CSCI 2110 Data Structures and Algorithms

Module 10: Graphs
Part 1 - Introduction

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Data Structures: Module 10 – Part 1 Srini Sampalli

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Topics

- Definition and motivation
- Graph terminology
- Graph Representation

} Part 1

- Graph Algorithms

} Part 2

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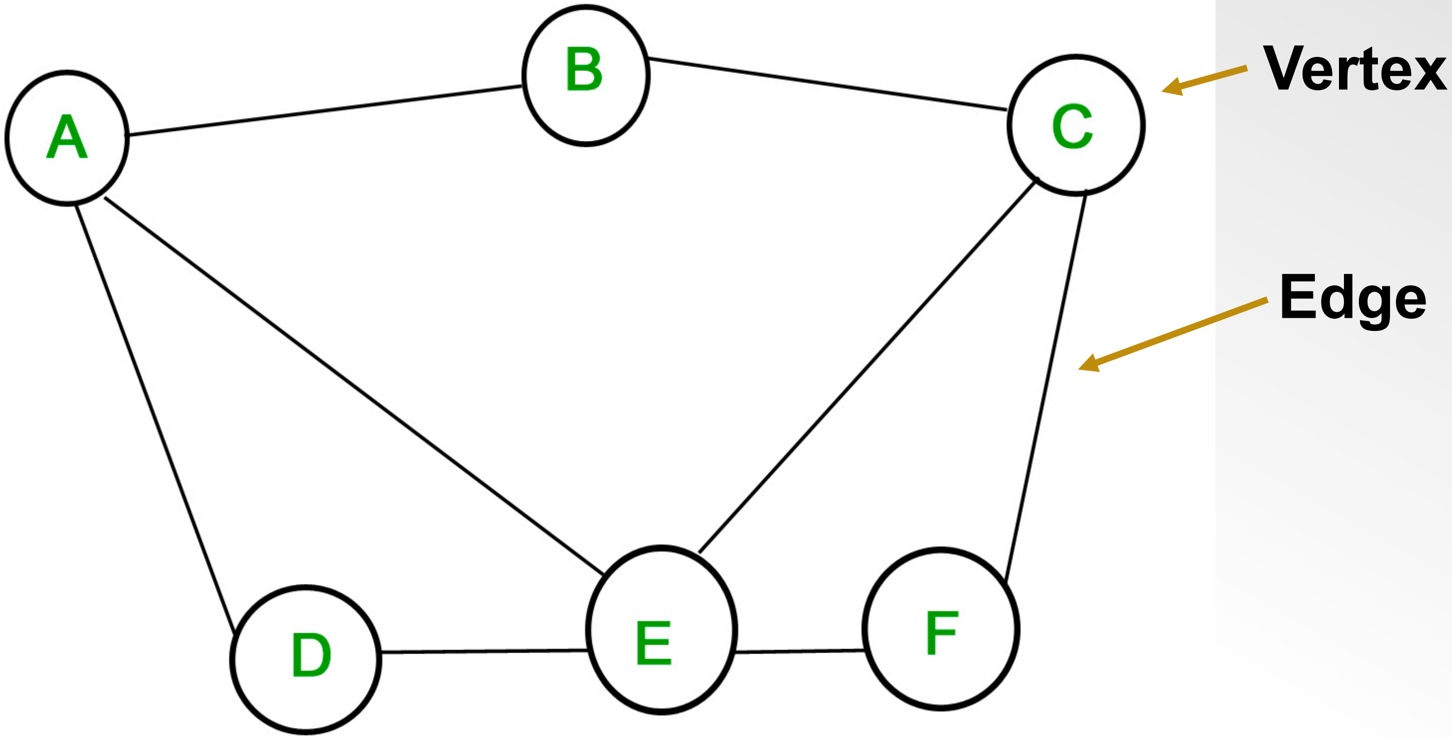
The Graph Data Structure

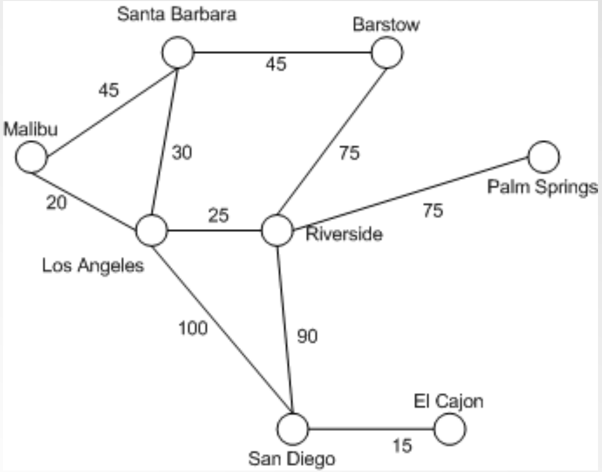
- A graph is a data structure that consists of a set of **nodes** (also called **vertices**) and a set of **edges**.
- Each node or vertex generally holds data.
- Each edge connects a pair of nodes that have some form of relationship between them.
- A graph can be used to represent any geometric pattern – that is, a list, a binary tree, binary search tree, etc. can all be considered as special cases of a graph data structure.
- This makes the graph one of the most widely used data structures.

