Electricity and Magnetism - Lecture 10 Notes

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Potential Due to a Charge Distribution

- **Potential of a Charge Distribution**: Can be calculated by dividing the distribution into small point charges and adding up their contributions.
- **Method 1**: Divide into point charges and add up contributions:

$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

- **Method 2**: Integrate the electric field along a path to find the potential:

$$V = -\int_{\infty}^{r} \vec{E} \cdot d\vec{l}$$

Potential Inside Conductors and Insulators

- **Inside a Conductor**:
  - **Equilibrium Condition**: The electric field inside a conductor is zero, meaning the electric potential is constant.
  - **Equipotential Surface**: The entire conductor is at the same electric potential in electrostatic equilibrium.
- **Inside an Insulator**:
  - The electric field inside an insulator is not necessarily zero.
  - **Potential Difference in an Insulator**: Includes contributions from the external field and the induced dipoles within the insulator.

Electric Potential of a Uniformly Charged Ring

- **Observation Point Along Axis**:

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{R^2 + z^2}}$$

  where Q is the total charge, R is the radius of the ring, and z is the distance from the center of the ring along its axis.

- **Far-Field Approximation** ($z \gg R$): The potential resembles that of a point charge:

$$V \approx \frac{1}{4\pi\epsilon_0} \frac{Q}{z}$$

Electric Potential of a Uniformly Charged Sphere and Shell

- **Uniformly Charged Shell**:
  - **Outside the Shell** ($r > R$):

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

  - **Inside the Shell** ($r \leq R$): The potential is constant.

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$

- **Uniformly Charged Sphere**:
  - **Outside the Sphere** ($r > R$): Similar to a point charge.

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

  - **Inside the Sphere** ($r < R$): The potential varies as:

$$V(r) = \frac{1}{4\pi\epsilon_0} \left(\frac{3Q}{2R} - \frac{Qr^2}{2R^3} \right)$$

Energy Stored in an Electric Field

- **Energy Density of an Electric Field**:

$$\frac{U}{Volume} = \frac{1}{2}\epsilon_0 E^2$$

  where u is the energy density, and E is the magnitude of the electric field.

- **Total Energy Stored**: For a volume containing an electric field, the total energy is:

$$U = \int_{\text{Volume}} u \, dV = \int_{\text{Volume}} \frac{1}{2} \epsilon_0 E^2 \, dV$$

Dielectric Constant and Insulators

- **Dielectric Constant** (K): The ratio of the applied electric field to the net electric field inside an insulator.

$$K = \frac{E_{\text{applied}}}{E_{\text{net}}}$$

- **Values for Common Materials**:
  - **Vacuum**: $K = 1$ (by definition)
  - **Air**: $K = 1.0006$
  - **Water**: $K = 80$
  - **Strontium Titanate**: $K = 310$