Modeling the Discharge of a Battery

Determining the Capacity Necessary to Power a Phone

Jacob Turner

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Abstract

The Runge-Kutta-Fehlberg (RKF) algorithm was used in a FORTRAN program to evaluate a system of differential equations that describe the movement of charge through a Lithium-ion Battery in a phone. The RKF algorithm uses 4th and 5th order estimates to solve a system of ordinary differential equations that describe complex problems. The program determined the capacity needed to power the device for a total of 14 hours. In order to account for and understand the uncertainty surrounding this solution a sensitivity analysis was performed on the initial charge distribution, voltage demanded from the phone components, and internal resistance of the battery. This analysis provided insight into how change in these parameters would affect the minimum capacity of the phone. The program produced results indicating that the phone battery needed to have 2713.42 (mAh) of charge in order to power the phone for the full period. The sensitivity analysis showed that reducing the voltage demand would be the most efficient use of research and development resources to increase run-time of phones.

Introduction

Lithium-ion batteries are an indispensable technology that have come to power a significant amount of the productivity in the 21st century. Hence, it will be useful to understand how a battery is depleted over time and what factors or parameters are the most prominent contributors. The Kinetic Battery Model describes the chemical depletion of a battery (Manwell & Mcgowan 1993). Specifically, it is very important to understand how long the device will last under a specific load and how much charge it should store, also known as the battery capacity.

Using the Kinetic Battery Model, a system of differential equations can be utilized to model the charge in a battery over time (Jongerden & Haverkort 2009). A program in FORTRAN was constructed to determine the minimum necessary capacity of the battery to power a phone for at least 14 hours. The program will use the Runge-Kutta-Fehlberg (RKF) method to solve the system, provided an initial capacity guess, to approximate the run-time of the battery. A search algorithm will then be used to solve the system in reverse for the minimum capacity needed to power the phone for the allotted 14 hours. A sensitivity analysis will be conducted on the initial distribution of charge, the voltage necessary to run CPU intensive applications, and the internal resistance of the battery. This will show how each of these parameters influence the capacity needed to power the phone for a full day of use in the model. Low resistance in the battery will allow for a higher current to pass from the battery to the device, which can shorten battery life (Gannet, Lab Lecture, 2018). Voltage is the power or strength behind the current, when increasing this value, one should expect a decrease in battery life (Gannet, Lecture, 2018). The charge distribution has the largest uncertainty when considering how it will affect the battery run-time. The parameter values used in the model for voltage and resistance are based on characteristics of a Lithium-ion battery used in a modern smart phones ("Li-Ion & LiPoly Batteries." & "Does Internal Resistance Affect Performance?"). The parameter value describing the charge distribution of the battery will be estimated from a peer-reviewed journal entry, “Battery Aging and the Kinetic Battery Model” (Jongerden & Haverkort 2017). Lastly, an investigation into the relationship between the battery run-time and the initial capacity required, will also be performed. This analysis will also provide insight into the factors that would be most efficient to invest in for research and development.

Methodology

The underlying principle used for this research that allows one to explore the discharge of a battery with time is the Kinetic Battery Model (KiBaM). The KiBaM is a simple and intuitive model that describes the movement of charge through two wells in a battery, and eventually to the continuously applied load (Jongerden & Haverkort 2017, Manwell & McGowan 1993). Originally developed to describe the discharge of lead-acid batteries, KiBaM was later shown to accurately model other types of batteries such as those of the Lithium-ion variety (Jongerden 2010). A diagram of this model can be seen below in Figure 1.

