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2.2.2.1. Designated PC

A 2015 Macbook Pro PC was selected as the designated development PC, as this was available to the researcher without the need to request additional funding.

2.2.2.2. Designated Operating System

macOS was selected as the designated operating system, as this was the only operating system installed on the designated PC. (*Version 10.12.5 was the most up to date version at the time of research.*)

Alternative Operating System Compatibility

Although the macOS operating system was used, alternative operating systems (*Windows and/or Linux*) would be equally acceptable.

Such a transition would primarily require an alternative Mathworks-supported compiler [18] which would be compatible with the new operating system. Slight alterations to the method of determining the test platform serial communication channelwould also be required.

It is not expected that such a transition would be preventatively difficult.

prohibatively

2.3. Selection of a Hardware Model

The physical plant model developed in Yamamoto [1] was used, due to:

- Use of state variables involving:
  - Body pitch angle α
  - Body yaw angle Ψ
  - Wheel angle θ
- Existing familiarity of the work by the advising professor.
- Existing knowledge of methods to measure nonintuitive model parameters by the advising professor.

The physical plant model is discussed in greater detail in ~~Section~~ ??.

Chapter 4.

Table 4.1.: [Simulink]: Root

Symbol	Definition		Unit
θ	Angular position: Wheel	$[\theta = \theta_{g,av}]$ [Measured from the wheel center of mass]	rad
θ_g, θ_b	Origin aligns with:	[global rejection vector ($orthogonal\ from\ earth's\ surface$)] [body pitch ϕ_x]	
$\theta_{av}, \theta_l, \theta_r$	Component:	[average of left and right wheels] [left wheel] [right wheel]	
ϕ	Angular position: Body	[Measured from the wheels center of mass.]	rad
ϕ_x, ϕ_y, ϕ_z	Dimension:	[X] [Y] [Z]	
p	Translational position	$[p = p_w]$ [Measured from the corresponding center of mass]	m
p_x, p_y, p_z	Dimension:	[X] [Y] [Z]	
p_w, p_{wl}, p_{wr}, p_b	Component:	[both wheels] [left wheel] [right wheel] [body]	
v_{mtr}	Voltage: Motor Input		V
$v_{mtr.l}, v_{mtr.r}$	Component:	[left-wheel] [right-wheel]	

Note: When a subscript is unspecified, assume the first option is used by default.

Table 4.2.: [Simulink]: Root

Symbol	Definition	Value	Unit	Source
a_g	Acceleration of gravity: Earth	9.81	$\frac{m}{s^2}$	-
m_w	Mass: Wheel [*Includes wheel axle.*]	0.018	kg	[5]
m_b	Mass: Body	0.381	kg	[5]
$l_{b.h}$	Length: Body: Height	??	m	-
$l_{b.w}$	Length: Body: Width	??	m	-
$l_{b.d}$	Length: Body: Depth	??	m	-
$l_{b.c2a}$	Length: Body: [Center of mass] to [Axis of Rotation]	??	m	Sec. 4.4.3
r_w	Length: Wheel: Radius	0.021	m	[5]
$J_{b.\phi_x}$	Moment of Inertia: Wheel	$7.40 \cdot 10^{-6}$	$kg \cdot m^2$	[5]
$J_{b.\phi_x}$	Moment of Inertia: Body: X-axis (pitch)	??	$kg \cdot m^2$	Sec. 4.4.1
$J_{b.\phi_y}$	Moment of Inertia: Body: Y-axis (yaw)	??	$kg \cdot m^2$	Sec. 4.4.2
R_{mtr}	Motor: Resistance	4.4	Ω	[5]
$k_{mtr.bEMF}$	Motor: Coefficient of Back EMF	0.495	$\frac{V \cdot s}{rad}$	[5]
$k_{mtr.T}$	Motor: Coefficient of Torque	0.470	$\frac{N \cdot m}{A}$	[5]
$k_{fr.m2w}$	Motor: Coefficient of friction: [DC Motor] to [Wheel]	??	-	Sec. ??

5.1.2.1. Discrete Additional Dynamics

Since the additional dynamics will be processed on a microcontroller, the additional dynamics will be digital; thus, a continuous-to-discrete conversion will be necessary. ~~An~~ A digital integrator is an established case which is exhibited in Equation (5.2).

$$
\begin{aligned}
\underset{nx1}{\dot{\mathbf{x}}} &= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \cdot \underset{nx1}{\mathbf{x}}[k] + \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \cdot \begin{bmatrix}e_\theta\\ e_{\phi_y}\end{bmatrix}\\
\underset{mx1}{\mathbf{y}}[k] &= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \cdot \underset{nx1}{\mathbf{x}}[k] + \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} \cdot \begin{bmatrix}e_\theta\\ e_{\phi_y}\end{bmatrix}
\end{aligned}
\qquad (5.2)
$$

$x[k+1]$

5.1.3.1.2. Results: Simulation

To demonstrate the capabilities of the device, a dynamic command is provided which attempts to move the device in the shape of an eight 8 on the ground while maintaining balancc.

Additionally, the device starts at a body angular position (pitch) ϕ_x of 0.03 $[rad]$. This represents the inability to start the device at a perfect angle. This causes additional transients in the initial milliseconds of operation.

Figures 5.11 - 5.17 depict the system state during its operation while completing its response to a figure-eight linear position command.

Give numerical values of weight matrices

$Q = \mathrm{diag}(*, *, \dots, *)$, $R = \mathrm{diag}(*, \dots, *)$

and the gain matrices

$K_1 = [\quad\quad]$

$K_2 = [\quad\quad]$