Electricity and Magnetism - Lecture 6 Notes

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Electric Field Calculations for Different Geometries

- **Points, Lines, and Planes**:
  - **Point Charge**: Electric field radiates outward from the point ($E = \frac{1}{r^2}$).
  - **Infinite Line Charge**: Electric field extends radially outward, decreases with distance ($E = \frac{1}{r}$).
  - **Infinite Plane Charge**: Electric field is constant at all points above or below the plane ($E = Constant$).

Electric Field Along the Axis of a Ring

- Goal: Find the electric field (E) along the **z-axis** of a ring with total charge Q and radius a.
- **Symmetry Considerations**:
  - Only the **z-component** of the electric field survives due to symmetry.
- **Electric Field Contribution** (ΔE_z) from a small charge element (Δq) on the ring:

$$\Delta E_z = \frac{1}{4\pi\epsilon_0} \frac{z\Delta q}{(a^2 + z^2)^{\frac{3}{2}}}$$

- **Total Electric Field**:

$$E_{\text{tot}} = \int_{\text{ring}} \Delta E_z = \frac{1}{4\pi\epsilon_0} \frac{Qz}{(a^2 + z^2)^{\frac{3}{2}}} \hat{z}$$

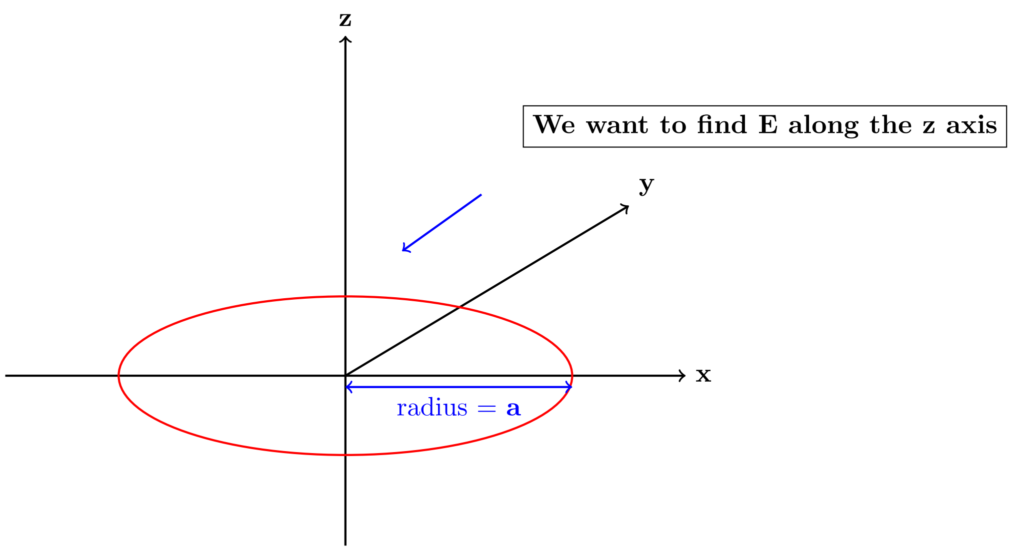

Figure 1: Illustration of finding the electric field along the z-axis of a charged ring.

Electric Field Along the Axis of a Disk

- Goal: Find the electric field (E) along the **z-axis** of a disk with total charge Q and radius R.
- **Symmetry Considerations**:
  - Only the **z-component** of the electric field survives due to symmetry.
- **Surface Charge Density** (σ):

$$\sigma = \frac{Q}{\pi R^2}$$

- **Electric Field Contribution** (ΔE_z):

$$\Delta E_z = \frac{1}{4\pi\epsilon_0} \frac{z\Delta q}{(a^2 + z^2)^{\frac{3}{2}}}$$

- **Total Electric Field**:

$$E_{\text{tot}} = \frac{1}{2\epsilon_0} \left(\frac{Q}{\pi R^2} \right) \left[1 - \frac{z}{\sqrt{R^2 + z^2}} \right] \hat{z}$$

Field of an Infinite Plane

- **Infinite Plane with Uniform Charge Density** (σ):

- The electric field is **constant** at all points on either side of the plane:

$$E = \frac{\sigma}{2\epsilon_0}$$

- This field is independent of the distance from the plane.

Field of Two Infinite Planes

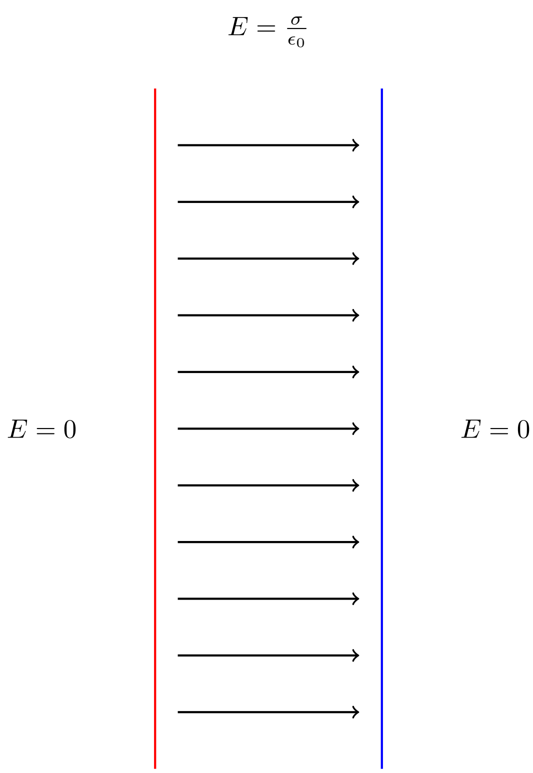

Figure 2: Electric field between two infinite parallel plates. Fields inside add up, while fields outside cancel.

- **Two Infinite Planes with Opposite Charge Density**:
- **Superposition Principle**:
  - The field between the planes **adds**, resulting in a net field of:

    $$E_{\text{net}} = \frac{\sigma}{\epsilon_0}$$

  - Outside the planes, the fields **cancel** each other.

Key Concepts

- **Symmetry** plays a crucial role in simplifying electric field calculations.
- For **rings and disks**, only the component along the axis survives.
- For **infinite planes**, the electric field is constant regardless of distance from the plane.
- **Superposition** helps determine the net field when dealing with multiple charge distributions.