Latent Gaussian Compression

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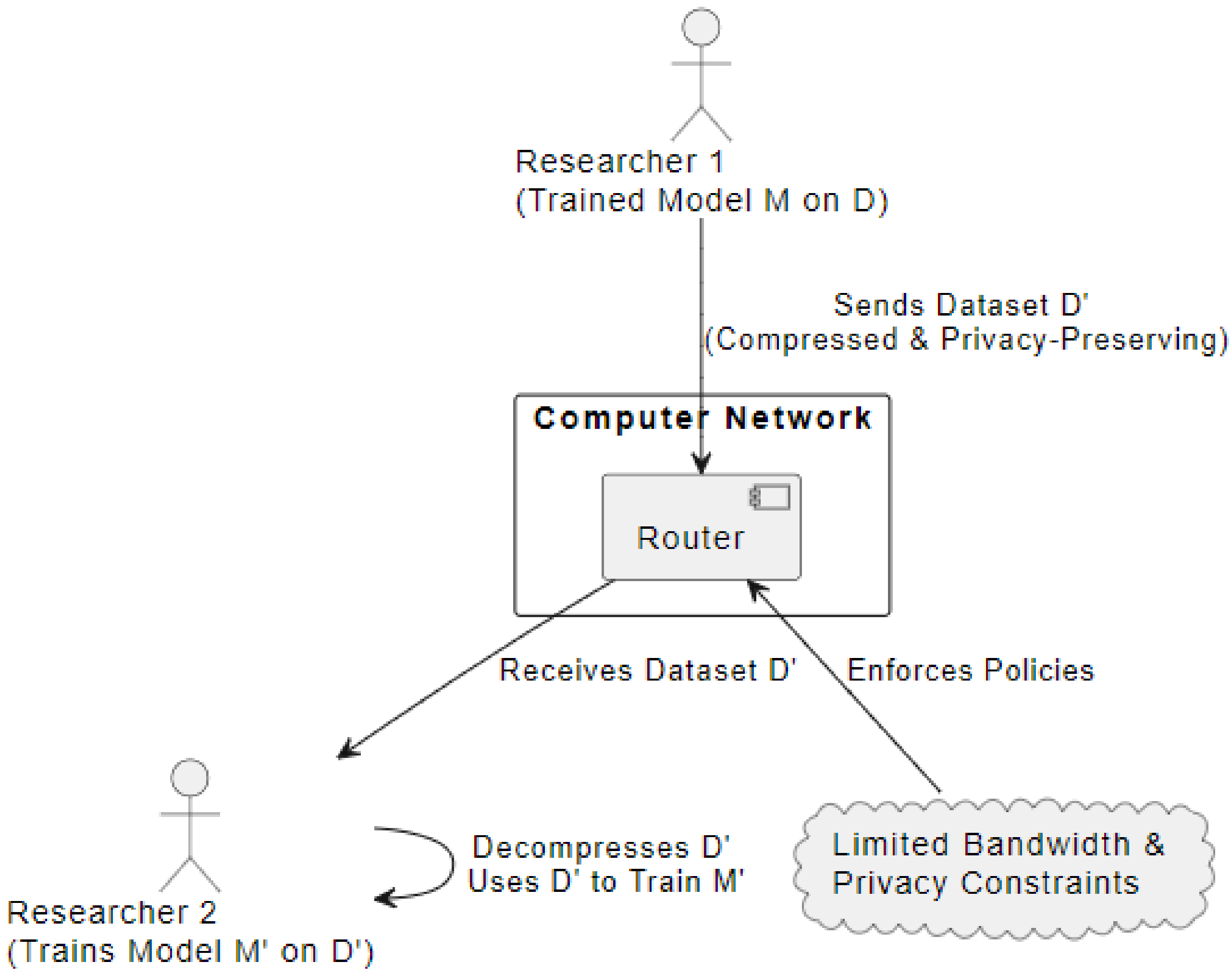
Problem Setup

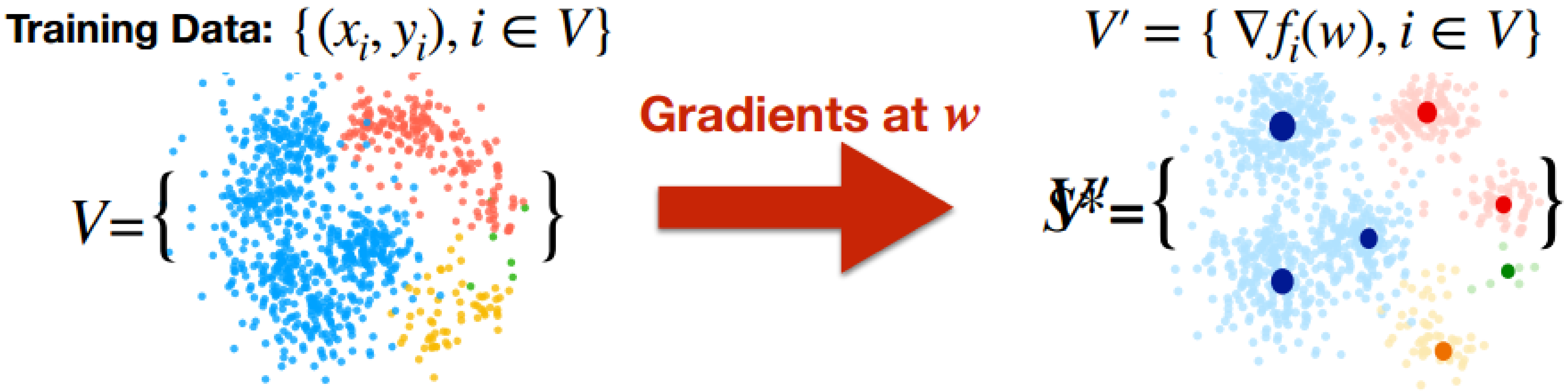
Suppose we have a dataset $D = \{Cat, Dog\}$ with two classes and we want to train a classifier.

- The Problem:
  - Cannot store or transmit full dataset D because of
    - Network bandwidth constraints
    - Space constraints
    - Privacy constraints
- Can we share compressed dataset D' (equivalent to coreset S_k) instead?

