Assignment 1

ECS 122B — Spring 2016

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Problem 1

Using induction to prove that $\sum_{i=1}^{n}\sum_{j=1}^{m} f(i)g(j) = [\sum_{i=1}^{n} f(i)][\sum_{j=1}^{m} g(j)]$

Base case: Starting with n+m = 2,
Doing left hand side, $\sum_{i=1}^{1}\sum_{j=1}^{1} f(i)g(j) = [\sum_{i=1}^{1} f(i)][g(1)] = f(1)g(1)$
Doing right hand side, we get $[\sum_{i=1}^{1} f(i)][\sum_{j=1}^{1} g(j)] = f(1)g(1)$
Hence, proved.

Inductive hypothesis:
Assume that $\sum_{i=1}^{n}\sum_{j=1}^{m} f(i)g(j) = [\sum_{i=1}^{n} f(i)][\sum_{j=1}^{m} g(j)]$ to be true.

Now we break into two cases:

For $n > 1$: $(n-1)+m = t$

Left Hand Side: $\sum_{i=1}^{n+1}\sum_{j=1}^{m} f(i)g(j) = [\sum_{i=1}^{n+1} f(i)][\sum_{j=1}^{m} g(j)]$
$= [\sum_{i=1}^{n} f(i) + f(n+1)][\sum_{j=1}^{m} g(j)]$ By inductive hypothesis.
$= \sum_{i=1}^{n+1}\sum_{j=1}^{m} f(i)g(j)$

For $m > 1$: $n+(m-1) = t$

Left Hand Side: $\sum_{i=1}^{n}\sum_{j=1}^{m+1} f(i)g(j) = [\sum_{i=1}^{n} f(i)][\sum_{j=1}^{m+1} g(j)]$
$= [\sum_{i=1}^{n} f(i)][\sum_{j=1}^{m} g(j) + g(m+1)]$ By inductive hypothesis.
$= \sum_{i=1}^{n}\sum_{j=1}^{m+1} f(i)g(j)$

Problem 2

Let $T(n)$ be the number of trees with n nodes, so $T(1) = 1, T(2) = 3$ and $T(3) = 12$

I started out with $T(4) = 60$ and $T(5) = 360$ This gives the recurrence relation to be

$T(n) = \sum_{k=0}^{n-1} \sum_{m=0}^{n-k-1} T(k)T(m)T(n-k-m-1)$ for $n > 3$

Problem 3

Part a.

Get the initial tour of G

Initialize the set of vertex V

for $\pi \in P_n$ **do**
  $W = \emptyset$
  for $i = 1$ to n **do**
    for $j = 1$ to n **do**
      $W(i, j) = min(\sum_{i=1}^{n-1} w(v_{\pi(i)}, v_{\pi(i+1)})$
      cost = cost + $W(i, j)$ // Update the edges of the minimum cost.
    end for
    $\pi(i)$ = cost
  end for
  print $\pi(i) +$ ""
end for

Part b.
$\theta(n!)$

Problem 4

Given a Bipartition problem.

Part a.
There are possible partition sets:
1. $\{\emptyset, \{x_1, x_2, x_3\}\}$
2. $\{x_1, \{x_2, x_3\}\}$
3. $\{x_2, \{x_1, x_3\}\}$
4. $\{x_3, \{x_1, x_2\}\}$

Those sets are in $BP(\{x_1, x_2, x_3\})$

Part b. Given that $A \cup B = X$ and $A \cap B = \emptyset$, describing $f^{-1}(\{A', B'\})$
First we would partition $\{A, B\}$ of X.
Since $A \cap B = \emptyset$, X_n cannot be in both A and B.
This concludes that $\{A' \cup \{x_n\}, B'\} \neq \{A', B' \cup \{x_n\}\}$

$\forall A', B', |f^{-1}(\{A', B'\})| \geq 2$
If $A \cap B = \emptyset$ then $|f^{-1}(\{A', B'\})| \leq 2$
Those sets can be partition of $\{A, B\}$ of X which implies $A \cup B = X$

Part c.
$|dom(f)| = |f^{-1}(\{A', B'\})|$
Can be obtained from B. That is $\{A', B'\} \in BP(X')$
$BP(X) => b(n)$
$BP(X') => b(n-1)$

Part d.

The recurrence relation from part c) is $B(0) = 1$, $B(1) = 1$,$B(2) = 2$, $B(n) = B(n-1)B(n-2)$ for $n > 2$

Problem 5

Given the geometric series $\sum_{N\geq 0} ar^N$ for any values a and r.

Part A. For $a = 1$ and $r = \frac{1}{4}$ We can get $S = \frac{1}{(1-\frac{1}{4})} = \frac{4}{3}$.

Part B. For $a = 3$ and $r = \frac{1}{9}$ We can get $S = \frac{3}{(1-\frac{1}{9})} = 3 \times \frac{9}{8} = \frac{27}{8}$.