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Review

Review of modelling and control of two-wheeled robots

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ABSTRACT

In the past decade, there has been much more research in two-wheeled robots which actively stabilize themselves. Various models and controllers have been applied both to explain and control the dynamics of two-wheeled robots. We explore the methods which have been investigated and the controllers which have been used, first for balancing and movement of two-wheeled robots on flat terrain, then for two-wheeled robots in other situations, where terrain may not be flat, where there may be secondary objectives and where the robots may have additional actuators.

Contents

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1. Introduction

Mobile robots are increasingly ubiquitous today, and are used in a variety of different applications, including exploration, search and rescue, materials handling and entertainment. While legged robots are able to step over obstacles, they are more complex to design and control due to the greater number of degrees of freedom. Wheeled robots are more energy efficient, tend to have a simpler mechanical structure, as well as simpler dynamics compared to that required by legged robots to make contact with the ground and provide a driving force. Robots with at least three wheels can achieve static stability, further simplifying dynamics. Another common wheel configuration is with four wheels, most visible in vehicles, because the larger supporting polygon increases stability at high speeds. However with more than three wheels, the system becomes over-constrained, and requires a suspension system unless the ground is flat. This paper focuses on two-wheeled robots, specifically of the statically unstable variety. These robots have two coaxial wheels mounted on either side of an intermediate body, with a centre of mass above the wheel axles, and therefore, must actively stabilize themselves to prevent toppling.

Two-wheeled robots (Fig. 1) have a number of advantages over other mobile robots. Although they are more difficult to control than statically stable wheeled robots, two-wheeled robots are still much easier to control than legged robots. This wheel configuration makes them highly manoeuvrable, because of their ability to turn on the spot, similar to differential drive robots. Although two-wheeled robots are non-holonomic, this is a minor issue, because of their ability to turn on the spot. Being actively stabilised means that the robot can correct for any disturbances which may otherwise cause a statically stable robot to tip over, even with a higher centre of mass. Two-wheeled robots can be taller, with a small footprint, making them quite suitable for indoor environments, because they can fit through narrow corridors and tight corners. Because two-wheeled robots can lean fore and aft to restabilize themselves, they are also able to maintain stability on inclines, by leaning into the slope. Similarly, even with a tall profile, two-wheeled robots can accelerate quickly without tipping over. Having no more than two wheels means more room for larger wheels, potentially allowing them to traverse rougher terrain.

Fig. 1. Two-wheeled robot.

This article reviews the research on modelling and control of two-wheeled robots thus far – particularly the controllers for balancing a two-wheeled robot. This includes the different models and controllers used, and how successful they were. The dynamics of two-wheeled robots have some particular features which complicate their control beyond being unstable in open loop. Specifically, they are non-minimum phase, and under-actuated. Because of the coupled dynamics of tilt and forward velocity, to move forwards from equilibrium, it is necessary to first move backwards, allowing the intermediate body to tilt forwards, before beginning forward movement.

We will also review research on two-wheeled robots in environments where there are obstacles in the terrain, or where the robots have additional actuators and objectives to meet. However, we mainly consider those actuators and objectives which affect the dynamics of balancing a two-wheeled robot significantly. Therefore, higher level path planning and navigation is out of scope. The sensors that may be used, and how they may be implemented is not considered. Although some sensors, such as cameras, may be affected by the tilting of the intermediate body of a two-wheeled robot, measurement, estimation and filtering of sensors have wide coverage, and is not uniquely affected by the dynamics of two-wheeled robots.

2. The two-wheeled robot – standard modelling and control

In this section, we consider a standard two-wheeled robot, with only two independently actuated drive wheels connected to an intermediate body, travelling on flat surfaces. We will review the different modelling approaches for this basic scenario, and the control strategies used for standard control objectives for mobile, two-wheeled robots – balancing and locomotion. Many approaches for modelling and control strategies have been attempted for this standard scenario and it is a natural starting point to review this area.

2.1. Modelling of system dynamics

2.1.1. Models based on dynamic equations

Generally, the non-linear dynamic model of a two-wheeled robot is based on derivation via the Euler-Lagrange equation, Newton's laws of motion or Kane's method. The natural assumptions are ideal rigid body dynamics, flat and horizontal ground surface, zero wheel slip and no friction. Though here we are mostly considering non-linear models of two-wheeled robots, they are often linearised in order to simplify the analysis and design of controllers.

The simplest model only allows for straight-line motion, sometimes referred to as 1-dimensional motion (as viewed from above). Only motion in the vertical plane (in Fig. 2) is modelled, involving the two degrees of freedom yielding two states for each degree – longitudinal displacement and tilt of the intermediate body. Without trying to be complete, there are many instances of models for 1-dimensional motion without yaw (Åkesson, Blomdell, & Braun, 2006; Ha & Yuta, 1994; Han, Zhao, Li, & Li, 2008; Kim, Lee, & Kim, 2011; Li, Gao, Huang, Du, & Duan, 2007; Lien, Tu, Ross, & Burvill, 2006; Nomura, Kitsuka, Suemitsu, & Matsuo, 2009; Ooi, 2003; Ruan & Cai, 2009). An early example which does not model yaw is as given by Ha and Yuta (1994). An instructive example of this non-linear model derived using the Euler–Lagrange equation or Lagrangian is presented by Kim et al. (2011). Because longitudinal position does not further affect robot dynamics (other states), sometimes only the other three states are modelled (Ha & Yuta, 1994; Ruan & Cai, 2009).

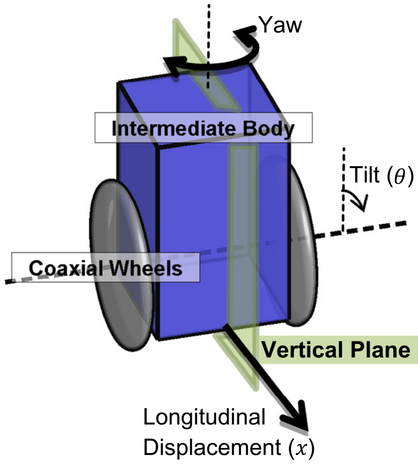


Fig. 2. Robot coordinates.

When turning, or yaw, is allowed, there exist two additional states for yaw angle. Another further state can be obtained if instead of longitudinal distance travelled, both x and y-coordinates in the Cartesian plane are used for position of the two-wheeled robot. The resulting non-linear model is much more complex. However, this is often simplified by disregarding some additional effects. By ignoring the change in moment of inertia in the yaw axis, as tilt angle changes, and also ignoring the effect of yaw rate on tilt and longitudinal motion, the yaw states are decoupled from the other states, simplifying derivation (Grasser, D'arrigo, Colombi, Ruffer, & Mobile, 2002; Hu & Tsai, 2008; Takei, Imamura, & Yuta, 2009; Tsai & Hu, 2007; Wu, Liang, & Wang, 2011).

There are two variations of model where moment of inertia around the yaw axis changes as tilt angle changes. Pathak, Franch, and Agrawal (2005) used a moment of inertia tensor matrix of the intermediate body to fully account for the dynamics. This can also be simplified to disregard the lesser change caused by rotation of the intermediate body, but taking into account the effect on moment of inertia caused by the change in distance of the centre of mass of the intermediate body from the yaw axis (Kim, Kim, & Kwak, 2005; Nawawi, Ahmad, & Osman, 2007; Muhammad, Buyamin, Ahmad, & Nawawi, 2011) – a more common alternative which avoids the inertia tensor, of which Muhammed et al.'s derivation (Muhammad et al., 2011) is a good example using Kane's method.

