Assignment 5 - Hady Ibrahim (400377576)
SFWRENG 2CO3: Data Structures and Algorithms–Winter 2023

Deadline: March 31, 2023

Department of Computing and Software
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Please read the *Course Outline* for the general policies related to assignments.

Plagiarism is a *serious academic offense* and will be handled accordingly.
All suspicions will be reported to the *Office of Academic Integrity*
(in accordance with the Academic Integrity Policy).

This assignment is an *individual* assignment: do not submit work of others. All parts of your submission *must* be your own work and be based on your own ideas and conclusions. Only *discuss or share* any parts of your submissions with your TA or instructor. You are *responsible for protecting* your work: you are strongly advised to password-protect and lock your electronic devices (e.g., laptop) and to not share your logins with partners or friends! If you *submit* work, then you are certifying that you have completed the work for this assignment by yourself. By submitting work, you agree to automated and manual plagiarism checking of all submitted work.

Late submission policy. Late submissions will receive a late penalty of 20% on the score per day late (with a five hour grace period on the first day, e.g., to deal with technical issues) and submissions five days (or more) past the due date are not accepted. In case of technical issues while submitting, contact the instructor *before* the deadline.

Problem 1. Consider a remote community of N houses $h_1, \dots h_N$. We want to provide each house with internet access at minimal cost.

We can make a local network between these houses by connecting them via long-range Wi-Fi using directional antennas. The cost to connect two houses h_i, h_j is given by $C(h_i, h_j)$ and will depend on their distance and the terrain in between (e.g., long distances and hills require strong radios and repeaters).

In addition, we can connect one or more houses directly to the internet via a new fiber connection. For house h_i, the cost of this fiber connection is $F(h_i)$. If a house has a direct internet connection, then it can share this connection with all other houses that are reachable via a path of long-range Wi-Fi connections.

The community wants to find how to connect all members of this community with internet at a minimal cost.

P1.1. Model the above problem as a graph problem: what are the nodes and edges in your graph, do the edges have weights, and what problem are you trying to answer on your graph?

P1.2. Provide an efficient algorithm to find a way to connect all members of this community with internet at minimal cost. Explain why your algorithm is correct, what the complexity of your algorithm is, and which graph representation you use.

Problem 2. A company has several distribution centers. To simplify logistics, the company wants to figure out which distribution center is the "most central": the maximum time it takes to transport freight from this distribution center to any other distribution center is *minimal*. Assume we know, for every pair of distribution centers X and Y, the time it takes to transport freight from X to Y.

P2.1. Model the above problem as a graph problem: what are the nodes and edges in your graph, do the edges have weights, and what problem are you trying to answer on your graph?

P2.2. Provide an efficient algorithm to find the most central distribution center. Explain why your algorithm is correct, what the complexity of your algorithm is, and which graph representation you use.

Problem 3. Let $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ be an undirected weighted graph with weight function *weight*.

P3.1. Consider a minimum spanning tree T of $\mathcal{G}$ such that edge $(m, n) \in \mathcal{E}$ is part of T. Prove that T is still a minimum spanning tree if we reduce the weight $weight(m, n)$ by $k \geq 0$ (hence, the new weight of edge (m, n) is $weight(m, n) - k$).

P3.2. We say that a set of edges $\mathcal{E}' \subseteq \mathcal{E}$ is a *connecting set* if there is a path between all pairs of nodes $(m, n) \in \mathcal{N} \times \mathcal{N}$ using only the edges in $\mathcal{E}'$. We say that a connecting set is minimal if the sum $\sum_{e \in \mathcal{E}'} weight(e)$ is minimal among all connecting sets. Argue in which cases a minimal connecting set is a minimal spanning tree.

Problem 4. Consider a communication network represented by a graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ in which the nodes represents network devices (e.g., computers, wireless access points, switches, and routers) and the edges represent a communication channel (e.g., a network cable or a wireless connection). In this graph, every edge $(m, n) \in \mathcal{E}$ has a weight $0 \leq weight(m, n) \leq 1$ that indicates the probability of *failure-free delivery* when a message is sent from m to n (the reliability of the communication channel).

You may assume that the probability that a message sent by m to n is delivered without failures is independent of the rest of the network. Provide an efficient algorithm that computes the most reliable communication-path between two nodes (such that the probability of failure-free delivery is maximal and the probability of failures is minimal).

Assignment Details

Write a report in which you solve each of the above problems. Your submission:

1. must be a PDF file;
2. must have clearly labeled solutions to each of the stated problems;
3. must be clearly presented;
4. must *not* be hand-written: prepare your report in LaTeX or in a word processor such as Microsoft Word (that can print or exported to PDF).

Submissions that do not follow the above requirements will get a grade of zero.

Grading

Each problem counts equally toward the final grade of this assignment.