



Stepwise versus Hierarchal Regression

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Abstract

Regression analysis is an essential tool in data analysis for predicting the value of a dependent variable based on one or more independent variables. Stepwise regression and hierarchical regression are two widely used techniques in regression analysis. Stepwise regression is a technique that involves adding or removing variables to a model based on their statistical significance, while hierarchical regression is a technique that involves adding variables to a model in a specific order based on theoretical or practical considerations. Both methods have their advantages and disadvantages, and the choice between the two depends on the research question and the underlying assumptions of the data.

This review paper provides an overview of stepwise regression and hierarchical regression and compares their advantages and disadvantages. We review the assumptions, advantages, and disadvantages of both techniques and discuss the factors that influence the choice of technique. We also highlight the potential problems associated with each technique and provide recommendations for their appropriate use. Overall, this review paper aims to help researchers and practitioners in selecting the most appropriate regression technique for their specific research question.

Introduction

Multiple regression is commonly used in social and behavioral data analysis (Fox, 1991; Huberty, 1989). In multiple regression contexts, researchers are very often interested in determining the "best" predictors in the analysis. This focus may stem from a need to identify those predictors that are supportive of theory. Alternatively, the researcher may simply be interested in explaining the most variability in the dependent variable with the fewest possible predictors, perhaps as part of a cost analysis. Two approaches to determining the quality of predictors are stepwise regression and hierarchical regression. This paper will explore the advantages and disadvantages of these methods and use a small SPSS dataset for illustration purposes.

Methodology

Stepwise Regression: Stepwise methods are sometimes used in educational and psychological research to evaluate the order of importance of variables and to select useful subsets of variables (Huberty, 1989; Thompson, 1995). Stepwise regression involves developing a sequence of linear models that, according to Snyder (1991), can be viewed as a variation of the forward selection method since predictor variables are entered one at a time, but true stepwise entry differs from forward entry in that at each step of a stepwise analysis the removal of each entered predictor is also considered; entered predictors are deleted in subsequent steps if they no longer contribute appreciable unique predictive power to the regression when considered in combination with newly entered predictors (Thompson, 1989). (p. 99) Although this approach may sound appealing, it contains inherent problems. These

problems include (a) use of degrees of freedom, (b) identification of best predictor set of a prespecified size, and (c) replicability (Thompson, 1995).

Degrees of Freedom: Using incorrect degrees of freedom results in inflated statistical significance levels when compared to tabled values, a phenomenon that was found to be substantial in a survey of published psychological research (Wilkinson, 1979). The most widely used statistical software packages do not correctly calculate the correct degrees of freedom in stepwise analysis, and they do not print any warning that this is the case (Thompson, 1995; Wilkinson, 1979). This point is emphasized by Cliff (1987) in his statement that "most computer programs for multiple regression are positively satanic in their temptation toward Type I errors in this context" (p. 185).

Discussion

How are these degrees of freedom incorrectly calculated by software packages during stepwise regression?

Essentially, stepwise regression applies an F test to the sum of squares at each stage of the procedure. Performing multiple statistical significance tests on the same data set as if no previous tests had been carried out can have severe consequences on the correctness of the resulting inferences. An appropriate analogy is given by Selvin and Stuart (1966): the fish which don't fall through the net are bound to be bigger than those which do, and it is quite fruitless to test whether they are of average size. Not only will this alter the performance of all subsequent tests on the retained explanatory model – it may destroy unbiasedness and alter mean-square-error in estimation." However, as noted by Thompson (1995), all applications of stepwise regression are "not equally evil regarding the inflation of Type I error". Examples include situations with (a) near zero sum of squares explained across steps, (b) small number of predictor variables, and/or (c) large sample size.