Problem Assumptions

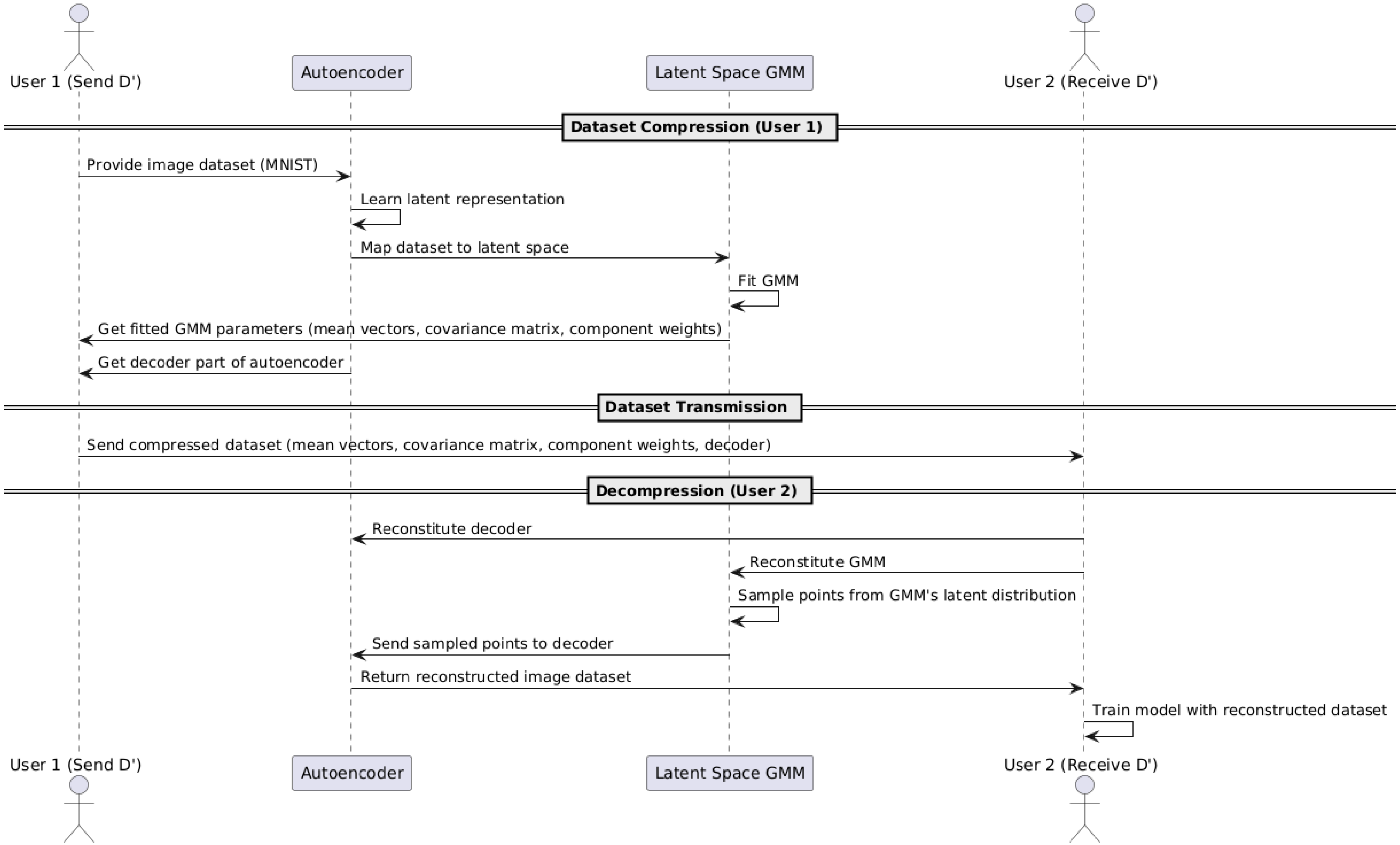


Existing Approaches



- Select subset $S*$ and obtain a $\frac{|V|}{|S*|}$ speedup and compression factor

Workflow



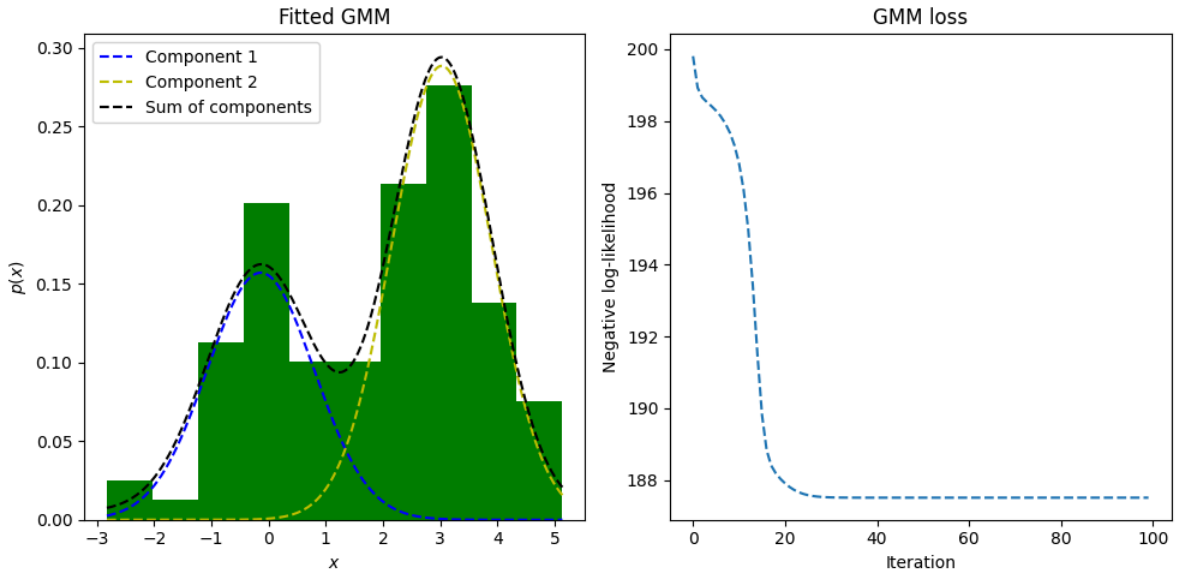
User 1 (Send D')
Autoencoder
Latent Space GMM
User 2 (Receive D')
Dataset Compression (User 1)
Provide image dataset (MNIST)
Learn latent representation
Map dataset to latent space
Fit GMM
Get fitted GMM parameters (mean vectors, covariance matrix, component weights)
Get decoder part of autoencoder
Dataset Transmission
Send compressed dataset (mean vectors, covariance matrix, component weights, decoder)
Decompression (User 2)
Reconstitute decoder
Reconstitute GMM
Sample points from GMM's latent distribution
Send sampled points to decoder
Return reconstructed image dataset
Train model with reconstructed dataset
User 1 (Send D')
Autoencoder
Latent Space GMM
User 2 (Receive D')

