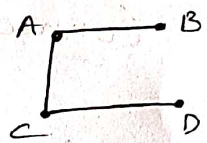
Module - 4.

a. Graph:-

A graph is a pair (V,E), where V is a non-empty set and E is a set of unordered pairs of elements taken from the set V

For a Graph (V,E), the elements of V are called vertices and the elements of E are called undirected edges. The set V is called Vertex set and the set E is called Edge set.

It is denoted by $G = (V,E)$ or $G = G(V,E)$



b. Simple Graph:-

A Graph which doesn't contain loops & multiple edges is called a simple Graph

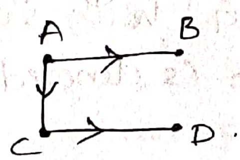
c. Multi Graph:-

A Graph which contains multiple edges ~~or~~ but no loops is called a Multigraph.

d. D~~iscrete~~rected Graph:-

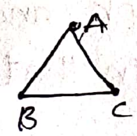
A directed graph is a pair (V,E), where V is a non empty set and E is a set of ordered pairs of elements taken from the set V

For a directed graph (V,E), the elements of V are called Vertices and the elements of E are directed edges. The set V is called the vertex set and the set E is called the directed edge set.



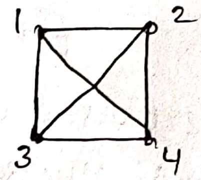
e. Complete Graph:-

A simple graph of order ≥ 2 in which there is an edge between every pair of vertices is called a complete graph.



f. Regular Graph:-

A Regular Graph is a graph where each vertex has the same number of neighbors i.e every vertex has the same degree or valency. A regular directed graph must also satisfy the stronger condition that the indegree and outdegree of each vertex are equal to each other.

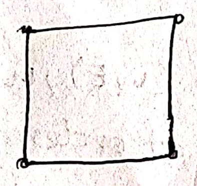


g. Connected Graph:-

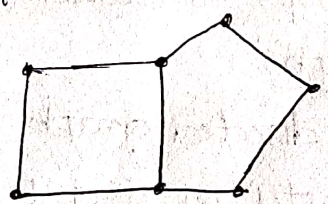
A graph is a connected graph if, for each pair of vertices there exists atleast one single path which joins them.

h. Subgraph:-

A graph whose vertices and edges are subsets of another graph



Sub Graph



Graph

i. Spanning subgraph:-

A Spanning subgraph of graph G is a Subgraph obtained by edge deletions only (so that a spanning subgraph is a subgraph of G with the same vertex set as G). With S set of deleted edges, the spanning subgraph is denoted G\S

j. Induced Subgraph:-

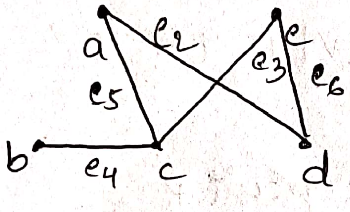
An Induced Subgraph of a graph is another graph formed from a subset of the vertice of the graph, and all of the edges connecting pairs of vertices in that subset.

k. Complement of Subgraph :-

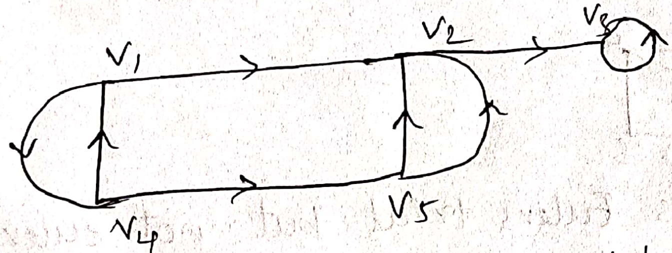
A Subgraph Complement of a graph G is a graph obtained from G by complementing all the edges of one of its induced subgraphs.

Graph Complement Graph.

b.

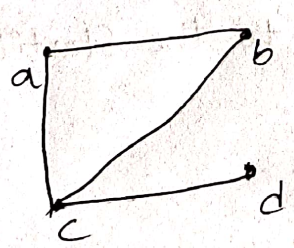


2 i.