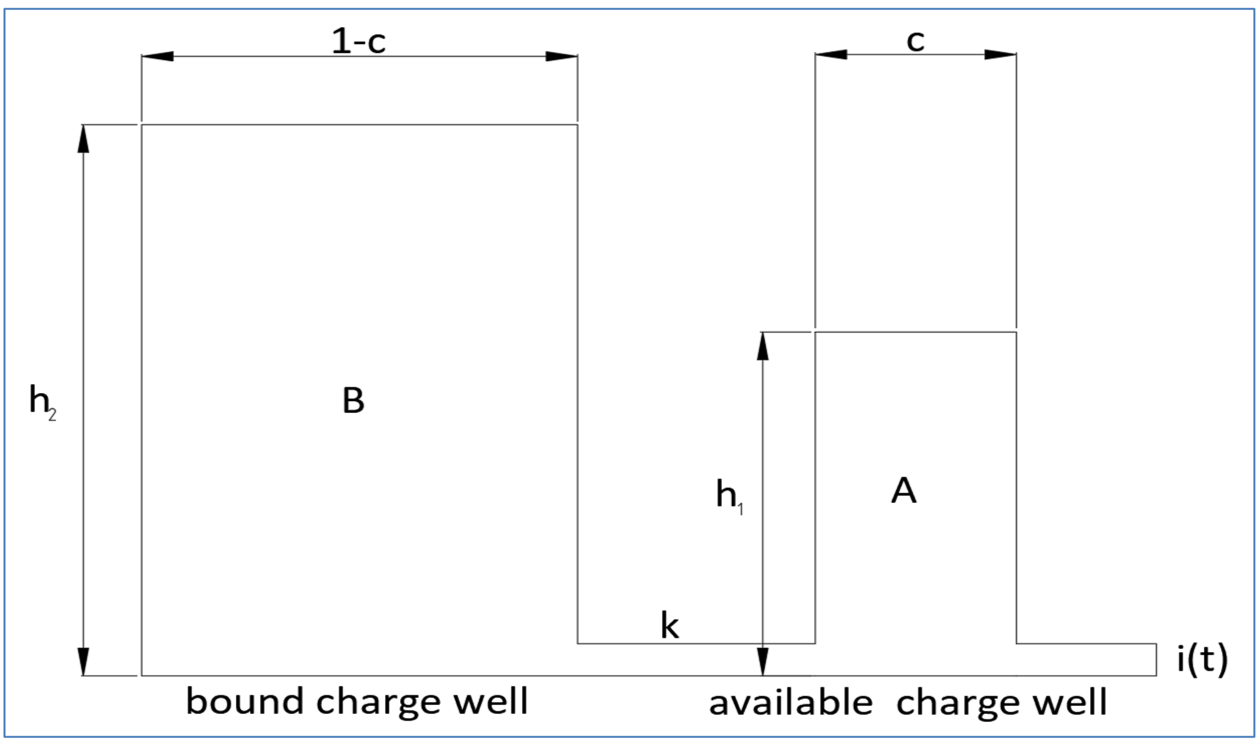

Figure 1. Model of the kinetic battery model with two wells. (inspiration: Jongerden & Haverkort 2009)

A system of differential equations [1 & 2] can be used to describe the behavior of charge in a lithium-ion battery from the KiBaM (Jongerden & Haverkort 2009). The charge moves from the bound well to the available well. This flow of charge is limited by the conductance (k), and eventually moves from the available well to the device as the load or current $[-i(t)]$ (Jongerden & Haverkort 2017). The battery is considered empty when the charge in the available well is zero (Jongerden & Haverkort 2017).

$$\frac{dA}{dt} = -i(t) + k(h_2 - h_1) \qquad A(0) = c \cdot C \qquad [1]$$

$$\frac{dB}{dt} = -k(h_2 - h_1) \qquad B(0) = (1 - c) \cdot C \qquad [2]$$

Where:

A(t) - charge in the available well of the battery (Coulombs)
B(t) - charge in the bound well of the battery (Coulombs)
t - time (minutes)
c - ratio of charge, distributed between the two wells
C - total charge capacity of the battery (Coulombs)
V - potential difference in the battery (Volts)
R - internal resistance of the battery (Ohms)
k - fixed conductance, defined as inverse the internal resistance (R)
h_1 - height of the charge within the available well, where $h_1(t) = \frac{A(t)}{c}$
h_2 - height of the charge within the bound well, where $h_2(t) = \frac{B(t)}{1-c}$
t_{goal} - length of run time needed for the battery (hours)
i_0 - initial current demand from the load (Amperes), where $i_0 = \frac{v}{R}$
i(t) - demand on the battery due to the applied load, where $i(t) = i_0 \cdot t \cdot e^{\left(-\frac{t}{t_{goal} \cdot 60}\right)}$

