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# Research on Control Strategy of Two-wheeled Self-balancing Robot

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***Abstract*—The structure of two-wheeled self-balancing robot is equivalent to the combination of the linear inverted pendulum and wheeled mobile robot. But compared to the linear inverted pendulum, two-wheeled self-balancing robot can move freely and turn flexibly. So the system is more complex and difficult to control. As the two-wheeled self-balancing robot is a high order and multi variable system, PID controller based on output feedback can't have a satisfactory control effect. The LQR control strategy is implemented, and the weight matrix Q and R of the LQR controller are optimized by using multi-population genetic ideas. Taking two wheeled self balancing robot as the test platform, the two control modes of LQR and PID are used to carry out experiments and analysis. In the simulation experiments of balance control, position control and speed control, the LQR controller has a better performance in the system's robustness and fast response.**

***Keywords-two-wheeled self-balancing robot; PID; LQR; Optimization of weighted matrix***

## I. INTRODUCTION

Two-wheeled self-balancing robot is a nonlinear and unstable system, which has aroused a lot of researchers' attention. The kinematics analysis and dynamic modeling are the basis of the research on the two wheeled self-balancing robot, and the key problem is the realization of real-time control. According to the mechanical characteristics of the two wheeled self-balancing robot, the mathematical model of the controlled system is established. The kinematic and dynamic characteristics of the robot are analyzed, and the theoretical basis for the design of the control system is obtained[1].

Self-balancing is the premise of robot standing, while standing is a necessary condition for robot motion. How to keep the balance and resist the external disturbance is the key point of the research of the two-wheeled self-balancing robot. Self-balancing algorithms include traditional, modern and intelligent control algorithms. The modern control theory is mainly focused on the multi-variable and strong-coupled nonlinear system which can not be solved by using the traditional control theory. It is based on space state equation and realized by using the theory of LQR control. PID control algorithm in the classical control theory and LQR control algorithm in the modern control theory are tested in the two-wheeled self-balancing robot. The optimization of weight matrix Q and R in LQR controller is carried out by using the idea of multi-population genetic algorithm. Simulation experiments are carried out on the aspects of balance control, position control and speed control using PID and LQR.

## II. MATHEMATICAL MODEL

Although the dynamic performance being more complex, it's similar to the vehicle mounted on the inverted pendulum system. The wheel and the pendulum are analyzed separately. It's easy to get the nonlinear equation of the two wheeled self balancing robot, after linearization to get state equation[2,3].

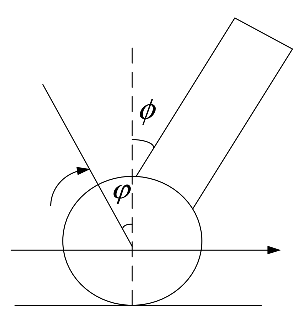


Figure 1. Dynamic model.

The nonlinear equations of the system are obtained by the Newton model.

$$\begin{cases} (M_S + 2M_L + \frac{2I_W}{r^2})\ddot{x} = \frac{C_R + C_L}{r} \\ -2M_S l \cos\phi\ddot{\phi} \\ M_S l\dot{x} + (I_S + M_S l^2 \cos\phi + M_S l^2)\ddot{\phi} = \\ M_S l^2 \sin\phi\dot{\phi}^2 + M_S gl\sin\phi \end{cases} \quad (1)$$

Assume that $X = \begin{bmatrix} x & \dot{x} & \phi & \dot{\phi} \end{bmatrix}^T$, hence nonlinear model can be obtained.

The nonlinear equations are linearized in the range of 5 degrees:

$$\dot{\phi}^2 \approx 0, \sin\phi \approx \phi, \cos\phi \approx 1 \quad (2)$$

So the linearized equation can be obtained after finishing the linearization.

$$\begin{aligned} A_1 &= -2M_S{}^2 l^2 R^2 g \\ A_2 &= (2M_S l^2 + I_S)(M_S r^2 + 2M_L r^2 \\ &+2I_L) - 2M_S{}^2 l^2 r^2 \\ A_3 &= M_S gl(M_S r^2 + 2M_L r^2 + 2I_L) \\ B_1 &= 2M_S l^2 r + I_S r \\ B_2 &= M_S lr \end{aligned} \quad (3)$$

Then the system's state equation can be obtained:

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{A_1}{A_2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{A_3}{A_2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{B_1}{A_2} & \frac{B_1}{A_2} \\ 0 & 0 \\ -\frac{B_2}{A_2} & -\frac{B_2}{A_2} \end{bmatrix} \begin{bmatrix} C_R \\ C_L \end{bmatrix} \quad (4)$$

$$y = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix}$$

# III. CONTROL STRATEGY

## A. PID Control Strategy for Two-wheeled Self-balancing Robot

PID is a classic way of automatic control theory and the general trial control mode. Two-wheeled self-balancing robot can be controlled by PID controller[4]. Two-wheeled self-balancing robot's key problem is the control of balance and motion, which includes two aspects. First, two-wheeled self-balancing robot should always maintain the status of the vertical. Second, while maintaining a two wheeled robot, it can move at the same time. As shown in Figure 2, it is the control loop of the two-wheeled self-balancing robot.

