Kungliga Tekniska Högskolan

SF2930 Regression Analysis

Report I

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Contents

1 Introduction and Project Goals

2 Analyses and Model Development

2.1 Residual analysis

2.1.1 Normality of residuals

The normality of residuals therefore ensures that the confidence intervals presented in section 3 are valid.

2.1.2 Fitted Against Residuals

2.1.3 Added Variable Analysis

2.2 Diagnostics and handling of Outliers

2.3 Transformations of variables

2.4 Diagnostics and handling of Multicolinearity

3 Results

3.1 Residual analysis

3.1.1 Normality of residuals

Figure 1 illustrates QQ plot of the model residuals. The observer may say that the points exhibit a pattern that indicates that the residuals come from a distribution with heavier tails than that of a normal distribution. [1]. Still, the deviations from the diagonal line is relatively small, and hence we conclude that the first Gauss-Markov condition is fulfilled. That is, the model errors seem to be normally distributed.

3.1.2 Fitted Against Residuals

Figure 2 illustrates the fitted values $\hat{y}_j$ against the R-student residuals. No apparent pattern is formed by the points, i.e. the points seem to be randomly scattered along the horizontal line. Hence we conclude that the second Gauss-Markov condition is fulfilled, that is the errors have a constant variance.

3.1.3 Added Variable Analysis

Partial regression plots are found in figure 3, 4, 5, and 6. All figures exhibits potential outliers (which will be further considered in section 2.2). More specifically, in figure 3 we note a few potential outliers on the right hand side of the plot for the `biceps` regressor, and on the right and left hand side for the `forearm` regressor. Moreover, in figure 4, we notice outliers on the right hand side of the `ankle` plot, and a group of potential outliers on the `thigh` plot. Finally, we notice a few potential outliers in figure 5 and 6.

Figure 4, 5, and 6 conveys important information about the information that `knee`, `height`, and `chest` adds to the model. These regressors seem to follow a horizontal band

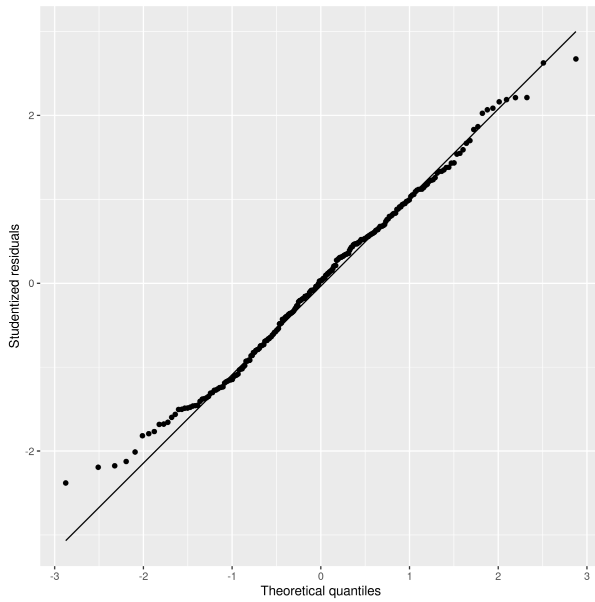

Figure 1: Normality plot of residuals.

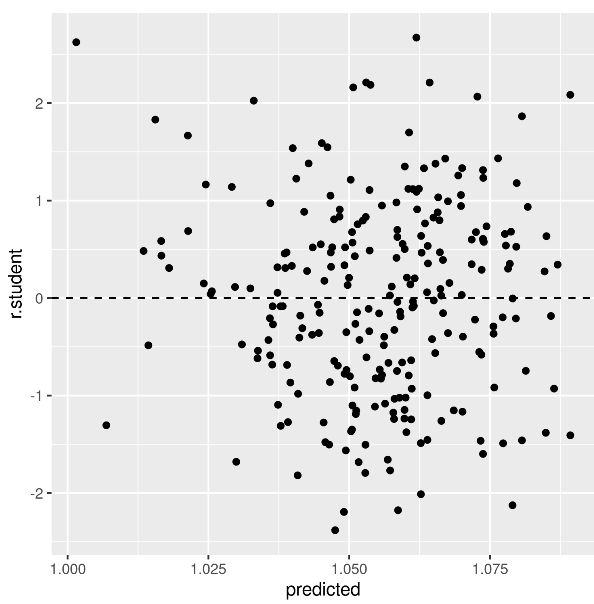

Figure 2: Fitted values against R-student residuals.

along a fitted line from the origin, which may suggest that none of the regressors adds additional information to the predictions.