Gaussian Mixture Modeling

- Map original data in $R^n(A, B)$ to simpler latent space $R^l(A', B')$ where $l << n$.
- We can approximate the class distributions using Gaussian Mixture Models (GMMs):
  - Represent each class distribution $C' \in (A', B')$ as linear combinations of k Gaussian distributions:

$$P(z) = \sum_{i=1}^{k} \pi_i \mathcal{N}(\mu_{k_{C'}}, \Sigma_{k_{C'}}), \quad z \in R^l$$

Visualizing GMM Distribution Learning

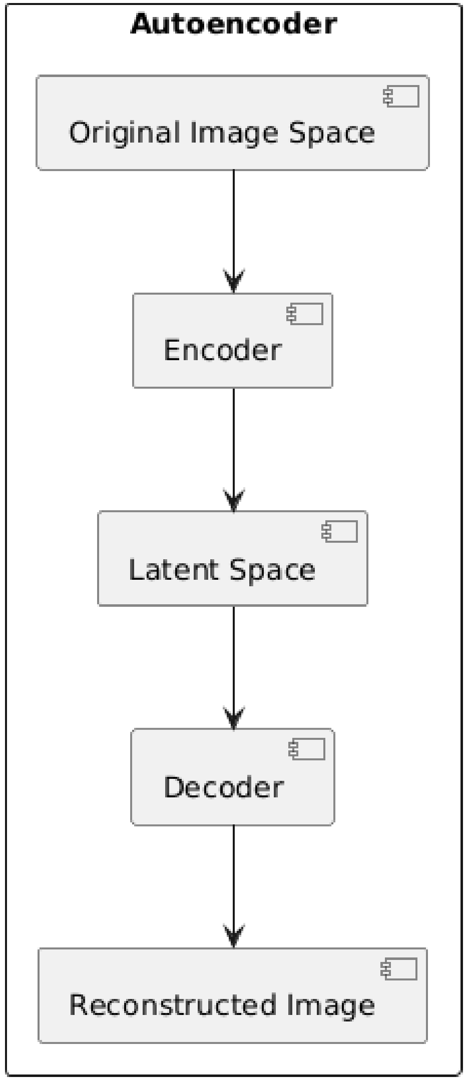


- The image shows the learning of a Gaussian Mixture Model (GMMs) with two components ($k = 2$).
- The distribution is a linear combination of the two components, but can be any integer number of components:

$$p(z) = \pi_1 \mathcal{N}(\mu_1, \Sigma_1) + \pi_2 \mathcal{N}(\mu_2, \Sigma_2)$$

Compression with Autoencoders (AE) and GMMs

- Different autoencoders are used to train up an encoder to transform images into lower dimensional latent space
- The decoder is also trained to recover the original image



Autoencoder Architectures and Losses

We experimented with vanilla AE with/without contrastive learning and the Variational AE with/without contrastive learning:

1. **Reconstruction Loss**:

$$L_{\text{recon}}(x, \hat{x}) = ||x - \hat{x}||^2$$

2. **KL Divergence (regularizer)**:

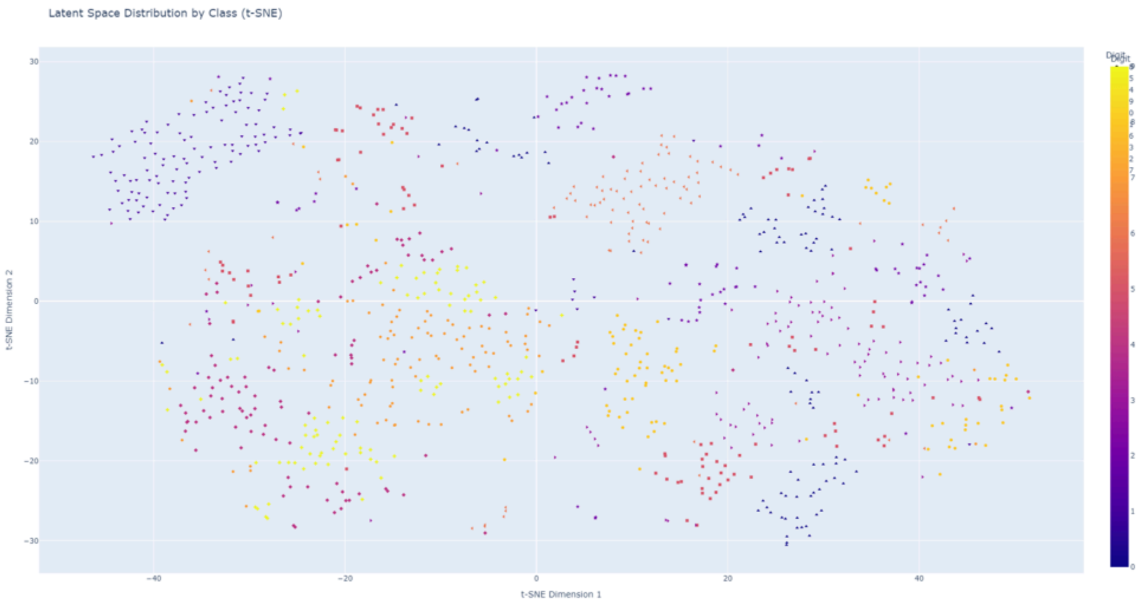
$$L_{\text{KL}} = D_{\text{KL}}(q(z|x)||p(z))$$

