Quadrotors and Multicopters:
Tutorial and Review of the State of the Art

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1 Introduction

Quadrotors and multicopters, officially both "multi-rotor rotorcraft", offer significant advantages as both research platforms and fully functional unmanned air vehicles (UAVs). Given their size, they are often also called micro air vehicles or MAVs.

2 Early History

Quoting Altug et al. [2002], "The idea of using four rotors is not new. A full-scale four-rotor helicopter was built by De Bothezat in 1921" (George De Bothezat Helicopter on YouTube). In the 60's and 70's there were also two experimental aircraft, the Ling-Temco-Vought (LTV) XC-142 (XC-142 on YouTube) and the Bell X-22A (X22-A on YouTube) that flew in quadrotor configuration during take-off and landing Weingarten [2003].

The unpublished white paper by Johan Borenstein Borenstein [1993] (see also the Hoverbot webpage) essentially developed the basic control scheme on which almost all modern quadrotors are based:

> Up/down motion is easily controlled by collectively increasing or decreasing the power to all 4 motors. [...] increasing the power to the two left rotors lifts the left side up and generates a thrust component to the left. [...] adding power to the two rear rotors causes the HoverBot to fly forward. The implementation of horizontal rotation control is less obvious: [...] If one set of rotors, for example the one that turns counter-clockwise [...] increase their rotational speed or their pitch, the resultant net induced moment will cause the HoverBot to rotate clockwise."

He mentioned achieving stable hover but experiments were abandoned because of lack of funding.

A few years later, Kroo and Kunz Kroo and Kunz [2000] described the development of a centimeter-scale quadrotor, focusing mostly on the aerodynamic design and fabrication of the rotors. They mention that "a linear model of the rotor aerodynamics was developed and combined with a nonlinear simulation of the vehicle dynamics [which] suggested that the vehicle was unstable, but could be stabilized with a moderate amount of rate feedback from a MEMS gyro."

3 Major Research Efforts

Research in modeling and controlling quadrotors was picked up again at the University of Pennsylvania by Altug, Ostrowski, and Mahony [2002], Altug, Ostrowski, and Taylor [2003, 2005], and at the Australian National University by Hamel, Mahony, Lozano, and Ostrowski [2002] and Pounds, Mahony, Hynes, and Roberts [2002], Pounds, Mahony, and Corke [2006, 2010].

Very shortly thereafter, this also became a major research effort at ETH, spearheaded by Bouabdallah and Siegwart Bouabdallah et al. [2004a,b, 2007], Bouabdallah [2007], Bouabdallah and Siegwart [2005, 2007], at Rogelio Lozano's group at CNRS Castillo et al. [2004, 2005], Romero et al. [2006, 2007], and at Claire Tomlin's group at Stanford Hoffmann et al. [2004, 2007], Aswani et al. [2012], Bouffard et al. [2012], Gillula and Tomlin [2012].

Since 2010, the GRASP lab at the University of Pennsylvania, led by Vijay Kumar Michael et al. [2010], Mellinger and Kumar [2010], Mellinger et al. [2010], Mellinger and Kumar [2011], Mellinger et al. [2011], Shen et al. [2011], Mahony et al. [2012], Mellinger et al. [2012], Powers et al. [2012], Michael et al. [2012], Shen et al. [2013a] and several labs at ETH, that of Siegwart Blosch et al. [2010], Achtelik et al. [2011b,a], Kneip et al. [2011], Weiss and Siegwart [2011], Achtelik et al. [2012b], Weiss et al. [2012b,a], Alonso-Mora et al. [2012], Achtelik et al. [2012a], Leutenegger and Siegwart [2012], Achtelik et al. [2013], Nikolic et al. [2013] and D'Andrea Lupashin et al. [2010], Lupashin and D'Andrea [2011], Oung et al. [2012], Ritz et al. [2012] have shown impressive advances in navigation, control, and autonomy.

4 Kinematics

The kinematics of quadrotors and multicopters are those of simple rigid 3D bodies. Many different coordinate frame conventions are used in the literature. I introduce one good choice below, and convert equations from the literature to this convention where needed.

The kinematics equations are most useful for navigation and control when expressed in the **navigation frame** $\mathcal{N}$, which for MAV applications is almost universally assumed to be non-rotating and aligned with gravity, but can otherwise be defined arbitrarily, e.g., North-East-Down (NED) or East-North-Up (ENU). The origin of the navigation frame is often chosen as the take-off point, but does not have to be.

We also define a **body frame** $\mathcal{B}$ as having its origin at the center of mass of the vehicle. Following convention in aerospace applications, we fix the the x-axis as pointing to the front of the vehicle (not always the direction of travel), the y-axis as pointing to the right, and the z-axis pointing down.

We then define, respectively,

- the vehicle's **position** $\mathbf{r}^n$,
- its linear **velocity** $\mathbf{v}^n$,
- the **attitude** $\mathbf{R} \triangleq [\mathbf{i}_b^n, \mathbf{j}_b^n, \mathbf{k}_b^n] \in SO(3)$, a 3×3 rotation matrix from $\mathcal{B}$ to $\mathcal{N}$,
- the **body angular velocity** ω^b.

Above the superscript n and b denote quantities expressed in the navigation and body frame, respectively.

Below I develop all attitude kinematics and dynamics directly on $SO(3)$, following Murray et al. [1994]. In particular, the vehicle's kinematics are given by

$$\dot{\mathbf{r}}^n = \mathbf{v}^n \quad (1)$$

$$\dot{\mathbf{R}} = \mathbf{R}\hat{\omega}^b \quad (2)$$

where the 3×3 skew-symmetric matrix $\hat{\omega}^b \triangleq \mathbf{R}^T\dot{\mathbf{R}}$ is obtained from the 3-vector ω^b as the as follows:

$$\hat{\omega}^b \triangleq \begin{bmatrix} & -\omega_z^b & \omega_y^b \\ \omega_z^b & & -\omega_x^b \\ -\omega_y^b & \omega_x^b & \end{bmatrix}.$$

Note $\hat{\omega}^b \in \mathfrak{so}(3)$, the Lie algebra associated with the 3D rotation group $SO(3)$. Also, the kinematics can also expressed using the *spatial* angular velocity $\omega^n \triangleq \mathbf{R}\omega^b$, as follows:

$$\dot{\mathbf{R}} \;=\; \hat{\omega}^n \mathbf{R}$$

However, it is much easier to visualize and calculate the angular velocity in the body frame (see Section 5).

4.1 DCMs, Quaternions, and Euler Angles

Euler-angle representation are very popular in the literature, but 3×3 rotation matrices (sometimes called discrete cosine matrices of DCMs) are a cleaner and singularity-free representation for vehicle attitude $\mathbf{R}$, in addition to being the native $SO(3)$ representation. In addition, because of the homorphism from $SO(3)$ to the group of unit quaternions, any $SO(3)$ based equations can easily be efficiently implemented using unit quaternions without any change in semantics (see e.g. Mahony et al. [2008], Lee et al. [2010b]).

