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① $\vec{w}\cdot(\vec{u}\times\vec{v}) = ?$

$\vec{u}\begin{pmatrix}1\\1\\2\end{pmatrix}, \vec{v}\begin{pmatrix}1\\2\\1\end{pmatrix}, \vec{w}\begin{pmatrix}-3\\1\\1\end{pmatrix}$

$$\vec{u}\times\vec{v} = \begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 2\\ 1 & 2 & 1\end{bmatrix} = \hat{i}((1\cdot1)-(2\cdot2)) - \hat{j}((1\cdot1)-(2\cdot1)) + \hat{k}((1\cdot2)-(1\cdot1))$$

$$= \hat{i}(1-4) - \hat{j}(1-2) + \hat{k}(2-1)$$

$$= -3\hat{i} + \hat{j} + \hat{k}$$

$$\vec{u}\times\vec{v} = \begin{pmatrix}-3\\1\\1\end{pmatrix} \rightarrow \vec{t}$$

$$\vec{w}\bullet\vec{t} = a_1b_1 + a_2b_2 + a_3b_3$$

$$= (-3\cdot(-3)) + (1\cdot1) + (1\cdot1)$$

$$= 9+1+1$$

$$= 11$$

② $A = \begin{pmatrix}1 & 2 & 0\\ -1 & -3 & -1\\ 0 & 2 & 1\end{pmatrix}$

$$a = \begin{pmatrix}1 & 2 & 0\\ -1 & -3 & -1\\ 0 & 2 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$b = \left(\begin{array}{ccc|c}1 & 2 & 0 & 2\\ -1 & -3 & -1 & -2\\ 0 & 2 & 1 & 1\end{array}\right) = \begin{cases} x_1 + 2x_2 = 2\\ -x_1 + 3x_2 - x_3 = -2\\ 2x_2 + x_3 = 1\end{cases}$$

$$B_{21}^{(1)} = \left(\begin{array}{ccc|ccc}1 & 2 & 0 & 1 & 0 & 0\\ 0 & -1 & -1 & 1 & 1 & 0\\ 0 & 2 & 1 & 0 & 0 & 1\end{array}\right) \qquad B_{21}^{(1)} = \left(\begin{array}{ccc|c}1 & 2 & 0 & 2\\ 0 & -1 & -1 & 0\\ 0 & 2 & 1 & 1\end{array}\right)$$

$$B_{23}^{(1)} = \left(\begin{array}{ccc|ccc}1 & 2 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 1 & 1 & 1\\ 0 & 2 & 1 & 0 & 0 & 1\end{array}\right) \qquad B_{23}^{(1)} = \left(\begin{array}{ccc|c}1 & 2 & 0 & 2\\ 0 & 1 & 0 & 1\\ 0 & 2 & 1 & 1\end{array}\right)$$

$$B_{32}^{(-2)} = \left(\begin{array}{ccc|ccc}1 & 2 & 0 & 1 & 0 & 0\\ 0 & 1 & 0 & 1 & 1 & 1\\ 0 & 0 & 1 & -2 & -2 & -1\end{array}\right) \qquad B_{32}^{(-2)} = \left(\begin{array}{ccc|c}1 & 2 & 0 & 2\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & -1\end{array}\right)$$

$$B_{12}^{(-2)} = \left(\begin{array}{ccc|ccc}1 & 0 & 0 & -1 & -2 & -2\\ 0 & 1 & 0 & 1 & 1 & 1\\ 0 & 0 & 1 & -2 & -2 & -1\end{array}\right) \qquad B_{12}^{(-2)} = \left(\begin{array}{ccc|c}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & -1\end{array}\right) = \begin{cases}x_1 = 0\\ x_2 = 1\\ x_3 = -1\end{cases}$$

maka diperoleh

$$A^{-1} = \begin{pmatrix}-1 & -2 & -2\\ 1 & 1 & 1\\ -2 & -2 & -1\end{pmatrix}$$

$x = 0$
$y = 1$
$z = -1$

③

$$S = \left\{ \vec{v}_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}, \vec{v}_2 = \begin{pmatrix}0\\1\\2\end{pmatrix}, \vec{v}_3 = \begin{pmatrix}-1\\-2\\1\end{pmatrix} \right\}$$

$$= \vec{v}_1 + \vec{v}_2 + \vec{v}_3 = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\begin{aligned} &= v_1 + (-v_3) &&= 0 \\ &v_1 + v_2 + (-2v_3) &&= 0 \\ &v_1 + 2v_2 + v_3 &&= 0 \end{aligned} = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right)$$

