1. $P(a|b \wedge a) = 1$

$$P(a|b \wedge a) = \frac{P(a \wedge b \wedge a)}{P(b \wedge a)}$$

$$= \frac{\cancel{P(a \wedge b)}}{\cancel{P(b \wedge a)}} = 1$$

Proved

2. $P(\text{toothache}) = $

$0.108 + 0.012 + 0.016 + 0.064$
$= 0.176$

$P(\text{cavity}) = 0.108 + 0.012 + 0.072 + 0.008 =$
0.20

$$P(\text{toothache}|\text{cavity}) = \frac{P(\text{Toothache} \wedge \text{cavity})}{P(\text{cavity})}$$

$$= \frac{0.12}{0.20} = 0.6$$

$$\frac{P(\text{Cavity} \wedge (\text{toothache} \vee \text{catch}))}{P(\text{toothache} \vee \text{catch})}$$

$$= \frac{0.108 + 0.012 + 0.072}{0.108 + 0.012 + 0.016 + 0.064 + 0.108 + 0.016 + 0.072 + 0.144 - 0.108 - 0.016}$$

$$= 0.4615$$

3. $P(a) = 0.025$

$P(b) = 0.015$

$P(a|b) = 0.6$

$P(a|\neg b) = 0.25$

(a) $P(a \wedge b) = P(a|b)\,P(b)$

$$= 0.6 \times 0.015$$

$$= 0.009$$

(b) $P(b|a) = \dfrac{P(b \wedge a)}{P(a)} = \dfrac{0.009}{0.025} = 0.36$

(c) $P(\neg b|a) = \dfrac{P(\neg b \wedge a)}{P(a)}$

$$P(a|\neg b) = 0.25$$

$$P(a \mid \neg b) = \frac{P(a \wedge \neg b)}{P(\neg b)}$$

$$0.25 = \frac{P(a \wedge \neg b)}{0.985}$$

$$0.25 \times 0.985 = P(a \wedge \neg b)$$

$$P(a \wedge \neg b) = 0.246$$

4 ⓐ yes, Burglary and Earthquake are indebendent.

By Tofological semantics:-

$$\begin{tikzcd} B \arrow[dr] & & E \arrow[dl] \\ & A & \end{tikzcd}$$

B, E have no parents so they don't depend on anything. Also Band E are not connected together so they won't depend on each other as well.

Numerical semantics:-

using the joint prior over B, E, A, J, M $\Rightarrow$

$$P(B,E,A,J,M) = P(B)\,P(E)\,P(A \mid B,E)\,P(J \mid A)\,P(M \mid A)$$

For $P(B,E)$

$$P(B,E) = P(B)\,P(E) \sum_A \overbrace{\cancel{P(A \mid B,E)}}^{1} \sum_J \overbrace{\cancel{P(J \mid A)}}^{1} \sum_M \overbrace{\cancel{P(M \mid A)}}^{1}$$

$$P(B,E) = P(B)\,P(E)$$

$\therefore$ They are conditionally independent.

4b) $P(B, E|A) = P(B|A)\,P(E|A)$ for conditional independence

Lets check if its true or false.

$$P(B,E|A) = \frac{P(A|B,E)\,P(B,E)}{P(A)}$$

for $B=T$ and $E=T$

$$P(A|B,E) = 0.95$$
$$P(B) = 0.01$$
$$P(E) = 0.02$$

$$= \frac{0.95 \times 0.001 \times 0.002}{P(A)}$$

$$P(A) = P(a|b \wedge e)\,P(b)\,P(e) + P(a|b \wedge \neg e)\,P(b)\,P(\neg e) + P(a|\neg b \wedge e)\,P(\neg b)\,P(e) + P(a|\neg b \wedge \neg e)\,P(\neg b)\,P(\neg e)$$

$\ldots (0.94)(0.01)(0.98) +$

$$P(a) = (0.95)(0.01)(0.02) + \ldots$$

$$(0.29)(0.99)(0.02) + (0.01)(0.99)(0.98)$$

$$= 0.024846$$

$$P(B, E \mid A) = \frac{0.95 \times 0.001 \times 0.002}{0.024846}$$

$$= 0.00019$$

$$P(B \mid A) = \frac{P(B \land A)}{P(A)}$$

$$P(B \land A) = \frac{P(a \mid b \land e)\, P(b \land e) + P(a \mid b \land \neg e)\, P(b \land \neg e)}{P(A)}$$

$$= \frac{0.95 \times 0.002 \times 0.001 + 0.94 \times 0.001 \times 0.98}{0.024846}$$

$$= 0.03715286$$

$$P(E \land A) = \frac{P(a \mid e \land \neg b)\, P(e)\, P(\neg b) + P(a \mid e \land b)\, P(e)\, P(b)}{P(A)}$$

$$= \frac{0.29 \times 0.02 \times 0.99 + 0.95 \times 0.02 \times 0.01}{0.024846}$$

$$= \quad 0.030757$$

$$P(B|A)\,P(E|A) = 0.0371S286 \times 0.030757$$

$$\approx 0.0011427$$

Clearly.

$$P(B,E|A) \simeq 0.00764711$$

$$P(B|A)\,P(E|A) \simeq 0.0011427$$

$$P(B,E|A) \neq P(B|A)\,P(E|A)$$

They are conditionally dependendent.