JAIN DEEMED-TO-BE UNIVERSITY
FACULTY OF ENGINEERING AND TECHNOLOGY

Department of Mathematics

Question Bank	
Subject: Discrete Mathematics and Graph Theory	**Subject Code:21CIDS31**

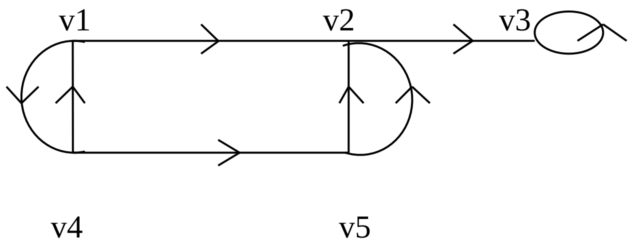
Module-4
GRAPH THEORY

1. Define the following terms with an example for each

   a) Graph
   b) Simple Graph
   c) Multigraph
   d) Discrete graph
   e) Complete graph
   f) Regular graph
   g) Connected graph
   h) Subgraph
   i) Spanning Subgraph
   j) Induced Subgraph
   k) Complement of a subgraph

   Draw a picture of the graph $G=(V,E,\nu)$ where $V=\{a,b,c,d,e\}$ $E=\{e_1,e_2,e_3,e_4,e_5,e_6\}$ and $\nu(e_1)=\nu(e_5)=\{a,c\}$ $\nu(e_2)=\{a,d\}$ $\nu(e_3)=\{e,c\}$ $\nu(e_4)=\{b,c\}$ $\nu(e_6)=\{e,d\}$

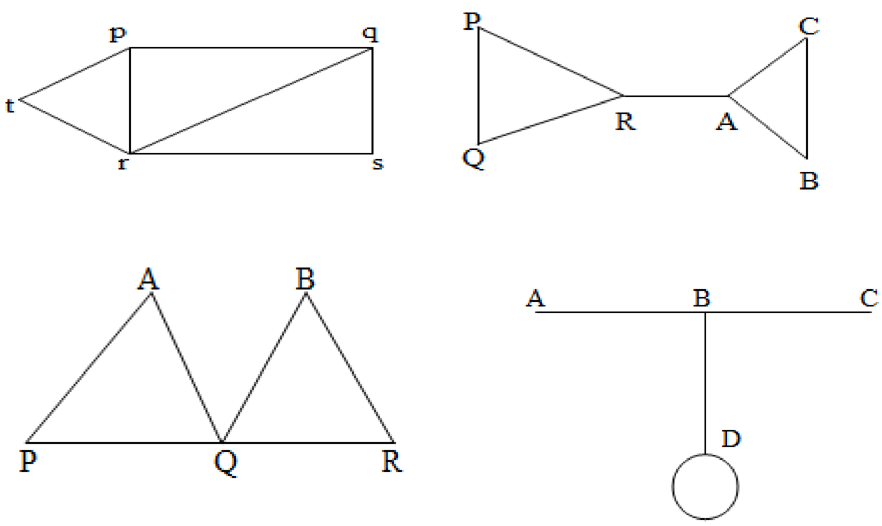
2. i. Find the In-degree and Out-degree Of the graph given below.



   ii. Draw a Graph $G=(V,E)$ where $V=\{a,b,c,d\}$ and $E=\{\{a,b\},\{a,c\},\{b,c\},\{c,d\}\}$.

3. For a graph $G(V, E)$ what is the largest possible value for $|V|$, if $|E|=19$ and $deg(v)= 4$ for all $v \in V$.

4. i. Determine whether the graph has Euler circuit, an Euler trail but no Euler circuit or neither. Give reasons for your choice



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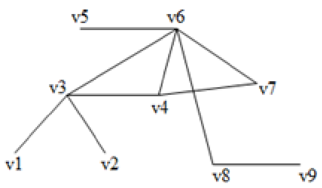
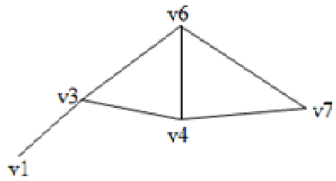
5. Consider the graph G, verify that the graph G1 is an induced sub graph of G. Is this a spanning sub graph of G. Draw the sub graph G2 of G induced by the set V={v3,v4,v6,v8,v9}

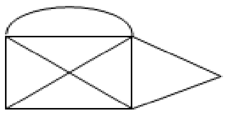


(G)

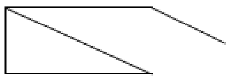


(G1)

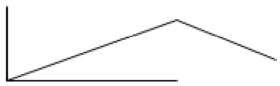
6. For the graph G and its subgraph G1 and G2 find the complement of its subgraph.



