Volatility Modeling with R

Measuring Volatility Using the ARCH and GARCH Models

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ARIMA vs. ARCH/GARCH Models

- **ARIMA-type models** are unable to explain some key features common in financial time series, such as volatility clustering.
- **ARIMA models** fail to capture volatility clustering in financial time series.
- **ARCH and GARCH models** are among the most widely used non-linear models to predict and model **volatility**.
- **ARCH** and **GARCH** models provide a robust framework for modeling time-varying volatility, a key feature in financial markets.
- Introduced by **Robert F. Engle** in the 1980s, ARCH models improved financial modeling by accounting for time-varying volatility, leading to his Nobel Prize in 2003.

ARCH and GARCH Effects

Why does volatility modeling matter?

- Financial time series often show non-constant variance (**heteroscedasticity**), with periods of high and low volatility.
- **ARCH models** account for conditional volatility based on past residuals, while **GARCH models** extend this by incorporating past variances.
- **Heteroscedasticity** is addressed using ARCH/GARCH models, which handle varying error variance over time.

ARCH Effect

- **Why does the ARCH effect matter?**
  - In financial time series, variance is often not constant (heteroscedasticity).
  - ARCH models incorporate **conditional volatility**, meaning the variance at any time depends on past squared residuals.
- **Heteroscedasticity** occurs when the error variance changes over time, and this is modeled with the ARCH framework.

ARCH Effect

- A time series shows the ARCH effect when there is **autocorrelation** in squared residuals, signaling **conditional heteroscedasticity**.

GARCH Effect

- The **GARCH model** generalizes ARCH by including **lagged variances**, accounting for both past errors and volatility persistence.

Testing for ARCH Effects

- **Before applying ARCH or GARCH models**, it is crucial to test for ARCH effects. Engle's (1982) test is a common approach.
- **Engle's ARCH Test:**
  - A **Lagrange Multiplier (LM) test** to assess the significance of ARCH effects in the residuals.
  - Tests the hypothesis:
    - **Null Hypothesis (H):** No ARCH effects.
    - **Alternative Hypothesis (H):** ARCH effects are present.
- **Steps:**
  - Fit an initial model (e.g., ARIMA).
  - Apply the **LM ARCH test** to the residuals of the model.
  - A **p-value** $<$ **0.05** suggests rejection of the null hypothesis, indicating the presence of ARCH effects.

Section 1

ARCH Model

ARCH(p) Model

- **What is ARCH?**
  - ARCH stands for Autoregressive Conditional Heteroskedasticity.
  - It was introduced by Robert F. Engle in 1982, focusing on time-series data where volatility tends to cluster.
  - Primarily used in **econometrics** and **quantitative finance**, ARCH models capture time-varying volatility, which traditional models like ARIMA fail to address.
- **Key Concept:**
  - **Volatility Clustering:** Large price changes are often followed by large changes (of either sign), and small changes are followed by small changes. This clustering effect is commonly seen in financial markets.

Real-Life Examples of ARCH Models

- **Stock Market Volatility:**
  - Financial markets like the **stock market** exhibit periods of high and low volatility.
  - For example, during a financial crisis (like 2008), stock prices experience extreme swings over short periods. ARCH models help quantify this volatility, improving risk assessment and portfolio management.
- **Foreign Exchange Rates:**
  - Exchange rates between currencies, such as USD/EUR, also show volatility clustering.
  - High volatility periods may follow geopolitical events or economic announcements. ARCH models are widely used by **currency traders** to adjust hedging strategies and predict risks during such times.
- **Interest Rates:**
  - Changes in **interest rates** often show periods of high variability. Central bank policies or unexpected market conditions can cause sudden spikes or drops, which ARCH models can capture, helping financial institutions in forecasting and risk mitigation.

Motivation Behind ARCH Model

- **Why ARCH?**
  - Traditional time-series models (e.g., ARIMA) assume constant variance (homoscedasticity).
  - ARCH models account for **time-varying variance** (heteroscedasticity), crucial for modeling asset returns, risk management, and option pricing.