The Graph Data Structure

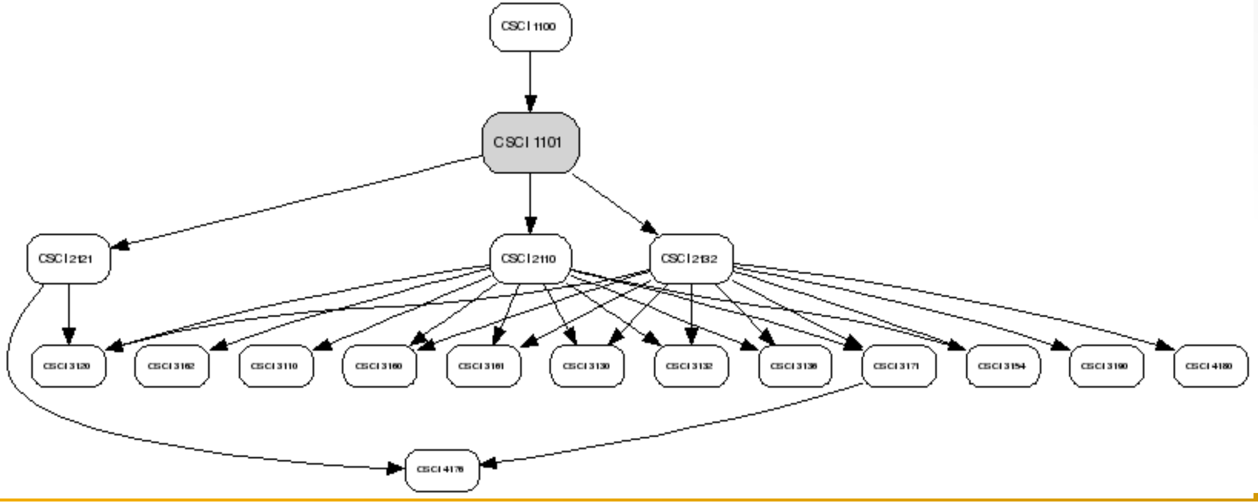


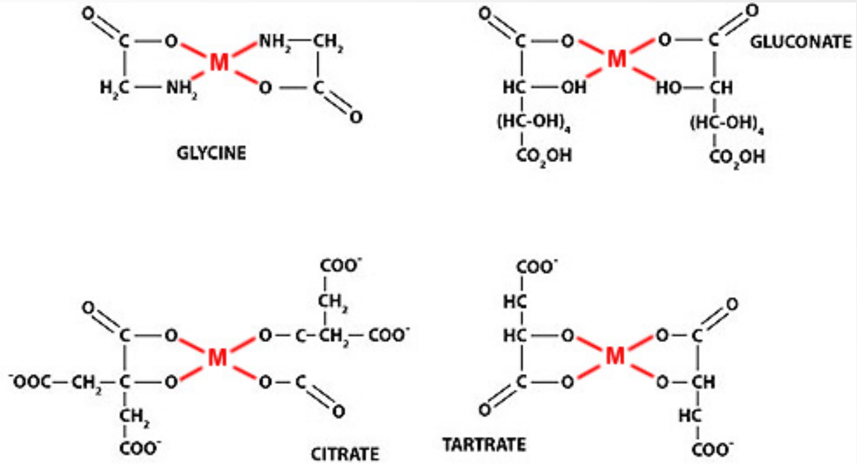
Examples:

