MATH 142: Mathematical Modeling, Homework 1

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```
library(readxl) #load this library to read in xls files
library(ggplot2) #load this library for plotting
library(gridExtra) #load this to nicely organize multiple graphs
```

Question 1

Part a.

$N_{k+1} = N_k$ *+ # of kakapo born in the wild*
- # of kakapo removed for captive breeding
+ # of kakapo reintroduced into the wild from captive breeding
- # of kakapo killed by rats

Part b.

N_t *= amount of ibuprofen in the patients blood after* $t - 1$ *hours*
+ amount of ibuprofen absorbed from the gut into the blood
- amount of ibuprofen absorbed into tissues
- amount of ibuprofen filtered from blood by kidneys
+ amount of ibuprofen reabsorbed into blood from tissues

Question 2

Part a.

(i) $(\frac{50}{100} * \frac{25}{100} * \frac{29}{100}) * N_k = .03625 * N_k$, as desired.

(ii) The corresponding number of deaths is $(\frac{1}{50}) * N_k = .02 * N_k$.

(iii)

```
population <- c()
for(i in 0:5){
  population <- c(population, (1.01625)^i*50)
}

print(data.frame("k" = 0:5, "Population.Size" = population), row.names = F)
```

```
##  k Population.Size
##  0        50.00000
##  1        50.81250
##  2        51.63820
##  3        52.47732
##  4        53.33008
##  5        54.19669
```

The predicted population size over the next five years is shown above.

(iv)

```
hundredbirds <- log(100/50)/log(1.01625)
hundredbirds
```

```
## [1] 43.00085
```

```
twohundredbirds <- log(200/50)/log(1.01625)
twohundredbirds
```

```
## [1] 86.00171
```

The explicit formula is $N_k = 50 * (1.01625)^k$. The population will reach **100 birds in 44 years**. The population will reach **200 birds in 87 years**.

Part b.

(i) The reccurence equation will be $N_k = N_{k-1} * (1 + (\frac{29}{100} * \frac{1}{3} * \frac{1}{2}) - \frac{1}{50}) = N_{k-1} * (\frac{617}{600})^k$.

```
methodOne <- c()
for(i in 0:5){
  methodOne <- c(methodOne, 50*(617/600)^i)
}
print(data.frame("k" = 0:5, "Population.Size" = methodOne), row.names = F)
```

```
##   k Population.Size
##   0        50.00000
##   1        51.41667
##   2        52.87347
##   3        54.37155
##   4        55.91208
##   5        57.49626
```

$N_0, N_1, ..., N_5$ is shown above.

(ii) The reccurence equation will be $N_k = N_{k-1} * (1 + (\frac{1}{2} * \frac{1}{4} * \frac{1}{2}) - \frac{1}{50}) = N_{k-1} * (\frac{417}{400})^k$.

```r
methodTwo <- c()
for(i in 0:5){
  methodTwo <- c(methodTwo, 50*(417/400)^i)
}
print(data.frame("k" = 0:5, "Population.Size" = methodTwo), row.names = F)
```