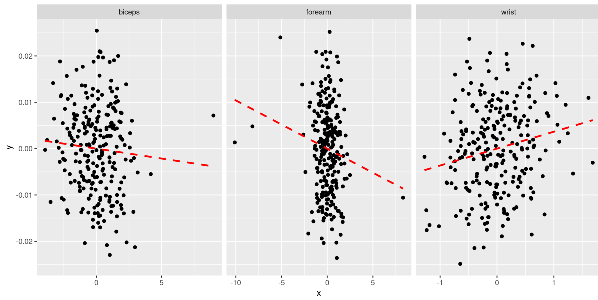

Figure 3: Partial regression plots of regressors `biceps`, `forearm`, and `wrist`.

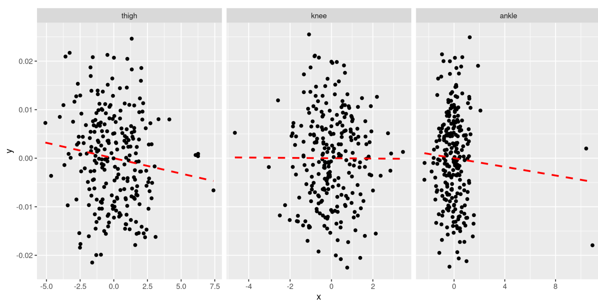

Figure 4: Partial regression plots of regressors `thigh`, `knee`, and `ankle`.

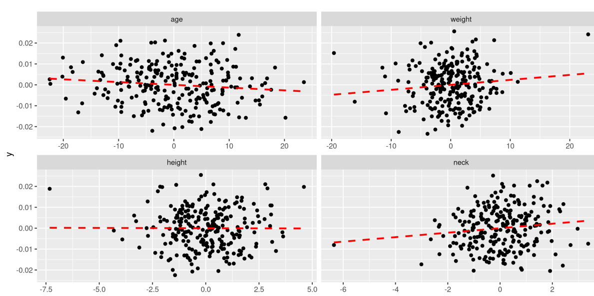

Figure 5: Partial regression plots of regressors `age`, `weight`, `height`, and `neck`.

3.2 Transformations of variables

Figure 7 displays the values of λ to be used in a potential Box-Cox transformation of the dependent variable `density`. The λ that maximized the log-likelihood is 0.9 (0.7-1.1 95% CI).

Using $\lambda = 0.9$ gives us the normal probability plot displayed on the right hand side in figure 7. We notice that this affects the distribution of residuals by making it more

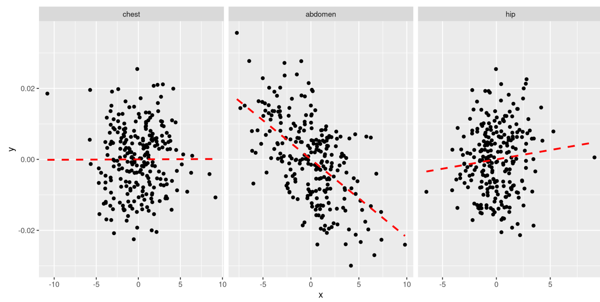

Figure 6: Partial regression plots of regressors `chest`, `abdomen`, and `hip`.

light-tailed. That is, the the tails of the distribution are too light for the distribution to be considered normal.

In section 2.1 we noted that there was no indication that a transformation was needed. Here, we see that the transformation of the response variable only makes matters worse.

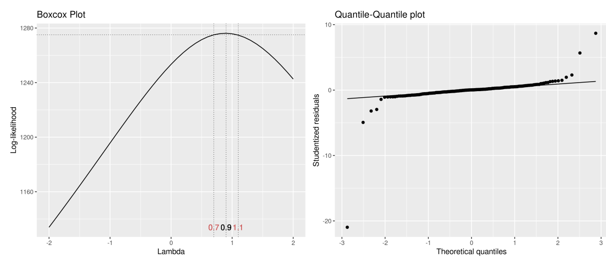

Figure 7: Values for lambda against the log-likelihood of `density` for Box-Cox transformations.

