MATH 142: Mathematical Modeling, Homework 5

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Part 1

The paper's main goal was to answer the following 5 questions:

(Q1) Is it possible to contain COVID-19?

(Q2) If COVID-19 can be contained, when will be the peak of the epidemic, and when will it end?

(Q3) How do the asymptomatic infections affect the spread of disease?

(Q4) If COVID-19 cannot be contained, what is the ratio of the population that needs to be infected in order to achieve herd immunity?

(Q5) How effective are the social distancing approaches?

To answer the questions above, the paper went through the process of modeling the spread of COVID-19, building from the basic SIR model to a more robust model.

In the beginning, the authors explained that the basic SIR model does not do a good job at modeling the spread because β and γ are being treated as a constant; they vary with time, which means they should be time dependent functions $\beta(t)$ and $\gamma(t)$. The authors continue by "breaking down" the differential equations $\frac{dS(t)}{dt}$, $\frac{dI(t)}{dt}$, and $\frac{dR(t)}{dt}$ into their respective discrete time differential equations. The functions $\beta(t)$ and $\gamma(t)$ are going to be tracked using ridge regression, one of the many machine learning methods out there. Then, the authors proceed to develop an algorithm $\hat{X}(T)$ and $\hat{R}(T)$.

Next, the authors discuss the fact that not everyone who is infected shows symptoms, which leads them to denote w_1 and w_2 to represent the probabilty that an infected person is detected and the probability than an infected person is not detected. $\beta_1(t)$, $\gamma_1(t)$, $\beta_2(t)$, and $\gamma_2(t)$ are also defined.

Then, the transtion matrix $\mathbf{A}$ is defined, and its eigenvalues are discussed. It states that if the spectral radius is less than 1, then the system is considered stable (ie. there will not be an outbreak). On the other hand, if the spectral radius is greater than 1, there will be an outbreak. There are also disucssions about how R_0, the reproduction number of a newly infected person, is related to the spectral radius.

There are then discussions about how to reach herd immunity, and the author concludes that it will be achieved once $1 - \frac{1}{R_0}$ fraction of the individuals are infected and become recovered.

The authors then proceed to discuss numerical results. There are multiple graphs that show their predicted vs actual values; overall, the authors' models seem to model the data well. For example, there is a graph that shows $\hat{X}(t), X(t), \hat{R}(t)$, and $R(t)$. Comparing the data that was known and the model the authors came up with, there is barely any difference. The model predicts that $\hat{R}(t)$ will level out around 80000 on 3/18, and $\hat{X}(t)$ will decrease to 0 on 3/25. Another interesting graph was the phase diagram they produced; it shows which combinations of w_2 and β_2 would result in an outbreak.

Lastly, the authors extend their study into other countries, such as the United States, France, Spain, etc. They try to estimate $R_0(t)$ starting from 14 to 42 days in intervals of 7 days. There is another phase diagram that shows which countries are going to have an outbreak; it predicts that South Korea, Italy, and Iran (while including deaths in the recovery rate) will have an outbreak.

Part 2

(1) Why was ridge regression chosen over the other estimation methods mentioned?

(2) What are some ways to improve the model? Would it require better data?

(3) What was your motivation behind writing this paper? I would assume that there has been many papers trying to model the spread of COVID-19; what makes yours different?

(4) Why did you not also include data from the beginning of COVID-19? It could still be valuable even though there may be noise in the data.

Part 3

(1) I thought the article was really interesting! It was really cool to see the math behind all the models I was seeing in the news.

(2) Before reading the paper, I thought I would have been really lost when reading it. Instead, I actually understood a lot of the parts in the paper.

(3) I think the authors did a pretty good job in explaining their intuition in developing their models.

(4) The graphs were all pretty nice; they did a good job in displaying valuable information.

Part 4

I recognized the SIR model that was dicussed in the beginning; it was essentially what we have been covering in class for the past week or so (it also helped that we used the same variable, β and γ). I also recognized the concept of having infected people who were and weren't diagnosed. Surprisingly, I recognized some of the parts talking about different machine learning methods; I am currently enrolled in STATS 102B where we have covered methods such as OLS, ridge, etc. Lastly, I also recoginzed the idea of using the spectral radius of a matrix to determine if there was or was not going to be an outbreak; it is a very similar idea to the game theory concepts we discussed in class.

I did not recognize the parts talking about the effects of social distancing. I do not believe we have discussed this factor in this class or any of my previous classes.