

## Tugas 1

①. SPL  $\Rightarrow$  Metode eliminasi Gauss

$$2I_1 - I_2 + 3I_3 + 4I_4 = 9$$

$$I_1 - 2I_3 + 7I_4 = 11$$

$$3I_1 - 3I_2 + I_3 + 5I_4 = 8$$

$$2I_1 + I_2 + 4I_3 + 4I_4 = 10$$

 $\Rightarrow$ 

$$\begin{bmatrix} 2 & -1 & 3 & 4 & 9 \\ 1 & 0 & -2 & 7 & 11 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 2 & -1 & 3 & 4 & 9 \\ 3 & -3 & 1 & 5 & 8 \\ 2 & 1 & 4 & 4 & 10 \end{bmatrix}$$

 $\Downarrow$ 

Eselon Terekursi

$$I_1 = -1$$

$$I_2 = 0$$

$$I_3 = 1$$

$$I_4 = 2$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & 1 & -7 & 10 & 13 \\ 0 & 0 & -14 & 14 & 14 \\ 0 & 0 & 15 & -20 & -25 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 & 7 & 11 \\ 0 & -1 & 7 & -10 & -13 \\ 0 & -3 & 7 & -16 & -25 \\ 0 & 1 & 8 & -10 & -12 \end{bmatrix}$$

$$\textcircled{2} \quad x + 2y - 3z = 4$$

$$3x - y + 5z = 2$$

$$4x + y + (a^2 - 4)z = a + 2$$

$$(a^2 - 16)z = a - 4$$

$$(a - 4)(a + 4)z = a - 4$$

$$(i) \rightarrow 0 = k, \quad k \text{ bukan } 0$$

jika

$$a^2 - 16 = 0 \quad \text{dan} \quad a - 4 \neq 0$$

$$(a - 4)(a + 4) = 0$$

$$\text{maka } a = 4 \text{ atau } a = -4$$

Sistem tdk memiliki Solusi jika  $a = -4$ 

$$a = -4 \Rightarrow (-4 - 4)(-4 + 4)z = -4 - 4$$

$$\rightarrow 0 = -8$$

$$(ii) \text{ jika } a^2 - 16 = 0 \quad \text{dan} \quad a - 4 = 0$$

$$(a - 4)(a + 4) = 0 \quad \text{dan} \quad a = 4$$

kedua kondisi terpenuhi jika  $a = 4$  $0 = 0 \rightarrow$  jika baris terakhir  $0 = 0$ , tdkhingga banyak Solusi  $(4 - 4)(4 + 4)z = 0(8)z$ 

$$0 = 0$$

$$(iii) \text{ nilai unik utk } z, \text{ koef } z \text{ tidak } 0$$

$$a^2 - 16 \neq 0$$

$$(a - 4)(a + 4) \neq 0$$

$$a \neq 4 \quad \text{dan} \quad a \neq -4$$

semua nilai kecuali 4 dan -4



③. SPL → metode eliminasi Gauss

$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= -15 \\ 5x_1 + 3x_2 + 2x_3 &= 0 \\ 9x_1 + x_2 + 3x_3 &= 11 \end{aligned} \Rightarrow \begin{bmatrix} 3 & 2 & -1 & -15 \\ 5 & 3 & 2 & 0 \\ 9 & 1 & 3 & 11 \end{bmatrix} \xrightarrow{\frac{1}{3} \cdot B_1} \begin{bmatrix} 1 & \frac{2}{3} & -\frac{1}{3} & -5 \\ 5 & 3 & 2 & 0 \\ 9 & 1 & 3 & 11 \end{bmatrix} \xrightarrow{\begin{matrix} B_2 - 5B_1 \\ B_3 - 9B_1 \end{matrix}} \begin{bmatrix} 1 & \frac{2}{3} & -\frac{1}{3} & -5 \\ 0 & -\frac{1}{3} & \frac{17}{3} & 25 \\ 0 & -1 & 12 & 46 \end{bmatrix}$$

$$\xrightarrow{-3B_2} \begin{bmatrix} 1 & \frac{2}{3} & -\frac{1}{3} & -5 \\ 0 & 1 & -11 & -75 \\ 0 & -1 & 12 & 46 \end{bmatrix} \xrightarrow{\begin{matrix} B_1 - \frac{2}{3}B_2 \\ B_3 + B_2 \end{matrix}} \begin{bmatrix} 1 & 0 & 7 & 45 \\ 0 & 1 & -11 & -75 \\ 0 & 0 & -7 & -49 \end{bmatrix} \xrightarrow{\frac{1}{-7}B_3} \begin{bmatrix} 1 & 0 & 7 & 45 \\ 0 & 1 & -11 & -75 \\ 0 & 0 & 1 & 7 \end{bmatrix} \xrightarrow{\begin{matrix} B_1 - 7B_3 \\ B_2 + 11B_3 \end{matrix}} \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 7 \end{bmatrix}$$

$$\begin{aligned} x_1 &= -4 \\ x_2 &= -2 \\ x_3 &= 7 \end{aligned} \Rightarrow \begin{aligned} 5(-4) + 3(-2) + 2(7) &= 0 \\ -20 + 6 + 14 &= 0 \\ -20 + 20 &= 0 \end{aligned}$$

④ a).  $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$  b).  $B = \begin{bmatrix} -1 & 3 & -4 \\ 2 & 4 & 1 \\ -4 & 2 & -9 \end{bmatrix}$

$$\begin{aligned} a) \det A &= 2(7 \cdot 7 - 6 \cdot 7) - 6(2 \cdot 7 - 6 \cdot 2) + 6(2 \cdot 7 - 7 \cdot 2) \\ &= 2(7) - 6(2) + 6(0) \\ &= 14 - 12 \end{aligned}$$

$$\begin{bmatrix} 2 & 6 & 6 & | & 1 & 0 & 0 \\ 2 & 7 & 6 & | & 0 & 1 & 0 \\ 2 & 7 & 7 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\frac{1}{2} \cdot B_1} \begin{bmatrix} 1 & 3 & 3 & | & \frac{1}{2} & 0 & 0 \\ 2 & 7 & 6 & | & 0 & 1 & 0 \\ 2 & 7 & 7 & | & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\begin{matrix} B_2 - 2B_1 \\ B_3 - 2B_1 \end{matrix}} \begin{bmatrix} 1 & 3 & 3 & | & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & | & -1 & 1 & 0 \\ 0 & 1 & 1 & | & -1 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{\begin{matrix} B_1 - 3B_2 \\ B_3 - B_2 \end{matrix}} \begin{bmatrix} 1 & 0 & 3 & | & \frac{7}{2} & -3 & 0 \\ 0 & 1 & 0 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & -1 & 1 \end{bmatrix} \xrightarrow{B_1 - 3B_3} \begin{bmatrix} 1 & 0 & 0 & | & \frac{7}{2} & 0 & -3 \\ 0 & 1 & 0 & | & -1 & 1 & 0 \\ 0 & 0 & 1 & | & 0 & -1 & 1 \end{bmatrix} A^{-1}$$

$$\begin{aligned} b) \det B &= -1(4 \cdot -9 - 1 \cdot 2) - 3(2 \cdot -9 - 1 \cdot 4) - 4(2 \cdot 2 - 4 \cdot -4) \\ &= -1(-36 - 2) - 3(-18 - 4) - 4(4 + 16) \\ &= -1(-38) - 3(-22) - 4(20) \\ &= 38 + 66 - 80 \\ &= 24 \neq \text{invers} \end{aligned}$$