Note that we can calculate K and L <u>separately</u> and then put things together to get an OBR. The use of an observer does not change the locations of zpoles. This is called the <u>separation principle</u>

⋮

K = place(phi, gamma, zpoles)

⋮

L = place(phi', C', zopoles)'

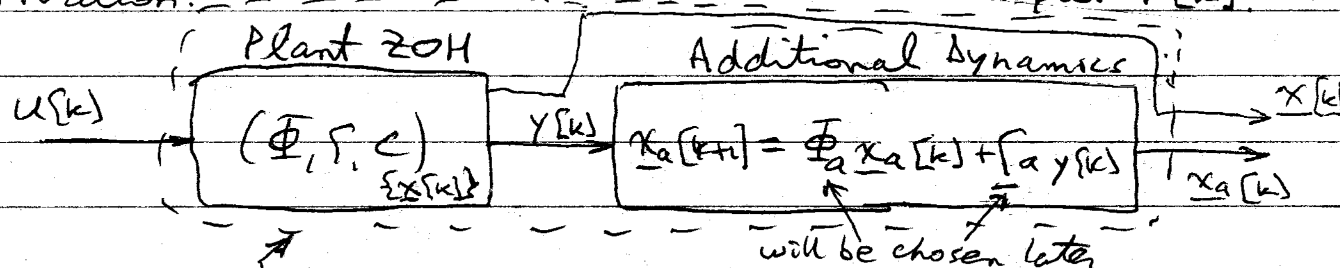
In this course you will learn why the separation principle is useful <u>only</u> for SISO systems. It is dangerous to use with MIMO systems. Most books do not tell you this! For single-input, multiple-output plant we use L = obg_reg(phi, gamma, C, K, ~~zopoles~~, T)

end 11/13

<u>Digital State-Feedback Tracking Systems for SISO Plants</u>

Want plant output $y[k]$ to track a ref. input $r[k]$.

Derivation:



Design Model, $\underline{x}_d[k] = \begin{bmatrix} \underline{x}[k] \\ \underline{x}_a[k] \end{bmatrix}$

$\underset{q \times q}{\Phi_a}, \quad \underset{q \times 1}{\Gamma_a}$

Write state-space description of the design model:

$$\underline{x}[k+1] = \Phi \underline{x}[k] + \Gamma u[k]$$
$$\underline{x}_a[k+1] = \Phi_a \underline{x}_a[k] + \Gamma_a C \underline{x}[k]$$

$$\underbrace{\begin{bmatrix} \underline{x}[k+1] \\ \underline{x}_a[k+1] \end{bmatrix}}_{\underline{x}_d[k+1]} = \underbrace{\begin{bmatrix} \Phi & 0 \\ \Gamma_a C & \Phi_a \end{bmatrix}}_{\substack{\Phi_d \\ (n+q)\times(n+q)}} \underbrace{\begin{bmatrix} \underline{x}[k] \\ \underline{x}_a[k] \end{bmatrix}}_{\underline{x}_d[k]} + \underbrace{\begin{bmatrix} \Gamma \\ 0 \end{bmatrix}}_{\substack{\Gamma_d \\ (n+q)\times 1}} u[k]$$

Choose $n+q$ pole locations and calculate $\underset{1\times(n+q)}{K_d}$ to regulate the design model *

```
>> spoles = choose n+q CL poles
>> zpoles = exp(T*spoles)
>> Kd = place (phid, gammad, zpoles)
```

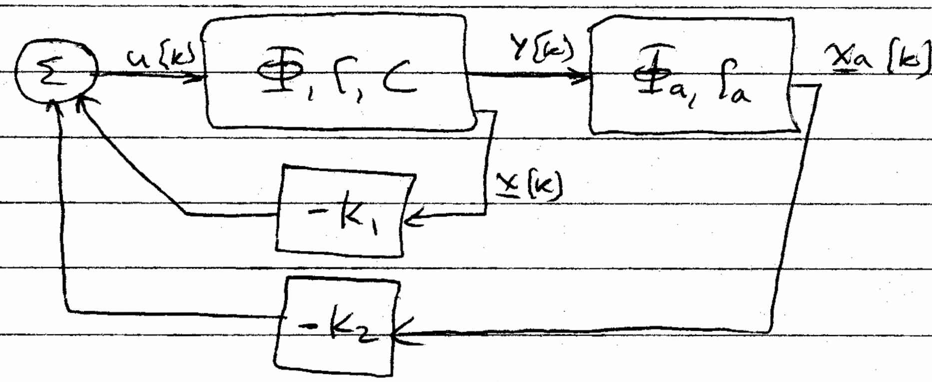
The regulator loop is closed as follows:

$$u[k] = -K_d \underline{x}_d[k] = \begin{bmatrix} \underbrace{-K_1}_{1\times n} & \underbrace{-K_2}_{1\times q} \end{bmatrix} \begin{bmatrix} \underline{x}_1[k] \\ \underline{x}_2[k] \end{bmatrix}$$
$$= -K_1 \underline{x}_1[k] - K_2 \underline{x}_2[k]$$