Graph of cities connected by highways



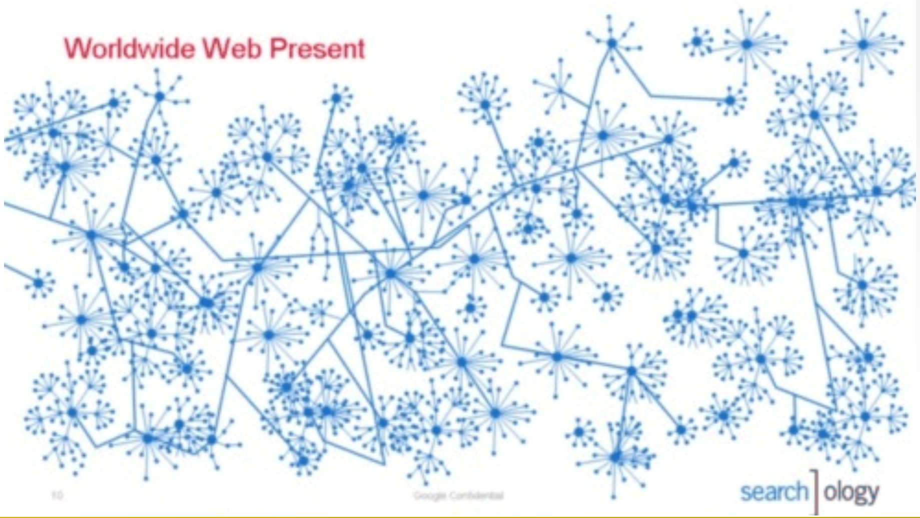
Graph of CS courses with prerequisites



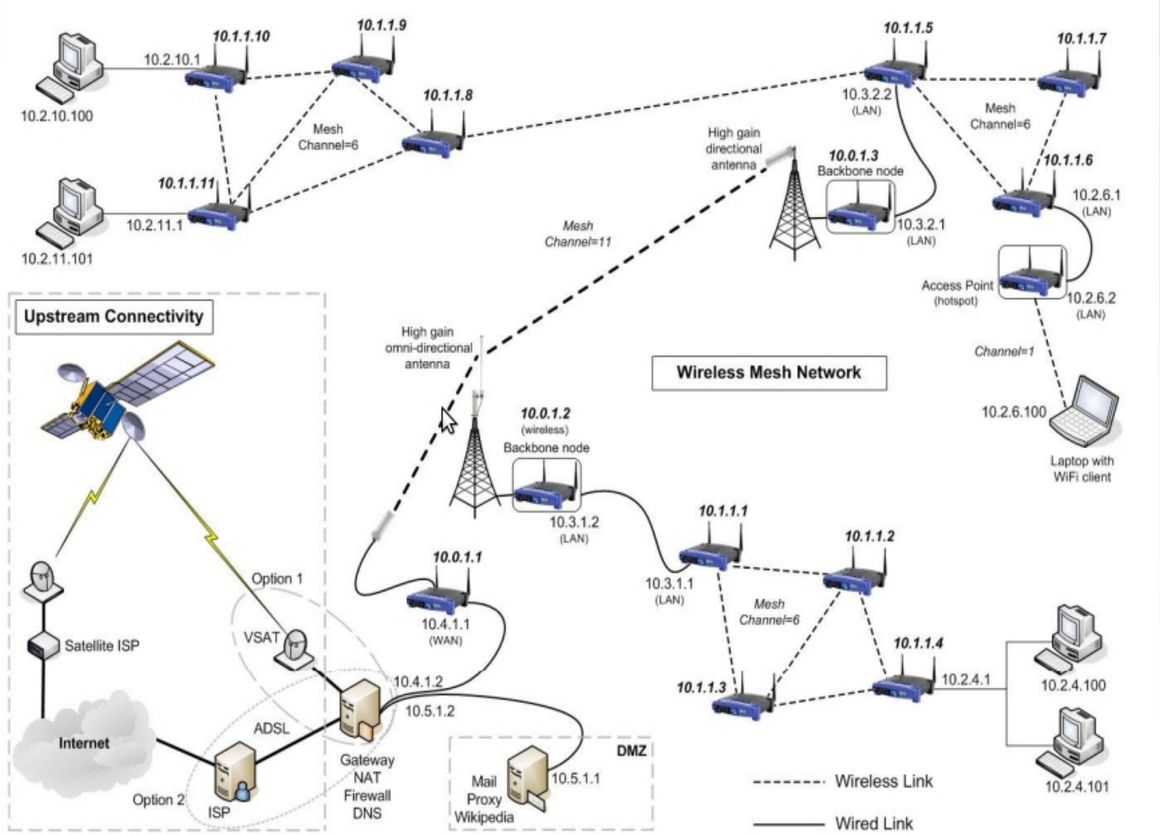


Graphs of molecular structures of organic compounds

Graph of web pages connected by hyperlinks



Graph of a wireless/wired network

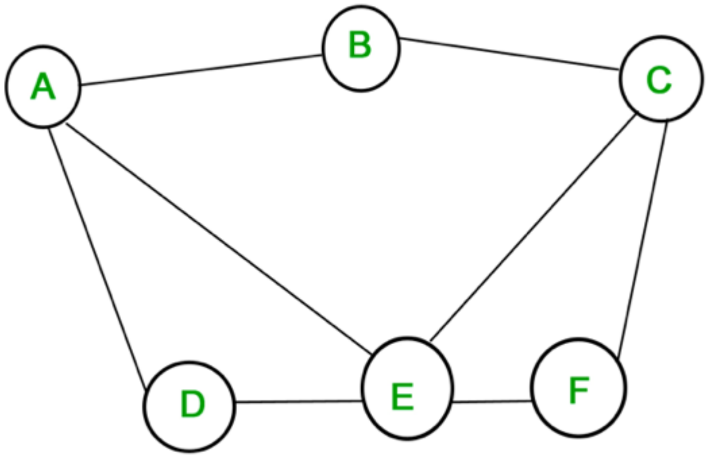


Source: http://www.nvtech.com.au/ProjCurr/LCMAN/LCMAN.html

Graph Terminology

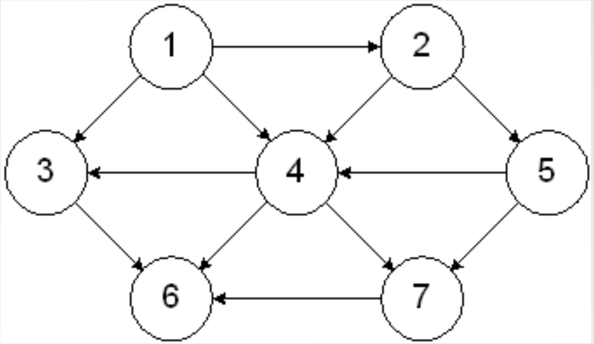
Undirected Graph

- An undirected graph is one in which there is a symmetric relationship among the vertices.
- An undirected graph has edges without arrows.
- It means that if A and B are two vertices connected by an edge in an undirected graph, you can get to B from A and vice versa.

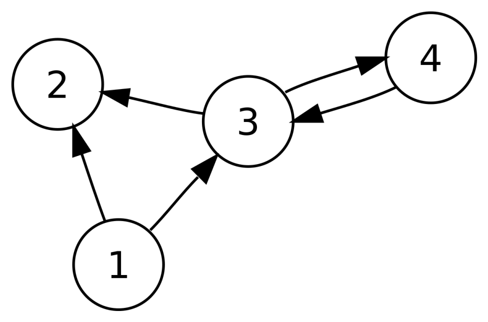


Directed Graph

- A directed graph is one in which there is an asymmetric relationship among the vertices.