The equivalent of the attitude kinematics (Equation 2) for unit quaternions is

$$\dot{q} = \frac{1}{2} q \cdot \left(0, \omega^b\right)$$

where $\left(0, \omega^b\right)$ is a pure quaternion corresponding to the angular velocity ω^b, and the operation above is quaternion multiplication:

$$(s, \mathbf{v}) \cdot (t, \mathbf{w}) = (st - \mathbf{v}^T \mathbf{w}, s\mathbf{w} + t\mathbf{v} + \mathbf{v} \times \mathbf{w})$$

If required by the application, it is easy to write down the kinematics using Euler angles as well. The most common approach defines the variables roll ϕ, pitch θ, and yaw ψ, for which

$$\mathbf{R}(\phi, \theta, \psi) = \mathbf{R}_z(\psi)\mathbf{R}_y(\theta)\mathbf{R}_x(\phi)$$

These Euler angles can be integrated using (see e.g. Beard and McLain [2012]) the following differential equation:

$$\begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix} = \begin{pmatrix} 1 & \sin\phi \tan\theta & \cos\phi \tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \frac{\sin\phi}{\cos\theta} & \frac{\cos\phi}{\cos\theta} \end{pmatrix} \omega^b = J(\phi, \theta)\omega^b$$

Above the 3×3 matrix $J(\phi, \theta)$ is *not* a rotation matrix but a Jacobian matrix to convert angular rates in the body frame into Euler angle rates. Note that, around hover, this Jacobian is equal to the identity matrix.

Roll, pitch, and yaw are useful concepts underlying much common-sense intuition about flight, and can be used regardless of which representation is used. As shown above, as $J(\phi, \theta)$ is identity around hover, one can still refer to the body angular velocities $\omega^b \triangleq [p, q, r]^T$ as roll, pitch, and yaw rates, even when using an $SO(3)$ representation. This is exact around hover, and even far away from hover we can think of rolling *relative* to the body frame, so the terminology remains useful even in that case.

4.1.1 Simulation aka Forward Integrating

If time histories $\mathbf{v}^n(t)$ and $\omega^b(t)$ for the linear and angular velocities are available, the vehicle's position $\mathbf{r}^n(t)$ and attitude $\mathbf{R}(t)$ can be integrated by Equations 1 and 2. The translation can easily be done using any integration scheme. The attitude part can be done in discrete time by

$$\mathbf{R}_{k+1} \approx \mathbf{R}_k e^{h\hat{\omega}^b} \tag{3}$$

where h is the sample time and $\exp\left(h\hat{\omega}^b\right)$ is the exponential map from $\mathfrak{so}(3)$ to $SO(3)$. This assumes ω^b is constant over the sample period, and hence this is equivalent to Euler's method for forward integration. Since $\exp\left(h\hat{\omega}^b\right) \approx I + h\hat{\omega}^b$ for small h, we can also approximate Equation 3 by

$$\mathbf{R}_{k+1} \approx \mathbf{R}_k \left(I + h\hat{\omega}^b\right) = \mathbf{R}_k \begin{bmatrix} 1 & -h\omega_z^b & h\omega_y^b \\ h\omega_z^b & 1 & -h\omega_x^b \\ -h\omega_y^b & h\omega_x^b & 1 \end{bmatrix}$$

but this will require re-normalizing to ensure that $\mathbf{R}_{k+1} \in SO(3)$.

5 Dynamics

5.1 Newton-Euler Equations

The *translational* dynamics are best expressed in the navigation frame $\mathcal{N}$. As we have assumed it non-rotating, in what follows the so-called Coriolis effect due to Earth's rotation is neglected, as is valid for MAVs [Beard and McLain, 2012]. The *attitude* dynamics are best expressed in the body frame $\mathcal{B}$, which aligns with moments and forces generated by the rotors.

We define, respectively,

- the vehicle's **mass** m,
- the net **force** $\mathbf{F}^n$ acting on the center of mass,
- the vehicle's **inertia tensor** $\mathbf{I}$, calculated in the body frame and assumed fixed
- the net **torque** τ^b

The **Newton-Euler equations** governing the vehicle's dynamics are then given by Murray et al. [1994]:

$$m\dot{\mathbf{v}}^n = \mathbf{F}^n \tag{4}$$

$$\mathbf{I}\,\dot{\omega}^b = \tau^b - \omega^b \times \mathbf{I}\omega^b \tag{5}$$

The last term, $-\omega^b \times \mathbf{I}\omega^b$, is due to gyroscopic effects , which are typically small except in extreme maneuvers. Euler's equations simplify, however, when the inertia tensor is examined more closely.

5.2 The Inertia Tensor

In general, the inertia tensor $\mathbf{I}$ in a given frame is a 3×3 matrix

$$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

where $I_{xx} = \int (y^2 + z^2) dm$, $I_{xy} = -\int (xy) dm$, and the other terms are similarly computed.

However, most multi-rotor rotorcraft have symmetries that restrict the form of the inertia tensor $\mathbf{I}$ in the body frame. In particular, we have the following properties:

- If a rigid body, like most quadrotors (and other airframes in fact, e.g., fixed-wing), displays reflective symmetry about the xz-plane (in the body frame), then the inertia tensor is of the form

$$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & I_{xz} \\ 0 & I_{yy} & 0 \\ I_{zx} & 0 & I_{zz} \end{bmatrix}$$

- If in addition, if there is reflective symmetry about the xy-plane or yz-plane, the inertia tensor becomes *diagonal*. This holds approximately for many platforms, and is almost universally assumed.

- Note that there is *always* a coordinate frame in which $\mathbf{I}$ is diagonal, but it might not coincide with the body frame. Hence, the statements above are about symmetries with respect to the body frame. One case where these symmetries are broken are when sensors are added one one side but not another.

Reference	Model	m	I_{xx}	I_{yy}	I_{zz}
Tayebi and McGilvray [2006]	DraganFlyer III	468 g	$4.9\,\mathrm{g}\cdot\mathrm{m}^2$	$4.9\,\mathrm{g}\cdot\mathrm{m}^2$	$8.8\,\mathrm{g}\cdot\mathrm{m}^2$
Pounds et al. [2006]	ANU X-4 Flyer	4.34 kg	$82\,\mathrm{g}\cdot\mathrm{m}^2$	$84.5\,\mathrm{g}\cdot\mathrm{m}^2$	$137.7\,\mathrm{g}\cdot\mathrm{m}^2$
Bouabdallah [2007]	OS4	650 g	$7.5\,\mathrm{g}\cdot\mathrm{m}^2$	$7.5\,\mathrm{g}\cdot\mathrm{m}^2$	$13\,\mathrm{g}\cdot\mathrm{m}^2$

Table 1: Parameters given for a number of experimental research platforms.

Some typical values for the mass and moments of inertia are given in Table 1 for various platforms.

It is clear that in many cases $I_{xx} \approx I_{yy}$, which stems from yet another symmetry. To explain this, we can explicitly calculate $\mathbf{I}$ for a simplified model. Beard [2008] gives an approximate calculation for the moments of inertia of a simple, symmetric quadrotor with motors at a length l from the CG. By treating the motor/rotor assemblies as point masses m and the body as a solid sphere with mass M and radius R, he obtains

$$\begin{aligned} I_{xx} = I_{yy} &= \frac{2}{5}MR^2 + 2ml^2 \\ I_{zz} &= \frac{2}{5}MR^2 + 4ml^2 \end{aligned}$$

We can generalize this to n rotors equally spaced in angle, by noting that

$$\sum_{j=1}^{n} \cos^2\left(j\frac{2\pi}{n}\right) = \sum_{j=1}^{n} \sin^2\left(j\frac{2\pi}{n}\right) = \frac{n}{2}$$

which yields

$$\begin{aligned} I_{xx} = I_{yy} &= \frac{2}{5}MR^2 + \frac{n}{2}ml^2 \\ I_{zz} &= \frac{2}{5}MR^2 + nml^2 \end{aligned}$$

The above, that $I_{xx} = I_{yy}$ for symmetric multi-rotor craft even when $n > 4$, and *even* for n odd, is not a coincidence and does not depend on the point-mass approximation. In fact, for any rigid body with n-fold *rotational* symmetry around the z-axis, with $n > 2$, the inertia tensor $\mathbf{I}$ is diagonal and with $I_{xx} = I_{yy}$. Clearly, this is approximately the case for many symmetrically built tri-rotors, quadrotors, hexacopters, etc...