Best Predictor Set of a Prespecified Size:

The novice researcher may believe that the best predictor set of a specific size s will be selected by performing the same s number of steps of a stepwise regression analysis. However, stepwise analysis results are is dependent on the sampling error present in any given sample and can lead to erroneous results (Huberty, 1989; Licht, 1995; Thompson, 1995). Stepwise regression will typically not result in the best set of s predictors and could even result in selecting none of the best s predictors. Other subsets could result in a larger effect size and still other subsets of size s could yield nearly the same effect size. Why is this so? The predictor selected at each step of the analysis is conditioned on the previously included predictors and thus yields a "situation-specific conditional answer in the context (a) only of the specific variables already entered and (b) only those variables used in the particular study but not yet entered" (Thompson, 1995, p. 528). The order of variable entry can be important. If any of the predictors are correlated with each other, the relative amount of variance in the criterion variable explained by each of the predictors can change "drastically" when the order of entry is changed (Kerlinger, 1986, p. 543). A predictor with a statistically nonsignificant b could actually have

a statistically significant b if another predictor(s) is deleted from the model (Pedhazur, 1997). Also, stepwise regression would not select a suppressor predictor for inclusion in the model when in actuality that predictor could increase the R2. The explained variance would be increased when a suppressor predictor is included because part of the irrelevant variance of the predictor on the criterion would be partialled out (suppressed), and the remaining predictor variance would be more strongly linked to the criterion. Thompson (1995) shared a literal analogy to this situation from one of his students of picking a five-player basketball team. Stepwise selection of a team first picks the best potential player, then in the context of the characteristics of this player picks the second best potential player, and then proceeds to pick the rest of the five players in this manner. Thompson further suggests an alternative strategy of all-possible-subsets, which asks "which five potential players play together best as a team?". The team that is picked via this method might not have any of the players from the stepwisepicked team, and could also perform much better than the stepwisepicked team A colleague of the present author noted that one could also imagine a different type of team being brought together to work on a common goal. For example, a team of the smartest people in an organization might be selected in a stepwise manner to produce a report of cutting edge research in their field. These highly intelligent people might be, for example, Professor B. T. Weight, Professor S. T. Coefficient, Professor E. F. Size, and Professor C. R. Lation. Although these people may be the most intelligent people in the organization, they may not be the group of people who could produce the best possible report if they do not work together well. Perhaps personality conflicts, varying philosophies, or egos might interfere with the group being able to work together effectively. It could be that using an all-possible-subsets approach, or a hierarchical regression approach (see subsequent discussion), would result in a totally different group of individuals since these approaches would also consider how different combinations of individuals work together as a team. This new team might then be the one that would produce the best possible report because they do not have the previously mentioned issues and as a result work together more successfully as a team. (Disclaimer: any resemblance of these fictional team members to actual people is purely a coincidence.)

Replicability: Stepwise regression generally does not result in replicable conclusions due to its dependence on sampling error (Copas, 1983; Fox, 1991; Gronnerod, 1006; Huberty, 1989; Menard, 1995; Pedhazur, 1991; Thompson, 1995). As stated by Menard (1995), the use of stepwise procedures "capitalizes on random variations in the data and produces results that tend to be idosyncratic and difficult to replicate in any sample other than the sample in which they were originally obtained" and therefore results should be regarded as "inconclusive". As variable determinations are made at each step, there may be instances in which one variable is chosen over another due to a small difference in predictive ability. This small difference, which could be due to sampling error, impacts each subsequent step. Thompson (1995) likens these linearseries decisions to decisions that are made when working through a maze. Once a decision is made to turn one way instead of another, a whole sequence of decisions (and therefore results) are no longer possible. This difficulty of sampling error, and thus the possible impact of sampling error on the analysis, could be estimated using cross-validation (Fox, 1991; Henderson & Valleman, 1981; Tabachnick & Fidell, 1996) or other techniques. Sampling error is less problematic with (a) fewer predictor variables, (b) larger effect sizes, and (c) larger

sample sizes (Thompson, 1995). Also, sampling error is less of an issue when the regressor values for the predicted data will be used "within the configuration for which selection was employed" (e.g., as in a census undercount).

