Solution for MQP.

III sem,

Discrete Mathematics & Graph theory

PART - A

1.

We have to prove that

$\{p \to (q \to r)\} \to [(p \to q) \to (p \to r)]$ is tautology.

We prove this by using truth table.

p	q	r	s $p \to q$	$q \to r$	t $p \to r$	a $p \to (q \to r)$	b $s \to t$	$a \to b$
1	1	1	1	1	1	1	1	1
1	1	0	1	0	1	0	1	1
1	0	1	0	1	0	1	1	1
1	0	0	0	1	0	1	1	1
0	1	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1
0	0	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1

From the above table, all the values in the last column are 1 (True)

∴ The given compound proposition is tautology

[If very small space is remaining, then start next question from next page.]

P.T.O

2. Given : $A = \{1, 2, 3, 4, 6, 12\}$.

and Relation $R \rightarrow aRb \Leftrightarrow a$ divides b.

then

$R = \{ (1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,6)\ (1,12)$
$(2,2)\ (2,4)\ (2,6)\ (2,12)$
$(3,3)\ (3,6)\ (3,12)$
$(4,4)\ (4,12)$
$(6,6)\ (6,12) \}$.

We need to prove that (R, A) is Reflexive, antisymmetric and transitive. (POSET)

We observe the following.

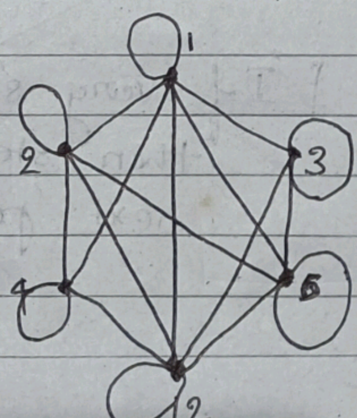
(i) for all $a \in A$, $(a,a) \in R$.
$\therefore$ R is reflexive.

(ii) when, $(a,b) \in R$ and $a \neq b$,
$(b,a) \notin R$.
$\therefore$ R is antisymmetric.

(iii) when $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R$
for ex:
$(1,2) \in R$ & $(2,4) \in R \Rightarrow (1,4) \in R$.

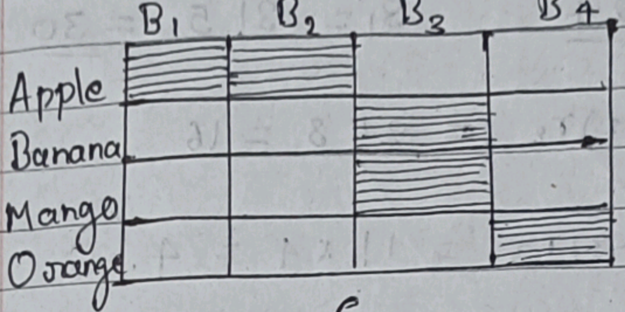
Hence we conclude that (R, A) is **POSET**.

Diagraph of R



3. Given data can be represented in terms of table (Board).

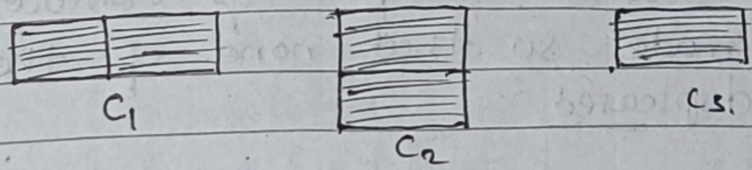
that is;



C

Shaded portion together represents forbidden places in the distribution.

We can observe that, C is formed by the mutually disjoint boards C_1, C_2, C_3



The rook polynomial of C is given by the formula

$$\boxed{r(C,x) = r(C_1,x)\, r(C_2,x)\, r(C_3,x)} \quad (1)$$

We find that

$r(C_1,x) = 1+2x, \quad r(C_2,x) = 1+2x$

$r(C_3,x) = 1+x$

$\therefore$ By eq (1)

$$r(C,x) = (1+2x)(1+2x)(1+x)$$
$$= (1+4x^2+4x)(1+x)$$
$$= 1+x+4x^2+4x^3+4x+4x^2$$
$$= 1+5x+8x^2+4x^3.$$

P.T.O

We have, $r_1 = 5$, $r_2 = 8$, $r_3 = 4$
then by formula

$S_0 = n! = 4!$, $S_1 = (n-1)! r_1$
$\underline{\underline{S_0 = 24}}$. $S_1 = 3! \cdot 5 = \underline{\underline{30}}$

$S_2 = (n-2)! r_2 = 2! \cdot 8 = 16$

$S_3 = (n-3) r_3 = 1! \times 4 = 4.$

$\therefore \bar{N} = S_0 - S_1 + S_2 - S_3 = 6$

This is the no. of ways distribution can be made so that none of the boy is displeased