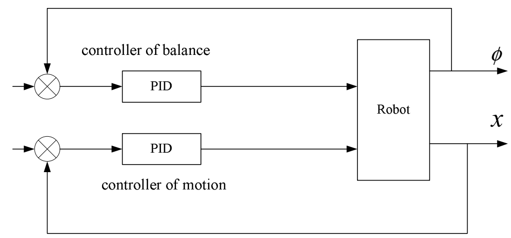


Figure 2. Motion balance control loop.

## B. LQR Control Strategy for Two-wheeled Self-balancing Robot

### 1) LQR controller

LQR is linear quadratic regulator, and the object is the linear system which is given by the state space form in the modern control theory[5]. The objective function is a quadratic function of the object state and control input. LQR optimal design is for equation of $\dot{x} = Ax + Bu$, through the determination of optimal control of $K$ matrix of $u(t) = -Kx(t)$, to achieve minimum control performance index of $J = \int_0^\infty (x^T Qx + u^T Ru)dt$. The real symmetric matrices Q and R represent the relative importance of each state tracking error and energy consumption. Each element of the diagonal matrix in the Q represents the relative importance of each index error.

Gain based on LQR is $u(t) = -Kx(t)$, and the control law is $K = LQR(A, B, Q, R)$. LQR controller is used to control the robot. Its purpose is that if the system state deviates from the equilibrium state, it is still able to maintain the system to be close to the equilibrium state, so that the system has good robustness, and the LQR optimal control can make the system achieve better performance[6,7].

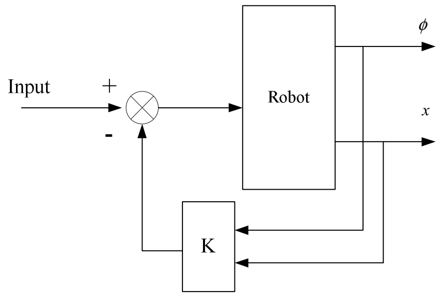


Figure 3. LQR control loop

*2) Optimization of weighting matrix*

When the weight matrix Q and R of the quadratic performance index values are determined, the optimal feedback K can be obtained by solving the Riccati equation. In the case of multi variable and multi constraints, the selection of the appropriate Q and R to make the system with optimal stability is relatively complicated.

Genetic algorithm is usually used to determine the weighting matrix. Genetic algorithm is a highly parallel, random and adaptive global optimization probability search algorithm based on natural selection and evolutionary mechanism. Traditional genetic algorithm has two major drawbacks: first, it is easy to be premature that the convergence ends ahead of time and it's trapped in local optimal solution; second, the late evolutionary search efficiency is low, which brings about that the final result is often not the global optimal solution. To avoid the above situation, based on multiple population genetic theory, a variety of groups are introduced into genetic manipulation. The genetic algorithm is carried out in a number of sub populations with different control parameters, and the information is exchanged among the sub populations by the immigration operator. The immigration operator is used to bring the best individual in each population to other populations, and each population uses different control parameters. By means of the artificial selection operator, the best individual in each group is preserved, and the parallel operation can improve the efficiency of the algorithm[8]. The main steps are as follows:

Step 1: Randomly generating N populations, each individual in each population are assigned to the weighted coefficient of the LQR controller, and the optimal control feedback gain matrix and the control signal are obtained.

Step 2: The criterion of termination of the algorithm is based on the minimum of the best individual in the elite population. The fitness function value of each individual in each population is calculated, and the conditions for the termination of the algorithm are judged. If the condition is met, exit the algorithm and the optimal individual is obtained; if not, then go to step 3.

Step 3: Based on the evolutionary mechanism of traditional standard genetic algorithm, a new population is generated by roulette wheel selection, single point crossover and site variation. Each specie chooses different crossover probability and mutation probability, which brings about that the multi populations co-evolve and simultaneously improve the global search and local search ability of the algorithm.

Step 4: By means of immigration operator, the worst individual in the target population is replaced by the best individual of the source population, and the exchange of the information between populations is realized.

Step 5: The best individual of each generation of the other populations is selected by the artificial selection operator. The best individual is put into the elite population and transferred to Step 2.

In order to improve the efficiency and performance of LQR design, the best two weighted Q and R are determined by the idea of the algorithm.

## IV. SYSTEM SIMULATION

On the platform of the two wheeled self balancing robot, the simulation experiments of PID control and LQR control are carried out respectively.