$$= \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & -2 \\ 1 & 2 & 1 \end{pmatrix}$$

$$B_{21}^{(-1)} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & 2 & 1 \end{pmatrix} \Rightarrow B_{31}^{(-1)} \Rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 2 & 2 \end{pmatrix} \Rightarrow B_{32}^{(-2)} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{pmatrix}$$

$$\Rightarrow B_3(\tfrac{1}{4}) = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow B_{23}(1) = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow B_{13}(1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Mendapatkan persamaan =

$$\left.\begin{aligned} v_1 &= 0 \\ v_2 &= 0 \\ v_3 &= 0 \end{aligned}\right\}$$ oleh karena itu, didapatkan bahwa himpunan S <u>bebas linier (linearly independent)</u> karena $v_1 = v_2 = v_3 = 0$

④ $$S = \left\{ v_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}, v_2 = \begin{pmatrix}0\\1\\2\end{pmatrix}, v_3 = \begin{pmatrix}-1\\-2\\1\end{pmatrix} \right\} \quad \bar{w} = \begin{pmatrix}1\\1\\2\end{pmatrix}$$

$$\begin{aligned} &v_1 + v_2 + v_3 = \bar{w} \\ &v_1 + 0 + v_3 &&= 1 \\ &v_1 + v_2 - 2v_3 &&= 1 \\ &v_3 + 2v_2 + v_1 &&= 2 \end{aligned} = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 1 & 1 & -2 & 1 \\ 1 & 2 & 1 & 2 \end{array}\right) \Rightarrow B_{21}(-1) \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & 2 & 1 & 2 \end{array}\right) \Rightarrow B_{31}(-1) = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 2 & 2 & 1 \end{array}\right)$$

$$\Rightarrow B_{32}^{(-2)} = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 4 & 1 \end{array}\right) \Rightarrow B_3(\tfrac{1}{4}) = \left(\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \tfrac{1}{4} \end{array}\right) \Rightarrow B_{13}(1) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & \tfrac{5}{4} \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \tfrac{1}{4} \end{array}\right) \Rightarrow B_{23}(1) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1\tfrac{1}{4} \\ 0 & 1 & 0 & \tfrac{1}{4} \\ 0 & 0 & 1 & \tfrac{1}{4} \end{array}\right)$$

Maka didapatkan persamaan =

$$\begin{aligned} v_1 &= 1\tfrac{1}{4} \\ v_2 &= \tfrac{1}{4} \\ v_3 &= \tfrac{1}{4} \end{aligned}$$

$$\vec{w} = 1\tfrac{1}{4} v_1 + \tfrac{1}{4} v_2 + \tfrac{1}{4} v_3$$

5. $B = \begin{pmatrix} -1 & 10 & 9 \\ 0 & 1 & 13 \\ 0 & 0 & -2 \end{pmatrix}$

a) $Det(B) = \begin{pmatrix} -1 & 10 & 9 & -1 & 10 \\ 0 & 1 & 13 & 0 & 1 \\ 0 & 0 & -2 & 0 & 0 \end{pmatrix}$

$= (2+0+0) - (0+0+0)$

$= 2-0$

$= 2$

$Det(B^4) = 2^4 = 16$

b) Karena $Det(B) = 2 > 0$, maka matriks ini bisa diinvers

Pembuktian menggunakan OBE =

$$\left[\begin{array}{ccc|ccc} -1 & 10 & 9 & 1 & 0 & 0 \\ 0 & 1 & 13 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \end{array}\right] \Rightarrow B_1(-1) \left[\begin{array}{ccc|ccc} 1 & -10 & -9 & -1 & 0 & 0 \\ 0 & 1 & 13 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \end{array}\right] \Rightarrow B_{12}(10) \left[\begin{array}{ccc|ccc} 1 & 0 & 121 & -1 & 10 & 0 \\ 0 & 1 & 13 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow B_3(-\tfrac{1}{2}) = \left[\begin{array}{ccc|ccc} 1 & 0 & 121 & -1 & 10 & 0 \\ 0 & 1 & 13 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} \end{array}\right] \Rightarrow B_{13}(-121) \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 10 & 60\frac{1}{2} \\ 0 & 1 & 13 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} \end{array}\right]$$

$$\Rightarrow B_{23}(-13) = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 10 & 60\frac{1}{2} \\ 0 & 1 & 0 & 0 & 1 & 6\frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{2} \end{array}\right]$$