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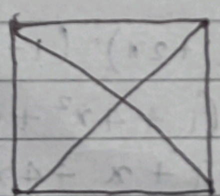
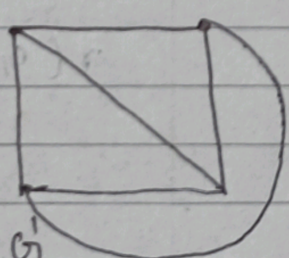
4. Isomorphism:
Consider a graph $G = (V, E)$ and $G' = (V', E')$. Suppose there exists $f : V \to V'$ such that
(i) f is one-one correspondence
(ii) for all vertices A, B of G, $\{A, B\}$ is an edge of G iff $\{f(A), f(B)\}$ is an edge of G'.
Then we say that G & G' are isomorphic Graph.

example:

[USE PENCIL TO DRAW ALL GRAPH]


G


G'

Given graphs are isomorphic to each other as.

(i) there is one-one correspondence between vertices & edges.

that is

$A \leftrightarrow A$, $B \leftrightarrow B$, $C \leftrightarrow D$, $D \leftrightarrow C$

$P \leftrightarrow P$, $Q \leftrightarrow Q$, $R \leftrightarrow R$, $S \leftrightarrow S$.

and

$AB \leftrightarrow AB$, $AC \leftrightarrow AD$, $AP \leftrightarrow AP$ .....

$PQ \leftrightarrow PQ$. $PS \leftrightarrow PS$ ....

and No. of vertices are 8 in both the graph

No. of edges are 12 in both the graph

Each vertex having degree is equal to 3 in both the graph.

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5. Given: {7, 3, 8, 4, 5, 10, 6, 2, 9}

First: We split the given list into subsequent lists as close as half.

{7, 3, 8, 4, 5, 10, 6, 2, 9}.

{7, 3, 8, 4, 5}   {10, 6, 2, 9}.

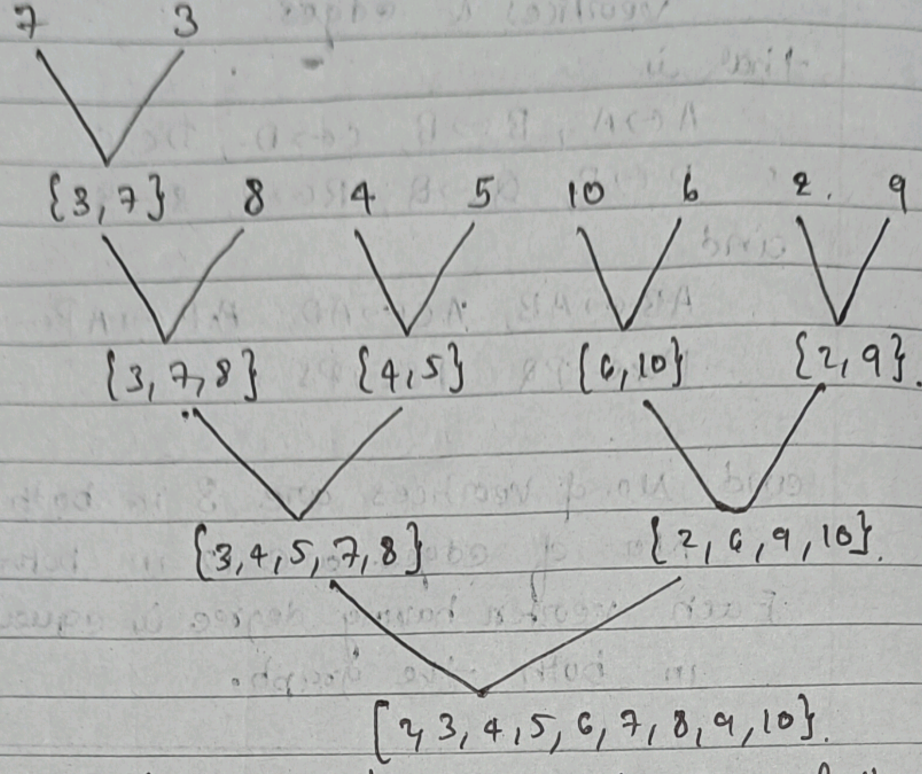
{7, 3, 8}   {4, 5}   {10, 6}   {2, 9}.

{7, 3}   {8}   {4}   {5}   {10}   {6}   {2}   {9}

{7}   {3}.

P.T.O

Second: Now we merge the sublist in non decreasing order.



Thus the sorted form of given list is

{2, 3, 4, 5, 6, 7, 8, 9, 10}

PART - B

6. a) Given:

$p \to q$
$q \to (r \wedge s)$
$\sim r \vee (\sim t \vee u)$
$p \wedge t$
$\therefore u$

To prove the validity of the argument we need to prove that if all the arguments are TRUE then the conclusion 'u' is TRUE.