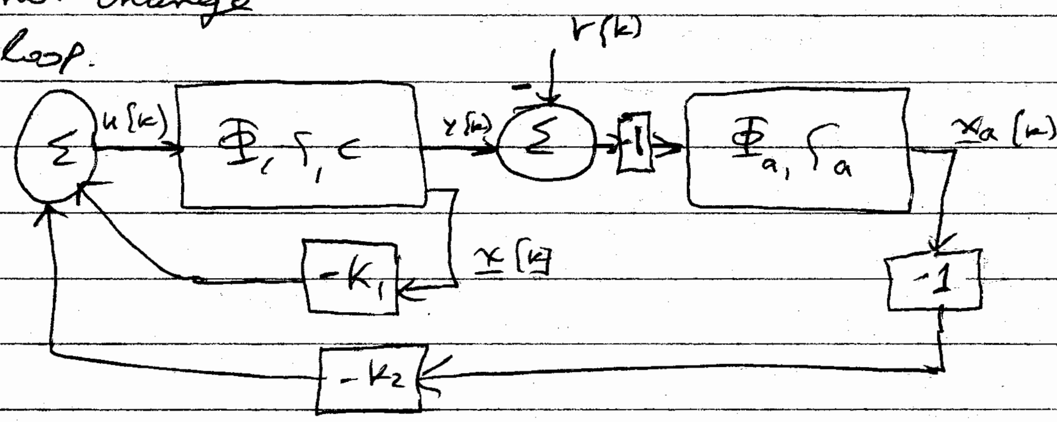
* In the rule-of-thumb for selecting the sampling interval, use $n+q$ (the order of the design model) in place of n.

Block diagram of regulated design model:

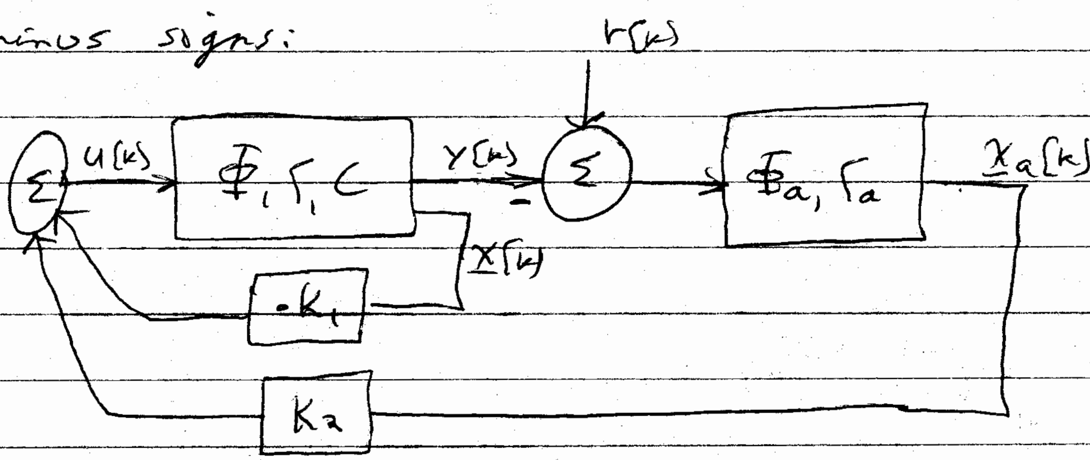
This is a stable CL system with poles specified by zpoles.



Insert a reference input. This does not change the CL pole locations. Insert two minus signs. Does not change the feedback loop.

