Magnetic Fields and Moments Notes

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Magnetic Field of a Straight Wire

- **Magnetic Field** ($\vec{B}$) of a long straight wire:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{IL}{r\sqrt{r^2 + \frac{L}{2}^2}} \hat{\theta}$$

  - In **cylindrical coordinates**, $\vec{B}$ points along concentric circles around the wire.
  - **Very close to the wire** ($r \ll L$), another right-hand rule can be applied to determine direction.

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{2I}{r} \hat{\theta}$$

  - If current changes in the wire, the magnetic field changes at the **speed of light**.
  - **Biot-Savart Law** is an approximation that does not account for **retardation** and **relativistic effects** ($v \ll c$).

Magnetic Field of a Current Loop

- **Magnetic Field on Axis** of a circular current loop:
  - **Biot-Savart Law**:

$$\vec{B}_{\text{loop}} = \frac{\mu_0}{4\pi} \frac{2\pi R^2 I}{(z^2 + R^2)^{3/2}} \hat{z}$$

This can be writen as: ($\mu = I(\pi R^2)$)

$$\vec{B}_{\text{loop}} = \frac{\mu_0}{4\pi} \frac{2\mu}{(z^2 + R^2)^{3/2}} \hat{z}$$

- **Far from the loop** ($z \gg R$, and $\mu = IA$) A being the area of the loop:

$$B_{\text{axis}} \approx \frac{\mu_0}{4\pi}\frac{2\vec{\mu}}{r^3}$$

$$B_{\perp} \approx \frac{-\mu_0}{4\pi}\frac{\vec{\mu}}{r^3}$$

- **Magnetic Dipole Moment** (μ): Defined as $\mu = IA$, where A is the area of the loop.
- Units of magnetic dipole moment: $[A \cdot m^2]$.

Magnetic Field of Thin and Long Coils (Solenoids)

- **Solenoid**: A coil of wire with N turns.
  - **Magnetic Field Inside** a long solenoid:

    $$B \approx \mu_0 n I$$

    where $n = \frac{N}{L}$ is the loop density. N = number of loops, L = length of solenoid.
  - The magnetic field inside is nearly **uniform** across the solenoid's cross-section (except near ends).

Atomic Magnetic Dipole Moments

- **Ferromagnetic Materials**:
  - Magnetic dipole moments of atoms align below a critical temperature called the **Curie Temperature**.
  - **Ferromagnet**: Material in which atomic magnetic dipole moments align, leading to a net magnetic field.
- **Atomic Magnetic Moments** are primarily due to:
  1. **Electron orbitals** (electrons orbiting nucleus).
  2. **Electron spin** (electrons spinning on their own axis).
  3. **Protons and neutrons** also contribute, but their magnetic moments are smaller than those of electrons.

Permanent Magnets and Domains

- **Magnetic Domains**: Regions in ferromagnetic materials where atomic dipoles are aligned.
  - Ferromagnetic materials like **iron, cobalt,** and **nickel** have permanent magnetic moments.
  - **Heating a ferromagnet** can **demagnetize** it by disturbing the alignment of magnetic domains.
- **Paramagnetic Materials**: Attracted by an external magnetic field (most atoms with incomplete orbitals).
- **Diamagnetic Materials**: Repelled by an external magnetic field (e.g., water, wood, organic compounds).