ENGR 325
Lab Assignment – Root finding: Secant Method

A irrigation district uses two storage reservoirs connected by a pipe (Figure 1). A pump (at elevation Z_2) is used to transfer water from the lower elevation reservoir (with water surface elevation point Z_1) to the higher elevation reservoir (water surface elevation Z_3). The energy equation governs the flow of water through the system:

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 - h_f + E_p = \frac{p_3}{\gamma} + \frac{v_3^2}{2g} + z_3 \tag{1}$$

where

$$\begin{aligned} p_i &= \text{pressure at point } i \\ \gamma &= \text{specific weight of the fluid} \\ v_i &= \text{fluid velocity at point } i \\ h_f &= \text{friction losses} \\ E_p &= \text{energy head supplied by the pump (P)} \end{aligned}$$

All of the terms in Equation 1 have units of length. This equation can be simplified by noting that $p_1 = p_3$ (both at atmospheric pressure). In addition, for reservoirs with large surface areas, $v_1 \approx v_3 \approx 0$. Defining $h = z_3 - z_1$ as the change in potential energy in the system from the high to the low elevation reservoir, then Equation 1 reduces to

$$E_p = h + h_f \tag{2}$$

or

$$h_f = E_p - h \tag{3}$$

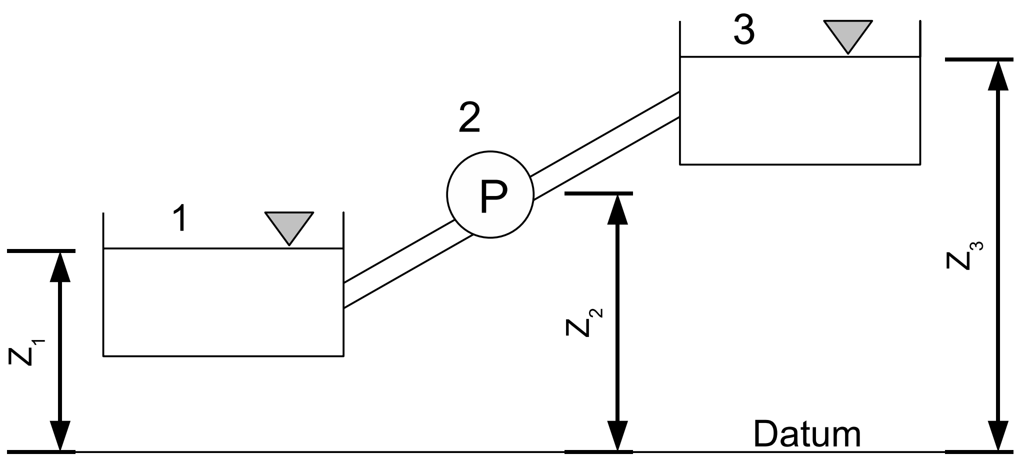

Figure 1. Irrigation water storage reservoir system.

In a typical design situation, the flowrate (Q), pipe length (L), and potential energy head change (h) are known. The design choices are either the pump power input or the pipe diameter. In a case where the pump has already been selected, the output energy head of the pump can be determined by the equation

$$E_p = \frac{76.04e \cdot h_p}{\rho Q} \tag{4}$$

where

$$Q = \text{flow rate (m}^3\text{/s)}$$
$$h_p = \text{pump input power (horsepower)}$$
$$\rho = \text{density (kg/m}^3)$$
$$e = \text{pump efficency}$$
$$E_p = \text{pump output head (m)}$$

With the potential energy head change (h) known, and the pump output head (E_p) specified, Equation 3 can be used to compute the maximum allowable friction headloss over the pipeline.

The most general equation for head loss (decrease in total head) in a pipe due to friction is the Darcy-Weisbach equation

$$h_f = f \frac{Lv^2}{2dg} \tag{5}$$

where

$$h_f = \text{friction head loss (m)}$$
$$f = \text{Darcy-Weisbach friction coefficient}$$
$$L = \text{pipe length (m)}$$
$$v = \text{fluid velocity (m/s)}$$
$$d = \text{pipe diameter (m)}$$
$$g = \text{acceleration of gravity (9.81 m/s}^2)$$

The friction coefficient f can be determined by the Colebrook and White equation (which is a numerical fit of experimental data)

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{k}{3.7d} + \frac{2.51}{\mathrm{Re}\sqrt{f}} \right) \tag{6}$$

where

$$k = \text{absolute roughness of the pipe (m)}$$
$$\mathrm{Re} = \text{Reynolds number } (vd/\nu)$$

The kinematic viscosity, ν, is a function of the fluid and the fluid temperature, and typical values for the roughness factor for common pipe materials are available in the literature. The pipe diameter for a specified maximum friction head loss can be found rearranging Equation 5 to solve for f as a function of d. Then this expression for f is substituted into Equation 6. The result is a nonlinear equation in the single unknown, pipe diameter d.

Write a general purpose program to determine the required minimum pipe diameter for a specified maximum head loss at a given flow rate. The secant root finding routine must be in a generic subroutine that can be used in another problem setting without any changes.

Determine the minimum pipe diameter for a cast iron pipe line ($k = 0.0002591$ m) that the irrigation district should use given $h = 10$ m, $L = 95$ m and $Q = 0.3$ m^3/s. Assume a water temperature of 20 degrees C ($\nu = 0.000001007$ m^2/s, $\rho = 998.2$ kg/m^3). The irrigation district will be using a 100 HP (input power) pump with an efficiency of 0.6.

Finally, suppose that the pipe size just identified is not available for purchase. Use the same program in a trial and error fashion to determine the minimum pump horsepower that would be required if the pipe diameter was instead 0.254 m (10 inches).

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