B**
y	$\frac{x+z}{2}$

2.

Quantity A	**Quantity B**
$(-3)^4$	$(-3)^{-3}$

3.



Quantity A	**Quantity B**
x	5

4.

Quantity A	**Quantity B**
$y^7 \times y^8 \times y^{-6}$	$3y^9$

5.

$xy > 0$ and $yz < 0$

Quantity A	**Quantity B**
xz	0

6. In 2011, it cost Tammy $1.30 to manufacture each copy of her magazine, which she sold for $2.30. In 2012, it cost Tammy $1.50 to manufacture each copy of the same magazine, which she sold for $3.00.

Quantity A	**Quantity B**
The percent by which Tammy's profit per copy of the magazine changed from 2011 to 2012	$33\frac{1}{3}\%$

7. List X: 4, 7, 9, 11, 24, 32

List Y (not shown) consists of 6 unique numbers, each computed from the corresponding term in List X by dividing the number in List X by 2, then adding 5 to the result.

Quantity A	**Quantity B**
The range of List Y	6 less than the greatest number in List Y

8. Which of the following represents the length of the diagonal d of a square with area a?

(A) $d = a^2$

(B) $d = \sqrt{2a}$

(C) $d = 2\sqrt{a}$

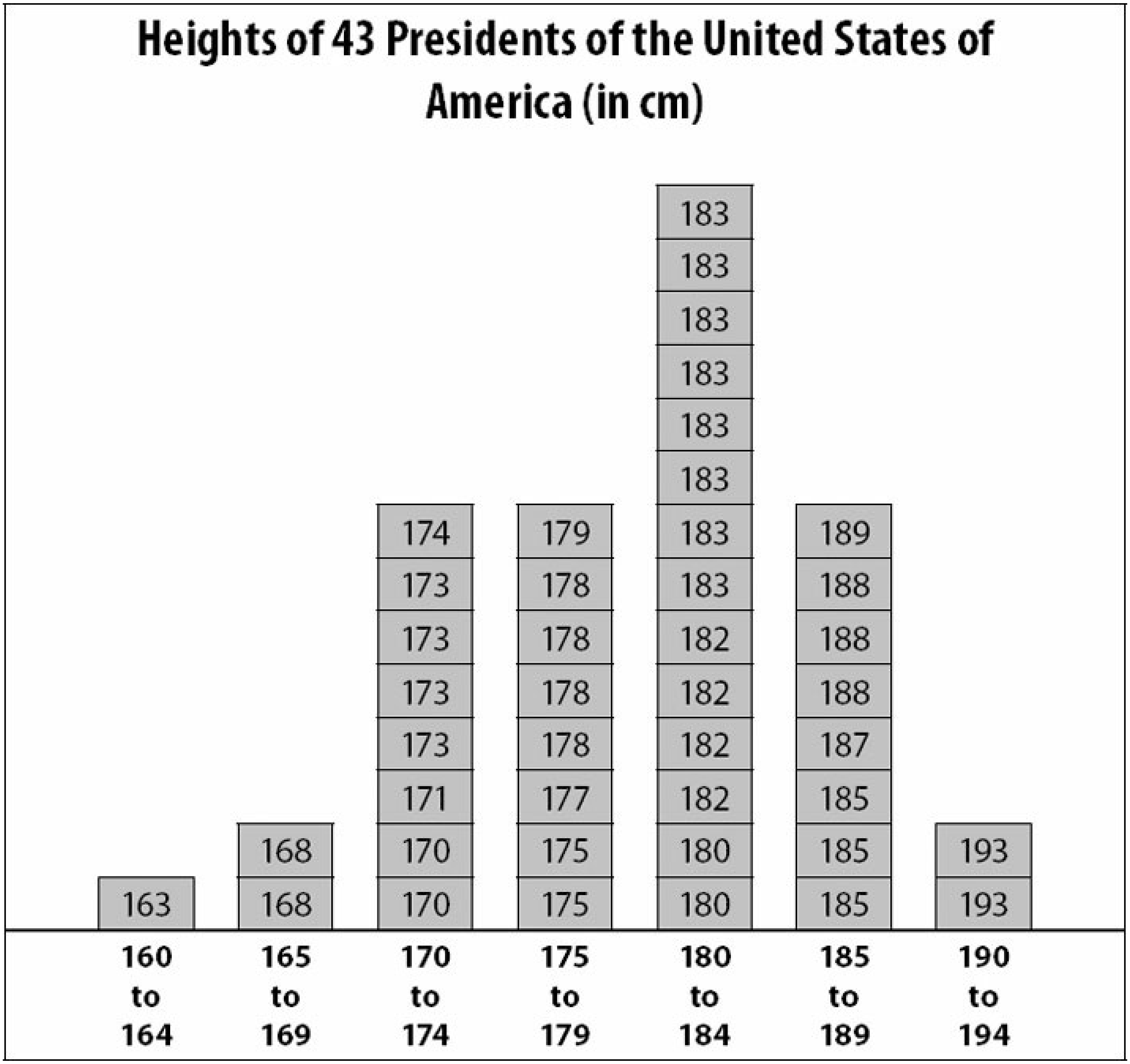
(D) $d = a\sqrt{2}$

(E) $d = a\sqrt{3}$

9. In an apartment complex, 60 percent of the apartments contain at least one television, and 20 percent of these apartments are equipped with cable. If every apartment that is equipped with cable contains at least one television, what percent of the apartments in the complex are not equipped with cable?

(A) 8%
(B) 12%
(C) 16%
(D) 88%
(E) 92%

Questions 10–12 are based on the following chart.



10. What is the range of heights of the 43 U.S. Presidents in the chart?

(A) 30 cm
(B) 34 cm
(C) 35 cm
(D) 163 cm
(E) 178 cm

11. What is the median height of the 43 U.S. Presidents in the chart, in centimeters?

(A) 175
(B) 177
(C) 178
(D) 180
(E) 182

12. Approximately what percent of U.S. Presidents have been 185 cm or taller?

(A) 10%
(B) 23%
(C) 29%
(D) 43%
(E) 50%

13. If $m + 5 < \frac{3}{2}$, which of the following scould be the value of *m*?

(A) $-\frac{15}{4}$
(B) $-\frac{7}{2}$
(C) -2
(D) $\frac{7}{2}$
(E) 2

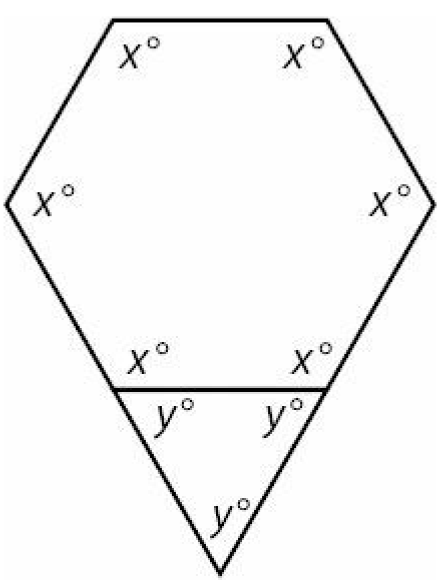
14. List M consists of the numbers 10, 20, 30, 40, 50.

Which of the following lists of numbers have an average (arithmetic mean) that is equal to the average of the numbers in List M?

Indicate all such lists.

☐ 0, 30, 60
☐ 10, 20, 30, 35, 50
☐ 10, 22, 30, 38, 50
☐ 0, 0, 0, 0, 150

15.



What is the value of xy?

16.

Buying Habits Of Customers Buying Toothpaste X At Chan's Grocery Store

Discount Type	Manufacturers' Coupon	Store Coupon	No Coupon
Percent of Customers	54%	43%	x%

The table above summarizes all possible discount types for customers buying Toothpaste X at a certain grocery store. No one used both types of coupon. If a person is selected randomly from among the customers buying Toothpaste X at Chan's Grocery Store, what is the probability that this customer did not use a coupon?

(A) 0.003
(B) 0.03
(C) 0.3
(D) 0.33
(E) 3.3

17. Company A can pave 500 feet of sidewalk in 6 hours, and Company B can pave 1,000 feet of sidewalk in 8 hours. At these rates, how many more *yards* of sidewalk can Company B pave in 9 hours than Company A can pave in 9 hours? (3 feet = 1 yard)

(A) 125
(B) 166
(C) 333
(D) 375
(E) 500

18. If the three sides of an equilateral triangle are equal to $4x$, $6y$, and 24, respectively, what is the ratio of x to y?

Give your answer as a fraction.

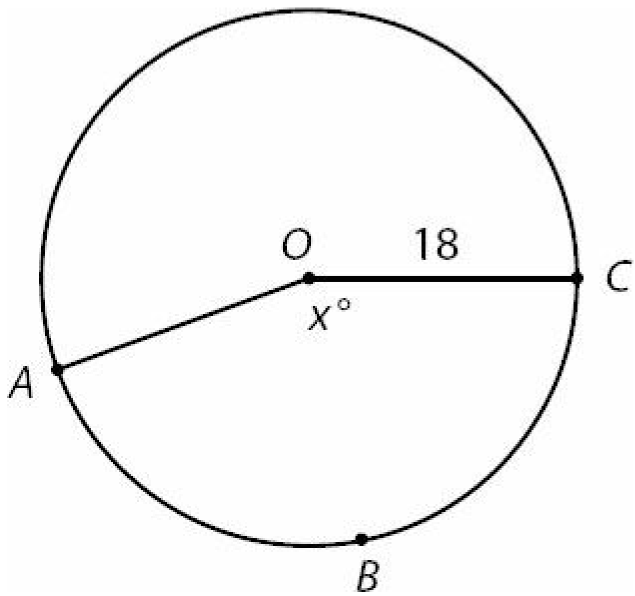
19. If the ratio of undergraduate students to graduate students is 7 to 4 and the ratio of graduate students to professors is 2 to 1, which could be the total number of undergraduate students, graduate students, and professors?

Indicate <u>all</u> such numbers.

☐ 520

☐ 640
☐ 2,600
☐ 10,000

20.



What is the perimeter of sector *ABCO* if $x = 160$?

(A) $18 + 8\pi$
(B) $18 + 16\pi$
(C) $36 + 8\pi$
(D) $36 + 16\pi$
(E) $36 + 24\pi$

Answers to Math Practice Section 1

1. **(C).** The average of three consecutive integers is always equal to the middle value, and is always equal to the average of the smallest and largest terms. Since Quantity B represents the average of the smallest and largest terms, it is equal to the middle term *y*.

Alternatively, pick numbers. If *x*, *y*, and *z* are 1, 2, and 3, the Quantity A = 2 and Quantity $B = \frac{1+3}{2} = 2$ quantities are equal. Any other example of three consecutive numbers will also yield equal quantities.

2. **(A).** In Quantity A, $(-3)^4 = (-3)(-3)(-3)(-3) = 81$. In Quantity B, $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27}$. Quantity A is greater. Note that you can stop calculating as soon as you realize that one quantity is positive and one is negative. The negative base in both quantities suggests that you should check whether the exponents are odd or even. Even exponents "hide the sign" of the base, so a negative base to a even exponent is positive. On the other hand, a negative base to an odd exponent remains negative (even if the exponent is a *negative* odd).

3. **(D).** If this were a right triangle, the Pythagorean theorem would indicate that

$$3^2 + 4^2 = x^2$$
$$9 + 16 = x^2$$
$$25 = x^2$$
$$5 = x$$

However, the triangle is not known to be right (the Pythagorean theorem only applies to right triangles), as none of the angles are labeled. The Third Side Rule, which applies to all triangles regardless of angle measures, states that the third side of any triangle must be greater than the difference between the other two sides and less than the sum of the other two sides. So, *x* must be greater than 4 - 3 = 1 and less than 4 + 3 = 7. *x* could be less than, greater than, or equal to 5, so it cannot be determined which quantity is greater.

4. **(D).** Since the terms in Quantity A have the same base and are multiplied together, simplify by adding the exponents:

$$y^7 \times y^8 \times y^{-6} = y^9$$

While y^9 may *seem* smaller than $3y^9$, this is only true if *y* is positive. If $y = 0$, the two quantities are equal. If *y* is negative, so is y^9, and $3y^9$ is more negative than y^9. Thus, it cannot be determined which quantity is greater.

5. **(B).** Since $xy > 0$, *x* and *y* have the same sign. Since $yz < 0$, *y* and *z* have opposite signs. Therefore, *x* and *z* have opposite signs. If *x* and *z* have opposite signs, their product is negative, which is less than 0. Quantity B is greater.

6. **(A).** In order to calculate the percent change in profit from 2011 to 2012, first calculate the profits in each year

based on the formula:

Profit = Revenues - Costs

Therefore,

Profit per each copy of the magazine in 2011 = \$2.30 - \$1.30 = \$1.00
Profit per each copy of the magazine in 2012 = \$3.00 - \$1.50 = \$1.50

To find the percent increase, use the percent change formula:

$$\text{Percent Change} = \left(\frac{\textit{Difference}}{\textit{Original}} \times 100\right)\%$$

$$\text{Percent Change} = \left(\frac{0.50}{1.00} \times 100\right)\% = 50\%$$

Be careful not to put the 2012 profit in the denominator. Mistakenly doing so would lead you to pick (C) erroneously. The "Original" profit is that for 2011.

Quantity A is greater.

7. **(B).** Since the terms in List Y are "each computed from the corresponding term in List X by dividing the number in List X by 2, then adding 5 to the result," List Y consists of 7, 8.5, 9.5, 10.5, 17, 21.