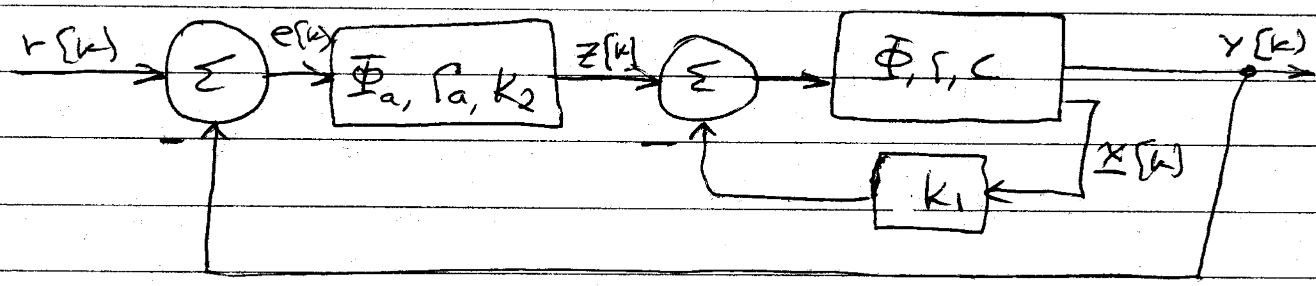


Combine minus signs:



Note that $k_2 \underline{x}_a$ is an "output" equation on the (Φ_a, Γ_a) state-space model.

Rotate the diagram counterclockwise to get $r[k]$ on the left and combine K_2 with (Φ_a, Γ_a):



Q: What are the poles of this tracking system?

A: The poles chosen to regulate the design model, resulting in the gains K_1 and K_2.

Q: For a step input $r[k]=1$, what condition must be met to guarantee that $\lim_{k\to\infty} y[k] = 1$?

A: Recall HW 5 Prob 5

The system from $e[k]$ to $y[k]$ must have (at least) one pole at $z=1$.

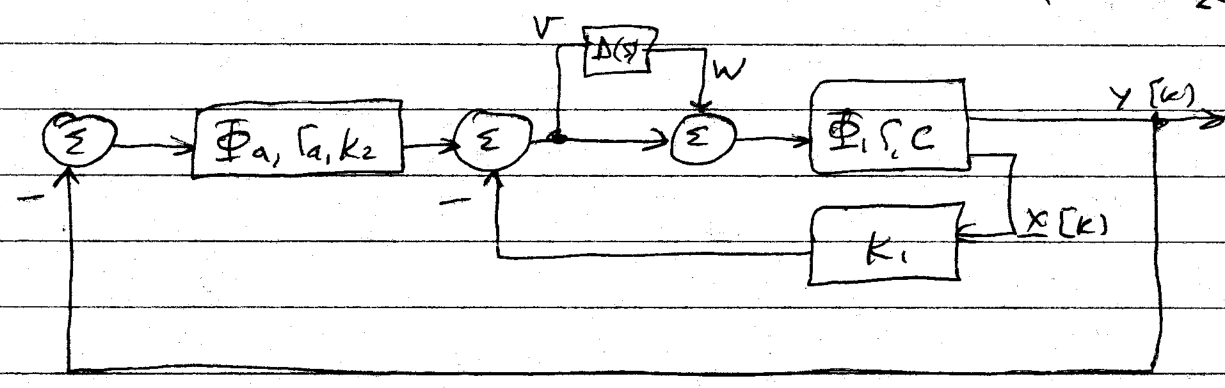
The system from $z[k]$ to $y[k]$ is stable and thus does not have a pole at $z=1$ (all of its poles are inside the unit circle).

The system from $e[k]$ to $z[k]$ will have a pole at $z=1$ if we choose $\Phi_a = 1$. The only condition on Γ_a is that (Φ_a, Γ_a) must be controllable, which implies that Γ_a is nonzero. The simplest choice is $\Gamma_a = 1$. With Φ_a and Γ_a specified, we can form the design model, choose $n+1$ closed-loop pole locations, and calculate the feedback gain vector for $K_d = [\underset{1\times n}{k_1} \quad \underset{1}{k_2}]$

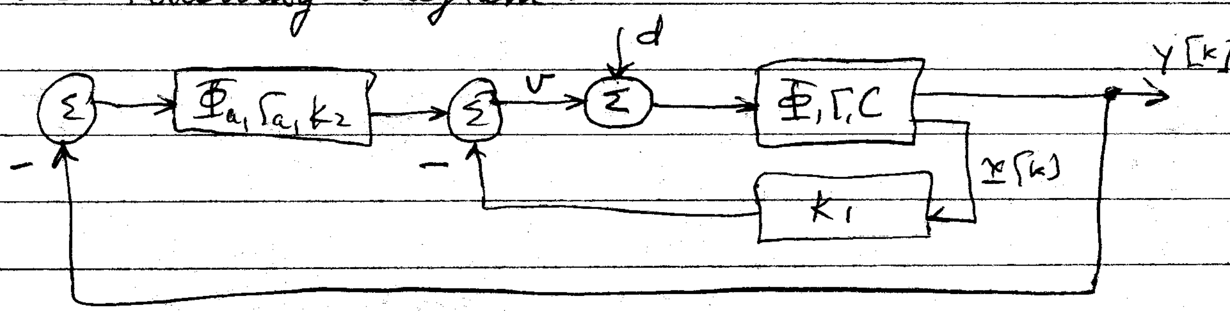
Note that (Φ_a, Γ_a, k_2) is a digital integrator with integral gain k_2.

