0.1 Additional Dynamics

Additional dynamics may be incorporated into a state feedback regulating system in order to beneficially alter the response of the system in various respects.

0.1.1 Background

A state feedback regulating system is depicted in Figure 0.1. It contains the hardware plant as well as the inverted system feedback gains.

The *additional dynamics* are added in Figure 0.2. In the figure, the output vector of the plant is demuxed into its individual output components such that certain outputs may additionally be used as inputs to the additional dynamics state-space representation. This is represented using flags; connections exist between flags with equivalent labels. The outputs of the plant as well as the outputs of the additional dynamics are then muxed to form an output vector representing the output of a larger system.

Thus, the larger system [*which includes the plant and the additional dynamics*] may temporarily be considered as new plant, as depicted in Figure 0.3. It may therefore be expressed as a single state-space representation containing both systems. Thus, state-feedback regulation techniques may be used to control the system; however, the system response will now include any benefits which the additional dynamics provide. A representation of the larger system is depicted in Figure 0.4. Note the increase in the output vector.

From this point, the system depiction may be rearranged such that the input to the controller is on the left, while the plant outputs remain on the right. A sequential description of the process, and the figure number which corresponds to each step is provided below:

1. The feedback gains are separated into those with respect to the original plant outputs x and those with respect to the additional dynamics $x.a$. [Figure 0.5]

2. The gains are shifted such that they are forward facing. [Figure 0.6]

    - In this configuration, the original plant may be separated from the components used to control it, including the additional dynamics. [Figure 0.7]

3. The plant is shifted to the right side of the system depiction. [Figure 0.8]

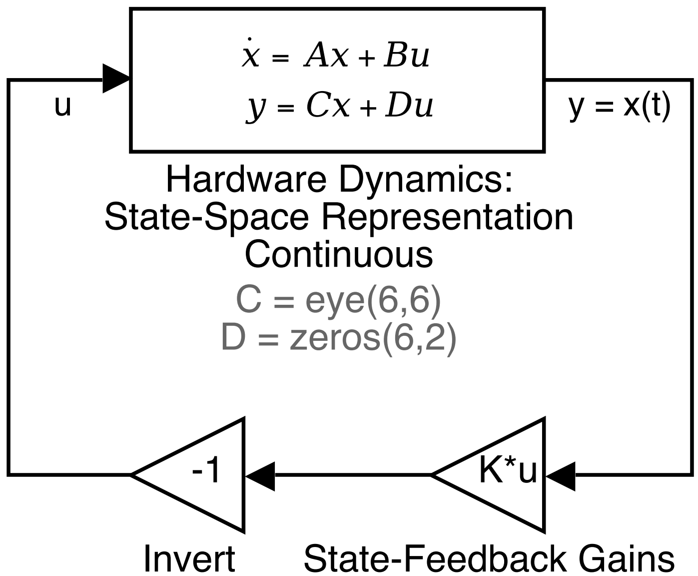


Figure 0.1: [Additional Dynamics]: State Feedback Regulator

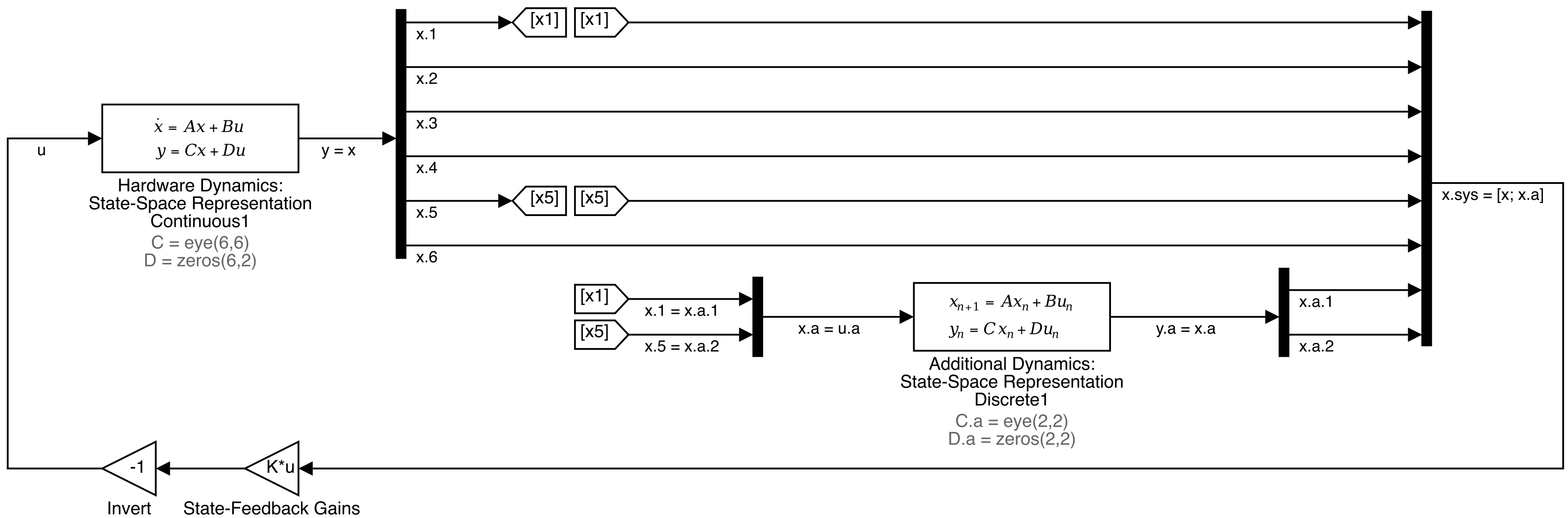


Figure 0.2: [Additional Dynamics]: 1.0 Additional Dynamics (Design View)