Quantity A: The range is 21 - 7 = 14.
Quantity B: 6 less than the greatest number in Set Y = 21 - 6 = 15.

Quantity B is greater.

8. **(B).** A square with area *a* has sides of $\sqrt{a}$. Use the Pythagorean theorem with $\sqrt{a}$ for each leg and *d* for the hypotenuse:

$$(\sqrt{a})^2 + (\sqrt{a})^2 = d^2$$
$$a + a = d^2$$
$$2a = d^2$$
$$\sqrt{2a} = d$$

This is a match with choice (B). Alternatively, plug in numbers. If a square has side length 4, the area *a* equals 16 and the diagonal would be:

$$4^2 + 4^2 = d^2$$
$$32 = d^2$$
$$\sqrt{32} = d$$

Plug $a = 16$ into each choice to see which yields $d = \sqrt{32}$. Only choice (B) works.

9. **(D).** The easiest way to solve this problem is to choose a smart number for the total number of apartments in the apartment complex. As this is a percents problem, choose a total of 100 apartments. Since 60% of these apartments have a television, 60 apartments contain a television (or more than one television—it doesn't matter how many—only television at all vs. no television matters) and 40 apartments do not contain a television.

Because 20% of the apartments that contain a television are equipped with cable, 20% of 60 = 12 apartments have both television and cable. By extension, 60 - 12 = 48 apartments have television, but are not equipped with cable.

"Every apartment that is equipped with cable contains at least one television" means that none of the 40 apartments without a television are equipped with cable. Thus, 40 apartments have neither a television nor cable.

In summary:

No TV, no cable = 40
TV, no cable = 48
No TV, cable = 0
TV and cable = 12

Only 12 apartments are equipped with cable, meaning 100 - 12 = 88 are not. Alternatively, 48 + 40 = 88 apartments are not equipped with cable.

Since 88 out of 100 apartments are not equipped with cable, the answer is 88%.

Alternatively, you can solve this problem by assigning the variable x to the total number of apartments in the apartment complex. Following the steps from above, $0.6x$ apartments contain a television and $(0.2)(0.6x) = 0.12x$ apartments are equipped with cable. From here, $x - 0.12x = 0.88x$ apartments, or 88% of the apartments in the complex, do not have cable.

10. **(A).** The shortest US President was 163 centimeters tall, and the tallest was 193 centimeters tall. The range is the difference between the highest and lowest value, and 193 cm - 163 cm = 30 cm.

11. **(E).** The median is the middle value if all the data points are arranged from least to greatest. With 43 data points, the median is the 22nd data point, because there are 21 data points that are less than or equal, and 21 data points that are greater than or equal, this median. Counting up from the least value (or down from the greatest value), the 22nd data point is 182 cm.

12. **(B).** From the chart, 10 U.S. Presidents have been 185 cm or taller, out of a total of 43. As a percent, this is $\left(\frac{10}{43} \times 100\right)\%$, or approximately 23%.

13. **(A).** Solve the inequality:

$$m+5<\frac{3}{2}$$
$$m<\frac{3}{2}-5$$
$$m<\frac{3}{2}-\frac{10}{2}$$
$$m<-\frac{7}{2}$$

(A) is the only answer choice that is less than $-\frac{7}{2}$. If needed, plug each answer choice into the calculator and compare decimal values to -3.5.

14. **I, III, and IV only.** Certainly, you could average the list 10, 20, 30, 40, 50 (the average is 30) and then average the lists in all the answer choices to see which also average to 30. However, you cannot afford to waste any time on the GRE.
Instead, note that the average of an evenly-spaced set is equal to the median. Thus, the average of 10, 20, 30, 40, 50 is the median, or middle term, 30. In Statement I, the list 0, 30, 60 is also evenly-spaced, so the average is 30.

In Statement II, the list 10, 20, 30, 35, 50 is the same as the original list (10, 20, 30, 40, 50) except for one number — the 40 has been changed to 35. Thus, the averages cannot be the same.

In Statement III, the list 10, 22, 30, 28, 50 is the same as the original list (10, 20, 30, 40, 50), but with 2 taken away from the fourth number and added to the second number. Since the sum didn't change, the average doesn't either.

In Statement IV, the average is simply the sum divided by the number of items, or 150/5 = 30.

15. **7,200.** Since every angle in the hexagon is labeled $x°$, the hexagon is equiangular. To find the sum of the degree measures in a polygon, use the formula $(n - 2)(180)$, where n is the number of sides. Since $n = 6$, $(6 - 2)(180) = 720$, and the sum of the degrees in the hexagon is 720. Thus, $6x = 720$ and $x = 120$.

Since the triangle is equiangular, $3y = 180$ and $y = 60$.

Thus, the value of $xy = 120 \times 60 = 7{,}200$.

16. **(B).** Add 54% + 43% = 97% to get the percent of customers who used a coupon. Only 100% - 97% = 3% of customers did not use a coupon. Thus, for a person selected randomly from among the customers buying Toothpaste X at Chan's Grocery Store, there is a 3%, or 0.03, probability that he or she did not use a coupon.

17. **(A).** Company A can pave 500 feet of sidewalk in 6 hours, and thus $\frac{500}{6}$ feet per hour. In 9 hours, Company A can pave $\frac{500}{6} \times 9 = 750$ feet of sidewalk.

Company B can pave 1,000 feet of sidewalk in 8 hours, and thus $\frac{1000}{8} = 125$ feet per hour. In 9 hours, Company B can pave 125 × 9 = 1,125 feet of sidewalk.

Thus, in 9 hours, Company B can pave 1,125 - 750 = 375 feet of sidewalk more than Company A. Since 3 feet = 1 yard, divide by 3 to get the answer in the correct units: 375 feet divided by 3 feet per yard = 125 yards.

18. $\frac{3}{2}$ **(or any equivalent fraction).** Since the sides of an equilateral triangle are all equal, $4x = 6y = 24$. With a three part equation, you can equate any two parts you wish.

For instance:

$4x = 24$
$x = 6$
$6y = 24$
$y = 4$

Thus, the ratio of x to y is 6 to 4, which reduces to 3 to 2. On the GRE, you do not need to reduce the answers to fraction numeric entry questions.

19. **I and III only.** If the ratio of undergraduate students to graduate students is 7 to 4 and the ratio of graduate students to professors is 2 to 1:

Undergraduate	Graduate	Professors
7	4	
	2	1

Equate the ratios by making the two numbers under "Graduate" equal. To do this, double the second ratio. (If you change one number in the ratio 2 : 1, you must perform the same operation to the other number in that ratio.)

Undergraduate	Graduate	Professors
7	4	
	4	2

Now, collapse the ratios onto one line:

Undergraduate	Graduate	Professors
7	4	2

The ratio is 7 to 4 to 2. Since 7 + 4 + 2 = 13 and numbers of people must be integers, the total number of people must be a multiple of 13. Only 520 and 2,600 qualify.

20. **(D).** If $x = 160$, then the sector is $\frac{160}{360} = \frac{4}{9}$ of the circle. Thus, arc *ABC* is $\frac{4}{9}$ of the circumference. Since the circumference $= 2\pi r = 2\pi(18) = 36\pi$, take $\frac{4}{9}(36\pi) = 16\pi$

Thus, the perimeter of the sector is equal to two radii plus 16π, or $36 + 16\pi$.

Math Practice Section 2: Medium Difficult

Math Practice Section: Medium
20 Questions
35 Minutes

For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box $\boxed{}$, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes $\frac{\boxed{}}{\boxed{}}$, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.

Set M consists of all the integers between -2 and 12, inclusive
Set N consists of all the integers between 9 and 15, inclusive

Quantity A	Quantity B
The smallest integer in Set M that is also in Set N	9

2.

17% of p is equal to 18% of q, where p and q are positive

Quantity A	Quantity B

p	q

3.

Circle A has area a

Semicircle B has area $\frac{a}{2}$

Quantity A	**Quantity B**
The circumference of Circle A	Twice the perimeter of semicircle B

4.

Quantity A	**Quantity B**
The standard deviation of the set 1, 5, 7, 19	The standard deviation of the set 0, 5, 7, 20

5.

An isosceles triangle has a perimeter of 28. The shortest side has length 8.

Quantity A	**Quantity B**
The length of the longest side of the triangle	12

6.

$$(3 - z)(z + 4) = 0$$

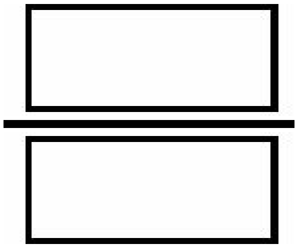
Quantity A	**Quantity B**
z	5

7.

$$a > b > c > d$$
$$ab > 0$$
$$ad < 0$$

Quantity A	**Quantity B**
ac	cd

8. If $12b = 2g$ and $4g - 3b = 63$, what is the value of $\frac{g}{b}$?
Give your answer as a fraction.

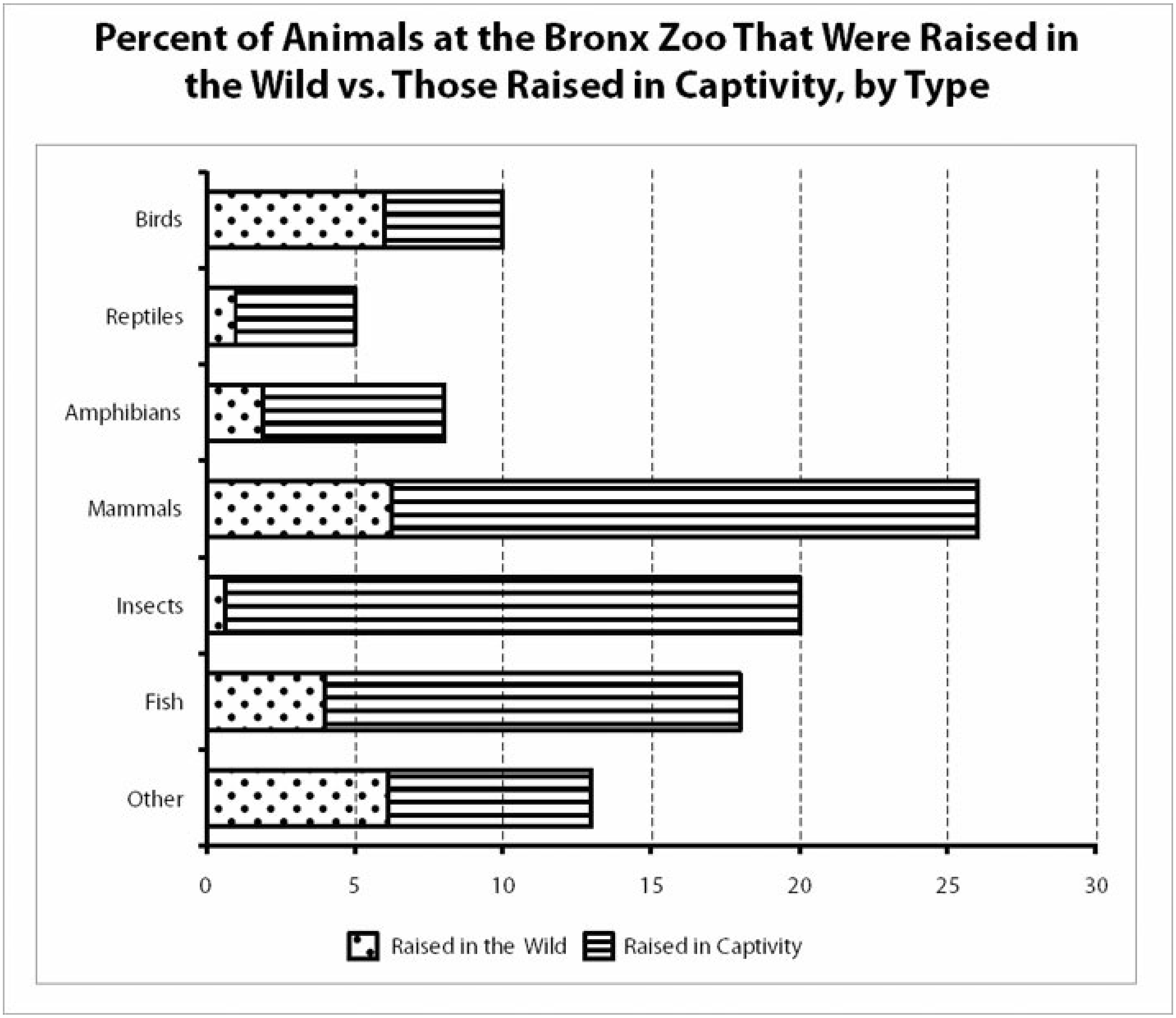


9. $81^3 + 27^4$ is equivalent to which of the following expressions?

Indicate <u>all</u> such expressions.

☐ $3^7(2)$
☐ $3^{12}(2)$
☐ $9^6(2)$
☐ 9^{12}
☐ 3^{24}

Questions 10–12 are based on the following chart.



10. Approximately what percent of all the zoo's animals are either mammals that were raised in the wild or amphibians raised in captivity?

(A) 8
(B) 12
(C) 18
(D) 34
(E) 100

11. If the Bronx Zoo donated all of its insects and fish to other zoos, approximately what percent of the animals in the zoo would be birds raised in the wild?

(A) 5
(B) 9
(C) 24
(D) 32
(E) 60

12. If the zoo currently has 80 total birds, what is the smallest number of birds that could be added such that at least 20% of the animals at the zoo would be birds?

