Trajectory Control of a Two-Wheeled Robot Using Optimal Control

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1 Statement of the Problem

The two-wheeled robot is a well-established control problem. The robot is topheavy and must continually work to balance itself. The robot is able to move freely on a two-dimensional plane; however, any movements performed by the robot create additional disturbances against its ability to balance itself.

Numerous approaches have been developed to control such a device; however, no one approach has been determined as a clear choice. Additionally, additional functionalities may be implemented in a controller beyond command regulators which may significantly improve performance.

This study intends to comparatively study multiple control approaches involving optimal controllers and to study the effects of additional functionalities which may be beneficial in general control cases.

2 Justification for Significance of the Study

2.1 Inverted Pendulum

In control theory, the balancing of an inverted pendulum is a well-established problem [1].

In such a problem, a rigid, column-like mass is used as a pendulum. One end of the pendulum is mounted to a motoring device. The mounted end of the pendulum is granted a degree of freedom to rotate. If the pendulum is inverted (positioned in a standing position), any disturbance will ultimately tip it such that it falls.

One such system is depicted in Figure 1. In this simple case, the wheels of the cart allow it to move along a one-dimensional plane (a linear path).

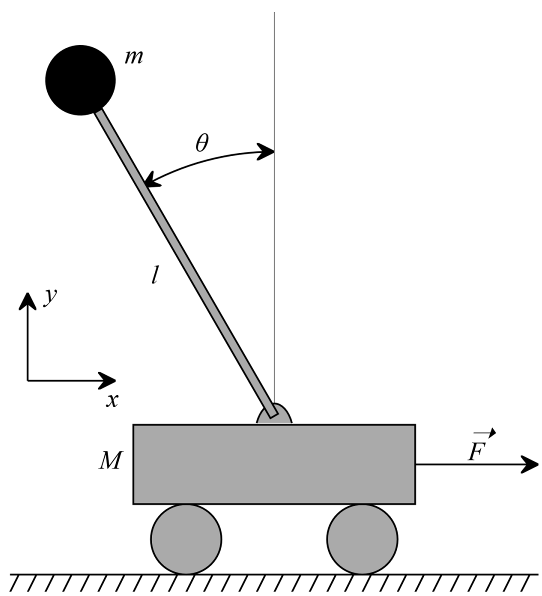

Figure 1: :

Inverted Pendulum on Cart] [Justification]: Inverted Pendulum on Cart[1]

[1]Source: "Inverted Pendulum." Wikipedia. Wikimedia Foundation, n.d. Web. 05 Feb. 2017.

The motoring device is used in such a setup to provide counterforces to the mounted end of the pendulum. These counterforces are intended to ultimately return the top of the pendulum to its inverted (standing) position.

For the actuator to successfully perform these actions, a controller (calculation device) is required. The controller, with the assistance of sensory data, is able to dynamically calculate (in real time) the exact forces needed to reestablish the positioning of the pendulum to a standing equilibrium. The controller then communicates the magnitude and direction of these forces to the motoring device which actuates the forces in the physical space. This in turn changes the state of the system, requiring that the controller continually recalculate the forces needed to return to equilibrium.

This problem may be further complicated by implementing trajectory control, in which the operator may command the device to move to one or more different locations. In such a scenario, the device must maintain its control of the balance of the inverted pendulum during and after moving.

2.2 Two-Wheeled Robot

The two-wheeled robot is a special case of the inverted pendulum model. In this case, the inverted pendulum model is reduced to only the pendulum and the wheels. The entirety of the robot hardware forms the pendulum, and the pendulum is coupled directly to the wheels.

One such device is depicted in Figure 2. In this case, the robot is being used in a medical application. The significance of the two-wheeled robot is not related to any one application; rather its ability to balance allows the added inclusion of top-heavy architectures in design options.

Figure 2: :

Two Wheeled Robot] [Justification]: Two Wheeled Robot[2]

The robot has two wheels, each of which is coupled to an individual motoring device (included in the robot hardware). The motoring devices are able to act independently; therefore, the device is capable of turning and moving across a two-dimensional plane. This transition from one shaft to two shafts creates additional complexity in the system which must be considered in the design of the controller.

[2]Source: Melanson, Donald. "Researchers Create Life-saving UBOT-5 Robot, Play Dress-up with It." Engadget. N.p., 14 July 2016. Web. 05 Feb. 2017.

2.3 Controller for a Two-Wheeled Robot

The control of a two-wheeled robot has been studied heavily in recent years. Several control approaches have been reviewed, as is depicted in Figure 3 [2].

