# RENSSELAER MECHATRONICS DC Motor Parameter Identification

# Part 1: Motor Steady state response

## **Objectives:**

• Perform different experiments on a DC motor to determine its motor parameters

## **Background Information:**

A DC motor can be modeled with an electrical system coupled to a mechanical system by a magnetic field. The equation for the electrical system is:

$$V = \frac{di}{dt} + Ri + K_b \omega$$

Where:

- $K_b$  back emf constant
- *R* armature resistance
- L inductance
- *i* the current through the motor windings

The coupling is seen from the voltage generated by the spinning motor  $K_b\omega$ . The equation for the electrical system is

$$K_t i = J \frac{d\omega}{dt} + \omega b$$

- $K_t$  torque constant
- *b* viscous damping coefficient
- *J* armature resistance (or combined motor and load Inertia if load is attached)
- $\omega$  the velocity of the motor shaft

In the steady-state (no change with time) the equations become:

$$V_{ss} = Ri_{ss} + K_b \omega_{ss}$$

$$K_t i_{ss} = b \omega_{ss}$$

These equations, with the steady state current and voltage measurements can be used to determine the motor parameters.

#### **Parameter Estimation Background**

Resistance: If there is access to the motor leads this can be measured with a multimeter.

#### **Torque and Back EMF constants:**

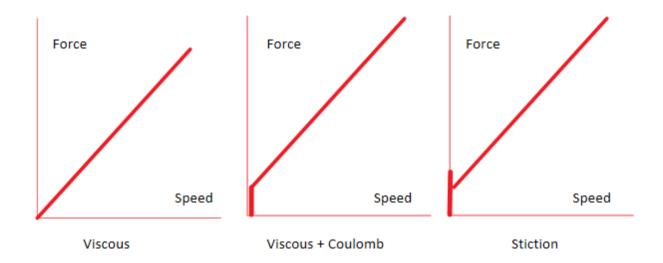
First assume that energy is conserved in the motor in which case  $K_b = K_t$  and only one needs to be estimated. From the steady-state electrical equation:

$$V_{ss} = Ri_{ss} + K_b \omega_{ss}$$

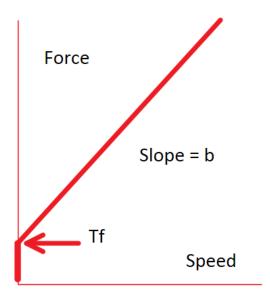
If resistance is known, speed is measured and current and voltage are measured these equations can be used to calculate  $K_b$  which, by assumption equal to  $K_t$ . For the Lego NXT motors, since they contain a gear train and have significant losses assume  $K_t$  is about 65% of  $K_b$ . This value can be obtained by an experiment where a known load is applied, and the current is measured.

#### Viscous and Coulomb Friction Estimation:

There are typically 3 different components of friction: viscous, coulomb, and static. This leads to three different types of friction models:

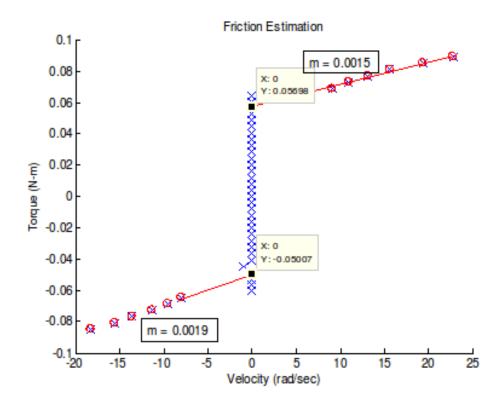


For a motor curves can be obtained by plotting steady state speed and measured torque  $K_t i$ .