Hierarchical Regression: One alternative to stepwise regression is hierarchical regression. Hierarchical regression can be useful for evaluating the contributions of predictors above and beyond previously entered predictors, as a means of statistical control, and for examining incremental validity. Like stepwise regression, hierarchical regression is a sequential process involving the entry of predictor variables into the analysis in steps. Unlike stepwise regression, the order of variable entry into the analysis is based on theory. Instead of letting a computer software algorithm "choose" the order in which to enter the variables, these order determinations are made by the researcher based on theory and past research. As Kerlinger (1986) noted, while there is no "correct" method for choosing order of variable entry, there is also "no substitute for depth of knowledge of the research problem . . . the research problem and the theory behind the problem should determine the order of entry of variables in multiple regression analysis". Stated another way by Fox (1991), "mechanical modelselection and modification procedures . . . generally cannot compensate for weaknesses in the data and are no substitute for judgment and thought". Simply put, "the data analyst knows more than the computer" (Henderson & Velleman, 1981, p. 391). Hierarchical regression is an appropriate tool for analysis when variance on a criterion variable is being explained by predictor variables that are correlated with each other (Pedhazur, 1997). Since correlated variables are commonly seen in social sciences research and are especially prevalent in educational research, this makes hierarchical regression quite useful. Hierarchical regression is a popular method used to analyze the effect of a predictor variable after controlling for other variables. This "control" is achieved by calculating the change in the adjusted R2 at each step of the analysis, thus accounting for the increment in variance after each variable (or group of variables) is entered into the regression model (Pedhazur, 1997). Just a few recent examples of hierarchical regression analysis use in research include:

- 1. Reading comprehension: To assess the unique proportion of variance of listening comprehension and decoding ability on first and second grade children's reading comprehension (Megherbi, Seigneuric, & Ehrlich, 2006).
- 2. Adolescent development: To assess the unique proportion of variance of parental attachment and social support to college students' adjustment following a romantic relationship breakup (Moller, Fouladi, McCarthy, & Hatch, 2003).
- 3. Reading Disability: To assess the unique proportion of variance of visual-orthographic skills on reading abilities (Badian, 2005).
- 4. School Counselor Burnout: To assess the unique proportion of variance of demographic, intrapersonal, and organizational factors on school counselor burnout (Wilkerson & Bellini, 2006).
- 5. College Student Alcohol Use: To assess the unique proportion of variance of sensation seeking and peer influence on college students' drinking behaviors (Yanovitky, 2006).

6. Children with Movement Difficulties in Physical Education: To examine effects of motivational climate and perceived competence on participation behaviors of children with movement difficulties in physical education (Dunn & Dunn, 2006).

Another reason that hierarchical regression is the analysis tool of choice in so many research scenarios is that it does not have the same drawbacks of stepwise regression regarding degrees of freedom, identification of best predictor set of a prespecified size, and replicability.

Degrees of Freedom: Degrees of freedom for hierarchical regression are correctly displayed in many of the statistical software packages that do not display the correct degrees of freedom for stepwise regression. This is because in hierarchical regression, the degrees of freedom correctly reflect the number of statistical tests that have been made to arrive at the resulting model. Degrees of freedom utilized by many software packages in stepwise regression analysis do not correctly reflect the number of statistical tests that have been made to arrive at the resulting model; instead the degrees of freedom are under calculated. Thus, statistical significance levels displayed in hierarchical regression output are correct and statistical significance levels displayed in stepwise regression output are inflated, resulting in inflated chances for Type I errors. Best Predictor Set of a Prespecified Size Hierarchical regression analysis involves choosing a best predictor set interactively between computer and the researcher. The order of variable entry is determined by the researcher before the analysis is conducted. In this manner, decisions are based on theory and research instead of being made arbitrarily, in blind automation, by the computer (as they are in stepwise regression; Henderson & Vellman, 1981).

Replicability: Like stepwise regression, hierarchical regression is also subject to problems associated with sampling error. However, the likelihood of these problems is reduced by interaction of the researcher with the data. For example, instead of one variable being chosen over another variable due to a small difference in predictive ability, the order of variable entry is chosen by the researcher. Thus, results from an arbitrary decision that is more likely to reflect sampling error (in the case of stepwise regression) are instead results based on researcher expertise (in the case of hierarchical regression). Of course, remaining sampling error can still be estimated via cross-validation or other techniques. And again, sampling error will be less of an issue the larger the sample size and effect size, and the fewer the predictor variables.

