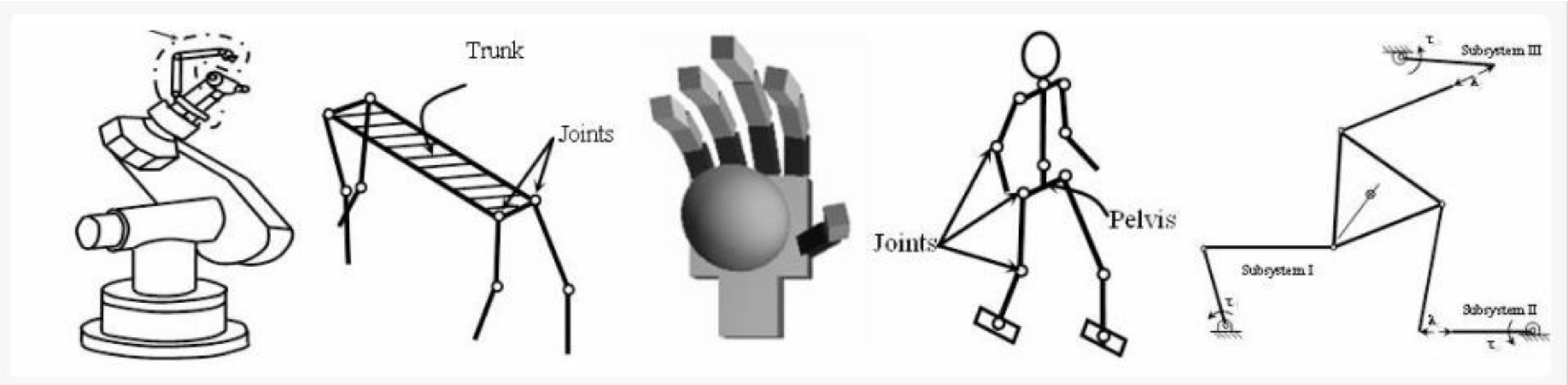
Recursive Dynamics Simulator (ReDySim)

- MATLAB-based recursive solver for dynamic analysis of robotic and multibody systems
- Uses the concept of Decoupled Natural Orthogonal Complement (DeNOC) based approach for dynamic modeling
- Dynamic analysis can be performed without creating solid model
- Considerable improvements in terms of the computational time and numerical accuracy.



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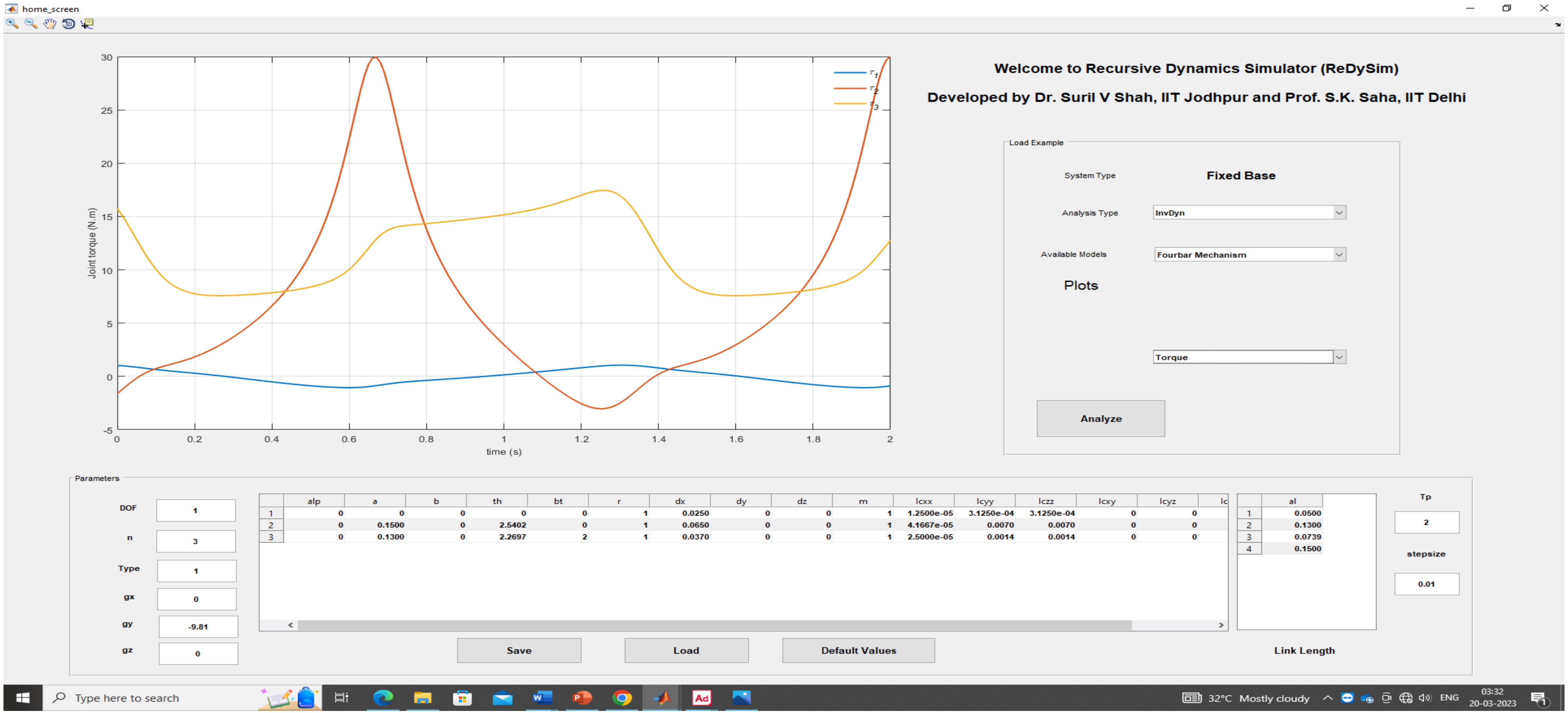

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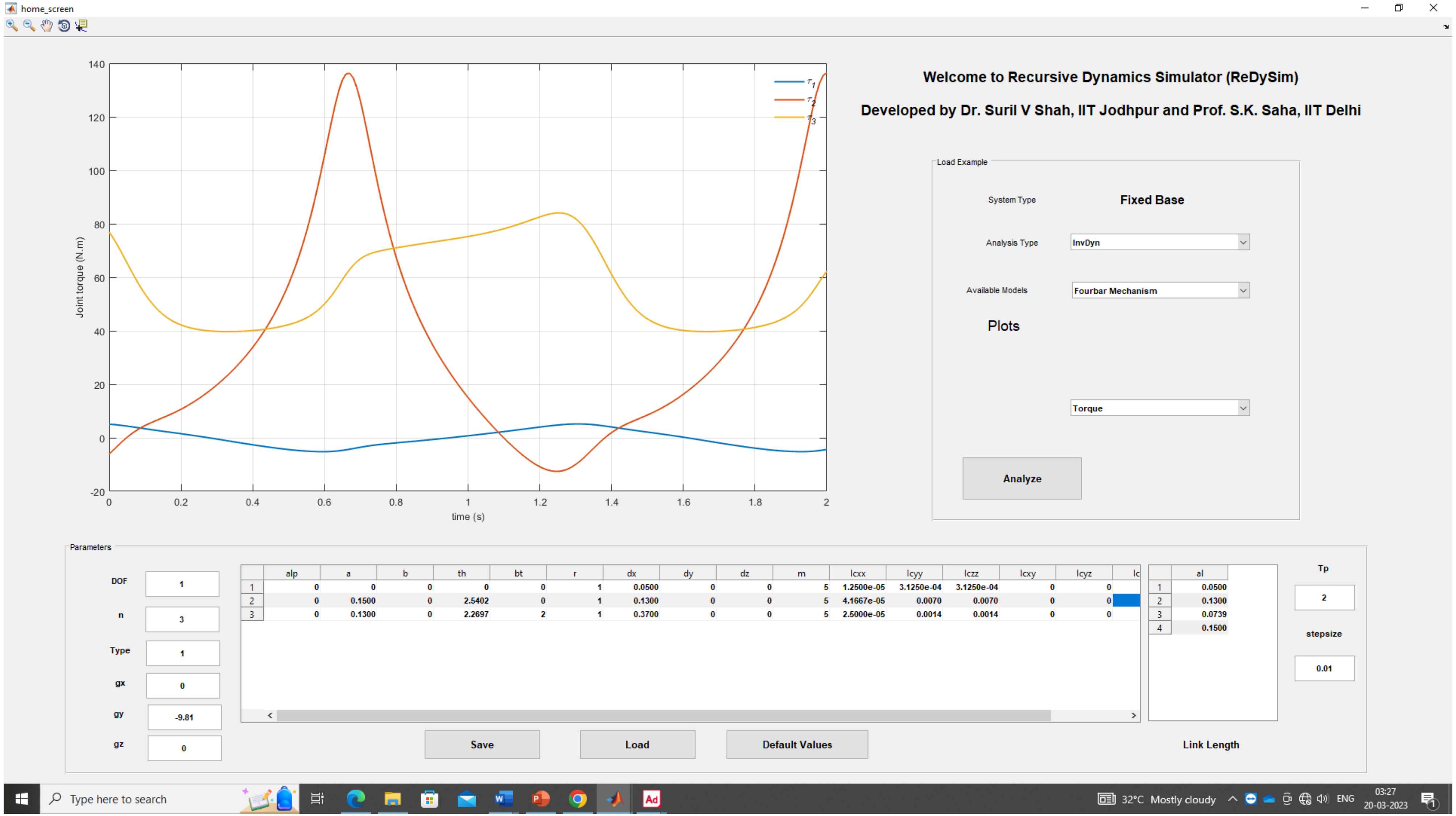