Why Use ARCH Models in Finance?

- **Risk Management:**
  - In finance, accurately forecasting future volatility is crucial for **derivative pricing**, **portfolio management**, and **risk assessment**.
  - ARCH models allow investors to anticipate volatility shocks and adjust their positions accordingly.
- **Value at Risk (VaR):**
  - ARCH models are often used to compute VaR, a metric to estimate the potential loss of an asset or portfolio under normal market conditions over a set time horizon.
  - By understanding volatility dynamics, financial firms use ARCH models to avoid underestimated risks, especially in unstable markets.

ARCH Process: Basic Equation

ARCH(1) Model

$$X_t = \mu + \epsilon_t, \quad \epsilon_t \sim N(0, h_t)$$
$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2$$

Parameters

- X_t: Observed time series
- ϵ_t: White noise error term with time-varying variance h_t
- h_t: Conditional variance depending on previous period's squared residuals
- α_0, α_1: Parameters with $\alpha_0 > 0$ and $\alpha_1 \geq 0$

Explanation

- The error variance h_t changes based on past squared errors ϵ_{t-1}^2.
- When large residuals occur (e.g., market shock), the model predicts a higher variance for future observations.

ARCH Model Assumptions

- **Key Assumptions:**
  - Time series X_t exhibits **heteroscedasticity** (variance changes over time).
  - Error terms are normally distributed with mean zero and a time-varying conditional variance.
  - $\alpha_1$determines how much past squared errors influence current volatility.

Conclusion

- **ARCH models are essential tools** in financial econometrics, offering insights into volatility dynamics in markets that display clustered volatility.
- Whether analyzing **stocks**, **currencies**, or **interest rates**, the ARCH framework enables better forecasting, risk management, and decision-making in uncertain environments.

Section 2

GARCH Model - an Extensions to ARCH Model

GARCH(p, q) Model

- **What is GARCH?**
  - GARCH stands for **Generalized Autoregressive Conditional Heteroskedasticity**.
  - Introduced by **Tim Bollerslev** in 1986 as an extension of the ARCH model.
  - GARCH models are widely used in **financial econometrics** to capture persistent volatility, which ARCH models may not fully address.
- **Key Concept:**
  - **Volatility Clustering**: Large changes tend to be followed by large changes, and small changes by small changes.
  - GARCH models build on ARCH by incorporating lagged conditional variances, making it more flexible for modeling volatility persistence.

Real-Life Examples of GARCH Models

- **Stock Market Volatility:**
  - GARCH models help model volatility during financial crises when there is a high degree of market fluctuation, such as the **2008 global financial crisis**.
- **Foreign Exchange Rates:**
  - GARCH is widely used by **currency traders** to model periods of increased risk and make adjustments to their hedging strategies, particularly during significant economic events or geopolitical tensions.
- **Interest Rates:**
  - Sudden shifts in **interest rates** caused by unexpected central bank announcements can create persistent volatility, which GARCH models capture, aiding in forecasting and risk mitigation.

Motivation Behind GARCH Model

Why GARCH?

- Like ARCH, GARCH models account for **time-varying variance** (heteroscedasticity), but they also allow volatility to persist over time, which is essential for financial time series data, where volatility can remain high or low for extended periods.

Why Use GARCH Models in Finance?

- **Risk Management:**
  - GARCH models are crucial for accurately forecasting volatility in **derivative pricing**, **portfolio management**, and **risk assessment**.
  - By incorporating both past volatility and past residuals, GARCH models provide better forecasts for periods where volatility persists.
- **Value at Risk (VaR):**
  - Similar to ARCH, GARCH models are used to compute **Value at Risk (VaR)**, an important measure for estimating the potential loss in asset or portfolio value under normal market conditions.