2.1.2. Black-box models

A black-box model of system dynamics can also be identified. This is a model where we are not concerned with the meaning of every term, only that the result approximates reality accurately enough. Alarfaj and Kantor (2010) experimentally estimated the parameters of a discrete time state-space model. Jahaya, Nawawi, and Ibrahim (2011) estimated various linear models in the discrete time domain with a sampling time of 0.1 s. These included the ARX (auto-regressive with extraneous input), ARMAX (includes moving average), Box-Jenkins, and Output-error models. Model coefficients can be solved via a suitable least-squares algorithm given system responses to independent white noise. The primary factor affecting the accuracy of these models is the order and sampling rate. An excessively high sampling rate can cause numerical problems – Moore, Lai, and Shankar (2007) is a good source for how to estimate ARMAX models, and suggests a sampling time around 10% of the settling time of the system.

A Takagi–Sugeno (T–S) fuzzy model was used by Qin, Liu, Zang, and Liu (2011) to approximate the non-linear model. A linear combination of linear state-space models weighted by a membership function can approximate the non-linear state-space equations. A suitable controller was designed based on the fuzzy model of the system.

2.2. Summary of models

The various models used can thus be categorised along two major dimensions – model fidelity and model type, as shown in Table 1. Model fidelity involves the degrees of freedom that are modelled, as well as whether their interactions are modelled. Model type is first classified by whether it is a theoretical or white-box approach, or if it is a black box model. For theoretical models, the resultant equations are generally the same, but may be categorised into the method by which they were derived. Black box models can be categorised by their structure.

2.3. Control of two-wheeled robots

The primary objective in the control of two-wheeled robots is always to remain balanced and avoid toppling. Secondary objectives may include tracking a certain speed or trajectory. The sensors and state measurements available vary, but most commonly include wheel encoders, and an inertial measurement unit (IMU), which includes gyroscopes and accelerometers. These basic sensors yield forward speed and angular rotation directly, as well as tilt angle over time – normally from a Kalman filter in the IMU. The accuracy of the tilt angle primarily depends on the gyroscopes, beyond which there is a trade-off between the effect of sensor noise, and acceleration. It is also possible to estimate tilt without an accelerometer, since the angular acceleration of the intermediate body can perform as a crude accelerometer.

2.3.1. Linearisation

Two-wheeled robots are often controlled by some variant of linear state feedback of a linearised model. Normally, Jacobian linearisation is used for its simplicity, and is quite successful since the two-wheeled robot system is quite linear for small tilt angles. Non-linearity in the system is due to trigonometric sine and cosine functions as well as gyroscopic forces (depending on angular speed). When tilted at 10°, the error in linearity of the cosine function is still only 1.5%. This amount of tilt is more reasonable for two-wheeled robots with a relatively high centre of mass. Note that Jacobian linearisation approximation ignores the non-linear effects which intertwine the tilt and yaw states, resulting in two apparently decoupled state-space systems. Many papers (Åkesson et al., 2006; Grasser et al., 2002; Takei et al., 2009; Kim et al., 2005) use Jacobian linearisation before designing linear controllers based on the linear approximation of the system.

While Jacobian linearisation approximates the non-linear system, feedback linearisation is an exact linearisation via a change of state and input variables. Non-linear effects are cancelled by feedback dependent on the state of the system. Because feedback linearisation does not approximate, it results in better performance for larger tilt angles, where there is increasing non-linearity. For two-wheeled robots, partial feedback linearisation is the most common approach (Pathak et al., 2005; Hatakeyama & Shimada, 2008; Shimada & Hatakeyama, 2007; Shimada & Hatakeyama, 2008; Shimizu & Shimada, 2010; Yongyai, Shimada, & Sonoda, 2009). Teeyapan, Wang, Kunz, and Stilman (2010) used a simpler input-output feedback linearisation. Normally, feedback linearisation can cause internal states to disappear, which can make the system unobservable. Because many states in the two-wheeled robot system are almost directly measurable from IMUs, the possibility of certain states becoming unobservable can be avoided. However, feedback linearisation is quite sensitive to the accuracy

Table 1
Models categorised by fidelity and type.

		Degrees of freedom (motion), including tilt angle			
		Longitudinal only	Longitudinal and yaw motion		
			Decoupled	Coupled: Simplified	Coupled: Inertia tensor
Dynamic equations	Newton's equations of motion	Li et al. (2007) Nomura et al. (2009) Ooi (2003) Ruan & Cai (2009)	Takei et al. (2009) Grasser et al. (2002) Wu et al. (2011)		
	Euler-Lagrange equations	Åkesson et al. (2006) Ha & Yuta (1994) Kim, Lee, et al. (2011) Han et al. (2008) Lien et al. (2006)	Hu & Tsai (2008) Tsai & Hu (2007)		Pathak et al. (2005)
	Kane's method			Kim et al. (2005) Nawawi et al. (2007) Muhammad et al. (2011)	
Black box model	Discrete parameter estimation		Alarfaj & Kantor (2010) Jahaya et al. (2011)		
	Takagi-Sugeno fuzzy models		Qin et al. (2011)		

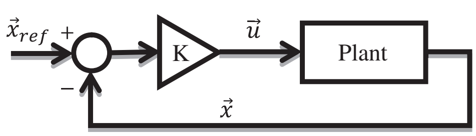


Fig. 3. Linear reference tracking controller.

of the model, to correctly cancel out non-linear effects. It is important to note that full state linearisation is not possible, because the system is under-actuated. Feedback with a single actuator (the drive wheels) can only be used to linearise either tilt or longitudinal position.

2.3.2. Linear controllers

Given a linearised model, a linear controller can be easily connected to track a reference state as shown in Fig. 3, requiring only a gain matrix K to be designed. One way is via pole placement, which can be used to control rise time and overshoot to obtain satisfactory performance. Grasser et al.'s JOE (Grasser et al., 2002) is an early paper using this technique, but it is also used in a number of other papers (Han et al., 2008; Nawawi et al., 2007; Feng et al., 2011; Li, Gao, Huang, & Matsumoto, 2008). Pole placement involves finding a gain matrix K such that the desired poles in a linear plant are:

$$eig(A - BK) \tag{2.3.1}$$

For optimal control, linear quadratic regulation (LQR) is a very common choice. This involves finding a controller to minimise the cost function:

$$J = \int_0^\infty (x^T Qx + u^T Ru)dt \tag{2.3.2}$$

Although it involves slightly more calculation, its optimality guarantees are attractive enough that many more researchers design their controllers using LQR (Åkesson et al., 2006; Ha & Yuta, 1994; Takei et al., 2009; Kim et al., 2005; Alarfaj & Kantor, 2010; Butler & Bright, 2008; Coelho, Liew, Stol, & Liu, 2008; Kim, Kim, & Kwak, 2006). This often yields better convergence to stability, and assuming knowledge of the system states, has a guaranteed 60° phase margin. A cross-term cost $x^T Nu$ has not been used for balancing of two-wheeled robots.

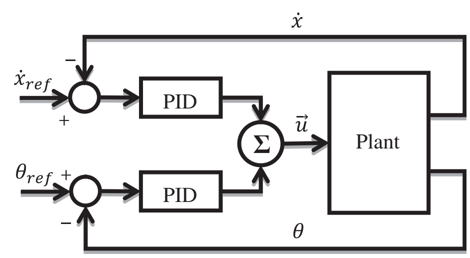


Fig. 4. PID controller for two-wheeled robot.

Comparisons between LQR and other linear controllers have been conducted (Lien et al., 2006; Ooi, 2003; Ghani, Ju, Othman, & Ahmad, 2010; Sun & Gan, 2010; Wu & Zhang, 2011a) by a few researchers. Their analysis is, at times, contradictory. Ghani et al. (2010) and Lien et al. (2006) found that pole placement is superior in settling time and smaller deviations. Wu and Zhang (2011a) found that pole placement has smaller overshoot but similar settling time. Sun and Gan (2010) found the PID controller to have a larger region of stability, but with vibrations at equilibrium. Ooi (2003) found pole placement has inferior robustness. These assessments are not very useful because they are specific to the particular tuning and system, and cannot be generalised. The controllers that can be obtained by pole placement or LQR can greatly differ depending on the poles selected, or the Q and R weighting matrices. In general, a higher gain (therefore better convergence, but perhaps larger vibrations at equilibrium) can be obtained by selecting poles further in the left half plane for pole placement, or larger Q gains for the LQR controller. However, LQR can be more useful in selecting those states for which faster settling time is desired, whereas this is not possible with pole placement. LQR therefore allows the trade-offs between states and actuators to be managed more precisely, and to conserve actuator work over time. The trade-off between multiple actuators is not straight forward with pole placement.