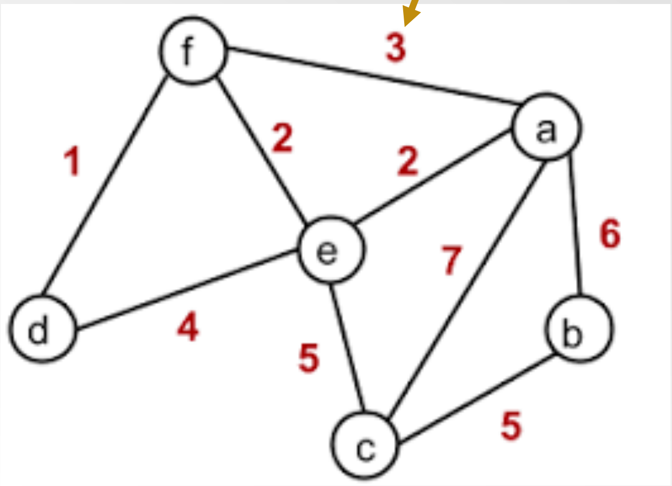
- In a directed graph, edges are represented by arrows starting at one vertex and ending at the neighbouring vertex.



- In the first figure, there is an edge from vertex 1 to 2. This means that 2 can be visited from 1 but 1 cannot be visited from 2 (directly).
- Edges can also be bidirectional as shown in the second figure (4 can be visited from 3 and vice versa).

Weighted Graph

Cost to traverse the link from a to f

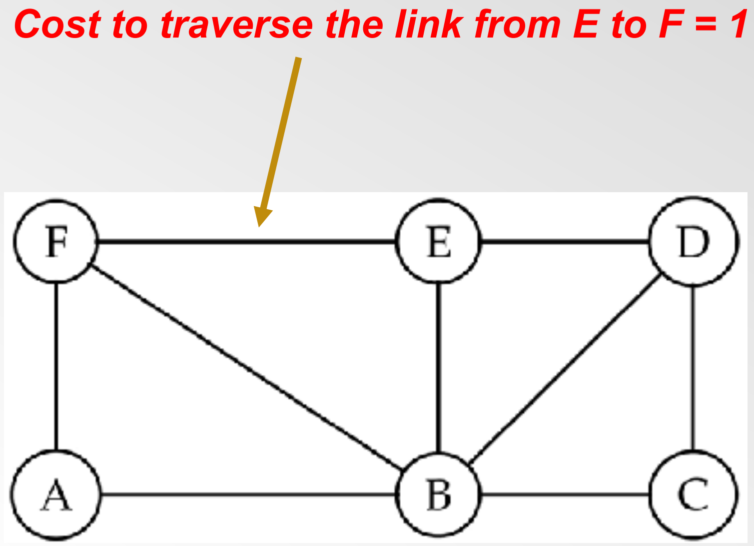


- Each edge has a number associated with it, called the cost or the weight. The number represents the "effort" it takes to traverse that edge.
- The weight could represent one parameter or an aggregate of multiple parameters.
- Weights play an important role in algorithms such as shortest path determination.

Shortest path from a to d is via f
The total cost of that path is 3 + 1 = 4.