5.3 Simplified Attitude Dynamics

A diagonal inertia tensor $\mathbf{I}$ allows us to simplify the Euler equations as:

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} l/I_{xx} \\ m/I_{yy} \\ n/I_{zz} \end{pmatrix} - \begin{pmatrix} qr\left(I_{zz} - I_{yy}\right)/I_{xx} \\ pr\left(I_{xx} - I_{zz}\right)/I_{yy} \\ pq\left(I_{yy} - I_{xx}\right)/I_{zz} \end{pmatrix}$$

where we defined the scalar components of the angular velocity $\omega^b \triangleq [p, q, r]^T$ and net torque $\tau^b \triangleq [l, m, n]$ as in Beard and McLain [2012]. It is clear that the moments of inertia indicate the resistance to roll, pitch, and yaw moments, respectively.

In the case that $I_{xx} = I_{yy}$, the Euler equations become even simpler:

$$\begin{pmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} l/I_{xx} \\ m/I_{yy} \\ n/I_{zz} \end{pmatrix} - \begin{pmatrix} qr\,(I_{zz}/I_{xx} - 1) \\ pr\,(1 - I_{zz}/I_{yy}) \\ 0 \end{pmatrix}$$

and we note that the so-called *body gyro effect no longer plays a role in yaw dynamics.* The yaw-rate r does affect roll and pitch, however, and the effect is non-negligible in maneuvers with large yaw rates in conjunction with large rotations outside the plane formed by the rotors.

5.4 Forces

For the force, Castillo et al. [2005] has

$$\mathbf{F}^n = m\mathbf{g}^n - \mathbf{k}_b^n \sum \mathbf{f}_i$$

where $\mathbf{k}_b^n$ is the body z-axis, and $\mathbf{f}_i = k_i w_i^2$ is the thrust produced by motor i, with k_i a constants and w_i the angular speed of the motor.

Thrust is developed in detail in Altug et al. [2005]:

$$\mathbf{f}_i = abc\frac{\rho}{4} w_i^2 R^3 (\theta_t - \phi_t)$$

where "a is the slope of the airfoil lift curve, b is the number of blades on a rotor, c is the lift coefficient, ρ is the air density, R is the rotor radius, θ_t is the pitch at the blade tip, and ϕ_t is the inflow angle at the tip." Setting $\phi_t = 0$ ignores the change in direction of the airflow due to the motion of the quadrotor, and assuming all other coefficients are constant yields $\mathbf{f}_i = kw_i^2$, although

Altug et al. [2005] work in the body frame:

$$\mathbf{F}^b = m\mathbf{R}\mathbf{g}^n - \sum \mathbf{f}_i - \mathbf{f}_{drag}$$

where $\mathbf{f}_{drag}$ is a drag force proportional to the squared velocity $\left\|\mathbf{v}^b\right\|^2$.

5.5 Torques

For the torques, they use

$$\tau^b = \begin{pmatrix} (f_3 - f_1)l \\ (f_2 - f_4)l \\ \sum \tau_i \end{pmatrix}$$

where l is the half the distance between the rotor centers. This is for a "+" configuration with motor 1 on the right and number CCW. For an "x" configuration with motor 1 forward left and numbered CW, we obtain

$$\tau^b = \begin{pmatrix} (f_1 - f_2 - f_3 + f_4)l \\ (f_1 + f_2 - f_3 - f_4)l \\ \sum \tau_i \end{pmatrix}$$

In both cases, and generalizing to multi-rotor Rotorcraft we can write

$$\tau^b = \mathbf{M}\mathbf{f}$$

with $\mathbf{M}$ an $3 \times n$ mixing matrix.

6 Chronological

6.1 Control/PVTOL

- Isidori et al. [1981] "Nonlinear decoupling via feedback: A differential geometric approach". The seminal paper on feedback linearization?
- Teel92scl "nested saturation control strategy" (Castillo05book)
- Teel96tac "nested saturation control strategy discussed for general nonlinear systems, including the PVTOL" (Castillo05book)
- Hauser et al. [1992] "Nonlinear control design for slightly non-minimum phase systems: Application to V/STOL aircraft". Castillo et al. [2005] say "dynamic model of a PVTOL." Hua et al. [2013] say "Early work [22], based on exact input-output linearization [31], has focused on the control of a planar vertical takeoff and landing (PVTOL) aircraft." Very nice explanation and illustration of feedback linearization (without the Lie derivatives) and the undesirable zero dynamics. These arise when parasitic translation-rotation coupling are modeled but cause exact FBL to fly based on those. Instead, a new controller is derived by neglecting the parasitic effects, which then arrives at the intuitive "thrust vectoring" control. Finally, a third controller is a slight modification to improve altitude tracking, although it requires knowledge of the coupling coefficient. Finally, these controllers necessitate controlling the second derivative of thrust, which has problems: Hua et al. [2013] say "For rotary-wing VTOL vehicles, such as ducted-fans and quadrotors, the thrust force is generated by propeller(s) and T is a function of the associated motor's angular velocity [...]. If, instead of this velocity, $\ddot{T}$ is used as a control variable this means that the motor's jerk becomes a control input which has to be monitored. This in turn induces serious complications."
- Martin94cdd "A different look at output tracking: control of a VTOL aircraft" (Altug02icra)
- **?** "Combined Feedback Linearization and Constrained Model Predictive Control for Entry Flight". Very interesting application of feedback linearization (FBL), coupled with MPC rather than PID as in their earlier work. Results are shown for the X-38 Crew Return Vehicle, in simulation, and show that the key advantage of MPC is that it knows about input and state constraints, and ensures the FBL stays valid. In addition, the control for the MPC never saturates, and the control is much smoother.
- Nielsen et al. [2010] "Path following using transverse feedback linearization: Application to a maglev positioning system".
- Bijnens et al. [2012] "Adaptive feedback linearization flight control for a helicopter UAV". Definitely intriguing, and there is some nice explanation of feedback linearization as well as *dynamic inversion*: given a system in affine form
$$\dot{x} = f(x) + g(x)u$$
we can linearize as follows:
$$u = [g(x)]^{-1}[\dot{x} - f(x)].$$
However, the application of these ideas in the rest of the paper leaves somewhat to be desired.

6.2 Multirotor History

"The idea of using four rotors is not new. A full-scale four-rotor helicopter was built by De Bothezat in 1921" (Altug02icra), see also this Youtube video

XC-142 Youtube video
Bell X-22A Youtube video and this webpage.

1. Beilman72ahs "An Integrated System of Airborne and Ground-Based Instrumentation for Flying Qualities Research with the X-22A Airplane", The 28th Annual National Forum of the American Helicopter Society, May 1972. "In hovering flight (Fig. 4), the X-22A employs fore and aft differential blade pitch for pitching moments, left and right differential blade pitch for rolling moments, left and right differential elevon deflection for yawing moments."