(A) 10
(B) 80
(C) 100
(D) 125
(E) 200

13. Trail mix is made by combining 3 pounds of nuts that cost x dollars per pound with 1 pound of chocolate that costs y dollars per pound and 2 pounds of dried fruit that costs z dollars per pound. What is the cost in dollars per pound for the trail mix?

(A) $\dfrac{3x+y+2z}{xyz}$

(B) $3x+y+2z$

(C) $\dfrac{3x+y+2z}{6}$

(D) $6(3x+y+2z)$

(E) $\dfrac{x}{3}+y+\dfrac{2}{z}$

14. If $z = 3^4$, then $\left(3^z\right)^z =$

(A) 3^{16}
(B) 3^{81}
(C) 3^{324}
(D) 3^{405}
(E) $3^{6,561}$

15. Maurice entered a number into his calculator and erroneously divided the number by 0.03 instead of 0.0003, resulting in an incorrect result. Which of the following is a single operation that Maurice could perform on his calculator to correct the error?

Indicate <u>all</u> such operations.

☐ Multiply the incorrect product by 100
☐ Divide the incorrect product by 100
☐ Multiply the incorrect product by 0.01
☐ Divide the incorrect product by 0.01

16. A company’s annual expenses are composed entirely of a fixed amount in costs, plus a variable amount that is directly proportional to the number of clients served. In 2009, the company served 450 clients and its total expense was \$830,000. In 2010, the company served 510 clients and its total expense was \$896,000. What is the company’s fixed annual expense, in dollars?

(A) 1,844
(B) 1,757
(C) 335,000

(D) 485,000
(E) 830,000

17. Which of the following lines is perpendicular to $4x + 5y = 9$ on the *xy* plane?

(A) $y = \frac{5}{4}x + 2$

(B) $y = -\frac{5}{4}x + 9$

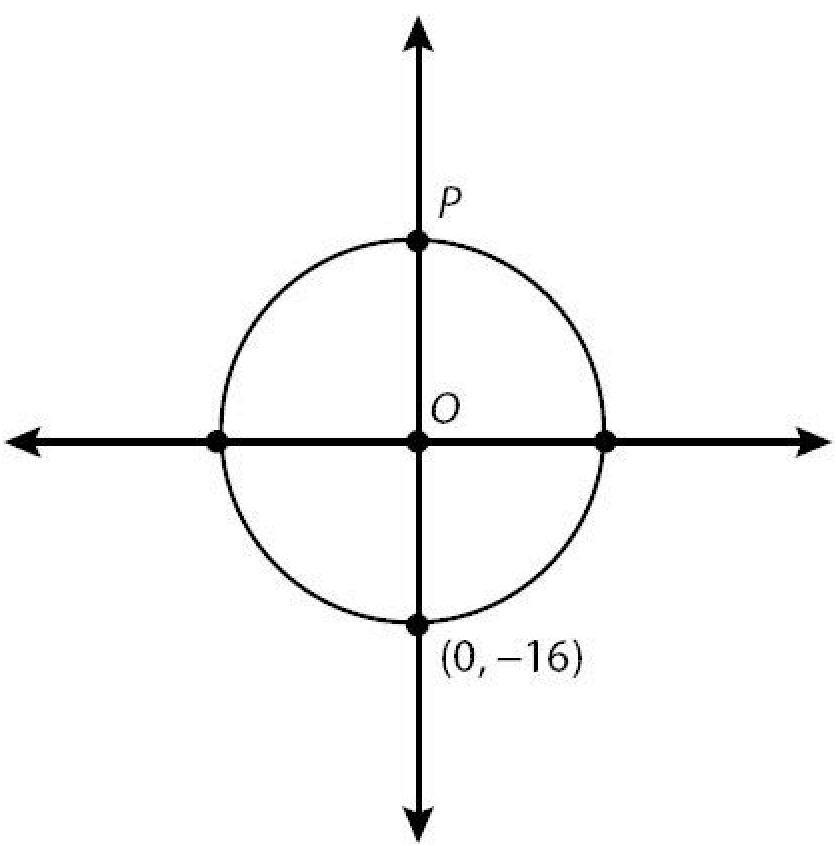
(C) $y = -4x + \frac{9}{5}$

(D) $y = \frac{4}{5}x - \frac{4}{5}$

(E) $y = -\frac{4}{5}x$

18. The tens digit is missing from the three-digit number 8 __ 9. If the tens digit is to be randomly selected from the ten different digits from 0 to 9, what is the probability that the resulting three-digit number will be a multiple of 9?

(A) 0.1
(B) 0.2
(C) 0.4
(D) 0.9
(E) 1



19. In the figure above, the circle is centered at (0, 0). What is the distance between point P and the point (-10, -8) (not shown on the graph)?

(A) 18
(B) 20
(C) 22
(D) 24
(E) 26

20. If $f(-0.5) = 0$, which of the following could be $f(x)$?

(A) $2x + 2$
(B) $4x - 2$
(C) $4x^2 - 1$
(D) $x^2 - 1$
(E) $(-x)^2 - 2.5$

Answers to Math Practice Section 2

1. **(C).** Set M consists of -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. Quantity A is the *least* of these integers that is also in Set N. The smallest integer in Set N is 9, which is also in Set M, so Quantity A is 9. The two quantities are equal.

2. **(A).** As algebra, "17% of p is equal to 18% of q" is:

$$\frac{17}{100}p = \frac{18}{100}q$$

Solve for p. The easiest way to do this is to first multiply both sides of the equation by 100, then divide both sides by 17:

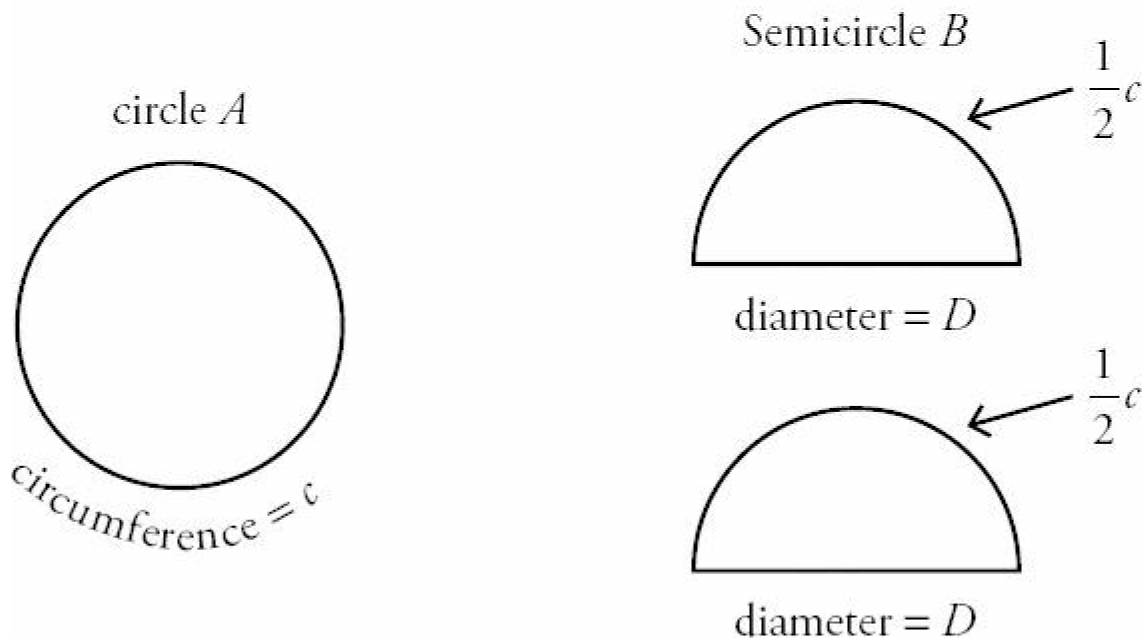
$$p = \frac{18}{17}q$$

Since $\frac{18}{17}$ is greater than 1 and both variables are positive, p is greater than q.

(Note that it was necessary to know that both variables were positive! If they were negative, $p = \frac{18}{17}q$ would imply that p is more negative than q, so q would have been greater than p. Without information about sign, the answer would have been (D).)

3. **(B).** If the given semicircle has half the area of the circle, then the Semicircle B is simply equal to half of Circle A. However, that does *not* mean that the semicircle has half the perimeter. Observe:

Quantity A **Quantity B**



The semicircle is drawn twice, as Quantity B refers to "twice the perimeter of Semicircle *B*." Note that Quantity A is equal to the circumference *c*, while Quantity B is equal to this same circumference, plus twice the length of the diameter. Quantity B is greater.

4. **(B).** Standard deviation measures the variance from the mean; the more spread out a set is, the higher the deviation. The set in Quantity B is the same as the one in Quantity A, but with the smallest number *even smaller* and the largest number *even larger*, so the set in Quantity B is more spread out, and has a greater standard deviation.

5. **(D).** An isosceles triangle has two sides that are equal and a third side that is a different length. The isosceles triangle in this question has a perimeter of 28 and shortest side of length 8. Now, suppose that the shortest side is the one that is repeated, such that the triangle has two sides of length 8 and one other side of length x. This would mean:

$8 + 8 + x = \text{Perimeter}$
$16 + x = 28$
$x = 12$

So, this triangle would have lengths 8, 8, and 12 as the three legs. Test this triangle via the Third Side Rule: the length of any side of a triangle must be greater than the difference between the other two sides and less than the sum of the other two sides. The third side (x) must be greater than $8 - 8 = 0$ and less than $8 + 8 = 16$. Since 12 is between 0 and 16, this is a legal triangle.

On the other hand, consider the possibility that the other side, x, is repeated and the length 8 is used only once. In this case:

$x + x + 8 = 28$
$2x = 20$
$x = 10$

The sides of this triangle are 10, 10, and 8. Test this triangle via the Third Side Rule: the third side (8) must be greater than $10 - 10 = 0$ and less than $10 + 10 = 20$. Since 8 is between 0 and 20, this is a legal triangle.

For one triangle, the quantities in Quantity A and Quantity B would be equal, but for the other, Quantity B would be greater than Quantity A. Therefore, the relationship cannot be determined from the given information.

6. **(B).** $(3 - z)(z + 4) = 0$, so either $(3 - z)$ or $(z + 4)$ must equal 0:

$$3 - z = 0$$
$$z = 3$$

OR

$$z + 4 = 0$$
$$z = -4$$

z is either 3 or -4. Either way, Quantity B is greater.

7. **(D).** If $ad < 0$, a and d have opposite signs. Because $a > d$, a must be positive and d must be negative. Similarly, if $ab > 0$, a and b have the same sign, so a and b are both positive. The remaining variable c can be positive, 0, or negative and still fall between b and d. If c is 0, the two quantities are equal. If c is positive, Quantity A is positive and Quantity B is negative. If c is negative, Quantity B is positive and Quantity A is negative. The relationship cannot be determined from the information given.

Alternatively, pick numbers. If $a = 4$, $b = 3$, $c = 2$, and $d = -1$, then all the criteria of the problem are fulfilled, and Quantity A is greater. But if $a = 4$, $b = 3$, $c = -5$, and $d = -10$, then all the criteria of the problem are still fulfilled, but Quantity B is greater.

8. $\frac{6}{1}$ **(or any equivalent fraction).** Solve one equation for a single variable, and substitute into the other equation:

Eq. (1): $12b = 2g$	Eq. (2): $4g - 3b = 63$
$12b = 2g$	
$6b = g$	Isolate g in Eq. (1). Divide by 2.
$4(6b) - 3b = 63$	Substitute $(6b)$ for g in Eq. (2).
$24b - 3b = 63$	Solve for b. Simplify.
$21b = 63$	Combine like terms.
$b = 3$	Divide by 21.
$12(3) = 2g$	Substitute (3) for b in Eq. (1). Solve for g.
$36 = 2g$	Simplify.
$g = 18$	Divide by 2.

$b = 3$ and $g = 18$, so $\frac{g}{b} = 6$.

9. **II and III only.** To simplify $81^3 + 27^4$, note that both bases are powers of 3. Rewrite the bases and combine.

$81^3 + 27^4 =$
$(34)^3 + (3^3)^4 =$
$3^{12} + 3^{12} =$
$3^{12}(1 + 1) =$
$3^{12}(2)$

Since $3^{12}(2)$ appears in the choices, this is one answer. However, this is an "indicate all" question, so you should check whether any other choices are equivalent. One other choice, $9^6(2)$, also qualifies, since $9^6(2) = (3^2)^6(2) = 3^{12}(2)$.

10. **(B).** 26% of the animals are mammals, and about a quarter *of those* were raised in the wild: $\frac{1}{4}$ of 26% = about 6.5%. 8% of all the animals are amphibians, and about three quarters *of those* were raised in captivity: $\frac{3}{4}$ of 8% = about 6 %. In total, these two categories account for about 12% of all the zoo's animals.

11. **(B).** To solve this question, imagine that there were originally 100 animals in the zoo. If the zoo gives away all the insects and fish, then there are 38 fewer animals (20 + 18) in the zoo, or 62. But there are still 10 birds, which now make up about 16% of the zoo's animals (use your calculator to find this if you don't feel comfortable estimating). Of those, a little more than half were raised in the wild. Among the choices, only 9% is a little more than half of 16%.

12. **(C).** If the zoo has 80 birds, which make up 10% of the total number of animals at the zoo, then there are 800 animals total. To correctly calculate how many birds must be added, realize that any birds added increases not only the subtotal of 80 birds but also the total of 800 animals. If adding new animals (rather than trading reptiles for birds, for example), you cannot simply double the number of birds to double the percent of the animals that are birds!