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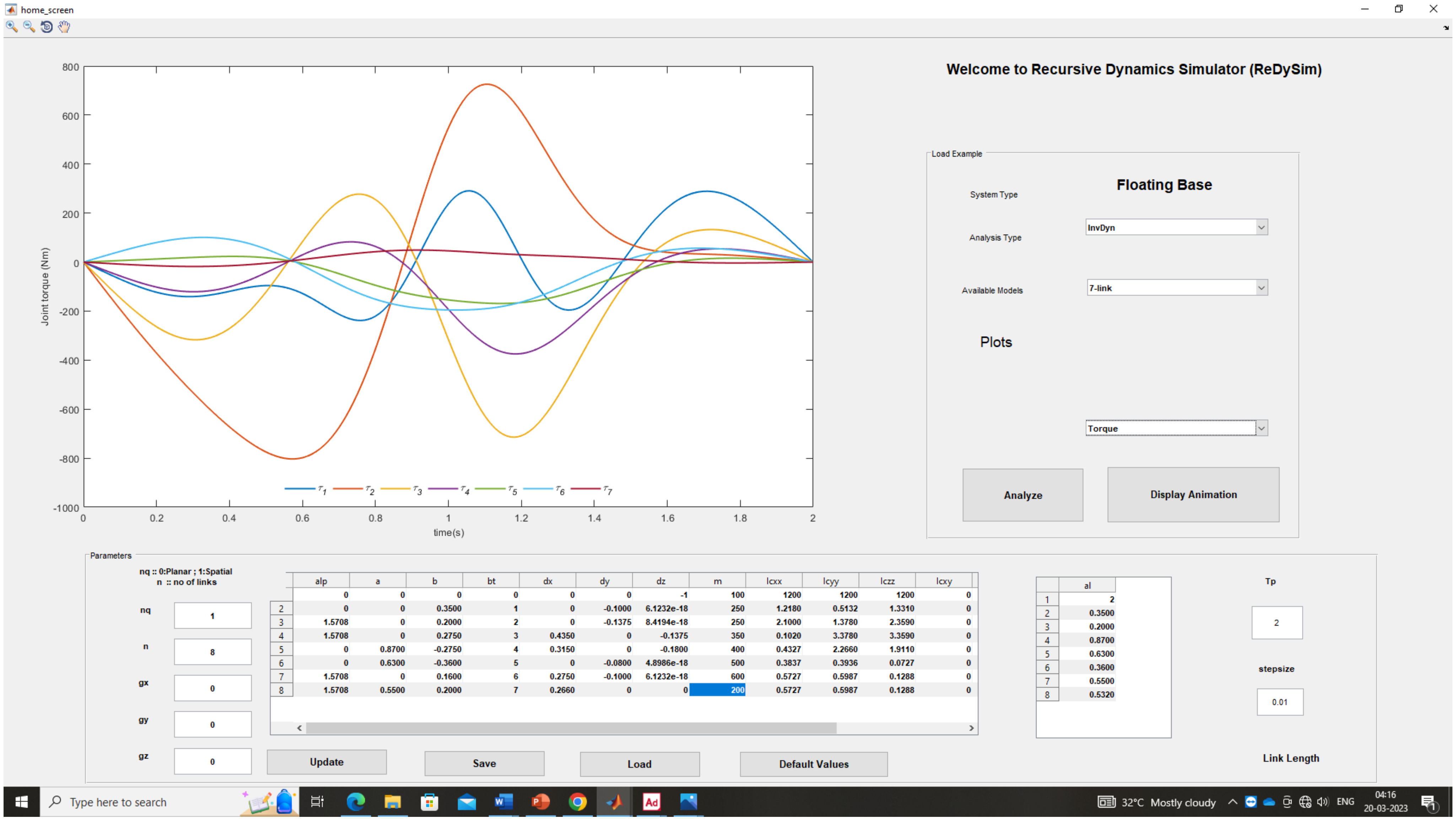

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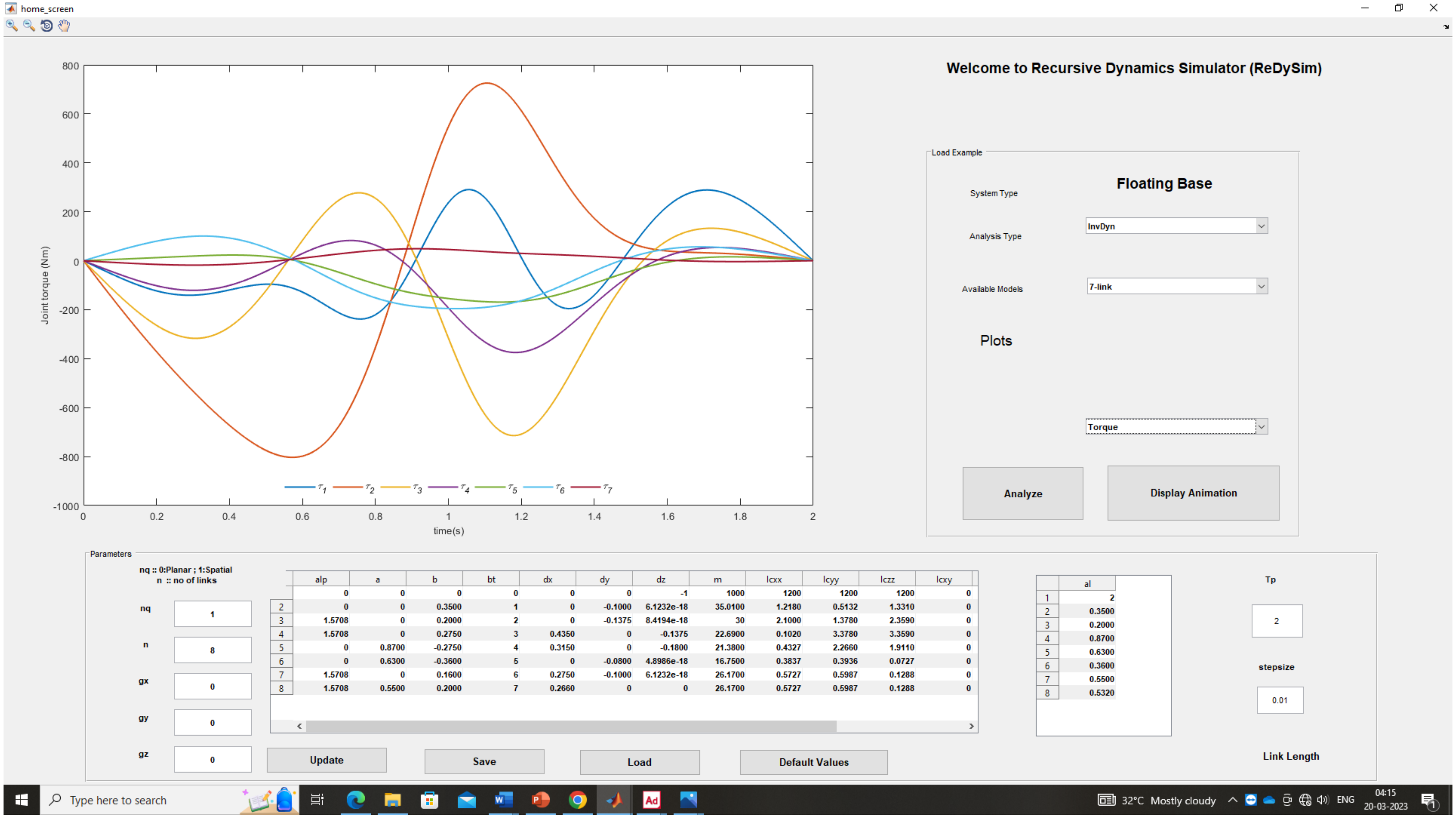