2. Smith76report "FLIGHT INVESTIGATION OF LATERAL DIRECTIONAL FLYING QUALITIES AND CONTROL POWER REQUIREMENTS FOR STOL LANDING APPROACH USING THE X-22A AIRCRAFT", "Briefly, the X-22A is a four-ducted-propeller V/ST0L aircraft with the capability of full transition between hover and forward flight. The four ducts are interconnected and can be rotated to change the duct angle and therefore the direction of the thrust vector to achieve the desired operating flight condition defined by a particular speed and duct angle combination. The thrust magnitude is determined by a collective pitch lever, very similar to a helicopter. Normal aircraft-type pitch, roll end yaw controls in the cockpit provide the desired control moments by differentially positioning the appropriate controls in each duct (propeller pitch and/or elevon deflection). A mechanical mixer directs and proportions the pilot's commands to the appropriate propellers and elevons as a function of the duct angle."

3. Duncan90thesis "Low-range Airspeed Sensors", discusses the LORAS system used in the X-22, among others.

4. Radford81report "AN EXPERIMENTAL INVESTIGATION OF VTOL FLYING QUALITIES REQUIREMENTS FOR SHIPBOARD LANDINGS", experiments with the X-22A.

5. Borenstein93unpublished "The HoverBot C An Electrically Powered Flying Robot", see also the Hoverbot webpage, developed the basic control scheme: "Up/down motion is easily controlled by collectively increasing or decreasing the power to all 4 motors. [...] increasing the power to the two left rotors lifts the left side up and generates a thrust component to the left. [...] adding power to the two rear rotors causes the HoverBot to fly forward. The implementation of horizontal rotation control is less obvious: [...] If one set of rotors, for example the one that turns counter-clockwise [...] increase their rotational speed or their pitch, the resultant net induced moment will cause the HoverBot to rotate clockwise." Unpublished White Paper. ftp://ftp.eecs.umich.edu/people/johannb/paper99.pdf

6. Kroo00vlad "Development of the Mesicopter: A Miniature Autonomous Rotorcraft", describes the development of a centimeter-scale quadrotor rotorcraft, focusing mostly on the aerodynamic design and fabrication of the rotors. They mention that "a linear model of the rotor aerodynamics was developed and combined with a nonlinear simulation of the vehicle dynamics [which] suggested that the vehicle was unstable, but could be stabilized with a moderate amount of rate feedback from a MEMS gyro."

7. Weingarten [2003] "HISTORY OF IN-FLIGHT SIMULATION & FLYING QUALITIES RESEARCH AT CALSPAN", discusses X-22, a "dual-tandem, tilting-ducted-propeller V/STOL" and its unique "very accurate low-airspeed sensor that was good down to vectorial-zero airspeed. This patented system was called LORAS (Linear Omnidirectional Resolving Airspeed System). The primary sensing device of LORAS was a differential pressure gauge in the hub of rotating arm which sensed any differential pressure between the tips of the arm (Figure 11). Each tip sensed its tangential rotational speed plus or minus a component of the relative wind or airspeed. The differential pressure – the

measure of airspeed – was very accurate down to very low magnitudes. Due to the rotating nature of the system, the output was a signwave whose phasing represented the direction of the relative wind or airspeed. Two airspeed sensors were used: one mounted on the vertical tail rotated about the z-axis to give x-y speeds, the other mounted on a nose boom rotated about the y-axis that yielded x-z speeds."

6.3 Multirotor Research Last Decade

1. Altug et al. [2002, 2003], Altug and Taylor [2004], Altug et al. [2005] "Control of a Quadrotor Helicopter Using Visual Feedback" and "Dual Camera Visual Feedback", "proposed a control algorithm to stabilize the quad-rotor using vision as the principal sensor. They studied two methods, the first uses a control algorithm of linearization and the other uses the technique of backstepping. They have tested the control laws in simulation. They also present an experience using vision to measure the yaw angle and the altitude." Castillo et al. [2005], Euler angles, **model drag coefficients in both Newton-Euler:**

$$\begin{aligned}
\ddot{x} &= \frac{(\sum_{i=1}^{4} F_i)(\cos\phi \sin\theta \cos\psi + \sin\phi \sin\psi) - K_1\dot{x}}{m} \\
\ddot{y} &= \frac{(\sum_{i=1}^{4} F_i)(\sin\phi \sin\theta \cos\psi - \cos\phi \sin\psi) - K_2\dot{y}}{m} \\
\ddot{z} &= \frac{(\sum_{i=1}^{4} F_i)(\cos\phi \cos\psi) - mg - K_3\dot{z}}{m} \qquad (2) \\
\ddot{\theta} &= l(-F_1 - F_2 + F_3 + F_4 - K_4\dot{\theta})/J_1 \\
\ddot{\psi} &= l(-F_1 + F_2 + F_3 - F_4 - K_5\dot{\psi})/J_2 \\
\ddot{\phi} &= (M_1 - M_2 + M_3 - M_4 - K_6\dot{\phi})/J_3
\end{aligned}$$

Note non-standard Euler angles, and the paper is full of typos! However, they then assume all zero, diagonal Inertia tensor, no Coriolis. They define inputs using mass and inertial moments:

$$\begin{aligned}
u_1 &= (F_1 + F_2 + F_3 + F_4)/m \\
u_2 &= (-F_1 - F_2 + F_3 + F_4)/J_1 \\
u_3 &= (-F_1 + F_2 + F_3 - F_4)/J_2 \\
u_4 &= C(F_1 - F_2 + F_3 - F_4)/J_3.
\end{aligned}$$

Has a hybrid controller with "hover" as the central mode. Develops both nonlinear controller and a backstepping controller for pitch and roll. E.g., for lateral y/roll ψ dynamics, they note that with $u_1 = 1$,$\phi = \theta = 0$ we can treat the UAV as a double integrator (ignores drag):

$$\ddot{y} = -\sin\psi$$

To drive y to zero, they use a simple PD controller where $(-\sin\psi)$ is considered the control input:

$$-\sin\psi = -K_p y - K_d\dot{y}$$

This in turn is then controlled using a (faster) PD controller:

$$u_2 = K_{p1}(\psi_d - \psi) + K_{d1}(\dot{\psi}_d - \dot{\psi})$$

where the desired roll angle ψ_d (and its derivative) is computed to satisfy $\psi_d = \arcsin(K_p y + K_d\dot{y})$. The backstepping controller is more involved and is given as is, referencing Sastry99book. A single camera and later a dual-camera system is then used as a mocap device in the experiments, which show stable hovering.

2. Hamel et al. [2002] "Dynamic modelling and configuration stabilization for an X4-Flyer". Describes a quaternion-based attitude controller. Unusual in that they also address individual rotor speed control. No experiments, purely theory.

3. Pounds et al. [2002] "conceived and developed a control algorithm for a prototype of an aerial vehicle having four rotors. They considered using an MIU (Measurement Inertial Unit) to measure the speed and the angular acceleration. They use a linearization of the dynamic model to conceive the control algorithm. The results of the control law have been tested in simulation." Castillo et al. [2005] A more detailed dynamic model discusses air velocity effects but states they approximately cancel out. Control is discussed but a bit confusing, and flight is not yet achieved.