Since $p \wedge t$ is TRUE(T) $\Rightarrow$ p is T and ~~t is~~ T

$p \to T$
$t \to T$
$q \to T$
$r \to T$
$s \to T$

Since p is T and $p \to q$ is T $\Rightarrow$ q is T

Since q is T and $q \to (r \wedge s)$ is T $\Rightarrow (r \wedge s)$ is T
$\Rightarrow$ r ~~is~~ T and s T

Since r is T and $[\sim r \vee (\sim t \vee u)]$ is True
$F \vee (F \vee u)$

Since $\sim r$ is False, $\sim t \vee u$ must be True.
Since $\sim t$ is False $\boxed{\text{u must True}}$

$\therefore$ The given argument is Valid

P.T.O

8.(b) Given:

The statement

(I) "Every real number has an additive inverse"

same as

"Given any real number x, there is a real no. y such that $x+y=y+x=0$"

that is $\forall x, \exists y, [(x+y)=(y+x)=0]$.

(II) The given statement is same as

"There exists a real no. x such that $xy=yx=y$"

that is $\exists x, \forall y, [xy=yx=y]$.

(III) The given statement is same as

"There exists a integer 'm' and 'n' such that $58=m^2+n^2$.

that is $\exists m, \exists n, \ m^2+n^2=58$

7. a) Given:

We have to use expansion formula to find the rook polynomial of Board 'C'.

		1*
	2	3
4	5	6
7	8	

'C'

Let us put * in square numbered 1.
then By deleting the row & column we get the Board $\boxed{D.}$
and by deleting only the square having * we get Board $\boxed{E}$

	2
4	5
7	8

D.

	2	3
4	5	6
7	8	

E.

Then By expansion formula.

$$\boxed{r(C,x) = x\, r(D,x) + r(E,x)} \longleftarrow (1)$$

Now for 'D'

$r_1 = 5,$

$r_2 = 4$ $\quad (2,4)\ (2,7)\ (4,8)\ (5,7)$

$r_3 = 0 = r_4 \cdots$

$\therefore\ r(D,x) = 1 + 5x + 4x^2$

and for E,

$r_1 = 7$,

$r_2 = 11$, (2, 4) (2,6) (2, 7) (3, 4) (3,5) (3,7) (3,8) (4,8) (5,7) (6,7) (6,8)

$r_3 = 3$, (2,6,7) (3,5,7) (3,4,8)

$$r(E,x) = 1 + 7x + 11x^2 + 3x^3.$$

$\therefore$ By eq^n (1)

$$r(C,x) = x\, r(D,x) + r(E,x)$$

$$= x[1 + 5x + 4x^2] + 1 + 7x + 11x^2 + 3x^3$$

$$= x + 5x^2 + 4x^3 + 1 + 7x + 11x^2 + 3x^3$$

$$\boxed{r(C,x) = 1 + 8x + 16x^2 + 7x^3}$$

This is the required polynomial for C.

(iv)

7b) We need to find the E.G.F for
i) MISSISSIPPI (ii) ISOMORPHISM (iii) ENGINEERING

i) Here, there are, 4 each I, and S
2 P's 1 – M.

therefore required E.G.F is

$$E(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}\right)^2 \left(1 + x + \frac{x^2}{2!}\right)(1+x)$$

ii) Here, 2 → I, S, O, M, 1 → R and P.

therefore E.G.F is

$$E(x) = \left(1 + x + \frac{x^2}{2!}\right)^4 (1+x)^2$$

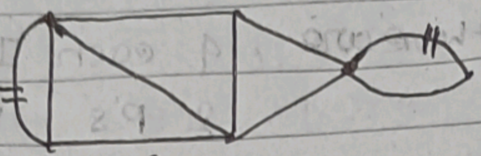
(iii) Here, 3 → E, N, 2 → G and I
1 → R

therefore, E.G.F is

$$E(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)^2 \left(1 + x + \frac{x^2}{2!}\right)^2 (1+x)$$

8.a) **Planarity by Reduction**

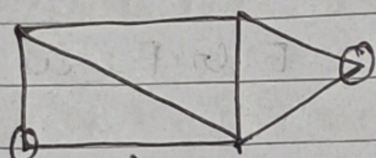
Given:



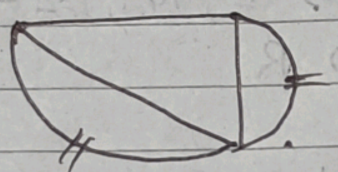
G.

Step 1: subdivide the graph if they are connected by single vertex.

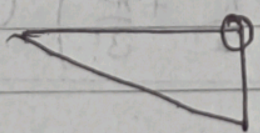
Step 2: Remove ~~R~~ loops & multiple edges.