The LQR controllers all use measured states. These are often filtered, and therefore the controller is, in a sense, a linear quadratic Gaussian (LQG) controller – consisting of the optimal LQR controller, and Kalman filter observer. However, supposing that the observer dynamics are much faster than system dynamics, LQR is an adequate design strategy mostly retaining its phase margin guarantees, particularly since the Kalman filter is often an integral part of an IMU, if it is used. Lupian and Avila (2008) explicitly designed an LQG controller. If an LQG controller is unstable, loop transfer recovery may be necessary. However, this does not appear to be a problem for two-wheeled robots, because of the comparatively fast dynamics that IMUs have in measuring absolute tilt angle.

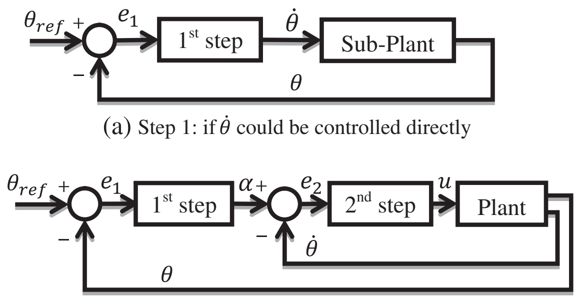
Other optimal linear controllers – H_∞ and H_2 controllers, which use different cost functions have been implemented. They are more robust because of their lower sensitivity to model errors and other disturbances. Hu and Tsai (2008) and Ruan and Chen (2010a) designed an H_∞ controller. Kanada, Watanabe, and Chen (2011) designed an H_2 controller. Kanada et al. compared the LQR and H_2 controller, showing that while the LQR controller achieves superior performance for the nominal plant, when there are model uncertainties, the H_2 controller can maintain its performance, while LQR performance degrades significantly.

PID controllers are very popular, whether by themselves or in conjunction with other controllers. They are widely used in many other areas in industry, because no model of the plant is required, with tuning of just three gains, with PD (Hatakeyama & Shimada, 2008) and PI (Liao & Li, 2010; Takahashi, Ishikawa, & Hagiwara, 2001) controllers appearing when one of the gains is zero. Two-wheeled robots have a number of states – therefore the control system will often have a PID controller for each degree of freedom – one for tilt, one for speed (shown in Fig. 4), and where applicable, one for yaw. The PID gains are often selected, and reasonable gains can be found easily just from experimentation. Otherwise, Ziegler–Nichols can be used for an initial set of gains as shown by Nasir, Ahmad, Ghazali, and Pakheri (2011). Nasir et al. used two separate PID control loops – for longitudinal position, and tilt, in an implementation of a purely PID controller.

2.3.3. Time-invariant non-linear controllers

Among non-linear control methods, backstepping is a common solution. Essentially, it involves first regulating just part of the system with a virtual input. For just balancing of a two-wheeled robot, a suitable control law using tilt rate can stabilize tilt error $e_1 = \theta_{ref} - \theta$ to the origin in a sub-plant, as shown in Fig. 5a. The virtual input cannot be controlled directly however, as the first step uses one state, for example tilt rate, to control another state, tilt angle. We therefore define an error between actual tilt rate and the first virtual input, using another input to regulate this error to zero. Each step until the real control involves controlling a new error between a virtual control, and the actual state (shown in Fig. 5b). Lyapunov stability can be proven by showing that the derivative of a candidate function at each step of the states is negative definite. Backstepping is used by a number of researchers, but often in conjunction with other control strategies.

A good initial backstepping example is provided by Thao, Nghia, and Phuc (2010), where backstepping is used to control tilt angle, along with a PD controller for longitudinal position, and a PI controller for yaw angle. Nomura et al. (2009) present another example of a controller designed via backstepping, though this time with an adaptive approach using a parameter update law to track



(b) Step 2: stabilize $\dot\theta$ to virtual control α using the real control u

Fig. 5. Backstepping control design.

Fig. 6. Adaptively estimating changing centre of mass height – robot from Li et al. (2010).

unknown system functions. Otherwise, the same steps are used to regulate tilt angle as in the first paper. Nomura et al. also compares the backstepping controller with an LQR controller in simulation, which illustrates the advantages and disadvantages of each approach – the LQR controller performs better at small tilt angles because it is locally optimal, and at small tilt angles, the system is approximately linear. However, at a large initial tilt angle of 35°, the backstepping controller remains stable, while the LQ controller is unstable.

Sliding mode controllers involve simplifying the dynamics by applying a large switching control input to force the system state to a desired hyper-surface. The remainder of the control input needs only to ensure asymptotic stability on the hyper-surface, rather than the full state-space. Usually, the control input is the sum of the switching input and a controller for the reduced-order system. This type of controller tends to be more robust to model uncertainties. However, due to switching around the sliding surface, there can be significant chattering. For the chosen hyper-surface:

$$\sigma(t) = Cx(t) = 0 \tag{2.3.3}$$

for some vector C, the sign of $\sigma(t)$ is used to determine that of the switching input. To mitigate the effect of chattering, instead of using the sign of σ, a saturating or more sophisticated function may be used, for example, $\frac{\sigma}{|\sigma|+\delta}$ by Ghani, Yatim, and Azmi (2010).

Other examples of the sliding mode controller, without the complications of other control strategies in a two-wheeled robot include Wu et al. (2011), Ghani et al. (2010) and Yau, Wang, Pai, and Jang (2009). In particular, Wu et al. simulated external disturbances as well as perturbation of system parameters to show the robustness of the sliding mode controller for two-wheeled robots. Nawawi, Ahmad, Osman, Husain, and Abdollah (2006) used a proportional integral sliding mode controller (PISMC) which aimed to improve tracking compared to the conventional sliding mode controller – they demonstrated less overshoot compared to pole placement. For a PISMC, the hypersurface is given by:

$$\sigma(t) = Cx(t) - \int_0^t (CA - CBK)x(\tau)d\tau \tag{2.3.4}$$

for chosen constant matrices C and K.

Sliding mode controllers are often used in conjunction with other control strategies. Nasrallah, Michalska, and Angeles (2007) used a sliding mode controller as an outer loop. Some papers combine backstepping with a sliding mode control approach, as shown by Tsai and Ju (2010). Disturbance observers may improve the performance of sliding mode controllers. Xiang-zhong, Xin-Wen, Ya-Jing, Jing, and Yu-Heng (2012) have compared the performance of the sliding mode controller with routine and dynamic switching surface, to a sliding mode controller with disturbance compensation. The compensation corrects any internal or external disturbance perpendicular to the sliding surface. They have demonstrated in simulation that using this method, chattering is suppressed.

Gain scheduling involves the selection of control gains depending on the system states, and can be used to maintain good performance in non-linear systems when the apparent linear dynamics changes. A fuzzy PID control strategy is presented by Wu, Ma, and Wang (2010), but is essentially gain scheduling of the PID controller. Like fuzzy control, gain scheduling can approximate other non-linear control laws, although the use of gains slightly alters the effects of interpolation.

Fuzzy control is another oft used strategy for non-linear systems. Depending on the number of fuzzy sets, it can approximate and approach any non-linear control law. Compared to backstepping, the theory is probably simpler. However, because of the piecewise nature of the control law, it would not be straightforward to prove Lyapunov stability. Because the two-wheeled system has a number of states (at least three even when considering straight line motion, and ignoring longitudinal position), the number of fuzzy sets needed to appropriately divide the state space can be large. Therefore, the system will normally consist of two fuzzy control loops, naturally one for balance, and one for speed/position control (similar to that for PID as shown in Fig. 4), so that each fuzzy controller is only affected by two states at once as in Chiu and Peng (2006), or with yaw rotation, three fuzzy control loops (Nasir et al., 2011). Wu and Zhang (2011b) used a fuzzy controller in the position control loop, and a PID controller for balancing.

