To simulate the dynamics of the hardware, a hardware-equivalent dynamics model was selected. The model was originally derived by Yamamoto [0] and has been successfully used in other control studies [0].

Figures 0.1 - 0.2, depict the physical model of the two-wheeled inverted pendulum as isometric and multiview projections. These figures use Yamamoto's original symbol notation; a legend is provided in Figure 0.2.

Yamamoto [0] makes the following assumptions in Figures 0.1 - 0.2:

- All mass geometries are uniform.
- All masses are uniformly distributed.
- The hardware consists of three principal masses:
    - · A rectangular cuboid [*The body.*]
    - · A cylinder [*The left wheel.*]
    - · A cylinder [*The right wheel.*]

Tables 0.1 - 0.2, define the variables and the parameters, respectively, that the physical model of the two-wheeled inverted pendulum will use.

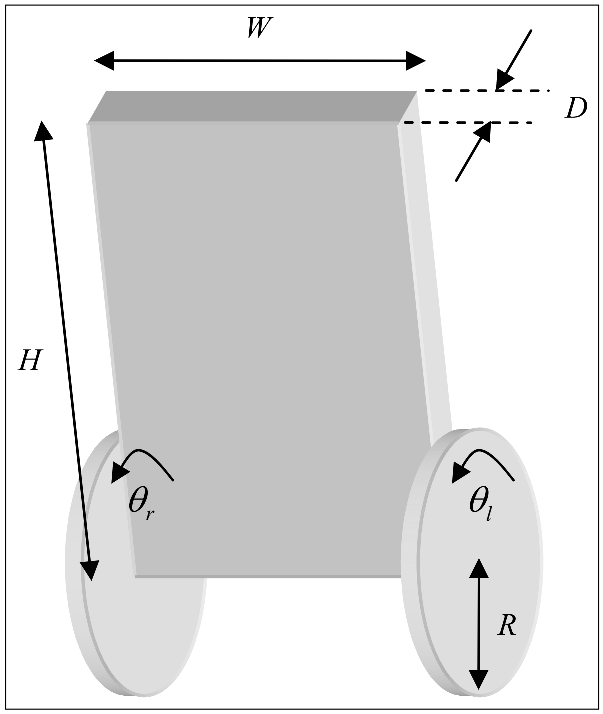

Figure 0.1: [Hardware-Equivalent Physical Dynamics Model]: Isometric[0]

