

Problem:

Check whether 7 divides:

$$1989 \times 1990 \times 1991 + (1992)^3$$

Problem

$$1989 = m = 7x + 1$$

$$1990 = m + 1$$

$$1991 = m + 2$$

$$1992 = m + 3$$

$$\therefore (m)(m+1)(m+2) + (m+3)^3$$

$$\Rightarrow (m^3 + 3m^2 + 2m) + (m^3 + 27 + 9m^2 + 27m)$$

$$\Rightarrow 2m^3 + 12m^2 + 29m + 27$$

$$\therefore (a+b)^3 = a^3 + b^3 + 3ab(a+b)$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$\checkmark 2(7x+1)^3 + 12(7x+1)^2 + 29(7x+1) + 27$$

$$\Rightarrow 2(7n+1) + 12(7y+1) + 29(7x+1) + 27$$

$$\Rightarrow 7(2n + 12y + 29x) + 2 + \underbrace{12 + 29 + 27}_{70}$$

$$\Rightarrow 7(2n + 12y + 29x + 10)$$

\therefore remainder is 0