MATH 151B Applied Numerical Methods, Homework 4

Question 1:

Find the coefficients a, b, c, and d so that

$$y(t_i) + h\left(af(t_i, y(t_i)) + bf(t_{i-1}, y(t_{i-1})) + cf(t_{i-2}, y(t_{i-2})) + df(t_{i-3}, y(t_{i-3}))\right)$$

approximates $y(t_{i+1})$. The resulting method is the 4-step Adams-Bashforth method. Give the coefficients exactly (as fractions, not decimals).

Question 2: Consider the following IVP:

$$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} - 4y(t) = 6\exp(-t), \quad y(0) = 0, \quad \left.\frac{\mathrm{d}y}{\mathrm{d}t}\right|_{t=0} = 0.$$

(a) Obtain the coefficients a and b in the one-step implicit method given as

$$w_{i+1} = w_i + af(t_{i+1}, w_{i+1}) + bf(t_i, w_i).$$

(b) Write a predictor-corrector method using the Midpoint method for the prediction step and the implicit method you derived in (a) for the correction step. Show your code.

(c) Estimate the value $y(1)$ of the solution of the IVP using $h = 0.1$.

Question 3: The Adams-Moulton 3-step implicit method is given by

$$w_{i+1} = w_i + \frac{h}{24}\left(9f(t_{i+1}, w_{i+1}) + 19f(t_i, w_i) - 5f(t_{i-1}, w_{i-1}) + f(t_{i-2}, w_{i-2})\right).$$

For a IVP of the form $y'(t) = yg(t)$ for some smooth function g, write out an explicit version of the Adams-Moulton 3-step implicit method, i.e., write w_{i+1} in terms of $g(t_{i+1})$, w_i , $g(t_i)$, $g(t_{i-1})$, w_{i-1}, $g(t_{i-2})$, and w_{i-2}.