G

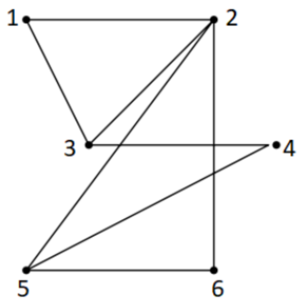


G1

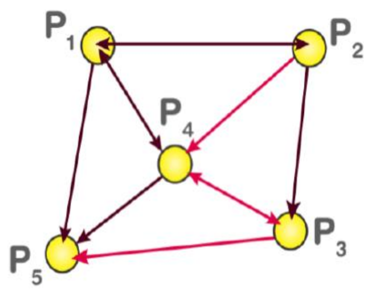


G2

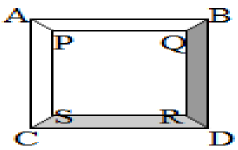
7. Write a adjacency matrix for the following graph

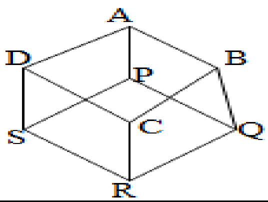


i.



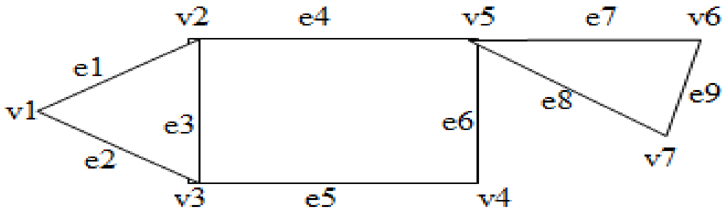
8. Define Isomorphism with example. Verify that the given graphs are isomorphic.





9. i. Define the following terms with an example for each
   - a) Walk
   - b) Closed walk
   - c) Open walk
   - d) Trail
   - e) Circuit
   - f) Path
   - g) Cycle.

   ii. For the graph, determine



a) A walk from v2 to v4 which is not a trail

b) v2-v4 trail which is not a path

c) A path from v2 to v4

d) A closed walk from v2 to v2 which is not a circuit.

e) A circuit from v2 to v2 which is not a cycle

f) A cycle from v2 to v2

g) The number of paths from v2 to v6

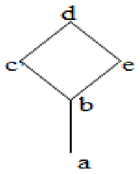
10. Prove that the sum of the degree of all the vertices in a graph is an even number and this number is equal to twice the number of edges in the graph.

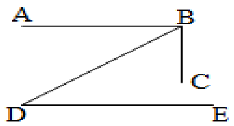
11. i. Define the terms with example for each:

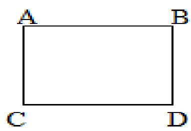
a) Hamiltonian path

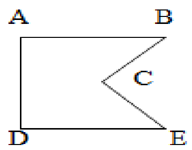
b) Hamiltonian circuit

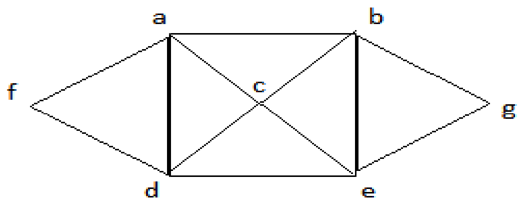
ii. Determine whether the graphs have Hamiltonian circuit, a Hamiltonian path but no Hamiltonian circuit or neither.