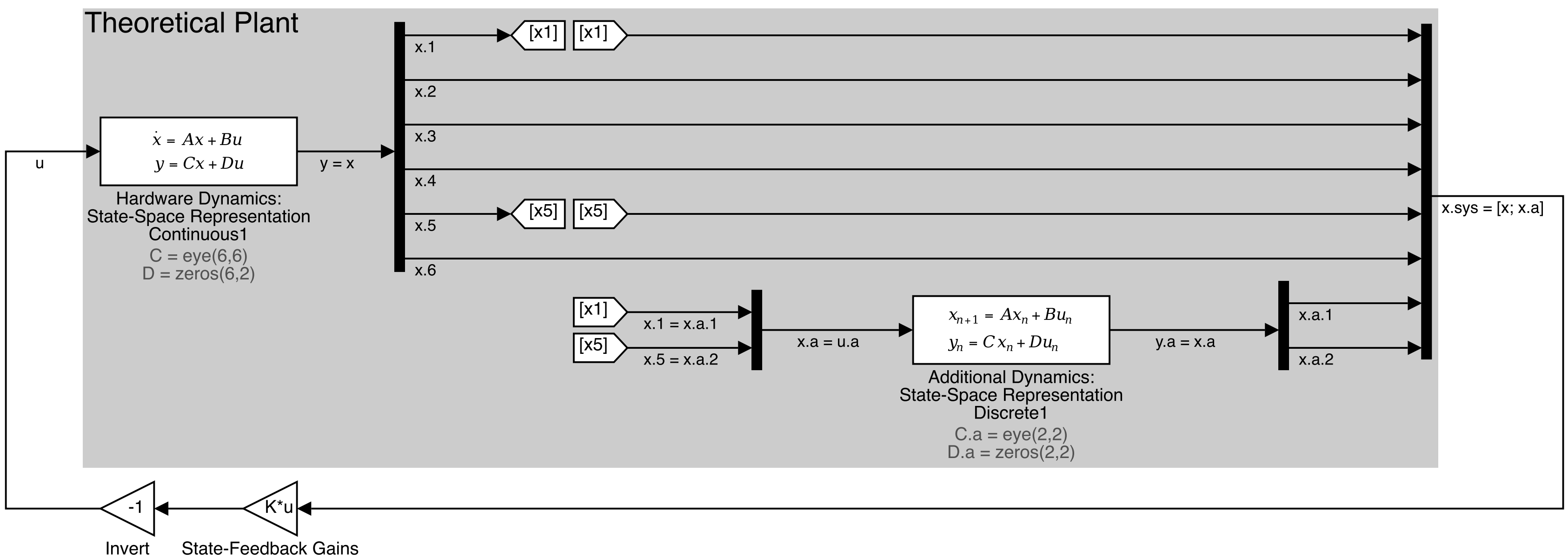


Figure 0.3: [Additional Dynamics]: 1.1 Additional Dynamics (Design View)

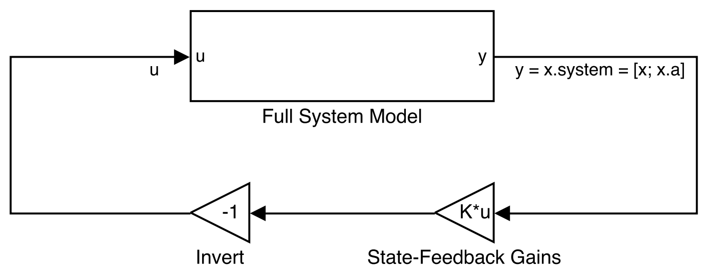


Figure 0.4: [Additional Dynamics]: 1.2 Additional Dynamics (State Feedback Regulator View)

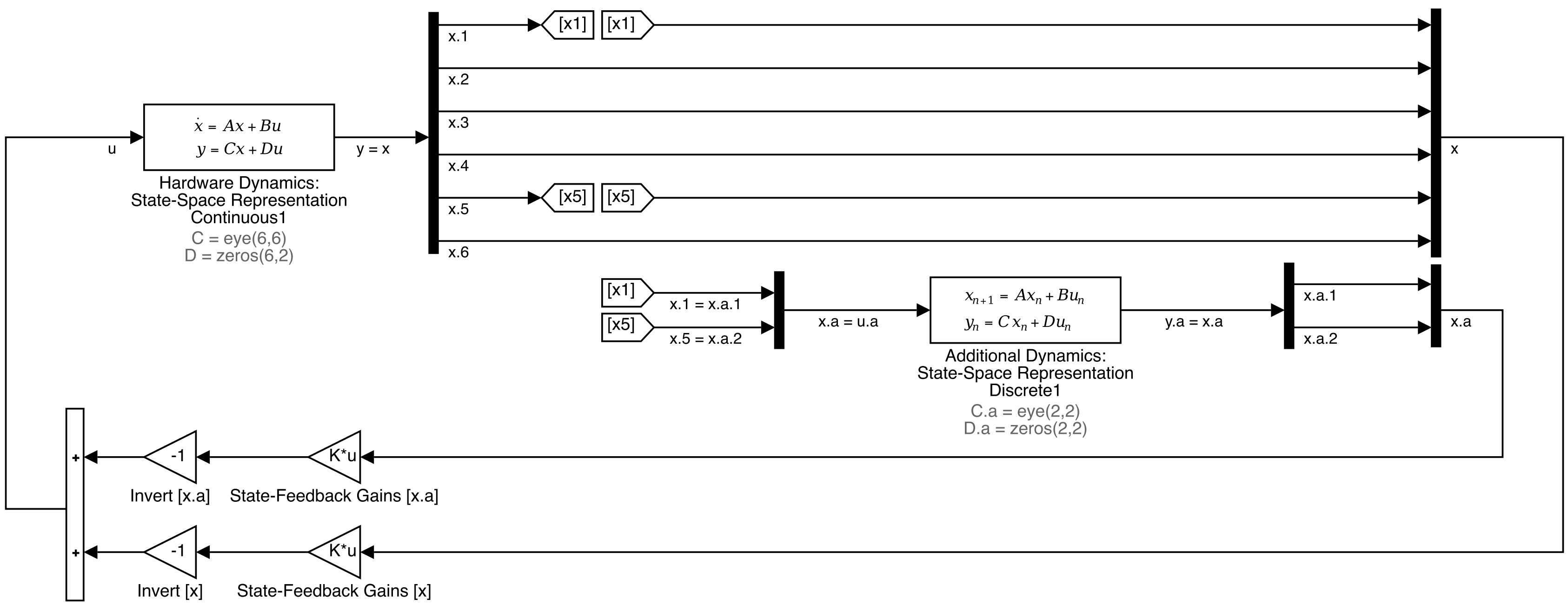


Figure 0.5: [Additional Dynamics]: 2.0 Additional Dynamics (Split Gains)

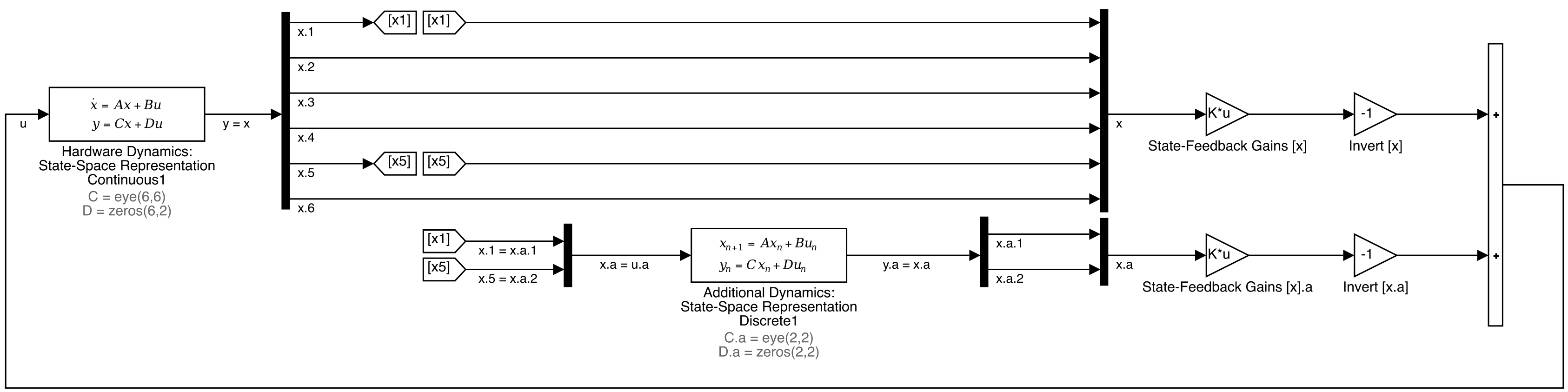


Figure 0.6: [Additional Dynamics]: 3.0 Additional Dynamics (Linear View: Plant)