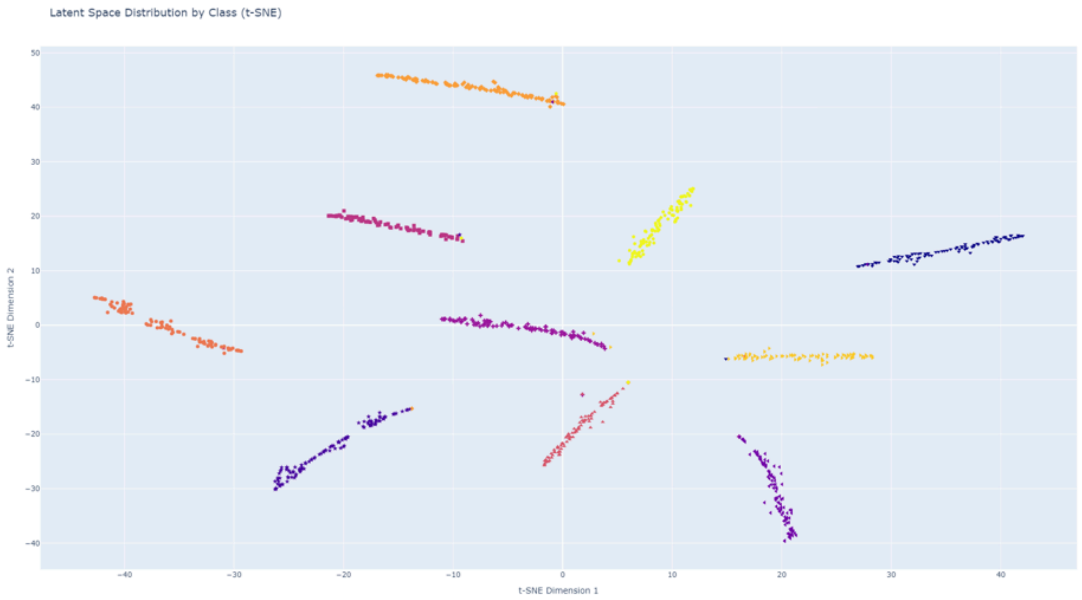
3. **Contrastive Learning Loss**:

$$L_{\text{CL}} = \frac{1}{2N}\sum_{i=1}^{N}(1 - y_i)D_i^2 + y_i \max(0, m - D_i)^2$$

Visualizing the AE Latent Space

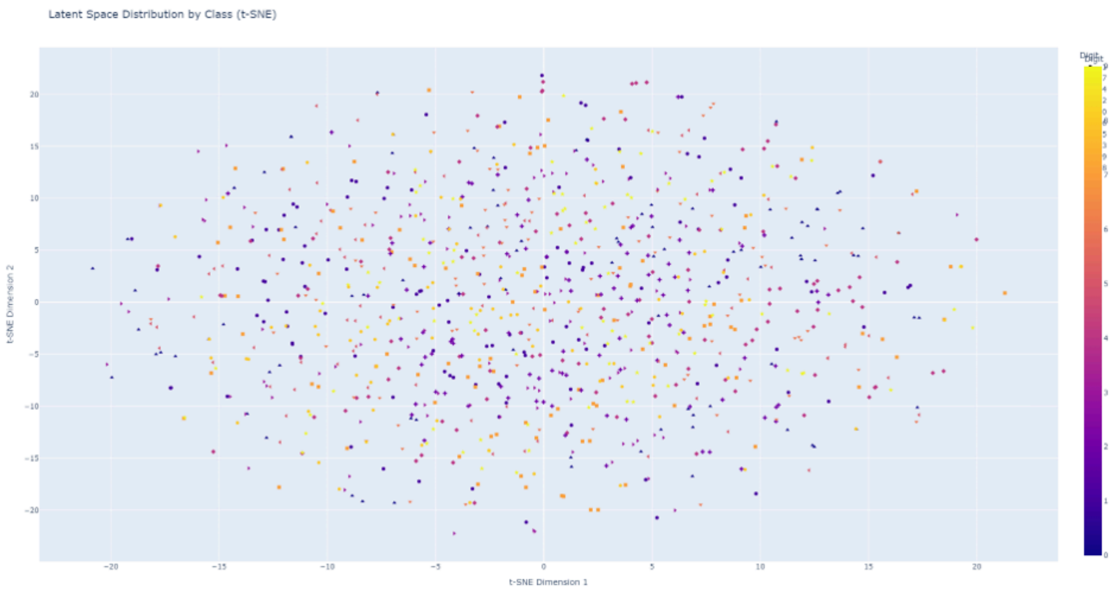
- t-SNE plots below show the AE encoding space without contrastive learning (left) and with contrastive learning (right)
- The effect of the contrastive loss can clearly be seen to pull examples within a class closer together and push examples outside of a class away from each other





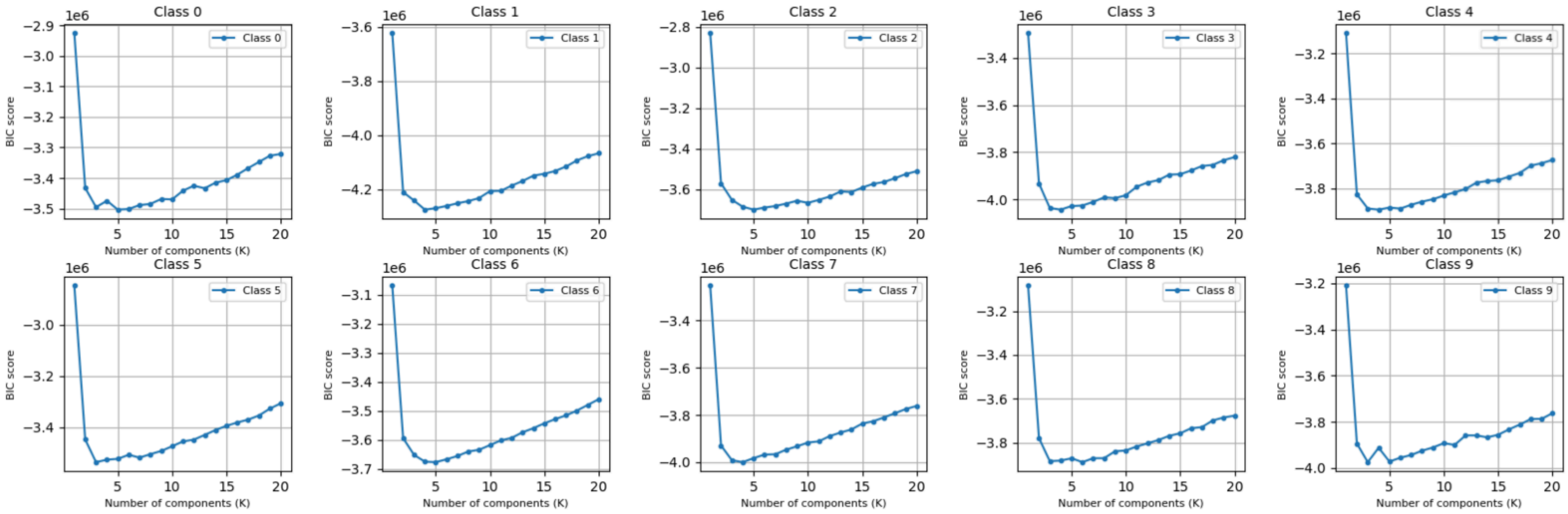
Visualizing the VAE Latent Space

- t-SNE plots below show the same latent space with VAE without contrastive learning (left), and with contrastive learning (right)
- Similarly, the VAE without contrastive loss sees the normalized latent space disitributions intermixed, while contrastive learning can be seen to separate classes