Q: How do you find the input-multiplicative stability robustness bound for this tracking system?

A: Insert an $I+\Delta$ perturbation on the plant input, find the system from w to v, and take the reciprocal of the system infinity norm. (Set $r[k]$ to zero)

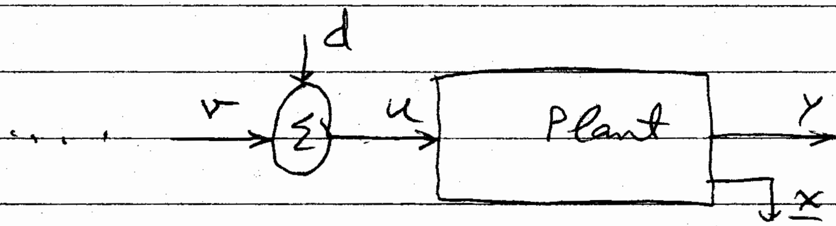


Note: to compute the robustness bound, the block containing $\Delta(s)$ does not have to be drawn. Consider the following diagram:



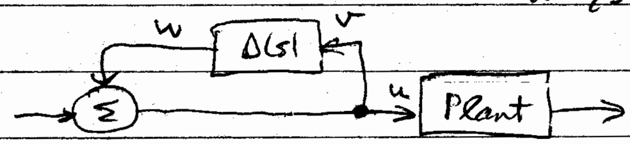
The system from d to v is the same as the system from w to v in the previous picture.

Conclusion: to find the stability robustness norms at the plant input, simply insert a summing junction at the plant input with a "disturbance" signal d:



Input-multiplicative bound: $1/\|\text{System from } d \text{ to } v\|_\infty$

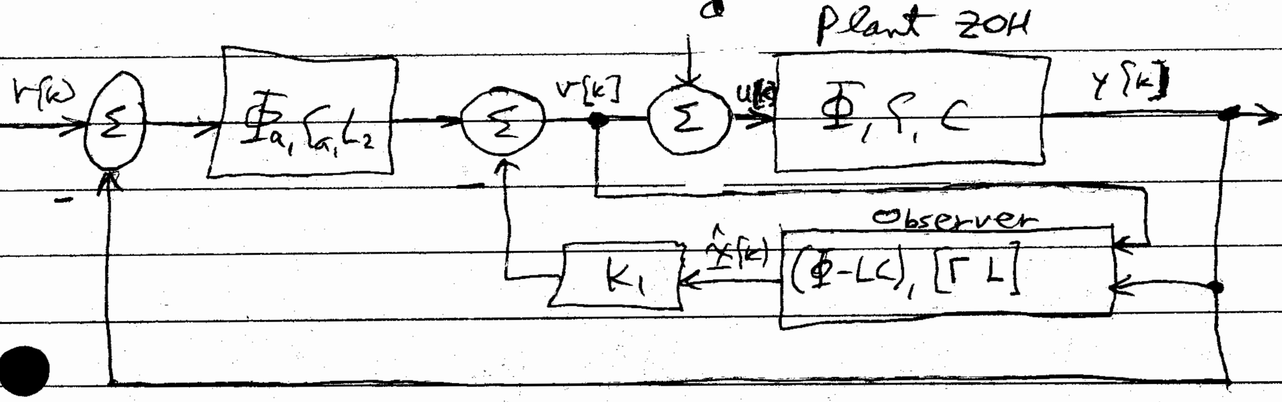
Input-feedback bound: $1/\|\text{System from } d \text{ to } u\|_\infty$