Li et al. (2011) have designed a fuzzy controller based on a Takagi–Sugeno model, demonstrating a fuzzy model-based approach to designing the controller. A control law based on such a model can be expressed as:

$$u = k\vec{x} + b \tag{2.3.5}$$

A Lyapunov function-based controller have been designed and simulated by Kausar, Stol, and Patel (2011, 2012a). The stability region and performance of an optimal LQR controller was compared to a controller designed using a constructive-Lyapunov function. Comparisons were made, first keeping settling time constant, then keeping initial control signal constant. Keeping one of these performance metrics constant allows fair comparisons based on other metrics, by removing bias introduced by arbitrarily choosing controller gains. Because there is more than one gain, further reduction of bias could use multiple performance metrics as control variables.

It is not possible to derive an optimal control analytically for non-linear systems in general. No researchers have investigated numerical methods of approximating the Hamilton–Jacobi–Bellman solutions to find a non-linear optimal controller.

There is the state-dependent Riccati equation (SDRE) for non-linear systems that finds a sub-optimal controller. Representing the state-space matrices $A(\vec{x}), B(\vec{x}), C(\vec{x}), D(\vec{x})$ as functions that depend on state, the Riccati equation can be solved for all states. While the LQR controller is optimal only at the equilibrium point, the SDRE further improves the cost of the controller for the region around the equilibrium. However, there is no unique set of state-space functions, and the SDRE does not guarantee global optimality. Kim and Kwon (2011) have shown that SDRE improves transient performance compared to LQR – the time when the system is most likely in its most non-linear region away from equilibrium. Controller performance is fairly comparable because control action at equilibrium involves the same gains.

2.3.4. Adaptive and self-learning non-linear controllers

Various adaptive control strategies have also been used. Li et al. (2010) demonstrates the use of an adaptive controller which tracks the height of the centre of mass. A change in the height of the centre of gravity can be detected by checking if the states deviate from what is expected, and if so, the height is re-estimated. This was experimentally tested to show that the adaptive controller can maintain stability when the height changes (Fig. 6), whilst becoming unstable without the adaptive controller.

An adaptive fuzzy controller has been used by Ruan, Chen, Cai, and Dai (2009). Because the precision of fuzzy controllers depends on the number of rules, an adaptive fuzzy controller can be used instead to reduce the number of rules required, where the output for each rule in a static set of membership functions can adapt to changes in the system. It also means that the fuzzy rules do not need to be designed initially. Nomura et al. (2009) used an adaptive integral backstepping controller because of limitations in transforming the system equations into a triangular form.

An adaptive fuzzy controller has been used for a sliding mode controller (Su, 2012), where the switching function is derived by using selecting fuzzy sets based on the distance from the sliding surface, to establish a boundary layer. The width of each fuzzy set is adaptively adjusted based on an estimated optimal width.

Artificial neural networks (NNs) are quite a popular adaptive control strategy for two-wheeled robots. Neural nets can approximate and control non-linear systems. However, to train the internal weights of the neurons, a supervisor is needed. A standard arrangement is shown in Fig. 7, the neural network is a feed-forward controller adding to the PD supervisor, to account for non-linear effects. The neural network learns by back-propagation of the error given by the PD supervisor. This arrangement allows the robot to balance somewhat even with an untrained neural network. However, balance will improve over time as the neural network learns and corrects the non-linearities of the system. The multi-layer perceptron (MLP) is a neural network for general use, but their use for two-wheeled robots has not been published. Cerebellar models are quite similar to the MLP and they have been used by Tanaka, Ohata, Kawamoto, and Hirata (2010) (with clear equations for the weighted sum and activation function), and Ruan and Chen (2010b).

A cerebellar model articulation controller (CMAC) can be used as a neural network. It uses much less processing power, but a large amount of memory use. Compared to other neural networks, it also learns more rapidly, with local generalisation abilities. Each memory location is associated with a region of state-space (often a hyper-rectangular shape), overlapping with other regions. The output is calculated by the average of memory locations associated with the current state. CMACs are quite common in other robotic

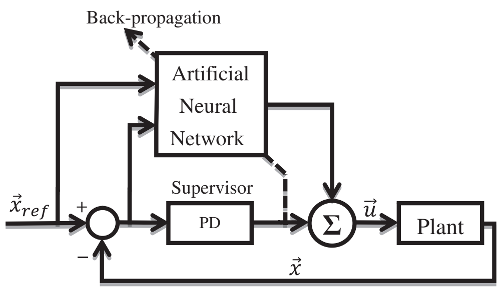


Fig. 7. Supervisor in artificial neural network controller.

fields where movement is to be controlled. For two-wheeled robots, their implementation is demonstrated by Li, Jiao, Qiao, and Ruan (2008). Chiu (2010) demonstrates the use of the adaptive output recurrent CMAC (AORCMAC), which adds a recurrent feedback path to the standard CMAC. Chiu experimentally compares the AORCMAC neural network with controllers derived from the Adaptive CMAC, the Elman NN, and the supervisory enhanced genetic algorithm controller (SEGAC). The conclusion is that the AORCMAC is most able to capture un-modelled system dynamics.

Another method of reinforcement learning uses internally recurrent neural networks, composed of a Critic Neural Network and an Action Neural Network, as shown in Fig. 8. This method was used to balance a two-wheeled robot by Sun and Gan (2011) and Ren, Ruan, and Li (2009). The Critic Neural Network approximates the Hamilton–Jacobi–Bellman equations, and learns the cost-to-go, so that the Action Neural Network can learn the control action. The Critic Neural Network can update its model using a temporal difference method. Compared to using a PD supervisor, this type of neural network does not rely on an external stabilizing controller, which may be important before the neural network is trained.

Fuzzy neural networks were demonstrated by Su and Chen (2010) and Wu and Jia (2011). In both cases, the simulation or experimental results show that they can be successful.

A radial basis function neural network (where the activation function is a radial basis function), was demonstrated by Tsai, Huang, and Lin (2010) for two-wheeled scooters. The radial basis function provides some degree of local generalisation. Tsai et al. used a NN for self-balancing, and another NN separately for yaw control. Currently, the NN provides appropriate control actions for slow speeds. It is unclear how the NN will perform if further developed for higher speeds.

The Boltzmann machine – a stochastic neural network, described as a Monte Carlo version of a Hopfield network, has also been demonstrated by Zhao and Ruan (2009) on a two-wheeled robot with a flexible lumbar.

In many cases, the researchers have only investigated a single neural network and control strategy, so it is unclear how each type of neural network compares with other neural networks and/or control strategies, though some general trends may be inferred from the use of these neural networks in other problems.

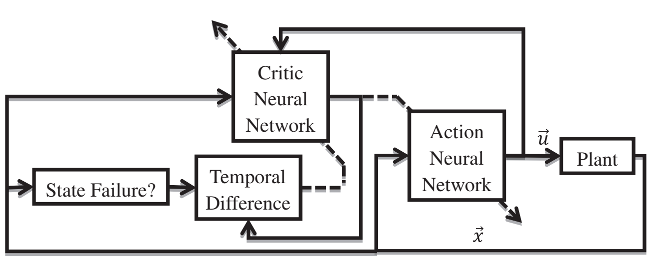


Fig. 8. Critic and action neural network.

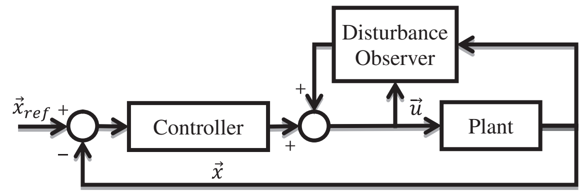


Fig. 9. Disturbance observer in the system.