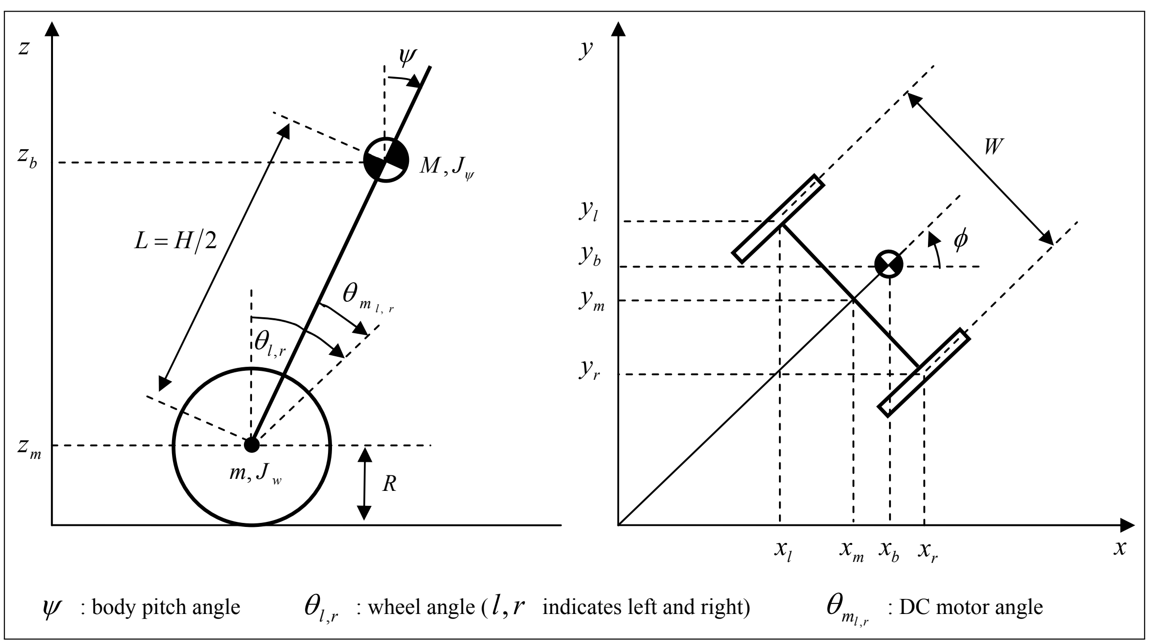

Figure 0.2: [Hardware-Equivalent Physical Dynamics Model]: Multiview[0]

Table 0.1: [Simulink]: Root

Symbol	Definition	Unit
θ	Angular position: Wheel [$\theta = \theta_{g,av}$] [Measured from the wheel center of mass]	rad
θ_g, θ_b	Origin aligns with: $\begin{bmatrix} \text{global rejection vector} \\ (orthogonal\ from\ earth's\ surface) \end{bmatrix}$ $\begin{bmatrix} \text{body pitch } \phi_x \end{bmatrix}$	
$\theta_{av}, \theta_l, \theta_r$	Component: [average of left and right wheels] [left wheel] [right wheel]	
ϕ	Angular position: Body [Measured from the wheels center of mass.]	rad
ϕ_x, ϕ_y, ϕ_z	Dimension: [X] [Y] [Z]	
p	Translational position [$p = p_w$] [Measured from the corresponding center of mass]	m
p_x, p_y, p_z	Dimension: [X] [Y] [Z]	
p_w, p_{wl}, p_{wr}, p_b	Component: [both wheels] [left wheel] [right wheel] [body]	
v_{mtr}	Voltage: Motor Input	V
$v_{mtr.l}, v_{mtr.r}$	Component: [left-wheel] [right-wheel]	

Note: When a subscript is unspecified, assume the first option is used by default.

Table 0.2: [Simulink]: Root

Symbol	Definition	Value	Unit	Source
a_g	Acceleration of gravity: Earth	9.81	$\frac{m}{s^2}$	-
m_w	Mass: Wheel [*Includes wheel axle.*]	0.018	kg	[0]
m_b	Mass: Body	0.381	kg	[0]
$l_{b.h}$	Length: Body: Height	0.2032	m	-
$l_{b.w}$	Length: Body: Width	0.0825	m	-
$l_{b.d}$	Length: Body: Depth	0.0635	m	-
$l_{b.c2a}$	Length: Body: [Center of mass] to [Axis of Rotation]	0.010	m	Sec. **??**
r_w	Length: Wheel: Radius	0.021	m	[0]
$J_{b.\phi_x}$	Moment of Inertia: Wheel	$7.46 \cdot 10^{-6}$	$kg \cdot m^2$	[0]
$J_{b.\phi_x}$	Moment of Inertia: Body: X-axis (pitch)	$1.5573 \cdot 10^{-5}$	$kg \cdot m^2$	Sec. **??**
$J_{b.\phi_y}$	Moment of Inertia: Body: Y-axis (yaw)	$4.2230 \cdot 10^{-4}$	$kg \cdot m^2$	Sec. **??**
R_{mtr}	Motor: Resistance	4.4	Ω	[0]
$k_{mtr.bEMF}$	Motor: Coefficient of Back EMF	0.495	$\frac{V \cdot s}{rad}$	[0]
$k_{mtr.T}$	Motor: Coefficient of Torque	0.470	$\frac{N \cdot m}{A}$	[0]
$k_{fr.m2w}$	Motor: Coefficient of friction: [DC Motor] to [Wheel]	0.0664	-	-