	Indegree	Outdegree
V_1	1	2
V_2	3	1
V_3	2	1
V_4	1	2
V_5	1	2

ii.



3. Since all vertices are of degree greater than or equal to 4, the sum of degrees of vertices is greater than or equal to 4n.

Given $|E| = 19$ $\deg(v) = 4$.

3. According to Handshaking property

$\sum d(v) = \deg(v) \times n$ where n is no of vertices

$\sum d(v) = 4n$

$2|E| = 4n$

$2(19) = 4n$

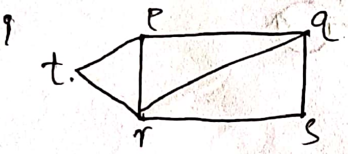
$n = 19/2$

$n = 9.5$

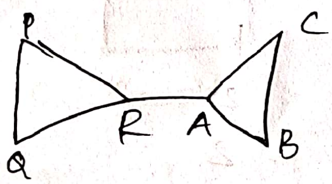
$\boxed{n \approx 9 \text{ or } 10}$

The largest possible value for $|V|$ is 9 or 10

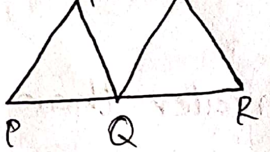
4. Euler's Circuit:- connected + Circuit



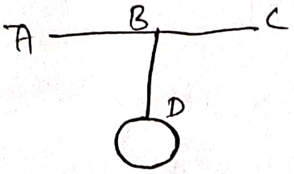
Here this is Euler's trial but not euler's circuit because we can't get back to the starting vertex without repeating the edge.



Here this is Euler's trial but not euler's circuit because we can't get back to the Starting vertex. without repeating the edge



Here this is Euler's Circuit and Euler's trail because without repetition we can get the starting vertex

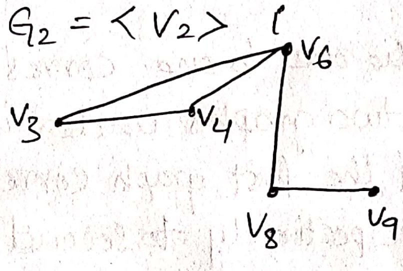
4 i 

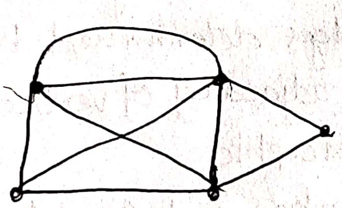
Here this is Euler's trail because without repetition we can't reach the starting vertex.

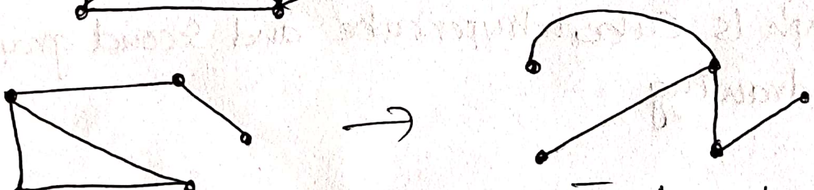
5. The vertex set of the Graph G_1 namely $V_1 = \{v_1, v_3, v_4, v_6, v_7\}$ is a subset of the vertex set $V = \{v_1, v_2 \dots v_9\}$ of G. Also all the edges of G_1 are in G further each edge in G_1 has the same end vertices in G as in G_1. Therefore G_1 is a subgraph of G.

We further check that every edge $\{v_i, v_j\}$ of G where $v_i, v_j \in V_1$ is an edge of G_1 $\therefore$ G_1 is an induced subgraph of G. Since $V_1 \neq V$, G_1 is not a spanning subgraph of G.

