0.0.1 Control Gains

Once the additional dynamics are established, state feedback gains must be calculated. Multiple methods exist to calculate these gains. The most established methods involve optimization or pole-placement.

0.0.1.1 Optimal

Several optimal control techniques exist [0]. This section will focus on linear quadratic regulation techniques.

0.0.1.1.1 Implementation

In order to determine the feedback gains of the system, the state-space representation of the system, [*the plant and the additional dynamics*], is input into a discrete linear quadratic regulator gain-calculation Matlab function, *dlqr*, which outputs state-feedback gains which best minimize the quadratic cost function. The Matlab function also requires quadratic cost function matrices Q and R as inputs.

The quadratic cost matrices Q and R were determined through trial and error; however, some constraints existed. The Q and R matrices were both diagonal matrices; [*thus, all indices which are not on the diagonal are equal to zero*]. Also, the R matrix was left as an identity matrix until the Q matrix established desirable behavior. Once desirable behavior was established, the option of multiplying the R matrix by a scalar value [*greater than one*] became a consideration.

Multiplying the R matrix by a scalar value decreases the response time of the controller; however, this also decreases the peak magnitude of the control output, [*in this case, motor voltage*]. While a decreased response time is generally undesirable, the reduction of the control output can be necessary in certain circumstances. For example, the maximum permissible value for the control output, motor voltage, is limited by the nominal voltage provided by the hardware power source.

0.0.1.1.2 Results: Simulation

To demonstrate the capabilities of the device, a dynamic command is provided which attempts to move the device in the shape of an eight 8 on the ground while maintaining balance.

The specific Q and R weighting matrices which were selected to calculate the control gains are exhibited in Equation (0.1).

$$\begin{aligned} Q &= \underset{6x6}{\mathrm{I}} \cdot \begin{bmatrix} 6 & 1 & 1 & 1 & 1 & 1 & 21 & 11 \end{bmatrix}^{\mathrm{T}} \\ R &= \underset{2x2}{\mathrm{I}} \cdot \begin{bmatrix} 1 & 1 \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{0.1}$$

Recall that Q is with respect to the states of the global system, [*plant and reference tracking dynamics*] and that R is with respect to inputs of the global system [*reference tracking input error*].

Note that the greatest weights have been applied to the reference tracking states, and also that additional weight has been provided to the wheel angular position state θ, in the cases of the plant states as well as the reference tracking states, respectively.

This results in the feedback gain matrices K exhibited in Equation (0.2).

Additionally, the device starts at a body angular position (pitch) ϕ_x of 0.03 $[rad]$. This represents the inability to start the device at a perfect angle. This causes additional transients in the initial milliseconds of operation.

Figures 0.1 - 0.7 depict the system state during its operation while completing its response to a figure-eight linear position command.

$$
\begin{aligned}
K_{plant} &= \begin{bmatrix} -50.3100392458413 & -67.5072785023266 & -6.36629920161377 & -1.55714590289198 & -31.3872481064794 & -0.0172137559799668 \\ -50.3100392458415 & -67.5072785023269 & -6.36629920161380 & -1.55714590289199 & +31.3872481064798 & +0.0172137559799669 \end{bmatrix}^{\mathrm{T}} \\
K_{referenceTracking} &= \begin{bmatrix} -2.29913264446660 & -2.07850353527147 \\ -2.29913264446662 & +2.07850353527155 \end{bmatrix}^{\mathrm{T}}
\end{aligned}
\tag{0.2}
$$