Heuristic SPSS Example

Stepwise Regression: As previously discussed, stepwise regression involves developing a sequence of linear models through variable entry as determined by computer algorithms. A heuristic SPSS dataset has been constructed (Appendix A) and will be analyzed for illustration purposes. Syntax is provided in Appendix B. Stepwise regression was used to regress mother's education level (ma_ed), father's education level (fa_ed), parent's income (par_inc), and faculty interaction level (fac_int) on years to graduation (years_grad). Inspection of correlations between the variables (Table 1) reveal (a) that mother's education, parent's income, and faculty interaction are all highly correlated with years to graduation and (b) that father's education is only slightly correlated with years to graduation. Also,

most of the predictor variables are correlated with each other, with one correlation coefficient as high as 0.747.

Variables	years_grad	ma_ed	fa_ed	par_inc	fac_int
years_grad		_	_	_	
ma_ed	-0.825*	_			
fa ed	-0.041	0.427*	-		
par_inc	-0.763*	0.480*	0.004	_	
fac_int	-0.834*	0.747*	0.038	0.651*	-
М	5.00	5.450	7.375	5.850	6.825
SD	.7116	1.509	1.072	1.631	1.621
*p < .001					

Table 1: Variable Correlations

Examination of the regression summary table as displayed in SPSS output (Table 2; outputs/tables are kept in close to original formats for illustrative purposes) provides a plethora of information. First, the miscalculation of degrees of freedom is apparent. The degrees of freedom indicated reflect the exact number of variables included in the model, not the number of comparisons that were made to arrive at the model.

		Sum of		Mean		
Model		Squares	df	Square	F	Sig.
1	Regression	27.829	1	27.829	178.356	0.000
	Residual	12.171	78	0.156		
	Total	40.000	79			
2	Regression	31.535	2	15.767	143.423	0.000
	Residual	8.465	77	0.110		
	Total	40.000	79			
3	Regression	34.992	3	11.664	177.027	0.000
	Residual	5.008	76	0.066		
	Total	40.000	79			
4	Regression	36.723	4	9.181	210.146	0.000
	Residual	3.277	75	0.044		
	Total	40.000	79			
5	Regression	36.659	3	12.220	277.951	0.000
	Residual	3.341	76	0.044		
	Total	40.000	79			

Model	Summary
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			Adjusted R	Std. E	Error of the	
Model	R R	Square	Square		Estimate	
1	0.834	0.696	0.692		0.395	
2	0.888	0.788	0.783		0.332	
3	0.935	0.875	0.870		0.257	
4	0.958	0.918	0.914		0.209	
5	0.957	0.916	0.913		0.210	
a	Predictors:	(Constan	t), Interactio	n with	Faculty	
b	Predictors:	(Constan	t), Interactio	n with	Faculty,	
	Mothers Educ	cation Le	vel			
С	Predictors:	(Constan	t), Interactio	n with	Faculty,	
	Mothers Educ	cation Le	vel, Parents I	ncome		
d	Predictors:	(Constan	t), Interactio	n with	Faculty,	
	Mothers Education Level, Parents Income, Fathers					
	Education Level					
е	Predictors:	(Constan	t), Mothers Ed	ucation	n Level,	
	Parents Income, Fathers Education Level					
f	Dependent Variable: years grad					

Table 2: Stepwise Regression Summary Table

Second, the predictor variable that has the highest R with the criterion variable, faculty interaction (fac_int), is the first variable entered into the analysis. However, the final model of the analysis (model 5/e) does not include the faculty interaction variable. Thus, stepwise regression egregiously results in a model that does not include the predictor variable that has the highest correlation with the criterion variable. Because the significance tests displayed in the output of the stepwise regression analysis do not approximate the probability that the resulting model will actually represent future samples, another method is needed to estimate replicability. Double cross-validation is performed to achieve this objective. The resulting double cross-validation coefficients are 0.999. Upon initial reflection, these findings may seem quite high, but in consideration of the unusually elevated R in these analyses (0.954 & 0.961), the findings are not so surprising. Had the R values been lower or had a larger number of predictor variables been included in the analysis, smaller doublecross validation coefficients would have been expected.

