1. Construct the truth tables for the following compound propositions (i) p ^ (~q) (ii) (~p) ˅ q (iii) p → (~q)

Ans;

p	q	~p	~q	p ^ (~q)	(~p) ∨ q	p → (~q)
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	1	0

2. Let p and q be primitive statements for which the conditional p → q is false. Determine the truth values of the following compound propositions:

(i) p ^ q (ii) (~p) ˅ q (iii) q → p (iv) (~q) → (~p)

SOLUTION:

Since p → q is given to be 0, p has to be 1 and q has to be 0. Consequently,

(~p) has to be 0 and (~q) has to be 1, therefore:

i) Since p is 1 and q is 0

By conjunction: T T =T , else F

the truth value of p ^ q is 0 (False)

ii) Since ~p is 0 and q is 0, then

By Disjunction; F F = F

hence the truth value of (~p) ˅ q is 0.

iii) Since q is 0 , p is 1

Then by conditional, F T = T,

so the truth value of q → p is 1.

iv) Since ~q is 1, ~p is 0

By conditional, T F= F

the truth value of (~q) → (~p) is 0.

3. Let p, q and r be propositions having truth values 0, 0 and 1 respectively. Find the truth values of the following compound propositions.

(1) $(p \vee q) \vee r$ (2) $(p \wedge q) \wedge r$ (3) $(p \wedge q) \rightarrow r$

(4) $p \rightarrow (q \wedge r)$ (5) $p \wedge (r \rightarrow q)$ (6) $p \rightarrow (q \rightarrow (\sim r))$

SOLUTION:

Since p, q and r having truth values 0, 0 and 1, ~r is 0

1) Since both p and q are 0,

By disjunction: "0 0 = 0"

=> $(p \vee q)$ is 0 Since r is 1,

By disjunction: 0 1 = 1

=> $(p \vee q) \vee r$ is 1

2) Since both p and q are 0,

By Conjunction: "0 0= 0"

$p \wedge q$ is 0.

Since $p \wedge q$ is 0 and r is 1,

By Conjunction: 0 1= 0

Thus, the truth value of $(p \wedge q) \wedge r$ is 0.

3) From (2) Since $p \wedge q$ is 0 and r is 1,

By Conditional: "1 0 = 0, else 1"

Hence $(p \wedge q) \rightarrow r$ gives "0 1 = 1" , $(p \wedge q) \rightarrow r$ is 1.

Thus, the truth value of $(p \wedge q) \rightarrow r$ is 1.

4) Since q is 0 and r is 1,

By conjunction: 1 1=1, else 0

Hence $q \wedge r$ gives 0 1 = 0. , $q \wedge r$ is 0

Also, p is 0.

Therefore, p → (q ^ r) gives 0 0

By conditional "1 0 = 0 else 1"

Thus, the truth value of p → (q ^ r) is 1.

5) Since r is 1 and q is 0

By conditional: "1 0 = 0", else 0

r → q gives "1 0" which is 0

Also, p is 0.

Hence, p ^ (r → q) gives 0 0

By conjunction: "0 0 = 0"

Thus, the truth value of p ^ (r → q) is 0.

6) Since r is 1, ~r is 0. Since q is 0,

q → (~r) gives 1 0

By conditional: 1 0 = 0

Therefore, q → (~r) is 0

Also, p is 0. Therefore, p → (q → (~r)) gives 0 0

By conditional: 1 0 =0 else 1

Thus, the truth value of p → (q → (~r)) is 1.

4. If the statement q has the truth value 1, determine all truth value assignments for the primitive statements p, r and s for which the truth value of the statement:
$(q \rightarrow [(\neg p \vee r) \wedge \neg s]) \wedge [\neg s \rightarrow (\neg r \wedge q)]$ is 1.

5. Prove that, for any proposition p, q, r the compound proposition {p → (q → r)} →{(p →q) →(p →r)} is a tautology.
SOLUTION ;

p	q	r	p →q	q → r	p →r	p → (q → r)	(p →q) →(p →r)	p → (q → r)} →{(p →q) →(p →r)}
0	0	0	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1
0	1	0	1	0	1	1	1	1
0	1	1	1	1	1	1	1	1
1	0	0	0	1	0	1	1	1
1	0	1	0	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	1	1	1	1	1	1	1	1

<u>Cond:</u>

→ --- T F – F , else T

6. Prove that, following are tautology.

i. p V [(¬p ∧ q)]   ii. (p V q) V ¬p

7. Prove that for any three propositions p, q and r: $[(p \vee q) \rightarrow r] \Leftrightarrow [(p \rightarrow r) \wedge (q \rightarrow r)]$.