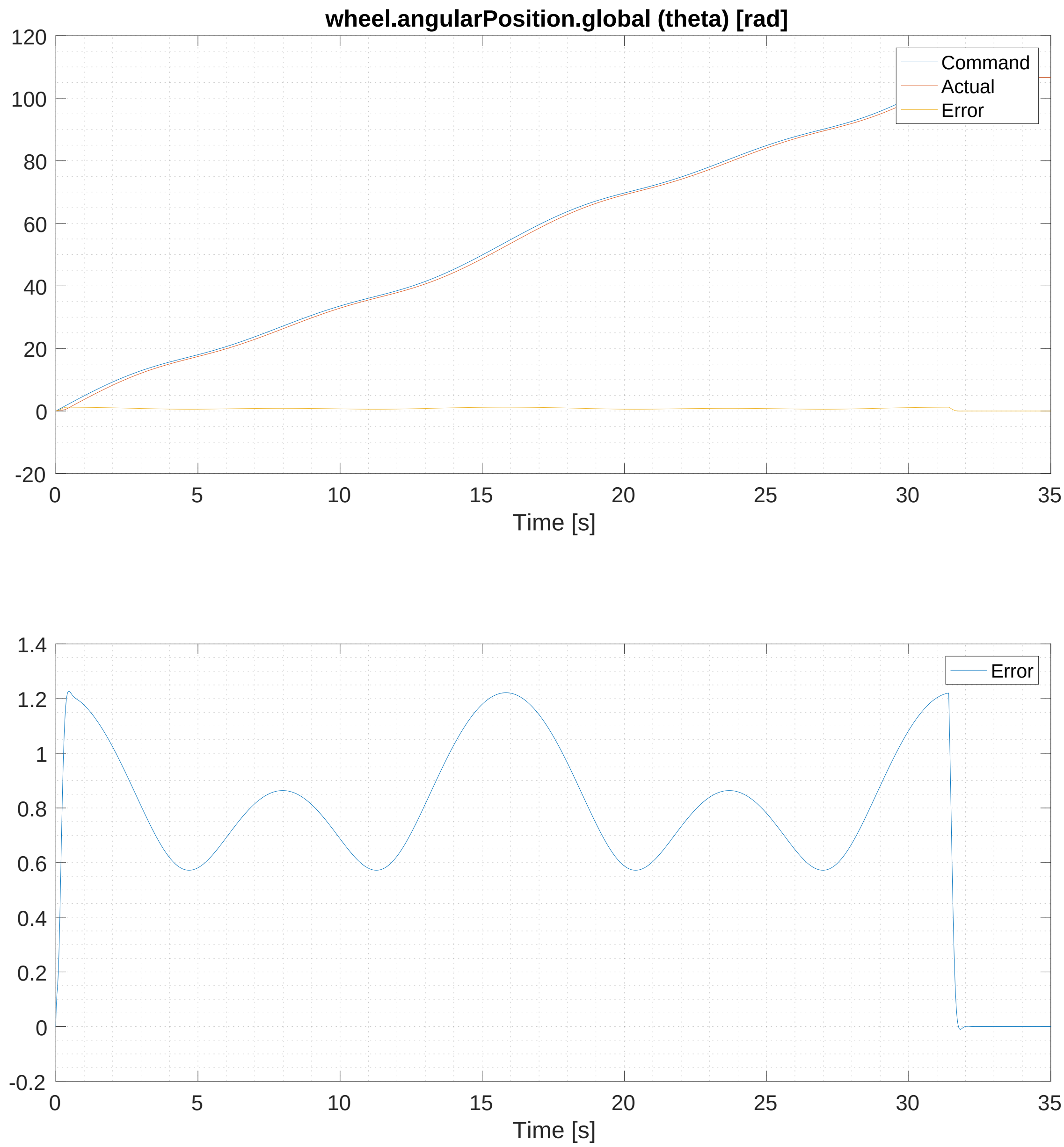

Figure 0.1: [Control Gains: LQR]: Simulation Results: Wheel Angular Position θ

