MATH 151B Applied Numerical Methods, Homework 2

Question 1: Consider the IVP

$$\frac{\mathrm{d}y}{\mathrm{d}t} = t^2 - 1$$

with $0 \leq t \leq 1$ and $y(0) = 0$.

(a) Write the Taylor Method of order 4 for this IVP (i.e. truncate the Taylor series at the quartic term).

(b) Use your method to approximate $y(1)$ by setting $h = 1$. Find the exact solution of the IVP and calculate the error in your approximation. Explain your results.

Question 2: In this question we investigate an alternative to the Taylor method of order 2 using two separate evaluations of $f(t, y)$.

(a) Pick the coefficients a_1, a_2, α and β so that $a_1 f(t, y) + a_2 f(t+\alpha, y+\beta f(t, y))$ approximates $T^{(2)}(t, y) = f(t, y) + hf_t(t, y)/2 + hf(t, y)f_y(t, y)/2$ (f_ξ refers to a partial derivative of f with respect to the variable ξ).

(b) You should find the solution is non-unique. Set $a_1 = 1/2$, which gives the Modified Euler Method.

(c) Find the truncation error of the Modified Euler Method. What is the order of this method?

(d) For the IVP from question 1, approximate $y(1)$ with $h = 0.5$ using the Modified Euler Method by hand (include your calculations).

Question 3: Heun's Method has error proportional to h^3, and the iterative step is,

$$w_{i+1} = w_i + \frac{h}{4}\left[f(t_i, w_i) + 3\left\{f\left(t_i + \frac{2h}{3}, w_i + \frac{2h}{3} f\left(t_i + \frac{h}{3}, w_i + \frac{h}{3} f(t_i, w_i)\right)\right)\right\}\right].$$

(a) Write code to implement Heun's Method. Include the code with your homework submission.

(b) Verify numerically that Heun's Method has error proportional to h^3 by comparing the error in to Euler's Method and the Modified Euler Method and produce a plot showing the errors in each as h is varied.