In order to solve for the minimum battery capacity needed to allow the device to run for the allotted time, the system of differential equations must be evaluated. The Runge-Kutta-Fehlberg algorithm was used in a FORTRAN program to solve the system and model the discharge of a battery with a capacity of 1000 (mAh). A search algorithm was then used to find the capacity necessary to power the device for the specified period. The results for the old capacity and the new capacity were written to a CSV file and plotted with R studio.

Solutions of differential equations describing complex systems can be approximated with high levels of precision using the Runge-Kutta-Fehlberg (RKF) algorithm. Both a 4th and 5th order estimate is used to determine the solution to the differential equations; the difference between the estimates is known as the truncation error (Chapra & Canale, 747, 2015). The truncation error was utilized in the constructed FORTRAN program to increase the precision of the solution by controlling the step size. The step size is controlled by allowing the RKF algorithm to increase its' value when the error is below a threshold of precision and decrease the step size when the error exceeds this threshold (Chapra & Canale, 748, 2015). Controlling the step size in this way provides the RKF method with a distinct advantage in being able to optimize precision, while also converging to the solution quickly. The governing iterative equations used in the RKF method are shown on the following page in equations [3 & 4]. The equations for K_1-K_6 describe the prediction of the slope at the midpoint of the function evaluated (Chapra & Canale, 740, 2015).

$$y_{i+1} = y_i + \left(\frac{37}{378}K_1 + \frac{250}{621}K_3 + \frac{125}{594}K_4 + \frac{512}{1771}K_6\right)h \qquad [3]$$

$$y_{i+1} = y_i + \left(\frac{2825}{27648}K_1 + \frac{18575}{48384}K_3 + \frac{13525}{55296}K_4 + \frac{277}{14336}K_5 + \frac{1}{4}K_6\right)h \qquad [4]$$

Where,

$$K_1 = f(x_i, y_i)$$

$$K_2 = f(x_i + \frac{1}{5}h, y_i + \frac{1}{5}K_1 h)$$

$$K_3 = f(x_i + \frac{3}{10}h, y_i + \frac{3}{40}K_1 h + \frac{9}{40}K_2 h)$$

$$K_4 = f(x_i + \frac{3}{5}h, y_i + \frac{3}{10}K_1 h - \frac{9}{10}K_2 h + \frac{6}{5}K_3 h)$$

$$K_5 = f(x_i + h, y_i - \frac{11}{54}K_1 h + \frac{5}{2}K_2 h + \frac{70}{27}K_3 h + \frac{35}{27}K_4 h)$$

$$K_6 = f(x_i + \frac{7}{8}h, y_i + \frac{1631}{55296}K_1 h + \frac{175}{512}K_2 h + \frac{575}{13824}K_3 h + \frac{44275}{110592}K_4 h + \frac{253}{4096}K_5 h)$$

Application

The RKF algorithm can be applied to find solutions to most problems that are described by a system of differential equations. The truncation error associated with the algorithm can also be easily solved for because there are two different order estimates, where the difference in the results will show how much less accurate the lower order estimate is.

The main parameters used to describe the model are shown in Table 1. The parameter *c* describes the ratio of the initial charge distribution between both the wells in the battery; the initial value chosen was based on research in “Battery Aging and the Kinetic Battery Model” (Jongerden & Haverkort 2017). The battery will be modeled initially with a capacity (*C*), this value will then be solved for in the program so that it will provide enough energy to power the device for the time required (t_{goal}). The values used for voltage (*V*) and resistance (*R*) are reasonable characteristics of a Lithium-ion battery used in a modern smart phone (“Li-Ion & LiPoly Batteries.” & “Does Internal Resistance Affect Performance?”). A sensitivity analysis will be performed on the parameters *c*, *V*, and *R* to determine how each parameter affects the required initial required capacity.