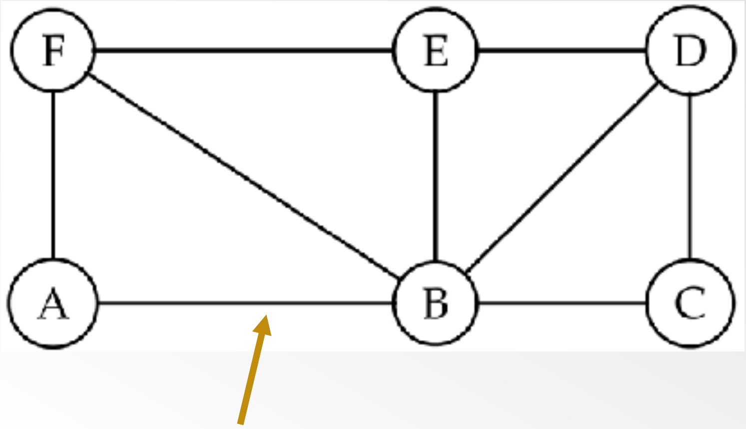
Unweighted Graph

- Edges don't have any costs associated with them.
- In unweighted graphs, all edges are treated alike, that is, it is not beneficial to prefer one path over the another.
- An unweighted graph can be considered as a special case of a weighted graph in which the weight of every edge = 1.



Representation of an Edge

- An edge between two vertices v1 and v2 is represented by the tuple (v1, v2).
- If the graph is undirected, if an edge (v1, v2) exists , then (v2, v1) also exists.
- If the graph is directed and there is an edge from v1 to v2 only, then only edge (v1, v2) exists.



Edge (A, B)

The edge can also be (B, A).

Degree of a vertex (Undirected Graph)

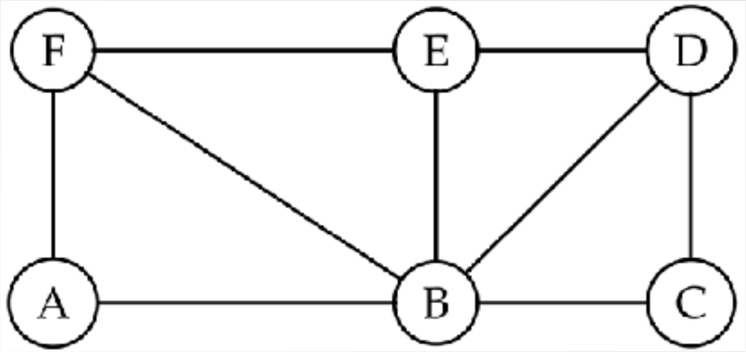
- Number of edges connected to the vertex

- In the figure:

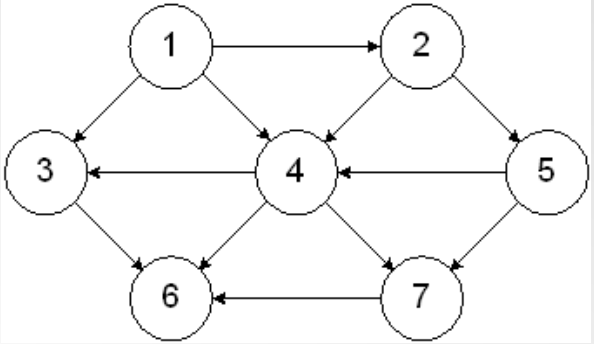
Degree of A = 2

Degree of B = 5

Degree of E = 3



Degree of a vertex (Directed Graph)



- **Indegree** = Number of edges terminating at the vertex
- **Outdegree** = Number of edges leaving the vertex

- In the figure:

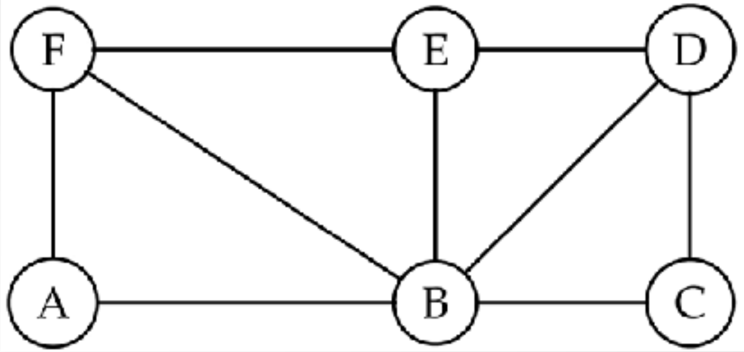
Indegree of vertex 1 = 0
Outdegree of vertex 1 = 3

Indegree of vertex 4 = 3

Outdegree of vertex 4 = 3

Path in a graph

- A path from vertex v1 to vertex vk is any sequence of edges to get from v1 to vk.
- Sequence is represented by tuples (v1, v2), (v2, v3), etc.



