Math 151a: HW #5, due on Friday, July 31st (no extensions)

[**1**] Find the constants c_0, c_1 and x_1 so that the quadrature formula

$$\int_0^1 f(x)dx \approx c_0 f(0) + c_1 f(x_1)$$

has the highest possible degree of precision.

[**2**] Use the Composite Trapezoidal, Simpson's and Midpoint Rules to approximate the integral

$$\int_1^2 x\ln(x)dx, \quad n = 4.$$

(For the Midpoint Rule use $n+2$ subintervals.)

[**3**] Determine the values of n and h required to approximate

$$\int_1^2 x\ln(x)dx$$

to within 10^{-5}. Use
(a) Composite Trapezoidal Rule.
(b) Composite Simpson's Rule.
(c) Composite Midpoint Rule.

[**4**] Find c_1, c_2, x_1 and x_2 such that the integration formula

$$\int_{-1}^1 f(x)dx \approx c_1 f(x_1) + c_2 f(x_2)$$

is exact for $f(x) = 1$, x, x^2 and x^3. (the resulting system of four equations has been derived in the lecture). Then show that the obtained formula has degree of precision 3 (you just need to choose $f(x) = x^4$ and check that the approximation no longer gives the exact integral for this polynomial).

[**5**] Use the result from [4] and change of variable to derive a quadrature formula for $\int_a^b f(x)dx$ of the same form.