Problem

Prove that if a and b are odd, positive integers then $a^2 - b^2$ is divisible by 8.

$a = 2k+1$
$b = 2x+1$

$\therefore \; a^2 - b^2 = (a+b)(a-b)$

$= (2k+1+2x+1)(2k+\cancel{1}-2x-\cancel{1})$

$= (2k+2x+2)(2k-2x)$

$= 4k^2 - \cancel{4kx} + \cancel{4kx} - 4x^2 + 4k - 4x$

$= 4k^2 - 4x^2 + 4k - 4x$

$a^2 - b^2 = 4(k^2 - x^2 + k - x)$

k^2 can be odd if k is odd
even if k is even

so $k^2 + k$ would be odd + odd or even + even
where in both cases $k^2 + k$ is even.
Same can be said for $x^2 + x$

$k^2 + k - (x^2 + x)$ would also be even and
it can be written as $2t$

$\therefore \; a^2 - b^2 = 4 \cdot 2t$
$= 8t$