Thus, use the following inequality: 80

$$\frac{80+x}{800+x} \geq \frac{20}{100}$$

$100(80 + x) \geq 20(800 + x)$

$8{,}000 + 100x \geq 160{,}000\ 20x$
$80x \geq 8{,}000$
$x \geq 100$

At least 100 birds must be added such that at least 20% of the animals at the zoo would be birds (check: There would be 180 birds among 900 animals, or 20% of the total).

13. **(C).** This question is a tricky one, because even though it never uses the word *average* or the word *ratio*, it's more or less a combined ratio and averages question. The trail mix is nuts, chocolate, and dried fruit in a ratio of 3 : 1 : 2. For every 6 pounds of trail mix, there are 3 pounds of nuts, 1 pound of chocolate, and 2 pounds of dried fruit.

The cost of 6 pounds of trail mix is $3x + y + 2z$. However, to solve for the cost of one pound, divide by 6. You could also think of this as a kind of average:

Average = (Sum)/(# of terms) = $(3x + y + 2z)/6$, where each "term" is a pound.

This is choice (C). Alternatively, pick numbers. For example:

$x = 6$
$y = 5$
$z = 2$

In this example, 3 lbs. of nuts that cost $x = 6$ dollars per pound plus 1 lb. chocolate that costs $y = 5$ dollars per pound plus 2 lbs. dried fruit that costs $z = 2$ dollars per pound would cost:

$3(6) + 1(5) + 2(2) = 27$

Thus, 6 pounds of trail mix (3 lbs. nuts + 1 lb. chocolate + 2 lbs. dried fruit) would cost \$27. So, 1 pound would cost one-sixth of that: 27/6 or 9/2 dollars, which is \$4.50.

Now, plug $x = 6$, $y = 5$, and $z = 2$ into the choices to see which answer yields \$4.50. Only (C) works.

14. **(E).** Since $3^4 = 81$, $z = 81$. So, $(3^z)^z = (3^{81})^{81} = 3^{81\times 81} = 3^{6,561}$.

15. **I and IV only.** Since 0.03 is 100 times greater than 0.0003, when Maurice accidentally divided by 0.03 instead of 0.0003, he divided by a number 100 times too big. Thus, multiplying by 100 will correct the error. Thus, Statement I is correct.

However, dividing by any quantity is the same as multiplying by its reciprocal. So, multiplying by 100 is the same as dividing by 0.01. Thus, Statement IV is also correct.

Alternatively, pick a number. Divide by both 0.03 and 0.0003, and then check each answer to see which correct the error. For instance, suppose the original number were 12.

12 divided by 0.03 = 400 ⟵ INCORRECT RESULT
12 divided by 0.0003 = 40,000 ⟵ CORRECT RESULT

Now, perform the operation in each answer choice on the incorrect product, 400, to see which operations turn that product into 40,000. Operations I and IV work.

16. **(C).** Begin by constructing a function describing the situation in the problem. Using E for expenses, x for the number of clients, c for the expense per client, and f for fixed costs:

$E(x) = xc + f$

In words, expense as a function of the number of clients equals the number of clients multiplied by the variable cost per client, plus the fixed cost.

In 2009, the company served 450 clients and its total expense was \$830,000. Thus:

$830{,}000 = 450c + f$

In 2010, the company served 510 clients and its total expense was \$896,000. Thus:

$896{,}000 = 510c + f$

Since it is easier to isolate *f* than *c* in each equation, get *f* by itself for each equation and then set the opposite sides equal:

$830{,}000 = 450c + f$
$f = 830{,}000 - 450c$

$896{,}000 = 510c + f$
$f = 896{,}000 - 510c$

$830{,}000 - 450c = 896{,}000 - 510c$
$830{,}000 + 60c = 896{,}000$
$60c = 66{,}000$
$c = 1{,}100$

Plug $c = 1{,}100$ into either equation to find *f*:

$f = 830{,}000 - 450(1{,}100)$
$f = 335{,}000$

Alternatively, subtract \$896,000 - \$830,000 to get \$66,000, which must be the cost difference between serving 450 clients and serving 510 clients (a difference of 60 clients). Divide \$66,000 by 60 clients to get \$1,100, the variable cost per client. Then, multiply \$1,100 × 450 = \$495,000 to get the variable cost of serving 450 clients, not counting the fixed cost. Finally, subtract this figure from the total cost of serving 450 clients to get the fixed cost. \$830,000 - \$495,000 = \$335,000. The numbers should look familiar; the point is that you can "reason through it" without strictly setting up equations.

17. **(A).** First, algebraically manipulate $4x + 5y = 9$ into $y = mx + b$ format, where *m* is the slope and *b* is the *y*-intercept.

$4x + 5y = 9$
$5y = -4x + 9$
$y = -\frac{4}{5}x + \frac{9}{5}$

Since $m = -\frac{4}{5}$, the slope is $-\frac{4}{5}$. Perpendicular lines have negative reciprocal slopes. Thus, the correct answer has a slope of $\frac{5}{4}$. Only choice (A) qualifies.

18. **(A).** If the tens digit is to be randomly selected from the digits 0 to 9, there are ten possibilities for the completed number. Using your calculator, divide each by 9 to see which ones are multiples of 9:

809	$\longleftarrow$	not a multiple of 9
819	$\longleftarrow$	MULTIPLE OF 9
829	$\longleftarrow$	not a multiple of 9
839	$\longleftarrow$	not a multiple of 9
849	$\longleftarrow$	not a multiple of 9
859	$\longleftarrow$	not a multiple of 9
869	$\longleftarrow$	not a multiple of 9
879	$\longleftarrow$	not a multiple of 9
889	$\longleftarrow$	not a multiple of 9
899	$\longleftarrow$	not a multiple of 9

The answer is 1/10, or 0.1.

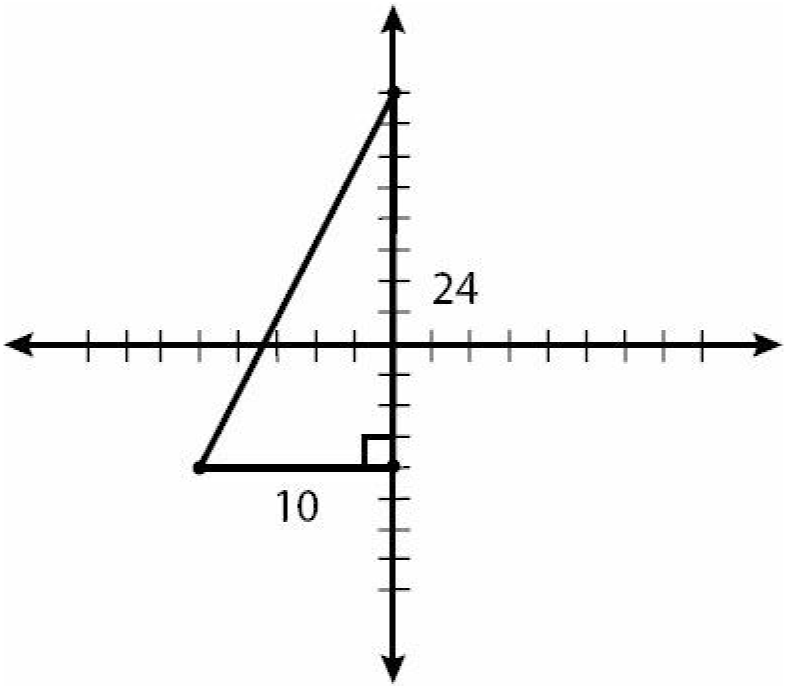
Alternatively, a number is divisible by 9 if the sum of its digits is a multiple of 9. The existing digits sum to 8 + 9 = 17, so the addition of 0 through 9 means that the sum of all three digits could be 17 through 26, inclusive. Only one multiple of 9 (i.e., 18) is found in this range.

19. **(E).** Because the circle is centered at (0, 0) and passes through (0, -16), the radius of the circle is 16. Point *P* lies on the circle and the *y*-axis, so it lies exactly one radius above the origin. Point *P*'s coordinates are therefore (0, 16). To find the distance between (0, 16) and (-10, -8), either use the distance formula, or draw a graph and make a right triangle on which you can use the Pythagorean theorem.

From the distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$d = \sqrt{(-10-0)^2 + (-8-16)^2}$$
$$d = \sqrt{(-10)^2 + (-24)^2}$$
$$d = \sqrt{676}$$
$$d = 26$$

To use the triangle method, plot (0, 16) and (-10, -8), then drop a line down from (0, 16) to make a right triangle. To do so, you will need to add the third point (0, -8).



Use the coordinates to determine the lengths of the legs, then use the Pythagorean theorem (the hypotenuse is d):

$$24^2 + 10^2 = d^2$$
$$576 + 100 = d^2$$
$$676 = d^2$$
$$d = 26$$

20. **(C).** If $f(-0.5) = 0$, then the answer is 0 when $x = -0.5$. For each choice, plug in -0.5 for x. Only if the result is 0 could the choice be $f(x)$.

(A) $2x + 2 = 2(-0.5) + 2 = -1 + 2 = 1$
(B) $4x - 2 = 4(-0.5) - 2 = -2 - 2 = -4$
(C) CORRECT. $4x^2 - 1 = 4(-0.5)^2 - 1 = 4(0.25) - 1 = 1 - 1 = 0$
(D) $x^2 - 1 = (-0.5)^2 - 1 = 0.25 - 1 = -0.75$
(E) $(-x)^2 - 2.5 = (-(-0.5))^2 - 2.5 = (0.5)^2 - 2.5 = 0.25 - 2.5 = -2.25$

Math Practice Section 3: Hard Difficult

Math Practice Section: Hard
20 Questions
35 Minutes

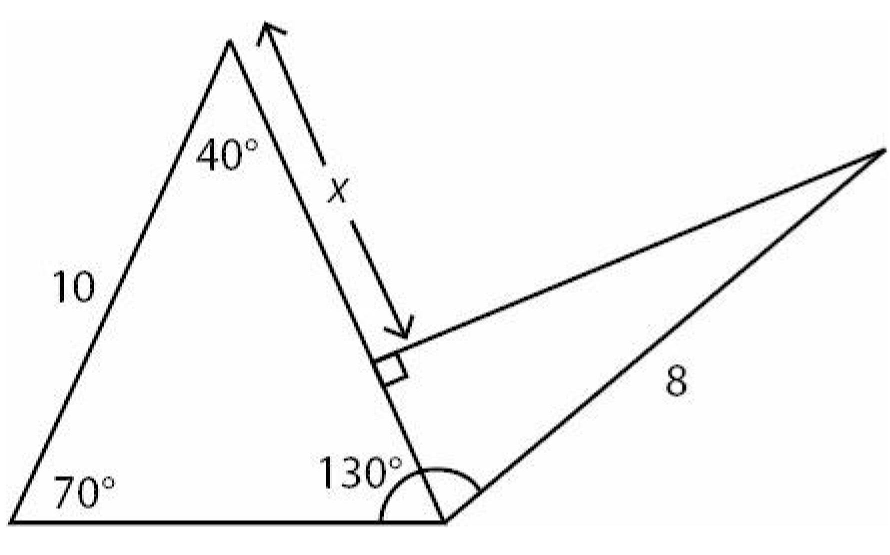
For questions in the Quantitative Comparison format ("Quantity A" and "Quantity B" given), the answer choices are always as follows:

(A) Quantity A is greater.
(B) Quantity B is greater.
(C) The two quantities are equal.
(D) The relationship cannot be determined from the information given.

For questions followed by a numeric entry box, you are to enter your own answer in the box. For questions followed by fraction-style numeric entry boxes, you are to enter your answer in the form of a fraction. You are not required to reduce fractions. For example, if the answer is 1/4, you may enter 25/100 or any equivalent fraction.

All numbers used are real numbers. All figures are assumed to lie in a plane unless otherwise indicated. Geometric figures are not necessarily drawn to scale. You should assume, however, that lines that appear to be straight are actually straight, points on a line are in the order shown, and all geometric objects are in the relative positions shown. Coordinate systems, such as *xy*-planes and number lines, as well as graphical data presentations such as bar charts, circle graphs, and line graphs, *are* drawn to scale. A symbol that appears more than once in a question has the same meaning throughout the question.

1.



Quantity A	Quantity B
x	6

2.

Quantity A	Quantity B
$(z^6)^x \times z^{3x}$	z^{9x}

3.

For a group of test takers, the scores on an aptitude test were normally distributed, had a mean of 154, and a standard deviation of 3.

Quantity A	Quantity B
The fraction of test takers in the group who scored greater than 158	$\frac{1}{3}$

4.

$$3x + 5y + 2z = 20$$
$$6x + 4z = 10$$

Quantity A	Quantity B
y by itself	2

5.

Romero Automobiles sells cars only from Manufacturer X and Manufacturer Y. The range of the list prices of the cars from Manufacturer X is $22,000. The range of the list prices of the cars from Manufacturer Y is $15,000.

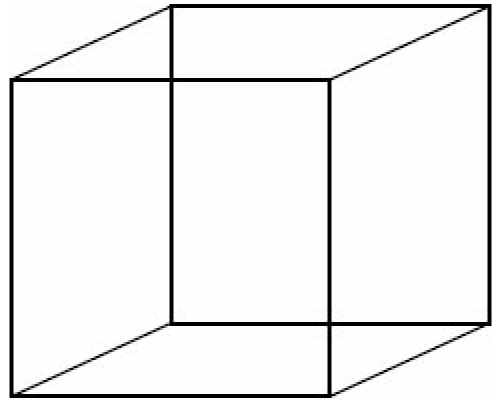
Quantity A	Quantity B
The range of the list prices of all automobiles sold by Romero Automobiles	$22,000

6.