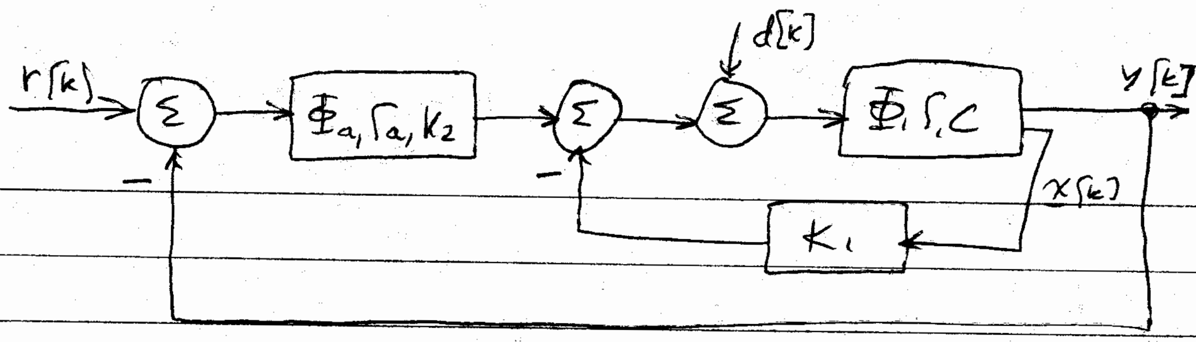
Insert pp 77-78 here

SISO Observer-Based Tracking System

Use an observer connected to plant input and output to produce estimate $\hat{x}[k]$ of plant state vector. Use $\hat{x}[k]$ in place of measured state feedback.



Go to pg 79.



Now suppose d is an actual disturbance signal. For example, if $d[k]$ is a constant disturbance, what effect does this have on the steady-state value of y?

We know that if $r[k] = c_1$ (step signal of amplitude c_1) and $d[k]=0$, $\lim_{k\to\infty} y[k] = c_1$ (zero steady-state error because the forward path contains an eigenvalue at $z=1$).

Consider now $r[k] = c_1$ and $d[k] = c_2$.
What is $\lim_{k\to\infty} y[k] = y_s$?

The control system now has two inputs, $r[k]$ and $d[k]$. Thus, $y_s = y_s^r + y_s^d$, where

y_s^r is the steady-state output due to r alone
and y_s^d " " " " " " " d alone

We know that $y_s^r = c_1$. Thus $y_s = c_1 + \underbrace{y_s^d}_{\text{we would like this number to be zero}}$

Write the system from $d[k]$ to $y[k]$:

$$\underline{x}[k+1] = \Phi\,\underline{x}[k] + \Gamma\left(d[k] + k_2 x_a[k] - K_1 \underline{x}[k]\right)$$

$$x_a[k+1] = \Phi_a x_a(k) - \Gamma_a C\,\underline{x}[k]$$

$$\begin{bmatrix} \underline{x}[k+1] \\ x_a[k+1] \end{bmatrix} = \begin{bmatrix} \Phi - \Gamma K_1 & \Gamma k_2 \\ -\Gamma_a C & \Phi_a \end{bmatrix} \begin{bmatrix} \underline{x}[k] \\ x_a[k] \end{bmatrix} + \begin{bmatrix} \Gamma \\ \underline{0} \end{bmatrix} d[k]$$

$$y[k] = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \underline{x}[k] \\ x_a[k] \end{bmatrix}$$

In the limit as $k \to \infty$, all functions of k become constant:

$$\begin{bmatrix} \underline{x}_s \\ x_a^s \end{bmatrix} = \begin{bmatrix} \Phi - \Gamma K_1 & \Gamma K_2 \\ -\Gamma_a C & \Phi_a \end{bmatrix} \begin{bmatrix} \underline{x}_s \\ x_a^s \end{bmatrix} + \begin{bmatrix} \Gamma \\ \underline{0} \end{bmatrix} c_2$$

$$y_s^d = C x_s$$

write equation for x_a^s; recall $\Phi_a = 1$, $\Gamma_a = 1$:

$$x_a^s = -C x_s + x_a^s$$

$$\cancel{x_a^s} = -y_s^d + \cancel{x_a^s} \Rightarrow y_s^d = 0 \; !$$

Thus, a tracking system designed to track a constant reference input will reject a constant disturbance at the plant input.

Go to bottom of page 76 for observer-based tracking system.

Rules for Choosing Observer Poles

1. For an observer-based regulator with desired settling time T_s, use Bessel poles scaled to T_{so}, where $T_{so} = T_s/f$ and $3 \le f \le 5$.

2. For an observer-based tracking system with desired settling time T_s, calculate the zeros of the analog plant (A, B, C) using >> tzero(A, B, C, 0). Consider using "slow, stable" plant zeros as observer poles. Use Bessel scaled by T_{so} for the remaining observer poles.

"stable zeros" have negative real parts
"slow zeros" are to the right of $5.1/T_s$.