0.0.1 Length From Center of Mass of Wheels To Center of Mass of Body $l_{b.c2a}$

The length from the body center of mass to the body axis of rotation $l_{b.c2a}$ may be determined using more than one method.

0.0.1.1 Yamamoto Method

As seen in Figure **??** [*on page* **??**], Yamamoto [0] assumes that the geometries of the wheels and the body are uniform. He also assumes that the masses of these geometries are uniform. He therefore defines the length from the center of mass of the wheels [*including the axels*] to the center of mass of the body l_{w2b}, as exhibited in Equation (0.1)

$$l_{b.c2a} = \frac{l_{b.h}}{2} \tag{0.1}$$

0.0.1.2 Vaccaro Method

Since the geometries of the actual hardware are assumed to significantly deviate from the assumption of uniform mass distribution, an alternative method is instead used to calculate the length from the center of mass of the wheels [*including the axels*] to the center of mass of the body l_{w2b}, as exhibited in Equation (0.1)

If the hardware is mounted at both wheel axels *along the axis which is shared by both wheel axels*, and if it is given a degree of freedom to rotate at along the wheel axel axis, then the hardware may be lifted slightly and then released to swing freely like a pendulum along that axis, *without rotating the actual wheel axels.*

Allowing the hardware to freely swing like a pendulum along the wheel axel axis significantly simplifies the dynamic equations of motion of the hardware. Furthermore, if friction at the newly added mount coupling points is negligible, then there will not be a need to model and implement the friction into the dynamics equations.

If the hardware is freely swung like a pendulum along the wheel axel axis as described above, then the relations exhibited in Equations (0.2) - (0.3) become true.

$$\theta = \phi_x \tag{0.2}$$

$$\mathbf{u} = \mathbf{0} \tag{0.3}$$

The effects of these changes are exhibited in Equation (0.5) [*on page* 3]. This results in two relations, which are exhibited in Equation (0.4).

$$\begin{aligned}
\ddot{\phi}_x \quad + \quad & 0 \quad = \quad 0 \\
\ddot{\phi}_x \quad + \quad & \underbrace{\frac{k_{1.5}}{k_{1.2} + k_{1.3}}}_{k_\omega} \cdot \phi_x \quad = \quad 0 \qquad \Leftarrow
\end{aligned} \tag{0.4}$$

Of the two resulting relations in Equation (0.4), the former cannot be true while the hardware is in motion; thus, the latter is selected, as depicted on the right with a left-facing arrow.

$$
\begin{aligned}
&\mathbf{K}_{1.\ddot{x}} \cdot \underbrace{\begin{bmatrix} \ddot{\theta} \\ \ddot{\phi}_x \end{bmatrix}}_{} + \mathbf{K}_{1.\dot{x}} \cdot \underbrace{\begin{bmatrix} \dot{\theta} \\ \dot{\phi}_x \end{bmatrix}}_{} + \mathbf{K}_{1.x} \cdot \underbrace{\begin{bmatrix} \theta \\ \phi_x \end{bmatrix}}_{} = \mathbf{K}_{1.v} \cdot \underbrace{\begin{bmatrix} v_{mtr.l} \\ v_{mtr.r} \end{bmatrix}}_{} \\
&\underbrace{\mathbf{K}_{1.\ddot{x}}}_{} \cdot \begin{bmatrix} \ddot{\phi}_x \\ \ddot{\phi}_x \end{bmatrix} + \underbrace{\mathbf{K}_{1.\dot{x}}}_{} \cdot \begin{bmatrix} \dot{\phi}_x \\ \dot{\phi}_x \end{bmatrix} + \underbrace{\mathbf{K}_{1.x}}_{} \cdot \begin{bmatrix} \phi_x \\ \phi_x \end{bmatrix} = \underbrace{\mathbf{K}_{1.v} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\oslash} \\
&\underbrace{\begin{bmatrix} k_{1.1} & k_{1.2} \\ k_{1.2} & k_{1.3} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\phi}_x \\ \ddot{\phi}_x \end{bmatrix}}_{} + \underbrace{2 \cdot k_{1.4} \cdot \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\phi}_x \\ \dot{\phi}_x \end{bmatrix}}_{\oslash} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & k_{1.5} \end{bmatrix} \cdot \begin{bmatrix} \phi_x \\ \phi_x \end{bmatrix}}_{} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
&\begin{bmatrix} \left(k_{1.1} + k_{1.2}\right) \cdot \ddot{\phi}_x \\ \left(k_{1.2} + k_{1.3}\right) \cdot \ddot{\phi}_x \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ k_{1.5} \cdot \phi_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{aligned}
\tag{0.5}
$$

The coefficient term, abbreviated as k_w, is expanded in Equation (0.6). It may be expanded further with the use of Equation (**??**), as exhibited in Equation (0.7).

$$k_\omega \quad = \quad \frac{k_{1.5}}{k_{1.2} + k_{1.3}} \quad = \quad \frac{-m_b \cdot a_g \cdot l_{b.c2a}}{\left(m_b \cdot r_w \cdot l_{b.c2a}\right) + \left(m_b \cdot l_{b.c2a}^2 + J_{b.\phi_x}\right)} \tag{0.6}$$

$$k_\omega \quad = \quad \frac{-m_b \cdot l_{b.c2a} \cdot a_g}{m_b \cdot l_{b.c2a} \cdot r_w \quad + \quad m_b \cdot l_{b.c2a}^2 \quad + \quad \left(m_b \cdot l_{b.c2a}^2 \cdot \frac{1}{3}\right)} \quad = \quad \frac{-a_g}{r_w + l_{b.c2a} \cdot \left(1 + \frac{1}{3}\right)} \tag{0.7}$$

Harmonic Oscillator

Notably, the selected relation in Equation (0.4) form-matches the equation for a harmonic oscillator [0, p. 119 - 120, 122 - 123], as is exhibited in Equation (0.8).

$$\begin{aligned} \ddot{y} \quad + \quad \omega^2 \quad \cdot \quad y \quad &= \quad \omega^2 \quad \cdot \quad u \\ \ddot{\phi}_x \quad + \quad k_\omega \quad \cdot \quad \phi_x \quad &= \quad k_\omega \quad \cdot \quad 0 \end{aligned} \tag{0.8}$$

This allows for the relation of the abbreviated term representing the system dynamics, k_w, to the natural angular frequency of the hardware [*a pendulum*] ω_p, as is exhibited in Equation (0.9).

$$\omega_p^2 \quad = \quad k_\omega \quad = \quad \frac{-a_g}{r_w + l_{b.c2a} \cdot \frac{4}{3}} \tag{0.9}$$

This proves significant since ω_p represents the angular frequency of the pendulum, which is a measurable value, and since k_ω includes the desired unknown term l_{w2b}. [*All other terms are known*]. The relation may rewritten to solve for the length from the center of mass of the wheels [*including the axels*] to the center of mass of the body l_{w2b} as is exhibited as Equation (0.10).

$$l_{b.c2a} \quad = \quad -\frac{3}{4} \cdot \left(\frac{a_g}{\omega_p^2} \quad + \quad r_w \right) \quad = \quad -\frac{3}{4} \cdot \left(\frac{a_g}{\left(2 \cdot \pi \cdot f_p\right)^2} \quad + \quad r_w \right) \tag{0.10}$$