The operation # is defined by $x\# = \frac{1}{x} + x$

Quantity A	Quantity B
(4#)#	4.5

7.



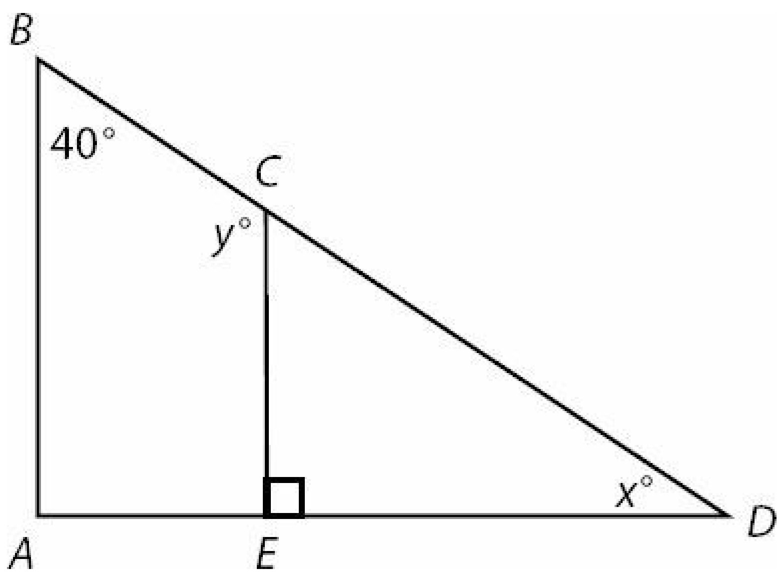
The cube above has side length of 4

Quantity A	**Quantity B**
After selecting one vertex of the cube, the number of straight line segments longer than 4 that can be drawn from that vertex of the cube to another vertex of the cube	When the cube is placed on a flat surface, the maximum number of edges of the cube that can be touching the flat surface at once

8. If $160^2 = 16x$, then x is equivalent to which of the following?

(A) 10
(B) $2^3 5$
(C) $2^2 5^2$
(D) $2^6 5^2$
(E) $2^6 5^3$

9.



In the triangle shown above, BA is parallel to CE. What is the value of $x + y$?

10. If the tick marks on the number line below are evenly spaced, what value is represented by x?

(A) 2^0
(B) 2
(C) $(-2)2^5$
(D) $(-3)2^5$
(E) $(-4)2^5$

11. If the volume of a cube is v, what is the surface area of the cube in terms of v?

(A) $6\sqrt{v}$
(B) $\left(\sqrt[2]{v}\right)^3$
(C) $6\left(\sqrt[2]{v}\right)^3$
(D) $\left(\sqrt[3]{v}\right)^2$
(E) $6\left(\sqrt[3]{v}\right)^2$

12. What is the area of an equilateral triangle with vertices at (-1, -3), (9, -3), and (m, n) where m and n are both positive numbers?

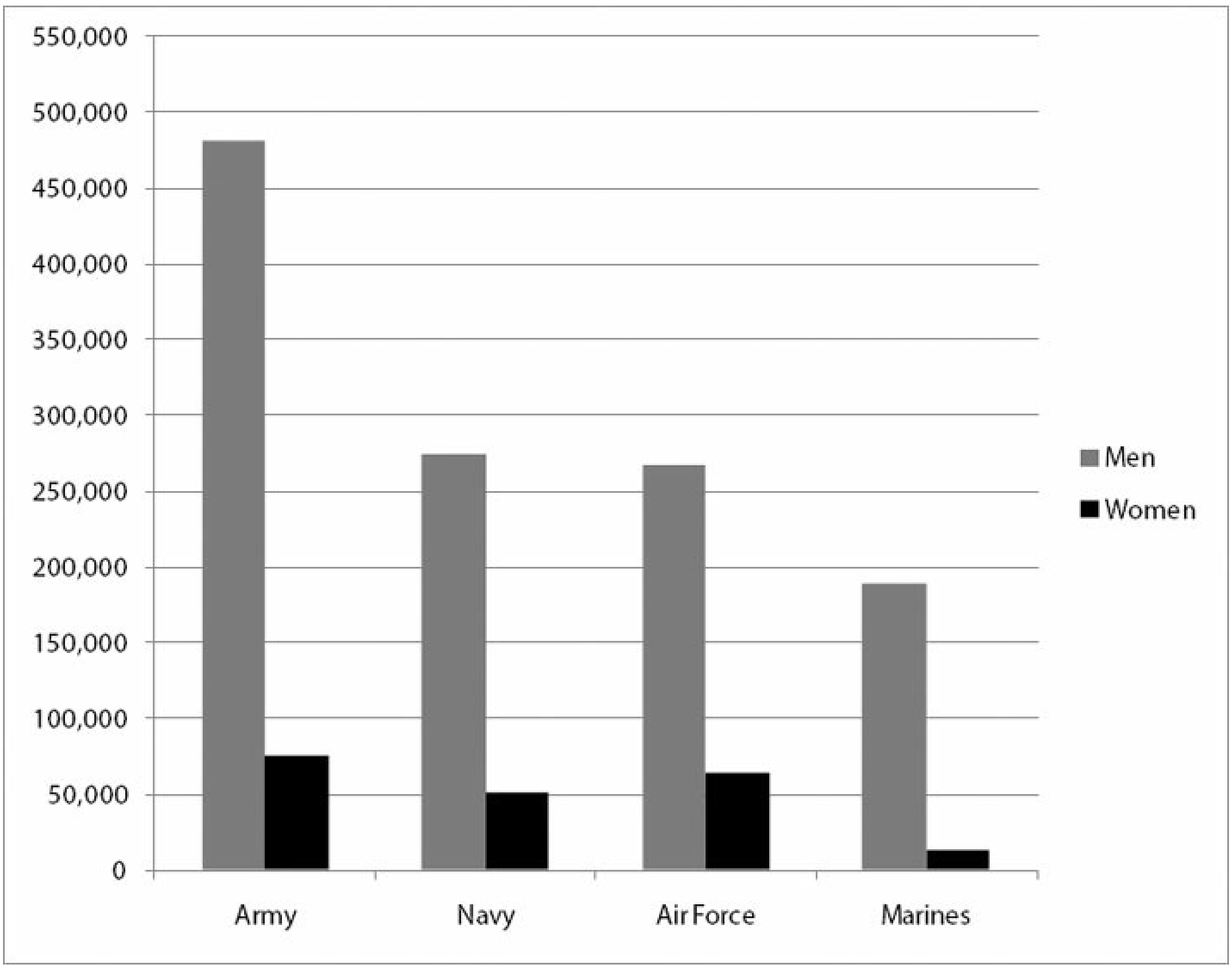
(A) $25\sqrt{2}$
(B) $50\sqrt{2}$
(C) $10\sqrt{3}$
(D) $25\sqrt{3}$
(E) $50\sqrt{3}$

Questions 13–15 are based on the following charts.

Marital Status of Military Personnel by Gender and Branch

Marital Status		Army	Navy	Air Force	Marines
Single, no children less than 18 years old	men	164,513	107,349	94,800	90,949
	women	27,492	24,757	25,247	6,338
Single, with children less than 18 years old	men	26,571	10,506	9,544	4,807
	women	11,037	5,859	6,313	1,263
Married, spouse is also military personnel or retired military	men	15,058	8,638	20,760	4,719
	women	14,633	8,832	18,574	3,676
Married, spouse is a civilian	men	275,953	147,255	142,573	88,233
	women	21,687	11,175	13,982	1,858

Number of Military Personnel by Gender and Branch



13. Which military branch has the greatest percentage of women who are single and have children under the age of 18?

(A) Army
(B) Navy

(C) Air Force
(D) Marines
(E) It cannot be determined from the information given.

14. If a man whose spouse is also military personnel or retired military were to be selected at random, what would the probability be that he was NOT in the Air Force?

(A) 72%
(B) 58%
(C) 42%
(D) 24%
(E) 13%

15. Which of the following expressions is equal to the approximate number of women who would have to enlist in the Army to make the fraction of Army personnel who are women equal the fraction of Air Force personnel who are women?

(Assume that the number of men in the Army and the number of men and women in the Air Force remain unchanged from what is shown in the tables above.)

(A) $\dfrac{482{,}000-268{,}000}{75{,}000-64{,}000}$

(B) $\dfrac{(482{,}000)(64{,}000)-(75{,}000)(268{,}000)}{482{,}000}$

(C) $\dfrac{(482{,}000)(75{,}000)-(64{,}000)(268{,}000)}{482{,}000}$

(D) $\dfrac{(482{,}000)(75{,}000)}{268{,}000}-64{,}000$

(E) $\dfrac{(482{,}000)(64{,}000)}{268{,}000}-75{,}000$

16. A cable car travels from City X to Resortville, making two stops in between. Between City X and the first stop, the cable car travels $\frac{1}{3}$ of the total distance between City X and Resortville. Between the first stop and the second stop, the cable car travels $\frac{3}{5}$ of the remaining distance between the first stop and Resortville. What fraction of the entire distance from City X to Resortville remains between the second stop and Resortville?

(A) $1-\frac{1}{3}-\frac{3}{5}$

(B) $1-\frac{1}{3}-\frac{3}{5}\left(\frac{1}{3}\right)$

(C) $1-\frac{1}{3}-\frac{3}{5}\left(1-\frac{1}{3}\right)$

(D) $1-\frac{1}{3}-\frac{1}{3}\left(1-\frac{3}{5}\right)$

(E) $1-\frac{1}{3}-\frac{1}{5}\left(1-\frac{1}{3}-\frac{1}{5}\right)$

17. If p and q are integers and $20p + 3q$ is odd, which of the following must be odd?

(A) $p - q$
(B) $p + 2q$
(C) $3p + q$
(D) $2p + q^2$
(E) $3p + 3q$

18. 4,400 participants in a study were surveyed regarding side effects of a new medication, and x percent reported experiencing drowsiness. If x is rounded to the nearest integer, the result is 8. Which of the following could be the number of survey participants who reported experiencing drowsiness?

Indicate all such values.

☐ 325
☐ 330
☐ 352
☐ 375

19. $\frac{5^3(4^{45}-4^{43})27}{225^2}$ is equivalent to which of the following?

(A) 4^{43}
(B) 4^{45}
(C) $4^{90}5^3$
(D) $4^{86}5^3 3^3$
(E) $4^{90}5^3\ 3^3$

20. Price of Plane Ticket for an April 1 Flight Based on Date of Purchase

Price	When Purchased By
$210	March 31

\$168	March 15
\$140	March 1

Harpreet purchased a ticket on March 1st. If he had purchased the ticket on March 2nd, he would have paid x percent more. If he had purchased the ticket on March 16th, he would have paid y percent more than he would have paid on March 2nd. What is the positive difference between x and y?

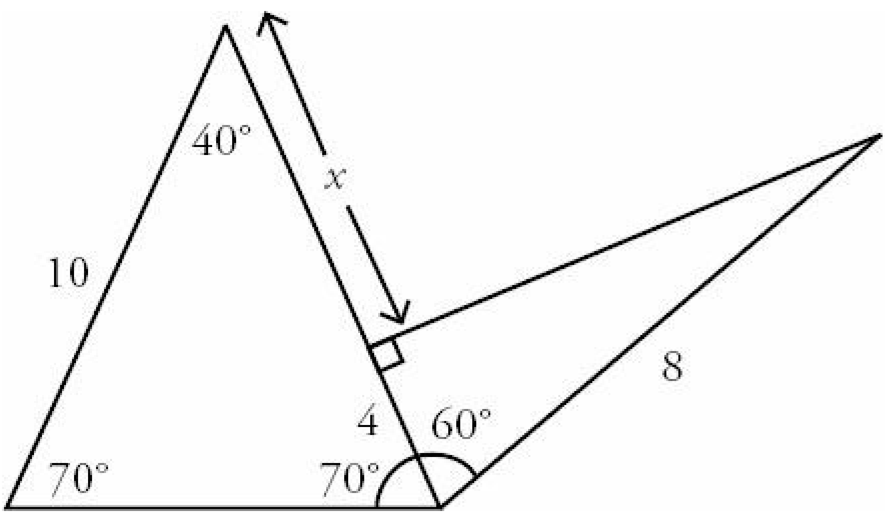
(A) 5
(B) 14
(C) 20
(D) 25
(E) 28

Answers to Math Practice Section 3

1. **(C).** The leftmost triangle has two angles labeled 40 and 70. Subtract these from 180 (the sum of the angles in any triangle) to determine that the third angle is 70. Subtract 130 - 70 = 60 to get the measure of the adjoining angle in the rightmost triangle.

Since the leftmost triangle is isosceles, the two long sides are each equal to 10.

Since the rightmost triangle is a 30–60–90 triangle, the sides are in the proportion $x : \sqrt{3}\,x : 2x$. Because the hypotenuse is 8 and also the $2x$ in the ratio, the shortest leg of this triangle is $x = 4$.



To calculate *x,* subtract: 10 - 4 = 6. The two quantities are equal.

