

Page 53

- 8) @ For all animals, if it's a rabbit then it hops.
 Ⓐ For all animals, they are rabbits and they hop.
 Ⓒ There exists an animal, if it's a rabbit then it hops.
 Ⓓ There exists an animal, it's a rabbit and it hops.

- 10) Ⓐ $\exists x (C(x) \wedge D(x) \wedge F(x))$
 Ⓑ $\forall x (C(x) \vee D(x) \vee F(x))$
 Ⓒ $\exists x (C(x) \wedge F(x) \wedge \neg D(x))$
 Ⓓ $\neg \exists x (C(x) \wedge D(x) \wedge F(x))$
 Ⓔ $\forall x (C(x) \vee D(x) \vee F(x))$

- 12) Ⓐ True Ⓑ False Ⓒ False Ⓓ False
 Ⓔ True Ⓕ True Ⓖ True

- 28) $P(x)$: x is in the correct place
 $T(x)$: x is a tool
 $E(x)$: x is in excellent condition

- Ⓐ $\exists x (\neg P(x))$ Ⓓ $\neg \exists x (P(x) \wedge E(x))$
 Ⓑ $\forall x (T(x) \wedge P(x) \wedge E(x))$ Ⓒ $\exists x (T(x) \wedge \neg P(x) \wedge E(x))$
 Ⓒ $\forall x (P(x) \wedge E(x))$

- 32) Ⓐ - Expression: $\forall x (D(x) \rightarrow F(x))$
 - Negation: $\exists x (D(x) \wedge \neg F(x))$
 - English: There exists a dog that does not have fleas.

- Ⓑ - Expression: $\exists x (H(x) \wedge A(x))$
 - Negation: $\forall x (H(x) \rightarrow \neg A(x))$
 - English: If it's a horse, it can't add.

① - Expression: $\forall x (K(x) \rightarrow C(x))$

- Negation: $\exists x (K(x) \wedge \neg C(x))$

- English: There exists a koala that can't climb

② - Expression: $\forall x (M(x) \rightarrow \neg F(x))$

- Negation: $\exists x (M(x) \wedge F(x))$

- English: There exists a monkey that can speak French

③ - Expression: ~~$\forall x (P(x) \rightarrow S(x) \wedge C(x))$~~

- Negation: ~~$\exists x (P(x) \wedge \neg (S(x) \wedge C(x)))$~~

- English: ~~$\forall x (\neg P(x) \vee \neg S(x) \vee \neg C(x))$~~

- English: All pigs can't swim or can't catch fish

Page 64

6) @ Randy Goldberg is enrolled in class CS252.

@ there exists a student who enrolled in class Math 695.

@ There exists a class that Carol Sitea is enrolled in.

@ There exists a student enrolled in both class Math 222 and class CS252.

@ There exists 2 different students x and y that, if x is enrolled in class Z , then y is also enrolled in class Z .

@ There exists 2 different students x and y that, x is enrolled in class Z if and only if y is enrolled in class Z and vice versa.

10) @ $\forall x (F(x, \text{Fred}))$

@ $\neg \exists x (F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$

@ $\forall y (F(\text{Evelyn}, y))$

@ $\neg \exists x (F(x, x))$

@ $\forall x \exists y (F(x, y))$

@ $\neg \exists x \forall y (F(x, y))$

@ $\forall x \exists y (F(y, x))$

②

Page 64

10) (g) $\exists x \exists y \forall z ((x \neq y \wedge x \neq z \wedge y \neq z) \wedge F(\text{Nancy}, x) \wedge F(\text{Nancy}, y) \wedge \neg F(\text{Nancy}, z))$

(h) $\exists x \forall y \forall z (F(y, x) \wedge (z \neq x) \wedge \neg F(y, z))$

(j) $\exists x \exists y \forall z (\neg F(x, x) \wedge F(x, y) \wedge (z \neq y) \wedge \neg F(x, z))$

12) (a) $\neg I(\text{Jerry})$

(b) $\neg C(\text{Rachel}, \text{Chelsea})$

(c) $\neg (C(\text{Jan}, \text{Sharon}) \wedge \neg C(\text{Sharon}, \text{Jan}))$

(d) $\neg \exists x (x \neq \text{Bob} \rightarrow \neg \exists x (C(x, \text{Bob})))$

(e) $\forall x (x \neq \text{Joseph} \wedge C(\text{Sanjay}, x))$

(f) $\exists x (\neg I(x))$

(g) $\neg \forall x (I(x))$

(h) $\exists x \forall y (I(y) \rightarrow (y = x))$

(i) $\exists x \forall y (\neg I(x) \wedge (I(y) \rightarrow (y \neq x)))$

(j) $\forall x \exists y (I(x) \rightarrow (I(y) \vee (y \neq x) \wedge C(x, y)))$

(k) $\exists x \forall y (I(x) \wedge \neg C(x, y))$

(l) $\exists x \exists y (\neg C(x, y) \wedge \neg C(y, x) \wedge (x \neq y))$

(m) $\exists x \forall y (C(x, y))$

(n) $\exists x \exists y \exists z ((x \neq y) \wedge \neg C(x, z) \wedge \neg C(y, z))$

(o) $\exists x \exists y \forall z ((x \neq z) \wedge (y \neq z) \wedge (x \neq y) \wedge C(x, z) \wedge C(y, z))$

28) (a) True

(b) False

(c) False

(d) True

(e) True

(f) False

(g) True

(h) False

(i) False

(j) True

37)

Ⓐ - Expression: $\forall x \exists y \exists z \forall w ((y \neq z) \wedge M(y) \wedge M(z)) \rightarrow (S(x))$

