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12) (a) If you don't miss the final exam, then you pass the course and conversely

(b) If you miss the final exam, then you don't pass the course

(c) If you have the flu and miss the exam, or you don't miss the final exam and pass the course

15) (a) $r \wedge \neg p$

(d) $\neg q \wedge \neg p \wedge r$

(b) $\neg p \wedge q \wedge r$

(e) $(\neg r \wedge \neg p) \rightarrow q$

(c) $r \rightarrow (q \leftrightarrow \neg p)$

(f) $(p \wedge r) \rightarrow \neg q$

28) (a) Converse: If I stay at home, it ~~won't~~ ^{will} snow tonight.

Contrapositive: If I don't stay at home, it won't snow tonight.

Inverse: If it doesn't snow tonight, I won't stay at home.

(b) Converse: If it is a sunny summer day, I ~~will~~ go to the beach.

Contrapositive: If it isn't a sunny summer day, I don't go to the beach.

Inverse: If I don't go to the beach ~~whenver~~ it isn't a sunny summer day.

(c) Converse: It is necessary that I sleep until noon if I stay up late.

Inverse ~~Contrapositive~~: If I don't stay up late, it isn't necessary that I sleep until noon

Contrapositive: It isn't necessary that I sleep until noon if I don't stay up late.

34) (a)

p	q	$\neg q$	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	T	F	T
F	F	T	F	T	T

35) (d)

p	$\neg p$	q	$(p \rightarrow q)$	$(\neg p \rightarrow \neg q)$	$(p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F

(f)

p	$\neg p$	q	$\neg q$	$(\neg p \leftrightarrow \neg q)$	$(p \leftrightarrow q)$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$
T	F	T	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	T	F	T	T	F	T

44) (a)

$$\begin{array}{r} 01011 \\ 11011 \\ \hline \text{OR: } 11011 \\ 11000 \\ \hline \text{AND: } 11000 \end{array}$$

(b)

$$\begin{array}{r} 01111 \\ 10101 \\ \hline \text{AND: } 00101 \\ 01000 \\ \hline \text{OR: } 01101 \end{array}$$

(c)

$$\begin{array}{r} 01010 \\ 11011 \\ \hline \text{XOR: } 10001 \\ 01000 \\ \hline \text{XOR: } 11001 \end{array}$$

(d)

$$\begin{array}{r} 11011 \\ 01010 \\ \hline \text{OR: } 11011 \end{array}$$

$$\begin{array}{r} 10001 \\ 11011 \\ \hline \text{OR: } 11011 \end{array}$$

$$\begin{array}{r} 11011 \\ 11011 \\ \hline \text{AND: } 11011 \\ \text{AND: } \end{array}$$

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- 8) (a) Kwame won't take a job in industry and won't go to graduate school
 (b) Yoshiko doesn't know Java or doesn't know calculus
 (c) James isn't young or isn't strong
 (d) Rita will not move to Oregon and will not move to Washington

1) ②	p	q	r	$(p \rightarrow q)$	$(q \rightarrow r)$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$(p \rightarrow r)$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
	T	T	T	T	T	T	T	T
	T	T	F	T	F	F	F	T
	T	F	T	F	T	F	T	T
	T	F	F	F	T	F	F	T
	F	T	T	T	T	T	T	T
	F	T	F	T	F	F	T	T
	F	F	T	T	T	T	T	T
	F	F	F	T	T	T	T	T

Therefore $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$ is a tautology

②	p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$[p \wedge (p \rightarrow q)] \rightarrow q$
	T	T	T	T	T
	T	F	F	F	T
	F	T	T	F	T
	F	F	T	F	T

Therefore $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology

$$\begin{aligned}
 12) \text{ ② } [p \wedge (p \rightarrow q)] \rightarrow q &\equiv \neg [p \wedge (p \rightarrow q)] \vee q \\
 &\equiv \neg p \vee \neg (p \rightarrow q) \vee q \\
 &\equiv (\neg p \vee q) \vee \neg (p \rightarrow q) \\
 &\equiv (\neg p \vee q) \vee \neg (\neg p \vee q) \equiv T
 \end{aligned}$$

Therefore $[p \wedge (p \rightarrow q)] \rightarrow q$ is a tautology

$$\text{③ } [(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$$

$$\text{LHS} \equiv (p \vee q) \wedge (\neg p \vee r) \wedge (\neg q \vee r) \equiv \cancel{(p \vee q) \wedge \neg (p \vee q)}$$

$$\equiv (p \vee q) \wedge [r \vee (\neg p \wedge \neg q)] \equiv (p \vee q) \wedge [\neg (p \vee q) \vee r]$$

$$\Rightarrow (p \vee q) \wedge [\neg (p \vee q) \vee r] \rightarrow r \equiv \neg (p \vee q) \vee \neg [\neg (p \vee q) \vee r] \vee r$$

$$\equiv [\neg (p \vee q) \vee r] \vee \neg [\neg (p \vee q) \vee r] \equiv T$$

Therefore $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)]$ is a tautology

26) Proof: $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

$$\begin{aligned} \text{LHS} &\equiv \neg \neg p \vee (\neg q \rightarrow r) \equiv p \vee \neg q \vee r \\ &\equiv \neg q \vee (p \vee r) \equiv q \rightarrow (p \vee r) \equiv \text{RHS} \end{aligned}$$

Therefore $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$

30) $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$

$$\equiv \neg(p \vee q) \vee \neg(\neg p \vee r) \vee (q \vee r)$$

$$\equiv \neg(p \vee q) \vee p \vee \neg r \vee q \vee r \equiv \neg(p \vee q) \vee (p \vee q) \equiv T$$

Therefore $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology

31) Proof: $(p \rightarrow q) \rightarrow r \not\equiv p \rightarrow (q \rightarrow r)$

$$\text{LHS} \equiv \neg(\neg p \vee q) \vee r \equiv (p \wedge \neg q) \vee r$$

$$\text{RHS} \equiv \neg p \vee (\neg q \vee r) \equiv \neg(p \wedge q) \vee r$$

~~Since~~ $(p \wedge \neg q)$ Suppose p, q, r are false

$$\Rightarrow \text{LHS} \equiv (F \wedge \neg F) \vee F \equiv F \vee F \equiv F$$

$$\text{RHS} \equiv \neg(F \wedge F) \vee F \equiv T \vee F \equiv T$$

Therefore $(p \rightarrow q) \rightarrow r \not\equiv p \rightarrow (q \rightarrow r)$