MATH 151B Applied Numerical Methods, Homework

Question 1: For the system of equations

$$\begin{aligned} x_1^2 + x_2 - 37 &= 0 \\ x_1 - x_2^2 - 5 &= 0 \\ x_1 + x_2 + x_3 - 3 &= 0. \end{aligned}$$

do one step of Newton's Method by hand with a starting value of $(1, 1, 1)^T$. Is your new approximation better than the starting guess? If not, why not? Numerically find approximations using 4 and 8 steps of Newton's Method and assess the accuracy of the solutions.

Question 2: Use the finite difference method for nonlinear BVPs to solve

$$y'' = 2y^3, \quad -1 \le x \le 0, \quad y(-1) = \frac{1}{2}, \quad y(0) = \frac{1}{3},$$

using Broyden's Method to solve the system of equations. Explain how to derive the system of equations, and provide your code (you can use the Broyden's Method function from CCLE). Using $N = 7$, give your approximation for $y(-1/2)$.

Question 3: Use the method of steepest descent to to approximate the solution of

$$\begin{aligned} x_1^3 + x_1^2 x_2 - x_1 x_3 + 6 &= 0 \\ e^{x_1} + e^{x_2} - x_3 &= 0 \\ x_2^2 - 2x_1 x_3 - 4 &= 0. \end{aligned}$$

with tolerance 0.01, and a starting guess $(1, 1, 1)^T$. Verify your answer by computing $\mathbf{F}(\mathbf{x})$.

If the tolerance is made much smaller, the algorithm will run for many more iterations. Describe how the approximations converge to the final answer if the tolerance is decreased to 10^{-5}.