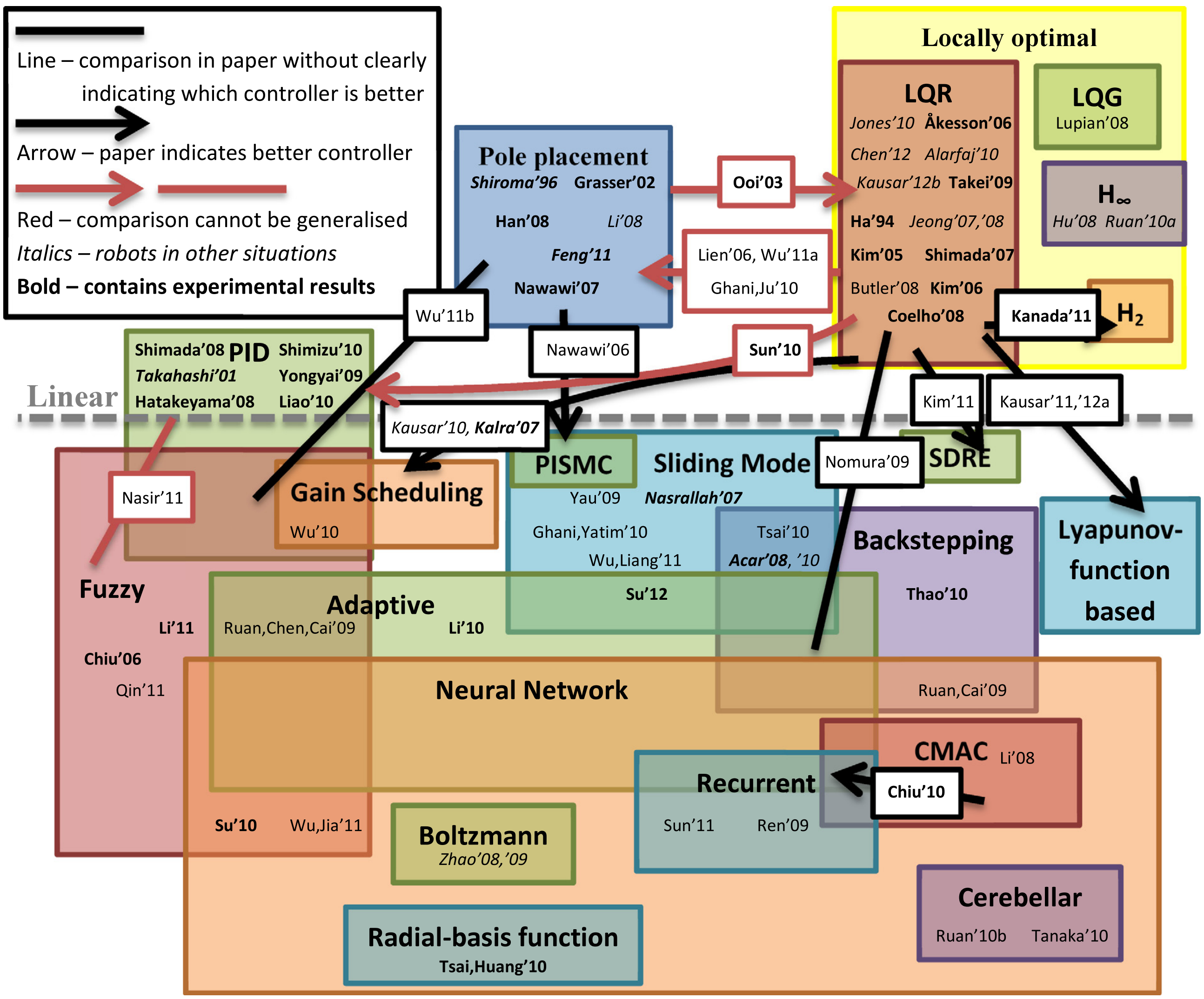

Figure 3: [Justification]: Control Methods Used for Two-Wheeled Robots

The top-half of the figure represents linear techniques whereas the bottom half of the figure represents non-linear techniques. It can be seen in the figure that in the category of linear control, pole-placement and optimization techniques have received considerable attention.

This study will focus on a high-precision control of the device. A study has already been performed on a comparison of different approaches for pole-placement controllers [3]. This study will focus on a comparison of different approaches for optimal controllers. Specifics are presented in Section 3.

Additionally, the study involving different approaches for pole-placement [3] did not fully address the digital tracking portion of its review. As such, this study will further review approaches to digital tracking, such as the inclusion of anti-windup capabilities and a command-shaping filter.

A comparison between simulated results and those implemented on the actual hardware will be made. This will involve a study of approaches to better emulate the system in a simulated enviro-ment.

3 Methodology or Procedures

- Hardware ultimately forming a two-wheeled robot will be selected and acquired.
- A physical model of the plant (two-wheeled robot hardware) will be developed in the Mathworks environment [1, 3, 4, 5, 6, 7].
  - The hardware will be characterized into measurable parameters.
  - The parameters will be populated into the physical model.
- High-precision controller algorithms will be developed in the Mathworks environment with the following control objectives [8, 9, 10, 6, 7]:
  - A multiple input, multiple output (MIMO), multiple state variable system will be used.
  - Closed-loop, optimal control approaches will be used.
  - The two wheel motors will be uncoupled with respect to the controller.
  - The robot hardware retains balance while stationary, even with slight disturbances.
  - The robot hardware retains balance while moving in accordance with operator commands.
  - The robot hardware does not incur continually increasing error during its operating time.
- A study of the best approaches to optimal controllers will be made, including:
  - A review of different approaches for choosing LQR matrices.
  - A comparison of the Vaccaro approach to robust observer design [11] with LQG, LTR, and use of differentiator/integrator feedback filters.
  - A study of the implementation of anti-windup functionalities.
  - A study of the implementation of a command-shaping filter [12].
- A simulation model of the complete system (plant and controller) will be developed in Simulink.

- Where practical, the simulated studies will be recreated on actual hardware.
- A comparison of the results between the simulated and actual systems will be made.
- A study of any approaches needed to most accurately simulate the actual system will be made.

4 Resources Required

Resource	**Status**
PC device	Personally owned
Mathworks software suite	Purchased
MinSeg robotic hardware	Purchased, (plus 1 redundancy)
MinSeg Mathworks drivers	Verified as functional
Cited references	Stored locally
IEEE Explore account, (for future references)	Available through Prof. Vaccaro

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