- In the figure:

Path from A to D: (A, B), (B, C), (C, D)

Another path from A to D: (A, F), (F, E), (E, B), (B, D)

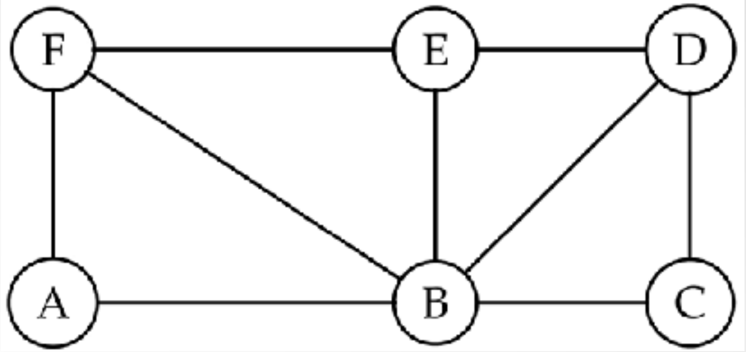
Yet another path from A to D:

(A, F), (F, B), (B, E), (E, B), (B, D)

Simple Path

- A simple path is a path in which no vertex is visited twice.

- The only exception for repetition is the first and last vertex (they can be the same).



- *Some simple paths in the figure:*

Simple path from A to D: (A,B) (B,C) (C,D)

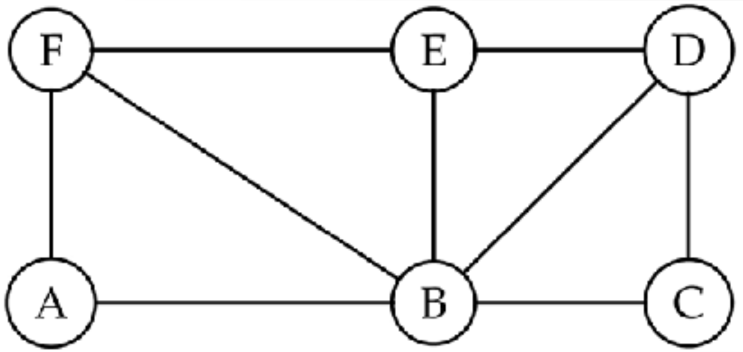
Simple path from B to B: (B,E) (E,F) (F, A) (A,B)

This is not a simple path: (A, B) (B, F) (F, B) (B, C)

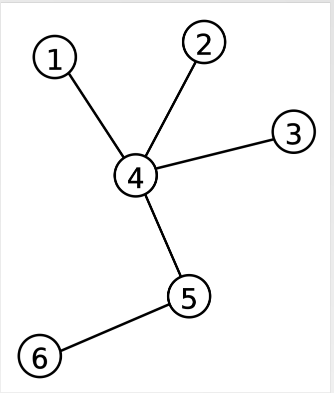
Cycle and Cyclic and Acyclic Graphs

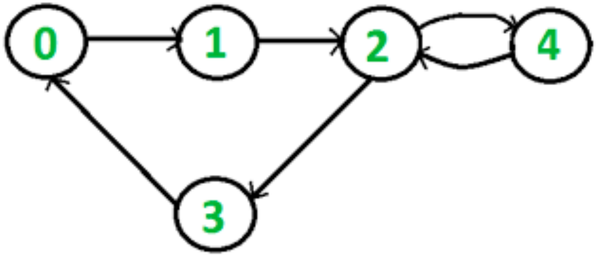
- A cycle is a simple path in which the first and last vertices are the same.
- A graph that has at least one cycle is a cyclic graph.
- A graph that has no cycles is an acyclic graph.

Undirected Cyclic Graph

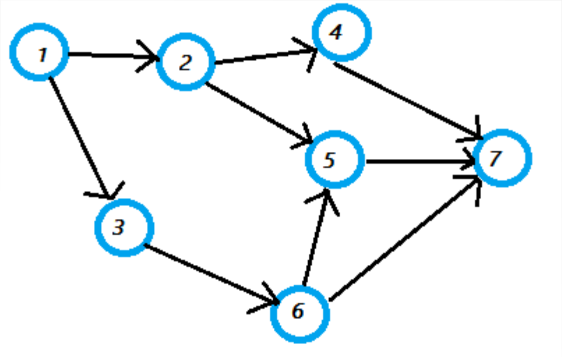


Undirected Acyclic Graph





Directed Cyclic Graph



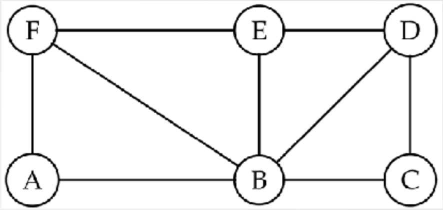
Directed Acyclic Graph

(No cycles!)

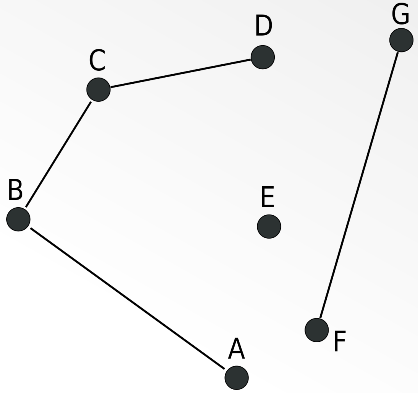
Connected and Unconnected Graphs

- Connected Graph: There is a path between every pair of vertices.

- Unconnected Graph: A collection of graphs, each of which is connected.