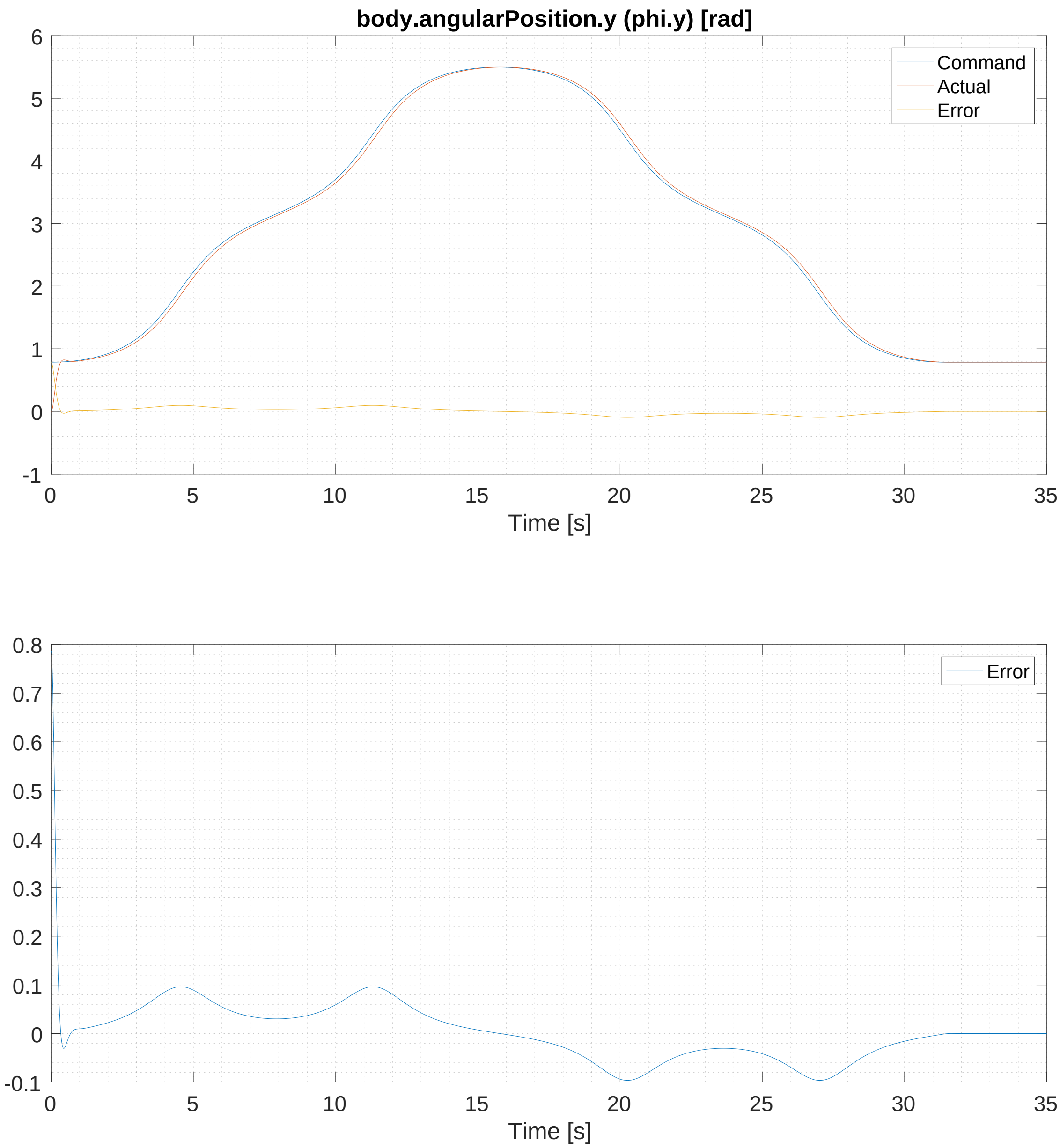

Figure 0.2: [Control Gains: LQR]: Simulation Results: Body Angular Position ϕ_y

