0.1 Nonlinear model

The dynamic motion equations of the two-wheeled robot are derived using the Lagrangian method. The equations are based on the coordinate system provided in Figure **??**.

0.1.1 Coordinate System

The coordinate system is explicitly defined in Equations (0.1) - (0.6).

$$\begin{bmatrix} \theta_{g.l} \\ \theta_{g.r} \\ \theta_{g.av} \\ \phi_y \end{bmatrix} = \begin{bmatrix} & & \theta_{b.l} & + & \phi_x \\ & & \theta_{b.r} & + & \phi_x \\ \frac{1}{2} & \cdot & (\theta_{g.l} & + & \theta_{g.r}) \\ \frac{r_w}{l_{b.w}} & \cdot & (\theta_{g.r} & - & \theta_{g.l}) \end{bmatrix} \tag{0.1}$$

$$\begin{bmatrix} \dot{p}_{w.x} \\ \dot{p}_{w.y} \\ \dot{p}_{w.z} \end{bmatrix} = \begin{bmatrix} r_w \cdot \dot{\theta}_{g.av} \cdot \cos(\phi_y) \\ r_w \cdot \dot{\theta}_{g.av} \cdot \sin(\phi_y) \\ 0 \end{bmatrix} \tag{0.2}$$

$$\begin{bmatrix} p_{w.x} \\ p_{w.y} \\ p_{w.z} \end{bmatrix} = \begin{bmatrix} \int \dot{p}_{w.x} \cdot \mathrm{d}t & + & p_{w.x}(0) \\ \int \dot{p}_{w.y} \cdot \mathrm{d}t & + & p_{w.y}(0) \\ \int \dot{p}_{w.z} \cdot \mathrm{d}t & + & p_{w.z}(0) \end{bmatrix} \tag{0.3}$$

$$\begin{bmatrix} p_{wl.x} \\ p_{wl.y} \\ p_{wl.z} \end{bmatrix} = \begin{bmatrix} p_{w.x} & - & \frac{l_{b.w}}{2} \cdot \sin(\phi_y) \\ p_{w.x} & + & \frac{l_{b.w}}{2} \cdot \cos(\phi_y) \\ & p_{w.z} & \end{bmatrix} \qquad \begin{bmatrix} p_{wr.x} \\ p_{wr.y} \\ p_{wr.z} \end{bmatrix} = \begin{bmatrix} p_{w.x} & + & \frac{l_{b.w}}{2} \cdot \sin(\phi_y) \\ p_{w.x} & - & \frac{l_{b.w}}{2} \cdot \cos(\phi_y) \\ & p_{w.z} & \end{bmatrix} \tag{0.4}$$

$$\begin{bmatrix} p_{b.x} \\ p_{b.y} \\ p_{b.z} \end{bmatrix} = \begin{bmatrix} p_{w.x} & + & l_{b.c2a} \cdot \sin(\phi_x) \cdot \cos(\phi_y) \\ p_{w.y} & + & l_{b.c2a} \cdot \sin(\phi_x) \cdot \sin(\phi_y) \\ p_{w.z} & + & l_{b.c2a} \cdot \cos(\phi_x) \end{bmatrix} \tag{0.5}$$

Typically, initial conditions are assumed to be as follows:

$$\begin{bmatrix} p_{w.x}(0) \\ p_{w.y}(0) \\ p_{w.z}(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ r_w \end{bmatrix} \tag{0.6}$$