2. **(C).** The terms in Quantity A have the same base, so add the exponents: $(z^6)^x \times z^{3x} = z^{6x} \times z^{3x} = z^{9x}$. The two quantities are equal. Note that $(z^6)^x$ is interchangeable with $(z^x)^6$ and z^{6x}.

3. **(B).** For a normal distribution, approximately two thirds of the values are within one standard deviation of the mean. Thus, roughly 1/6 of the population is more than a deviation above the mean, and 1/6 is more than a deviation below. Thus, about 1/6 of the test takers would score greater than 157 (154 + 3 = 157, one standard deviation above the mean), so an even smaller fraction of the test takers would score greater than 158.

4. **(A).** In order to isolate y, eliminate both x and z. Because there are only two equations, both x and z must be eliminated at the same time if the value of y is to be determined.

Notice that the coefficients for x and z in the second equation (6 and 4, respectively) are exactly double their coefficients in equation 1 (3 and 2, respectively). Divide the second equation by 2, making the coefficients the same.

$$3x + 5y + 2z = 20 \quad \longrightarrow \quad 3x + 5y + 2z = 20$$
$$6x + 4z = 10 \quad \longrightarrow \quad 3x + 2z = 5$$

Now subtract the second equation from the first.

$$\begin{array}{l} 3x + 5y + 2z = 20 \\ \underline{-(3x + 2z = 5)} \\ 5y = 15 \\ y = 3 \end{array}$$

Quantity A is greater.

5. **(D).** The range of list prices of automobiles is found by subtracting the price of the least expensive automobile from the price of the most expensive automobile. Given just the range, there is not enough information to determine the maximum and minimum list price vehicles from either manufacturer. Before selecting (D), though, you should try to prove (D). Construct two examples in which the list prices of the cars from Manufacturer X have a range of \$22,000 and the list prices of the cars from Manufacturer Y have a range of \$15,000, but the overall ranges are drastically different.

EXAMPLE 1:
List prices of Manufacturer X's cars range from \$10,000 to \$32,000
List prices of Manufacturer Y's cars range from \$10,000 to \$25,000
Here, the overall range is the same as X's range, which is \$32,000 - \$10,000 = \$22,000

EXAMPLE 2:
List prices of Manufacturer X's cars range from \$10,000 to \$32,000
List prices of Manufacturer Y's cars range from \$100,000 to \$115,000
Here, the overall range is \$115,000 - \$10,000 = \$105,000

In Example 1, the range = \$22,000 and the quantities are equal. In Example 2, Quantity A is much greater than Quantity B. It is not possible to make the range any smaller than \$22,000 (the minimum range of all the prices cannot be *smaller* than the larger of the two ranges of each manufacturer's prices), but it can get much, much larger.

Note that the testing done above was very important! If Quantity B had read "\$21,999," the answer would be (A) rather than (D).
The correct answer is (D).

6. **(B).** Start inside the parentheses (according to PEMDAS, always deal with parentheses first).

$$4\# = \frac{1}{4} + 4, \text{ or } \frac{17}{4}.$$

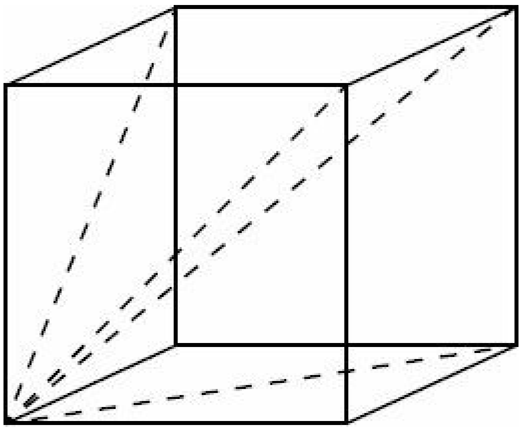
Since $4\# = \frac{17}{4}$, plug $\frac{17}{4}$ in for x to get $(4\#)\#$.

$$\text{Thus, } (4\#)\# = \frac{1}{\frac{17}{4}} + \frac{17}{4} = \frac{4}{17} + \frac{17}{4}$$

While you could find a common denominator, it is more efficient to ballpark the value or simply use the calculator.

Ballparking, $\frac{4}{17}$ is less than 0.25 and $\frac{17}{4}$ is exactly 4.25, so the sum is less than 4.5. Using the calculator, $\frac{4}{17}+\frac{17}{4}$ is about 4.485. Quantity B is greater.

7. **(C).** If a cube has side length of 4, all of the "straight line segments" connecting vertices of the cube *along an edge of the cube* will have length of 4. The only straight line segments between vertices that are longer than 4 are those that go diagonally through the cube or diagonally across a face. From a selected vertex of the cube, there are 3 diagonals across the adjacent faces of the cube, and 1 diagonal through the cube to the opposite vertex.



Thus, Quantity A is 4.

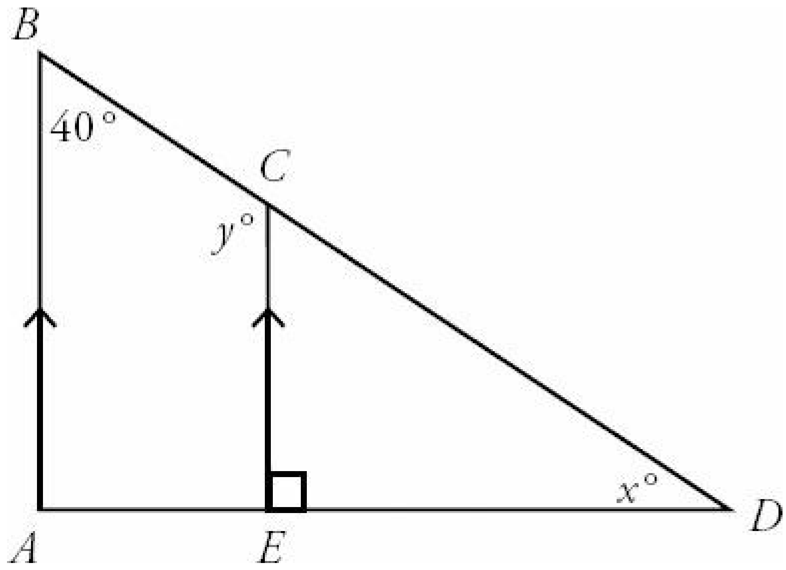
If a cube is placed on a flat surface, the maximum contact occurs when one cube face abuts the surface—and thus 4 cube edges touch the surface. There is no way to make more than 4 cube edges touch the flat surface at once. The two quantities are equal.

8. **(D).** The easiest first step is to divide both sides by 16. To do that, make sure you separate out 160^2 first. Notice that $160^2 = 160 \times 160 = 16 \times 10 \times 160$:

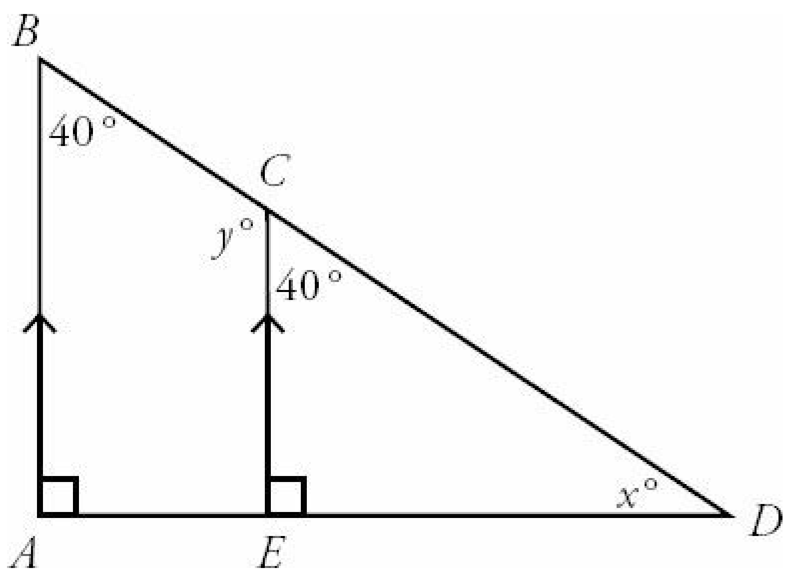
$16 \times 10 \times 160 = 16x$
$10 \times 160 = x$
$1{,}600 = x$

None of the answer choices match this, so break 1,600 down into its primes ($1{,}600 = 100 \times 16 = 25 \times 4 \times 16 = 5^2 \times 2^2 \times 2^4 = 5^2 \times 2^6$) and see which choice is equivalent. Alternatively, multiply out the answer choices to see which equals 1,600. The correct choice is (D).

9. **190.** Redraw the figure, labeling all information given:



Since *BA* and *CE* are parallel, angle *B* and minor angle *C* are equivalent, as shown:



The two angles that meet at *C* make up a straight line, so they sum to 180 degrees:

$180 = y + 40$
$y = 140$

The three angles of triangle *CDE* must sum to 180 degrees, and so

$180 = 40 + 90 + x$
$180 = 130 + x$
$x = 50$

Therefore, $x + y = 140 + 50 = 190$.

10. **(D).** At first glance, you might be tempted to think that each tick mark on this number line corresponds to a power of 2, but remember that powers grow exponentially (i.e. the distance between 2^5 and 2^6 is not the same as the distance between 2^1 and 2^2), whereas the tick marks in the diagram are evenly spaced. So, start by finding the distance between 2^5 and 2^6.

$2^5 = 32$, and $2^6 = 64$. The difference between them is 32. That means the distance between each tick mark on the number line is 32. So to get from 2^5 to x, "walk back" or subtract four intervals of 32: 32 - 4(32) = -96.

Multiply out the answers to see which one equals -96. Only choice (D) works.

11. **(E).** There are two ways to solve this question, with smart numbers or algebra. Start with plugging-in. First, set a value for the volume. In this case, pick a perfect cube, so the side length and all other values will be integers. The smallest perfect cube (other than 1, which you should try never to use when doing plug-in questions) is 8.

A cube with a volume of 8 has a side length of 2, meaning each side has an area of 4. A cube has 6 sides, making the total surface area 24. (The equation for surface area is Surface Area = $6s^2$). The answer to this question is 24, based on these numbers.

Immediately eliminate any answer choices that have the square root of 8, as the result will not be an integer. The answer must be either (D) or (E). The cube root of 8 is 2. Answer choice (D) simply squares it, yielding 4. In answer choice (E), that result is multiplied by 6, producing 24, which is the required answer. Thus, the answer is (E).

If you wanted to solve with algebra, you'd need to start by solving for a side of a cube with volume v:

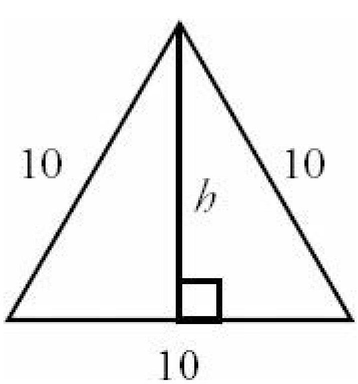
$$V = s^3 \quad \text{so} \quad s = (\sqrt[3]{v})$$

The equation for the surface area of a cube is $6s^2$. In this case, substitution for s results in exactly the expression written in answer choice (E).

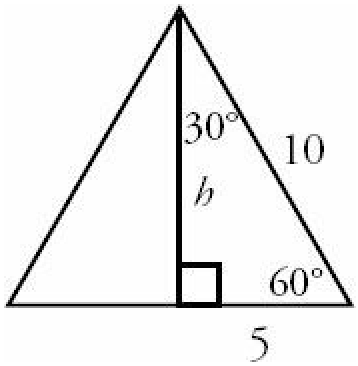
12. **(D).** To find the area of an equilateral triangle with vertices at (-1, -3), (9, -3), and (m, n), you do not need to find the values of m and n. To find the area of an equilateral triangle, you only need one side. So, you should first find the distance between (-1, -3) and (9, -3).

Since these two points are on a horizontal line together (they share a y-coordinate), the distance is just the difference between their x-coordinates: 9 - (-1) = 10.

An equilateral triangle with side 10 will have the same area regardless of where it is placed on an xy-coordinate plane, so the location of m and n is irrelevant. Instead, draw an equilateral triangle with sides equal 10. Drop a height down the middle. 10



Dividing a 60–60–60 triangle in this way creates two 30–60–90 triangles. The bottom side of the triangle is bisected by the height:



Using the properties of 30–60–90 triangles, *h* is equal to the shortest side multiplied by the square root of 3. Thus, $h = 5\sqrt{3}$. (You may also wish to memorize that the height of an equilateral triangle is *always* equal to half the side multiplied by $\sqrt{3}$.)
Find the area of the triangle, using 10 as the base:

$$A = \frac{bh}{2} = \frac{10(5\sqrt{3})}{2} = 25\sqrt{3}$$

13. **(A).** The percentage of women in a service who are single mothers is:

$$\frac{\text{\# single women with children}}{\text{\# women}}$$

Find the number of single mothers in each of the four services by looking at the first table, ***Martial Status of Military Personnel by Gender and Branch.*** The number of women who are single with children is given as:

Army	11,037
Navy	5,859
Air Force	6,313
Marines	1,263

There are two ways to find the total number of women in each service, though. Either sum the exact number of women in each branch of the service across each of the marital status' given in the first chart, or read an approximate number of women from the second bar chart, ***Number of Military Personnel by Gender and Branch***, then only bother to sum from the detailed chart if two answers are very close to each other.

Since using the chart will be faster and GRE problems are designed to be solved quickly, try approximating from the bar chart first. The total number of women in each of the four services is approximately

Army	75,000
Navy	50,000
Air Force	60,000

Marines	10,000

Calculate the approximate percent of women who are single mothers in each branch of the service.

