0.1 Calculation of Nonintuitive Parameters

Most of the hardware-equivalent dynamic model parameter values (*with respect to the MinSeg hardware*) were publicly available [0, 0], or were intuitive to obtain [*Example:* l_h, l_w, l_d].

Methods to determine those parameters which were not considered easily obtained are defined in the following sections.

0.1.1 Moment of Inertia: Body: X-axis (Pitch) J_{ϕ_x}

The moment of inertia of the body with respect to pitch, is assumed to be sufficiently equivalent to the moment of inertia of "an ideal thin rectangular plate with length l_h, width $l_w = 0$, an axis of rotation at one end of the plate".

This relation is exhibited in Equation (0.1).

$$J_{\phi_x} = \frac{m_b \cdot l_{b.c2a}^2}{3} \tag{0.1}$$

0.1.2 Moment of Inertia: Body: Y-axis (Yaw) J_{ϕ_y}

The moment of inertia of the body with respect to pitch, is assumed to be sufficiently equivalent to the moment of inertia of "an ideal thin rectangular plate with length l_h, width $l_w = 0$, an axis of rotation at one end of the plate".

This relation is exhibited in Equation (0.2).

$$J_{\phi_y} = \frac{m_b \cdot (l_{b.w}^2 + l_{b.d}^2)}{12} \tag{0.2}$$

0.1.3 Length From Body Center of Mass to Body Axis of Rotation $l_{b.c2a}$

The length from the body center of mass to the body axis of rotation $l_{b.c2a}$ may be determined using more than one method.

0.1.3.1 Yamamoto Method

As seen in Figure **??** [*on page* **??**], Yamamoto [0] assumes that the geometries of the wheels and the body are uniform. He also assumes that the masses of these geometries are uniform. He therefore defines length from the body center of mass to the body axis of rotation $l_{b.c2a}$, as exhibited in Equation (0.3)

$$l_{b.c2a} = \frac{l_{b.h}}{2} \tag{0.3}$$

0.1.3.2 Vaccaro Method

Since the geometries of the actual hardware are assumed to significantly deviate from the assumption of uniform mass distribution, an alternative method is instead used to calculate length from the body center of mass to the body axis of rotation $l_{b.c2a}$, as exhibited in Equation (0.3)

If the hardware is mounted at both wheel axels *along the axis which is shared by both wheel axels*, and if the hardware is given a degree of freedom to rotate about the wheel axel axis, *without rotating the actual wheel axels*, then the hardware may be lifted slightly and then released to swing freely like a pendulum along that axis.

Allowing the hardware to freely swing like a pendulum along the wheel axel axis significantly simplifies the dynamic equations of motion of the hardware. Furthermore, if friction at the newly added mount coupling points is negligible, then there will not be a need to model and implement the friction into the dynamics equations.

If the hardware is freely swung like a pendulum along the wheel axel axis as described above, then the relations exhibited in Equations (0.4) - (0.5) become true.

$$\theta = \phi_x \tag{0.4}$$

$$\mathbf{u} = \mathbf{0} \tag{0.5}$$

The effects of these changes are exhibited in Equation (0.7) [*on page* 4]. This results in two relations, which are exhibited in Equation (0.6).

$$\begin{array}{ccccccc} \ddot{\phi}_x & + & & 0 & = & 0 & \\ \ddot{\phi}_x & + & \underbrace{\dfrac{k_{1.5}}{k_{1.2}+k_{1.3}}}_{k_\omega} \cdot & \phi_x & = & 0 & \Leftarrow \end{array} \tag{0.6}$$

Of the two resulting relations in Equation (0.6), the former cannot be true while the hardware is in motion; thus, the latter is selected, as depicted on the right with a left-facing arrow.