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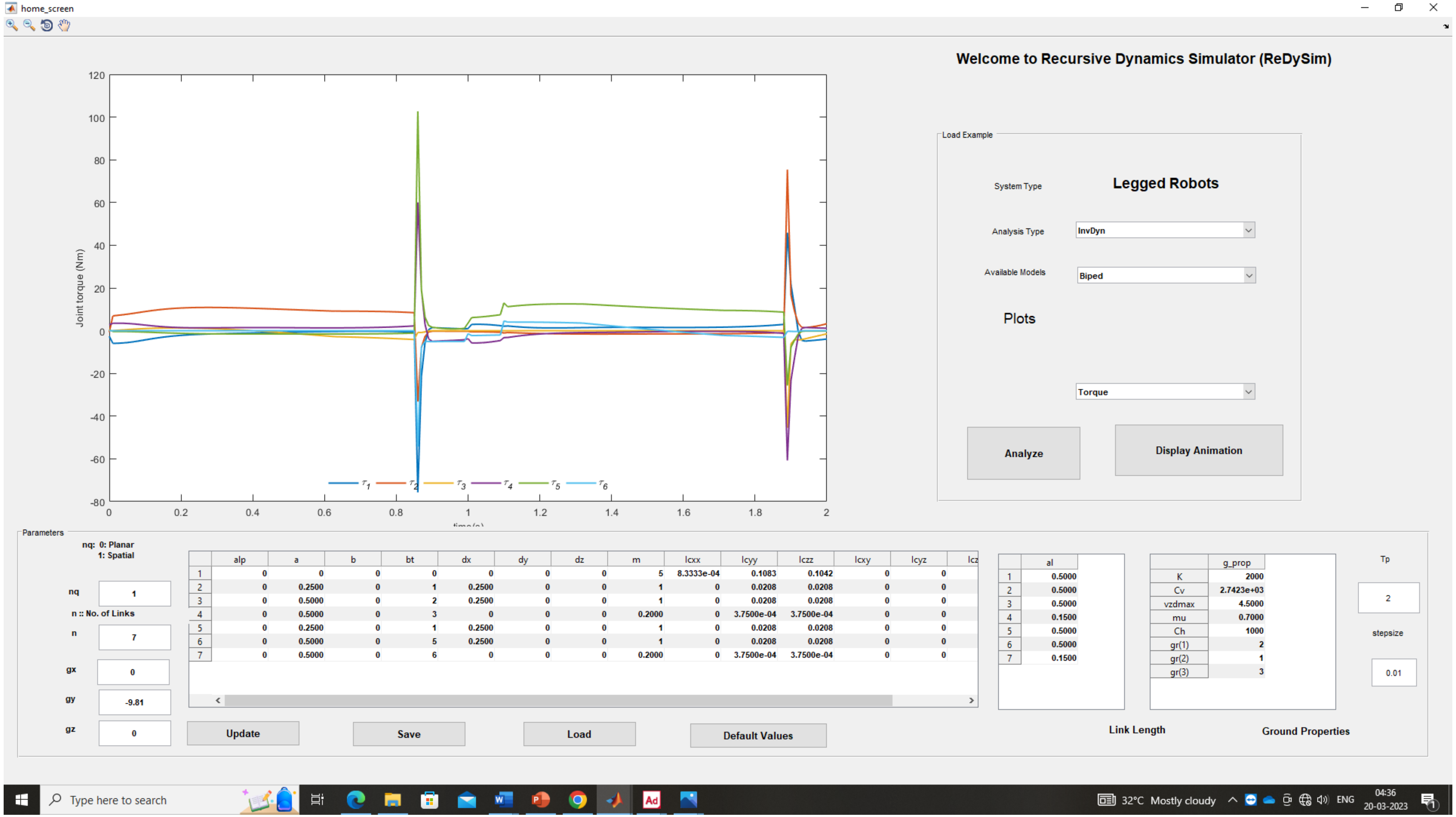

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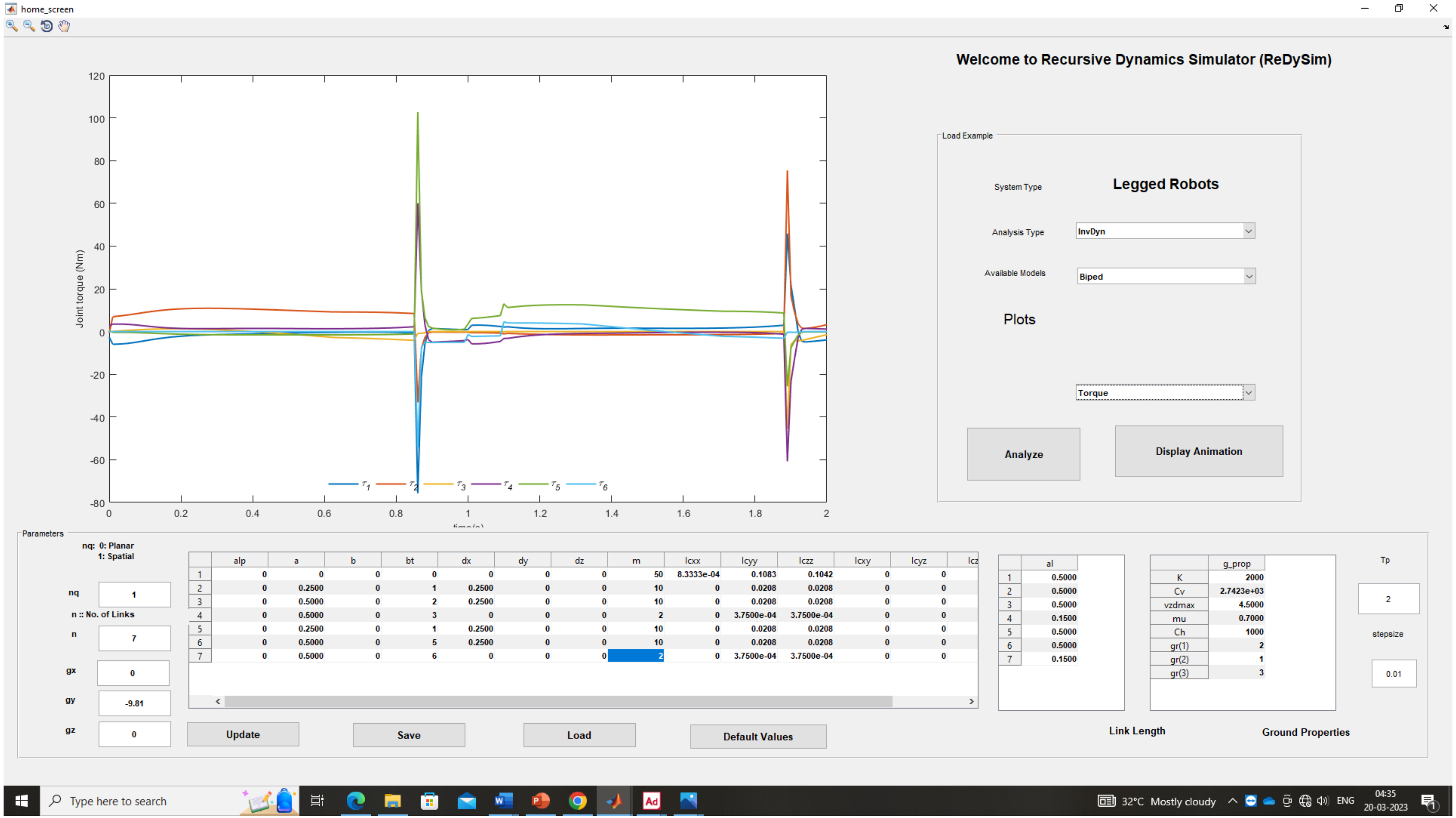