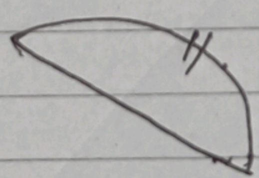


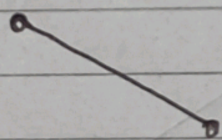
Step 3: Remove the vertices of degree 2.



Step 2 & 3 repeat.



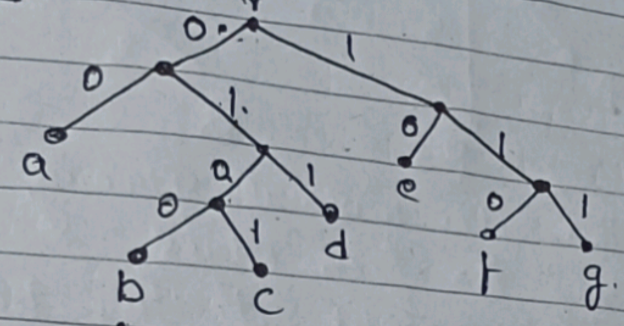




As the reduced graph is single edge which proves that the given graph is planar.

8 b) Prefix Code

Given:



Vertices	a	b	c	d	e	f	g.
Code	00	0100	0101	011	10	110	111..

PART-C

9 a) Recurrence Relation.

Given: $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$,

$F_0 = 0,\ F_1 = 1$

Let $F_{n+2} - F_{n+1} - F_n = 0$

The characteristic eqn is

$$k^2 - k - 1 = 0. \quad [a=1,\ b=-1,\ c=-1]$$

$$k = \frac{1 \pm \sqrt{1+4}}{2} \qquad \left[k = \frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$$

$$k = \frac{1 \pm \sqrt{5}}{2} = \frac{1}{2}(1 \pm \sqrt{5}).$$

∴ The general solution is

$$\boxed{F_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n} - (i)$$

Now put $F_0 = 0$, that $n = 0$, in (1).

$$F_0 = A + B \Rightarrow \boxed{0 = A + B} - (ii)$$

$$\boxed{A = -B} - (a)$$

P.T.O

and $F_1 = 1$, that is, $n = 1$ in (1)

$$F_1 = A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right)$$

$$\boxed{1 = A\left(\frac{1+\sqrt{5}}{2}\right) + B\left(\frac{1-\sqrt{5}}{2}\right)} \quad \text{— (b)}$$

solving (a) & (b) equation.

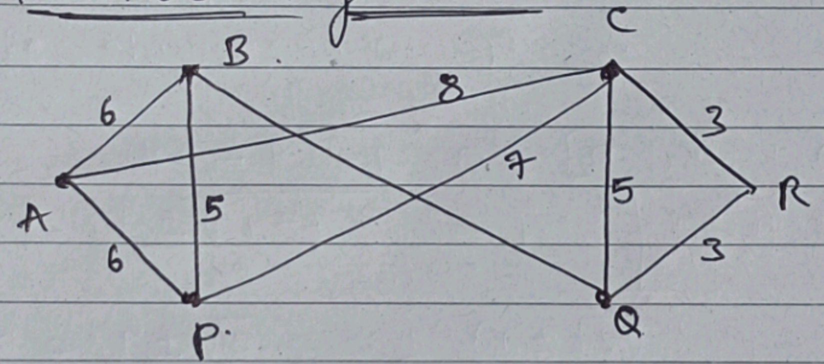
$$A = 1/\sqrt{5} \qquad B = -1/\sqrt{5}$$

then eqn (1) becomes.

$$\boxed{F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n}$$

This is the required soln.

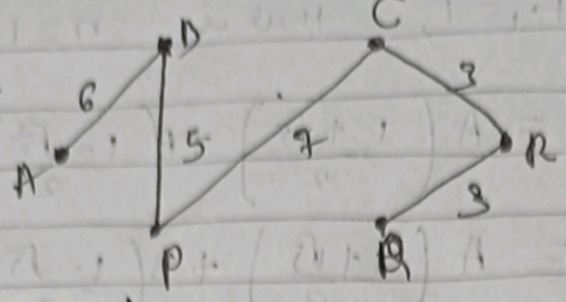
9, b). **Kruskal's Algorithm**



We observe that the given graph has 6 vertices.;
hence a spanning tree will have 5 edges.

Let us put the edges of the graph in the non decreasing order of their weights.

Edge	CR	QR	BP	CQ	AB	AP	CP	AC	BQ
Weight	3	3	5	5	6	6	7	8	10
	Y	Y	Y	No	Y	No	Y.		



This the minimal spanning tree of the graph contains the five edges.

The weight of this tree is <u>24.</u>

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mistakes - TRY TO WRITE PART-C Questions.

1 Question — one page

Wrong ~~PART C D~~ 24