Electricity and Magnetism - Lecture 5 Notes

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October 7, 2024

Electric Field of a Charge Distribution

- **Charge Density** (λ): Charge per unit length for a linear distribution.
- **Electric Field of a Charged Rod**:
  - To determine the electric field, break the rod into small elements (Δq).
  - Use the **superposition principle**: The total electric field is the sum of the contributions from each small charge element.

Steps to Solve Charge Distribution Problems

1. **Understand the Geometry**: Identify the shape and distribution of the charged object.
2. **Choose** dq: Define an infinitesimal charge element (dq).
3. **Evaluate** dE: Determine the electric field contribution from dq.
4. **Exploit Symmetry**: Use symmetry to simplify calculations.
5. **Set Up the Integral**: Integrate over the entire length/volume of the charge distribution.
6. **Solve the Integral**: Compute the electric field by solving the integral.
7. **Check Limiting Cases**: Ensure results are consistent for simpler configurations.

Electric Field of a Finite Charged Rod in the Bisecting Plane

- Consider a rod of length L with a uniform positive charge distribution.
- We calculate the electric field only in the plane that **bisects** the rod.

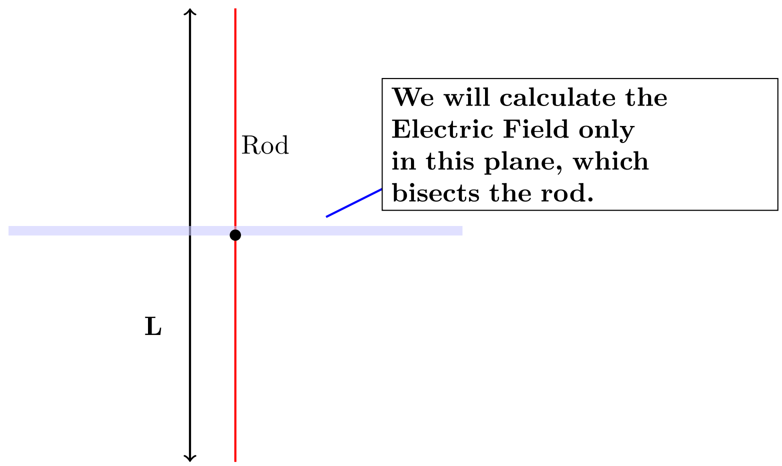

Figure 1: Illustration of calculating the electric field in the bisecting plane of a charged rod.

- **y-components cancel**, and only **x-components** contribute to the net electric field.
- Use the **superposition principle** to sum the contributions from each element (Δq).
- **Integral Representation**:

$$\vec{E}_{\text{tot}} = \int_{-L/2}^{L/2} \frac{Qx}{4\pi\epsilon_0 L(x^2+y^2)^{3/2}}\,dy$$

- **None Integral Representation lol idk what its actually called**:

$$\vec{E}_{\text{tot}} = (\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+(\frac{L}{2})^2}})\hat{x}$$

Electric Field of an Infinite Rod

- For an **infinite rod**, the electric field is defined everywhere in space.
- **Linear Charge Density** ($\lambda = \frac{Q}{L}$): The electric field is dependent on the charge density.
- **Electric Field** at a distance r from the rod:

$$E(r) = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$$

- The electric field decreases with distance ($\frac{1}{r}$).

Key Takeaways: Charged Rods

- **Finite Rod**: The electric field is only calculated in the **bisecting plane**.
- **Infinite Rod**: The electric field extends **everywhere in space**.
- For a point far away from a finite rod ($r \gg L$), the electric field resembles that of a **point charge**.