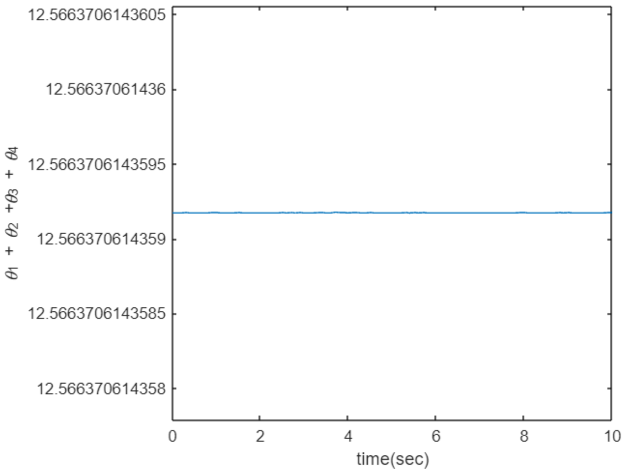
Recursive Dynamics Simulator (ReDySim)

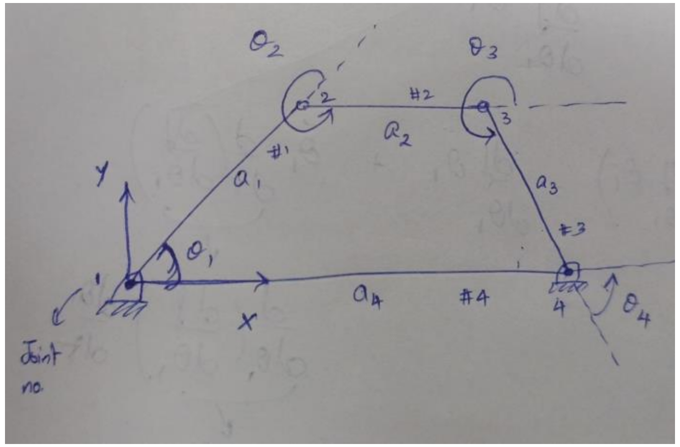
- MATLAB-based recursive solver for dynamic analysis of robotic and multibody systems
- Uses the concept of Decoupled Natural Orthogonal Complement (DeNOC) based approach for dynamic modeling
- Dynamic analyses can be performed without creating solid model
- considerable improvement in terms of the computational time and numerical accuracy.

Inverse Kinematics : 4 Bar Mechanism

- a_1=0.06m
- a_2=0.14m
- a_3=0.16m
- a_4=0.2m

- Initial and Given conditions :
  - $\theta_1 = 0\ and \theta_1 = pi/4$
  - $\dot{\theta}_1 = 1 \frac{rad}{sec}$
  - $\ddot{\theta}_1 = 0 \frac{rad}{sec^2}$

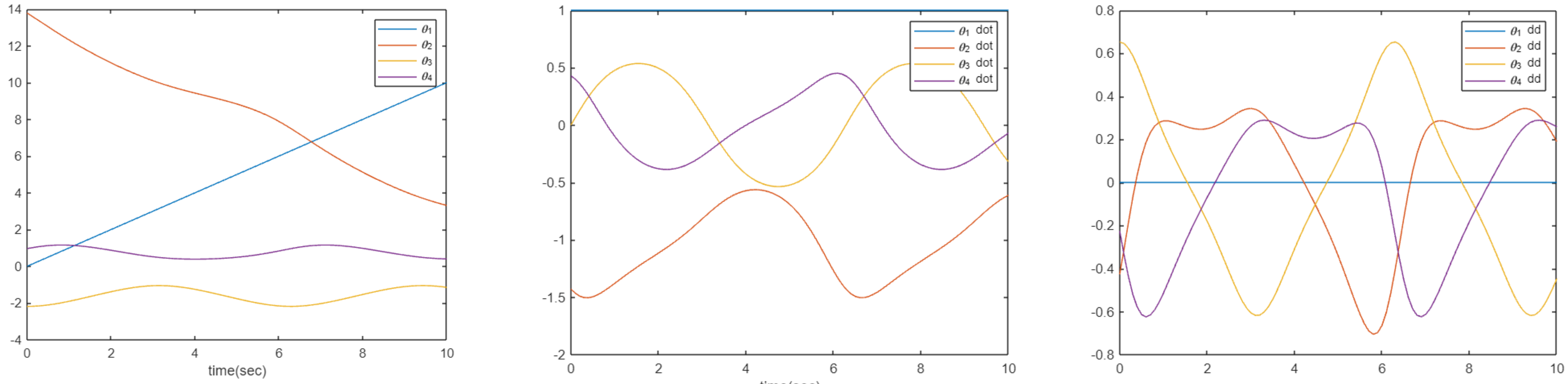




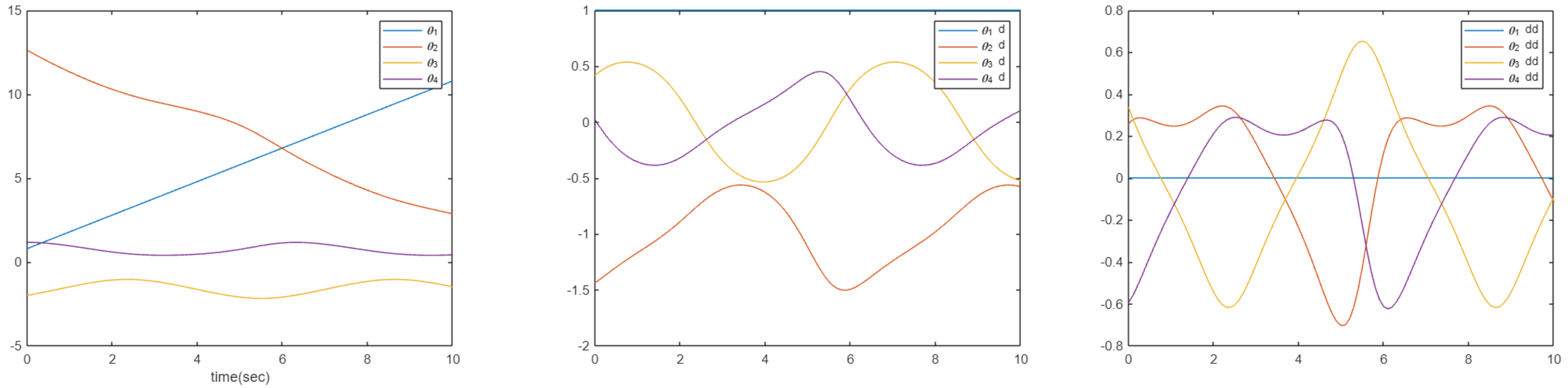

```
%Constraints
eqn1 = (l1*cos(theta_1)) + (l2*cos(theta_1+theta_2)) +
(l3*cos(theta_1+theta_2+theta_3)) - l4;    % x-coord of J-4
eqn2 = (l1*sin(theta_1)) + (l2*sin(theta_1+theta_2)) +
(l3*sin(theta_1+theta_2+theta_3));
eqn3 = theta_1 + theta_2 + theta_3 + theta_4 - (4*pi); % y-coord of J-4

solution = solve([eqn1,eqn2,eqn3],[theta_2,theta_3,theta_4]);
```