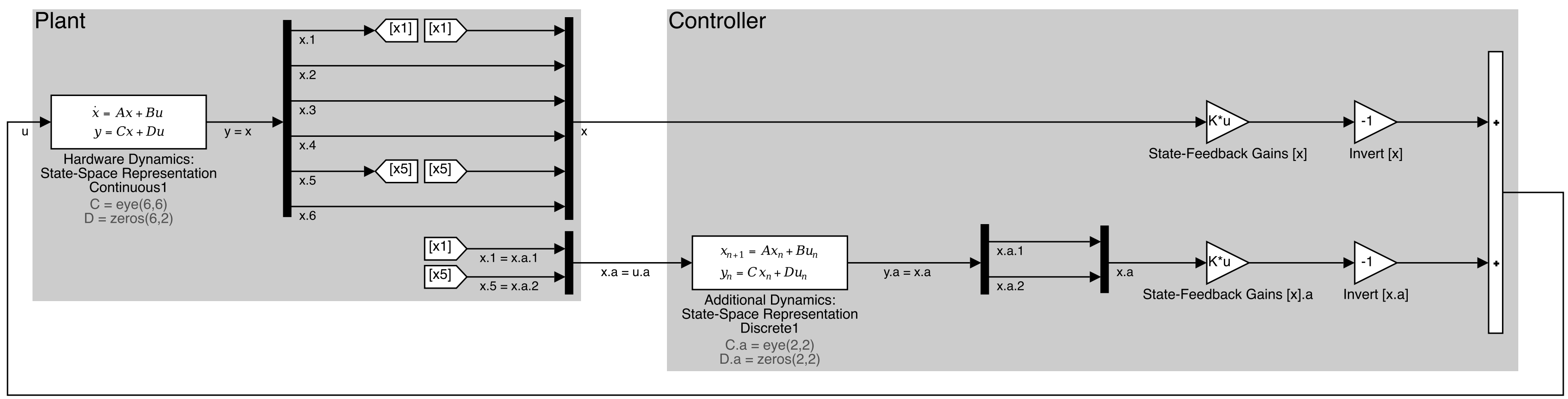


Figure 0.7: [Additional Dynamics]: 3.1 Additional Dynamics (Linear View: Plant)

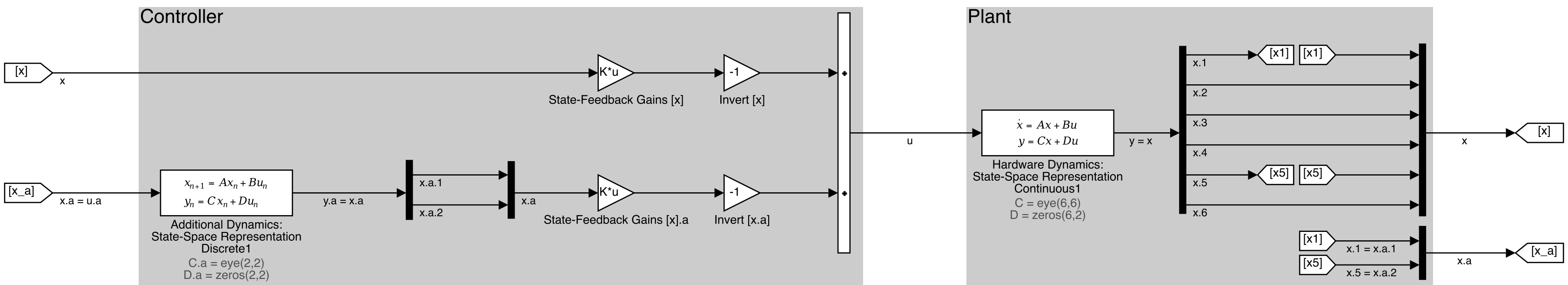


Figure 0.8: [Additional Dynamics]: 4.0 Additional Dynamics (Linear View: Controller)

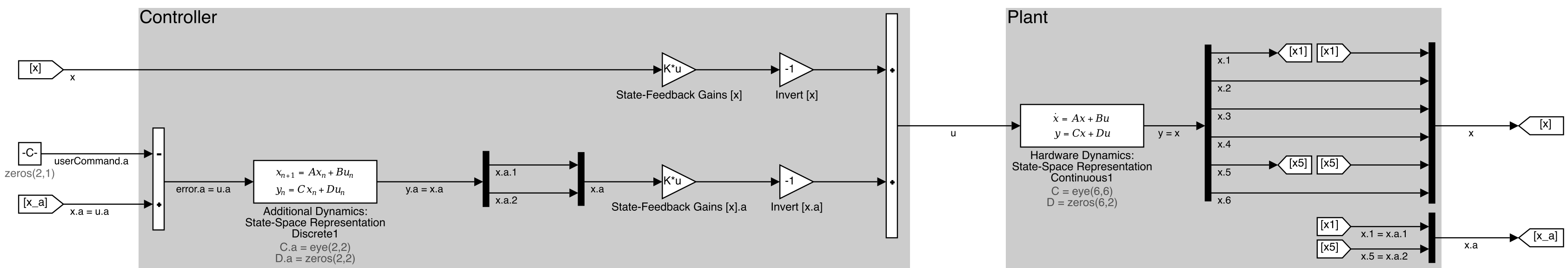


Figure 0.9: [Additional Dynamics]: 4.0 Additional Dynamics with Reference Signal (Linear View: User)

