```
//1D Max Sum
//Algorithm : Jay Kadane
//Complexity : O(n)

main() {
    int n;
    scanf("%d", &n);
    int A[n+1];
    for(int i = 0; i < n; i++)
        scanf("%d", &A[i]);

    //Main part of the code
    int sum = 0, ans = 0;
    for(int i = 0; i < 9; i++) {
        sum += A[i];
        ans = max(sum, ans);            // Always take the larger sum
        if(sum < 0)
            sum = 0;                    // If sum is negative, reset it (greedy)
    }
    printf("1D Max Sum : %d\n", ans);
}


//2D Max Sum
//DP, Inclusion Exclusion
//Complexity : O(n^4)

int main() {
    int row_column, A[100][100];                //A square matrix
    scanf("%d", &row_column);

    for(int i = 0; i < row_column; i++)         //input of the matrix/2D array
        for(int j = 0; j < row_column; j++) {
            scanf("%d", &A[i][j]);
            if(i > 0)
                A[i][j] += A[i-1][j];       // Take from right
            if(j > 0)
                A[i][j] += A[i][j-1];       // Take from left
            if(i > 0 && j > 0)
                A[i][j] -= A[i-1][j-1];     // Inclusion exclusion
        }

    int maxSubRect = -1e7;

    for(int i = 0; i < row_column; i++)         // i & j are the starting coordinate of sub-rectangle
        for(int j = 0; j < row_column; j++)
            for(int k = i; k < row_column; k++)     // k & l are the finishing coordinate of sub-rectangle
                for(int l = j; l < row_column; l++) {
                    int subRect = A[k][l];
                    if(i > 0)
```

```
                subRect -= A[i-1][l];
            if(j > 0)
                subRect -= A[k][j-1];
            if(i > 0 && j > 0)
                subRect += A[i-1][j-1];        //due to inclusion exclusion
            maxSubRect = max(subRect, maxSubRect);
        }
    printf("2D Max Sum : %d\n", maxSubRect);
    return 0;
}
```

// 0-1 Knapsack
// Dynamic Programming

//Note : val array contains element values starting from 1 index, 0 index is empty

```
int Knapsack(int totalWeight, int val[], int totalElements) {
    int dp[50001][101];                          // DP Table [BagWeight][TotalElements]
    //int track[101] = {0};                      // Use this if you want to print the taken elements
    for(int i = 0; i <= totalWeight; i++)
        dp[i][0] = 0;                            // Base Case

    // Calculating best weight(that will be taken) for every possible element
    for(int i = 1; i <= totalElements; i++) {    // Element starts from 1
        for(int weight = 1; weight <= totalWeight; weight++) {
            if(val[i] > weight)                  // If elements weight is greater than available weight
                dp[weight][i] = dp[weight][i-1]; // Skip this element
            else {                        // If enough space is available for this element *
                if(dp[weight][i-1] >= dp[weight-val[i]][i-1] + val[i])
                    dp[weight][i] = dp[weight][i-1];     // If ignoring this element causes good outcome, ignore this
                else {
                    dp[weight][i] = dp[weight - val[i]][i-1] + val[i];     // Otherwise take this element
                    //track[dp[weight][i]] = i;                            // This tracks the taken element
            }  }  }  }

            /* These code outputs the taken values
            int sumWeight = dp[totalWeight][totalElements];
            int lastTakenElement = track[sumWeight];
            while(lastTakenElement != 0) {                  // lastTakenElement is the index of element
                    printf("%d ", val[lastTakenElement]);   // val[lastTakenElement] is taken value
                    sumWeight -= val[lastTakenElement];
                    lastTakenElement = track[sumWeight];
            }
            printf("\n"); */

            return dp[totalWeight][totalElements];
}
```

		capacity j					
	i	0	1	2	3	4	5
	0	0	0	0	0	0	0
$w_1 = 2,\ v_1 = 12$	1	0	0	12	12	12	12
$w_2 = 1,\ v_2 = 10$	2	0	10	12	22	22	22
$w_3 = 3,\ v_3 = 20$	3	0	10	12	22	30	32
$w_4 = 2,\ v_4 = 15$	4	0	10	15	25	30	**37**

FIGURE 8.5 Example of solving an instance of the knapsack problem by the dynamic programming algorithm.

// 0-1 knapsack top down method
// Left side code runs from higher limit to zero, right side (commented code) runs from 0 to higher limit
// Left side code contains element values in array starting from index 1, in right side code element values in array starts from 0

```
// Index of elementWeight[] and cost[] starts from 1
int Knapsack(int weight, int i) {                         // Runs Knapsack from limit to 0
    if(i == 0 || weight == 0)                             // If weigh is zero or nothing is taken
        return 0;
    if(dp[weight][i] != -1)
        return dp[weight][i];
    if(elementWeight[i] > weight)                         // If weight if this element is more than available
       return dp[weight][i] = Knapsack(weight, i-1);      // Ignore the element
    else                                                  // Skip, Take
        return dp[i][w] = max(Knapsack(weight, i-1), Knapsack(weight – elementWeight[i], i-1) + cost[i]);
}

// Index of elementWeight[] and cost[] starts from 0
int Knapsack(int weight, int i) {                         // Runs Knapsack from 0 to limit
    if( i == element_number) {
        if(weight > weight_limit)                         // If weight is somehow more than the limit
            return -INF;                                  // Returning an INF so that, this value is ignored
        return 0; }
    if(dp[weight][i] != -1)
        return dp[weight][i];
    return dp[i][w] = max(Knapsack(weight, i-1), Knapsack(weight – elementWeight[i], i-1) + cost[i]);
}
```