2.3.5. Disturbance observers

A number of researchers use a disturbance observer (Fig. 9) in their work to reject low frequency disturbances, caused by unmodelled system dynamics or external disturbances. While not a controller by itself, a disturbance observer helps to improve the control action of an existing controller.

System parameters can influence the relative importance of unmodelled dynamics, which can be seen as a disturbance. Linearity of the two-wheeled robot depends on the height of the centre of mass, the inertia of the system and motor power.

A disturbance observer is simply an estimator of a state – often representing the size of a step disturbance. Disturbance estimation can be achieved by comparing expected system states (based on an internal model of the system) to the measured system states. The resulting estimated disturbance can be added to the output of the controller.

Shimada and Hatakeyama (2008), Shimizu and Shimada (2010) and Yongyai et al. (2009) give more detailed equations of how a disturbance observer can be implemented in a two-wheeled robot.

2.3.6. Summary

As shown in Fig. 10, a variety of controllers has been investigated for two-wheeled robots. Though many papers have experimental results, more such results would be preferable. Experimental results are particularly lacking for the non-linear controllers. Though simulations can be used to show robustness to disturbances and model uncertainties, there may be problems implementing them in practice, including sensor noise, sampling time and the total system lag time. Experimental data would be better able to show the robustness of the controllers to these effects. It is also uncertain how training of artificial neural network controllers can be done in practice, particularly where the NN may be unstable before training.

There is also a lack of comparison between different controller strategies, particularly for the non-linear controllers. There exist a number of papers making some comparison, but often they cannot be generalised. This is because the performance of a controller is influenced not only by the strategy, but also by the gains chosen. Moreover, it may be possible to obtain a range of similar gain matrices of a linear full-state feedback controller, via different methods, such as pole placement, LQR or H_∞, given appropriate input poles or cost matrices, making it difficult to make conclusive comparisons. Good comparisons between different control strategies are able to choose the gains of the different controllers fairly, for example, by selecting gains to match a particular metric to prevent bias.

A numerical solution to the Hamilton–Jacobi–Bellman for two-wheeled robots, to test the effectiveness of an optimal non-linear controller, is still an open topic. It would be useful to investigate how much performance is improved, compared to non-optimal controllers or optimal linear controllers.

Model predictive control has not been investigated. This approach may be a way to approximate an optimal controller, after a finite horizon. The challenge of model predictive control is in how it can be implemented, and what trade-offs might be necessary, to find an optimal solution at each time step, in a fast dynamic situation suitable for balancing a two-wheeled robot.

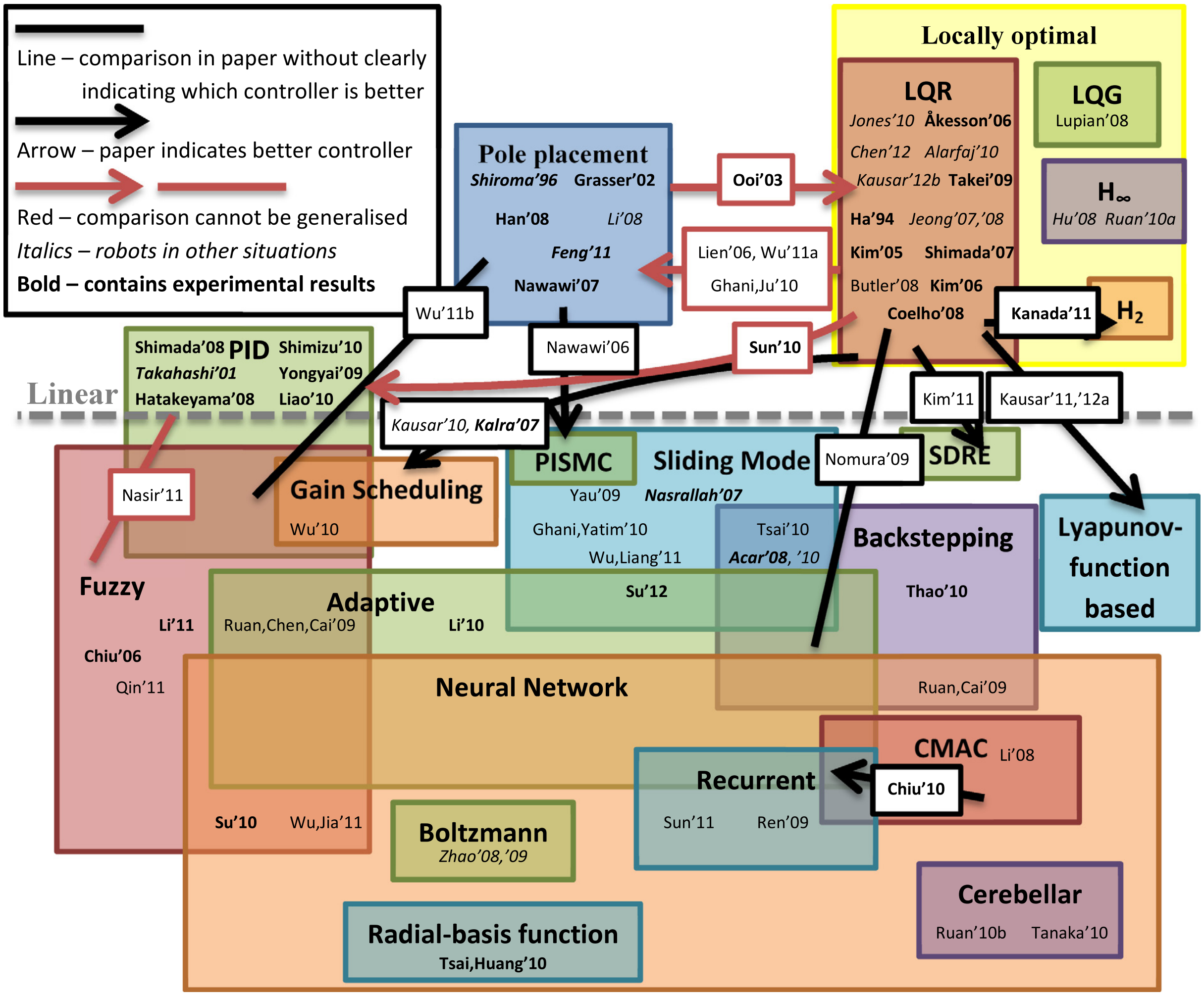


Fig. 10. Map of controllers used by researchers on two-wheeled robots.

3. Two-wheeled robots in other situations

In this section, we focus on variations of the basic two-wheeled robot model and other control objectives. The additional complexities are either due to changes in the environment which affect the robot, increased capability from additional actuators or trying to use the robot to manipulate the objects in the environment. However, a common theme is the control of a non-minimum phase, and usually under-actuated, system to remain balanced. We will look at the different extensions that have been made to the model of the system, and the control strategies which have been demonstrated. Because these models are less well-researched, the control strategies applied are often limited.

Research can be categorised into changes in assumptions about the environment, improvements in locomotion and balance by using additional actuators and manipulation of objects.

3.1. Terrain and obstacles in the environment

Some terrain properties affect the mathematical model of the two-wheeled robot, while a vertical obstacle can complicate the strategy for control.

3.1.1. Inclined terrain

A common assumption when deriving equations of the system dynamics is that the terrain is a flat, horizontal surface. For inclines

Fig. 11. Climbing slope reproduced from Li et al. (2007).

(Fig. 11) and steps, models and controllers designed for flat surfaces may have limited success (Li et al., 2007). However, modelling of these unusual environments and designing controllers with this in mind may be more robust and give better performance.

Geometrical parameters can affect whether a two-wheeled robot can ascend a slope. To remain in equilibrium, the robot must lean far enough that its centre of mass is above its point of contact with the incline. Thus, the height of the centre of mass compared to wheel radius can be a limiting factor.