GARCH Process: Basic Equation

GARCH(1,1) Model

$$X_t = \mu + \epsilon_t, \quad \epsilon_t \sim N(0, h_t)$$
$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1}$$

Parameters

- X_t: Observed time series
- ϵ_t: White noise error term with time-varying variance h_t
- h_t: Conditional variance, depending on both previous period's squared residuals (ϵ_{t-1}^2) and previous variance (h_{t-1})
- α_0: Constant term, must be positive
- α_1: Coefficient of the previous squared residuals (ϵ_{t-1}^2)
- β_1: Coefficient of the previous variance (h_{t-1}), capturing volatility persistence

Explanation

- The variance h_t is influenced by both the previous squared residuals ϵ_{t-1}^2 (as in ARCH models) and the prior period's variance h_{t-1}.
- The inclusion of h_{t-1} in the model accounts for **volatility persistence**, enabling GARCH to model long periods of high or low volatility that ARCH models may fail to capture.

GARCH(p,q)?

- **Generalized GARCH Model:**
  - GARCH(p,q) extends the GARCH(1,1) model by allowing for more lags in both past squared residuals (ARCH terms) and past variances (GARCH terms).
  - This generalization provides greater flexibility in capturing complex volatility patterns.
- **Key Parameters:**
  - **p**: The number of lagged squared residuals $(\epsilon_{t-1}^2, \dots, \epsilon_{t-p}^2)$ used in the model (ARCH terms).
  - **q**: The number of lagged variances $(h_{t-1}, \dots, h_{t-q})$ included in the model (GARCH terms).

GARCH(p,q) Equation

$$X_t = \mu + \epsilon_t, \quad \epsilon_t \sim N(0, h_t)$$

$$h_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j h_{t-j}$$

Explanation of Parameters:

- h_t: Conditional variance of X_t, influenced by both past squared residuals and past variances.
- α_0: Constant term (must be positive), determining the baseline variance.
- α_i: Coefficients of the past squared residuals (ARCH terms), capturing the immediate impact of volatility shocks.
- β_j: Coefficients of the past variances (GARCH terms), modeling the persistence of volatility over time.

Understanding p and q

- **ARCH Terms (p):**
  - The parameter p represents the number of previous squared residuals affecting current volatility. Higher p allows the model to capture immediate volatility shocks over multiple time periods.
- **GARCH Terms (q):**
  - The parameter q determines how much of the previous periods' variance impacts the current period's variance. A higher q models prolonged periods of high or low volatility, capturing volatility persistence.

Real-Life Application of GARCH(p,q)

- **Stock Market Volatility:**
  - Complex market conditions, like **prolonged periods of volatility**, can be better modeled with GARCH(p,q) than simpler GARCH(1,1) models.
- **Interest Rates:**
  - **Monetary policy announcements** can cause volatility patterns that exhibit persistence over multiple periods, requiring a model with more lag terms for both ARCH and GARCH components.
- **Portfolio Risk Management:**
  - GARCH(p,q) models are often used by financial institutions to **forecast risk** in highly volatile markets, allowing for a more nuanced understanding of how shocks and volatility persist over time.

Why Use GARCH(p,q)?

- **Flexibility in Volatility Modeling:**
  - GARCH(p,q) offers more flexibility than the basic GARCH(1,1) by allowing multiple lag terms, making it ideal for capturing complex volatility dynamics.
  - It's particularly useful in scenarios where volatility patterns are influenced by both immediate shocks and long-term persistence.
- **Risk Forecasting:**
  - GARCH(p,q) models improve the accuracy of **Value at Risk (VaR)** calculations, especially in portfolios that exhibit complex volatility patterns over time.

Conclusion

- **GARCH models** are an extension of ARCH models, capturing both **volatility clustering** and **volatility persistence** in financial markets.
- Whether analyzing **stocks**, **currencies**, or **interest rates**, GARCH provides more robust modeling for **risk forecasting** and **financial decision-making**, especially when volatility is expected to persist over time.
- **GARCH(p,q)** models provide a robust framework for capturing both short-term volatility shocks and long-term volatility persistence, making them essential for **financial risk management** in complex markets.
- By adjusting the values of p and q, analysts can fine-tune the model to best capture the volatility dynamics of specific time series, whether in **stock prices**, **exchange rates**, or **interest rates**.