// Coin Change
// All Possible Types

```
int main() {
        int n, coin_amount = 3;                   // n = value to produce
        int coin[] = {1, 2, 3}, test[1000];       // coin[] = coin values

        // Solution for producing amount with coins. Without any co-occurance and
        // coins can be used more than once
        // Bottom Up solution

        memset(test, 0, sizeof(test));
        test[0] = 1;      // Base case
        for(register int i = 0; i < coin_amount; i++)          // This will NOT produce co-occurrence
                for(register int j = 1; j<=n; j++)             // Solution for 4 if there is present 1 & 2 coins would be 3
                        if(j >= coin[i])                       // 1+1+2, 2+2, 1+1+1+1
                                test[j] += test[j - coin[i]];
        printf("Solution without co-occurance : %d\n", test[n]);

        // Solution for producing amount with coins. With co-occurance and
        // Coins can be used more than once
        // Bottom Up solution

        memset(test, 0, sizeof(test));
        test[0] = 1;                                           // Base case
        for(int j = 1; j <= n; j++)                            // This will produce co-occurrence
                for(int i = 0; i < coin_amount; i++)           // Solution for 4 if there is present 1 & 2 coins would be 5
                        if(j >= coin[i])                       // 1+1+2, 2+2, 1+1+1+1
```

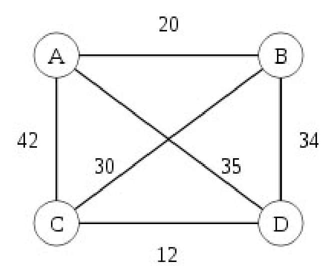
Coins \ Amount	0	1	2	3	4	5
0	1	0	0	0	0	0
1	1	1	1	1	1	1
2	1	1	2	2	3	3
3	1	1	2	3	4	5

Fig: Coin Change Table

```
                test[j] += test[j - coin[i]];        // and also 2+1+1, 1+2+1

    printf("Solution with co-occurrence : %d\n", test[n]);

    //  Solution for producing amount with coins. With co-occurrence and
    //  Coins can be used more than once
    //  Bottom up solution

    for(int i = 0; i <= 1000; i++)
        test[i] = inf;                  // Normal case
    test[0] = 0;                        // Base case
    for(int i = 0; i < coin_amount; i++)                          // this will produce co-occurrence
        for(int j = n; j > 0; j--)                   // solution for 4 if there is present 1, 2 & 3 coins would be 2
            if(j >= coin[i] && (test[j - coin[i]] + 1) < inf)        // 1+3, and 3+1
                test[j] = test[j-coin[i]] + 1;
    printf("Solution by using coins only once with co-occurrence : %d\n", test[n]);

    //  Solution for producing amount with coins. With co-occurrence and
    //  coins can be used more than once
    //  Bottom up solution

    for(int i = 0; i <= 1000; i++)
        test[i] = inf;                  // Normal case
    test[0] = 0;                        // Base case
    for(register int i = n; i > 0; i--)                           // this will NOT produce co-occurrence
        for(register int j = 0; j < coin_amount; j++)  // solution for 4 if there is present 1, 2 & 3 coins would be 1
            if(i >= coin[j] && (test[i - coin[j]] + 1) < inf)       // 1+3 only
                test[i] = test[i - coin[j]] + 1;

    printf("Solution by using coins only once without co-occurrence : %d\n", test[n]);
    return 0;
}
```

// Traveling Salesman
// Time Complexity : O(2^n * n^2)

//dist[u][v] = distance from u to v
//dp[u][bitmask] = dp[node][set_of_taken_nodes] (saves the best(min/max) path)
//call with tsp(starting node, 1)
// Best solution may be more than one