One generalisation is when the terrain is an inclined plane. If sloped only in the direction of movement, the equilibrium tilt angle from vertical changes to one leaning into the slope and the equilibrium wheel torque becomes non-zero. Otherwise the dynamics are almost the same, and after adjusting for the changes in equilibrium, controllers designed for flat surfaces can be used in

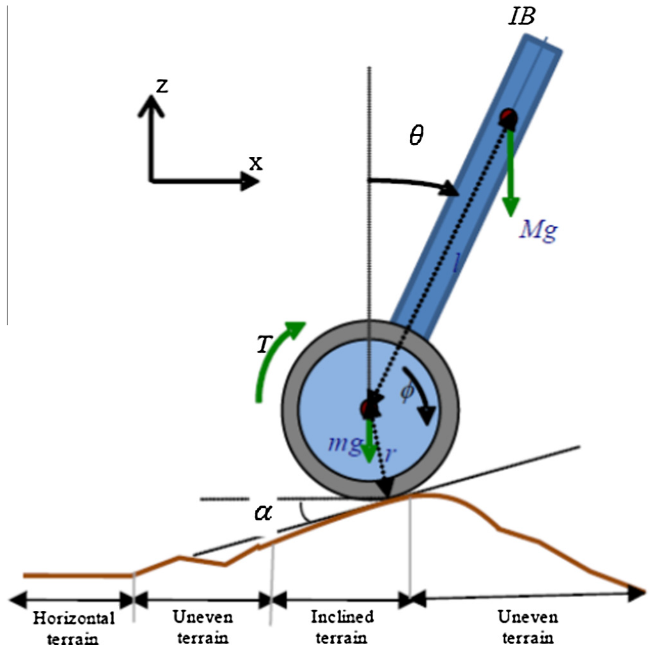


Fig. 12. Uneven terrain reproduced from Kausar et al. (2012b).

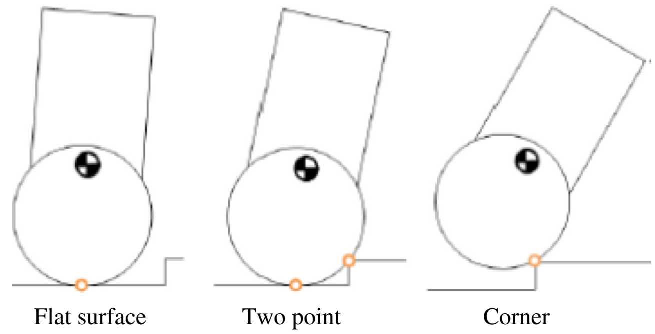


Fig. 13. Modes of behaviour – reproduced from Chen et al. (2012).

this situation. Kausar, Stol, and Patel (2012b) has evaluated the performance and stability region of two-wheeled robots on terrain with different inclination angles.

If the slope may be inclined in any direction, the incline will affect yaw movement, and tends to cause the robot to turn towards the downhill direction. This problem has been studied by Nasrallah et al. (2007), and a nested triple loop controller has been demonstrated in simulation. However, the experiment results have been solely on flat, horizontal terrain.

3.1.2. Smooth uneven terrain

More general terrain may have changing incline, or an uneven surface as shown in Fig. 12. As a simplification, the terrain may be inclined only in the direction of motion, so that the dynamics of motion are restricted to a 2-dimensional vertical plane. Kausar, Stol, and Patel (2010) have derived equations of motion to model this situation. Simulation results have been obtained for a baseline LQR controller compared to a gain-scheduled controller on a smooth bump given by an analytical equation, demonstrating much improved performance by the gain-scheduled controller – most notably halving traversal time. Experiments have not been done thus far – it would be interesting to see how it performs in practice.

3.1.3. Steps in the terrain

A step change in terrain height is one case of a sudden change in the height of the terrain. For simplification of the situation, other than step changes in height, the terrain is otherwise flat. Many two-wheeled robots can already descend a step, or ascend a step by approaching it at speed. For example, Li et al. (2007) showed in an experiment that a step can be traversed at a high speed. However, step traversal remains somewhat uncontrolled, as excess speed on descent can cause a robot to bounce, while to ascend, a higher than required approach speed would be utilised by a simple controller.

There are two geometrical constraints which can prevent a robot from ascending a step quasi-statically. First, the wheel radius must be greater than the step height. Second, the ratio between the height of centre of mass above the wheel axle to the wheel radius must be above a certain value depending on step height. There is therefore sometimes the conundrum where increasing wheel radius (which decreases the ratio of centre of mass height to wheel radius), as well as decreasing wheel radius, can reduce the step height that can be ascended.

Chen, Hazelwood, and Stol (2012) has modelled the system dynamics of step traversal. Step traversal is divided into three modes of behaviour (shown Fig. 13) – when the robot is balancing on flat terrain, when it has two points of contact – with the step corner as well as the ground below the wheel, and when it is balancing on the step corner. The advantage of modelling and designing controllers based on this is that it can enable a slower, more controlled ascent, keeping the robot near quasi-equilibrium, instead of blindly rushing the manoeuvre. The paper focuses on the corner model phase of step traversal, and simulations have demonstrated that a simple linearised LQR controller can ascend a step, taking 4.5 s for at 30 kg robot to ascend a 5-cm step.

3.1.4. Low-traction terrain

Because two-wheeled robots depend heavily on their wheels to balance, terrain traction can be a factor affecting their performance. A significant proportion of the motor torque to the wheels must be applied to the ground to obtain full control effectiveness. Thus, limited ground traction can limit the tilt angles from which they can recover, and therefore their maximum acceleration and speed.

Jones and Stol (2010) have investigated this topic, and derived a system of equations to model two-wheeled robots for a low-traction ground surface, where motion is restricted in a 2-dimensional vertical plane, with the addition of a reaction wheel actuator, which can also help to balance the robot. An additional state models wheel slip, and the equations incorporate a friction coefficient. Based on a chosen model of friction, Jones and Stol have designed a series of LQR controllers – a baseline controller for a surface with unlimited traction as well as a surface with no traction at all. By mixing the controllers, one which can remain stable on frictionless terrain, while being able to control forward speed when traction is present. Simulation results show that the hybrid controller can indeed control forward velocity, and by experimentation, remain balanced when there is no traction. A limitation of this approach is that the hybrid controller's performance is reduced even when there is sufficient ground traction.

The authors of this review have investigated whether conventional traction control techniques, as used in cars, would be useful for two-wheeled robots when ground traction is limited (Chan, Stol, & Halkyard, 2012). Conventional traction control limits wheel torque when wheel slip increases beyond an amount assumed to be optimal – when the friction force is at a maximum. Using the maximum speed from which a robot can stop as a measure of performance, simulation of traction control applied to a baseline con-

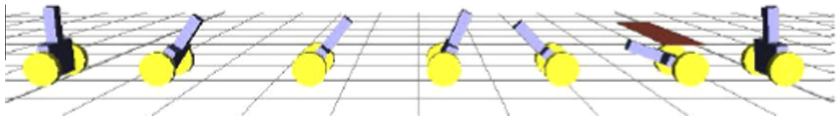


Fig. 14. Passing under a vertical obstacle – reproduced from Teeyapan et al. (2010).

troller on surfaces of varying traction properties shows no significant benefit in using traction control. The primary reason is due to the non-minimum phase nature of the system, which sometimes requires initially moving away from the intended direction of motion. This can cause the traction required to exceed the traction available when attempting to recover.

3.1.5. Passing under vertical obstacles

A vertical obstacle limits the height of the free space in which a robot must remain to traverse the obstacle. Teeyapan et al. (2010) have considered how a balancing two-wheeled robot can pass underneath (for example, as shown in Fig. 14), either by tilting forwards or backwards to lower its height.

Because such a robot is under-actuated, it is not possible to simultaneously control both tilt angle and position over an extended period, thus no single controller is able to complete the manoeuvre. Teeyapan et al. demonstrated the use of Particle Swarm Optimization to select a sequence of different controllers to perform the manoeuvre. Simulation results have been obtained for traversal by tilting forwards as well as tilting backwards. Preliminary experiments also demonstrate the practicality of their approach.

