Math 151a: HW #4. Due on Thursday, July 23

[1] (a) Use the most accurate three-point formula to determine each missing entry in the following table:

x	$f(x)$	$f'(x)$
-0.3	-0.27652	
-0.2	-0.25074	
-0.1	-0.16134	
0	0	

(b) The data in the table was taken from the function $f(x) = e^{2x} - \cos 2x$. Compute the actual errors, and find error bounds using the error formulas.

[2] Let $f(x) = 3xe^x - \cos x$. Use the following data and the Second Derivative Midpoint Formula to approximate $f''(1.3)$ with $h = 0.1$ and with $h = 0.01$.

x	1.20	1.29	1.30	1.31	1.40
$f(x)$	11.59006	13.78176	14.04276	14.30741	16.86187

Compare your results to $f''(1.3)$.

[3]

(a) The forward-difference formula can be expressed as

$$f'(x_0) = \frac{1}{h}[f(x_0 + h) - f(x_0)] - \frac{h}{2}f''(x_0) - \frac{h^2}{6}f'''(x_0) + O(h^3).$$

Use extrapolation to derive an $O(h^3)$ formula for $f'(x_0)$.

(b) Apply the result obtained in (a) to the function from Exercise [1], $f(x) = e^{2x} - \cos 2x$, to obtain an approximation to $f'(x_0)$ when $x_0 = -0.2$ and $h = 0.1$. Compute the actual error.

[4] Approximate the integral $\int_0^1 x^2 e^{-x} dx$ using the Trapezoidal, Simpson's and Midpoint Rules. Find a bound for the error using the error formula in each case and compare this to the actual error.

[5] The Trapezoidal Rule applied to $\int_0^2 f(x)dx$ gives the value 4, and Simpson's Rule gives the value 2. What is $f(1)$?