Week 01

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12.

b. $\neg q \leftrightarrow r$

If you don't miss the final exam, then you pass the course and conversely.

c. $q \rightarrow \neg r$

If you miss the final exam, then you don't pass the course.

f. $(p \wedge q) \vee (\neg q \wedge r)$

You have the flu and miss the final exam, or you don't miss the final exam and pass the course.

15.

a. $r \wedge \neg p$

b. $\neg p \wedge q \wedge r$

c. $r \rightarrow (q \leftrightarrow \neg p)$

d. $\neg q \wedge \neg p \wedge r$

e. $(\neg r \wedge \neg p) \rightarrow q$

f. $(p \wedge r) \rightarrow \neg q$

28.

a. If it snows tonight, then I will stay at home.

Converse: If I stay at home, then it snowed tonight.

Contrapositive: If it didn't snow tonight, then I didn't stay at home.

Inverse: If I didn't stay at home, then it didn't snow tonight.

b. I go to the beach whenever it is a sunny summer day.

Converse: If it is a sunny summer day, then I go to the beach.

Contrapositive: If I don't go to the beach, then it isn't a sunny summer day.

Inverse: If I don't go to the beach, then it isn't a sunny summer day.

c. When I stay up late, it is necessary that I sleep until noon.

Converse: If I sleep until noon, then I stayed up late.

Contrapositive: If I didn't stay up late, then I didn't sleep until noon.

Inverse: If I didn't sleep until noon, then I didn't stay up late.

34.

e.

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T

35.

d.

p	$\neg p$	q	$(p \rightarrow q)$	$(\neg p \rightarrow q)$	$(p \rightarrow q) \wedge (\neg p \rightarrow q)$
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F

f.

p	$\neg p$	q	$\neg q$	$(\neg p \leftrightarrow \neg q)$	$(p \leftrightarrow q)$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
T	F	T	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	T	F	T	T	T	T

44.

a. $1\ 1000 \land (0\ 1011 \lor 1\ 1011)$

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0 1011              1 1000
1 1011         >    0 1011
1 1011 (OR)         0 1000 (AND)
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b. $(0\ 1111 \land 1\ 0101) \lor 0\ 1000$

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0 1111              0 0101
1 0101         >    0 1000
0 0101 (AND)        0 1101 (OR)
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c. $(0\ 1010 \oplus 1\ 1011) \oplus 0\ 1000$

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0 1010              1 0001
1 1011         >    0 1000
1 0001 (XOR)        1 1001 (XOR)
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d. $(1\ 1011 \lor 0\ 1010) \land (1\ 0001 \lor 1\ 1011)$

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1 1011              1 0001              1 1011
0 1010         >    1 1011         >    1 1011
1 1011 (OR)         1 1011 (OR)         1 1011 (AND)
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8.

a. Kwame will take a job in industry or go to graduate school.

Negation: Kwame will not take a job in industry and will not go to graduate school.

b. Yoshiko knows Java and calculus.

Negation: Yoshiko does not know Java or does not know calculus.

c. James is young and strong.

Negation: James is not young or not strong.

d. Rita will move to Oregon or Washington.

Negation: Rita will not move to Oregon and will not move to Washington.

10.

b. $[(p \to q) \wedge (q \to r)] \to (p \to r)$

p	q	r	$(p \to q)$	$(q \to r)$	$(p \to q) \wedge (q \to r)$	$(p \to r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	F	T
T	F	F	F	T	F	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	T	T

$(p \to q) \wedge (q \to r)$	$(p \to r)$	$(p \to q) \wedge (q \to r) \to (p \to r)$
T	T	T
F	F	T
F	T	T

Therefore, $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

c. $[p \wedge (p \to q)] \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Therefore, $[p \wedge (p \to q)] \to q$ is a tautology.

12.

c.

$$\begin{aligned} [p \wedge (p \to q)] \to q &\equiv \neg[p \wedge (p \to q)] \vee q \\ &\equiv \neg p \vee \neg(p \to q) \vee q \\ &\equiv (\neg p \vee q) \vee \neg(\neg p \vee q) \\ &\equiv T \end{aligned}$$

Therefore, $[p \wedge (p \to q)] \to q$ is a tautology.

d.

$$[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

$$\begin{aligned} \text{LHS} &\equiv (p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r) \\ &\equiv (p \vee q) \wedge [r \vee (\neg p \wedge \neg q)] \\ &\equiv (p \vee q) \wedge [\neg(p \vee q) \vee r] \\ \Rightarrow (p \vee q) \wedge [\neg(p \vee q) \vee r] \rightarrow r &\equiv \neg(p \vee q) \vee \neg[\neg(p \vee q) \vee r] \vee r \\ &\equiv [\neg(p \vee q) \vee r] \vee \neg[\neg(p \vee q) \vee r] \\ &\equiv T \end{aligned}$$

Therefore, $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$ is a tautology.

26.

$$\text{Proof that } \neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$$

$$\begin{aligned} \text{LHS} &\equiv \neg\neg p \vee (\neg q \vee r) \\ &\equiv p \vee \neg q \vee r \\ &\equiv \neg q \vee (p \vee r) \\ &\equiv q \rightarrow (p \vee r) \end{aligned}$$

Therefore, $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$.

30.

$$(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$$

$$\begin{aligned} &\equiv \neg(p \vee q) \vee \neg(\neg p \vee r) \vee (q \vee r) \\ &\equiv \neg(p \vee q) \vee p \vee \neg r \vee q \vee r \\ &\equiv \neg(p \vee q) \vee p \vee q \equiv T \end{aligned}$$

Therefore, $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology.

31.

$$\text{Proof that } (p \rightarrow q) \rightarrow r \not\equiv p \rightarrow (q \rightarrow r)$$

$$\begin{aligned} \text{LHS} &\equiv \neg(\neg p \vee q) \vee r \\ &\equiv (p \wedge \neg q) \vee r \\ \text{RHS} &\equiv \neg p \vee (\neg q \vee r) \\ &\equiv (p \wedge q) \vee r \end{aligned}$$

Since $(p \wedge \neg q) \not\equiv (p \wedge q) \Rightarrow$ therefore $(p \rightarrow q) \rightarrow r \not\equiv p \rightarrow (q \rightarrow r)$.