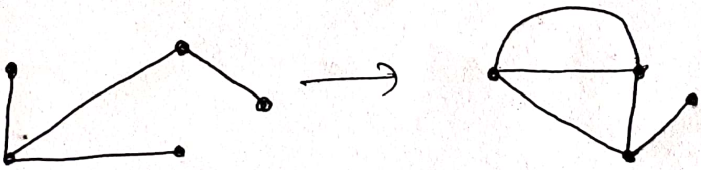
b The subgraph $G_2 = \langle V_2 \rangle$ (



6. G 



G1 $\overline{G1}$ (complement)

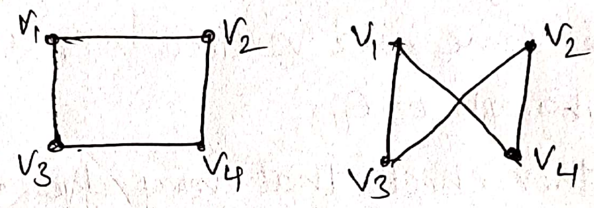


G2 $\overline{G2}$ (complement)

8. <u>Isomorphism</u>:-

Consider two graphs $G=(V,E)$ and $G'=(V',E')$ Suppose there exist a function $f: V \to V'$ such that (i) f is a one to one correspondence and (ii) for all vertices A, B of G, $\{A, B\}$ is an edge of G if and only if $\{f(A), f(B)\}$ is an edge of G' then f is called an Isomorphism b/w G and G'

Example



$u_1 \leftrightarrow V_1$; $u_2 \leftrightarrow V_4$; $u_3 \leftrightarrow V_3$; $u_4 \leftrightarrow V_2$

$\{u_1, u_2\} \leftrightarrow \{V_1, V_4\}$, $\{u_1, u_3\} \leftrightarrow \{V_1, V_3\}$

$\{u_2, u_4\} \leftrightarrow \{V_4, V_2\}$ $\{u_3, u_4\} \leftrightarrow \{V_3, V_2\}$

---x---

Let us consider the one to one correspondence between the vertices of the two graphs under which the vertices A, B, C, D, P, Q, R, S of the first graph correspond to the vertices $A', B', C', D', P', Q', R', S'$ respectievely to second graph and viceversa In this correspondence the edges determined by the corresponding vertices correspond so that the adjacency of vertices retained As such two graphs are isomorphic first graph is Cube or hypercube and second graph is just another drawing

9 ii.

a. $v_2 e_4 v_5 e_7 v_6 e_9 v_7 e_8 v_5 e_4 v_2 e_3 v_3 e_5 v_4$

b. $v_2 e_3 v_3 e_2 v_1 e_1 v_2 e_4 v_5 e_6 v_4$

c. $v_2 e_3 v_3 e_5 v_4$

d. $v_2 e_4 v_5 e_7 v_6 e_9 v_7 e_8 v_5 e_4 v_2$

e. $v_2 e_3 v_3 e_5 v_4 e_6 v_5 e_7 v_6 e_9 v_7 e_8 v_5 e_4$

f. $v_2 e_1 v_1 e_2 v_3 e_3 v_2$

g. six

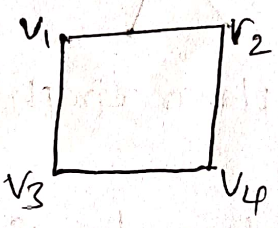
10.

$$\sum d(v_i) = 2|E|$$

$$d(v_1)+d(v_2)+d(v_3)+d(v_4) = 2(4)$$

$$2+2+2+2 = 2(4)$$

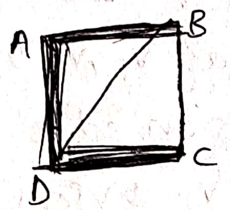
$$\boxed{8 = 8}$$



11. a. Hamiltonian path:-

A path in a connected graph which includes every vertex (but not necessarily every edge) of the graph is called Hamilton path

Ex:-



b. Hamiltonian Circuit:-

It is a circuit that visits every vertex at once with no repeats.