Bayesian Information Criterion (BIC) Curve

- BIC metric is used to determine the appropriate number of k GMMs to decompose into to represent the latent distribution
- The below plots show the BIC plots for each class's distribution, each showing that 2 to 3 GMMs satisfy the criterion of model simplicity and goodness-of-fit



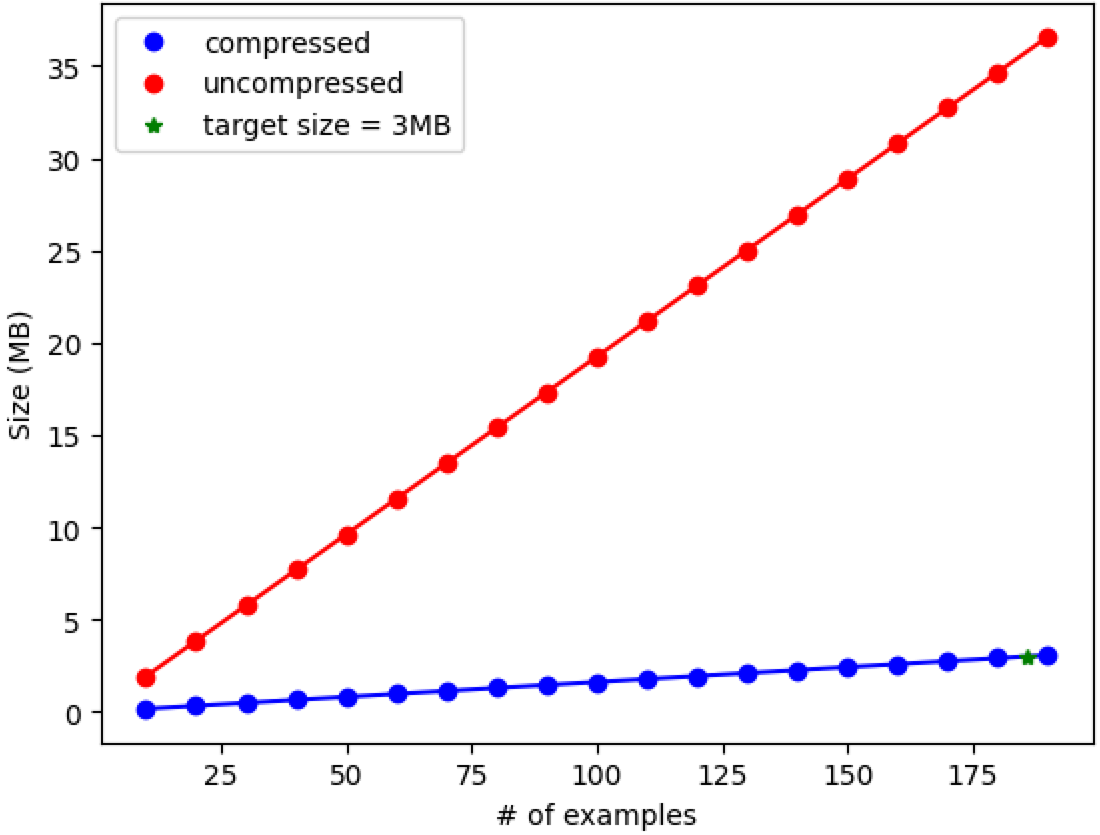
$$\mathrm{BIC} = k\ln(n) - 2\ln(\widehat{L})$$

Baseline Comparison

As a baseline comparison for the performance, subsets of size equal to the compressed model were extracted from the MNIST dataset

- Gradient-Based Clustering
- Random Subset Selection

Each model was evaluated using a CNN classifier

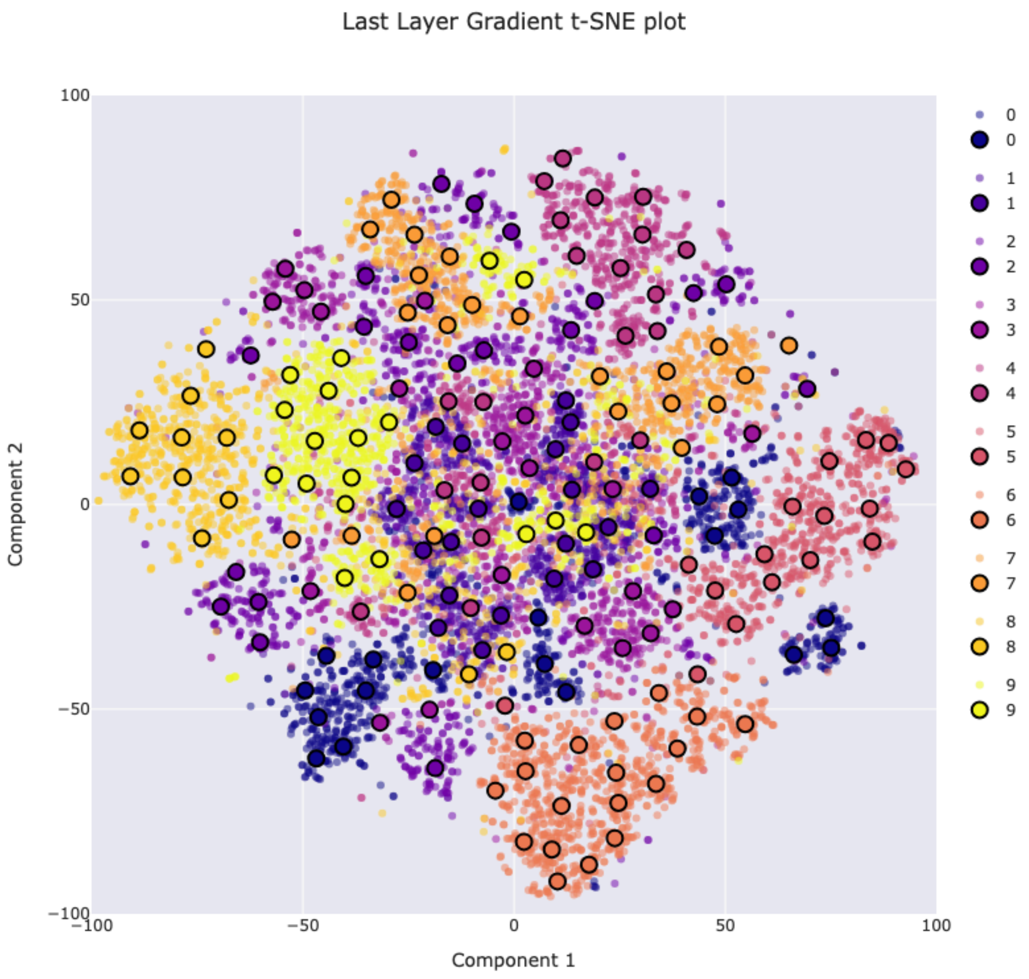


Gradient-Based Exemplar Clustering

Optimization problem:

$$\arg\min_{S,\gamma_j \geq 0} |S| \quad \text{s.t.} \quad \max_{w \in W} ||\nabla_w F(w,V) - \nabla_w F(w,S)|| \leq \epsilon$$