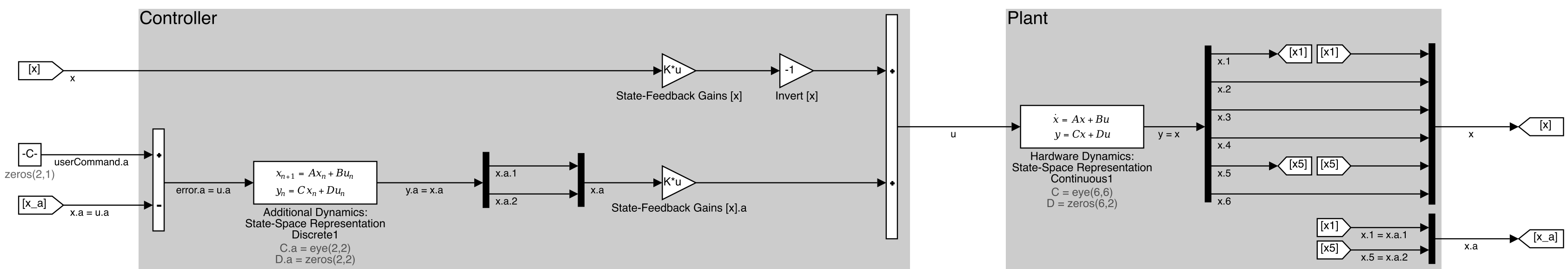


Figure 0.10: [Additional Dynamics]: 4.0 Additional Dynamics with Reference Signal (Linear View: User)

0.1.1.1 Reference Signal

Recall that a standard state-feedback regulator simply brings its inputs, [*in this case, system states x and x_a*], to zero. If it is desired that a controller input be brought to a value other than zero, a reference signal may be implemented.

In these cases, rather than input the controller with a state which the controller will bring to zero, the controller is input with the difference between the state value and the reference [*desired*] value. This difference is commonly known as the error signal. Once the error signal is brought to zero for a given state, the state will be equivalent to the desired reference value.

Returning to system depiction, a reference command is implemented, as depicted in Figure 0.9.

Note that the reference signal receives the negative. Inverting the system state alters the system equation, and could cause the system to become unstable.

Despite this fact, it is sometimes more common to see the system state subtracted from the reference signal. To correctly achieve this, once the reference signal is implemented, either side of the difference equation is multiplied by -1. The negative on the input side is distributed to both inputs. The output of the difference equation is the input of the additional dynamics; thus, when the negative appears on the output side of the difference equation, a negative exists on either side of the additional dynamics equation.

Recall that all state-space representations are linear; therefore, the input and the output may be multiplied by the same value. In this case, the negative may be divided out on both sides.

These changes are depicted in Figure 0.10.

0.1.2 Tracking System

Additional dynamics may be incorporated to improve reference tracking. When implemented for this purpose, the additional dynamics are known as a tracking system.

In the case of a tracking system, an *integrator* may be implemented as the additional dynamics to track a constant reference exactly, or to track a slowly varying reference approximately.

Integrators are also able to mitigate constant disturbances. Incidentally, the MinSeg M2V3 system uses gyroscopes as body angular velocity ψ sensors. Bias is inherent in the output of a gyroscope; therefore, the use of such an integrator as a tracking system has an additional benefit: it will mitigate the effects of bias from a gyroscope output, whether directly or within terms which are derivative of the gyroscope output.

Thus, in the case of the two-wheeled robot, integrators are implemented as additional dynamics for the states representing wheel angular position θ and body angular position (yaw) ϕ_y. This establishes a tracking system, [*an augmented method of state feedback regulation*], for the system. The state-space representation of the integrator is exhibited in Equation (0.1).

$$\begin{aligned} \underset{nx1}{\dot{\mathbf{x}}} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \underset{nx1}{\mathbf{x}} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} e_\theta \\ e_{\phi_y} \end{bmatrix} \\ \underset{mx1}{\mathbf{y}} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \underset{nx1}{\mathbf{x}} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} e_\theta \\ e_{\phi_y} \end{bmatrix} \end{aligned} \tag{0.1}$$

0.1.2.1 Discrete Additional Dynamics

Since the additional dynamics will be processed on a microcontroller, the additional dynamics will be digital; thus, a continuous-to-discrete conversion will be necessary. An integrator is an established case which is exhibited in Equation (0.2).

$$
\begin{aligned}
\underset{nx1}{\dot{\mathbf{x}}} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \underset{nx1}{\mathbf{x}} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} e_\theta \\ e_{\phi_y} \end{bmatrix} \\
\underset{mx1}{\mathbf{y}} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \underset{nx1}{\mathbf{x}} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} e_\theta \\ e_{\phi_y} \end{bmatrix}
\end{aligned}
\tag{0.2}
$$

0.1.3 Control Gains

Once the additional dynamics are established, state feedback gains must be calculated. Multiple methods exist to calculate these gains. The most established methods involve optimization or pole-placement.

0.1.3.1 Optimal

Several optimal control techniques exist [0]. This section will focus on linear quadratic regulation techniques.

0.1.3.1.1 Implementation

In order to determine the feedback gains of the system, the state-space representation of the system, [*the plant and the additional dynamics*], is input into a discrete linear quadratic regulator gain-calculation Matlab function, *dlqr*, which outputs state-feedback gains which best minimize the quadratic cost function. The Matlab function also requires quadratic cost function matrices Q and R as inputs.

The quadratic cost matrices Q and R were determined through trial and error; however, some constraints existed. The Q and R matrices were both diagonal matrices; [*thus, all indices which are not on the diagonal are equal to zero*]. Also, the R matrix was left as an identity matrix until the Q matrix established desirable behavior. Once desirable behavior was established, the option of multiplying the R matrix by a scalar value [*greater than one*] became a consideration.

Multiplying the R matrix by a scalar value decreases the response time of the controller; however, this also decreases the peak magnitude of the control output, [*in this case, motor voltage*]. While a decreased response time is generally undesirable, the reduction of the control output can be necessary in certain circumstances. For example, the maximum permissible value for the control output, motor voltage, is limited by the nominal voltage provided by the hardware power source.