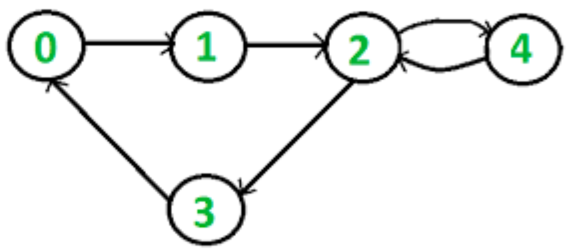
Connected Graph



Unconnected Graph

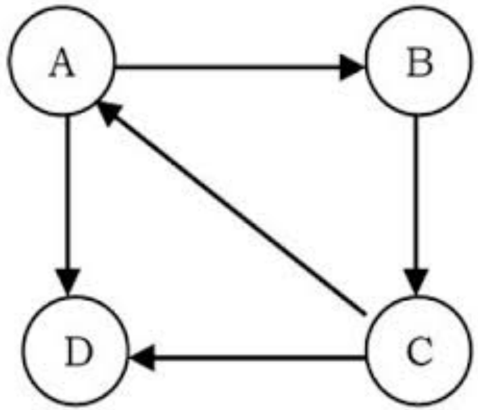
Directed Graphs:
Strongly Connected and Weakly Connected

- A directed graph is strongly connected if there is a path from every vertex to every other vertex.



Strongly Connected

- A directed graph is weakly connected if, ignoring the directions of edges, the resulting undirected graph is connected.



Weakly Connected
(No path from D to A, B or C)

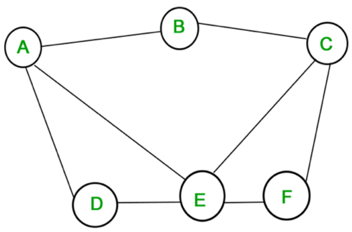
GRAPH REPRESENTATION

Graphs can be represented in one of two ways for implementation purposes:
ADJACENCY MATRIX REPRESENTATION or
ADJACENCY LIST REPRESENTATION.

2-d array

Adjacency Matrix Representation: The graph is represented as a 2-d matrix of n rows and n columns, where each row or column represents a vertex. The element G[i,j] = 1 if there is an edge between vertex i and vertex j, and 0 otherwise.
(Note: If the graph is weighted, G[i,j] will be equal to the weight of the edge (i,j))

Example 1 (Adjacency Matrix Representation of an undirected, unweighted graph):



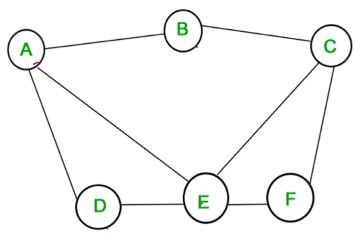
		A 0	B 1	C 2	D 3	E 4	F 5
A	0	0	1	0	1	1	0
B	1	1	0	1	0	0	0
C	2	0	1	0	0	1	1
D	3	1	0	0	0	1	0
E	4	1	0	1	1	0	1
F	5	0	0	1	0	1	0

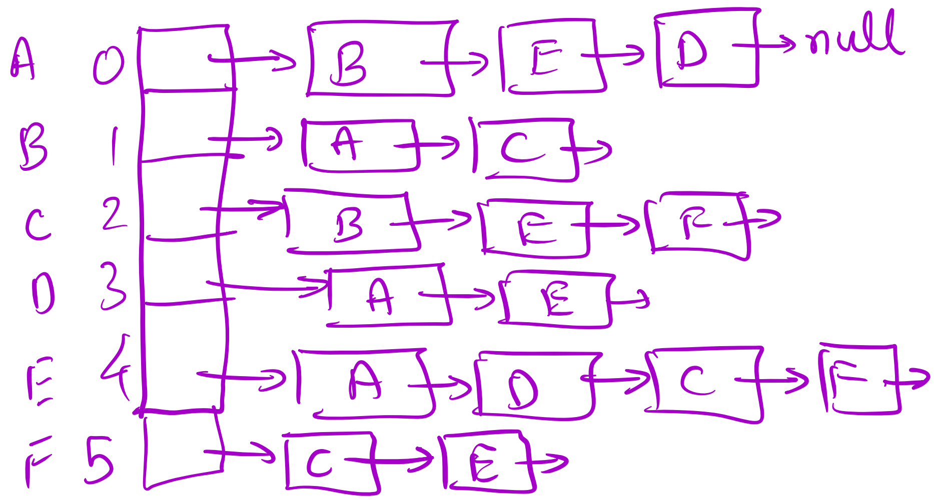
Advantages: Given an edge, it takes O(1) time to find if it exists.

Drawbacks: 1. Waste of space (sparse matrices)
2. Search for all edges connected to a vertex: O(n)

Adjacency List Representation: The graph is represented by an array of linked lists. Each linked list represents a vertex, with the entries in the linked list representing the neighbors of that vertex. (Order of storing the elements in the linked list doesn't matter).