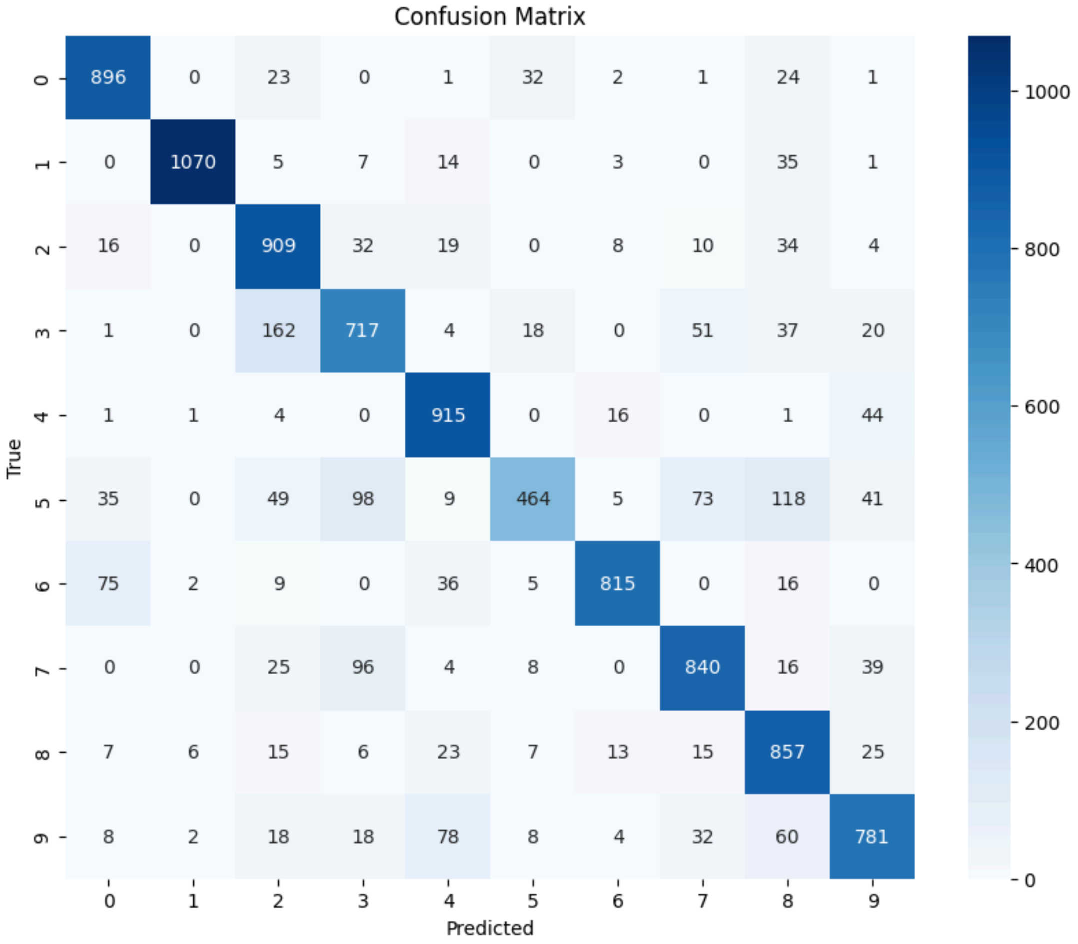
1. Train a model (1-3 epochs)
2. Extract last layer gradients
3. k-medoids++ algorithm for exemplar cluster selection



Baseline Results

Random Subset

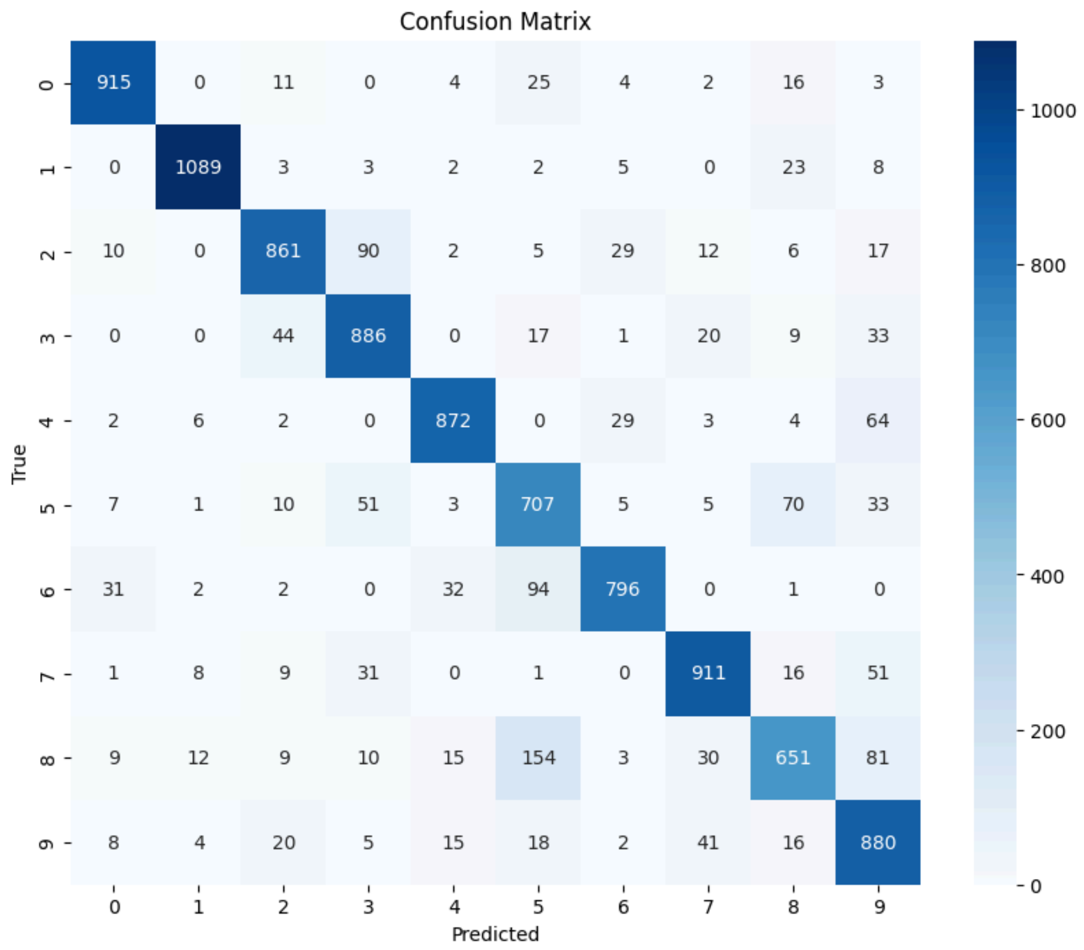
- Test Accuracy on the 10000 test images: 82.64%



