UCLA Quiz 1, Math 164, Summer 2020

Student name and ID number: _____

Instructions:

- This is a 24 hours open-notes exam.
- Clarity will also be considered in grading.
- Please upload directly your solutions to gradescope.

Question	Points	Score
1	17	
2	15	
Total:	32	

1. Consider the function $f(x) = f(x_1, x_2) = (x_1 + x_2^2)^2$.
   (a) (3 points) Derive the gradient of $f(x)$.
   (b) (3 points) At the point $x^{(0)} = (0, 1)^T$, we consider the search direction $d = (1, -1)^T$. Show that $f(x^{(0)})$ decreases into the direction d.
   (c) (3 points) Find a step-size α that minimizes $f(x^{(0)} + \alpha d)$. What is $f(x^{(0)} + \alpha d)$?
   (d) (3 points) Independent of the work before, what is the minimum of $f(x)$ and characterize the points $S \subset \mathbb{R}^2$ that obtain the minima.
   (e) (2 points) Show that $(0, 0)^T$ does not satisfy the SONC.
   (f) (3 points) Does $(-1, 1)$ satisfy the SOSC? Please justify your answer.

2. (15 points) Consider Newton's method to find the root of $f(x) = x^2$ with starting point $x_0 = 10^{20}$. How many steps are required to obtain the root with accuracy 10^{-10}? You can solve the problem either experimentally (if so, you have to code the Newton's method explicitly and attach the code), or analytically. If solving the problem analytically, here is a hint: restart the Newton method if the error is less than 1).