ECS 132

Homework 3

Problem A

a. $\binom{82}{5} \times (1-0.05)^{82-5} \times (0.05)^5$

$= binom(5, 82, 0.05) = 0.1642468$

b. The mean for binomial distribution should be np.

Then $EX = \lambda = np = 82 \times 0.05 = 4.1 < 5$

Then to find $P(X = 5)$, we evaluate $dpois(5, 4.1) = 0.1600039$

The error is 0.0042429. Therefore, the approximation works well.

c. We need to add up all probabilities from 3 to 42 components and there are 3 defectives. $dpois(3 : 42, 50)$ is the Poisson distribution probabilities

and $binom(3, 3 : 42, 50)$ is the binomial distribution probabilities.

So $P = \sum_{i=3}^{42}(dpois(i, 50) \times binom(3, i, 0.05))$

$P = \sum_{i=3}^{42}(\frac{e^{-50} \times 50^i}{i!} \times \binom{i}{3}(1-0.05)^{i-3} \times 0.05^3)$

$= 0.0258277$

Problem B

$E[(X - EX)^3]$

$= E[(X - p)^3]$

$= E(X^3 - 3X^2p + 3Xp^2 - p^3)$

$= EX^3 - 3pEX^2 + 3p^2EX - p^3$

X is an indicator variable, thus

$= (EX)^3 - 3p(EX)^2 + 3p^2EX - p^3$

$= p^3 - 3pp^2 + 3p^2p - p^3$

$= 0$

Problem C

Given $g_{B1}(S) = \sum_{n=1}^{\infty} P(B_1 = i)S^i$

$g_{B1}(1) = P(B_1 = 0) + P(B_1 = 1) + P(B_1 = 2)$

$g_w(1) = g_{B1+B2+B3}(1) = g_{B1}(1)g_{B2}(1)g_{B3}(1) = [g_{B1}(1)]^3$

We can say that

$g_w(1) = (P(B_1 = 0) + P(B_1 = 1) + P(B_1 = 2))^3$

$= P(B_1 = 0)^3 + P(B_1 = 1)^3 + P(B_1 = 2)^3 + 3P(B_1 = 0)^2P(B_1 = 1) + P(B_1 = 0)^2P(B_1 = 2)$

$+3P(B_1 = 1)^2P(B_1 = 0) + 3P(B_1 = 1)^2P(B_1 = 2) + 3P(B_1 = 2)^2P(B_1 = 0)$

$+3P(B_1 = 2)^2P(B_1 = 1) + 6P(B_1 = 0)P(B_1 = 1)P(B_1 = 2)$

$= P(B_1 = 0)^3 + 3P(B_1 = 0)^2P(B_1 = 1) + 3P(B_1 = 1)^2P(B_1 = 0)$

$+3P(B_1 = 0)^2P(B_1 = 2)+$

$P(B_1 = 1)^3 + 6P(B_1 = 0)P(B_1 = 1)P(B_1 = 2) + 3P(B_1 = 1)^2P(B_1 = 2)$

$+3P(B_1 = 2)^2P(B_1 = 0) + 3P(B_1 = 2)^2P(B_1 = 1) + P(B_1 = 2)^3$

$= P(W = 0)+P(W = 1)+P(W = 2)+P(W = 3)+P(W = 4)+P(W = 5)+P(W = 6) = 1$

regroup them, we can obtain our results

$P(W = 0) = P(B_1 = 0)^3 = 0.125$

$P(W = 1) = 3P(B_1 = 0)^2P(B_1 = 1) = 0.3$

$P(W = 2) = 3P(B_1 = 1)^2P(B_1 = 0) + 3P(B_1 = 0)^2P(B_1 = 2) = 0.315$

$P(W = 3) = P(B_1 = 1)^3 + 6P(B_1 = 0)P(B_1 = 1)P(B_1 = 2) = 0.184$

$P(W = 4) = 3P(B_1 = 1)^2P(B_1 = 2) + 3P(B_1 = 2)^2P(B_1 = 0) = 0.063$

$P(W = 5) = 3P(B_1 = 2)^2P(B_1 = 1) = 0.012$

$P(W = 6) = P(B_1 = 2)^3 = 0.001$