Sol:

p	q	r	pVq	$(p \vee q) \rightarrow r$	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \wedge (q \rightarrow r)$
0	0	0	0	1	1	1	1
0	0	1	0	1	1	1	1
0	1	0	1	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1

8.Prove following logical equivalence without using Truth table.

a. $[(P \vee Q) \wedge (P \vee \neg Q)] \vee Q \Leftrightarrow P \vee Q$

Solution: $[(p \vee q) \wedge (p \vee \neg q)] \vee q$

$\equiv [p \vee (q \wedge p) \vee (q \wedge \neg q)] \vee q$ Distributive

$\equiv [p \vee (q \wedge \neg q)] \vee q$ Commutative

$\equiv [p \vee (F)] \vee q$ inverse law

$\equiv p \vee q$ Absorption Law and identity law $(p \vee F) = p$

b. $[p \vee q \vee (\neg p \wedge \neg q \wedge r)] \Leftrightarrow (p \vee q \vee r)$

2. $[p \vee q \vee (\neg p \wedge \neg q \wedge r)]$	$\Leftrightarrow$	$(p \vee q \vee r)$	
$\Rightarrow [p \vee q \vee (\neg p \wedge \neg q \wedge r)]$	$\equiv$	$\{p \vee q \vee [\neg (p \vee q) \wedge r\}$	de Morgan's Laws
$\Rightarrow$	$\equiv$	$\{(p \vee q) \vee [\neg (p \vee q) \wedge r\}$	Associative Laws
$\Rightarrow$	$\equiv$	$[(p \vee q) \vee \neg (p \vee q)] \wedge [(p \vee q) \vee r]$	Distributive law
$\Rightarrow$	$\equiv$	$[T \wedge [(p \vee q) \vee r]$	
$\Rightarrow$	$\equiv$	$[(p \vee q) \vee r]$	

c. $p \rightarrow (q \rightarrow r) \Leftrightarrow (p \wedge q) \rightarrow r$

Solution $p \rightarrow (q \rightarrow r) \Leftrightarrow p \rightarrow (\neg q \vee r)$	**(Equivalence of Conditional)**
$\Leftrightarrow \neg p \vee (\neg q \vee r)$	**(Equivalence of Conditional)**
$\Leftrightarrow (\neg p \vee \neg q) \vee r$	**(Associative law)**
$\Leftrightarrow \neg(p \wedge q) \vee r$	**(De Morgan's law)**
$\Leftrightarrow (p \wedge q) \rightarrow r$	**(Equivalence of Conditional**

d. $(p \vee q) \vee (\neg p \wedge \neg q \wedge r) \Leftrightarrow (p \vee q \vee r)$

Solution:

$(p \vee q) \vee (\neg p \wedge \neg q \wedge r) \Leftrightarrow (p \vee q) \vee (\neg p \wedge \neg q) \wedge r$,	**by Associative Law**
$\Leftrightarrow (p \vee q) \vee [\neg(p \vee q) \wedge r]$	**by De Morgan Law.**
$\Leftrightarrow [(p \vee q) \vee \neg(p \vee q)] \wedge [(p \vee q) \vee r]$	**(Distributive)**
$\Leftrightarrow T0 \wedge [(p \vee q) \vee r]$	**($\because$ $u \vee \neg u \Leftrightarrow T$, Here, $u : (p \vee q)$)**
$\Leftrightarrow (p \vee q) \vee r$	**($\because$ $T \wedge v \Leftrightarrow v$, Here $v : (p \vee q) \vee r$)**

9. a. $(p \rightarrow r) \vee (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$

Solution:

$(p \rightarrow r) \vee (q \rightarrow r) \Leftrightarrow (\neg p \vee r) \vee (\neg q \vee r)$	**(Equivalence of conditional)**
$\Leftrightarrow (\neg p \vee \neg q) \vee (r \vee r)$	**(Associative)**
$\Leftrightarrow (\neg p \vee \neg q) \vee r$	**($\because$ $r \vee r \Leftrightarrow r$)**
$\Leftrightarrow \neg(p \wedge q) \vee r$	**(De Morgan's law)**
$\Leftrightarrow (p \wedge q) \rightarrow r$	**(Equivalence of conditional**

b. $\neg (p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

11.

Q-4: Write converse, inverse and contrapositive of
(1) If a quadrilateral is a parallelogram, then its diagonals bisect each other.
(2) If a real number x^2 is greater than zero, then x is not equal to zero.
(3) If a triangle is not isosceles, then it is not equilateral. (4) If two lines are parallel, then they are equidistant.

Solution :

(1)	converse	: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.
	inverse	: If a quadrilateral is not a parallelogram, then its diagonals do not bisect each other.
	contrapositive	: If the diagonals of a quadrilateral do not bisect each other, then it is not a parallelogram.
(2)	Converse	: If a real number x is not equal to zero, then x^2 is greater than zero.
	inverse	: If a real number x^2 is not greater than zero, then x is equal to zero.
	contrapositive	: If a real number x is equal to zero, then x^2 is not greater than zero.
(3)	converse	: If a triangle is not equilateral, then it is not isosceles.
	inverse	: If a triangle is isosceles, then it is equilateral.
	contrapositive	: If a triangle is equilateral, then it is isosceles

12.