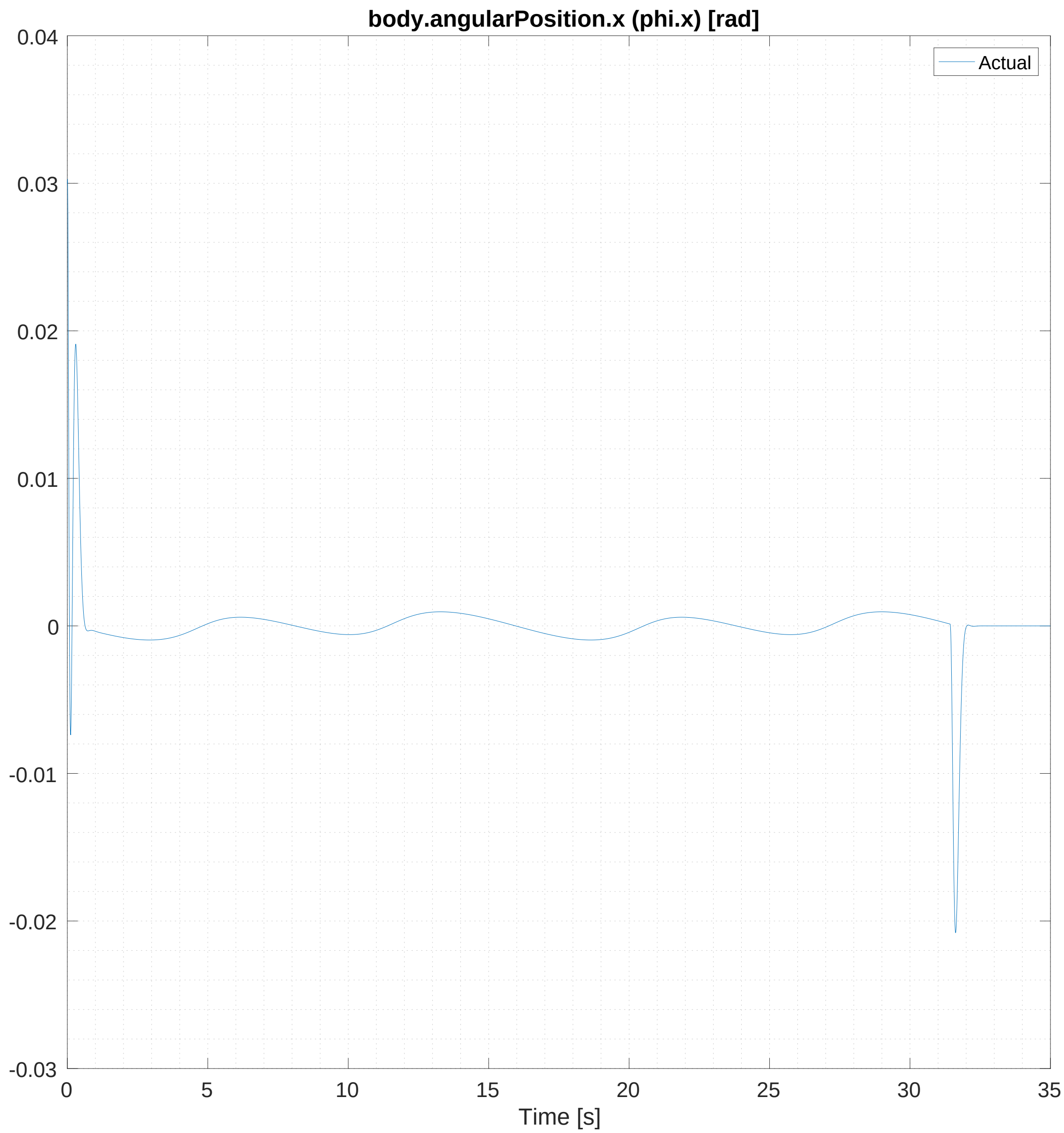

Figure 0.3: [Control Gains: LQR]: Simulation Results: Body Angular Position ϕ_x