Example 2 (Adjacency List Representation of an undirected, unweighted graph):





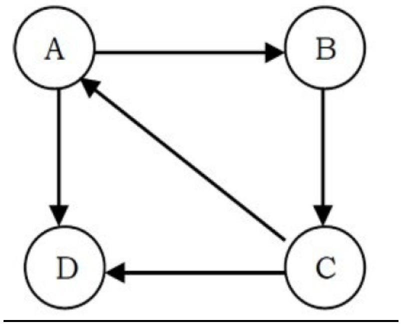
Advantages:

1. Space efficient
2. Search for all edges connected to a vertex.
   - → O(1) Best case
   - → O(n) Avg or worst case.

Drawbacks:

Search for a given edge
- → O(1) Best case
- → O(n) Avg & worst case

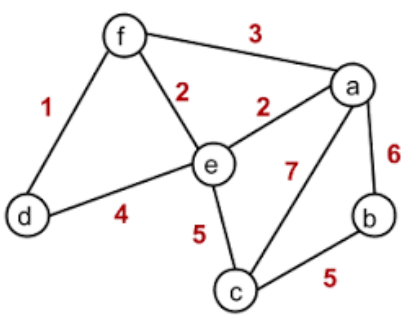
Example 3 (Representation of an directed graph):



	A	B	C	D
A	0	1	0	1
B	0	0	1	0
C	1	0	0	1
D	0	0	0	0

A → B → D →
B → C →
C → A → D →
D →

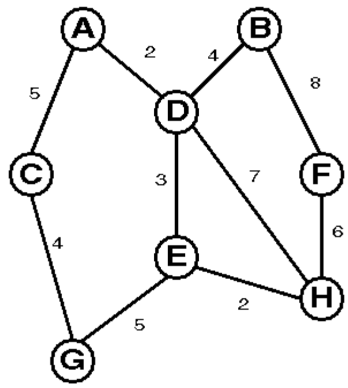
Example 4 (Representation of a weighted graph):



	a	b	c	d	e	f
a	0	6	7	0	2	3
b	6	0	5	0	0	0
c	7	5	0	0	5	0
d	0	0	0	0	4	1
e	2	0	5	4	0	2
f	3	0	0	1	2	0

a → b,6 → c,7 → e,2 → f,3
b → a,6 → c,5
c → a,7 → b,5 → e,5
d → e,4 → f,1
e → a,2 → c,5 → d,4 → f,2
f → a,3 → d,1 → e,2

Exercise: Study the graph shown in the diagram below and answer the following questions



a) Classify the graph as undirected/directed, unweighted/weighted, unconnected/connected, cyclic or acyclic.

Undirected, weighted, connected, cyclic

b) List all simple paths from vertex A to vertex E.

ADE, ACGE, ADBFHE, ADHE

c) What is the largest degree in the graph? Which nodes have the largest degree?

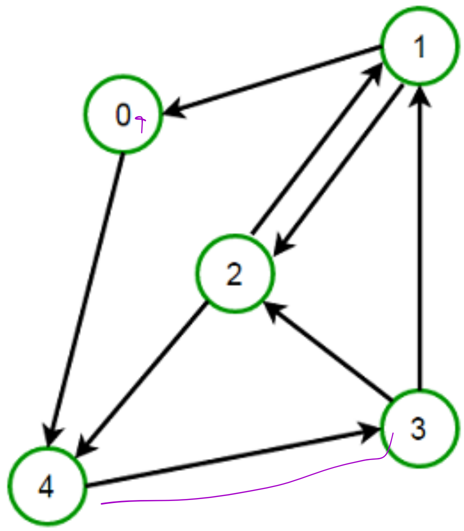
4 D

d) What is the smallest degree in the graph? Which nodes have the smallest degree?

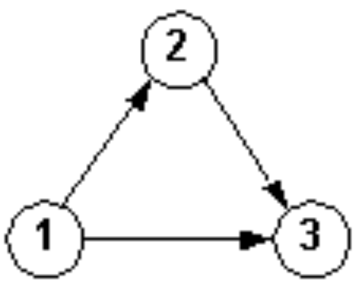
2 A, B, F, G, C

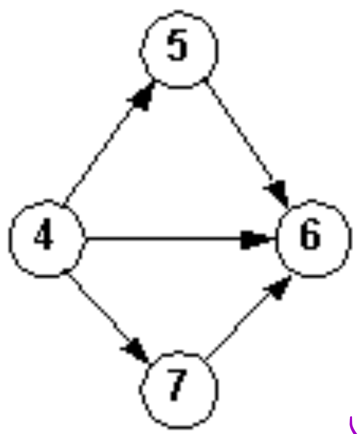
e) Write the adjacency matrix and the adjacency list representations of the graph.

Exercise 2: Determine if each of the following directed graphs are strongly connected or weakly connected. Give reasons.



Strongly connected!





Weakly connected

E.g. 2 → 1 is not possible

Weakly connected

E.g. 4 cannot be visited from other nodes.