4. Lozano et al. [2004], Castillo et al. [2004] "Global stabilization of the PVTOL: real-time application to a mini-aircraft" and "Real-Time Stabilization and Tracking of a Four-Rotor Mini Rotorcraft", design a control for *planar* VTOL based on PID for altitude and a "nested saturation" cascade of controllers to bound horizontal displacement and pitch. Experiments are done with a pilot in the loop for roll and yaw, Polhemus tracker for estimation. The latter paper develops a system model from the Lagrangian, a bit overkill, and adds yaw and roll control. Claim that, "to the best of our knowledge, this is the first successful real-time control applied to a four-rotor rotorcraft," although Altug was also doing this at the time, I think.

5. Nice [2004], "Design of a four rotor hovering vehicle", Lee10tac says "Linear control systems such as proportional-derivative controllers or linear quadratic regulators are widely used to enhance the stability properties of an equilibrium". A student of Raffaello D'Andrea while he was still at Cornell, designed the hardware and did some hover tests, but not big on theory/control/estimation.

6. McKerrow [2004] "Modelling the Draganflyer four-rotor helicopter" Detailed description of moments of inertia calculation (5 masses), and discussion of gyroscopic torques and Coriolis acceleration. Developed a flight simulator.

7. Bouabdallah et al. [2004b] "PID vs LQ Control Techniques Applied to an Indoor Micro Quadrotor"

8. Hoffmann et al. [2004] "THE STANFORD TESTBED OF AUTONOMOUS ROTORCRAFT FOR MULTI AGENT CONTROL (STARMAC)"

9. **Castillo et al. [2005] "Modelling and Control of Mini-Flying Machines", nice history, simple PVTOL controller,**

10. Bouabdallah05ar, "Towards autonomous indoor micro VTOL", Lee10tac says "Linear control systems such as proportional-derivative controllers or linear quadratic regulators are widely used to enhance the stability properties of an equilibrium"

11. Castillo05csm, "Stabilization of a mini rotorcraft with four rotors", Lee10tac says "Linear control systems such as proportional-derivative controllers or linear quadratic regulators are widely used to enhance the stability properties of an equilibrium"

12. Bouabdallah05icra, "Backstepping and Sliding-mode Techniques Applied to an Indoor Micro Quadrotor", Lee10tac says "Backstepping and sliding mode techniques are applied"

13. Guenard05icca ""Dynamic modeling and intuitive control strategy for an "X4-flyer",

14. Tayebi and McGilvray [2006] "Attitude stabilization of a VTOL quadrotor aircraft"

15. Cowling et al. [2010] "Direct Method Based Control System for an Autonomous Quadrotor"

16. Pounds et al. [2006] “Modelling and Control of a Quad-Rotor Robot” Has some good data on pitch and roll damping.

17. Romero et al. [2006] “Stabilization and location of a four rotor helicopter applying vision”

18. Madani and Benallegue [2006] “Backstepping Control for a Quadrotor Helicopter”

19. Benallegue et al. [2006] “Feedback Linearization and High Order Sliding Mode Observer For A Quadrotor UAV”

20. Cowling et al. [2006] “MBPC for Autonomous Operation of a Quadrotor Air Vehicle”

21. Valenti06aiaa, “Indoor Multi-Vehicle Flight Testbed for Fault Detection, Isolation, and Recovery”, Lee10tac says “Linear control systems such as proportional-derivative controllers or linear quadratic regulators are widely used to enhance the stability properties of an equilibrium”

22. Bouabdallah [2007] “Design and control of quadrotors with application to autonomous flying” Detailed model (too scalar for my taste, uses Euler angles, too).

23. Guenard et al. [2007, 2008] “A Practical Visual Servo Control for an Unmanned Aerial Vehicle”. Servos to a particular visual target, fairly complex control math, uses same high-gain idea and only talks about position control.

24. Bouabdallah and Siegwart [2007] “Full Control of a Quadrotor” Makes (fairly trivial) two sub-system statement: “One can ideally imagine the overall system described by (12) as constituted of two subsystems, the angular rotations and the linear translations.”

25. Romero et al. [2007] “Modelling and real-time control stabilization of a new VTOL aircraft with eight rotors”

26. Cowling et al. [2007] “A Prototype of an Autonomous Controller for a Quadrotor UAV”

27. Hoffmann et al. [2007] “Quadrotor helicopter flight dynamics and control: Theory and experiment” Highly cited. Tomlin’s group. Has load cell for individual propeller. Three separate aerodynamic effects are investigated as they pertain to quadrotor flight, due to vehicular velocity, angle of attack, and airframe design. STARMAC II vehicle. Lee10tac says “Linear control systems such as proportional-derivative controllers or linear quadratic regulators are widely used to enhance the stability properties of an equilibrium”

28. Cowling [2008] “Towards Autonomy of a Quadrotor UAV” Has good related work/intro on differential flatness, trajectory generation, MBPC, and Tarnenko’s method.

29. Mahony et al. [2008] “Nonlinear Complementary Filters on the Special Orthogonal Group”

30. He et al. [2008] ”Planning in Information Space for a Quadrotor Helicopter in a GPS-denied Environment”. Hokuyo localization with known map.

31. Herissé et al. [2008, 2010, 2012] “Landing a VTOL Unmanned Aerial Vehicle on a Moving Platform Using Optical Flow”. Very interesting decoupling of translation and orientation dynamics: “For the orientation dynamics of (3) and (4), a high-gain controller is used to ensure that the orientation R of the UAV converges to the desired orientation Rd. The resulting control problem is simplified to [translation dynamics only].” Blosch et al. [2010] say “use an optical flow based PI-controller to stabilize a hovering MAV, they also implemented an automatic landing routine by 2 contemplating the divergent optical flow.”

32. Raffo et al. [2008, 2010] "Backstepping/nonlinear $\mathcal{H}_\infty$ control for path tracking of a quadrotor unmanned aerial vehicle" Extensive review of control. Quoted by Alexis et al. [2011b] as example of neglecting aerodynamic effects.

33. Angeletti et al. [2008] "Autonomous Indoor Hovering with a Quadrotor". Use Hokuyo for hovering in place.

34. **Beard [2008] "Quadrotor Dynamics and Control"**

35. Ahrens [2008], Ahrens et al. [2009] "Vision-Based Guidance and Control of a Hovering Vehicle in Unknown, GPS-denied Environments", uses monocular features to estimate a local map based on pose filter and most recent measurements (older measurements are gradually forgotten). Has some obstacle avoidance heuristics. Blosch et al. [2010] say "An approach with offboard vehicle tracking equipment was implemented by Ahrens et al. Based on the visual SLAM algorithm of Davison et al., they build a localization and mapping framework that is able to provide an almost drift-free pose estimation. With that they implemented a very efficient position controller and obstacle avoidance framework. However, due to the simplification they used in their feature tracking algorithm a non-negligible drift persists. Also, they used an external Vicon localization system to control the aerial vehicle with millimeter precision (a system of external cameras that tracks the 3D pose of the vehicle). So far, they did not use the output of the visual SLAM based localization system for controlling the vehicle."

36. Bourquardez et al. [2009] "Image-based visual servo control of the translation kinematics of a quadrotor aerial vehicle".

37. Bachrach et al. [2009, 2011] "Autonomous flight in unknown indoor environments". Laser-based, mirror in laser fan for height. Fast scan matching, iSAM or gMapping, 2D. Journal paper has excellent related work, and impressive experimental results.

38. Das et al. [2009] "Backstepping Approach for Controlling a Quadrotor Using Lagrange Form Dynamics". They talk about the Lagrangian form of the dynamics, but the formulas they give (7 and 8, in the paper) are pretty much identical to everyone else's. The rest of the paper describes a backstepping controller with some neural network components.