*A. Simulation Experiment of PID Control Strategy*

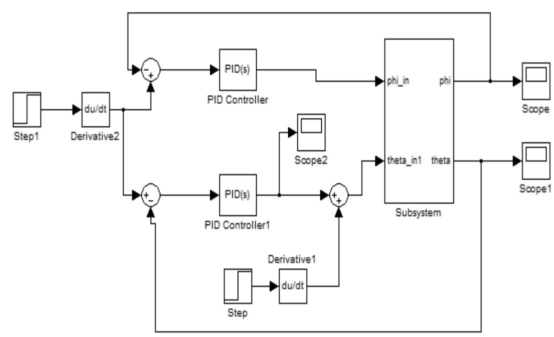


Figure 4. PID control simulation model

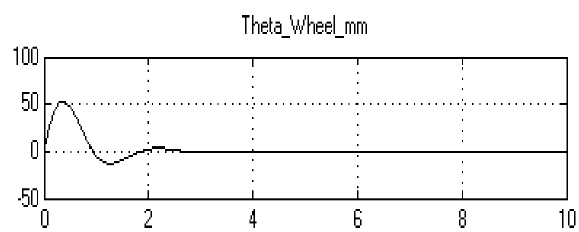


Figure 5. Position control based on PID

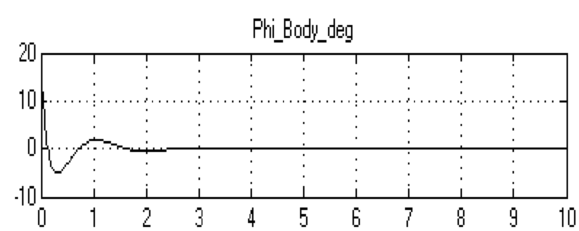


Figure 6. Body tilt angle based on PID

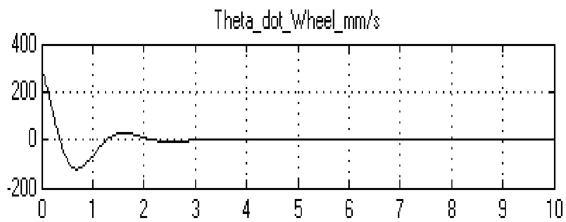


Figure 7. Speed control based on PID

From three aspects: position control, control of body angle and speed control, PID control algorithm can achieve the balance control and motion control.

### B. Simulation Experiment of LQR Control Strategy

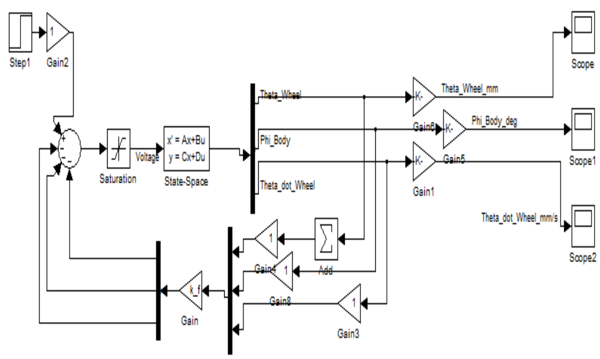


Figure 8. LQR control simulation model

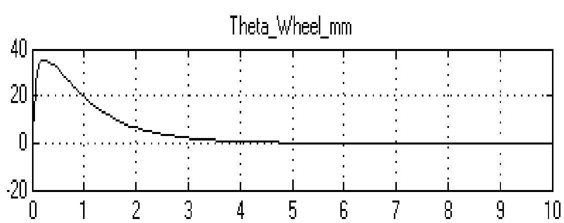


Figure 9. Position control based on LQR

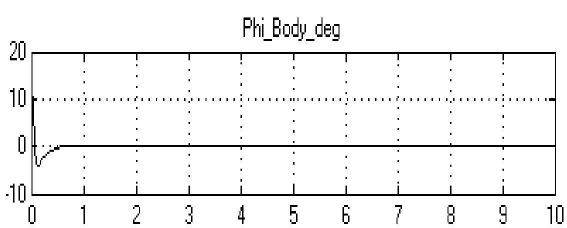


Figure 10. Body tilt angle based on LQR

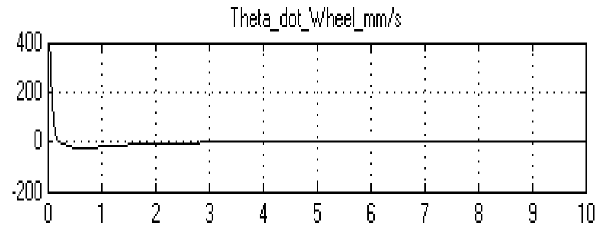


Figure 11. Speed control based on LQR

It is not difficult to find that the LQR controller can effectively control the two-wheeled self-balancing robot. Compared to the PID controller, the overshoot of the LQR controller is smaller and the response speed is faster. For position control, body angle and speed control, it has better performance.

## V. CONCLUSION

In this paper, the dynamic model of two-wheeled self-balancing robot is presented. Based on the mathematical model, the classical PID control strategy and LQR control strategy are simulated respectively. The results show that the LQR control strategy can achieve better performance in robustness and fast response. But the simulation can not reflect the actual situation completely. Therefore, the actual verification based on of the control strategy is the next important problem.

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