0.1.3.1.2 Results: Simulation

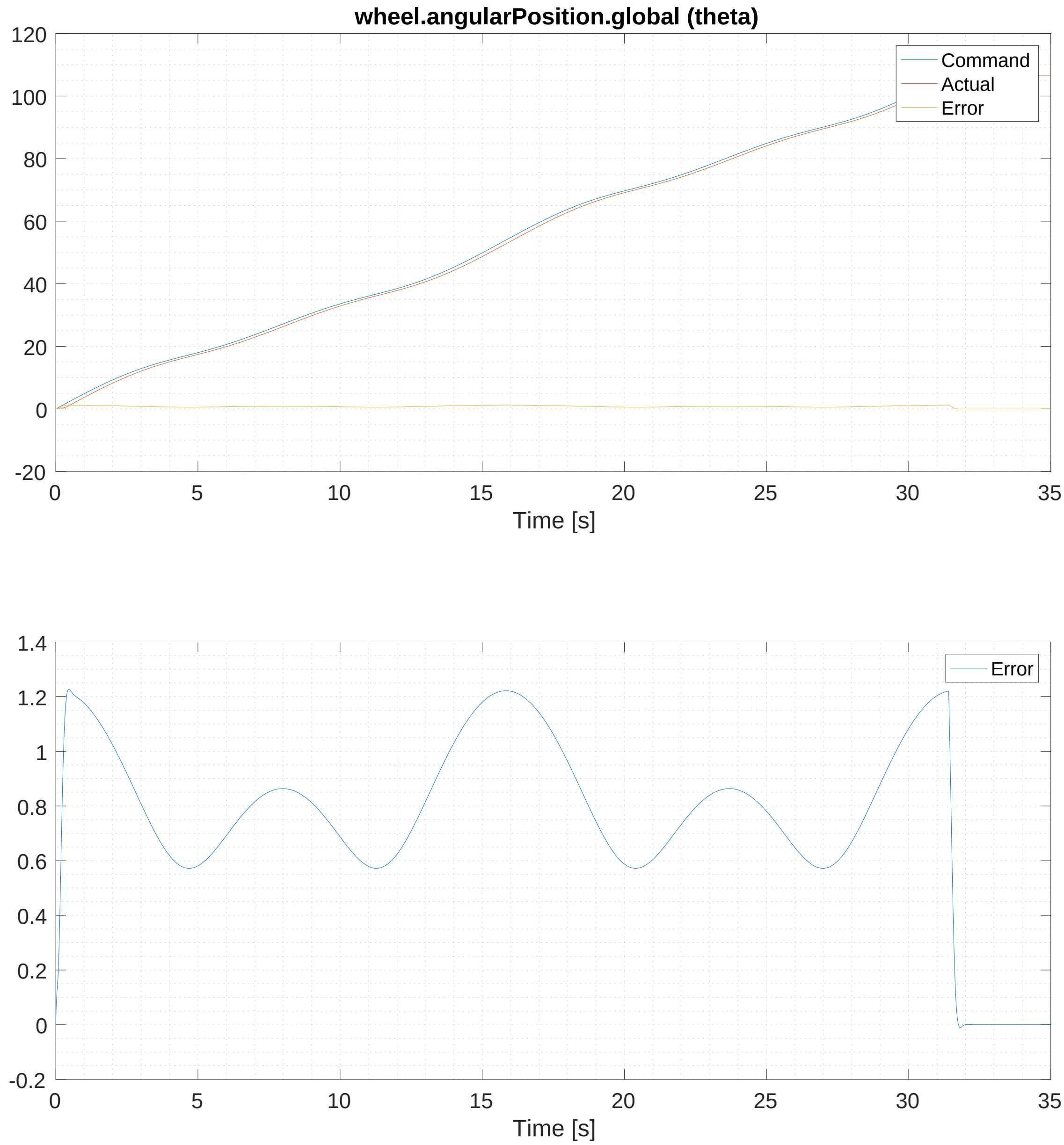


Figure 0.11: [Control Gains: LQR]: Simulation Results: Wheel Angular Position θ