Simulation Results

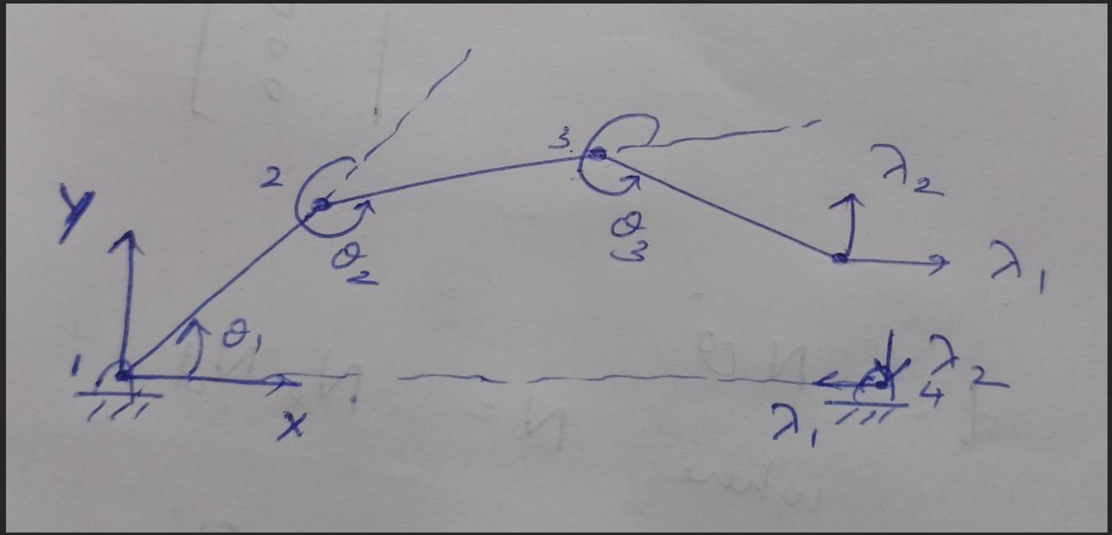
- For 0 initial angle :



- For 45^o initial angle :



Inverse Dynamics Methodology using DeNOC



$A_{i-1,I}$' matrix:

```
A1_2d = [Id vec_cross(-a1_2);ZO Id];
A2_3d = [Id vec_cross(-a2_3);ZO Id];
A3_4d = [Id vec_cross(-a3_4);ZO Id];
```

N_l and N_d matrices:

```
A1_2d_2 = [Id vec_cross(a1_2);ZO Id];
A2_3d_2 = [Id vec_cross(a2_3);ZO Id];
A_d2 = [Id_1 -1*A1_2d_2 ZO_1;ZO_1 Id_1 -1*A2_3d_2;ZO_1 ZO_1 Id_1];
Nu = inv(A_d2);
Nl = transpose(Nu);

Nd = [p1 ZO_c ZO_c;ZO_c p2 ZO_c;ZO_c ZO_c p3];
N = Nl*Nd;
```

Mass matrix:

```
%% COM positions
d1 = [l1*cos(theta_1)/2;l1*sin(theta_1)/2;0];
d2 = [l2*cos(theta_1+theta_2_1)/2;l2*sin(theta_1+theta_2_1)/2;0];
d3 =
[l3*cos(theta_1+theta_2_1+theta_3_1)/2;l3*sin(theta_1+theta_2_1+theta_3_1)/2;0];

M1 = [I1 m1*vec_cross(d1);-m1*vec_cross(d1) m1*Id];
M2 = [I2 m2*vec_cross(d2);-m2*vec_cross(d2) m2*Id];
M3 = [I3 m3*vec_cross(d3);-m3*vec_cross(d3) m3*Id];
```

Wrench Matrix and Final equation

Angular Velocity Cross product matrix:

```
%Finding ang velocities
ang1 = diff([0;0;theta_1],theta_1)*th1_d_i;
ang2 = diff([0;0;theta_1+theta_2_1],theta_1)*th1_d_i;
ang3 = diff([0;0;theta_1+theta_2_1+theta_3_1],theta_1)*th1_d_i;

angC_1 = [vec_cross(ang1) ZO;ZO vec_cross(ang1)];
angC_2 = [vec_cross(ang2) ZO;ZO vec_cross(ang2)];
angC_3 = [vec_cross(ang3) ZO;ZO vec_cross(ang3)];
```

External wrench:

```
w_1_e = [0;0;tau;0;-m1*g;0];
w_2_e = [0;0;0;0;-m2*g;0];
w_3_e = [0;0;0;0;-m3*g;0];
w_e = [w_1_e;w_2_e;w_3_e];
```

Wrench due to Lagrange multipliers:

```
w_1_lam = zeros(6,1);
w_2_lam = zeros(6,1);
w_4_lam = [0;0;0;lambda_1;lambda_2;0];
w_3_lam = A3_4d*w_4_lam;

w_lam = [w_1_lam;w_2_lam;w_3_lam];
```

Twist Matrix:

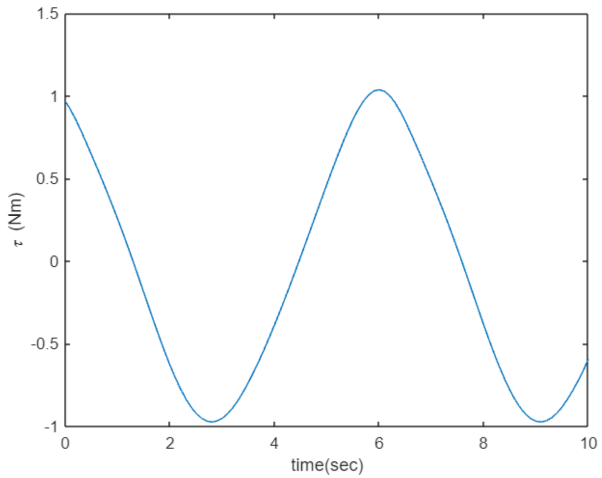
```
twist = Nl*Nd*diff(th_vec,theta_1)*th1_d_i;
```

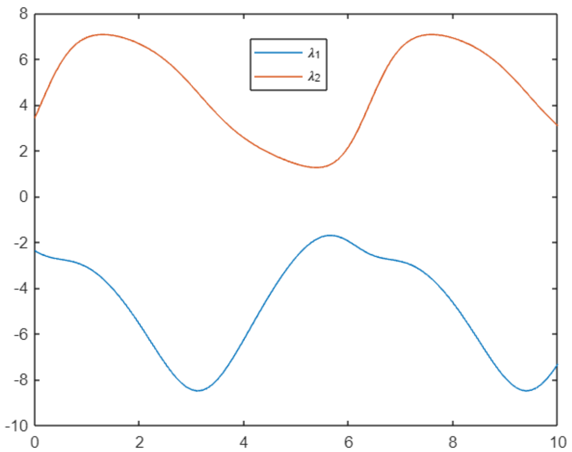
Final equation:

```
f_eqn = transpose(N)*((M_f*twist_d)+(W_f*M_f*E_f*twist)-(w_lam+w_e));
```

