

If  $i(0) = 0$ , find the current  $i$  for the values  $t = 0.1j$ , where  $j = 0, 1, \dots, 100$ .

17. In a book entitled *Looking at History Through Mathematics*, Rashevsky [Ra], pp. 103–110, considers a model for a problem involving the production of nonconformists in society. Suppose that a society has a population of  $x(t)$  individuals at time  $t$ , in years, and that all nonconformists who mate with other nonconformists have offspring who are also nonconformists, while a fixed proportion  $r$  of all other offspring are also nonconformist. If the birth and death rates for all individuals are assumed to be the constants  $b$  and  $d$ , respectively, and if conformists and nonconformists mate at random, the problem can be expressed by the differential equations

$$\frac{dx(t)}{dt} = (b - d)x(t) \quad \text{and} \quad \frac{dx_n(t)}{dt} = (b - d)x_n(t) + rb(x(t) - x_n(t)),$$

where  $x_n(t)$  denotes the number of nonconformists in the population at time  $t$ .

- a. Suppose the variable  $p(t) = x_n(t)/x(t)$  is introduced to represent the proportion of nonconformists in the society at time  $t$ . Show that these equations can be combined and simplified to the single differential equation

$$\frac{dp(t)}{dt} = rb(1 - p(t)).$$

- b. Assuming that  $p(0) = 0.01$ ,  $b = 0.02$ ,  $d = 0.015$ , and  $r = 0.1$ , approximate the solution  $p(t)$  from  $t = 0$  to  $t = 50$  when the step size is  $h = 1$  year.
- c. Solve the differential equation for  $p(t)$  exactly, and compare your result in part (b) when  $t = 50$  with the exact value at that time.