Army	11,000/75,000 = about 14.7%
Navy	5,900/50,000 = about 11.8%
Air Force	6,300/60,000 = about 10.5%
Marines	1,300/10,000 = about 13%

The percent looks highest in the Army. At least, reason that the number of single mothers in the Army is about double the number of single women in either the Navy or Air Force, yet the total number of women in the Army is definitely less than double the total number of women in either the Navy or Air Force, making their percentage of women greater in the Army.

Just quickly check the actual totals for the Army and the Marines.

Army: single mothers = 11,037 and total women = 74,849. The percent is 14.7%.
Marines: single mothers = 1,263 and total women = 13,135. The percent is 9.6%.

Thus, the Army has the greatest percentage of women who are single and have dependents under the age of 18.

14. **(B).** The probability that a man whose spouse is also military personnel or retired military is NOT in the Air Force is given by the formula:

$$\frac{\text{\# of men in the "married, military spouse" category who are NOT in the Air Force}}{\text{total \# men in the "married, military spouse" category}}$$

All of the information needed to calculate both of these numbers is in the first table, ***Martial Status of Military Personnel by Gender and Branch.***

The total number of men married to a military spouse or retired military in each of the four services:

15,058 + 8,638 + 20,760 + 4,719 = 49,175

Then just subtract the number of Air Force men in this category to get the number of men in such marriages who are not in the Air Force:

49,175 - 20,760 = 28,415

And finally:

$$\frac{28,415}{49,175} = 0.5778 \approx 58\%$$

15. **(E).** In order to solve this problem, make the two ratios equal. The ratio in question is Women/Total, but Women/Men is simpler and works also, because Total depends only on Women and Men):

$$\frac{\text{resulting \# of women in Army}}{\text{\# of men in Army}} = \frac{\text{\# of women in Air Force}}{\text{\# of men in Air Force}}$$

There are two ways to find the number of women and men in the Army and Air Force. Either sum the exact number of women and men in each marital status for each branch of the service in question, or read an approximate number from the second bar chart, ***Number of Military Personnel by Gender and Branch.***

Since the problem says to approximate and gives numbers in the answer choices that can serve as guidelines, approximation from the bar chart will be good enough.

The important thing is to focus on the *structure* of the math. Since adding women to the Army will change the number of women in the Army, use a variable to represent the additional women. Let x represent the number of women who would have to enlist in the Army in order to make the ratios equal.

$$\frac{\text{current \# of women in Army} + x}{\text{\# of men in Army}} = \frac{\text{\# of women in Air Force}}{\text{\# of men in Air Force}}$$

From the bar chart, look up the approximate numbers:

	Army	AF
# of women	75,000	60,000
# of men	475,000	270,000

The next step is to plug these approximate numbers into the equation and solve for x.

$$\frac{75{,}000 + x}{475{,}000} = \frac{60{,}000}{270{,}000}$$

$$\rightarrow 75{,}000 + x = 475{,}000 \times \frac{60{,}000}{270{,}000}$$

$$\rightarrow x = 475{,}000 \times \frac{60{,}000}{270{,}000} - 75{,}000$$

Looking at the answer choices, structurally, the answer must be (D) or (E), and the numbers in (E) are a better fit to the numbers approximated from the chart.

16. **(C).** Since the question concerns the "fraction of the entire distance from City X to Resortville," think of the entire distance as equal to 1. Between City X and the first stop, the cable car travels $\frac{1}{3}$, leaving $\frac{2}{3}$ left to travel.

Between the first stop and the second stop, the cable car travels $\frac{3}{5}$ of the remaining $\frac{2}{3}$, or $\frac{3}{5}\times\frac{2}{3}=\frac{2}{5}$.

So far, the cable car has gone $\frac{1}{3}+\frac{2}{5}=\frac{11}{15}$. Thus, the remaining distance is $1-\frac{11}{15}=\frac{4}{15}$. Only choice (C) is equal to $\frac{4}{15}$, although this takes some manipulation of the choices to check.

Alternatively, construct a formula. The first leg of the journey leaves $1-\frac{1}{3}$ left to travel. The second leg of the journey subtracts another $\frac{3}{5}$ of the remaining $1-\frac{1}{3}$, or $\frac{3}{5}\left(1-\frac{1}{3}\right)$. Thus, the correct expression is $1-\frac{1}{3}-\frac{3}{5}\left(1-\frac{1}{3}\right)$.

17. **(D).** If p and q are integers, then $20p$ is even regardless of whether p is even or odd. Since $20p + 3q$ is odd, $3q$ must be odd. If $3q$ is odd, then q is odd. Thus, q is odd, but p could be odd or even. The correct answer must be odd regardless of whether p is odd or even.

If p is odd, (A) is even, (B) is odd, (C) is even, (D) is odd, and (E) is even. Since the correct answer choice is the one that *must* be odd, only (B) and (D) are possibilities.

If p is even, (B) is even and (D) is odd. Thus, choice (D) is definitely odd and is the correct answer.

18. **II and III only (330 and 352).** Using your calculator, convert each choice to a percent, and determine whether that percent would round up or down to 8%.

The first choice $=\frac{325}{4,400}\times 100=7.386\ldots\%$ This number would round down to 7%, not up to 8%.

The second choice $=\frac{330}{4,400}\times 100=7.5\%$ This number rounds up to 8%, and thus this choice is correct.

The third choice $=\frac{352}{4,400}\times 100=8\%$ exactly, and thus this choice is correct.

The fourth choice $=\frac{375}{4,400}\times 100=8.522\ldots\%$ This number would round up to 9%, not down to 8%.

19. **(A).** When dealing with exponents, try to get (almost) everything in terms of common prime bases. Since all the answer choices have a base 4, leave those terms alone for now.

5^3 is already simplified
$27 = 3^3$
$225^2 = 25^2 \times 9^2 = (5^2)^2 \times (3^2)^2 = 5^4 \times 3^4$

Replacing all of these in the equation, you get this: $\dfrac{5^3(4^{45}-4^{43})3^3}{5^4 3^4}$

Cancel 5's and 3's in the top and bottom: $\dfrac{(4^{45}-4^{43})}{5\times 3}=\dfrac{(4^{45}-4^{43})}{15}$

Factor 4^{43} out of both terms in the numerator and simplify: $\dfrac{4^{43}(4^2-4^0)}{15}=\dfrac{4^{43}(16-1)}{15}=\dfrac{4^{43}(15)}{15}=4^{43}$

20. **(A).** On March 1st, the ticket cost \$140. If he had purchased it on March 2nd, Harpreet would have paid \$168, which is \$28 more. To find x, use the percent change formula:

$$\text{Percent Change }=\left(\frac{\textit{Difference}}{\textit{Original}}\times 100\right)\%=\left(\frac{28}{140}\times 100\right)\%=20\%$$

Thus, $x = 20$. If he had purchased the ticket on March 16th, he would have paid \$210, which is \$42 more than the \$168 he would have paid on March 2nd.

$$\text{Percent Change }=\left(\frac{\textit{Difference}}{\textit{Original}}\times 100\right)\%=\left(\frac{42}{168}\times 100\right)\%=25\%$$

Thus, $y = 25$ and the positive difference between x and y is 5. ("Positive difference" just means to subtract the smaller one from the bigger one, or to subtract either one from the other and then take the absolute value.)

Appendix A
of

Vocabulary List

In This Chapter…

Vocabulary List

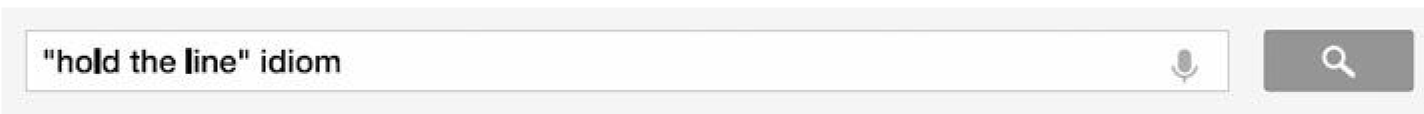
Vocabulary List

The following is a list of all the vocabulary words — as well as some expressions — used in this book.

A few words are marked (n.) for noun, (adj.) for adjective, or (v.) for verb. We have done this in cases where a word has a common meaning (*pan*, noun – a metal container for cooking food in) as well as a less-common one (*pan*, verb – to criticize harshly), and we want you to learn the less common one.

While you can look up words on online dictionary sites such as dictionary.com, m-w.com, and thefreedictionary.com, we also like learnersdictionary.com, which offers simple definitions intended for English Language Learners, as well as wordnik.com, which offers sample sentences and in-context usage for each word.

If you need to look up a phrase, try idioms.thefreedictionary.com, or simply Google the phrase in quotation marks, followed by the word "idiom." For instance:



Note for English Language Learners: In selecting which words to include, we've assumed you already know words like *simultaneous*, *considerate*, *dampen*, *wit*, *merge*, *saturate*, *mandatory*, *innovation*, *delusion*, *juvenile*, *dictate*, *catastrophe*, *hysteria*, *enduring*, *generalize*, *superstitious*, *penalize*, *detest*, *suitable*, *controversial*, *contemporary*, *dismiss*, *insightful*, *naive*, *mystified*, *peculiar*, *literally*. If not, please visit www.manhattanprep.com/gre, create a free membership, and download our Basic Vocab List.

abate
aberrant
abet, abettor
abhorrent, abhorrence
abide
abjure
abode
abound
abreast of
abrogate
abscond
abstruse
acclaim
accord
accrue, accrual
acculturation
acerbic
acuity
acumen
acute
addled

addlepated
adduce
adept
adherence
adjudicator
adjuvant
admonish
adroit
adulatory
adumbrate
aerial
aesthetics
affinity
aforementioned
agglomerate
aggrandize
aggravate
aggrieved
aghast
agog
agonistic
agrarian
akimbo
alacrity
albatross
aleatory
alias
alienate
allegorical
altruistic
amateur
ambiguous
ambivalent
ameliorate
amenable
amend
amiable
amicable
amity
amorphous
amortize
anachronism,
anachronistic
analeptic
anathema
anecdote
anemic
annihilate
anodyne
anomaly, anomalous
anomie
antagonism

antediluvian
antipathy
antiquated
antithetical to
apathy, apathetic
ape (v.)
apex
aphorism
apocalyptic
apocryphal, apocrypha
apollonian
apologia
apologist
appellant
appellations
appendage
apportion
apposed
apposite
apprenticeship
apprising
approbation
appropriate (v.)
aptitude
arbitrary
arcadian
arcane
arch (adj.)
archaic
ardor
arduous
arid
aristocratic
arrant
arresting
artful
articulate
artifact
artifice
artisan
artless
ascent
ascertain
ascetic
asinine
askance to
asperity
aspersion
assay
assent
asseveration
assiduousness

assimilation
assuage
astute
asymmetrical
atrocious
attest to
auspicious
austere
autodidact
autonomy
auxiliary
avail
avant-garde
avarice
averred
bald
balkanize
ballast
banal
baneful
banter
barbaric
barbarous
baroque
base
bask
baying
bedazzle
bedizen
bedlam
beguile
belie
belletristic
bellicose
belligerent
beneficent
benign, benignant
berate
berserk
beseech
besmirch
besotted
biased
bifurcate
bile
blacklist
blandishments
blatant
blather
bleat (v.)
blithe
bloodthirsty

blunt (verb)
bogged down
boisterous
bolster
bombastic
boom (n.)
boon
boondoggle
boor, boorish
boosterism
bootless
botch
boycott
brawl
breach
breadth
brusque
bucolic
bureaucrat
burgeon
bustling
buttress
cabal
cad
cajole
calibrated
callous
callow
calumny
candid, candor
canon
canonization
capital
capitulatory
capricious
careen
carnage
castigate
catalyst
catharsis
caustic
cavalier
celerity
censorious
censure
chagrin
champion (v.)
chary
chastity
chauvinism
chicanery
chide

chilly
choleric
chronic
chronology
chump
clamor
clandestine
claptrap
cleave to
coalesce
coarse
coda
cogent
cognitive
collide
collude, collusive
color (v.)
coltish
commercial (adj.)
communal
compendious
complaisant
complement
composure
compound (v.)
con artist
concatenate
concede
conceit
concentrated (adj.)
concessions
conciliatory
condescending
condone
confabulation
confounded,
confounding
congenial
congruent
conjectural
conjure
consanguineous
consecrate
considerable
considered (adj.)
conspiratorial
construe
contemporary,
contemporaries
contempt
contemptible
contentious

contiguous
contingent
contortion
contraband
contrapuntal
contretemps
contrite
contrivance
contrived
conundrum
conventional
conversance
convivial
convoke
convoluted
copious
cornerstone
corroborate
cosmopolitan
cosseted
costive
couched in
counterargument
counterfactual
countless
covey
crack (adj.)
crackpot
credence
credibility
credulous
crescendo
crucible
crushed
cryptic
culmination
culpable
cumbersome
cunning
curb (v.)
cursory
curt
curtail
cynical
daft
dally
dated
daunting
dearth
debacle
debase
debauchery, debauched

debunk
decipherable
decor
decry
deface
defame
defeasible
defector
defer
defile
deflect
deft
degrade
deification, deify
dejected
deleterious
delimit
delineate
deluge
delve
demarcate
dementia
demonize
denigrate
denomination
denouement
depose
depreciate
depredate
derivation
derogate
descent
desecrate
desiccated
despot
desuetude
deter
detonation
detractor
detriment
devoid of
devotee
devout
dexterous
dialect
dialectical
dicey
didactic
diffident
digress
dilapidated
dilettante