Table 1. Parameters used in description of model.

Parameter	*c*	*C*	*V*	*R*	t_{goal}
Value	0.65	1000	4.0	100	14
Units	N/A	mAh	Volts	mOhms	hours

Several relationships, shown in Table 2, were used in the model of the charge in the battery. The initial conditions [*A(0) & B(0)*] describe the charge in each well at time zero. The total charge of the battery is distributed in each of the wells according to the fraction *c*. The heights of each of the wells is important because the rate that the charge flows from the bound well to the available well depends on the difference in height (Jongerden & Haverkort 2009). Hence, the heights of each well [h_1 *(t)* & h_2*(t)*] are functions of the charge in the respective well at time *t*. The conductance [*k*] can be characterized as the limiting factor or resistance to the charge traveling from the bound well to the available well (Jongerden & Haverkort 2017). The initial current [i_0] depends on both the voltage and resistance of the battery. Lastly, a decaying current will be used to model change in demand of the device with time [*i(t)*].

Table 2. Variable relationships used to model charge of the battery.

Variable	*A(0)*	*B(0)*	h_1 *(t)*	h_2*(t)*	*k*	i_0	*i(t)*
Equation	c·C	(1-c)·C	A(t)/c	B(t)/(1-c)	1/R	v/R	$i_0 \cdot t \cdot e^{\left(-\frac{t}{t_{goal} \cdot 60}\right)}$

Results and Discussion

The FORTRAN program produced a value of 9768.3 coulombs of charge, which can be converted to a minimum capacity of 2713.42 (mAh) needed to power the device the desired 14 hours. This result makes sense because most phones have battery capacities in the range from 2500 – 3500 (mAh) (Android Authority 2018). The battery life for every device is subject to variation in usage by the consumer, which is not taken into account by the program. The program also approximated a battery lifetime of 8 hours 27 minutes, or about 60% of the needed time for the 1000 (mAh) battery. This shows that to reach the run-time goal of 14 hours, the battery size must nearly be tripled. Showing a diminishing return in increasing the capacity to achieve longer battery lifetimes. This trend helps explain the recent plateauing of battery capacity in newer phones (Android Authority 2018). The program then creates a table of the charge in each well vs. time, this table was plotted and is shown in Figure 1.