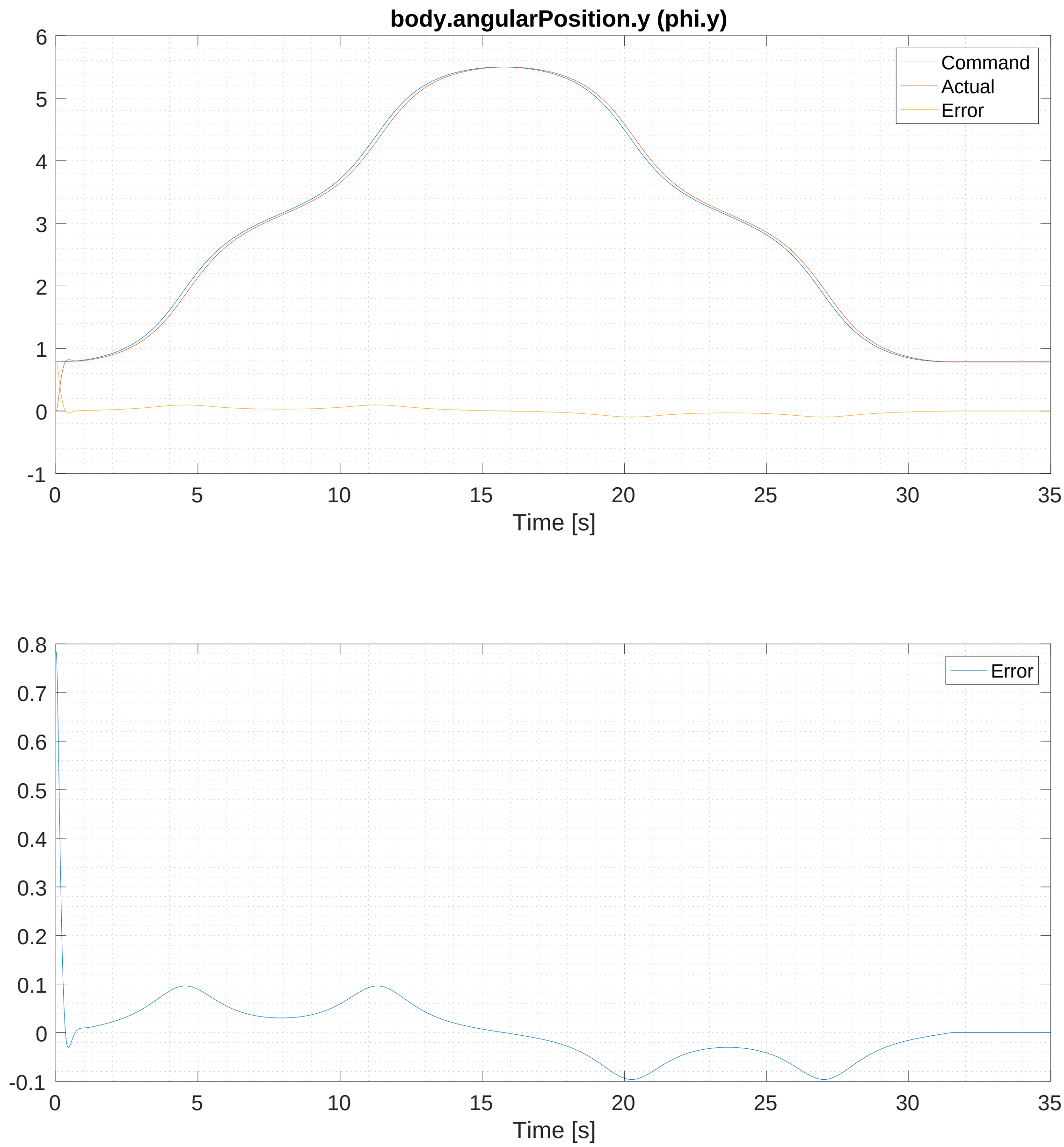


Figure 0.12: [Control Gains: LQR]: Simulation Results: Body Angular Position ϕ_y

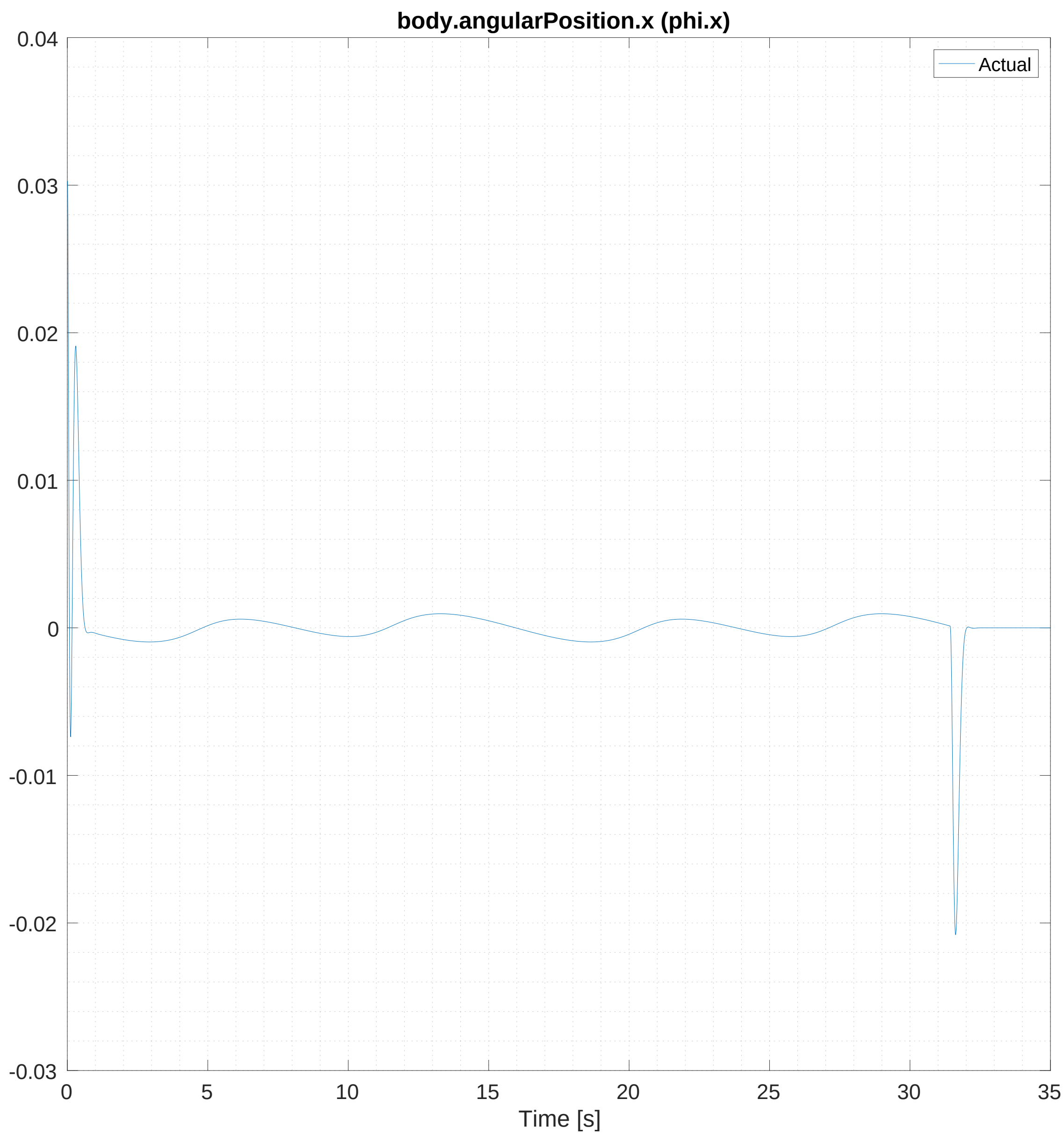