Viscous + Coulomb

#### An experimental example is:



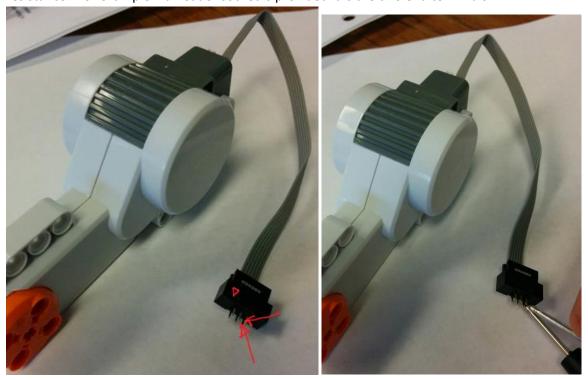
This will require steady state measurements of voltage and current at a couple different speeds. Once this graph is obtained then the viscous damping coefficient b is the slope of the line, and the constant coulomb friction is the y-intercept.

## **System Measurements**

#### Resistance:

First you can measure the motor resistance by using a multimeter and measuring the resistance between the two motor leads.

If an adapter cable is available you can disconnect the motor and us the adapters to measure the resistance. For example if a header socket is provided it is the two end terminals:



• If no tools or cables are provided you can make the measurement while the motor is connected.

First make sure your board is completely powered off – it is not connected to the USB cable, and

there are NO batteries installed. To do this find where you have access to the motor leads; shown below:

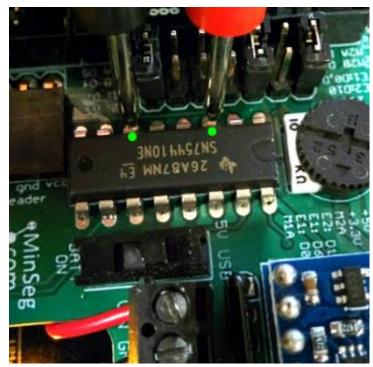




Figure 1: Motor wire connections on M1V4 board – resistance and voltage measurement from pins 3 and 6 of the motor driver.



Figure 2: Alternate location for resistance or voltage measurements

You need to measure the resistance between between the two motor leads; M1 and M2. You can do this from the last jumper (connected to M1 – one of the motor leads, also connected to the  $3^{rd}$  motor driver chip) and the  $6^{th}$  pin on the driver chip. Alternately you can simply measure

between pin 3 and pin 6 on the driver chip:

With the powered completely off (usb disconnected) and the motor connected this will allow measurement of the resistance.

Record the value of the motor resistance:

R=

#### **Driver Supply Voltage:**

Before you run tests you need the driver supply voltage (you specify this in the driver block). For the SN754410 this is the voltage supply minus 1.2 volts which are the losses from the driver. For the DRV8833 driver the loss is about .6 volts. You can measure the voltage supply by measuring the voltage at the 5v pin (if you are going to use the usb voltage), or from the power supply terminals (if you are going to use battery voltage). For the M1V4 board you can directly measure the pin from pin #8 of the driver:

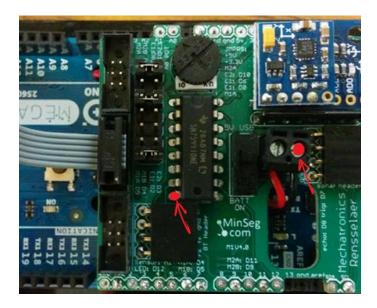


Figure 3: Measurement Locations for driver supply voltage

Take this measurement and subtract 1.2 for the SN754410 (shields) and subtract .6 for DRV8833 (MinSegMega). (You can directly determine the voltage drop by turning the motor fully ON and measuring the voltage across the motor terminals compared to the voltage supply shown in figure 3).

#### Speed, Voltage and current:

The current and voltage need to be measured at different speeds. The Simulink diagram below can be used in external mode at 0.03 seconds to record the steady state velocity. Make sure "send single Port0" is commented out when using external mode. This same diagram can be used later to capture the step response in normal mode.