3.3 Diagnostics and handling of Outliers

Figure 8 illustrates Cook's distance for all points, where the three observations with the largest Cook's distance are labelled.

Figure 9 reports the $DFFITS$ values. We label observations as in figure 8. We observe that the three largest absolute $DFFITS$ correspond to the same observations as in the Cook's distance plot.

Figure 10, 11, 12, and 13 presents $DFBETA$ values for groups of regressors. Observation 39 is present in a number of these figures.

4 Conclusion

References

[1] Douglas C Montgomery, Elizabeth A Peck, and G Geoffrey Vining. *Introduction to Linear Regression Analysis*. Wiley-Interscience, 5 edition, 2012.

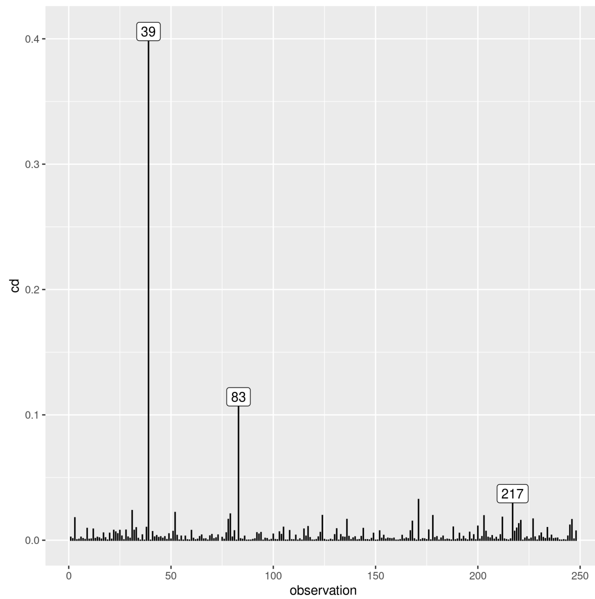

Figure 8: Plot of Cook's distance for all observations.

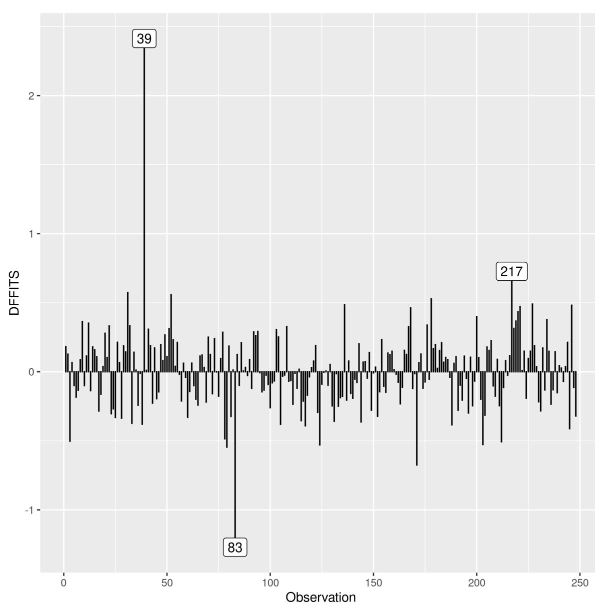

Figure 9: $DFFITS$ for all observations.

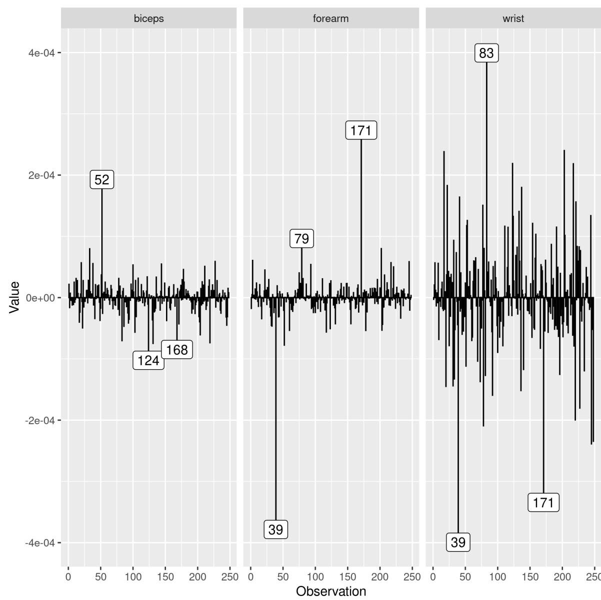

Figure 10: $DFBETA$ for regressors `biceps`, `forearm`, and `wrist`.