- Expression: $\forall x \exists y \exists z \forall w (S(x) \rightarrow (M(y) \wedge M(z) \wedge (y \neq z)))$

Ⓑ - Expression: $\forall x \exists y \exists z \forall w (S(x) \rightarrow (T(x, y) \wedge T(x, z)))$

$\wedge (y \neq w \wedge z \neq w \wedge y \neq w) \wedge \neg T(x, w))$

- Negation: $\exists x \forall y \forall z \exists w (S(x) \wedge (\neg T(x, y) \vee \neg T(x, z) \vee T(x, w) \vee (y \neq z \wedge z \neq w \wedge y \neq w)))$

- English: There exists a student in this class that has ^{not} taken ~~every~~ exactly two math classes at this school. ~~except for two math classes.~~

Ⓒ - Expression: $\exists x \forall y ((C(y) \wedge V(x, y)) \rightarrow (y \neq \text{Libya}))$

- Negation: $\forall x \exists y ((C(y) \wedge V(x, y)) \wedge (y = \text{Libya}))$

- English: Everyone has visited Libya

Ⓓ - Expression: $\neg \exists x \forall y ((P(x) \wedge M(y)) \rightarrow C(x, y))$

- Negation: $\exists x \exists y ((P(x) \wedge M(y)) \wedge \neg C(x, y))$

- English: Someone hasn't climbed ^{some} ~~any~~ mountains in the Himalayas.

Ⓔ - Expression: $\forall x \exists y ((A(x) \wedge A(y) \wedge M(y, \text{Kevin Brown})) \rightarrow (M(x, \text{Kevin Brown}) \vee M(x, y)))$

- Negation: $\exists x \forall y ((A(x) \wedge A(y) \wedge M(y, \text{Kevin Brown})) \wedge \neg M(x, \text{Kevin Brown}) \wedge \neg M(x, y))$

- English: Some ^{movie} actors ~~hasn't~~ haven't been in a movie with Kevin Brown nor been in a movie with someone who has been in a movie with Kevin Brown.

Page 78

4) Ⓐ Simplification

Ⓓ Addition

Ⓑ Disjunctive syllogism

Ⓒ Hypothetical syllogism

Ⓓ Modus ponens

④

Page 78

15) a) $S(x)$: x is a student in this class $U(x)$: x understands logic

Scope: all students

$$\forall x (S(x) \rightarrow U(x))$$

$$S(\text{Xavier})$$

$$\therefore U(\text{Xavier})$$

This is correct, based on universal modus ponens. If all students in this class can understand logic and Xavier is one of them, he must also understand logic.

b) $C(x)$: x is a computer science major $M(x)$: x is taking discrete mathematics

Scope: people

$$\forall x (C(x) \rightarrow M(x))$$

$$M(\text{Natasha})$$

$$\therefore C(\text{Natasha})$$

This is incorrect, based on fallacy of affirming the conclusion. There can be other majors that also required people to take discrete mathematics, and Natasha isn't necessarily a computer science major.

c) $P(x)$: x is a parrot $F(x)$: x likes fruit

Scope: birds

$$\forall x (P(x) \rightarrow F(x))$$

$$\neg P(\text{pet bird})$$

$$\therefore \neg F(\text{pet bird})$$

This is incorrect, based on fallacy of denying the hypothesis. We only know that parrots ~~don't~~ like fruit, not about other bird species. There can be a bird species that like fruit and the pet bird can belong to that species.

d) $G(x)$: x eats granola everyday $H(x)$: x is healthy

Scope: people

$$\begin{array}{l} \forall x (G(x) \rightarrow H(x)) \\ \neg H(\text{Linda}) \\ \hline \therefore \neg G(\text{Linda}) \end{array}$$

This is correct, based on universal modus tollens.
Since everyone who eats granola everyday are healthy, Linda can't be one of them since she isn't healthy.

- 9) (b) S : I eat spicy food
T : there is thunder while I sleep
D : I have strange dream

$$\begin{array}{l} S \rightarrow D \quad (1) \\ T \rightarrow D \quad (2) \\ \neg D \quad (3) \\ \hline \therefore \neg(S \vee T) \end{array}$$

$$\begin{array}{l} \text{From (1), (3): } S \rightarrow D \\ \neg D \\ \hline \therefore \neg S \quad (\text{modus tollens}) \quad (4) \end{array}$$

$$\begin{array}{l} \text{From (2), (3): } T \rightarrow D \\ \neg D \\ \hline \therefore \neg T \quad (\text{modus tollens}) \quad (5) \end{array}$$

$$\begin{array}{l} \text{From (4), (5): } \neg S \\ \neg T \\ \hline \therefore \neg(S \vee T) \quad (\text{conjunction}) \end{array}$$

Conclusion: I didn't eat spicy food and there is no thunder while I sleep.

- (c) C: good for corporations
U: good for United States
Y: good for you
B: you buy a lots of stuff

$$\begin{array}{l} C \rightarrow U \quad (1) \\ U \rightarrow Y \quad (2) \\ B \rightarrow C \quad (3) \\ \hline B \rightarrow Y \end{array} \quad \begin{array}{l} \text{From (1), (2): } C \rightarrow U \\ U \rightarrow Y \\ \hline \therefore C \rightarrow Y \quad (\text{hypothetical syllogism}) \quad (4) \\ \text{From (3), (4): } B \rightarrow C \\ C \rightarrow Y \\ \hline \therefore B \rightarrow Y \quad (\text{hypothetical syllogism}) \quad (5) \end{array}$$

Conclusion: ^{You}Buy lots of stuff is good for yourself.