

## Tugas 1

(1). SPL  $\Rightarrow$  Metode eliminasi Gauss

$$2I_1 - I_2 + 3I_3 + 4I_4 = 9$$

$$I_1 - 2I_3 + 7I_4 = 11$$

$$3I_1 - 3I_2 + I_3 + 8I_4 = 8$$

$$2I_1 + I_2 + 4I_3 + 4I_4 = 10$$

$$\begin{array}{l} \left[ \begin{array}{cccc|c} 2 & -1 & 3 & 4 & 9 \\ 1 & 0 & -2 & 7 & 11 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{array} \right] \xrightarrow{\text{R1} \leftrightarrow \text{R2}} \left[ \begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 2 & -1 & 3 & 4 & 9 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{array} \right] \\ \xrightarrow{\text{R2} \leftarrow \text{R2} - 2\text{R1}, \text{R3} \leftarrow \text{R3} - 3\text{R1}, \text{R4} \leftarrow \text{R4} - 2\text{R1}} \left[ \begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & 0 & -14 & 14 & 14 \\ 0 & 0 & 15 & -20 & -25 \end{array} \right] \end{array}$$

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$$I_1 = 1$$

$$I_2 = 0$$

$$I_3 = 1$$

$$I_4 = 2$$

$$\left[ \begin{array}{cccc|c} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right] \xleftarrow{\text{R1} \leftrightarrow \text{R2}} \left[ \begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & 0 & -14 & 14 & 14 \\ 0 & 0 & 15 & -20 & -25 \end{array} \right] \xleftarrow{\text{R2} \leftarrow \text{R2} - 7\text{R1}, \text{R3} \leftarrow \text{R3} - 14\text{R1}, \text{R4} \leftarrow \text{R4} - 15\text{R1}} \left[ \begin{array}{cccc|c} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 8 & -10 \end{array} \right]$$

$$(2) \begin{array}{l} X + 2y - 3z = 4 \\ 3x - y + 5z = 2 \\ 4x + y + (a^2 - 14)z = a + 2 \end{array} \Rightarrow \left[ \begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & a^2 - 14 & a + 2 \end{array} \right] \xrightarrow{\text{R2} \leftarrow 3\text{R1}} \left[ \begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 4 & 1 & a^2 - 14 & a + 2 \end{array} \right] \xrightarrow{\text{R3} \leftarrow 4\text{R1}} \left[ \begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & a^2 - 2 & a + 2 \end{array} \right] \xrightarrow{\text{R3} \leftarrow \text{R3} - \text{R2}} \left[ \begin{array}{ccc|c} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & 0 & a^2 - 16 & a + 12 \end{array} \right]$$

$$(a^2 - 16)z = a - 4$$

$$(a-4)(a+4)z = a - 4$$

$$(i) \rightarrow 0 = k, k \text{ bukan } 0$$

Jika

$$a^2 - 16 = 0 \text{ dan } a - 4 \neq 0$$

$$(a-4)(a+4) = 0$$

$$\text{Maka } a=4 \text{ atau } a=-4$$

Sistem fdk memiliki solusi jika  $a = -4$ 

$$a = -4 = (-4-4)(-4+4)z = -4 \cdot 0$$

$$\rightarrow 0 = -8$$

$$(ii) \text{ Dari } a^2 - 16 = 0 \text{ dan } a - 4 = 0$$

$$(a-4)(a+4) = 0 \text{ dan } a = 4$$

Kedua kondisi terpenuhi jika  $a=4$ 

$$0 = 0 \rightarrow \text{Jika baris terdiri } 0=0, \text{ tdk}$$

hingga banyak solusi  $(4-4)(4+4)z \rightarrow 0(8)_2$ 

$$0=0$$

$$(iii) \text{ nilai unik utk } z, \text{ kec } z=0 \text{ tdk } 0$$

$$a^2 - 16 \neq 0$$

$$(a-4)(a+4) \neq 0$$

$$a \neq 4 \text{ dan } a \neq -4$$

Semua nilai kecuali 4 dan -4

③. SPL → Metode eliminasi Gauss

$$\begin{array}{l} 3x_1 + 2x_2 - x_3 = -15 \\ 5x_1 + 3x_2 + 2x_3 = 0 \\ 3x_1 + x_2 + 3x_3 = 11 \end{array} \Rightarrow \left[ \begin{array}{ccc|c} 3 & 2 & -1 & -15 \\ 5 & 3 & 2 & 0 \\ 3 & 1 & 3 & 11 \end{array} \right] \xrightarrow{\frac{1}{3}B_1} \left[ \begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{1}{3} & -5 \\ 5 & 3 & 2 & 0 \\ 3 & 1 & 3 & 11 \end{array} \right] \xrightarrow{B_2 - 5B_1} \left[ \begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{1}{3} & -5 \\ 0 & -\frac{15}{3} & \frac{15}{3} & 25 \\ 3 & 1 & 3 & 11 \end{array} \right] \xrightarrow{B_2 - 3B_1} \left[ \begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{1}{3} & -5 \\ 0 & -15 & 15 & 25 \\ 3 & 1 & 3 & 11 \end{array} \right]$$

$$\xrightarrow{3B_2} \left[ \begin{array}{ccc|c} 1 & \frac{2}{3} & -\frac{1}{3} & -5 \\ 0 & 1 & -1 & -5 \\ 0 & 1 & 4 & 26 \end{array} \right] \xrightarrow{B_3 - B_2} \left[ \begin{array}{ccc|c} 1 & 0 & \frac{4}{3} & -5 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & -7 & 49 \end{array} \right] \xrightarrow{\frac{1}{7}B_3} \left[ \begin{array}{ccc|c} 1 & 0 & \frac{4}{3} & -5 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 7 \end{array} \right] \xrightarrow{B_1 - \frac{4}{3}B_3} \left[ \begin{array}{ccc|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 7 \end{array} \right]$$

$$x_1 = -4 \quad 5(-4) + 3(-2) + 2(7) = 0$$

$$x_2 = -2 \quad -20 + 6 + 14 = 0$$

$$x_3 = 7 \quad -20 + 20 = 0$$

④ a). A.  $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$

b). B  $\begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$

$$a) \det A = 2(7 \cdot 7 - 6 \cdot 7) - 6(2 \cdot 7 - 6 \cdot 2) + 6(2 \cdot 7 - 7 \cdot 2)$$

$$= 2(7) - 6(2) + 6(0)$$

$$= 14 - 12$$

$$\left[ \begin{array}{ccc|ccc} 2 & 6 & 6 & 1 & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\frac{1}{2}B_1} \left[ \begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 2 & 7 & 6 & 0 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array} \right] \xrightarrow{B_2 - 2B_1} \left[ \begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 2 & 7 & 7 & 0 & 0 & 1 \end{array} \right] \xrightarrow{B_3 - 2B_1} \left[ \begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{array} \right]$$

$$\xrightarrow{B_1 - 3B_3} \left[ \begin{array}{ccc|ccc} 1 & 0 & 3 & \frac{1}{2} & -3 & 0 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right] \xrightarrow{B_1 - 3B_3} \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & 0 & -3 \\ 0 & 1 & 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \end{array} \right] \xrightarrow{A^{-1}}$$

$$b) \det B = -1(4 \cdot -9 - 1 \cdot 2) - 3(2 \cdot -9 - (-4)) - 4(2 \cdot 2 - 4 \cdot 4)$$

$$= -1(-36 - 2) - 3(-18 + 4) - 4(4 + 16)$$

$$= -1(-38) - 3(-14) - 4(20)$$

$$= 36 + 42 - 80$$

$\neq 0 \neq$  invers