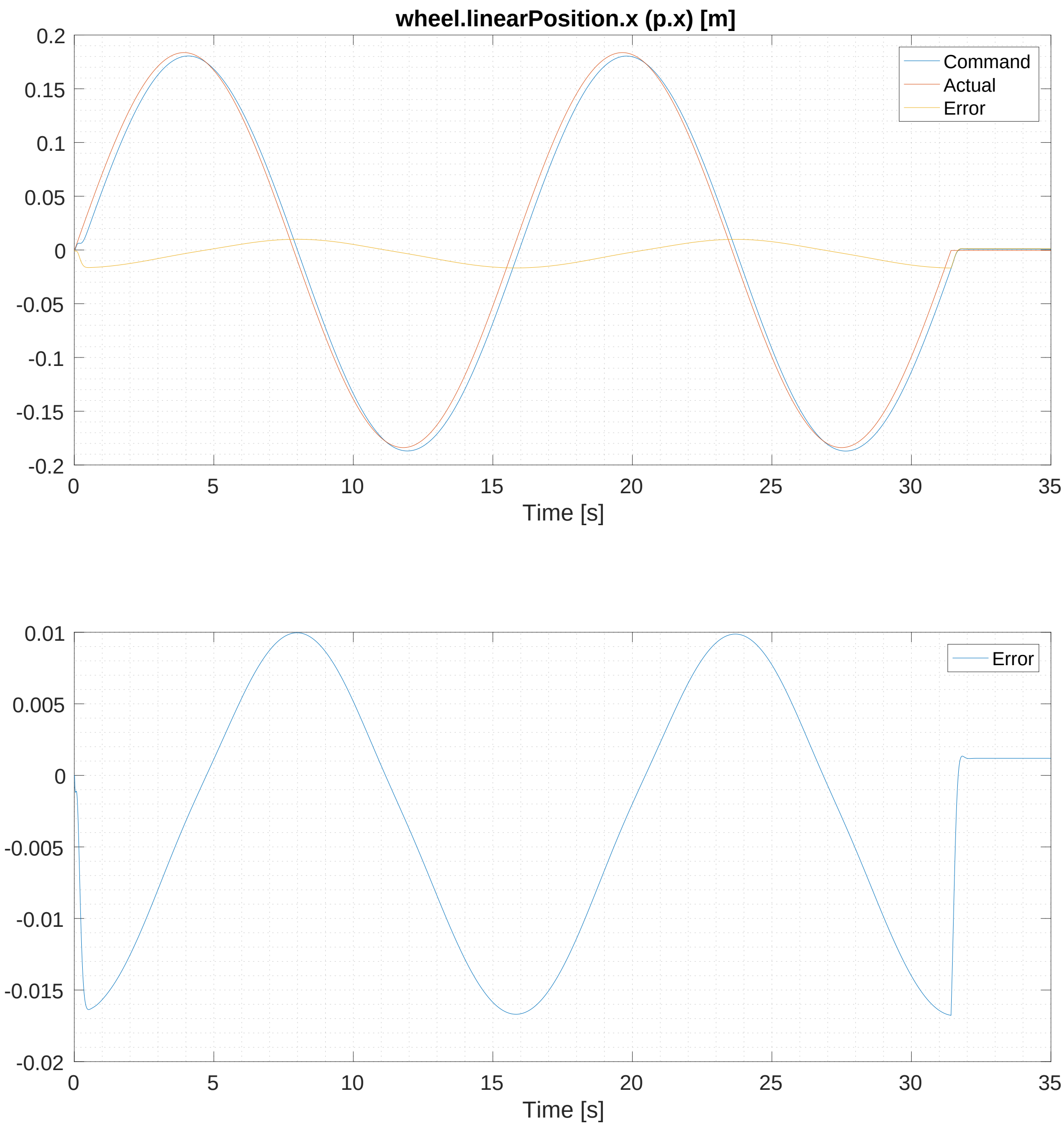

Figure 0.4: [Control Gains: LQR]: Simulation Results: Wheel Linear Position p_x

