MATH 142: Mathematical Modeling, Quiz 1

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Question 1

Part A

$births = (\frac{1}{2} * \frac{1}{4} * 6 * 2 * \frac{1}{2})N_{k-1} = .75 * N_{k-1}$
$deaths = \frac{1}{4} * N_{k-1} = .25 * N_{k-1}$

Part B

$N_k = (1 + .75 - .25)N_{k-1} = 1.5 * N_{k-1} = (1.5)^k * N_0$

Part C

We start with $328 = (1.5)^k * 1.5$. Then, we rearrange to solve for k: $k = \frac{log(\frac{328}{1.5})}{log(1.5)} = 13.287 \approx 13$ years

Part D

$N_k = (1 + .75 - .25h)N_{k-1} + \alpha - \beta = (1.75 - .25h)N_{k-1} + \alpha - \beta$

Question 2

Part A

$$L = \begin{bmatrix} (1-a_1)(1+r) & a_2(1-c) \\ a_1(1-c) & (1-a_2)(1+r) \end{bmatrix}$$

Part B

$$L = \begin{bmatrix} (1-\frac{2}{5})(1+\frac{1}{4}) & \frac{2}{5}(1-c) \\ \frac{2}{5}(1-c) & (1-\frac{2}{5})(1+\frac{1}{4}) \end{bmatrix}$$

We want $(1-\frac{2}{5})(1+\frac{1}{4}) + \frac{2}{5}(1-c) = .75 + .4 - .4c = 1$. Solving for c, we get $c = \frac{1-.75-.4}{-.4} = .375$

Thus, the maximum value for the population to persist is $c = .375$. Any value of c greater than .375 will result in the population going extinct.

Part C

$$L = \begin{bmatrix} (1-\frac{2}{5})(1+\frac{1}{4}) & \frac{2}{5}(1-.375) \\ \frac{2}{5}(1-.375) & (1-\frac{2}{5})(1+\frac{1}{4}) \end{bmatrix} = \begin{bmatrix} .75 & .25 \\ .25 & .75 \end{bmatrix}$$

$$det(L-\lambda I) = det(\begin{bmatrix} .75-\lambda & .25 \\ .25 & .75-\lambda \end{bmatrix}) = (.75-\lambda)^2 - .25^2 = \lambda^2 - 1.5\lambda + .5 = (\lambda-1)(\lambda-.5) \to \lambda_1 = 1, \lambda_2 = .5$$

$$L - 1I = \begin{bmatrix} -.25 & .25 \\ .25 & -.25 \end{bmatrix} \to \overrightarrow{u_1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

The stable fraction will be $\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$