

Department of Computer Science and Engineering

Question Bank

Subject: Discrete Mathematics & Graph theory

Subject Code:21CIDS31

Module-3

Generating Functions and Recurrence Relations

1. Find the Sequences generated by the following functions:

i) $(3+x)^3$

ii) $2x^2(1-x)^{-1}$

iii) $3x^3 + e^{2x}$

2. In each of the following, $f(x)$ is a generating function for the sequences $\langle a_r \rangle$ and $g(x)$ is the generating function for the sequence $\langle b_r \rangle$. Express $g(x)$ in terms of $f(x)$.

i) $b_3 = 3, b_7 = 7, b_n = a_n$ for $n \neq 3, 7$

3. Find the generating function for the following sequences

i) $0, 1, 2, 3, 4, \dots$

ii) $0^2, 1^2, 2^2, 3^2, 4^2, \dots$

4. Determine the coefficient of

i) x^n in the expansion of $(x^2 + x^3 + x^4 + \dots)^4$

ii) x^n in the expansion of $(1 + x^2 + x^4 + \dots)^7$

iii) x^{18} in the expansion of $(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + \dots)^5$

5. Find the number integer solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ under the constraints $x_i \geq 0$ for $i = 1, 2, 3, 4, 5$ and further x_2 is even and x_3 is odd.

6. Find the generating function for the number of integer solutions to the equation $c_1 + c_2 + c_3 + c_4 = 20$, where $-3 \leq c_1, -3 \leq c_2, -5 \leq c_3 \leq 5$ and $0 \leq c_4$. Also find the number of such solutions.

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7. In how many ways can 12 oranges be distributed among three children A, B, and C, so that A gets at least four, B and C get at least two, but C gets no more than five?
8. Using generating function, find the number of partitions of $n = 6$ into distinct summands.
9. Using exponential generating function, find the number of ways in which 4 of the letters in ENGINE be arranged.
10. A company appoints 11 software engineers, each of whom is to be assigned to one of four offices of the company. Each office should get at least one of these engineers. In how many ways can these arrangements be made?
11. Solve the recurrence relation $a_n = na_{n-1}$, for $n \geq 1$ given that $a_0 = 1$.
12. There are 3 pegs fixed vertically on a table top, and n circular disks having holes at their centers and having increasing diameters are slipped onto one of these pegs, with the largest disk at the bottom. The disks are to be transformed one at a time on to another peg with the condition that at no time a larger disk is put on a smaller disk. Determine the no of moves for the transfer of all the disks, so that at the end the disks are in the original order.
13. Solve the recurrence relation $a_n + a_{n-1} - 6a_{n-2} = 0$ for $n \geq 2$, given that $a_0 = -1$ and $a_1 = 8$.
14. Solve the recurrence relation $a_n = 2(a_{n-1} - a_{n-2})$ for $n \geq 2$, given that $a_0 = 1$ and $a_1 = 2$.
15. Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$ for $n \geq 0$, given that $F_0 = 0$ and $F_1 = 1$.
16. Solve the recurrence relation $a_n + 4a_{n-1} + 4a_{n-2} = 8$ for $n \geq 2$, given that $a_0 = 1$ and $a_1 = 2$.
17. Solve the recurrence relation $a_{n+2} + 3a_{n+1} + 2a_n = 3^n$ for $n \geq 0$, given that $a_0 = 0$ and $a_1 = 1$.

*** ALL THE BEST ***