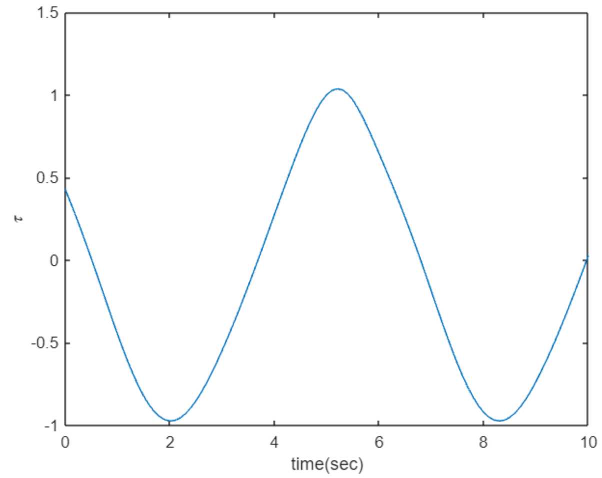
Simulation results

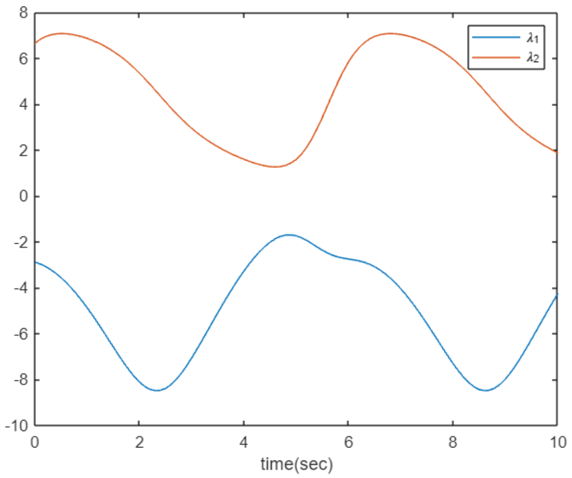
- For initial position of crank = 0 rad



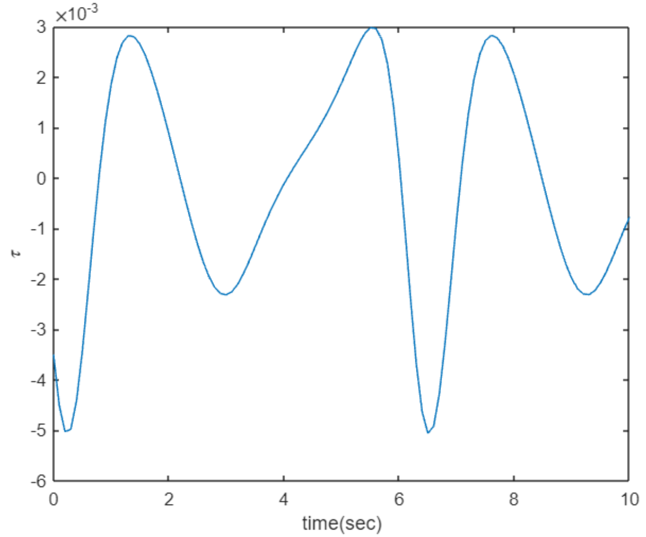


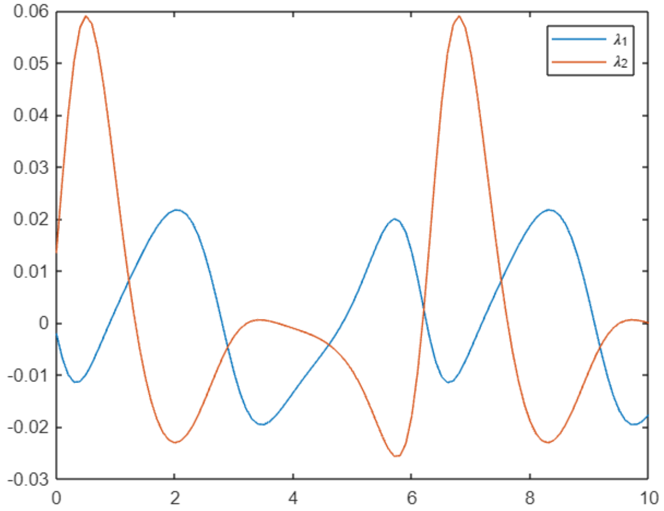
- For initial position of crank = pi/4 rad





- For initial position of crank = 0 rad without gravity





```
f_eqn =
```

$$
\left(\begin{array}{c}
\frac{3\,\mathrm{th}_{\mathrm{fdd}}}{2500} - \sigma_8 + \sigma_{14} + \sigma_{22}\,\sigma_4 + \sigma_7 - \sigma_2 - \sigma_5 + \sigma_1 + \sigma_{18}\,\sigma_3 + \sigma_{10} + \frac{3\cos(\mathrm{th}_f)\left(\frac{7\cos(\sigma_{25})\,\sigma_{12}}{100} - \sigma_{26} + \sigma_{27} - \frac{7\sin(\sigma_{25})\,\sigma_{11}}{100} + \frac{49}{5}\right)}{50} + \sigma_9 + \frac{3\sin(\mathrm{th}_f)\left(\frac{3\,\mathrm{th}_{\mathrm{fd}}^2\cos(\mathrm{th}_f)}{50} + \frac{7\sin(\sigma_{25})\,\sigma_{12}}{100} + \sigma_{24} + \frac{7\cos(\sigma_{25})\,\sigma_{11}}{100}\right)}{50} + \sigma_{13} - \sigma_6 \\
-\sigma_8 + \sigma_{14} + \frac{7\sin(\sigma_{31})\,\sigma_4}{50} + \sigma_7 - \sigma_2 - \sigma_5 + \sigma_1 + \frac{7\cos(\sigma_{31})\,\sigma_3}{50} + \sigma_{10} + \sigma_9 + \sigma_{13} - \sigma_6 \\
\frac{32\,\sigma_{33}}{1875\,\sigma_{49}} - \sigma_2 + \sigma_1 + \sigma_{10} + \sigma_9 + \frac{32\,\sigma_{48}\,\sigma_{47}}{1875\,\sigma_{49}^2}
\end{array}\right)
$$

Forward Dynamics Methodology and problem faced

- Unknown variables : $\theta_1, \theta_2, \theta_3, \theta_4, \lambda_1, \lambda_2$
- We can find $\theta_2, \theta_3, \theta_4$ after we find θ_1 using loop closure equations.
- $\theta_1, \lambda_1, \lambda_2$ can be determined by solving the 3 equations obtained from the DeNOC matrix.
- We need to find λ_1, λ_2 in terms of θ_1 and its derivatives.
- Finally, we must solve the second order differential equation of θ_1.

Forward Dynamics using Euler Lagrange formulation : Methodology

- $[\boldsymbol{M}(\mathbf{q})]\{\ddot{\mathbf{q}}\} + [\boldsymbol{C}(\mathbf{q},\dot{\mathbf{q}})]\{\dot{\mathbf{q}}\} + [\boldsymbol{G}(\mathbf{q})] = \boldsymbol{\tau} + (\boldsymbol{J}_{\eta q})^{\boldsymbol{T}}[\boldsymbol{\lambda}]$
- $\boldsymbol{M}(\mathbf{q}) = \sum \boldsymbol{J}_{viq}{}^{\boldsymbol{T}} m_i \boldsymbol{J}_{viq} + \sum \boldsymbol{J}_{\omega iq}{}^{\boldsymbol{T}} \boldsymbol{I}_i \boldsymbol{J}_{\omega iq}$ $\longrightarrow$

```
%Constructing matrices
syms a b c;
eqn_theta = subs(eqn_theta,[diff(theta_dot,t),diff(phi_dot,t),diff(psi_dot,t)],[a,b,c]);
eqn_phi = subs(eqn_phi,[diff(theta_dot,t),diff(phi_dot,t),diff(psi_dot,t)],[a,b,c]);
eqn_psi = subs(eqn_psi,[diff(theta_dot,t),diff(phi_dot,t),diff(psi_dot,t)],[a,b,c]);
eqn_theta = simplify(subs(eqn_theta,[diff(theta,t),diff(phi,t),diff(psi,t)],[dtht,dphi,dpsi]));
eqn_phi = simplify(subs(eqn_phi,[diff(theta,t),diff(phi,t),diff(psi,t)],[dtht,dphi,dpsi]));
eqn_psi = simplify(subs(eqn_psi,[diff(theta,t),diff(phi,t),diff(psi,t)],[dtht,dphi,dpsi]));

M1 = simplify(formula([(diff(eqn_theta,a)) (diff(eqn_theta,b)) (diff(eqn_theta,c))]));
M2 = simplify(formula([(diff(eqn_phi,a)) (diff(eqn_phi,b)) (diff(eqn_phi,c))]));
M3 = simplify(formula([(diff(eqn_psi,a)) (diff(eqn_psi,b)) (diff(eqn_psi,c))]));
M = [M1;M2;M3];
```