Figure 0.13: [Control Gains: LQR]: Simulation Results: Body Angular Position ϕ_x

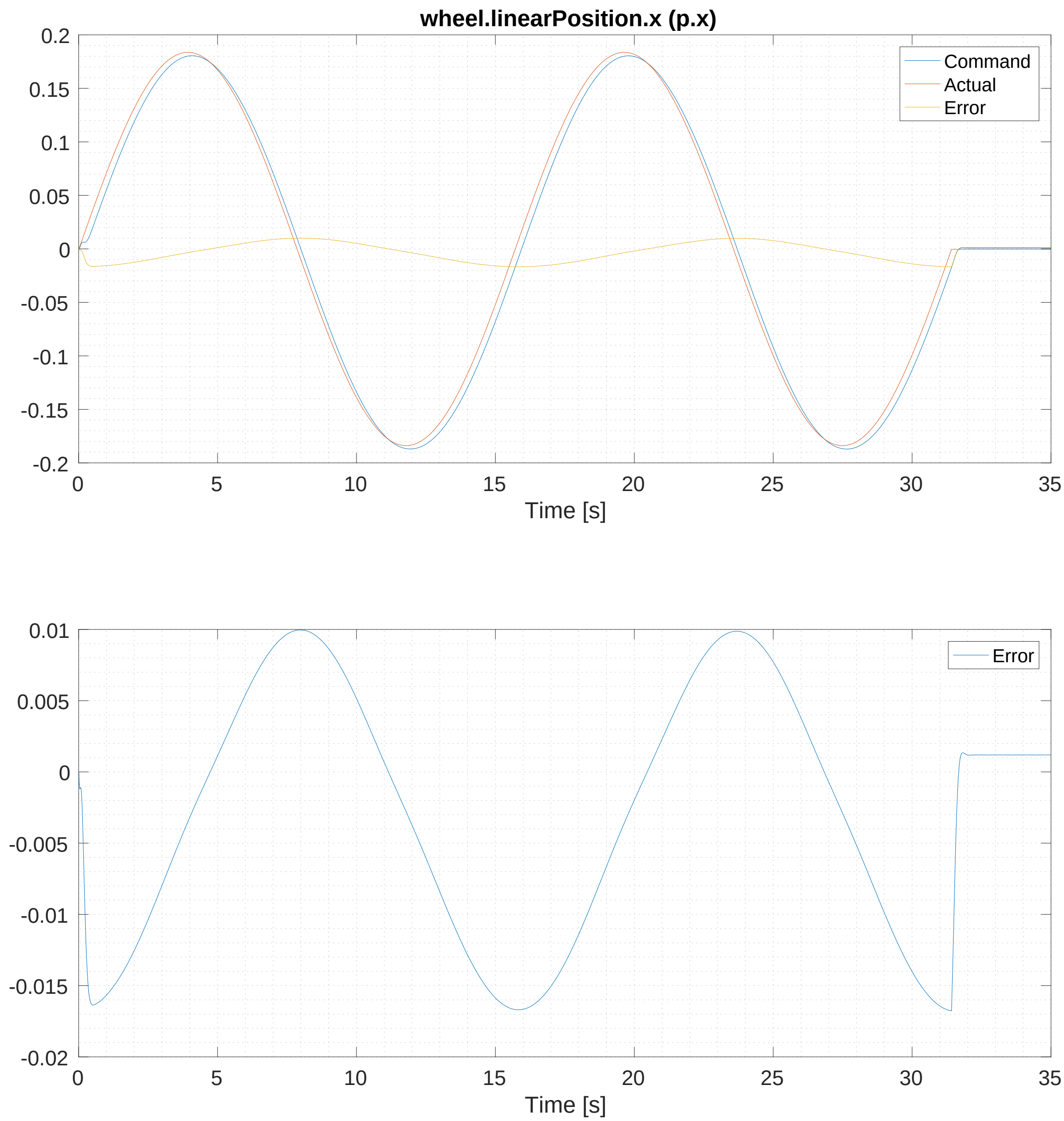
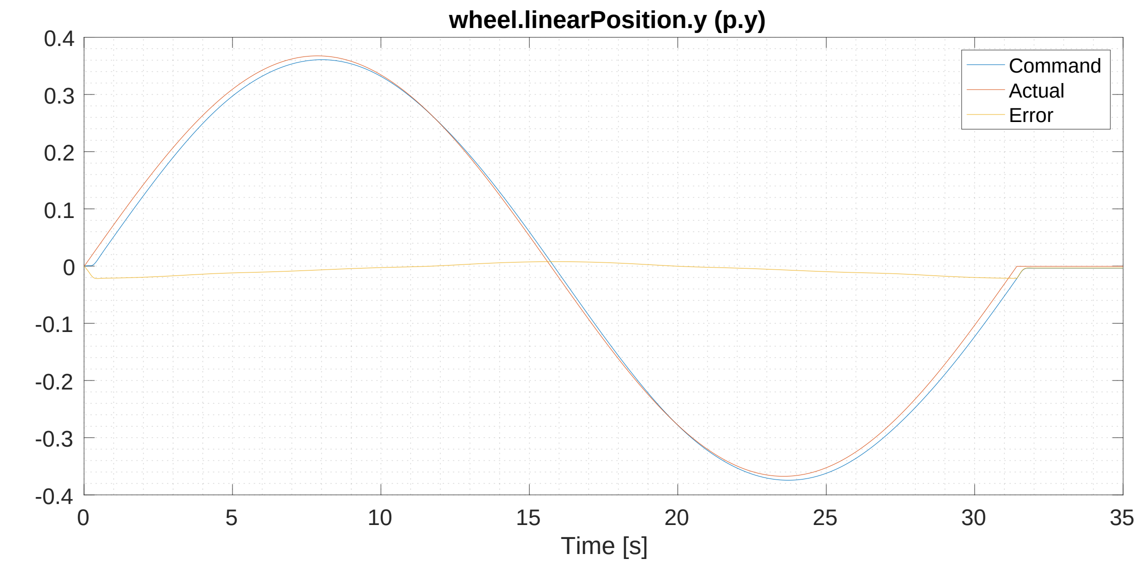