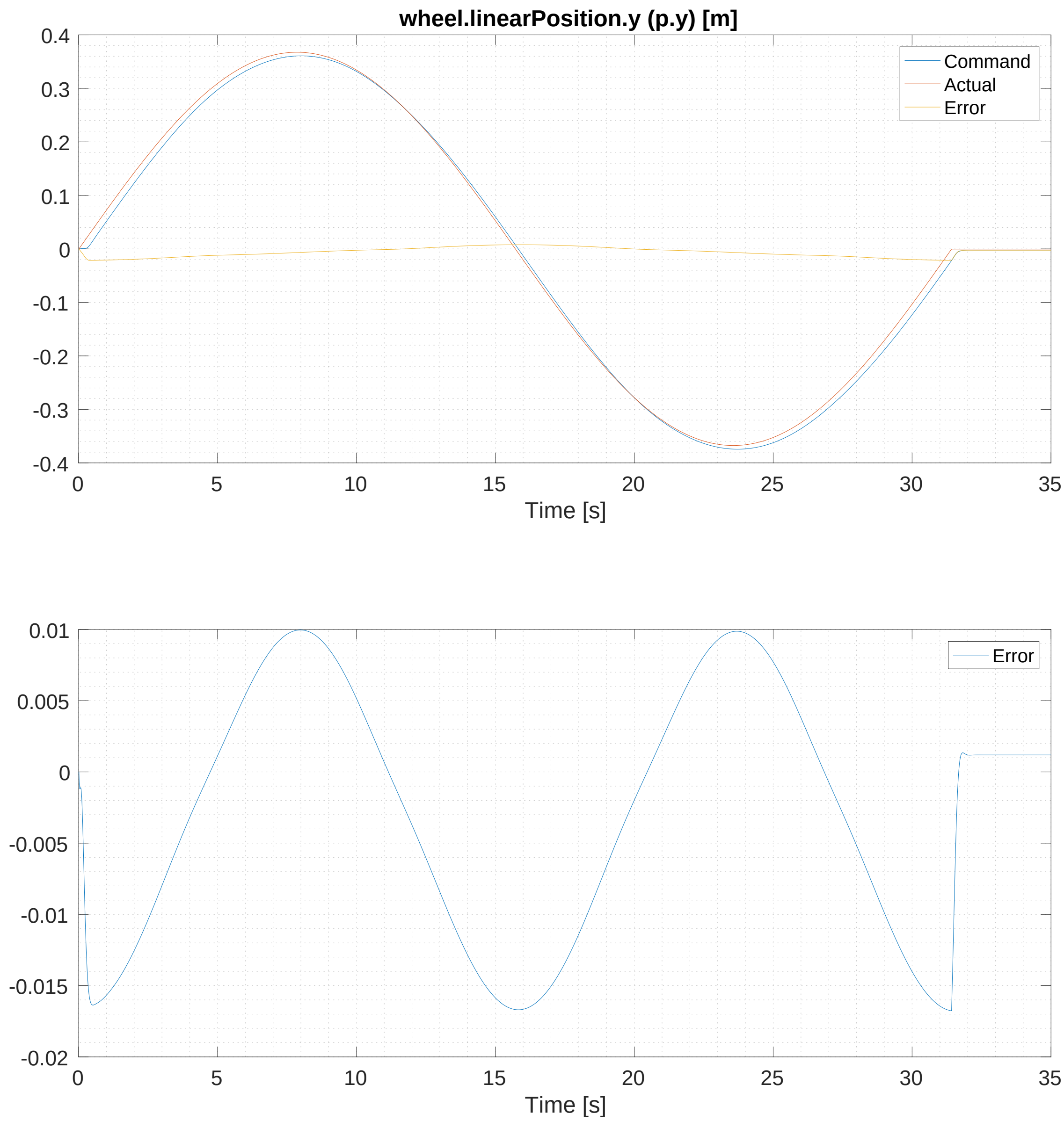

Figure 0.5: [Control Gains: LQR]: Simulation Results: Wheel Linear Position p_y

