

## Math 142 - Spring 2020

### Super Quiz 1

Please read the following instructions carefully before taking this quiz:

1. You have a 24 hour window to complete this quiz, from 1:00 PM Pacific time on Monday, April 13th to 1:00 PM Pacific time on Tuesday, April 14th. You **must** upload the quiz to Gradescope before the deadline.
2. This quiz is open book, open note, open internet, but it **must** be completed on your own. You may not ask other students/tutors for help or use forums such as Stack Exchange to ask questions. You must show work on each problem to receive full credit.
3. If you have any clarification questions about the quiz, you may post your questions on Piazza, but you may not use Piazza to discuss how to do quiz problems.
4. You may scan or take pictures of the quiz in order to upload. Regardless of what option you are using, make sure that your work is legible and that pictures/scans are in focus. I would recommend that you use a free scanning app such as Scannable.
5. Please select the correct pages of your solution associated with each question when you upload to Gradescope.
6. If you wish, you may print out the quiz and write the answers on the test paper. You may also write answers on your own paper, on a tablet, or on a computer.

By signing below, you acknowledge that you have read the instructions on the previous page and that you agree to the following statement:

*"I assert, on my honor, that I have not received assistance of any kind from any other person while working on this quiz and that I have not used any non-permitted resources during the period of this evaluation."*

(If you are answering the questions on separate paper, please copy the statement above onto your paper, and write your name, student ID, and signature on the same page.)

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STUDENT ID: 405 433 124

SIGNATURE: 

## SHOW ALL WORK AND BOX YOUR ANSWERS

1. [13 points] Feral hogs are an invasive species that are of major concern in the United States because they damage crops, spread parasites, and occasionally attack humans. Suppose that you are measuring the population of feral hogs in Texas once per year. Use the following assumptions:
- Half of the population is female, and each female reproduces every six months, on average.
  - Each time a female hog reproduces, she bears an average of six offspring. Half of these offspring survive their first year.
  - A quarter of the female hogs are of reproductive age (capable of reproducing).
  - Life expectancy for the entire hog population is about four years. Hence, we say that one in four hogs alive at the beginning of year of  $k$  dies during year  $k$ .

Let  $N_k$  be the total number of hogs (male and female) in Texas in year  $k$ .

- (a) (4 pts) Using the assumptions above, find the number of hog births and number of hog deaths each year. *You must show work for how you calculated number of hog births.*
- (b) (3 pts) Write down a recurrence (difference) equation for hog population  $N_k$ . Then find an explicit formula for  $N_k$ , starting from an initial population of  $N_0$ .
- (c) (3 pts) There are currently 1.5 million feral hogs in Texas. According to your model, how many years will it take for the feral hog population in Texas to reach the current United States human population (328 million)? You may round your answer to the nearest year.
- (d) (3 pts) Now suppose that, in addition to the assumptions described above, you want to incorporate immigration, emigration, and hunting into your model. Assume that  $\alpha$  hogs enter Texas each year,  $\beta$  hogs leave Texas, and a fraction  $h$  of hogs in Texas are killed by hunters, where  $\alpha$ ,  $\beta$ , and  $h$  are constants. Adjust your recurrence equation from part (b) to incorporate this new information.

(Space to continue your answer to Question 1)

2. [12 points] A population of meerkats lives in two separate burrows. Consider the following:

- Each day, a fraction  $a_1$  of meerkats at burrow 1 travel to burrow 2, and a fraction  $a_2$  of meerkats at burrow 2 travel to burrow 1.
- Unfortunately, a fraction  $c$  of traveling meerkats are consumed by predators (such as snakes, jackals, and eagles) during their journey.
- The meerkats that do not travel (remain at the burrow during the day) have a growth rate of  $r$  per meerkat per day.

Suppose that a census is taken each night (when all meerkats are in their burrows) of the number of meerkats at each burrow. Let

- $N_1^{(k)}$  = population of meerkats at burrow 1 on night  $k$ ,
- $N_2^{(k)}$  = population of meerkats at burrow 2 on night  $k$ .

(a) (4 pts) You want to write down a Leslie matrix model of the form

$$\begin{pmatrix} N_1^{(k+1)} \\ N_2^{(k+1)} \end{pmatrix} = L \begin{pmatrix} N_1^{(k)} \\ N_2^{(k)} \end{pmatrix}$$

for the situation described above. Specify the matrix  $L$ .

- (b) (5 pts) Suppose that you measure  $a_1 = 2/5$ ,  $a_2 = 2/5$ , and  $r = 1/4$ . What is the maximum fraction of meerkats  $c$  that can be eaten such that the meerkat population will persist (not decay to zero) long-term?
- (c) (3 pts) For the value of  $c$  you found in part (b), find the stable fraction of meerkats at each burrow.

(Space to continue your answer to Question 2)