Electricity and Magnetism - Lecture 12 Notes

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Key Topics

- **Bar Magnets as Sources of Magnetic Field**
- **Electron Current and Conventional Current**
- **Biot-Savart Law in a Wire**
- **Magnetic Field of a Straight Wire**
- **Magnetic Monopoles and Special Relativity**

Bar Magnets as Magnetic Dipoles

- **Magnetic Field Direction**: From **North (N)** to **South (S)** outside the magnet.
- **Field Lines**: Similar to electric field lines, they indicate direction and density of the field.
- **Interactions**:
  - **Like Poles** (N and N, S and S): **Repel**.
  - **Unlike Poles** (N and S): **Attract**.

Magnetic Monopoles

- No magnetic monopoles have been observed in nature, but their existence is theoretically possible.

- A magnetic monopole would have a single magnetic charge $(+$ or $-)), similar to an electric charge.$

Conventional Current and Electron Current

- **Electron Current**: Electrons move from the negative ($-$) terminal to the positive ($+$) terminal of a battery.
- **Conventional Current**: Defined as the movement of positive charges, from the positive ($+$) terminal to the negative ($-$) terminal.
- **Difference in Convention**:
  - **Benjamin Franklin's Assumption**: Current was carried by positive charges, leading to the definition of **conventional current**.
  - Despite being incorrect, conventional current remains a useful model.
- **Equation for Conventional Current**

$$I = |q|nA\vec{v}$$

Calculating Electron Current

- **Electron Current** (i) depends on:
  - **Density of Mobile Electrons** in the wire (n).
  - **Cross-Sectional Area** of the wire (A).
  - **Drift Velocity** of the electrons (v_d).
- **Equation for Electron Current**:

$$i = nA\vec{v}e$$

  where e is the charge of an electron.

Biot-Savart Law

- **Definition**: The Biot-Savart Law gives the magnetic field produced by a small segment of current-carrying wire or a moving point charge.
- **Point Charge**:

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{|r|^2}$$

  where q is the charge, $\vec{v}$ is the velocity, $\hat{r}$ is the unit vector from the charge to the observation point, and r is the distance between them.
- **Current Element in a Wire**:

$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{|r|^2}$$

  where I is the current and $d\vec{l}$ is a small segment of the wire.

Magnetic Field of a Straight Wire

- **Goal**: Calculate the magnetic field (B) at a point in the bisecting plane of the wire.
- **Method**:
  - Divide the wire into small segments.
  - Apply the Biot-Savart Law to find the field contribution from each segment.
  - Integrate over the entire length of the wire.
- **Result for a Long Straight Wire**:

$$|B_y| = \frac{\mu_0}{4\pi} \frac{IL}{x\sqrt{x^2 + (\frac{L}{2})^2}}$$

  This is B of a Long Straight Wire where r is the perpendicular distance from the wire.

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{IL}{r\sqrt{r^2 + (\frac{L}{2})^2}} \hat{\theta}$$

  B of a Long Straight Wire (Cylindrical Coordinates)

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{2I}{r} \hat{\theta}$$

  CLOSE TO THE WIRE VALID ANYWHERE FOR AN INFINITE WIRE!

Special Relativity and Magnetic Fields

- **Magnetic Fields Depend on Reference Frame**:
  - The observation of a magnetic field depends on the relative motion of the observer.
  - A moving charge can produce both electric and magnetic fields depending on the frame of reference.
- **Connection to Electric Fields**: There is a deep relationship between electric and magnetic fields, as shown by Einstein's special theory of relativity.