3.2. Actuators for locomotion and balance

Additional actuators are sometimes added to improve balancing of the robot, or to allow manoeuvres which may otherwise be difficult. Balancing can be divided into two categories depending on the direction – tilt angle, and roll angle. Other actuators which are added allow a robot that is lying down to stand up.

3.2.1. Improvements in control of tilt angle

Two types of actuators have been used to improve the control of tilt angle. The robot can either change its centre of mass, or incorporate a reaction wheel. The robot centre of mass may change due to a number of factors – including the transport of objects, or the movement of manipulators attached to the robot. While manipulators may affect balancing, in this section, we are more interested in actuators that can be controlled independently of any other control objective, to assist in balancing.

Probably the most relevant research of a moving centre of mass on two-wheeled robots is by Takahashi et al. (2001) to assist in balancing for a wheelchair, particularly when raising its front wheels off the ground. The advantage is that a shock can be avoided when initially switching from a statically stable to a balancing mode of behaviour, which is desired when transporting humans.

Another method to improve balancing is to add a reaction wheel. A study by Kalra, Patel, and Stol (2007) shows reduced energy consumption when a reaction wheel is used to assist in balancing in a hybrid gain scheduling scheme, compared to when it is not used. The advantage is that a reaction wheel is more efficient for temporary application of torque compared to drive wheels, so may be more appropriate for corrections around equilibrium. It is important to note that a reaction wheel does add weight and may demand an increase in energy to balance, even when unused, which is not accounted for in the study. A reaction wheel may also be used to balance a robot (Jones, 2011), even without the use of the main drive wheels, as mentioned in the section on low-traction terrain.

3.2.2. Control of roll

Two-wheeled robots are statically stable in the roll direction, which means that they only actively balance in one direction. However, if they corner too quickly, the centrifugal force could cause one wheel to lose contact with the ground. Because they are statically stable in this direction, balance cannot be maintained unless the rate of cornering is limited – either increasing the radius of curvature, or slowing down. To mitigate this problem, two groups of researchers have investigated the possibility of a robot leaning into the corner.

Alarfaj and Kantor (2010) use a four-bar linkage mechanism (a parallelogram looking from the front) to allow the robot to tilt sideways. They have shown in simulation that using a simple LQR controller, without leaning, the maximum centrifugal force that can be resisted is 54.6 N, increasing to 70 N with leaning.

Kim, Seo, and Kwon (2011) take a different approach, using a single actuator between two halves of a robot. This allows one half to be vertically displaced relative to the other half – effectively tilting the robot sideways, as shown Fig. 15.

3.2.3. Actuators for standing up

Two-wheeled robots must actively stabilize themselves constantly to avoid falling down. However, this will constantly drain energy. For practical use, a two-wheeled robot may have to lie down, or otherwise adopt a statically stable position. However, two-wheeled robots often are not powerful enough to stand up using only the power of their drive wheel motors.

A number of different methods have been investigated for standing up. Jeong and Takahashi (2007, 2008) used a four-bar link (Fig. 16) to allow a robot I-PENTAR to stand up. While standing up, it can adopt a statically stable posture with four wheels on the ground, until the end when it lifts its roller wheels off the ground. The researchers decided to control the centre of mass to ensure it follows a straight line trajectory while standing. They have experimentally demonstrated that it is possible to stand and sit, taking around 15–20 s for a 32 kg robot.

More recently, Miao and Cao (2011) have demonstrated a flywheel mounted on a vertical axis, to exploit precession. When

Fig. 15. Mobile tilting balancing robot reproduced from Kim et al. (2011).

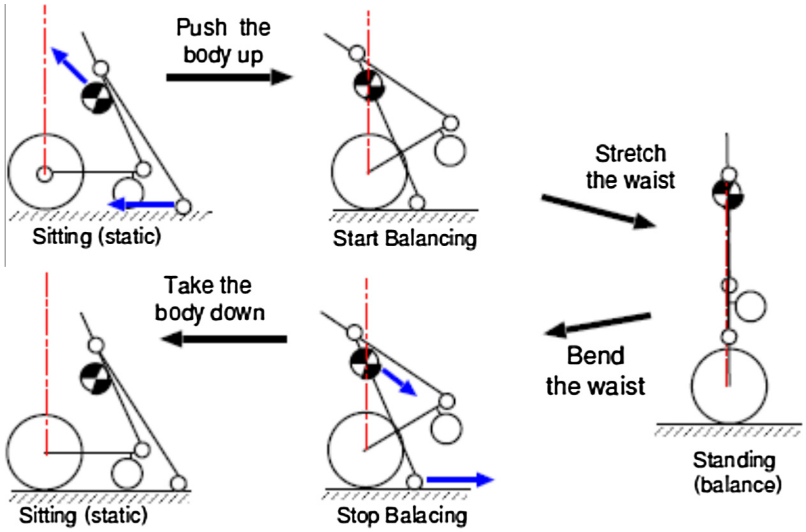


Fig. 16. I-PENTAR standing motion reproduced from Jeong and Takahashi (2007).

the flywheel spins at a high speed, the robot can maintain the height of its centre of mass by spinning, and therefore gradually tilt up. They have verified both in simulation and experimentally that the robot can tilt up in just a few seconds, and a few turns.

Finally, the two-wheeled robot by Feng et al. (2011) uses two supporting arms, allowing the robot to adopt a static position without tilting too far down. Thus, the robot does not require as much force to begin balancing again. It can be controlled to stand up again easily simply by accelerating in the direction it was resting in, and can balance in just half a second.

3.3. Manipulating objects in the environment

A two-wheeled robot may be used to manipulate objects in the environment. However, because it is dynamically stable, any force a two-wheeled robot exerts on the objects around it will immediately affect its own body. Some researchers have considered control of a two-wheeled robot when it is used to push other objects. In other cases, manipulator arms or other moving parts may be attached to the robot.

3.3.1. Kicking and pushing

Without any additional actuators, a two-wheeled robot can use its own body to push on objects in its environment. In this case, while in contact with the object, a disturbance force will be exerted on the robot, and affect its dynamics.

Takahashi, Nonoshita, Nakamura, and Maeda (2010) have developed a motion suitable for kicking a soccer ball (Fig. 17), by using a policy gradient method to learn improved trajectories. Learning involves testing in simulation a number of similar policies to determine their effectiveness – optimised by reducing running distance, while maximising ball velocity. After 300 iterations of learning, a running distance of less than a metre is able to produce ball velocities of 1.2 m/s.

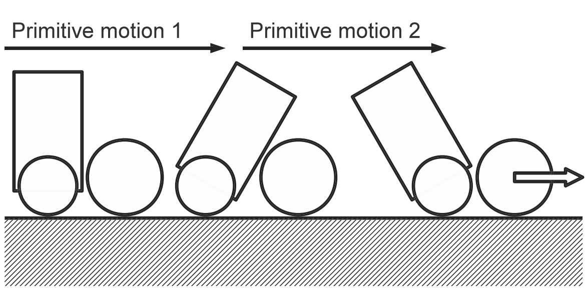


Fig. 17. Motion to kick ball reproduced from Takahashi et al. (2010).

The idea of using two robots to cooperatively transport an object, by pushing on either side to hold it up, has been investigated. A very early study by Shiroma, Matsumoto, Kajita, and Tani (1996) considers how a robot can be controlled to sustain an external force. A simple controller designed by pole placement, was tested against a force exerted by a string and pulley system. The force sustained by the 1kg robot varied 0.3–1.5 N for a 0.8 N reference. It also demonstrated that the two-wheeled robot could transport an object with a human (as a substitute for the other robot) cooperatively. Lee and Jung (2011a) has recently experimentally demonstrated the use of two ROBOKER two-wheeled robots (as well as force control between a robot and a human Lee, Yeong Geol, & Seul, 2012), to transport an object in this way, using a simple PID controller. They designate a master robot that leads the way (position tracking), while a slave robot pushes the object against the master using a force sensor to track force. Using a force sensor allows more accurate force control – tracking 7–13 N for an 8 N reference. Neither study shows how an object can be lifted from the ground, which a useful implementation in the future will need to solve.