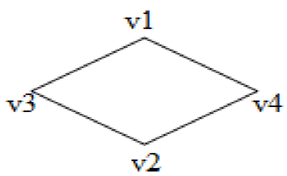


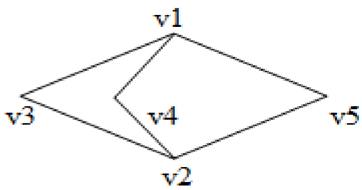
12. i. Define the terms with example for each:

a) Planar Graph

b) Non planar graph

ii. Show that the bipartite graph K2,2 and K2,3 are planar graph





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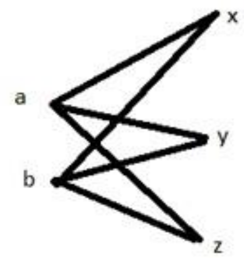
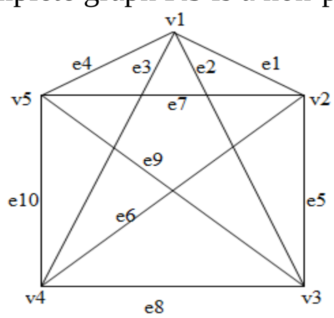
13. Consider the graph K2,3 shown in the below Figure. Let A denote the number of colors available to properly color the vertices of this graph. Find:

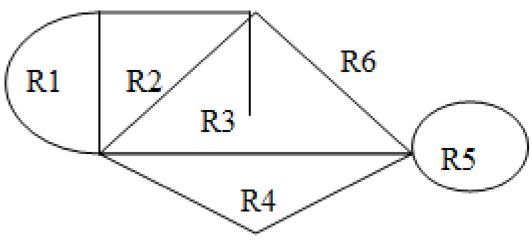
   i. how many proper colorings of the graph have vertices a, b colored the same?
   ii. how many proper colorings of the graph have vertices a, b colored differently?
   iii. the chromatic polynomial of the graph.



14. Show that the complete graph K5 is a non-planar graph

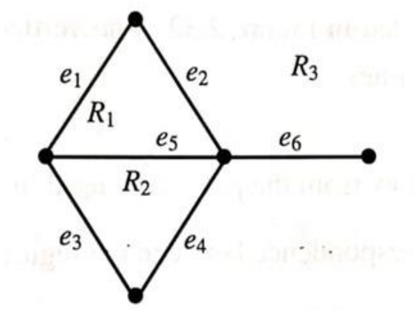


15. Verify Euler's formula for the planar graph. Find the degree of each Region and show that sum of degree of regions is equal to twice the number of edges.



ii. A connected planar graph G has 9 vertices with degree 2,2,3,3,3,4,5,6,6. Find the number of regions of G.

16. Construct the duals for the following planar graphs.



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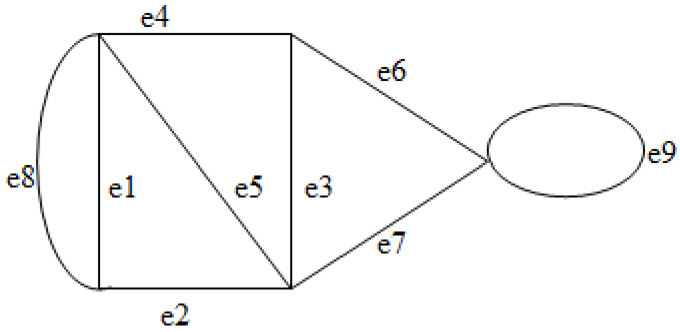
<tr><td></td><td>Steps for detection of planarity by Elementary Reduction. Find the Planarity of the graph given.

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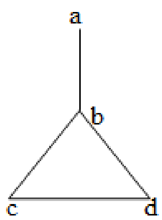
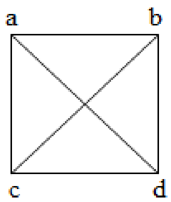
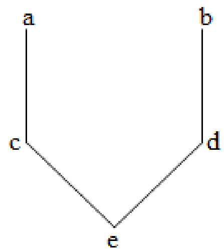
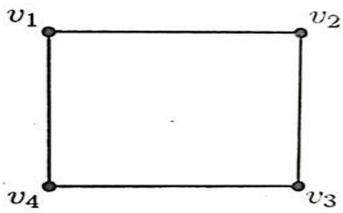
<tr><td>17.</td><td>Find the chromatic polynomial for the graph.




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