12 a. Planar Graph:-

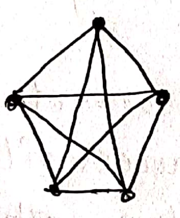
A graph which can be represented by atleast one plane drawing (drawing done on a plane surface) in which the edges meet only at the vertices is called a Planar Graph

Ex:-

b. Non planar Graph:-

A graph which cannot be represented by a plane drawing in which the edges meet only at the vertices is called a Non planar Graph

Ex:-



ii) In $K_{2,2}$ the vertex set is made up of two bipartites V_1 & V_2 with V_1 containing two vertices, say v_1, v_2 & V_2 containing two vertices say v_3, v_4 and there is an edge joining every vertex in V_1 with every vertex in V_2 & viceversa in this figure edges meet only at vertices $\therefore K_{2,2}$ is planar Graph.

In $K_{2,3}$ the vertex set is made up of two bipatites V_1 & V_2 with V_1 containing two vertices say $v_1 v_2$ & V_2 containing three vertices say v_3, v_4, v_5 and there is an edge joining every vertex in V_1 with every vertex in V_2 & viceversa $\therefore K_{2,3}$ is a planar Graph

13.

i. If the vertices a and b are to have the same colour then there are λ choices for colouring the vertex a and only one choice for the vertex b. Consequently, there are $\lambda-1$ choices for each of the vertices x, y, z. Hence the no of proper colourings is $\lambda(\lambda-1)^3$.

ii. $\lambda(\lambda-1)(\lambda-2)^3$.

If the vertices a and b are to have different colours then there are λ choices for colouring the vertex a and $(\lambda-1)$ choices for the vertex b. Consequently there are $(\lambda-2)$ choices for each of vertices x, y, z. Hence the no of proper colouring is

iii. Since the two cases of vertices a and b having the same colour or diff colours are exhaustive and mutually exclusive the chromatic polynomial of graph is.

$$P(K_{2,3}, \lambda) = \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2)^3.$$

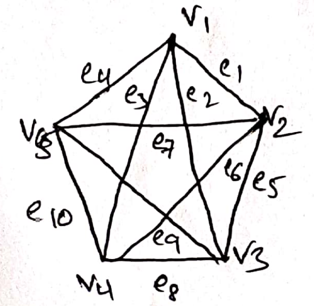
14. K_5 having 5 vertices & edge b/w every pair of vertices.

$V = \{v_1 \dots v_5\}$ $E = \{e_1, \dots e_{10}\}$

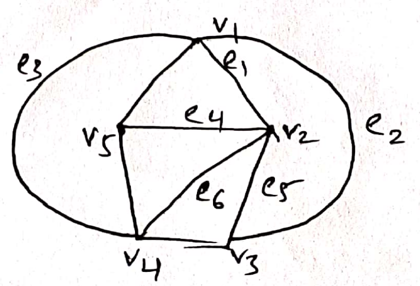
The edges $e_1, e_5, e_8, e_{10}, e_4$ form pentagonal cycle remaining edges e_2, e_3, e_6, e_7, e_9 are all wide he cycle & intersect at points other than the vertices.

Let us try to ~~know~~ draw K_5 in which edge meets only at vertices the edge $e_9 = [v_8 v_5]$ If we draw this edge outside the pentagon it intersect e_3, & ~~if~~ we draw inside it it will intersect e_6 (PPT: 115)

$\therefore K_5$ is nonplanar



K_5



15 $d(R_1) = 2$ $d(R_2) = d(R_4) = 3$ $d(R_3) = 5$ $d(R_5) = 1$ $d(R_6) = 6$.

$$d(R_1) + d(R_2) + d(R_3) + d(R_4) + d(R_5) + d(R_6) = 20$$

$$2+3+5+1+6+3 = 20$$

(PPT : 118

$$20 = 20$$

ii. The given graph has $n=9$ vertices let m be the no of edges and r be the no of regions.

By the hand shaking property we have

$$2m = \text{sum of degree of vertices}$$

$$= 2+3+2+3+3+4+5+6+6 = 34$$

$\therefore m = 17$.

By using Euler's formula we find that

$$r = m - n + 2 = 17 - 9 + 2 = 10.$$

$\therefore$ the given graphs has 10 regions.

16. PPT – 134

PPT - 126

17.