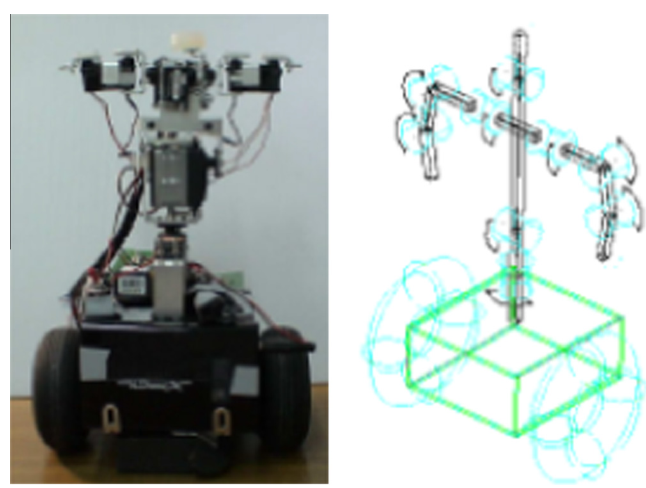


Fig. 18. Glove puppet robot reproduced from Hu et al. (2009).

Fig. 19. Two-wheeled mobile manipulator of Keio University.

It is worthwhile to note that a constant force by an object mimics the dynamics of a robot on an inclined slope (where gravity exerts a force in one direction). In both cases, the equilibrium tilt angle of the robot is offset from vertical. Thus, both situations can be controlled by a simple controller normally suitable for flat terrain after accounting for the offset in equilibrium state.

3.3.2. Limbs and manipulators

Manipulators or other moving parts of a robot affect its dynamics as they move from position to position. These differ from actuators for the sole purpose of balancing, because they may be used to affect the environment around them. Thus, while additional actuators to improve balance can assist in balancing, two-wheeled robot controllers must conversely resist and sustain the effects of motion of its limbs. In some cases, the movement of other parts of the two-wheeled robot may be ignored, such as by Lee and Jung (2011b). However, if the impulse imparted is significant, balancing may be affected significantly. Hu, Wang, and Sun (2009) used a simple computed torque method to resist the effects of its upper body – a glove puppet robot (Fig. 18). A two-wheeled robot with a flexible torso has been modelled and a neural network controller

Fig. 20. Golem Krang of the Georgia Institute of Technology, reproduced from Stilman et al. (2010).

Table 2
Two-wheeled robots in modified situations or with additional actuators.

Situation	Research articles	Control strategy
Terrain & obstacles		
Inclined terrain	Nasrallah et al. (2007)	3-Loop nested lyapunov, sliding mode
	Kausar et al. (2012b)	LQR
Uneven terrain	Kausar et al. (2010)	LQR and Gain Scheduling
Steps	Chen et al. (2012)	LQR with selected reference
Low-traction terrain	Jones and Stol (2010)	LQR
	Chan et al. (2012)	LQR with conventional traction control
Vertical obstacles	Teeyapan et al. (2010)	Control sequence Particle Swarm Optimization
Balancing		
Moving centre of mass	Takahashi et al. (2001)	PI
Reaction wheel	Kalra et al. (2007)	Gain scheduling
Leaning sideways	Alarfaj and Kantor (2010)	LQR on four-bar linkage
	Kim et al. (2011)	
Precession to stand	Miao and Cao (2011)	Tilt up using flywheel
Support arms to stand	Jeong and Takahashi (2007, 2008)	LQR
	Feng et al. (2011)	Pole placement
Manipulating objects		
Kicking	Takahashi et al. (2010)	Policy gradient method to learn state feedback
Pushing	Shiroma et al. (1996)	Pole placement
	Lee and Jung (2011a, 2012)	PID
Flexible/moving limbs	Hu et al. (2009)	Computed torque method
	Zhao and Ruan (2009), Zhao and Ruan (2008)	Boltzmann machine neural network
Manipulators	Hu et al. (2009)	Computed torque method
	Abeygunawardhana and Murakami (2007)	Multiple modes – Linear state feedback
	Abeygunawardhana and Murakami (2010)	Resonance ratio
	Abeygunawardhana et al. (2010)	Sliding mode
	Acar and Murakami (2008, 2010)	Backstepping to sliding model control
	Acar and Murakami (2011)	Task priority control

demonstrated in simulation by Zhao and Ruan (2009) and Zhao and Ruan (2008).

A reasonably well-studied system (using a number of different controllers) is a two-wheeled mobile manipulator at Keio University (Fig. 19). The robot has, attached to its intermediate body, a three-link manipulator. All the links are in the same vertical plane. Thus, it is modelled as a four link pendulum with two wheels. The additional links can be simplified and modelled as a single virtual link to get a double pendulum model. Abeygunawardhana et al. demonstrated control by switching between a trajectory mode and a balance mode controller (Abeygunawardhana & Murakami, 2007), vibration suppression with resonance ratio control (Abeygunawardhana & Murakami, 2010) and a sliding mode controller (Abeygunawardhana, Defoort, & Murakami, 2010). Acar and Murakami have demonstrated a non-linear backstepping controller (Acar & Murakami, 2008) experimentally and through simulation control via centre of mass manipulation (Acar & Murakami, 2010) using a backstepping controller and task priority control for multitasking (Acar & Murakami, 2011) to handle the competing objectives of balancing, centre of gravity of the multiple links, and the position of the end-effector (see Fig. 19).

The control of a double-link two-wheeled robot with fuzzy controllers has been investigated. This is simpler system than the two-wheeled manipulator above. Ahmad, Siddique, and Tokhi (2011) and Ahmad, Aminnuddin, and Shukor (2012) has demonstrated two approaches in simulation. Because the number of rules in fuzzy controllers increases exponentially with the number of states, a modular approach can be used to design separate fuzzy controllers, with a coordinator between the fuzzy controllers for each link. The second approach demonstrated is a fuzzy-PD approach for the lower link, and a conventional PID controller for the upper link.

A 4° of freedom manipulator with joints oriented in different axes was mounted on a two-wheeled robot base (Fig. 20). Stilman et al. (2010) experimentally demonstrated control of such a two-wheeled robot by taking an un-modelled approach to system dynamics, and using a cascade of PID controllers for balancing and control of velocity. To account for the manipulator on its upper body, the balance controller uses an optimal pitch value based on manipulator position.

3.4. Summary of two-wheeled robots in other situations

Many extensions and control strategies to the two-wheeled robot model have been studied, as shown in Table 2 Two-wheeled robots in modified situations or with additional actuators. Because the models are often simplified, further extensions can be made to accommodate more factors. For example, the model of uneven terrain and step traversal is only for 1-dimensional motion. The controllers which have been applied to these are also often limited. Other controllers may be able to better control the two-wheeled robot. Models of two-wheeled robots with attached limbs or manipulators which affect their balance can be further extended for different configurations. Current models have only considered manipulators in the vertical plane.

4. Conclusion

There has been a large amount of increased research in two-wheeled robots in the past decade. The two-wheeled robot has been modelled by a number of approaches theoretically. However, there is a lack of experimental verification. It is not known how actual system parameters differ from those used to design controllers.

A number of controllers have also been applied to the two-wheeled robot, showing varying performance. However, it is quite clear that simple linear controllers can effectively control the two-wheeled robot, even for a largely un-modelled system. Although it has been shown that more advanced controllers have improved performance or robustness, the extent to which it is improved is not well quantified.

In the second half of this review, we looked at extensions to the standard model of a two-wheeled robot, whether due to terrain or other obstacles, or additional actuators on a two-wheeled robot.

Based on the research in the last decade, many additional problems have been considered, with control solutions demonstrated. There is room for further experiments, particularly for some controllers which have only been tested in simulation, more objective comparisons can be made between different controllers, and further model extensions can be considered (particularly where simplifications have been made).

Acknowledgements

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