diminutive
din
dionysian
diplomatic
dire
dirge
disabuse
disaffection
disclaim
disconcerting
discount (v.)
disdain
disenfranchisement
disincentive
disinclination
disingenuous
disinterested
dispensation
disquiet
dissembling
disseminate
dissent
dissimulation
dissipate, dissipation
dissolute, dissolution
dissuade
distort
divagate
divergence
doctrinaire
doggedly
dogmatic
dotage
doting
doyenne
draconian
dramaturgy
dross
drum
dulcet
dumbfounded
dupe
duplicitous, duplicity
dwindle
dyed-in-the-wool
dynamic
dynamo
dystopian
earmark
earnest
earthy
ebb

ebullient
eccentric
eclectic
ecumenical
edifice
eerie
efface
effect (v.)
effectual
effeminate
effete
efficacious
effusive
egoist, egoistic
egregious
elicit
emblazon
emblematic
eminence
empathetic
empirical
enamored
encumbered,
encumbrances
endemic
enervate
enhance
enigmatic, enigma
enshrine
ensure
enthralled
ephemera
ephemeral
epicurean
epoch
equable
equanimity
equity
equivocal
equivocate
eremitic
erode
erratic
erroneous
ersatz
erstwhile
erudite, erudition
eschew
esoteric
espouse
esteemed
estrangement

ethos
eulogize, eulogy
evanescent
evergreen
evocative
evoke
exacerbate
excogitate
excoriate
excruciating
exculpate
executive
exegesis, exegetic
exhaustive
exorbitant
expedite
expeditious
explicate
explicit
exploits
extant
extemporaneous
extenuate
extirpate
extort, extortionate
extraneous
fabulous
facetious
facile
faction
factotum
faculties
fad, faddish
fallacious
fallacy
famine
fanatical
fantastical
farce, farcical
fascist
fastidious
fatuous
faux-naif
fawning
fealty
feasible
feckless
fecund
fester
fetid
fettle
fib

fickle
fictive
field (v.)
figurative
finite
fissile
flabbergasted
flag (v.)
flagging
flagitious
flagrant
flamboyant
flashy
flawless
fledgling
fleeting
flimsy
floe
flout
flowery
fluke
flummox
foibles
forebears
foresee
forfeit
forgery
forgo, forwent
forte
fortitude
foster
fractured
frank
frantic
fraught
fruition
full-bore
fulmination
furnish (v.)
gadfly
gaffe
gainsay
gallivant
galvanizing
garlanded
garner
garnish (v.)
gauche
genesis
genteel
germane
gibe

gilded
ginger, gingerly
gist
glamorize
glean
glib
gore
grandiloquent
grave (adj.)
gregarious
grievous
grizzled
groaning
guesswork
guileless
hackneyed
hale
hallmark
hallow
hampered
hankering
haphazard
hapless
harbingers
hedonism
hegemony, hegemonic
heinous
hemorrhage
hence
herald
heretofore
hermeneutic
heuristic
hierarchy, hierarchical
high-handed
hinder
hinterland
histrionics
hodgepodge
hoi-polloi
homage to
homely
homily
homogeneous
hone
hortatory
host of
humanize
humbling
husband (v.)
hyperbole
hypocrisy

idealistic
idealogues
ideological
idiom
idiosyncratic
idle
idyllic
ignominious
immaterial
immortalize
impartial
impasse
impeccable
impecunious
impede
impenetrable
imperil
imperious
impious
implacability
implicated, implication
implicit
importunate
improbity
impromptu
impudent
imputation
inadvertently
inane
incendiary
incensed
incentive, incentivize
inchoate
incipient
incomprehensible
incongruous
inconsequential
inconsolable
incorrigible
indefatigable
indelicate
indemnify
indeterminate
indict
indifferent
indigenous
indignant
indiscretion
indistinct
indolent
industrious
inexorable

infamy
infinitesimal
infirmity
inform
ingénue
ingenuity
ingenuous
inhibited
inimical
initially
inkling
innocuous
innuendo
inquisition
inscrutable
insidious
insipid
insofar
insolvent
insurgent
inter
interlocutor
interlude
interpolate
intransigent
intrepid
intrinsic
inundate
inured to
invalidate
investiture
iota
irascible
irate
ironclad
irony
irreconcilable
irredeemable
irredentist
irrefutable
irrelevancies
iteration, iterative
jaundiced
jejune
jeopardize
jingoistic
jocular
jouissance
joviality
jurisprudence
keen
klutzy

labile
labyrinthine
lackadaisical
lackey
laconic
lacunae
lament
lapse
lascivious
lassitude
laud
lavish
lax
layperson, layman
leery
legions of
leitmotifs
leniency
libertine, libertinous
limpid
lissome
literati
lithe
loaf (v.)
loathsome
lobbying
lofty
logistical
loquacious
lubricious
lucidity
lull
lummox
lurid
lurk
macabre
magnanimity
mainstream
makeshift
malfeasance
malice, malicious
malign
manifest
manumit
markedly
marrow
martial
masquerade
material (adj.)
maudlin
maxim
mayhem

means (n.)
meddling
melancholy
melange
mendacious
mercenary
mercurial
meretricious
messianic
metastasize
meticulous
militant
mimetic
minuscule
minutiae
mired
mirror (v.)
mirth
misanthrope,
misanthropy
miser
mishap
mishmash
misnomer
mitigate
mock
modicum
modish
mollify
monarchy
moneyed
monological
moot
moralist
mordant
mores
morose
mortifying
mote
motley
muckraking
mundane
munificent
mushroom
muster
mute, muted
mutiny, mutinous
myopic
myriad
myrmidon
nadir
nascent

naysayer
nebulous
nemesis
neologism
nominal, nominally
nonchalant
nonplussed
nostrum
notional
notorious
notwithstanding
novel
novelty
novice
noxious
nugatory
nuisance
objective, objectivity
obliging
obloquy
obsequious
obsolete
obtuse
obviate
odious
officious
oft
oligarchy
olio
ominous
omnipotent,
omnipresence
opine
opportunism
opprobrium
orthodox
ostentatious
ostracize
otherworldly
oust
outdone
outlaw
outpace
outright
outstrip
override
overrun
overt
overtake
overture
pacific
pacifist

paean
painstaking
palliating
pan (v.)
panacea
panache
pandemic
pander
panegyric
pantheon
parable
paradigm, paradigmatic
paradox, paradoxical
paragon
parameter
paramount
paramour
parlous
parochial
parody
parsimony, parsimonious
partial, partiality
partisan
parvenu
passé
pastime
patent (adj.)
pathogen
patrimony, patrimonial
patronize, patronizing
paucity
peccadillo
pecuniary
pedantic, pedantry
peddle
pell-mell
pellucid
penchant
penetrating
penitential
penumbra
pepper (v.)
peppery
perennial
perfidy
peril
perilous
peripheral
pernicious
perpetuity
perquisites
personable

perspicacious
pert
pertinent
peruse
pervade
petty
phased in
philippic
philistine
physiology
picayune
pidgins
piecemeal
piety
pilfering
pillory
pine
pious
piques
pith, pithy
placate
placid
plangent
plateau
platitude
plaudits
plenary
plethora
plum
plunder
poignant
polemic
politicize
polymath
ponderous
pontificate
populist
posit
posterity
postulate
potable
poultice
pragmatic
prattle
precedent
precocious
predicated on
premeditated
preoccupied
preposterous
prerequisites
presage

prescient
presumptuous
pretentious
preternatural
prevalence
priggish
probity
proclivity
procure
prodigal
prodigy
profane
profligate
profundity
progeny
progressive
prologue
promulgate
proponent
propound
proscribe
protagonist
protean
proverbial
providential
provincial
prowess
prude, prudish
prudent
prurient
puckish
puerile
puffery
puissant
pulchritude
pun
punctilious
pundit
puritanical
purported
putative
putsch
pyrrhic
quack
quagmire
quantum
quarrelsome
quiescent
quietus
quisling
quixotic
quotidian

rabble
ramifications
rampage
ramshackle
rancor
rapscallion
rapture
rash (adj.)
ratify
ravish
raze
reactionary
recant
recapitulate
recede
recidivist
reciprocal
recondite
recount
recrimination
rectitude
red-blooded
redoubtable
redress
redundant
refinement
regalia
regressive
reign
rejoinder
rejuvenation
reminiscences, reminisce
remorse
remuneration
rend
render
renege
renown, renowned
replicate
reportorial
republican
reputability
requisite
requite
resolute
resplendent
restive
restless
resurrect
retch
retort
retract

retrograde
retrospective
reverent
revisionist
revivification
rhetoric, rhetorical
rife with
rift
rigor
risible
risqué
robust
rote
roundly
rousing
rout
rudder
rue
ruminate
rustic
sabotage
saccharine
safeguard
sagacity
sage
said (adj.)
salubrious
salutation
salvo
sanctimonious
sangfroid
sanguinary
sanguine
sap (n.), (v.)
sardonic
sate, satiate
satire, satirical
saturnine
scanty
scapegoat
scintillating
scorn
scotch (v.)
scruples, scrupulous
scrutiny
scurvy
seasoned
sectarian
sedulous
seemly
seething
self-styled

semiotic
senescent
sentries
sequelae
seraph
sere
serendipity
shabby
shortcomings
shorthand
shrewd
shun
shutter (v.)
shyster
sidereal
similes
simpatico
simulacrum
sinuous
siren
skeptical
skitter
skulk
slack
slight (n.)
slogan
smitten
snub
sobriquets
soi-disant
solace
solecisms
solicitous
solipsistic
somber
sop
sophistry
sophomoric
spare
sparing
spartan
specious
specter
spendthrift
spooked
sporadically
spree
sprint
spurious
squalid
stalemate
stanchion

standstill
stanza
stasis
staunch
steadfast
stew (v.)
steward
stigmatize
stipulate
stoic
stolid
stonewall (v.)
stopgap
storied
strong suit
stygian
stymie
subjective
subsequent
subservient to
subside
subsidize
subterfuge
suffrage
sumptuous
superannuated
supersede
supine
supplicate
surplus
surreptitious
suspend
sweeping
swindle
sycophant
syncopated
synergetic
tacit
taciturn
tactful
tactical, tactics
tangential
tangible
tantamount to
tarry
taut
taxonomy
teeming
temperate
tenacious
tenacity
tendentious

tenet
tepid
tergiversate
tether
theses
thwart
timorous
tirade
titillation
titular
toady
toil
tome
tony
topographical
torpid, torpor
tortuous
torturous
totalitarian
touchstone
tout
tractable
trademark
traipse
traitorous
transgressor
transient
transmogrify
transmute
travesty
treachery
tremulous
trenchant
triage
tribulation
trite
trove
truculent
trumpet
trundle (v.)
tumescence
turgid
turpitude
tweak
tyrannical
ubiquitous
ululation
umbrage
unbeknownst
unbridled
unconscionable
uncouth

unctuous
undercut
undergirds
undermine
underpin
underscore
underserve
undeterminable
undue
unforeseen
unnerve
unorthodox
unpropitious
unscrupulous
unseemly
unstrung
unwavering
upend
upstanding
urbane
utilitarian
utility
utopia
vacillate
vainglorious
vanquish
vapid
variegated
vaunted
venerable, venerate
vent (v.)
veracity
verdant
versatile
vexed
viable
vice
vicissitude
vigorous
vindicate
vitiation
vitreous
vociferous
volatile
volley
voluble
vulgar
wag
wail
wallop
wallowing
wanderlust

wane
ward off
warily, wary
wax (v.)
well-heeled
wend
whet
whimsical
whitewash
wholly
wizened
wracked
wrongdoing
wry
zealot
zealous
“accepted as gospel”
“at loggerheads”
“bode well”
“circular logic”
“common cause”
“fellow feeling”
“fit the bill”
“flights of fancy”
“for naught”
“gain currency”
“given teeth”
“gloss over”
“gray area”
“hinge on”
“hold the line”
“in a jiffy”
“in cahoots”
“in lieu of”
“kept at bay”
“laid at the altar of”
“lay down arms”
“limping along”
“lose currency”
“no holds barred”
“of two minds”
“on par with”
“order of magnitude”
“paid dearly”
“part and parcel”
“rack and ruin”
“rail against”
“raking in”
“run the gamut”
“run the gauntlet”
“sea change”
“separate the wheat from

the chaff”
“single-handedly”
“stems from”
“takes pains to”
“tipping point”
“tit-for-tat”
“to send up”
“under the guise of”
“vim and vigor”
[something is] “lost on”
[someone]

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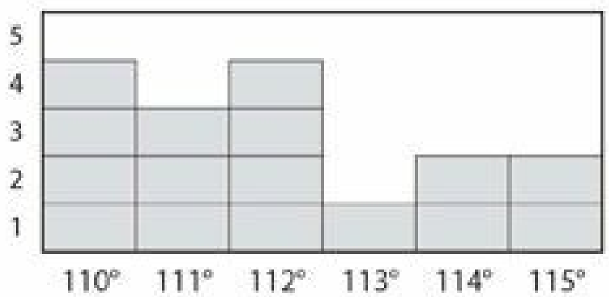
34. The bar graph below displays the number of temperature readings at each value from a sample, measured in degrees Fahrenheit. What was the average temperature reading?



[ ] degrees Fahrenheit

The answer is on page 755.

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