

# Week 01

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Name: Đào Minh Đức

Student ID: 2159003

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12.

b.  $\neg q \leftrightarrow r$

If you don't miss the final exam, then you pass the course and conversely.

c.  $q \rightarrow \neg r$

If you miss the final exam, then you don't pass the course.

f.  $(p \wedge q) \vee (\neg q \wedge r)$

You have the flu and miss the final exam, or you don't miss the final exam and pass the course.

15.

a.  $r \wedge \neg p$

b.  $\neg p \wedge q \wedge r$

c.  $r \rightarrow (q \leftrightarrow \neg p)$

d.  $\neg q \wedge \neg p \wedge r$

e.  $(\neg r \wedge \neg p) \rightarrow q$

f.  $(p \wedge r) \rightarrow \neg q$

28.

a. If it snows tonight, then I will stay at home.

**Converse:** If I stay at home, then it snowed tonight.

**Contrapositive:** If it didn't snow tonight, then I didn't stay at home.

**Inverse:** If I didn't stay at home, then it didn't snow tonight.

**b. I go to the beach whenever it is a sunny summer day.**

**Converse:** If it is a sunny summer day, then I go to the beach.

**Contrapositive:** If I don't go to the beach, then it isn't a sunny summer day.

**Inverse:** If I don't go to the beach, then it isn't a sunny summer day.

**c. When I stay up late, it is necessary that I sleep until noon.**

**Converse:** If I sleep until noon, then I stayed up late.

**Contrapositive:** If I didn't stay up late, then I didn't sleep until noon.

**Inverse:** If I didn't sleep until noon, then I didn't stay up late.

**34.**

**e.**

$p$	$q$	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T

**35.**

**d.**

$p$	$\neg p$	$q$	$(p \rightarrow q)$	$(\neg p \rightarrow q)$	$(p \rightarrow q) \wedge (\neg p \rightarrow q)$
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F

**f.**

$p$	$\neg p$	$q$	$\neg q$	$(\neg p \leftrightarrow \neg q)$	$(p \leftrightarrow q)$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
T	F	T	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	T	F	T	T	T	T

44.

a.  $1\ 1000 \wedge (0\ 1011 \vee 1\ 1011)$

0 1011		1 1000
1 1011	>	0 1011
1 1011 (OR)		0 1000 (AND)

b.  $(0\ 1111 \wedge 1\ 0101) \vee 0\ 1000$

0 1111		0 0101
1 0101	>	0 1000
0 0101 (AND)		0 1101 (OR)

c.  $(0\ 1010 \oplus 1\ 1011) \oplus 0\ 1000$

0 1010		1 0001
1 1011	>	0 1000
1 0001 (XOR)		1 1001 (XOR)

d.  $(1\ 1011 \vee 0\ 1010) \wedge (1\ 0001 \vee 1\ 1011)$

1 1011		1 0001		1 1011
0 1010	>	1 1011	>	1 1011
1 1011 (OR)		1 1011 (OR)		1 1011 (AND)

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8.

a. Kwame will take a job in industry or go to graduate school.

**Negation:** Kwame will not take a job in industry and will not go to graduate school.

b. Yoshiko knows Java and calculus.

**Negation:** Yoshiko does not know Java or does not know calculus.

c. James is young and strong.

**Negation:** James is not young or not strong.

d. Rita will move to Oregon or Washington.

**Negation:** Rita will not move to Oregon and will not move to Washington.

10.

b.  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

$p$	$q$	$r$	$(p \rightarrow q)$	$(q \rightarrow r)$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$(p \rightarrow r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

$(p \rightarrow q) \wedge (q \rightarrow r)$	$(p \rightarrow r)$	$(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$
T	T	T
F	F	T
F	T	T

Therefore,  $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$  is a tautology.

c.  $[p \wedge (p \rightarrow q)] \rightarrow q$

$p$	$q$	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Therefore,  $[p \wedge (p \rightarrow q)] \rightarrow q$  is a tautology.

12.

c.

$$\begin{aligned}[p \wedge (p \rightarrow q)] \rightarrow q &\equiv \neg[p \wedge (p \rightarrow q)] \vee q \\ &\equiv \neg p \vee \neg(p \rightarrow q) \vee q \\ &\equiv (\neg p \vee q) \vee \neg(\neg p \vee q) \\ &\equiv T\end{aligned}$$

Therefore,  $[p \wedge (p \rightarrow q)] \rightarrow q$  is a tautology.

d.

$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

$$\begin{aligned}\text{LHS} &\equiv (p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r) \\ &\equiv (p \vee q) \wedge [r \vee (\neg p \wedge \neg q)] \\ &\equiv (p \vee q) \wedge [\neg(p \vee q) \vee r] \\ \Rightarrow (p \vee q) \wedge [\neg(p \vee q) \vee r] \rightarrow r &\equiv \neg(p \vee q) \vee \neg[\neg(p \vee q) \vee r] \vee r \\ &\equiv [\neg(p \vee q) \vee r] \vee \neg[\neg(p \vee q) \vee r] \\ &\equiv T\end{aligned}$$

Therefore,  $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$  is a tautology.

26.

Proof that  $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$$\begin{aligned}\text{LHS} &\equiv \neg\neg p \vee (\neg q \vee r) \\ &\equiv p \vee \neg q \vee r \\ &\equiv \neg q \vee (p \vee r) \\ &\equiv q \rightarrow (p \vee r)\end{aligned}$$

Therefore,  $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$ .

30.

$$\begin{aligned}(p \vee q) \wedge (\neg p \vee r) &\rightarrow (q \vee r) \\ &\equiv \neg(p \vee q) \vee \neg(\neg p \vee r) \vee (q \vee r) \\ &\equiv \neg(p \vee q) \vee p \vee \neg r \vee q \vee r \\ &\equiv \neg(p \vee q) \vee p \vee q \equiv T\end{aligned}$$

Therefore,  $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$  is a tautology.

31.

Proof that  $(p \rightarrow q) \rightarrow r \not\equiv p \rightarrow (q \rightarrow r)$

$$\begin{aligned}\text{LHS} &\equiv \neg(\neg p \vee q) \vee r \\ &\equiv (p \wedge \neg q) \vee r \\ \text{RHS} &\equiv \neg p \vee (\neg q \vee r) \\ &\equiv (p \wedge q) \vee r\end{aligned}$$

Since  $(p \wedge \neg q) \not\equiv (p \wedge q) \Rightarrow$  therefore  $(p \rightarrow q) \rightarrow r \not\equiv p \rightarrow (q \rightarrow r)$ .