39. Grzonka et al. [2009] "Towards a navigation system for autonomous indoor flying". Localization using Hokuyo in known map, limited SLAM evaluation.

40. Huang et al. [2009] "Aerodynamics and control of autonomous quadrotor helicopters in aggressive maneuvering". Referenced by Mahony et al. [2012] for aerodynamics.

41. Lange et al. [2009] "A Vision Based Onboard Approach for Landing and Position Control of an Autonomous Multirotor UAV in GPS-Denied Environments"

42. Bristeau et al. [2009] "Stabilization and location of a four rotor helicopter applying vision"

43. Lee et al. [2009] "Robust tracking control of an underactuated quadrotor aerial-robot based on a parametric uncertain model".

44. Voos [2009] "Nonlinear Control of a Quadrotor Micro-UAV using Feedback-Linearization"

45. Tarhan and Altu [2009], "Control of a Quadrotor Air Vehicle by Vanishing Points in Catadioptric Images"

46. Pounds et al. [2010] "Modelling and control of a large quadrotor robot"

47. Lee et al. [2010a,b] "Control of Complex Maneuvers for a Quadrotor UAV using Geometric Methods on SE(3)" Three flight modes: attitude, position, velocity. First, they describe a nonlinear attitude controller that tracks desired rotation $\mathbf{R}$, desired angular velocity ω_d^b , and desired angular acceleration $\dot{\omega}_d^b$, which still allows for, as an example, maintaining a desired altitude via conventional PID control of total thrust. The second flight tracks track a desired $\mathbf{r}_d^n(t)$ by converting a PID command to into a computed attitude $\mathbf{R}_c$. There is still some freedom to choose a desired body direction in that computation, which constitutes the fourth desired output. Finally, the final velocity-controlled mode has a similar structure. Simulation experiments are shown.

48. Michael et al. [2010] "The GRASP Multiple Micro UAV Testbed"

49. Alexis et al. [2010b] "Design and Experimental Verification of a Constrained Finite Time Optimal Control Scheme for the Attitude Control of a Quadrotor Helicopter Subject to Wind Gusts"

50. Blosch et al. [2010], Weiss et al. [2010, 2011] "Vision Based MAV Navigation in Unknown and Unstructured Environments" and "Monocular-SLAM–Based Navigation for Autonomous Micro Helicopters in GPS-Denied Environments". Bachrach et al. [2011] say "extended monocular vision SLAM to a quadrotor helicopter using lower quality acceleration estimates from a more realistic MAV-scale IMU, but this work also uses a downward pointing camera and makes strong assumptions about the environment." Use PTAM from Klein and Murray [2007]. Quote: "To the best of our knowledge, this is the first implementation of a vision-based MAV controller that can be used in an unknown environment without the aid of any infrastructure based localization system, any beacons, artificial features, or any prior knowledge on the environment." Use an LQG/LTR method to obtain the pose controller, after they modeled and identified the faster attitude controller as a second order system (one for pitch and one for roll). Have hover example and waypoint navigation. JIRS paper adds "also reconstruct in real-time a textured 3D mesh of the environment for efficient exploration. In this textured mesh we can also recognize obstacles and avoid them using standard obstacle avoidance routines and path planners in 3D. Moreover, the texture on the mesh map may be used to facilitate interaction with human operators."

51. Kim et al. [2010] "Multiple Relative Pose Graphs for Robust Cooperative Mapping"

52. Lupashin et al. [2010] "A simple learning strategy for high-speed quadrocopter multi-flips"

53. Martin and Salaün [2010] "The True Role of Accelerometer Feedback in Quadrotor Control". Quote: "A revisited quadrotor model is proposed, including in particular the so-called rotor drag." Very nice paper identifying that people are incorrectly neglecting drag, achieving constant velocity at a given pitch angle. The conclusions are simple: rather than control pitch, as most people do, try to estimate both pitch and velocity, and control both. The resulting controllers are much faster in reaching the desired velocity. Basically last chapters of Salaün [2009].

54. Zingg et al. [2010] "MAV Navigation through Indoor Corridors Using Optical Flow". Uses a sparse optical flow method and IMU state estimates to calculate depth to obstacles.

55. Alexis et al. [2010a] "Constrained Optimal Attitude Control of a Quadrotor Helicopter subject to Wind-Gusts: Experimental Studies"

56. Mellinger et al. [2010, 2012] "Trajectory Generation and Control for Precise Aggressive Maneuvers with Quadrotors". Uses $\Delta\omega$ control.

57. Abeywardena and Munasinghe [2010] “Performance Analysis of a Kalman Filter Based Attitude Estimator for a Quad Rotor UAV”

58. Al-Younes et al. [2010] “Linear vs. nonlinear control techniques for a quadrotor vehicle”.

59. Harrington [2011] “Optimal Propulsion System Design for a Micro Quad Rotor”

60. Chamberlain [2011] “System Identification, State Estimation, and Control of Unmanned Aerial Robots”

61. Bertrand et al. [2011] “A hierarchical controller for miniature VTOL UAVs: Design and stability analysis using singular perturbation theory”. Takes a “time-scale separation” approach to control, formalizing the intuition that the attitude dynamics are much faster than the translational dynamics. They have experimental results with a (rather complicated) attitude controller running at 166Hz, and a position controller at about 15Hz. Despite the theoretical appeal, the resulting trajectories seem to have a lot of overshoot and oscillations. Also forms the basis of control strategy by Herissé et al. [2012].

62. Achtelik et al. [2011b] “Onboard IMU and Monocular Vision Based Control for MAVs in Unknown In- and Outdoor Environments”

63. Bills et al. [2011] “Autonomous MAV flight in indoor environments using single image perspective cues”. Fraundorfer et al. [2012] say “implemented an approach for higher level navigation using a Parrot AR.Drone. Utilizing image classification, the MAV was able to follow corridors, and even make turns, but no notion of a metric map was involved.”

64. Lee et al. [2011] “MAV Visual SLAM with Plane Constraint”

65. Meier et al. [2011, 2012] “PIXHAWK: A System for Autonomous Flight using Onboard Computer Vision”

66. Mellinger and Kumar [2011] “Minimum Snap Trajectory Generation and Control for Quadrotors”. Shows that the quadrotor dynamics with the four inputs is differentially flat van Nieuwstadt and Murray [1998]. Uses quadratic velocities for mixing (plus config):

$$u = \begin{bmatrix} k_F & k_F & k_F & k_F \\ & k_F L & & -k_F L \\ -k_F L & & k_F L & \\ k_M & k_M & k_M & k_M \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$

67. Milford et al. [2011] “Aerial SLAM with a Single Camera Using Visual Expectation”

68. Lupashin11cra: “The flying machine arena as of 2010”

69. Shen et al. [2011] “Autonomous Multi-Floor Indoor Navigation with a Computationally Constrained MAV”. Fraundorfer et al. [2012] say “show multi-floor navigation based on the exclusive use of on-board processing. The MAV pose is estimated by laser scan matching, and optimized using pose-graph optimization SLAM with an iterative Kalman filter approach. Loops in the pose-graph are detected using a camera and vocabulary tree matching. In their approach, camera images are only used for loop detection. Pose estimation and SLAM run on-board the MAV. The pose graph includes full 6-DOF poses so that it is possible to distinguish poses at different altitudes, thus allowing multi-floor mapping. They demonstrate autonomous operation and mapping of the MAV where the flight path is defined by manual setting of waypoints in the currently estimated map.”