- $[C_{ij}] = 0.5 \times \sum_{k=1}^{n} \left(\frac{\partial M_{ij}}{\partial q_k} + \frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{kj}}{\partial q_i} \right) \dot{q}_k$ $\longrightarrow$

```
C_matrix = zeros(3,3);
C_matrix = symmatrix(C_matrix);
C_matrix = symmatrix2sym(C_matrix);

for i=1:3
    for j=1:3
        for k=1:3
            temp = simplify(formula(0.5*((diff(M(i,j),q(k,1)) + ( ...
                diff(M(i,k),q(j,1))) - (diff(M(k,j),q(i,1))))*q_dot(k,1)));
            C_matrix(i,j) = C_matrix(i,j) + temp;
        end
    end
end
```

- $[G_i] = \frac{\partial V}{\partial q_i}$ $\longrightarrow$

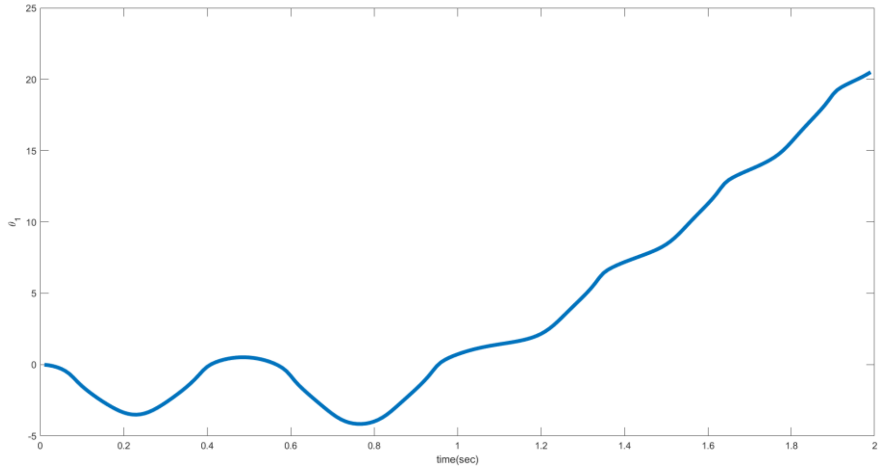
```
G_matrix = zeros(3,1);
G_matrix = symmatrix(G_matrix);
G_matrix = symmatrix2sym(G_matrix);
PE = PE1 + PE2 + PE3;

for i=1:3
    G_matrix(i,1) = diff(PE,q(i,1));
end
```

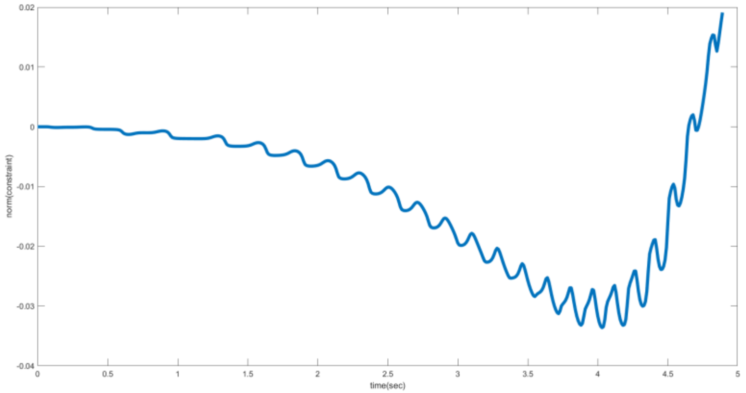
- $\boldsymbol{J}_{\eta q}$ $\longrightarrow$

```
psi_Matrix = formula([diff(cons1,q(1,1)) diff(cons1,q(2,1)) diff(cons1,q(3,1)); ...
    diff(cons2,q(1,1)) diff(cons2,q(2,1)) diff(cons2,q(3,1))]);
psi_Matrix_dot = formula(diff(psi_Matrix,t));
```

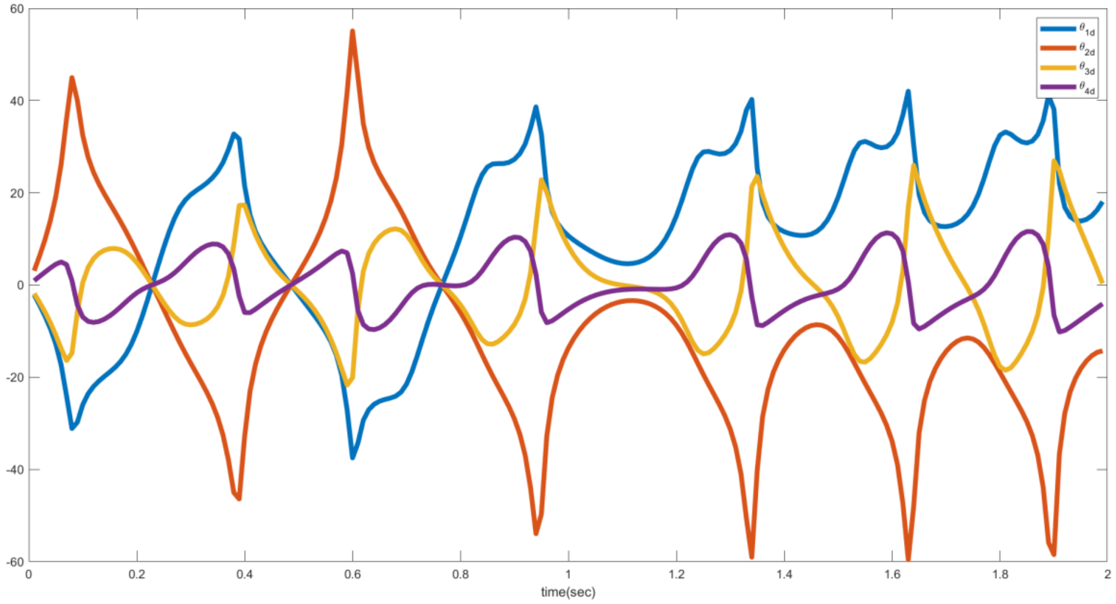
$$\boxed{\boldsymbol{\lambda} = -\left(\boldsymbol{J}_{\eta q}\boldsymbol{M}^{-1}\boldsymbol{J}_{\eta q}{}^{\boldsymbol{T}}\right)^{-1}\left(\dot{\boldsymbol{J}}_{\eta q}\{\dot{\mathbf{q}}\} + \boldsymbol{J}_{\eta q}\boldsymbol{M}^{-1}(\boldsymbol{\tau} - \boldsymbol{C}\{\dot{\mathbf{q}}\} - \boldsymbol{G})\right)}$$

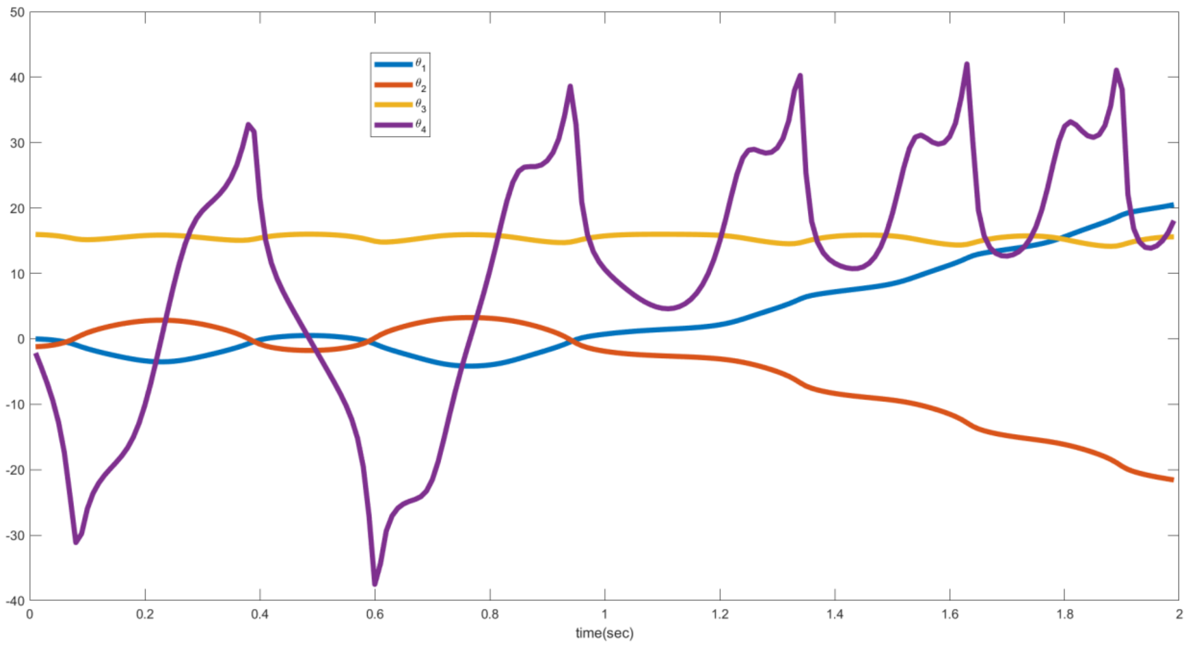
Forward Dynamics using Euler Lagrange formulation