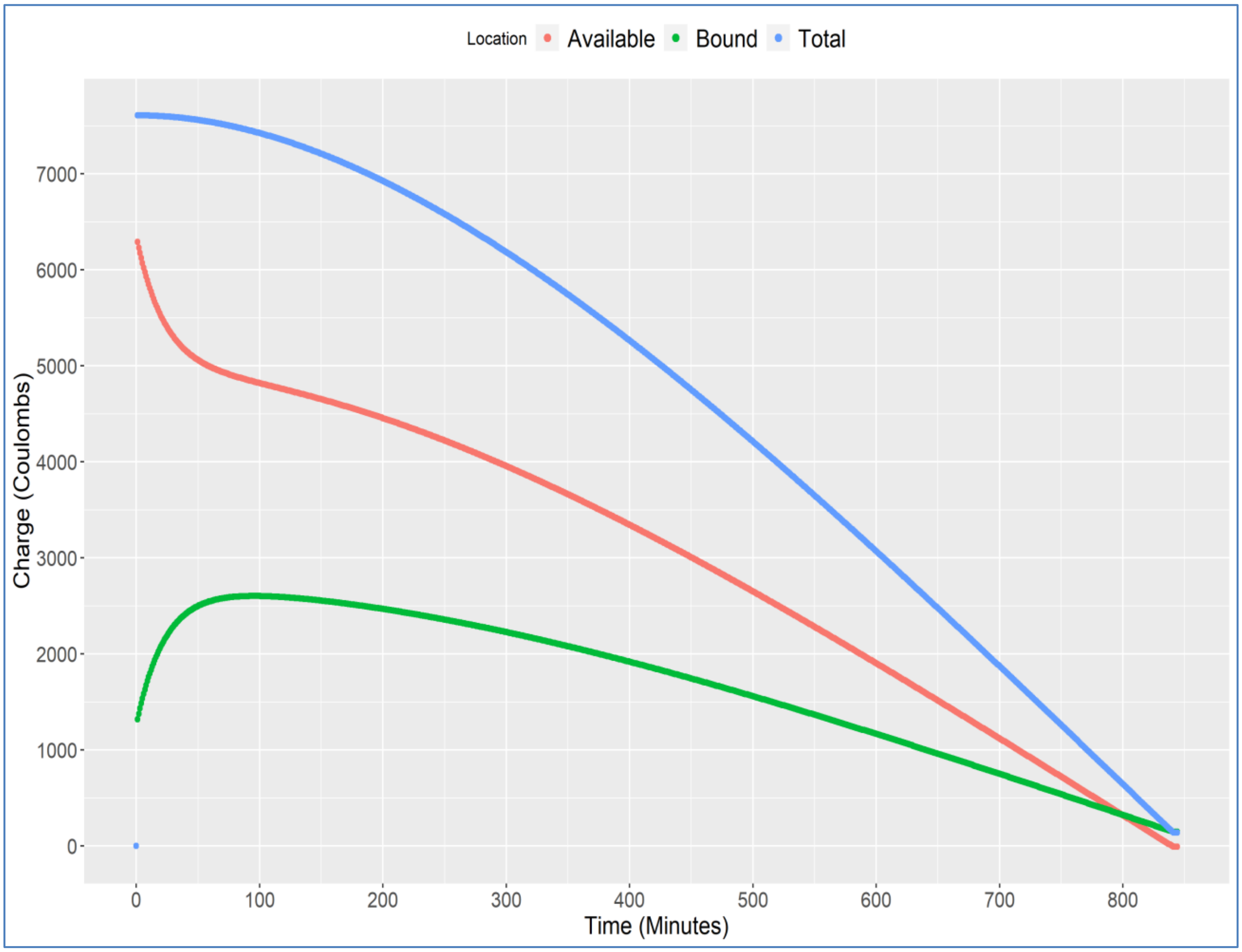

Figure 1: The charge (Coulombs) in each well as a function of time.

The plot above shows the charge in the available well, bound well, and the total charge in the battery. Since the battery is determined to be dead when the available well runs out of its charge, this is the most important value to consider. After the initial startup of the device, the battery experiences its fastest rate of discharge. The discharge rate then slows down as it stabilizes from the constant usage of the device. The rate of discharge then slowly increases until the battery is fully drained, which occurs when the available well runs out of charge, after the 14-hour (840 minute) time period.

A sensitivity analysis was conducted to test the charge distribution (c), the internal resistance (R), and the voltage (V) on a scale from -50% to +50%. The plot for this analysis can be seen in Figure 2.