$$
\begin{aligned}
&\mathbf{K}_{1.\ddot{x}} \cdot \underbrace{\begin{bmatrix} \ddot{\theta} \\ \ddot{\phi}_x \end{bmatrix}} + \mathbf{K}_{1.\dot{x}} \cdot \underbrace{\begin{bmatrix} \dot{\theta} \\ \dot{\phi}_x \end{bmatrix}} + \mathbf{K}_{1.x} \cdot \underbrace{\begin{bmatrix} \theta \\ \phi_x \end{bmatrix}} = \mathbf{K}_{1.v} \cdot \underbrace{\begin{bmatrix} v_{mtr.l} \\ v_{mtr.r} \end{bmatrix}} \\
&\underbrace{\mathbf{K}_{1.\ddot{x}}} \cdot \begin{bmatrix} \ddot{\phi}_x \\ \ddot{\phi}_x \end{bmatrix} + \underbrace{\mathbf{K}_{1.\dot{x}}} \cdot \begin{bmatrix} \dot{\phi}_x \\ \dot{\phi}_x \end{bmatrix} + \underbrace{\mathbf{K}_{1.x}} \cdot \begin{bmatrix} \phi_x \\ \phi_x \end{bmatrix} = \underbrace{\mathbf{K}_{1.v} \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{\oslash} \\
&\underbrace{\begin{bmatrix} k_{1.1} & k_{1.2} \\ k_{1.2} & k_{1.3} \end{bmatrix} \cdot \begin{bmatrix} \ddot{\phi}_x \\ \ddot{\phi}_x \end{bmatrix}} + \underbrace{2 \cdot k_{1.4} \cdot \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\phi}_x \\ \dot{\phi}_x \end{bmatrix}}_{\oslash} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & k_{1.5} \end{bmatrix} \cdot \begin{bmatrix} \phi_x \\ \phi_x \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\
&\begin{bmatrix} \left(k_{1.1} + k_{1.2}\right) \cdot \ddot{\phi}_x \\ \left(k_{1.2} + k_{1.3}\right) \cdot \ddot{\phi}_x \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ k_{1.5} \cdot \phi_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{aligned}
\tag{0.7}
$$

The coefficient term, abbreviated as k_w, is expanded in Equation (0.8). It may be expanded further with the use of Equation (0.1), as exhibited in Equation (0.9).

$$k_\omega = \frac{k_{1.5}}{k_{1.2} + k_{1.3}} = \frac{-m_b \cdot a_g \cdot l_{b.c2a}}{\left(m_b \cdot r_w \cdot l_{b.c2a}\right) + \left(m_b \cdot l_{b.c2a}^2 + J_{b.\phi_x}\right)} \tag{0.8}$$

$$k_\omega = \frac{-m_b \cdot l_{b.c2a} \cdot a_g}{m_b \cdot l_{b.c2a} \cdot r_w + m_b \cdot l_{b.c2a}^2 + \left(m_b \cdot l_{b.c2a}^2 \cdot \frac{1}{3}\right)} = \frac{-a_g}{r_w + l_{b.c2a} \cdot \left(1 + \frac{1}{3}\right)} \tag{0.9}$$

Harmonic Oscillator

Notably, the selected relation in Equation (0.6) form-matches the equation for a harmonic oscillator [0, p. 119 - 120, 122 - 123], as is exhibited in Equation (0.10).

$$\begin{aligned} \ddot{y} + \omega^2 \cdot y &= \omega^2 \cdot u \\ \ddot{\phi}_x + k_\omega \cdot \phi_x &= k_\omega \cdot 0 \end{aligned} \tag{0.10}$$

This allows for the relation of the abbreviated term representing the system dynamics, k_w, to the natural angular frequency of the hardware [*a pendulum*] ω_p, as is exhibited in Equation (0.11).

$$\omega_p^2 = k_\omega = \frac{-a_g}{r_w + l_{b.c2a} \cdot \frac{4}{3}} \tag{0.11}$$

This proves significant since ω_p represents the angular frequency of the pendulum, which is a measurable value, and since k_ω includes the desired unknown term $l_{b.c2a}$. [*All other terms are known*]. The relation may rewritten to solve for length from the body center of mass to the body axis of rotation $l_{b.c2a}$, as is exhibited as Equation (0.12).

$$l_{b.c2a} = -\frac{3}{4} \cdot \left(\frac{a_g}{\omega_p^2} + r_w \right) = -\frac{3}{4} \cdot \left(\frac{a_g}{\left(2 \cdot \pi \cdot f_p\right)^2} + r_w \right) \tag{0.12}$$

0.1.4 Motor: Coefficient of Friction: Wheel to DC Motor k_{mtr}