Hierarchical Regression: The dataset utilized to illustrate some of the concepts involved with stepwise regression can also be used to demonstrate hierarchical regression. Variable selection for the hierarchical regression analysis will be based on theory. It is generally understood that a number of factors contribute to the level of college student success (years grad), including parent's education level (ma ed and fa ed), socioeconomic status (par_inc), and amount of interaction with faculty members (fac int). Hierarchical regression will be employed to determine if the amount of student interaction with faculty members contributes a unique proportion of variance to student success (years grad). To "control" for student characteristics of parent's education level and socioeconomic status, these variables will be entered into the first block of the analysis. Fac int will be entered into the second block of the analysis to determine its unique contribution to variance explained of years to graduation. Note that (a) variable entry into these "blocks" can occur one variable at a time or as a group (or block) or variables and (b) these determinations are made by the researcher. Examination of the regression summary table (Table 3) again provides much information. First, since the researcher selected the specific variables for analysis, the degrees of freedom correctly reflect the number of comparisons that were made to arrive at the models. Second, the model summary provides (a) the change in R2 that occurred as a result of including the additional predictor variable (fac int) in the model and (b) the statistical significance of the change in R2. In the example provided, the additional variable only produced a very small change in R2 and this change was not statistically significant. If the dataset had been actual data instead of fabricated data, the change in explained variance of years to graduation by level of student/faculty interaction would be expected to be larger and statistically significant.

			Sum of		Mean			
Model		5	Squares	df	Square	F	Sig.	
1	Regression	1	36.659	3	12.220	277.951	0.000	
	Residual		3.341	76	0.044			
	Total		40	79				
2	Regression	1	36.723	4	9.181	210.146	0.000	
	Residual		3.277	75	0.044			
	Total		40	79				
Model	Summary							
	Std. Error							
		R	Adjuste	ed	of the	R Square	Sig. F	
Model	R	Square	R Squar	ce	Estimate	Change	Change	
1	0.957	0.916	0.91	L3	0.210	0.916	0.000	
2	0.958	0.918	0.91	L 4	0.209	0.002	0.228	
a	a Predictors: (Constant), Parents Income, Fathers							
	Education Level, Mothers Education Level							
b	b Predictors: (Constant), Parents Income, Fathers							
Education Level, Mothers Education Level, Interaction								
	with Faculty							

Table 3: Hierarchical Regression Summary Table

Again, the adjusted R2 would indicate that sampling error does not have much impact on the present scenario, probably because of the high effect size and the small number of predictor

variables. If the effect size were lower and/or the number of predictor variables increased, the adjusted R2 would probably provide a larger theoretical correction for these issues, and this correction could be further examined by cross-validation or other techniques.

Conclusion

Selecting the appropriate statistical tool for analysis is dependent upon the intended use of the analysis. As Pedhazur (1997) stated, Practical considerations in the selection of specific predictors may vary, depending on the circumstances of the study, the researcher's specific aims, resources, and frame of reference, to name some. Clearly, it is not possible to develop a systematic selection method that would take such considerations into account. This rationale is in conflict with the automated, algorithm based analysis of stepwise regression. Nonetheless, there are still instances where stepwise regression has been recommended for use: in exploratory, predictive research (Menard, 1995). Even in this case, stepwise regression might not yield the largest R2 because it would ignore suppressor variables. Therefore, while intended use is a critical factor for choosing a statistical analysis tool, the problems associated with stepwise regression suggest that extreme caution should be taken if it is selected. Specifically, one could lessen the issues connected with stepwise regression analysis if it were not selected in instances with smaller samples, smaller effect sizes, and more predictor variables. Even then, interpretation of results should only be preliminary and they should not include (a) assigning meaningfulness to the order of variable entry and selection or (b) assuming optimality of the resulting subset of variables. To emphasize, Pedhazur (1997) noted "the pairing of model construction, whose very essence is a theoretical framework . . . with predictor-selection procedures that are utterly atheoretical is deplorable".

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