Electricity and Magnetism - Lecture 11 Notes

Joshua Clement

October 7, 2024

Key Topics

- **Electric Potential of a Conductor**
- **Sharp Point Effect**
- **Sources of Magnetic Field**
- **Magnetic Field due to Moving Charges**
- **Biot-Savart Law for a Point Charge**
- **Cross Products: Right-Hand Rule and Mathematical Representation**

Electric Potential of a Spherical Conductor with Net Charge Q

- **Electric Potential** V for a spherical conductor of radius R and charge Q:

$$V(r) = \begin{cases} \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = ct & \text{if } r \leq R \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r} & \text{if } r > R \end{cases}$$

- **Potential Inside Conductor**: Constant, equal to $\frac{1}{4\pi\epsilon_0}\frac{Q}{R} = ct$

Sharp Point Effect

- **Electric Field Enhancement**: The electric field is stronger at sharp points due to higher surface charge density.
- **Practical Example**: Lightning rods use this effect to direct lightning strikes to a sharp point, where the field is strongest.

Magnetic Field Key Concepts

- **Moving Charges Create a Magnetic Field**: A magnetic field is generated when charges are in motion.
- **Magnetic Dipole**:
  - A current-carrying loop and a bar magnet are examples of magnetic dipoles.
  - Even a single atom can be a magnetic dipole.
- **Magnetic Field Units**:
  - **Tesla (T)**: Standard unit for magnetic field.
  - **Gauss (G)**: Another unit for magnetic field, where $1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$.

Compass Needle and Magnetic Field

- **Key Idea**: The needle of a compass aligns with the net magnetic field at its location, regardless of the source of the field.
- **Examples**:
  - When isolated, the compass points to Earth's geomagnetic pole.
  - When near a magnet or current-carrying wire, the needle deflects according to the net magnetic field.

Biot-Savart Law

- **Key Idea**: The Biot-Savart Law calculates the magnetic field generated by a moving charge.
- **Formula**:
$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$
where μ_0 is the vacuum permeability, q is the charge, $\vec{v}$ is the velocity, and $\hat{r}$ is the unit vector pointing from the charge to the observation point.

Right-Hand Rule for Cross Products

- **Right-Hand Rule**: Used to determine the direction of the magnetic field resulting from a moving charge.
- **Magnitude of Cross Product**:
$$|\vec{A} \times \vec{B}| = AB \sin\theta$$
where θ is the angle between vectors $\vec{A}$ and $\vec{B}$.

Interesting Facts About Magnetic Fields

- **Earth's Magnetic Field**:
  - Strength ranges from 0.25 G to 0.65 G (25 to 65 μT).
  - Generated by electric currents in the Earth's molten iron and nickel core.
  - North geomagnetic pole is actually the South pole of Earth's magnet.
- **Magnetic Field Strengths**:
  - Typical Nd magnet: 1 T.
  - Strongest permanent magnets: 4.5 T.
  - Neutron stars: Magnetic fields up to $10^4 - 10^{11}$ T.