```
##   k Population.Size
##   0        50.00000
##   1        52.12500
##   2        54.34031
##   3        56.64978
##   4        59.05739
##   5        61.56733
```

$N_0, N_1, ..., N_5$ is shown above.

(iii) We can see that **strategy 2** gives the biggest increase in population size.

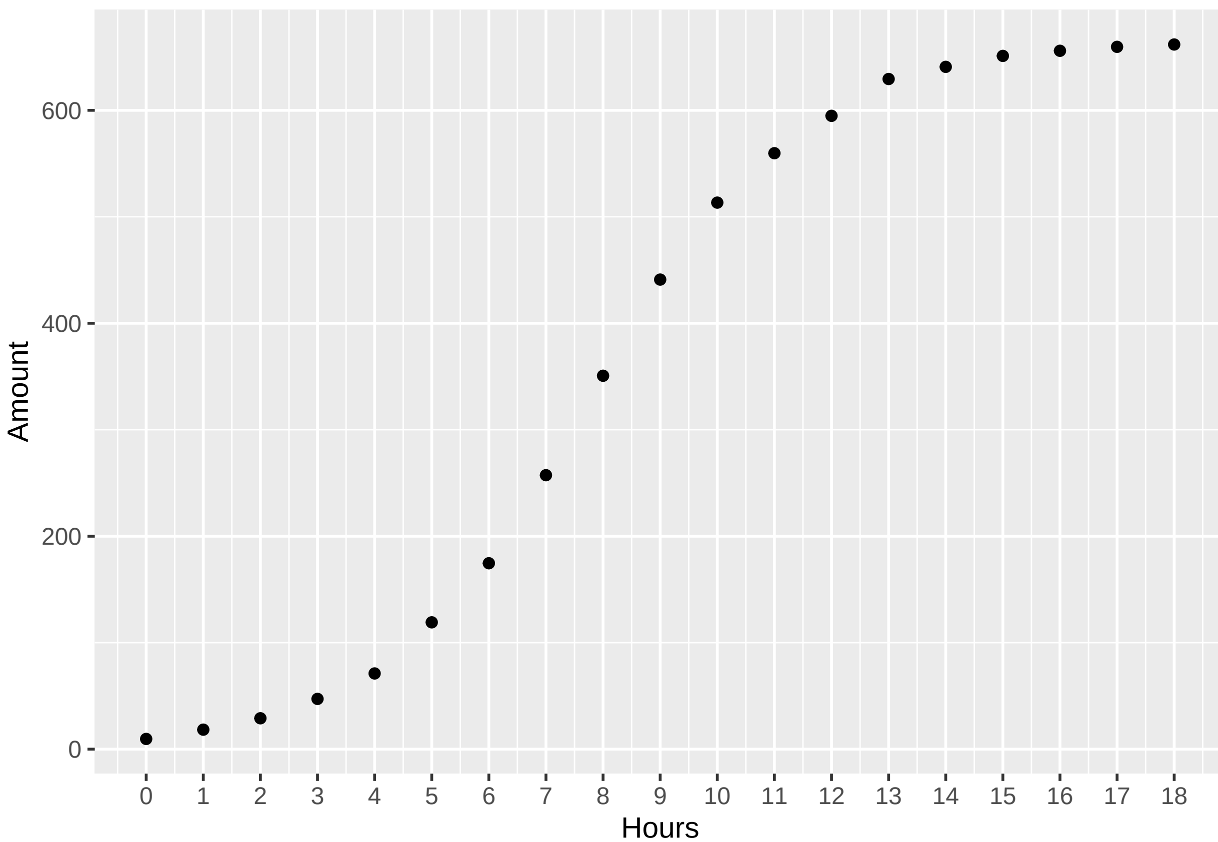
Question 3

Part a.

If you were to watch the cells for time t and observe n deaths, then the death rate, m, would be $\frac{n}{t}$.

Part b.

```
cells <- read_excel("31395_Carlson_Yeast_Data.xls")
ggplot(aes(x = Hours, y = Amount), data = cells) +
  geom_point() +
  scale_x_continuous(breaks = seq(0, 18, 1))
```



The population size does not grow exponentially because from the graph, you can see that the rate of growth decreases after around the 10^{th} hour. For example, the rate of growth from the 14^{th} to 15^{th} hour is less than the rate of growth from the 7^{th} hour to the 8^{th} hour. Thus, the population does not grow exponentially.

Part c.

To see that the the growth rate is approximately exponential for the first few hours, you would calculate the rate of growth between each hour in the first few hours. Then, you'd notice that the rate of growth is growing approximately exponentially.

Part d.

