Math 151A

HW #3, due on Thursday, July 16

[**1**] Use Newton's method to find solutions accurate to within 10^{-5} for the problem:

$$1 - 4x\cos x + 2x^2 + \cos 2x = 0 \qquad \text{for } 0 \le x \le 1.$$

Repeat using the modified Newton's method for the case of multiple roots (Section 2.4).

For the output, give the final answer and the number of steps required in practice.

[**2**] Let $f(x) = \sin(\pi x)$ and $x_0 = 1$, $x_1 = 1.25$, and $x_2 = 1.6$.

(a) Construct interpolation polynomials of degree at most one and at most two to approximate $f(1.4)$ and find the absolute error.

(b) Use the theorem expressing the error in Lagrange interpolation to find an error bound for the approximations.

[**3**] Let $x_0, x_1, ..., x_n$ be $n+1$ distinct points with given values $f(x_0)$, $f(x_1)$,..., $f(x_n)$. Let P_n be the Lagrange interpolating polynomial defined using all these points.

(a) Give the formulas for the divided differences $f[x_0]$, $f[x_0, x_1]$, and $f[x_0, x_1, x_2]$.

(b) Given

$$\begin{aligned} P_n(x) &= f[x_0] + a_1(x - x_0) + a_2(x - x_0)(x - x_1) \\ &+ a_3(x - x_0)(x - x_1)(x - x_2) + ... + a_n(x - x_0)(x - x_1)...(x - x_{n-1}), \end{aligned}$$

use $P_n(x_1)$ to show that $a_1 = f[x_0, x_1]$.

(c) Given

$$\begin{aligned} P_n(x) &= f[x_0] + f[x_0, x_1](x - x_0) + a_2(x - x_0)(x - x_1) \\ &+ a_3(x - x_0)(x - x_1)(x - x_2) + ... + a_n(x - x_0)(x - x_1)...(x - x_{n-1}), \end{aligned}$$

use $P_n(x_2)$ to show that $a_2 = f[x_0, x_1, x_2]$.

[**4**] Use Neville's method to obtain the approximations for Lagrange interpolating polynomials of degrees one, two and three to approximate the following:

$f(0.43)$ if $f(0) = 1$, $f(0.25) = 1.64872$, $f(0.5) = 2.71828$, $f(0.75) = 4.48169$.

[5] Suppose $x_j = j$ for $j = 0, 1, 2, 3$ and it is known that

$$P_{0,1}(x) = 2x + 1,\ P_{0,2}(x) = x + 1,\ \text{and } P_{1,2,3}(2.5) = 3.$$

Find $P_{0,1,2,3}(2.5)$.

[6] Use Newton's divided difference formula to construct interpolating polynomials of degree one, two and three for the following data. Approximate the specified value $f(8.4)$ using each of the polynomials if

$f(8.1) = 16.94410,\ f(8.3) = 17.56492,\ f(8.6) = 18.50515,\ f(8.7) = 18.82091.$