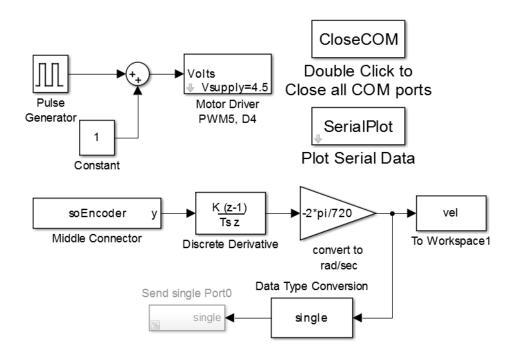


Figure 4: Simulink diagram for speed measurements

#### **Voltage Measurement:**

When the motor is spinning at steady state the voltage can be measured between the motor leads the same way the resistance was measured. from the last jumper that connects to pin #6 on the driver chip: There will be current flowing so be VERY careful where you place the multimeter leads. Alternatively you can measure between pin #6 and #3 on the driver chip.



Figure 5: Voltage Measurement location for M1V4

#### **Current Measurement:**

- First ensure the motor is spinning at a constant velocity
- Remove the last jumper (it will stop spinning)
- Then put the multimeter in current mode and connect the two leads that were connected with the jumper to the multimeter. The motor will start to spin as the multimeter completes the circuit and you can measure the current. There will be current flowing so be VERY careful where you place the multimeter leads so you do not short them.



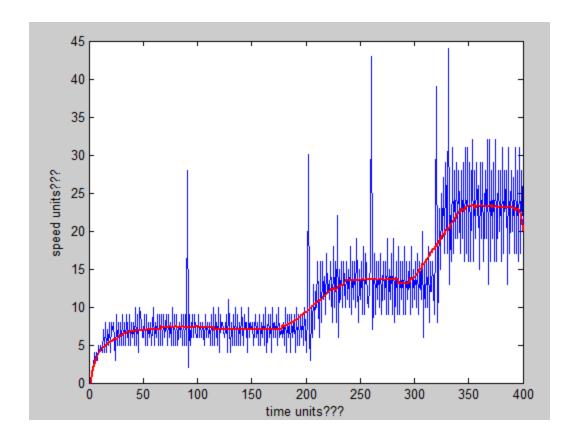
Figure 6: Current measurement with jumper removed.

#### Data Collection:

- Run the motor at 3 different steady state speeds, and record the speed data (1.5, 2, 3, and 3.5 volts would be good values to start with)
  - At the steady state speeds measure the velocity and current. This may require multiple tests. Record voltage and current at each speed.

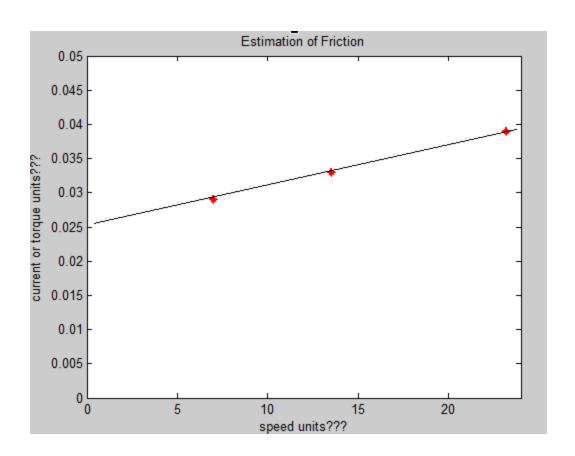
$$\omega_1 = V_1 = I_1 = 0$$
 $\omega_2 = V_2 = I_2 = 0$ 
 $\omega_3 = V_3 = 0$ 
 $\omega_3 = 0$ 
 $\omega_4 = 0$ 
 $\omega_5 = 0$ 
 $\omega_7 = 0$ 
 $\omega_8 = 0$ 
 $\omega_8 = 0$ 
 $\omega_8 = 0$ 
 $\omega_9 = 0$ 
 $\omega_9$ 

 You may need to smooth the velocity data to help determine the steady state speed if external mode is used:



#### Parameter Estimation:

- $\circ$  Use the collected data and the steady state equation to determine  $K_b$  from measured speed, current and voltage at the 3 different speeds:
  - $\bullet \quad V_{ss} = Ri_{ss} + K_b \omega_{ss}$
- Use the data to plot the speed vs. torque curve (with the correct units) to determine the coulomb friction value, and the damping coefficient. An example graph is shown below (units and data are not correct):



#### **Questions:**

- Create a plot of Current (milliamps, yaxis) versus speed (rad/sec. xasix) Clearly label the axes and provide a title for the graph
- Do the measured voltages match the voltage set in the Simulink diagram when you use the multimeter to measure the voltage?
- What are your experimental values for (with units!!):
  - o  $R, K_b, b, T_f, K_t$ ?