```r
print(data.frame(cells), row.names = F)
```

```
##  Hours Amount
##      0    9.6
##      1   18.3
##      2   29.0
##      3   47.2
##      4   71.1
##      5  119.1
##      6  174.6
##      7  257.3
##      8  350.7
##      9  441.0
##     10  513.3
##     11  559.7
##     12  594.8
##     13  629.4
##     14  640.8
##     15  651.1
##     16  655.9
##     17  659.6
##     18  661.8
```

I would estimate that $R_0 = \frac{513.3-9.6}{10-0} = 50.37$ cells/hr.

Question 4

Part a.

We see that $\frac{1}{2} * \frac{6}{35} = \frac{3}{35}$, as desired.

Part b.

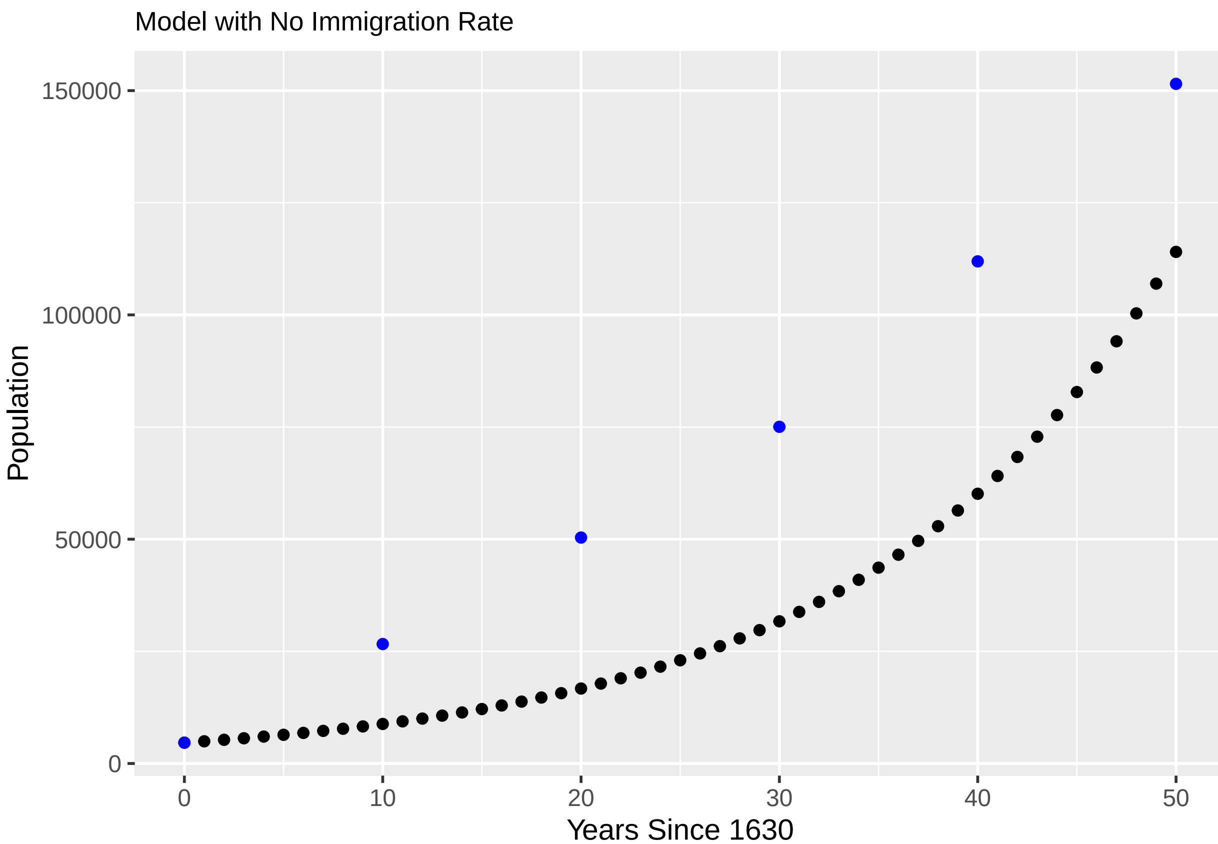
Solving for x in $\frac{N}{2} = N * x^{35}$, we get $\frac{1}{\sqrt[35]{2}}$, which means the death rate is $1 - \frac{1}{\sqrt[35]{2}}$.

Part c.

We are assuming that every female is physically able to give birth, which is not true. A possible solution to this is considering that only half of all females can give birth. Another problem is that we are assuming that the life expectancy is constant. This is not necessarily true because medicine can get better and will increase the life expectancy as time progresses. A possible solution is to have life expectancy be a function of time.

Part d.

