

Math 151B, Homework 5. Submission deadline: Tuesday 9.8, by 8pm

Question 1:

Plot the region of absolute stability for Euler's Method and the Midpoint method. Explain how to derive the equations that you plot.

Question 2:

Solve question 2 in Section 11.1 in the textbook.

Question 3:

Solve question 8 in Section 11.1 in the textbook.

Question 4:

Consider a linear BVP,

$$y'' = 4y - 4x, 0 \leq x \leq 1 \quad (1)$$

with boundary conditions $y'(0) = 0$ and $y(1) = 1$, that is, the slope of the function at $x = 0$ should be zero, rather than the value. Use the finite difference method for linear BVPs to solve this problem. For an approximation with N grid-points, set up a system of $(N + 1) \times (N + 1)$ equations, where the first equation come from a backward difference approximation to $y(0)$ that is accurate to $O(h^2)$, and the last equation is simply $w_N = 1$. Then $i = 2, 3, \dots, N-1$ equations should come from the central-finite-difference discretisation of the differential equation. Derive the formula you use for the backward finite difference approximation to $y'(0)$ (i.e. use the values w_0, w_1 and w_2 approximate $y'(0)$). Show and explain how to set up the matrix equation.