Q-5 Test Whether following is a valid argument

If Ravi goes out with friends, he will not study.
If Ravi does not study, his father becomes angry
His father is not angry

Ravi has not gone out with friends

p : Ravi goes out with friends
q: Ravi will not study
r : his father becomes angry

Then argument reads

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \sim r \\ \hline \therefore \sim p \end{array}$$

Here we have three premises, so we club two premises to simplify by using rues of inference to make conclusion

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$ By Rule of Syllogism

$$\begin{array}{l} p \rightarrow r \\ \sim r \\ \hline \therefore \sim p \end{array}$$ By Rule of Modus Tollens

14.

Q-1 For the universe of all integers, let p(x), q(x), r(x), s(x) and t(x) denote the following open statements
p(x) : x > 0,
q(x): x is even,
r(x): x is a perfect square,
s(x) : x is divisible by 3,
t(x) : x is divisible by 7.
Write the following symbolic statements in words and indicate its truth value:
i) $\forall x, [r(x) \rightarrow p(x)]$ ii) $\exists x, [s(x) \wedge \sim q(x)]$ iii) $\forall x, \sim[r(x)]$ iv) $\forall x,[r(x) \vee t(x)]$

i)	$\forall x, [r(x) \rightarrow p(x)]$	: For any integer x, if x is a perfect square then x > 0	[false take x=0]
ii)	$\exists x, [s(x) \wedge \sim q(x)]$	: For some integer, x is divisible by 3 and x is not even	[true take x=9]
iii)	$\forall x, \sim[r(x)]$	: for any integer, x is not a perfect square	[false]
iv)	$\forall x,[r(x) \vee t(x)]$	: For any integer x, x is a perfect square or divisible by 7	[false take x=8]

Caption

15.

Example 1: Let p(x, y) and q(x, y) denote the following open statements.

p(x, y): $x^2 \geq y$, **q(x, y): (x+2) < y**

If the universe for both of x, y is the set of all real numbers, determine the truth value of each of the following statements:

(i) $p(2,4)$ **(ii)** $q(1,\pi)$ **(iii)** $p(-3,8) \wedge q(1,3)$
(iv) $p(1/2,1/3) \vee \sim q(-2,-3)$ **(v)** $p(2,2) \rightarrow q(1,1)$ **(vi)** $p(1,2) \leftrightarrow \sim q(3,8)$.

(i) $p(2,4) = 4 \geq 4$, which is true.

(ii) $q(1,\pi) = (1 + 2) < \pi$, which is true.

(iii) $(p(-3,8) \wedge q(1,3)) = [(-3)^2 \geq 8] \wedge [(1 + 2) < 3]$, which is false.

(iv) $(p(1/2,1/3) \vee \neg q(-2,-3)) = [(1/2)^2 \geq (1/3)| \vee [(-2 + 2) \geq -3]$ which is true

(v) $(p(2,2) \rightarrow q(1,1)) = (2^2 \geq 2) \rightarrow ((1+2) < 1)$, which is false.

(vi) $(p(1,2) \leftrightarrow \sim q(3,8)) = (1^2 \geq 2) \leftrightarrow (3 + 2 \geq 8)$, which is true.

16.

Example 3: Write down the following statements in symbolic form using quantifiers:

(1) Every real number has an additive inverse

(2) The set of real numbers has a multiplicative identity.

(3) The integer 58 is equal to the sum of two perfect squares.

(1) The statement
"Every real number has an additive inverse" is the same as:
"Given any real number x, there is a real number y such that x + y = y + x = 0".
In symbols, this reads **∀ x, ∃ y, (x + y = y + x = 0].**
Here, the set of all real numbers is the universe.

(2) The statement
"The set of real numbers has a multiplicative identity" is the same as:
"There exists a real number x such that xy= yxy for every y".
In symbols, this reads $\exists\, x, \forall\, y, \quad [xy = yx = y]$
Here, the set of all real numbers is the universe.

(3) The given statement is the same as ***"There exist integers m and n such that $58 = m^2 + n^2$***
"In symbols, this reads **∃ m, ∃ n, $58 = m^2 + n^2$.**
Here, the set of all integers is the universe.

17.

Example 4 : Determine the truth value of each of the following quantified statements , the universe being the set of all non-zero integers.

(i) $\exists\, x, \exists\, y, [xy = 1]$

(ii) $\exists\, x, \forall\, y, [xy = 1]$

(iii) $\forall\, x, \exists\, y, [xy = 1]$

(iv) $\exists\, x, \exists\, y, [(2x + y = 5) \Lambda (x - 3y = -8)]$

(v) $\exists\, x, \exists\, y, [(3x - y = \ 17) \Lambda (2x + 4y = 3)]$

Solution:

(i) True. (Take x = 1,y= 1).

(ii) False. (For a specified x, xy= 1 for every y is not true).

(iii) False. (For x = 2, there is no integer y such that xy = 1).

(iv) True. (Take x = 1,y= 3).

(v) False. (Equations 3x - y = 7 and 2x+4y = 3 do not have a common integer solution).