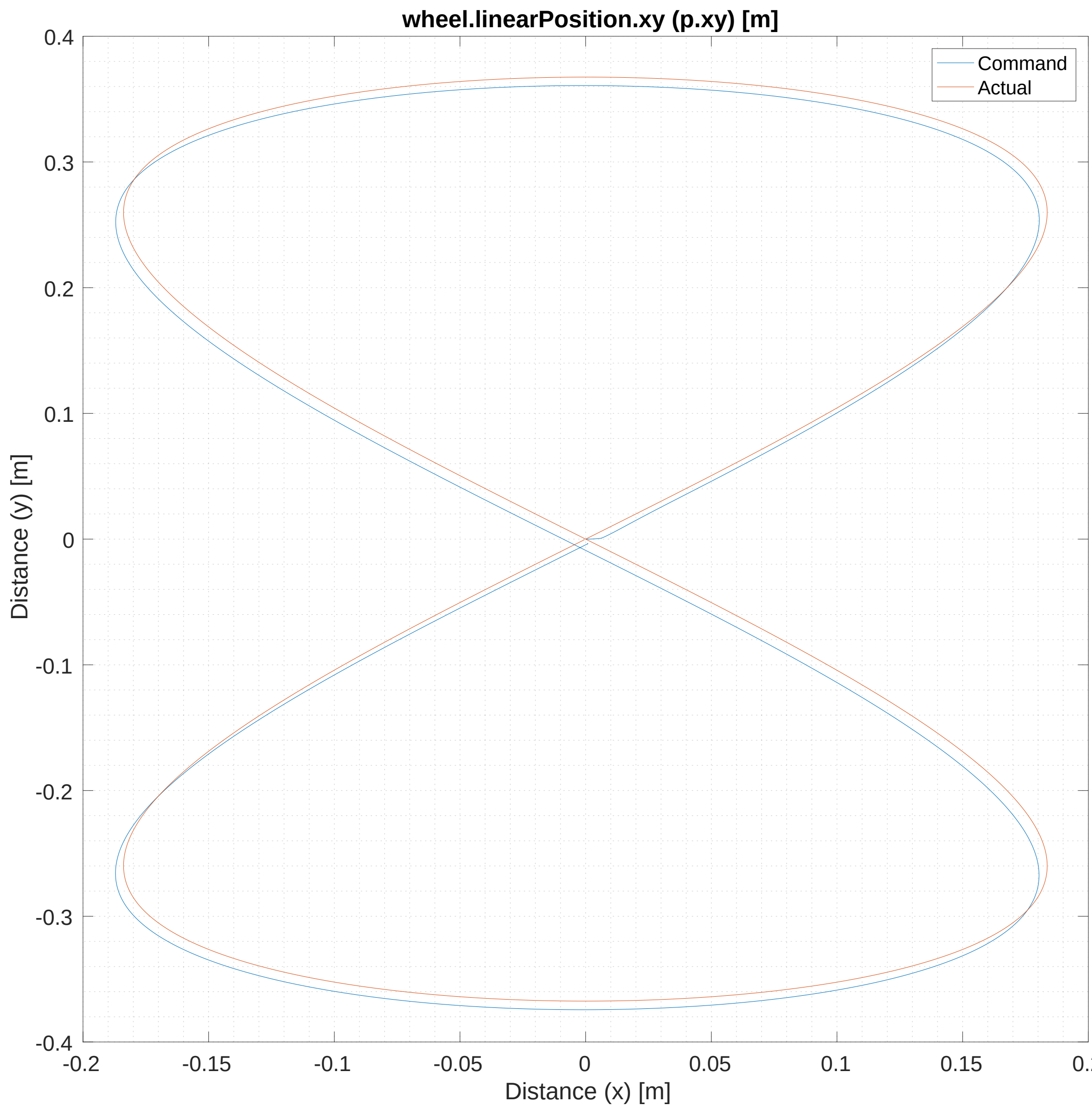

Figure 0.6: [Control Gains: LQR]: Simulation Results: Wheel Linear Position p_{xy}