Baseline Results

Gradient Clustering

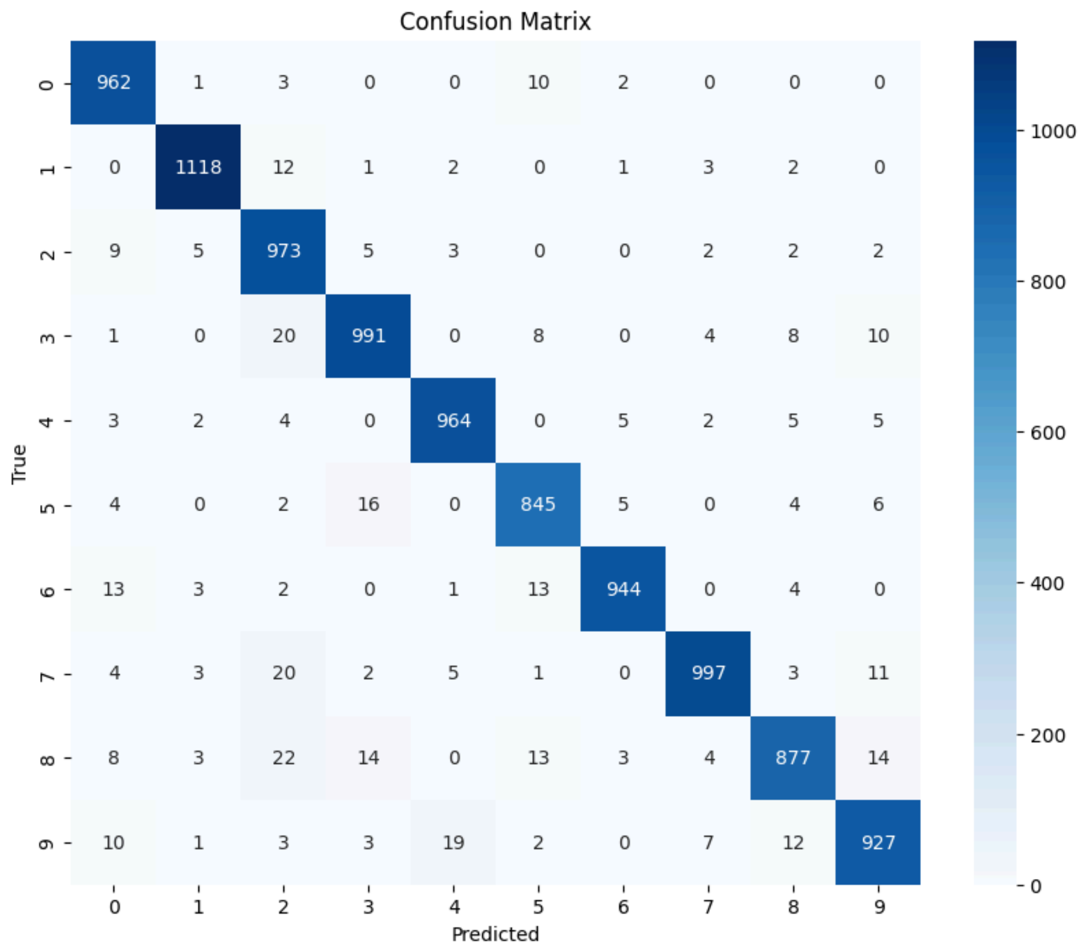
- Test Accuracy on the 10000 test images: 85.68%