Figure 0.14: [Control Gains: LQR]: Simulation Results: Wheel Linear Position p_x



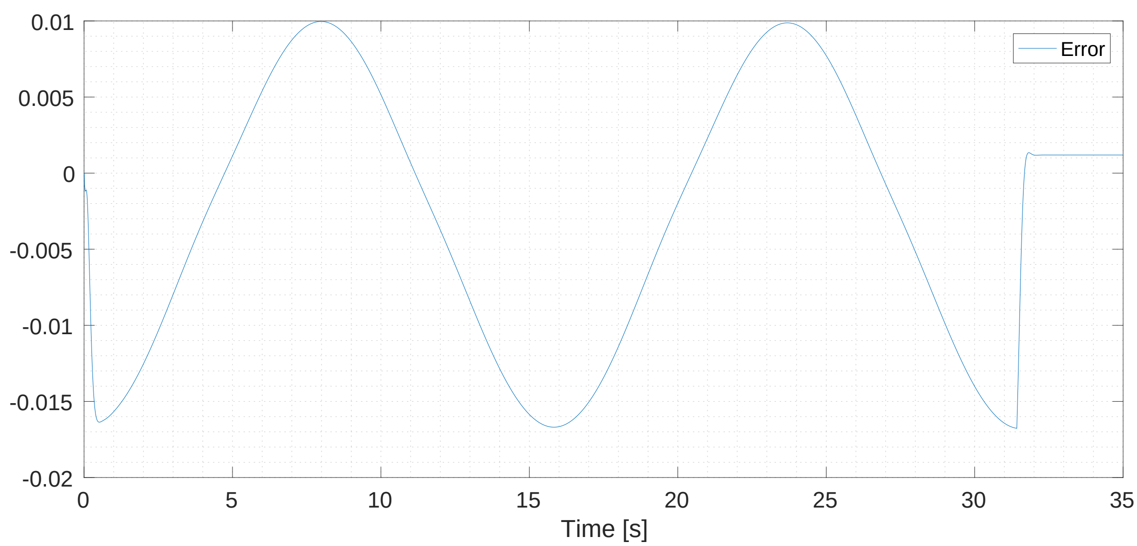


Figure 0.15: [Control Gains: LQR]: Simulation Results: Wheel Linear Position p_y

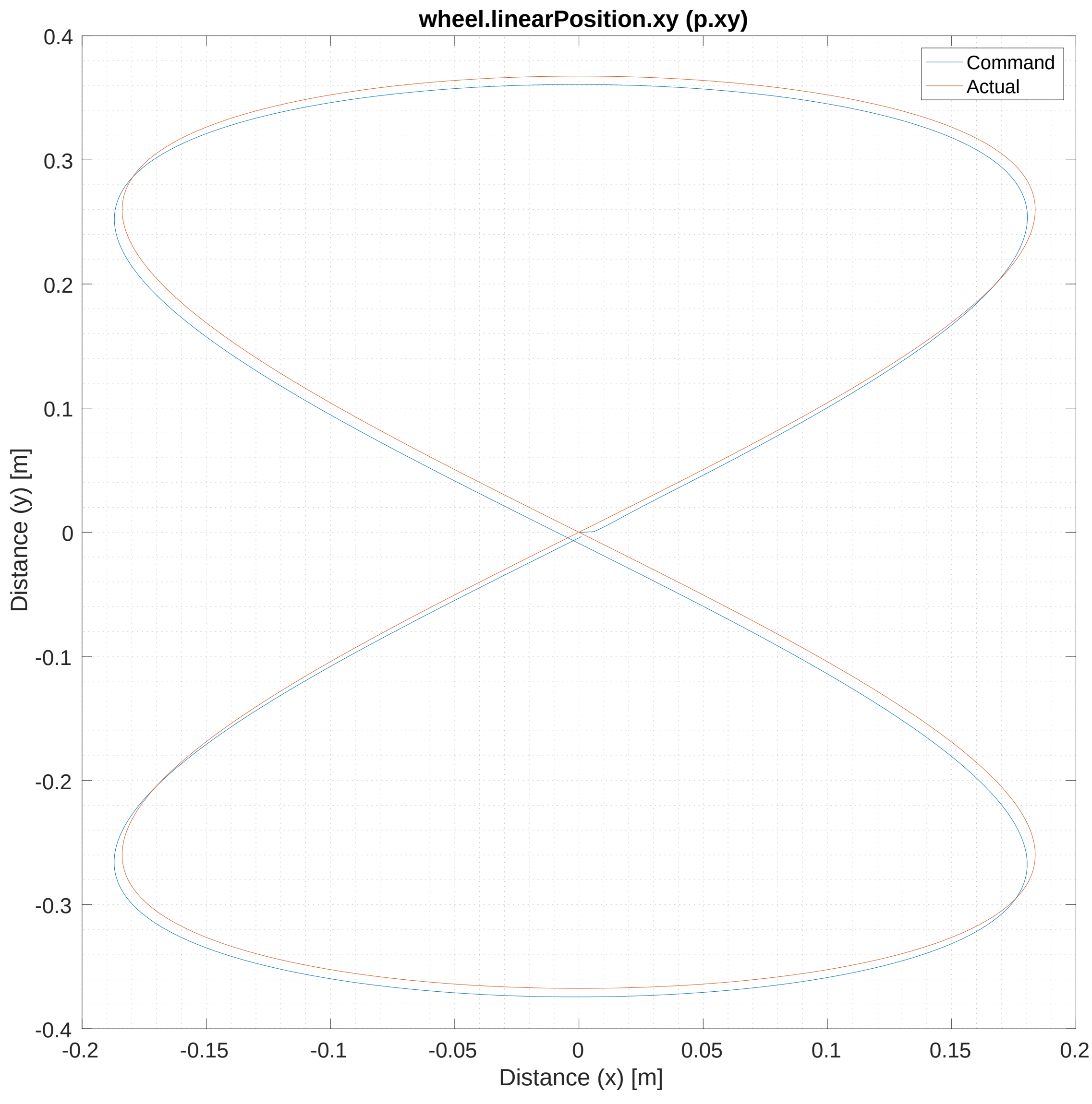


Figure 0.16: [Control Gains: LQR]: Simulation Results: Wheel Linear Position p_{xy}

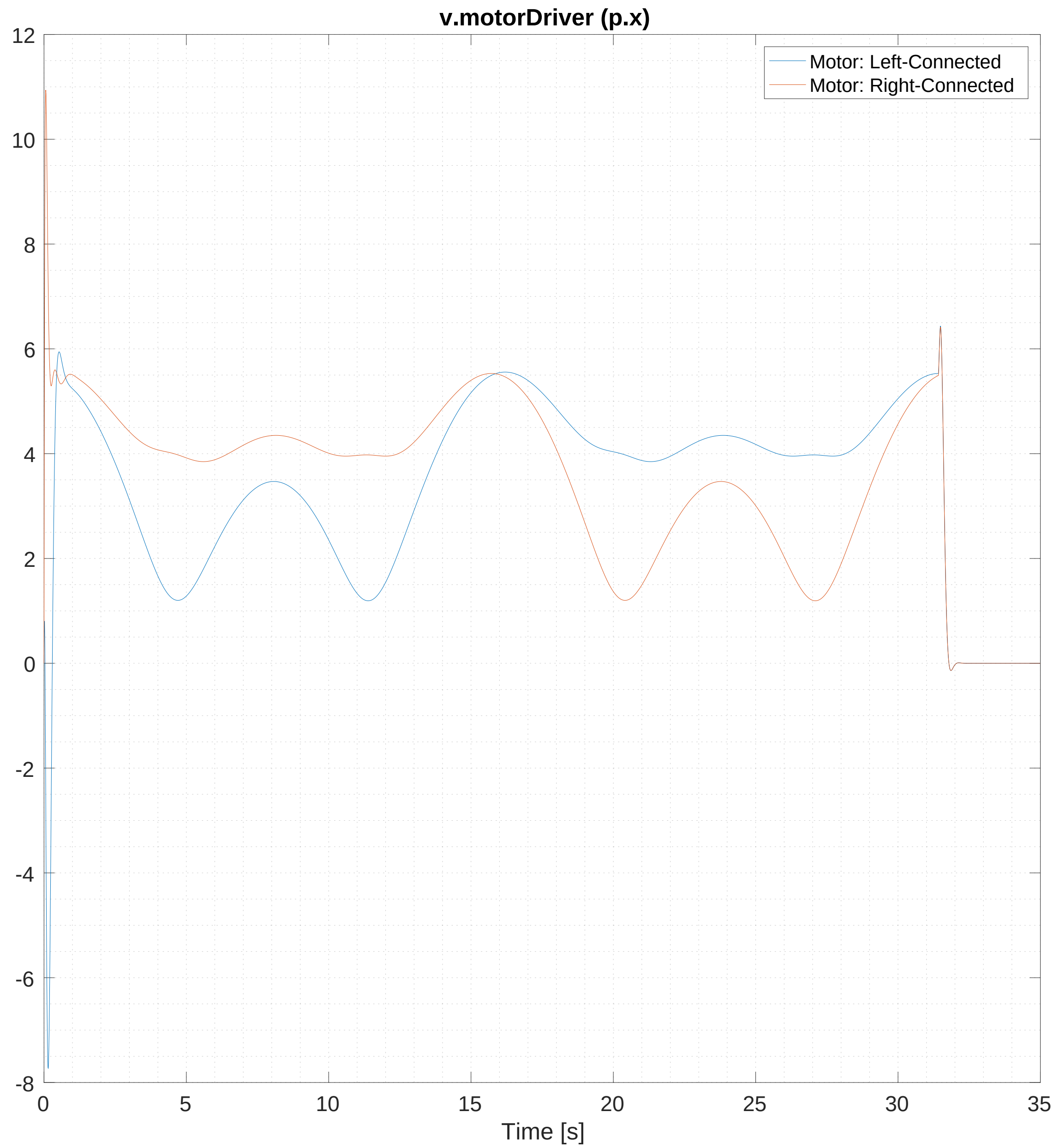


Figure 0.17: [Control Gains: LQR]: Simulation Results: Motor Driver Commanded Voltage $v_{motorDriver}$