0.1 Differential Equations

After creating a nonlinear model using the Lagrangian method, and then linearizing that model, Yamamoto [0] provides the differential equations (0.1) and (0.12), [*and their abbreviated term definitions*].

0.1.1 Wheel Angular Position θ and Body Pitch ϕ_x

Equation (0.1) corresponds to wheel angular position θ and body pitch ϕ_x.

$$\begin{aligned}
\mathbf{K}_{1.\ddot{x}} \cdot \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi}_x \end{bmatrix} + \mathbf{K}_{1.\dot{x}} \cdot \begin{bmatrix} \dot{\theta} \\ \dot{\phi}_x \end{bmatrix} + \mathbf{K}_{1.x} \cdot \begin{bmatrix} \theta \\ \phi_x \end{bmatrix} &= \mathbf{K}_{1.v} \cdot \begin{bmatrix} v_{mtr.l} \\ v_{mtr.r} \end{bmatrix} \\
\begin{bmatrix} \ddot{\theta} \\ \ddot{\phi}_x \end{bmatrix} &= -\mathbf{K}_{1.\ddot{x}}^{-1} \cdot \mathbf{K}_{1.\dot{x}} \cdot \begin{bmatrix} \dot{\theta} \\ \dot{\phi}_x \end{bmatrix} + -\mathbf{K}_{1.\ddot{x}}^{-1} \cdot \mathbf{K}_{1.x} \cdot \begin{bmatrix} \theta \\ \phi_x \end{bmatrix} + \mathbf{K}_{1.\ddot{x}}^{-1} \cdot \mathbf{K}_{1.v} \cdot \begin{bmatrix} v_{mtr.l} \\ v_{mtr.r} \end{bmatrix}
\end{aligned} \tag{0.1}$$

$$\mathbf{K}_{1.\ddot{x}} = \begin{bmatrix} +k_{1.1} & +k_{1.2} \\ +k_{1.2} & +k_{1.3} \end{bmatrix} \tag{0.2}$$

$$\mathbf{K}_{1.\dot{x}} = \begin{bmatrix} +k_{1.4} & -k_{1.4} \\ -k_{1.4} & +k_{1.4} \end{bmatrix} \cdot 2 \tag{0.3}$$

$$\mathbf{K}_{1.x} = \begin{bmatrix} 0 & 0 \\ 0 & +k_{1.5} \end{bmatrix} \tag{0.4}$$

$$\mathbf{K}_{1.v} = \begin{bmatrix} +k_{1.6} & +k_{1.6} \\ -k_{1.6} & -k_{1.6} \end{bmatrix} \tag{0.5}$$

$$k_{1.1} = (2 \cdot m_w + m_b) \cdot r_w + J_w \tag{0.6}$$

$$k_{1.2} = m_b \cdot l_w \cdot l_{w2b} \tag{0.7}$$

$$k_{1.3} = m_b \cdot l_{w2b}^2 + J_{\phi_x} \tag{0.8}$$

$$k_{1.4} = \frac{k_{mtr.T} \cdot k_{mtr.bEMF}}{R_{mtr}} + k_{fr.m2w} \tag{0.9}$$

$$k_{1.5} = m_b \cdot a_g \cdot l_{w2b} \tag{0.10}$$

$$k_{1.6} = \frac{k_{mtr.T}}{R_{mtr}} \tag{0.11}$$

0.1.2 Body Yaw ϕ_y

Equation (0.12) corresponds to body yaw ϕ_y.

$$\begin{aligned} k_{2.\ddot{x}} \quad &\cdot \begin{bmatrix} \ddot{\phi}_y \end{bmatrix} \quad + \quad k_{2.\dot{x}} \quad \cdot \begin{bmatrix} \dot{\phi}_y \end{bmatrix} \quad = \quad k_{2.v} \quad \cdot \begin{bmatrix} v_{mtr.r} - v_{mtr.l} \end{bmatrix} \\ \begin{bmatrix} \ddot{\phi}_y \end{bmatrix} \quad &= \quad -k_{2.\ddot{x}}^{-1} \cdot k_{2.\dot{x}} \quad \cdot \begin{bmatrix} \dot{\phi}_y \end{bmatrix} \quad + \quad k_{2.\ddot{x}}^{-1} \cdot k_{2.v} \quad \cdot \begin{bmatrix} v_{mtr.r} - v_{mtr.l} \end{bmatrix} \end{aligned} \tag{0.12}$$

$$k_{2.0} = \frac{l_w}{r_w} \tag{0.13}$$

$$k_{2.\ddot{x}} = \frac{1}{2} \cdot m_w \cdot l_w^2 + \frac{1}{2} \cdot k_{2.0}^2 \cdot J_{\phi_y} \tag{0.14}$$

$$k_{2.\dot{x}} = \frac{1}{2} \cdot k_{2.0}^2 \cdot k_{1.4} \tag{0.15}$$

$$k_{2.v} = \frac{1}{2} \cdot k_{2.0} \cdot k_{1.6} \tag{0.16}$$

0.2 State-Space Representation

The general form of state-space representation is exhibited in Equation (0.17).

$$\begin{aligned}
\underset{nx1}{\dot{\mathbf{x}}} &= \underset{nxn}{\mathbf{A}} \cdot \underset{nx1}{\mathbf{x}} + \underset{nxp}{\mathbf{B}} \cdot \underset{px1}{\mathbf{u}} \\
\underset{mx1}{\mathbf{y}} &= \underset{mxn}{\mathbf{C}} \cdot \underset{nx1}{\mathbf{x}} + \underset{mxp}{\mathbf{D}} \cdot \underset{px1}{\mathbf{u}}
\end{aligned} \tag{0.17}$$

The designated p inputs and n states are exhibited in Equations (0.18) - (0.19).

$$\underset{px1}{\mathbf{u}} = \begin{bmatrix} v_{mtr.l} \\ v_{mtr.r} \end{bmatrix} \tag{0.18}$$

$$\underset{nx1}{\mathbf{x}} = \begin{bmatrix} \theta \\ \phi_x \\ \dot{\theta} \\ \dot{\phi_x} \\ \phi_y \\ \dot{\phi_y} \end{bmatrix} \tag{0.19}$$