ECS 132

Homework 2

Problem 1

One in box 2: (11111) (1121) (1211) (2111) (221) (311) (131)

$P_1 = (\frac{1}{3})^5 + 3 \times (\frac{1}{3})^4 + 3 \times (\frac{1}{3})^3 = \frac{1+9+27}{243} = \frac{37}{243}$

Two in box 2: (11112) (1122) (1212) (2112) (222) (312) (132) (1112) (122) (212) (32)

$P_2 = (\frac{1}{3})^5 + 4 \times (\frac{1}{3})^4 + 5 \times (\frac{1}{3})^3 + (\frac{1}{3})^2 = \frac{1+12+45+27}{243} = \frac{85}{243}$

Three in box 2: (11113) (1123) (1213) (2113) (223) (313) (133) (1113) (123) (213) (33) (113) (23)

$P_3 = (\frac{1}{3})^5 + 4 \times (\frac{1}{3})^4 + 6 \times (\frac{1}{3})^3 + 2 \times (\frac{1}{3})^2 = \frac{1+12+54+54}{243} = \frac{121}{243}$

Part a.

$E(X) = \sum cP(X = c) = P_1 \times 1 + P_2 \times 2 + P_3 \times 3 = \frac{37}{243} + 2 \times \frac{85}{243} + 3 \times \frac{121}{243} = \frac{570}{243} \approx 2.34$

Part b.

$Var(X) = E(X^2) - E(X)^2 = \sum c^2 P(X = c) = 1^2 \times \frac{37}{243} + 2^2 \times \frac{85}{243} + 3^2 \times \frac{121}{243} - (\frac{570}{243})^2$

$= \frac{243(37+340+1{,}089-570^2}{570^2} = \frac{31338}{59049} = \frac{3482}{6561} \approx 0.53$

Problem 2

People who alight should be $E(0.2 \times L_1)$

$EL_2 = E(L_1 + B_2 - E(0.2 * L1))$

Some basic facts $EL_1 = EB_2 = 1 \times 0.4 + 2 \times 0.1 = 0.6$

$E(L_1^2) = 1^2 \times 0.4 + 2^2 \times 0.1 = 0.8$

Therefore:

$EL_2 = E(L_1 + B_2 - E(0.2 \times L_1)) = E(L_1) + E(B_2) - 0.2 \times E(L_1)$

$= 0.6 + 0.6 - 0.2 * 0.6$

$= 1.08$

$E(L_1 \times L_2) = E((L_1 + B_2 - E(0.2 \times L_1)) \times L_1)$

$= E(L_1^2 + B_2 \times L_1 - 0.2 \times L_1^)$

Because L_1 and B_2 is independent and L_1 is independent with itself. Therefore:

$= E(L_1^2) + EL_1 \times EB_2 - 0.2 \times E(L_1^2)$

$= 0.8 + 0.6 \times 0.6 - 0.2 \times 0.8$

$= 1$

Therefore:

$Cov(L_1, L_2) = E(L_1 \times L_2) - EL_1 \times EL_2$

$= 1 - 0.6 \times 1.08$

$= 0.352$

Problem 3

$Var(X + Y) = Var(X) + Var(Y) + 2 \times cov(X, Y)$

Since X and Y are indicator random variables, by mailing tube (3.73) and (3.79), we use $cov(X, Y) = E(XY) - E(X)E(Y)$

$Var(X + Y) = p \times (1 - p) + q \times (1 - q) + 2[E(XY) - p \times q]$

$= (p - p^2) + (q - q^2) + 2r - 2pq$

Problem 4

$E(D_4) = P(attaches\ to\ v_1) \times Deg(v_1) + P(attaches\ to\ v_2) \times Deg(v_2) + P(attaches\ to\ v_3) \times Deg(v_3)$

$= P(v_3\ attaches\ to\ v_2) \times P(attaches\ to\ v_1 \mid v_3\ attaches\ to\ v_2) \times Deg(v_1) + P(v_3\ attaches\ to\ v_1) \times P(attaches\ to\ v_1 \mid v_3\ attaches\ to\ v_1) \times Deg(v_1)$

$+P(v_3\ attaches\ to\ v_2) \times P(attaches\ to\ v_2 \mid v_3\ attaches\ to\ v_2) \times Deg(v_1) + P(v_3\ attaches\ to\ v_1) \times P(attaches\ to\ v_2 \mid v_3\ attaches\ to\ v_1) \times Deg(v_1)$

$P(v_3\ attaches\ to\ v_2) \times P(attaches\ to\ v_3 \mid v_3\ attaches\ to\ v_2) \times Deg(v_1) + P(v_3\ attaches\ to\ v_1) \times P(attaches\ to\ v_3 \mid v_3\ attaches\ to\ v_1) \times Deg(v_1)$

$= \frac{1}{2} \times \frac{1}{4} \times 1 + \frac{1}{2} \times \frac{2}{4} \times 2$

$+\frac{1}{2} \times \frac{1}{2} \times 2 + \frac{1}{2} \times \frac{1}{4} \times 1$

$+\frac{1}{2} \times \frac{1}{4} \times 1 + \frac{1}{2} \times \frac{1}{4} \times 1$

$= \frac{1}{8} + \frac{4}{8} + \frac{4}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$

$= 1.5$