Electricity and Magnetism - Lecture 9 Notes

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Electric Potential (Voltage Relative to Infinity)

- **Electric Potential** (V): The potential energy per unit charge due to the presence of other charges.
- **Potential at Infinity**: Defined as zero by convention, consistent with electric potential energy being zero for two charges infinitely separated.
- **Potential Due to One Particle**:

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

where q is the charge and r is the distance from the charge.

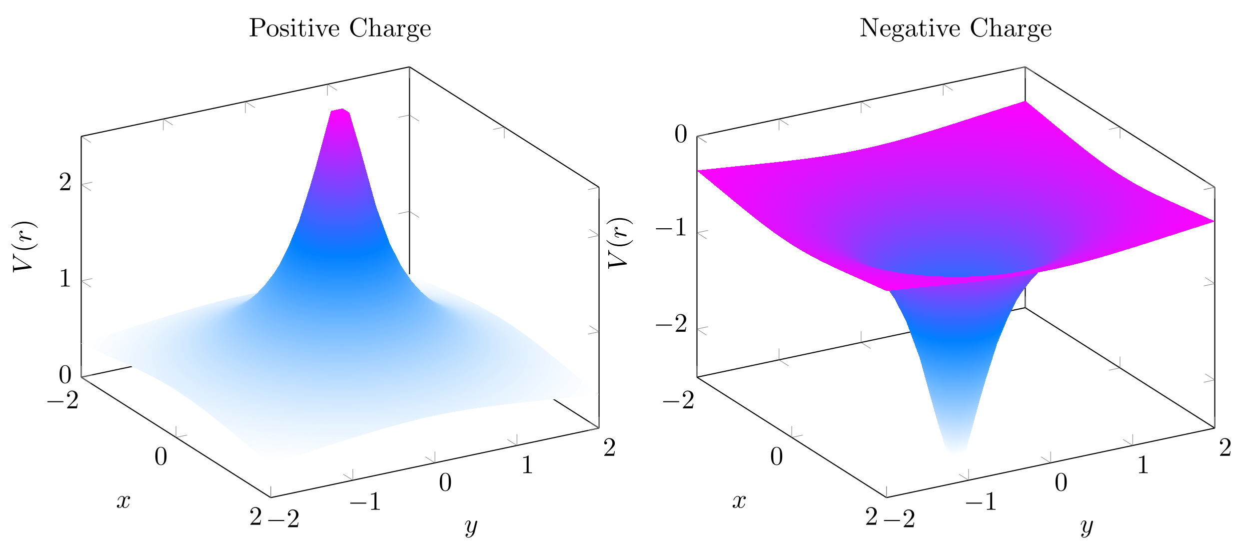

Figure 1: Electric field always points in the direction of steepest descent of V (steepest slope) and its magnitude is the slope.

Potential Difference and Electric Field

- **Potential Difference** (ΔV): Difference in electric potential between two points.
$$\Delta V = V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{l}$$

- **Uniform Electric Field** ($E \parallel x$):
$$\Delta V = -E\Delta x$$

- **Non-Uniform Electric Field**: The potential difference is obtained using the line integral of the electric field along the path.

Path Independence of Potential Difference

- **Path Independence**: The potential difference between two points does not depend on the path taken, only on the endpoints.
- **Round Trip Path**: For a complete loop, the potential difference is zero ($\Delta V = 0$).
- **Example**: Moving from point A to B and back results in $\Delta V = 0$, similar to changes in height for a round trip up and down a mountain.

Electric Potential and Electric Field

- **Electric Field Direction**: The electric field points in the direction where the potential decreases most rapidly.
- **Relation Between Potential and Field**:
$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

- **Gradient**: The electric field is the negative gradient of the electric potential:
$$\vec{E} = -\nabla V$$

Electric Potential of a Uniformly Charged Shell

- **Outside the Shell** ($r > R$):
$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

  where Q is the total charge and R is the radius of the shell.

- **Inside the Shell** ($r \leq R$):

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$$

The potential inside is constant.

Calculating Electric Field from Potential

- **Direction of Electric Field**: Points in the direction of the steepest descent of the potential (V).
- **Magnitude of Electric Field**: Related to the slope of the potential function.
- **Example**:
  - **Positive Charge**: Electric potential decreases with increasing distance, and the field points away from the charge.
  - **Negative Charge**: Electric potential increases (becomes less negative) with increasing distance, and the field points toward the charge.