## **Part 2: Motor Transient Response**

From the mechanical and electrical DC motor equations the 2<sup>nd</sup> order transfer function for a DC motor is:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K_t}{LJs^2 + (JR + Lb)s + (Rb + K_tK_b)}$$

If the inductance is "small" it can be neglected (L=0) and the first order transfer function is obtained:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K_t}{IRs + (Rb + K_t K_h)}$$

Where the motor parameters are:

- $K_t$  torque constant
- K<sub>b</sub> − back emf constant
- b viscous damping coefficient
- R armature resistance
- *J* armature resistance (or combined motor and load inertia if load is attached)
- *L* inductance

The equation in time constant form is:

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{K_t/(Rb + K_tK_b)}{JR/(Rb + K_tK_b)s + 1} = \frac{K}{\tau s + 1}$$

- Time Constant:  $\tau = \frac{JR}{Rb + K_t K_b}$
- Steady State Gain:  $K = \frac{K_t}{Rb + K_t K_b}$

From this equation the time constant and steady state gain can be identified. Calculate K from the parameters you identified (include units):

• Steady State Gain: 
$$K = \frac{K_t}{Rb + K_t K_b} = \underline{\hspace{1cm}}$$
 (Units \_\_\_\_\_)

#### **Questions:**

• What is your calculated values for the steady state gain *K*?

#### **Parameter Estimation from step response**

Obtain the step response of the motor in normal mode. An example step response is shown below (you may already have this data from a previous lab). You can modify the previous Simulink diagram to write the data to the serial port and plot the step response.

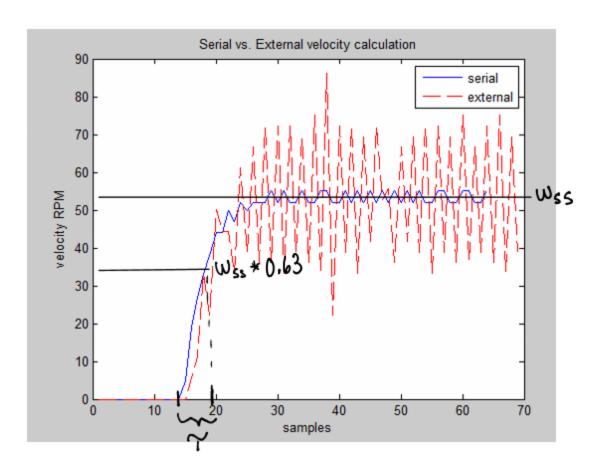


Figure 7: Sample Step response data obtained with 0.03 second sample time, 3 volt step

Take note of what voltage you used to generate this step response  $V_{in}$  (this should be the actual measured voltage):

$$V_{im} =$$

Calculate the experimental steady state gain (the step must be from zero speed as in the graph above):

$$K_{exp} = \omega_{ss}/V_{in} =$$

Where  $\omega_{ss}$  is the steady-steady state value from the step reponse graph, and  $V_{in}$  is the voltage used to generate the step response.

Next experimentally compute  $\tau$  by finding how long it takes for the step response to reach 63% of its steady state value:

$$\tau_{exp} =$$

The only unknown in the formula f	or the time constant $ au$ is the inertia	<ul> <li>Use the formula for the step</li> </ul>
response to calculate the inertia J:		

#### **Questions:**

- What is your experimental values for the time constant and steady state gain  $\tau$ , K?
- What is the inertia identified from the time constant?
- ullet What is the percent error from the calculated K versus the  $K_{exp}$  value from the step response?
- Kt can be determined by applying known loads and measuring the current. For this motor it is determined that Kt is approximately 65% of Kb. Why?
- If all the linear motor parameters are determined from separate experiments, the computed time constant for the resulting linear model is approximately  $\tau = .042$ . How does your value for the time constant compare to this why?