MATH 151B Applied Numerical Methods, Homework 1

Question 1: Determine whether the following IVP's are well-posed.

(a)

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1+y}{t}, \quad 1 \le t \le 2, \quad y(1) = 2$$

(b)

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y\cos(t), \quad 0 \le t \le 1, \quad y(0) = 1$$

Question 2: Consider the IVP

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1+t}{1+y}$$

with $1 \le t \le 2$ and $y(1) = 2$.

(a) By hand, compute an approximation to $y(2)$ using Euler's method with $h = 0.5$.

(b) Using code, approximate $y(2)$ using Euler's method with $h = 0.5, 0.2, 0.1, 0.01$ and record your results. (You do not have to submit your code).

(c) The exact solution to the IVP is $y(t) = \sqrt{t^2 + 2t + 6} - 1$. Compare your approximations with the exact result $y(2)$, and interpret your results.

Question 3: To derive Euler's method, we truncated the Taylor series expan-sion of y_{i+1} at the linear term. We could truncate at the quadratic term instead, giving the so-called Taylor method of order 2. This method approximates y_{i+1} by

$$w_{i+1} = w_i + hf(t_i, w_i) + \frac{h^2}{2}\frac{\mathrm{d}f}{\mathrm{d}t}\bigg|_{(t_i,w_i)}.$$

(a) For the IVP $y'(t) = y^2 e^{-t}$, $0 \le t \le 1$, $y(0) = 1$, calculate $\frac{\mathrm{d}f}{\mathrm{d}t}$. (Remember $\frac{\mathrm{d}f}{\mathrm{d}t}$ is different to $\frac{\partial f}{\partial t}$ since y is a function of t!)

(b) By hand, use both Euler's method and the Taylor method of order 2 to approximate $y(1)$ with $h = 0.5$.

(c) Modify your code from 2(b) to implement the Taylor method of order 2 to approximate $y(1)$ using $h = 0.5, 0.1, 0.01$. Also use Euler's method to calculate the same approximations. Provide the code your wrote for this question.

(d) The exact solution to the IVP is $y(t) = e^t$. Use this to calculate the errors (where error $e = |w_n - y(1)|$) in your approximations from part (c). Summarise your results, comparing Euler's method to the Taylor method of order 2.