```r
base_model <- function(x){
  (1+3/35-(1-2^(-1/35)))^x * 4646
}

x_actual <- c(0, 10, 20, 30, 40, 50)
y_actual <- c(4646, 26634, 50368, 75058, 111935, 151507)

p1 <- ggplot(aes(x = 0:50, y = base_model(c(0:50))), data = NULL) +
  geom_point() +
  geom_point(aes(x = x_actual, y = y_actual), data = NULL, color = "blue") +
  labs(x = "Years Since 1630",
       y = "Population",
       title = "Model with No Immigration Rate") +
  theme(plot.title= element_text(size = 10))
p1
```



We can see that the model and the actual population start at the same number, but the actual population quickly begins to be much higher than the population from our model.

Part e.

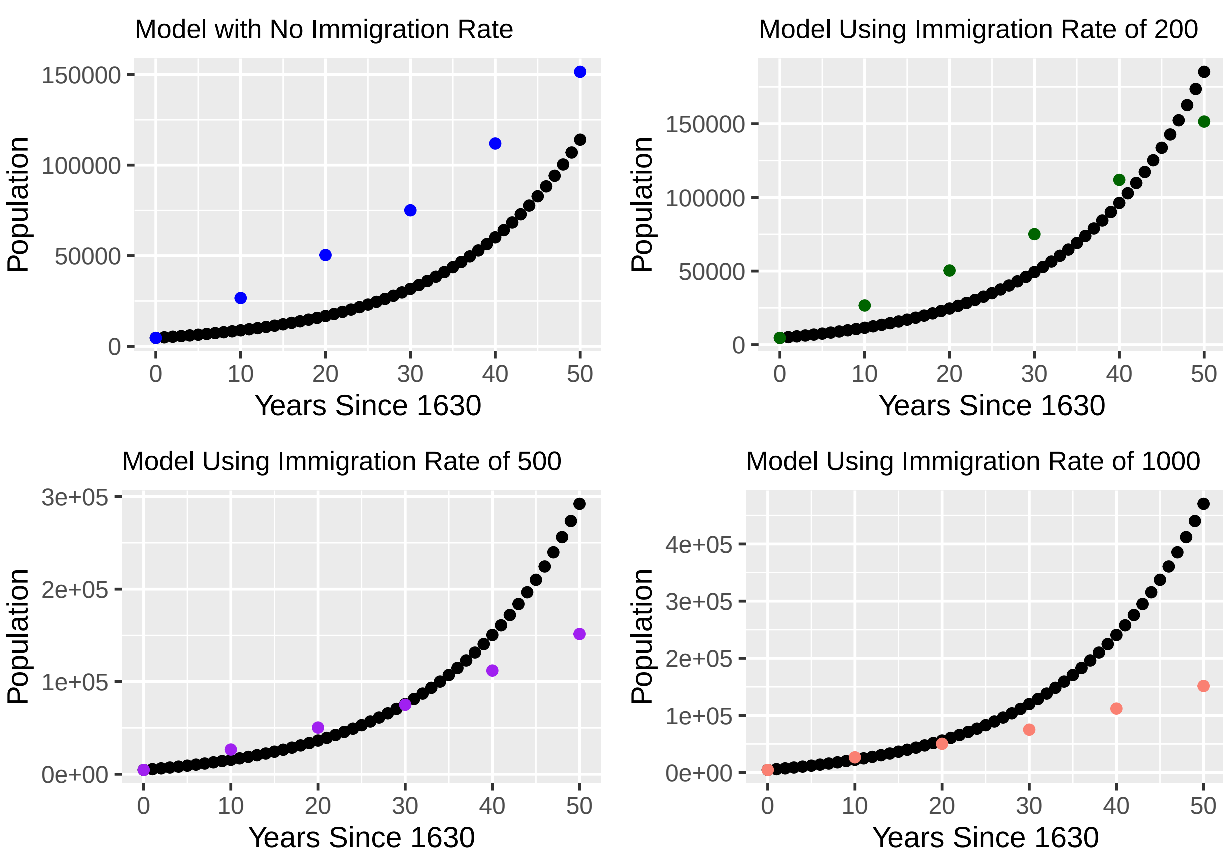
```
from_model2 <- c(4646, rep(0, 50))
for(i in seq(1:50)){
  from_model2[i + 1] = (1 + 3/35 - (1-2^(-1/35))) * from_model2[i] + 200
}