70. Achtelik et al. [2011a] “Collaborative stereo”

71. Heng et al. [2011] “Real-Time Photo-Realistic 3D Mapping for Micro Aerial Vehicles”.

72. Leishman et al. [2011] “Utilizing an Improved Rotorcraft Dynamic Model in State Estimation”. Quote: “By using an improved dynamic model we show how to more accurately estimate the aircraft states than can be done with the traditional approach of integrating IMU measurements. The estimation is done with relatively infrequent corrections from accelerometers (40Hz) and even less frequent updates from a vision-based SLAM algorithm (2-5 Hz).”

73. Formentin and Lovera [2011] “Flatness-based control of a quadrotor helicopter via feedforward linearization”. Split the dynamics into two subsystems, translational and attitude, and then show that the translational subsystem is differentially flat, with position the flat output. They then apply exact feedback linearization, defining three new control inputs that roughly correspond to x, y, and z velocities. Only results in simulation. My take: interesting, but since they do not model translational drag, might not work as well in practice.

74. Wang et al. [2011] “Novel Dynamic Inversion Architecture Design for Quadrocopter Control”

75. Abeywardena et al. [2011] “A Virtual Odometer for a Quadrotor Micro Aerial Vehicle”

76. Alexis et al. [2011a] “Switching model predictive attitude control for a quadrotor helicopter subject to atmospheric disturbances”

77. Alexis et al. [2011b] “Model Predictive Control Scheme for the Autonomous Flight of an Unmanned Quadrotor”. Quote: “development of a PID controller in Bouabdallah et al. [2004b], a Sliding Mode controller in Benallegue et al. [2006], a Backstepping controller in Bouabdallah and Siegwart [2007] and a Constrained Finite Time Optimal Control Scheme Alexis et al. [2010b,a].” “thrust and drag forces are proportional to the square of propeller’s speed.” Use dynamics model from Bouabdallah [2007], decoupled. Develops a discrete-time error state space model, which is decoupled into planar and altitude. Both are cast as PWA (piecewise affine) by having multiple operating points (planar depends on thrust, altitude on orientation). Similarly, the attitude dynamics are modeled as a set of PWA dynamics as in Alexis et al. [2010b]. Then solves for a minimum-cost MPC controller given a reference trajectory. Validated in simulation only on a simple trajectory.

78. Alexis et al. [2011c] “Model Predictive Quadrotor Indoor Position Control”

79. Bristeau et al. [2011] “The Navigation and Control technology inside the AR.Drone micro UAV”

80. Chaudhari “Aggressive maneuvers using differential flatness for a quadrotor”

81. Huang et al. [2011], Bachrach et al. [2012] “Visual Odometry and Mapping for Autonomous Flight Using an RGB-D Camera” and “Estimation, planning, and mapping for autonomous flight using an RGB-D camera in GPS-denied environments”, Visual odometry by putative features in RGB, depth used only in matching, then integrated with off-board RGBD-SLAM [Henry et al., 2010]. The ISRR results are quite preliminary.

82. Michael et al. [2012] “Collaborative Mapping of an Earthquake-Damaged Building via Ground and Aerial Robots”

83. **Mahony et al. [2012] “Multirotor Aerial Vehicles”** Very nice overview, if not with most elegant coordinate frame math. Good discussion of blade flapping and induced drag. Discussion of differential flatness. State estimation: SLAM (not discussed), Visual servoing to a snapshot.

84. Lim et al. [2012a] "Build your own quadrotor"

85. Tomic et al. [2012] "Toward a Fully Autonomous UAV: Research Platform for Indoor and Outdoor Urban Search and Rescue". Fairly boring system paper. Does not seem to have any absolute position component.

86. Lim et al. [2012b] "Real-time Image-based 6-DOF Localization in Large-Scale Environments"

87. Gillula and Tomlin [2012] "Reducing Conservativeness in Safety Guarantees by Learning Disturbances Online: Iterated Guaranteed Safe Online Learning"

88. Alexis and Tzes [2012] "Revisited Dos Samara Unmanned Aerial Vehicle: Design and Control"

89. Bry et al. [2012] "State Estimation for Aggressive Flight in GPS-Denied Environments Using Onboard Sensing"

90. Leishman et al. [2012] "Relative Navigation and Control of a Hexacopter". Quote: "We depart from the common practice of using a globally referenced map, preferring instead to keep the position and yaw states in the EKF relative to the current map node."

91. Mellinger et al. [2011] "Recent Advances in Quadrotor Capabilities"

92. Ryll et al. [2012] "Modeling and Control of a Quadrotor UAV with Tilting Propellers"

93. Sa and Corke [2012] "System Identification, Estimation and Control for a Cost Effective Open-Source Quadcopter"

94. Shen12icra, "Autonomous indoor 3d exploration with a micro-aerial vehicle", laser scanner, Microsoft Kinect, and IMU.

95. Weiss et al. [2012a] "Versatile Distributed Pose Estimation and Sensor Self-Calibration for an Autonomous MAV". Quote: "versatile framework to enable autonomous flights of a Micro Aerial Vehicle (MAV) which has only slow, noisy, delayed and possibly arbitrarily scaled measurements available." Systems paper, nothing new except applying Weiss and Siegwart [2011] on hexacopter.

96. Weiss et al. [2012b], Weiss [2012] "Real-time Onboard Visual-Inertial State Estimation and Self-Calibration of MAVs in Unknown Environments". Quote: "The main focus here is on the proposed speed-estimation module which converts the camera into a metric body-speed sensor using IMU data within an EKF framework. We show how this module can be used for full self-calibration of the sensor suite in real-time." About observability: "[The] analysis shows, that not only the visual scale factor L is observable, but also all 6DoF of the inter-sensor calibration states between IMU and camera. Obviously, L and the distance between IMU and camera have to be non-zero to be observable. Additionally, and less obvious, the system needs non-zero accelerations and non-zero angular velocities in at least 2 axes to render the mentioned states observable." Shen et al. [2013b] say "In [22], an optical flow-based velocity estimator, in conjunction with a loosely coupled filtering framework, successfully enables autonomous quadrotor flight via a downward-facing camera. However, this approach assumes a slowly-varying visual scale, which can be difficult to enforce during fast motions at low-altitudes with potentially rapid changes in the observed environment and large variations in scene depth (height). A downward-facing camera also severely limits the application of vision-based obstacle detection for planning and control purposes." Bottom-line: a filter that tracks average scene depth L by fusing velocity direction (from vision) and IMU. Like many others, rotation is removed based on gyros. The real measurement of scale is *very* subtle: angular velocity and the IMU-camera arm make part of the velocity direction predictable.

97. Achtelik et al. [2012b] "Visual-Inertial SLAM for a Small Helicopter in Large Outdoor Environments"

98. Engel et al. [2012] "Camera-Based Navigation of a Low-Cost Quadrocopter"

99. Fraundorfer et al. [2012] "Vision-Based Autonomous Mapping and Exploration Using a Quadrotor MAV". Schmid et al. [2013] say "use a forward looking stereo camera as the main sensor for 3D mapping and obstacle avoidance, velocity measurements are based on an optical flow sensor." Use downward looking flow (corrected for rotation) and ultrasound altimeter to get absolute velocity, the forward looking *stereo* camera for VO. Build a 3D occupancy map using techniques from Heng et al. [2011]. Use G2O for pose-SLAM, with frontier-based exploration and vocabulary-tree loop detection.

