## 0.1 Differential Equations

After creating a nonlinear model using the Lagrangian method, and then linearizing that model, Yamamoto [0] provides the differential equations (0.1) and (0.12), [*and their abbreviated term definitions*].

### 0.1.1 Wheel Angular Position $\theta$ and Body Pitch $\phi_x$

Equation (0.1) corresponds to wheel angular position $\theta$ and body pitch $\phi_x$.

$$\mathbf{K}_{1.\ddot{x}} \cdot \begin{bmatrix} \ddot{\theta} \\ \ddot{\phi}_x \end{bmatrix} + \mathbf{K}_{1.\dot{x}} \cdot \begin{bmatrix} \dot{\theta} \\ \dot{\phi}_x \end{bmatrix} + \mathbf{K}_{1.x} \cdot \begin{bmatrix} \theta \\ \phi_x \end{bmatrix} = \mathbf{K}_{1.v} \cdot \begin{bmatrix} v_{mtr.l} \\ v_{mtr.r} \end{bmatrix} \tag{0.1}$$

$$\mathbf{K}_{1.\ddot{x}} = \begin{bmatrix} +k_{1.1} & +k_{1.2} \\ +k_{1.2} & +k_{1.3} \end{bmatrix} \tag{0.2}$$

$$\mathbf{K}_{1.\dot{x}} = \begin{bmatrix} +k_{1.4} & -k_{1.4} \\ -k_{1.4} & +k_{1.4} \end{bmatrix} \tag{0.3}$$

$$\mathbf{K}_{1.x} = \begin{bmatrix} 0 & 0 \\ 0 & +k_{1.5} \end{bmatrix} \tag{0.4}$$

$$\mathbf{K}_{1.v} = \begin{bmatrix} +k_{1.6} & +k_{1.6} \\ -k_{1.6} & -k_{1.6} \end{bmatrix} \tag{0.5}$$

$$k_{1.1} = \left(2 \cdot m_w + m_b\right) \cdot r_w + J_w \tag{0.6}$$

$$k_{1.2} = m_b \cdot r_w \cdot l_{b.c2a} \tag{0.7}$$

$$k_{1.3} = m_b \cdot l_{b.c2a}^2 + J_{\phi_x} \tag{0.8}$$

$$k_{1.4} = 2 \cdot \left(\frac{k_{mtr.T} \cdot k_{mtr.bEMF}}{R_{mtr}} + k_{fr.m2w}\right) \tag{0.9}$$

$$k_{1.5} = -m_b \cdot a_g \cdot l_{b.c2a} \tag{0.10}$$

$$k_{1.6} = \frac{k_{mtr.T}}{R_{mtr}} \tag{0.11}$$

### 0.1.2 Body Yaw $\phi_y$

Equation (0.12) corresponds to body yaw $\phi_y$.

$$k_{2.\ddot{x}} \quad \cdot \left[\ddot{\phi}_y\right] \quad + \quad k_{2.\dot{x}} \quad \cdot \left[\dot{\phi}_y\right] \quad = \quad k_{2.v} \quad \cdot \left[v_{mtr.r} - v_{mtr.l}\right] \tag{0.12}$$

$$k_{2.0} = \frac{l_{bw}}{r_w} \tag{0.13}$$

$$k_{2.\ddot{x}} = \frac{1}{2} \cdot m_w \cdot l_{bw}^2 + \frac{1}{2} \cdot k_{2.0}^2 \cdot J_{\phi_y} \tag{0.14}$$

$$k_{2.\dot{x}} = \frac{1}{2} \cdot k_{2.0}^2 \cdot k_{1.4} \tag{0.15}$$

$$k_{2.v} = \frac{1}{2} \cdot k_{2.0} \cdot k_{1.6} \tag{0.16}$$