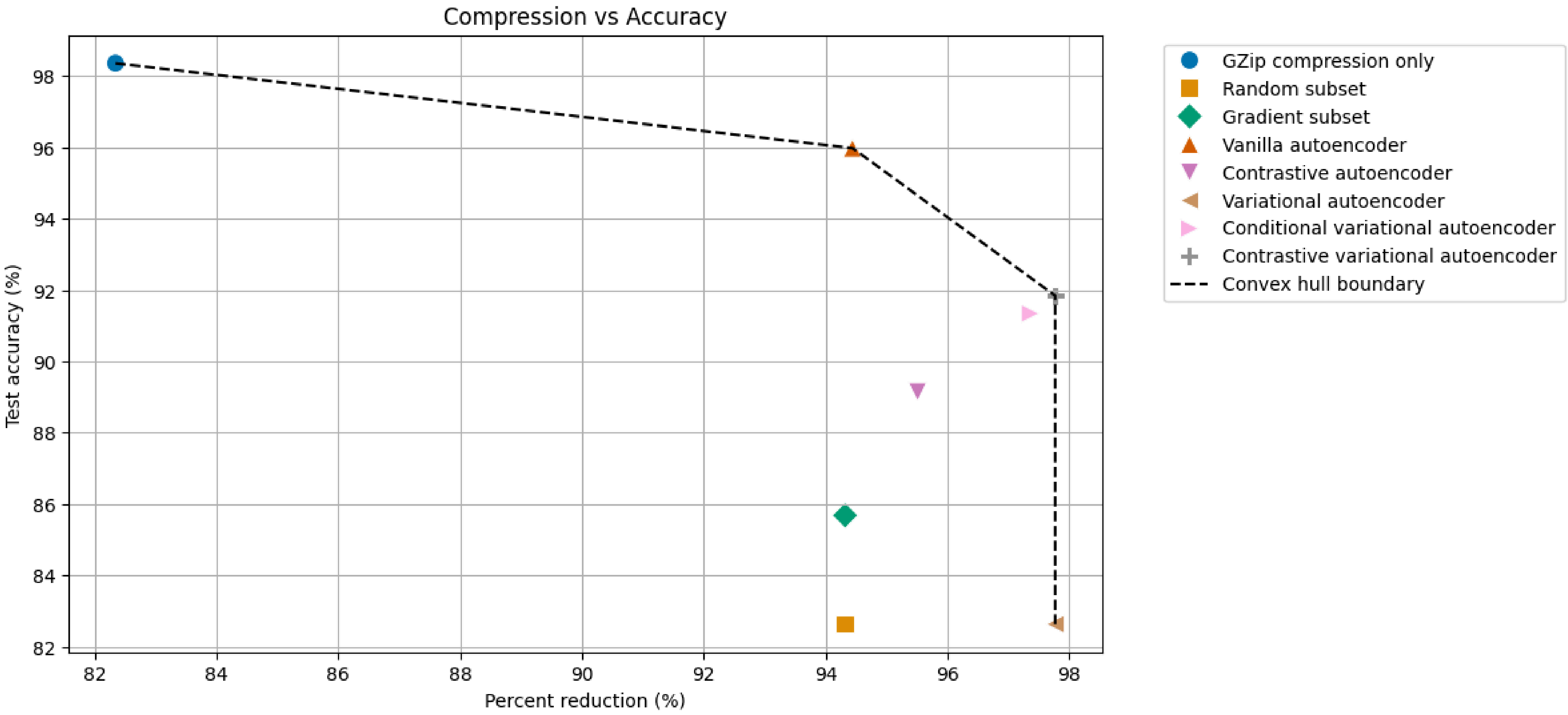
GMM Compression Results

Auto-Encoder

- Test Accuracy on the 10000 test images: 95.98%



Overall Results: Compression vs Accuracy



Information Bottleneck Principle

- A theoretical framework for compression in neural networks.
- Balances:
  - Compression: Reduce information from x to z.
  - Relevance: Ensure z retains information about y.

Connection Between VAEs and Information Bottleneck

- VAEs implicitly optimize an information bottleneck objective by minimizing the following loss:

$$\mathcal{L} = I(x;z) - \beta I(z;y)$$

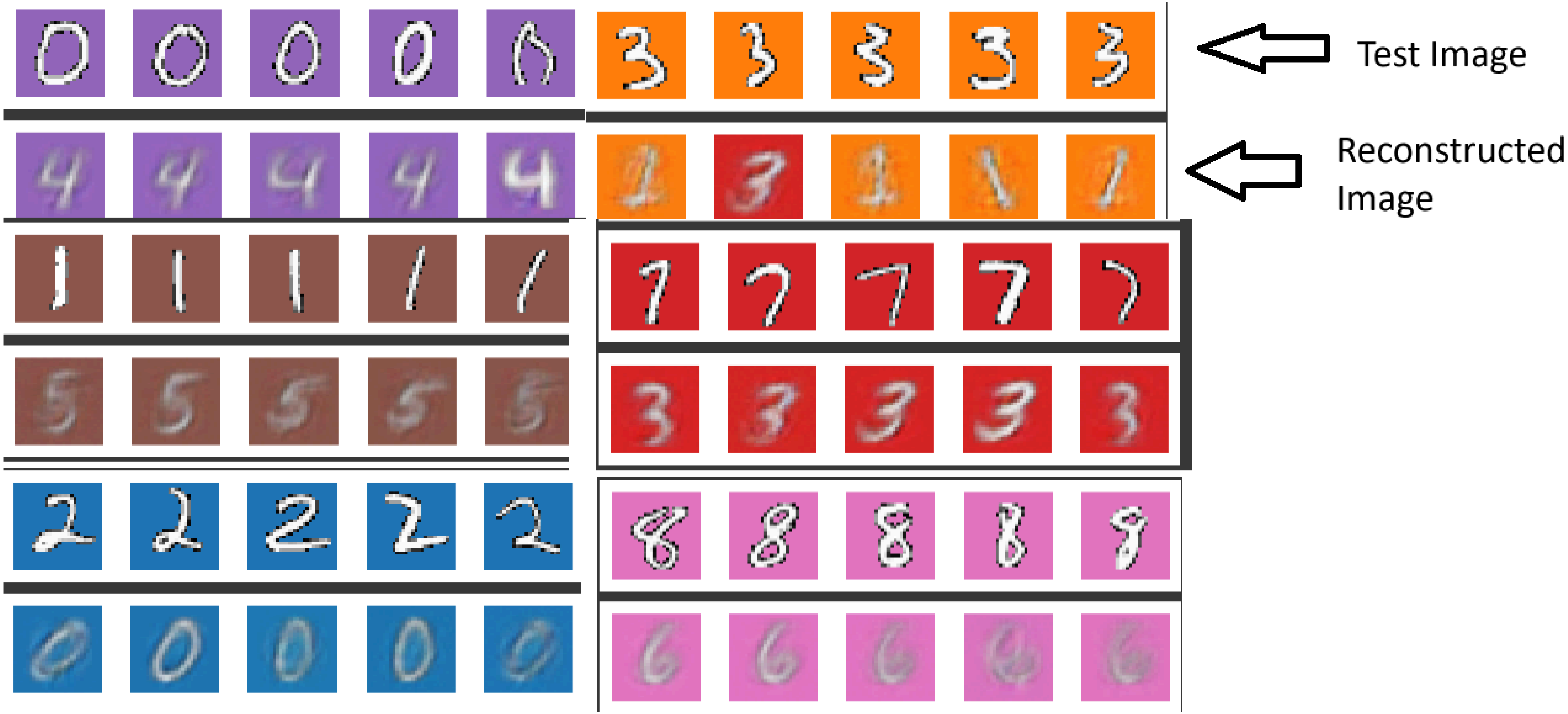
Where:

- $I(x;z)$: Mutual information between x and z.
- $I(z;y)$: Mutual information between z and y.
- β: Controls the trade-off.

Spurious Correlations

Spuco Dataset (Large Spurious Feature Difficulty)

Vanilla VAE Reconstruction (No Upsampling)



Vanilla VAE Reconstruction (With Upsampling)

Convolutional VAE Reconstruction (No Upsampling)

Convolutional VAE Reconstruction (With Upsampling)

References

Bishop, Christopher M., and Nasser M. Nasrabadi. Pattern recognition and machine learning. Vol. 4. No. 4. New York: springer, 2006.

Hinton, Geoffrey E., and Ruslan R. Salakhutdinov. "Reducing the dimensionality of data with neural networks." science 313.5786 (2006): 504-507.

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