Math 151A HW #2. Due on Thursday, July 9

#1(ab) Use algebraic manipulations to show that each of the following functions has a fixed point at p precisely when $f(p) = 0$, where

$$f(x) = x^4 + 2x^2 - x - 3.$$

(a) $g_1(x) = (3 + x - 2x^2)^{1/4}$
(b) $g_2(x) = \left(\frac{x+3-x^4}{2}\right)^{1/2}$

#2
(a) Perform four iterations, if possible, on each of the functions g defined in Exercise 1 ((a) and (b)). Let $p_0 = 1$ and $p_{n+1} = g(p_n)$, for $n = 0, 1, 2, 3$.
(b) Which function do you think gives the best approximation to the solution ?

#7 Use the Fixed Point Theorems from Section 2.2 to show that $g(x) = \pi + 0.5\sin(x/2)$ has a unique fixed point on $[0, 2\pi]$. Use fixed-point iteration to find an approximation to the fixed point that is accurate to within 10^{-2}. Use the theoretical result to estimate the number of iterations required to achieve 10^{-2} accuracy, and compare this theoretical estimate to the number actually needed.

#9 Use a fixed-point iteration method to find an approximation to $\sqrt{3}$ that is accurate to within 10^{-4}. Compare your result and the number of iterations required with the answer obtained using the Bisection Algorithm from the previous homework.

2.3, #2 Let $f(x) = -x^3 - \cos x$ and $p_0 = -1$. Use Newton's method to find p_2. Could $p_0 = 0$ be used ?

2.3, #4(a) Let $f(x) = -x^3 - \cos x$. With $p_0 = -1$ and $p_1 = 0$, find p_3 using the Secant method.

2.3, #6(a) Use Newton's method to find solutions accurate to within 10^{-5} for the problem:

$$e^x + 2^{-x} + 2\cos x - 6 = 0 \quad \text{for} \quad 1 \leq x \leq 2.$$

Repeat this problem using the Secant method.