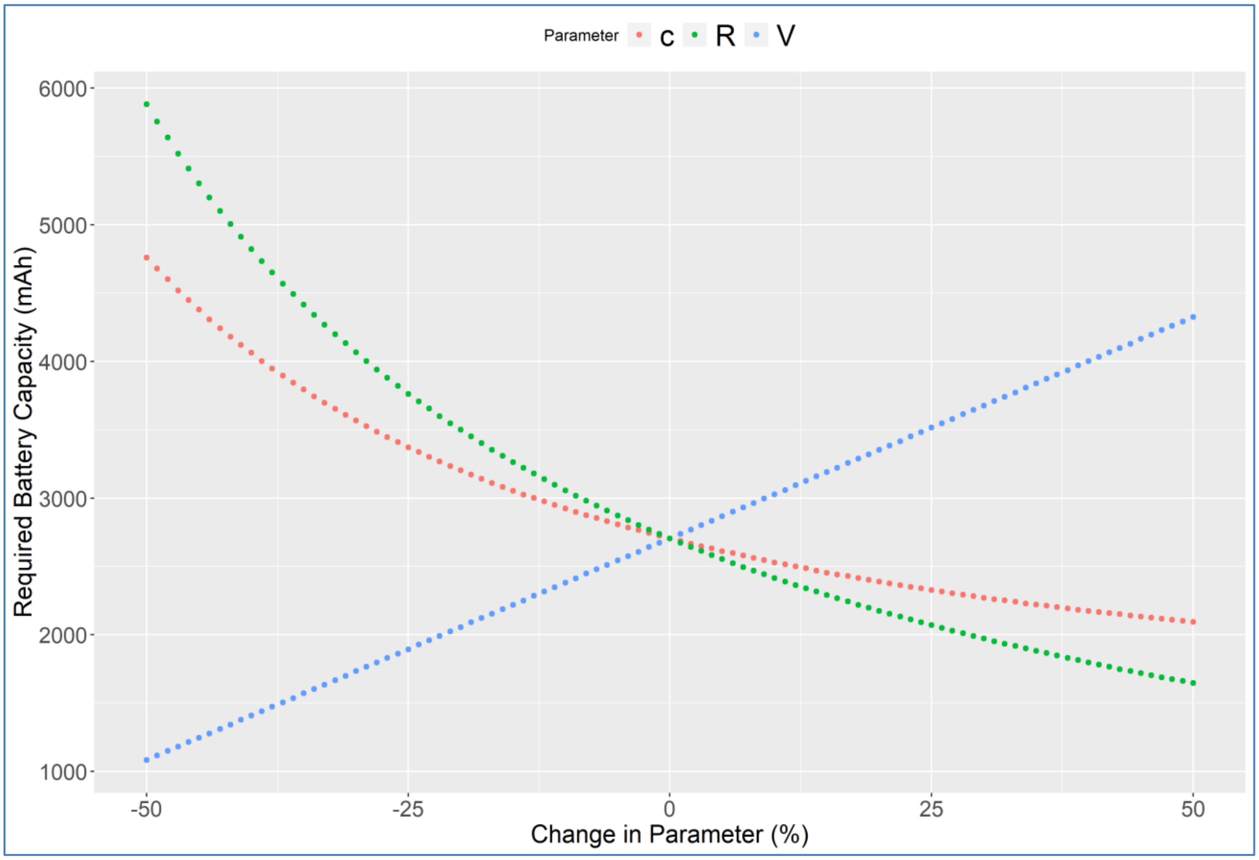

Figure 2: How the charge distribution (c), resistance (R), and the voltage (V) affect the minimum necessary capacity of the battery to keep the device powered for 14 hours.

The relationship for the charge distribution (c) and the resistance (R) to the minimum battery capacity is very similar as both appear to be logarithmic. The primary difference between the parameters is that the resistance has a more pronounced effect on the capacity than the charge distribution. However, both (c) and (R) appear to be approaching an asymptote, where increasing their original values above 50% has drastically diminishing returns. The effect of the voltage of the device on battery life is linear. This indicates that the effect on the capacity needed will remain constant no matter the value of voltage needed for the phone components. This shows that reducing the voltage demand of devices would be the most efficient use of research and development resources if one aims to decrease the needed battery size of phones or increase the average lifetime of batteries. Lastly, an additional analysis was performed on the time required (t_{goal}) to understand how the needed runtime of the battery affects the required capacity, this relationship can be seen in Figure 3.