p2 <- ggplot(aes(x = 0:50, y = from_model2), data = NULL) +
  geom_point() +
  geom_point(aes(x = x_actual, y = y_actual), data = NULL, color = "darkgreen") +
  labs(x = "Years Since 1630", y = "Population") +
  labs(x = "Years Since 1630",
       y = "Population",
       title = "Model Using Immigration Rate of 200") +
  theme(plot.title= element_text(size = 10))
```

```
from_model3 <- c(4646, rep(0, 50))
for(i in seq(1:50)){
  from_model3[i + 1] = (1 + 3/35 - (1-2^(-1/35))) * from_model3[i] + 500
}

p3 <- ggplot(aes(x = 0:50, y = from_model3), data = NULL) +
```

```
  geom_point() +
  geom_point(aes(x = x_actual, y = y_actual), data = NULL, color = "purple") +
  labs(x = "Years Since 1630", y = "Population") +
  labs(x = "Years Since 1630",
       y = "Population",
       title = "Model Using Immigration Rate of 500") +
  theme(plot.title= element_text(size = 10))
```

```
from_model4 <- c(4646, rep(0, 50))
for(i in seq(1:50)){
  from_model4[i + 1] = (1 + 3/35 - (1-2^(-1/35))) * from_model4[i] + 1000
}

p4 <- ggplot(aes(x = 0:50, y = from_model4), data = NULL) +
  geom_point() +
  geom_point(aes(x = x_actual, y = y_actual), data = NULL, color = "salmon") +
  labs(x = "Years Since 1630",
       y = "Population",
       title = "Model Using Immigration Rate of 1000") +
  theme(plot.title= element_text(size = 10))
```

```
grid.arrange(p1, p2, p3, p4,
             ncol = 2, nrow = 2)
```



From the above graphs, I would say the model with 200 for the value of I_0 is the best.

Question 5

Part a.

$C_0 = 0$
$C_1 = 40$
$C_2 = 40 + 0 - (\frac{24.25}{100} * 40) = 30.3$
$C_3 = 30.3 + 0 - (\frac{24.25}{100} * 30.3) = 22.95225$
$C_4 = 22.95225 + 0 - (\frac{24.25}{100} * 22.95225) = 17.38633$
$C_5 = 17.38633 + 0 - (\frac{24.25}{100} * 17.38633) = 13.17014$
$C_6 = 13.17014 + 0 - (\frac{24.25}{100} * 13.17014) = 9.97638$
$C_7 = 9.97638 + 40 - (\frac{24.25}{100} * 9.97638) = 47.55711$

Part b.

The equation is $C_n = 40 + C_{n-1} * (.7575)^6$.

Part c.

```r
concentration <- rep(0, 5)
for(i in seq_along(concentration)){
  concentration[i + 1] <- 40 + concentration[i] * (.7575)^(6)
}

print(data.frame("n" = 0:5, "C_n" = concentration), row.names = F)
```

```
##  n      C_n
##  0  0.00000
##  1 40.00000
##  2 47.55711
##  3 48.98486
##  4 49.25460
##  5 49.30556
```

Part d.

To find the fixed point we set $C_n = C_{n-1} = C$ in the equation from Part b and solve for C:

$$C = 40 + C(.7575)^6$$

$$40 = C(1 - (.7575)^6)$$

$$C = \frac{40}{1 - (.7575)^6} = 49.31743$$

Part e.

$$C_n = \sum_{k=1}^{n+1} C_1 * (.7575)^{6(k-1)}$$

Thus,

$$\lim_{x\to\infty} C_n = \lim_{x\to\infty}(\sum_{k=1}^{n+1} C_1 * (.7575)^{6(k-1)}) = \frac{40}{1-(.7575)^6} = 49.31743$$

This agrees with our answer from Part e.