100. Honegger et al. [2012] "Real-time Velocity Estimation Based on Optical Flow and Disparity Matching"

101. Lenz et al. [2012] "Low-Power Parallel Algorithms for Single Image based Obstacle Avoidance in Aerial Robots"

102. Oung et al. [2012] "A Parameterized Control Methodology for a Modular Flying Vehicle"

103. Schmid et al. [2012, 2013] "State estimation for highly dynamic flying systems using key frame odometry with varying time delays" and "Towards Autonomous MAV Exploration in Cluttered Indoor and Outdoor Environments". Use a stereo setup with semi-global matching at about 15Hz. Use a keyframe based VO fused with IMU, detailed in IROS paper.

104. Ritz et al. [2012] "Cooperative Quadrocopter Ball Throwing and Catching"

105. Roza and Maggiore [2012] "Path Following Controller for a Quadrotor Helicopter"

106. Sánchez et al. [2012] "Nonlinear and Optimal Real-Time Control of a Rotary-Wing UAV"

107. Beall et al. [2012] "Attitude Heading Reference System with Rotation-Aiding Visual Landmarks"

108. Powers et al. [2012] "Influence of Aerodynamics and Proximity Effects in Quadrotor Flight". Models ground effect, ceilimg effect, and fits a parametric function to predict thrust for different airspeed vectors. Improves tracking performance quite a bit.

109. Engel et al. "Accurate Figure Flying with a Quadrocopter Using Onboard Visual and Inertial Sensing"

110. Baránek and Solc [2012] "Modelling and Control of a Hexa-copter"

111. Akhtar et al. [2012] "Path following for a quadrotor using dynamic extension and transverse feedback linearization". Presents path following as alternative to nonlinear control. Note path following is different from trajectory following in that no timing law is pre-specified. Similar to Roza and Maggiore [2012], both based on Nielsen et al. [2010]. A typical control paper in that it obfuscates what is really going on; what follows is my take on what they do, in simpler notation: first, they specify the path Γ as the zero level set of a smooth map $\alpha : \mathbb{R}^3 \to \mathbb{R}^2$. This function is then part of a virtual output to be kept to zero, as this will keep the quadrotor on the path. Strangely, later in the paper they further restrict this to a function $\alpha_1(x, y)$ and a function $\alpha_2(z)$, i.e., this implies level trajectories? They specify two other virtual outputs, a $\pi_1(x, y, z)$ and a $\pi_2(x, y, z, \psi)$, which respectively indicate where on the path the quadrotor is, and a desired constraint on yaw. These four virtual outputs each form a feedback-linearized system stabilized with four auxiliary controls: one to move in the horizontal plane, one to move up and down, one that moves along the path, and another that changes yaw (Eq. 17).

112. Bouffard [2012] "On-board Model Predictive Control of a Quadrotor Helicopter: Design, Implementation, and Experiments"

113. Lange and Protzel [2012] "Cost-Efficient Mono-Camera Tracking System for a Multirotor UAV Aimed for Hardware-in-the-Loop Experiments"

114. Roberts et al. [2012] "Saliency Detection and Model-based Tracking: a Two Part Vision System for Small Robot Navigation in Forested Environments"

115. Hua et al. [2013] "Introduction to Feedback Control of Underactuated VTOL Vehicles: A Review of Basic Control Design Ideas and Principles". Great review paper, if not exclusively quadrotor-focused. Discussed both linear and nonlinear controllers. In the latter, the disadvantage of dynamic extension (using $\ddot{T}$ as a control, as done by Hauser et al. [1992]) is discussed, and hierarchical nonlinear control is advocated.

116. Abeywardena et al. [2013] "Visual-Inertial Fusion for Quadrotor Micro Air Vehicles with Improved Scale Observability", Vision-Inertial Fusion (VIF), high-level state model which includes scale (??) and with orientation and acceleration modeled as zero mean noise. Show that all vehicle state is directly observable.

117. Honegger et al. [2013] "An Open Source and Open Hardware Embedded Metric Optical Flow CMOS Camera for Indoor and Outdoor Applications"

118. Kalantari and Spenko [2013] "Design and Experimental Validation of HyTAQ, a Hybrid Terrestrial and Aerial Quadrotor"

119. Ross et al. [2013] "Learning Monocular Reactive UAV Control in Cluttered Natural Environments"

120. Shen et al. [2013a,b] "Vision-Based State Estimation for Autonomous Rotorcraft MAVs in Complex Environments" and "Vision-Based State Estimation and Trajectory Control Towards High-Speed Flight with a Quadrotor". RSS paper adds "1) an orientation estimation approach to reduce drifting; 2) online scale recovery using low-rate stereo measurements; and 3) system optimizations." Results are excellent, but both papers are rather hard-to-follow system papers. Essentially, rotation is estimated using essential matrix with some accelerometer aiding, then pose using 2-point ransac, a local map is updated in some fashion, and finally stereo is used for combatting scale drift.

121. Lynen13iros "A Robust and Modular Multi-Sensor Fusion Approach Applied to MAV Navigation". With Markus.

122. Goodarzi et al. [2013], "Geometric Nonlinear PID Control of a Quadrotor UAV on SE(3)", Extension of Lee10tac to PID rather than just PD.

123. Khorani13icrm, "The true role of accelerometers in quadrotor's Inertial Navigation System", probably misguided.

124. Abeywardena14iros "Model-aided state estimation for quadrotor micro air vehicles amidst wind disturbances"

6.4 Non-quadrotor VINS

1. Mourikis and Roumeliotis [2007] "A Multi-State Constraint Kalman Filter for Vision-aided Inertial Navigation". Quote: "derivation of a measurement model that is able to express the geometric constraints that arise when a static feature is observed from multiple camera poses."

2. Kelly et al. [2011] “Simultaneous Mapping and Stereo Extrinsic Parameter Calibration Using GPS Measurements”. Weiss et al. [2012a] say “a major step towards power-on-and-go systems. In the latter work, the authors used all information from the two sensors to statistically optimally estimate the 6DoF IMU pose, the inter-sensor calibration, the visual scale factor arising in monocular systems, and the gravity vector in the Vision frame.”

3. Weiss and Siegwart [2011] “Real-Time Metric State Estimation for Modular Vision-Inertial Systems”. Quote: “We treat the visual framework as a black box and thus the approach is modular and widely applicable to existing monocular solutions.” Error state KF with error quaternions. Very confusing paper: does not adequately describe the visual measurement assumptions.

4. Li et al. [2013] “Real-time Motion Tracking on a Cellphone using Inertial Sensing and a Rolling-Shutter Camera”. Quote: “[A] computationally tractable approach for taking into account the rolling-shutter effect, incurring only minimal approximations.”

5. Li and Mourikis [2013] “High-precision, consistent EKF-based visual-inertial odometry”. Quote: “This algorithm, which we term MSCKF 2.0, is shown to achieve accuracy and consistency higher than even an iterative, sliding-window fixed-lag smoother, in both Monte Carlo simulations and real-world testing.”

6. Leutenegger et al. [2013] “Keyframe-Based Visual-Inertial SLAM Using Nonlinear Optimization”. Quote: “[A] novel approach to tightly integrate visual measurements with readings from an Inertial Measurement Unit (IMU) in SLAM.”

None of these are ideal: we need model from Martin and Salaün [2010], Leishman et al. [2011] together with MSC-KF idea (just iSAM?) for vision integration. Read Weiss to see whether anything there.

References

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