0.0.1 Tracking System

Additional dynamics may be incorporated to improve reference tracking. When implemented for this purpose, the additional dynamics are known as a tracking system.

In the case of a tracking system, an *integrator* may be implemented as the additional dynamics to track a constant reference exactly, or to track a slowly varying reference approximately.

Integrators are also able to mitigate constant disturbances. Incidentally, the MinSeg M2V3 system uses gyroscopes as body angular velocity ψ sensors. Bias is inherent in the output of a gyroscope; therefore, the use of such an integrator as a tracking system has an additional benefit: it will mitigate the effects of bias from a gyroscope output, whether directly or within terms which are derivative of the gyroscope output.

Thus, in the case of the two-wheeled robot, integrators are implemented as additional dynamics for the states representing wheel angular position θ and body angular position (yaw) ϕ_y. This establishes a tracking system, [*an augmented method of state feedback regulation*], for the system. The state-space representation of the integrator is exhibited in Equation (0.1).

$$\begin{aligned}
\underset{nx1}{\dot{\mathbf{x}}(t)} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \underset{nx1}{\mathbf{x}(t)} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} e_\theta \ (t) \\ e_{\phi_y}(t) \end{bmatrix} \\
\underset{mx1}{\mathbf{y}(t)} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \underset{nx1}{\mathbf{x}(t)} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} e_\theta \ (t) \\ e_{\phi_y}(t) \end{bmatrix}
\end{aligned} \tag{0.1}$$

0.0.1.1 Discrete Additional Dynamics

Since the additional dynamics will be processed on a microcontroller, the additional dynamics will be digital; thus, a continuous-to-discrete conversion will be necessary. A digital integrator is an established case which is exhibited in Equation (0.2).

$$
\begin{aligned}
\underset{nx1}{\dot{\mathbf{x}}[k+1]} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \underset{nx1}{\mathbf{x}[k]} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} e_{\theta}\ [k] \\ e_{\phi_y}[k] \end{bmatrix} \\
\underset{mx1}{\mathbf{y}[k]} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \underset{nx1}{\mathbf{x}[k]} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} e_{\theta}\ [k] \\ e_{\phi_y}[k] \end{bmatrix}
\end{aligned}
\tag{0.2}
$$