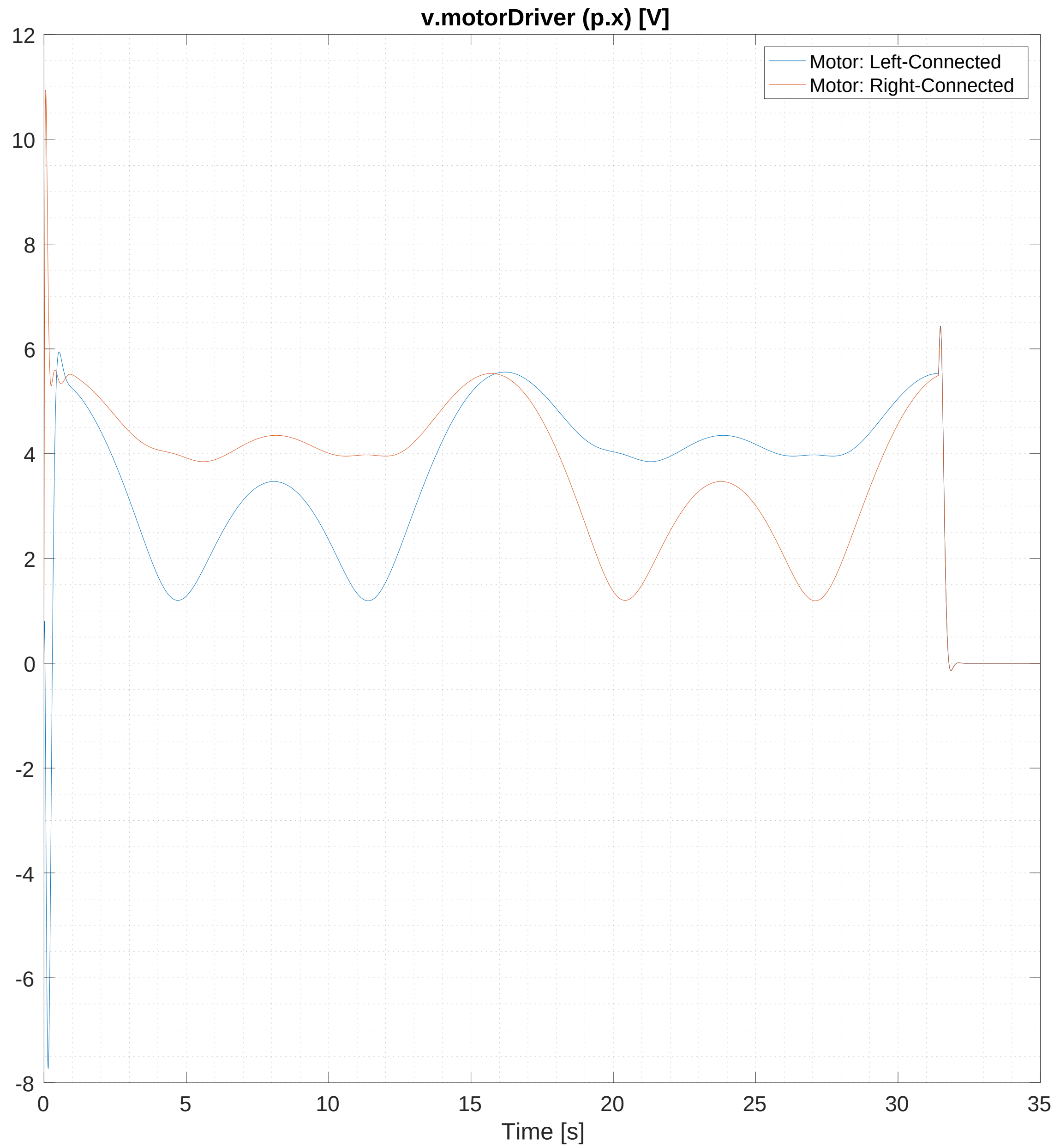

Figure 0.7: [Control Gains: LQR]: Simulation Results: Motor Driver Commanded Voltage $v_{motorDriver}$