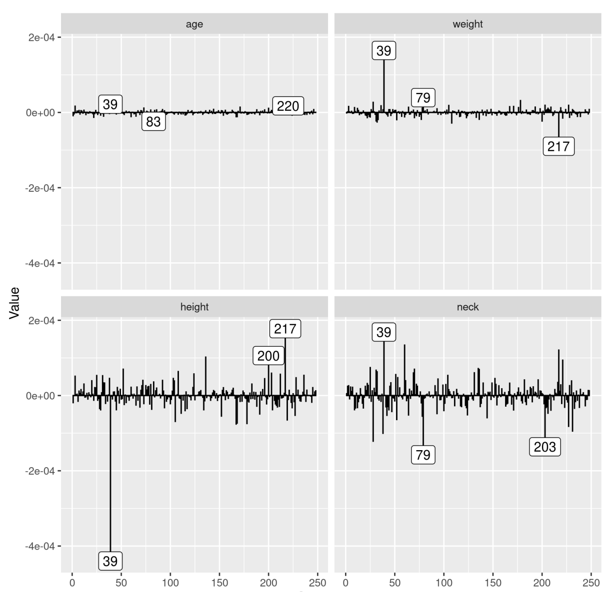

Figure 11: $DFBETA$ for regressors `age`, `weight`, `height` and `neck`.

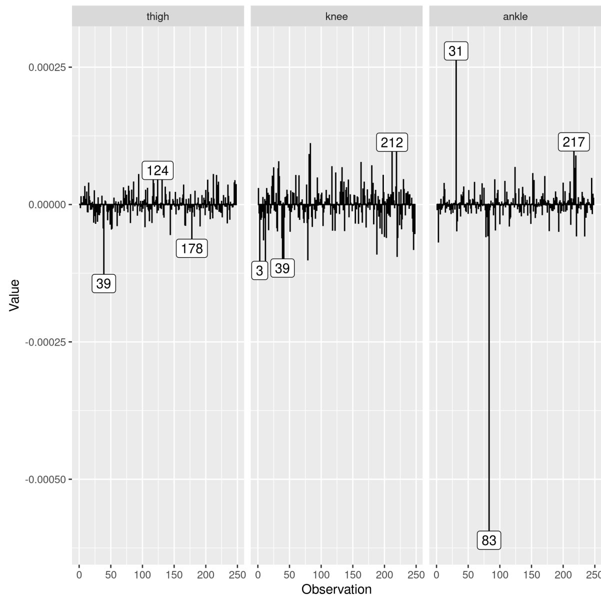

Figure 12: $DFBETA$ for regressors `thigh`, `knee`, and `ankle`.

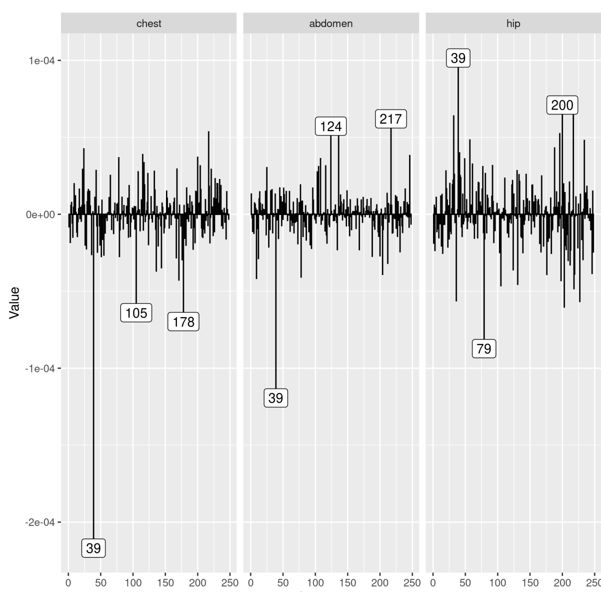

Figure 13: $DFBETA$ for regressors `chest`, `abdomen`, and `hip`.