	A	B	C	D
A	0	20	42	35
B	20	0	30	34
C	42	30	0	12
D	35	34	12	0

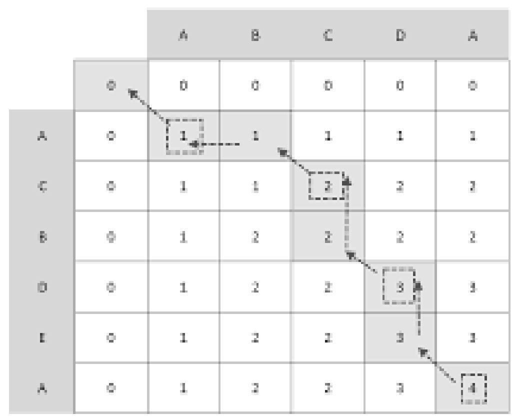
Fig: Traveling Salesman Problem State (A-B-C-D-A)

```
int n, x[11], y[11], dist[11][11], memo[11][1 << 11], dp[11][1 << 11];     //This example is for 11 routes/nodes
int tsp(int u, int bitmask) {               // Starting node and bitmask of taken nodes
    if(bitmask == ((1 << (1+n)) - 1))       // When it steps in this node, if all nodes are visited
        return dist[u][0];                  // Then return to 0'th (starting) node [as the path is Hamiltonian]

    //or use return dist[u][start]  if starting node is not 0
    if(dp[u][bitmask] != -1)                        // If we have previous value set up
        return dp[u][bitmask];                      // Use that previous value
```

```
        int ans = 1e9;                                   // Set an infinite value
        for(int v = 0; v <= n; v++)                      // For all the nodes
                if(u != v && !(bitmask & (1 << v)))      // if this node is not the same node, and if this node is not used
                                                         // yet(in bitmask)

                        ans = min(ans, dist[u][v] + tsp(v, bitmask | (1 << v)));
                                                         //min(past_val, dist u->v + dist(v->all other untaken nodes))
        return dp[u][bitmask] = ans;                     //save in dp and return
}
```

//Longest Common Sub Sequence
//Dynamic Programming

```
char a[210], b[210];
int dp[210][210], len_a, len_b;

//LCS is the same sequence in two strings:  a s x z  and  s x z a. Here LCS is 3 {a, sx, z}
int LCS(char a[], char b[], int len_a, int len_b) {
   dp[210][210] = 0;
   for(register int i = 1; i <= len_a; i++)
      for(register int j = 1; j <= len_b; j++) {
         if(i == 0 || j == 0)              //base case
            dp[i][j] = 0;
         else if(a[i-1] == b[j-1])         //if a match found
            dp[i][j] = dp[i-1][j-1] + 1;
         else
                  Fig: Longest Common Subsequence
            dp[i][j] = max(dp[i-1][j], dp[i][j-1]);       // dp[i][j] = max(ignoring
b[j-1] (taking b[j]), ignoring a[i-1] (taking a[i]))
      }
   return dp[len_a][len_b];
}
```



LCS - "ACDA"

// Longest Increasing Subsequence
// Time Complexity : n logn

```
int LIS(vector<int> &val) {
   vector<int>lis;
   for(int i = 0; i < (int)val.size(); i++) {               // Use upper_bound for longest non decreasing subsequence
      vector<int>::iterator it = lower_bound(lis.begin(), lis.end(), val[i]);
      if(lis.end() == it)                                   // 'it' points to the end, val[i] is bigger
         lis.push_back(val[i]);
      else
         lis[it - lis.begin()] = val[i];
   }
   return lis.size();
}
```

// In case if some tweak needed in binary search, use this instead

```
int lowerBound(std::vector<int>&v, int low, int hi, int key) {          // Less than-equal to the key
   while(hi - low > 1) {                                                // If low <= hi was used
      int m = (low+hi)/2;
```

```
        if(v[m] >= key)
            hi = m;                                         // hi = m - 1
        else
            low = m;                                        // low = m + 1
    }
    return hi;
}
```

arr[ ]	10	22	9	33	21	50	41	60	80
LIS	1	2		3		4		5	6

Fig : LIS Dynamic Solve

Dynamic Solve, **Complexity : n^2**

// Use this solve for **path printing**

```
int lis(int val[], int size) {
    int lis[size+10] = {1};                              // Default LIS value

    for(int i = 1; i < size; ++i)                        // This is the finish point
        for(int j = 0; j < i; ++j)                       // For all past points
            if(val[i] > val[j] && lis[i] < lis[j]+1)     // If finish point is higher than past point and taking it
                lis[i] = lis[j] + 1;                     // optimizes the result, then take it

    return max_element(lis, lis+size);                   // Find the maximum element
}
```

// Game Theory (UVa 847)

// Player 1 starts from number 1, and have to multiply it with [2-9] and pass it to player2, one who crosses the limit (lim) wins

// Everyone searches for optimal position, from which position the opponent has no option of winning

```
bool canWin(long long n, bool person) {                  // Returns who wins, 0 (player1), 1 (player2)
    for(long long i = 2; i <= 9; ++i) {
        long long tmp = n*i;
        if(dp.find(tmp) != dp.end() && dp[tmp] == person)   // Here dp is a map, array can also be used
            return person;
        if(tmp >= lim) {                                  // Player wins if tmp >= limit
            dp[tmp] = person;
            return person;                                // Winning position
        }
        if(canWin(tmp, !person) == person) {              // Let the second person play, if he looses
            dp[tmp] = person;
            return person;                                // Then this person wins
        }
    }                                 // This line only runs when the second person always wins
    return !person;                   // So, this person doesn't win
}
```

// In fig one player can remove 1 or 3 element, who reaches 0 will win. If player1 starts the game from 11, he will somehow try to reach an optimal position from where he will sure win. Here node 2 is an optimal position for player1 as it isn’t possible for player2 to win. Also node 5 is a winning position for player1.
A position is winning for player1 if : There is no way for player2 to win from that move also every player gives the optimal move.