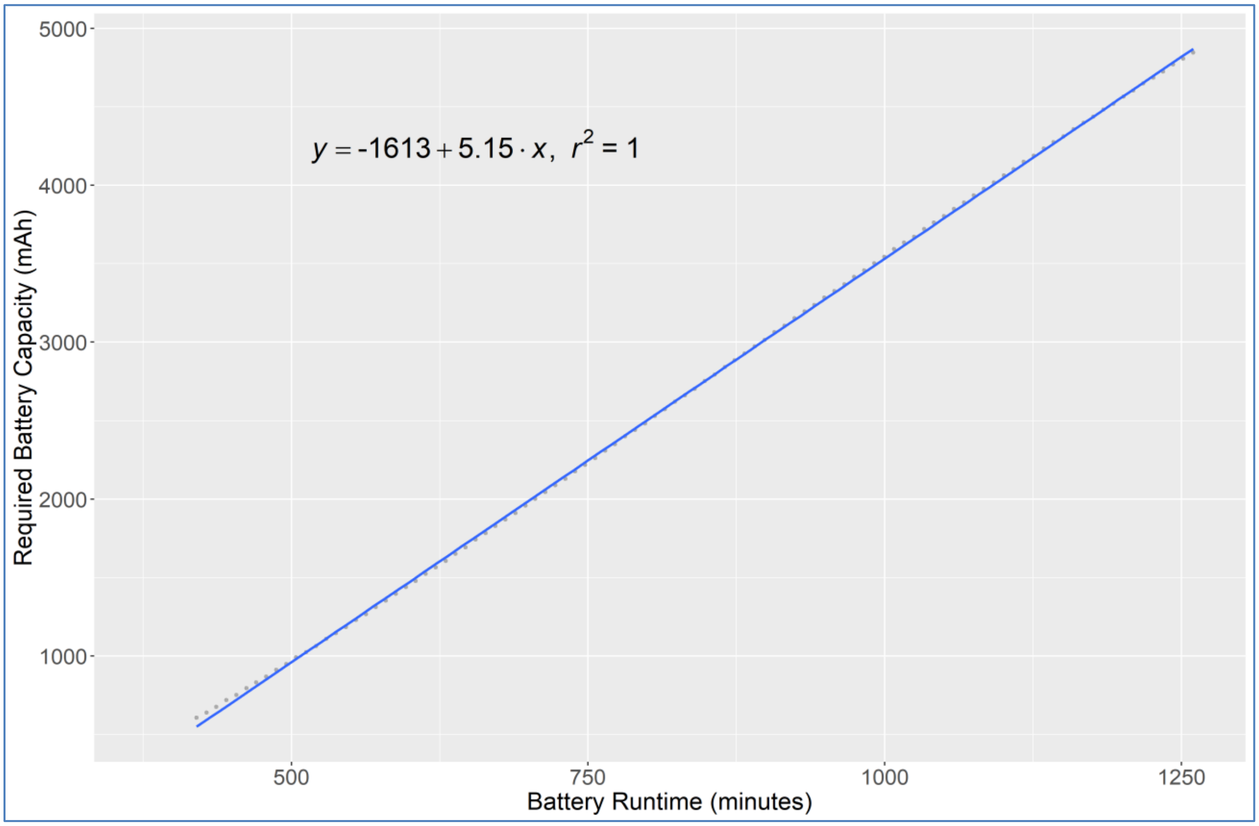

Figure 3: The relationship between the runtime of the battery and the required capacity necessary to power the phone.

The needed capacity appears to have a linear relationship with the battery runtime, however the slope of the relationship is fairly high at 5.15. This indicates that 5.15 (mAh) of additional capacity are needed to power the phone for every additional minute of battery run-time. This result shows that simply increasing the capacity of a phone is a rather inefficient way to increase the battery lifetime because of the large increase in capacity required per minute increase in run-time.

Conclusions

The RKF algorithm was used in a FORTRAN program to accurately model the discharge of a lithium-ion battery with time. The program was able to provide results indicating that the minimum capacity of the battery would need to be 2713.46 (mAh) in order to power the battery for the required time of 14 hours (t_{goal}). This result appeared to be reasonable as the majority of modern phones have batteries in the range of 2500-3500 (mAh). The sensitivity analysis was able to provide an understanding of how the uncertainty of each parameter (c, V, and R) would affect the minimum battery capacity. In addition, it showed that in order to most efficiently increase battery life, research and development resources would be best spent on lowering the voltage demand of the internal components of phones. Lastly, through investigation of the battery run-time it was shown that about 5.15 (mAh) of capacity would be needed in order to power the phone each additional minute. This is the main reason for the recent ceiling in battery capacities in new phones and also why increasing the capacity is the least efficient way to increase battery run-time. Further research could include adjusting the program to account for additional discharge profiles, this would provide insight into the effect of different consumer behavior on capacity requirements. In addition, including battery age and temperature in the model would improve the accuracy of the results.

References

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