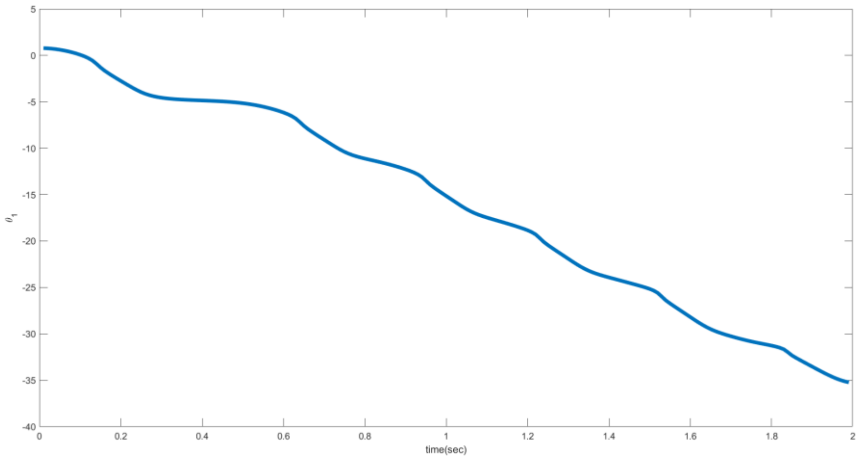


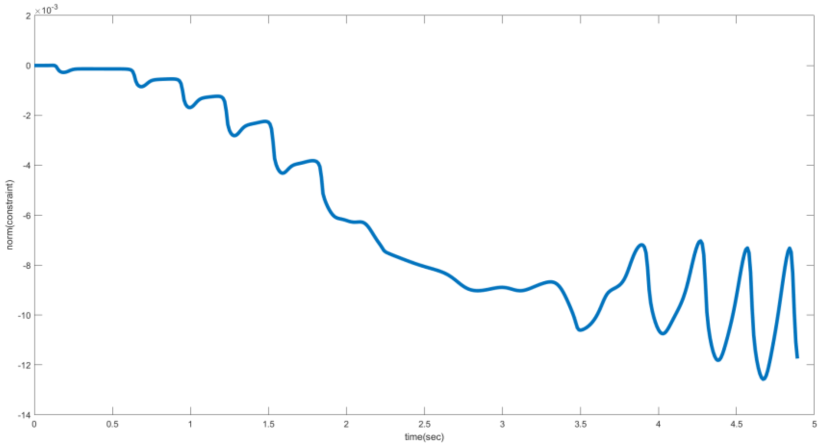
- When the Crank is horizontal :



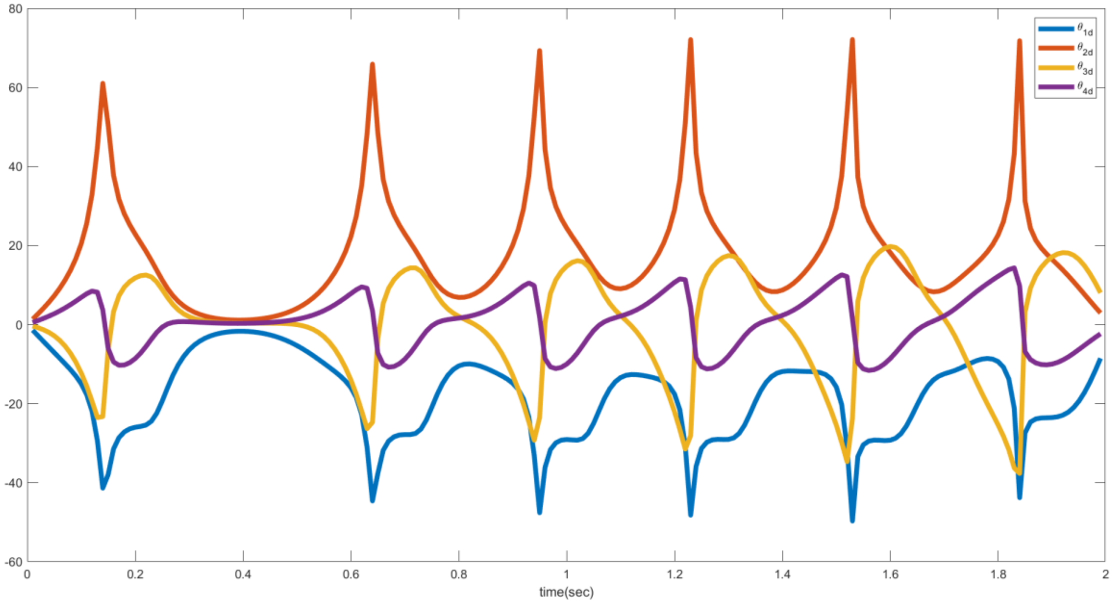


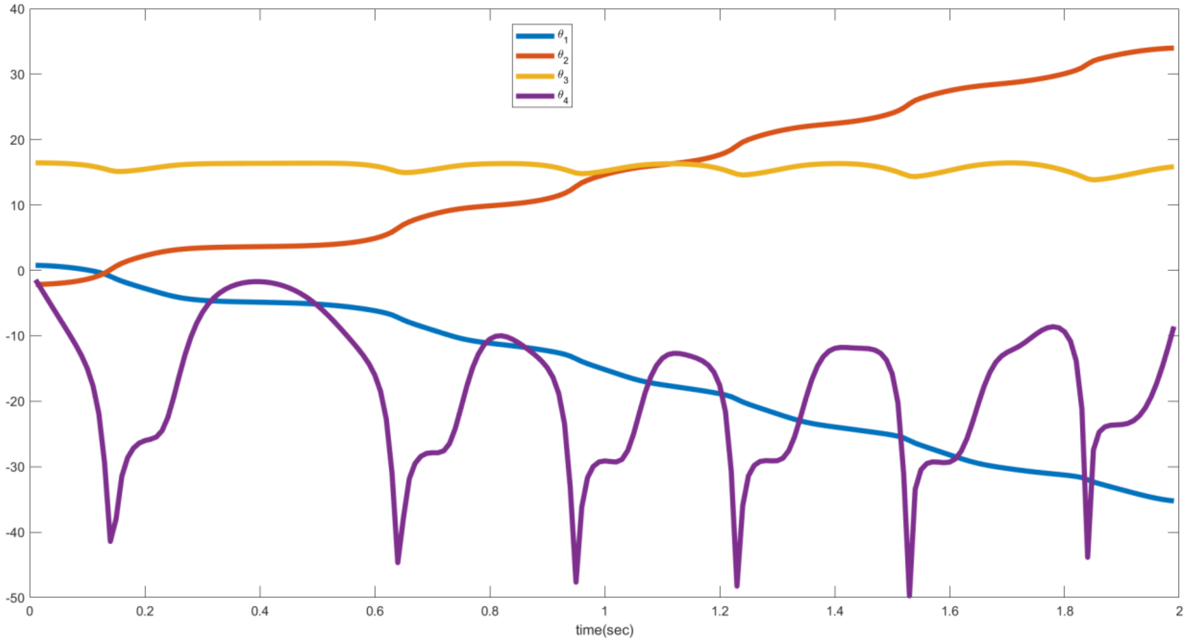
Forward Dynamics using Euler Lagrange formulation





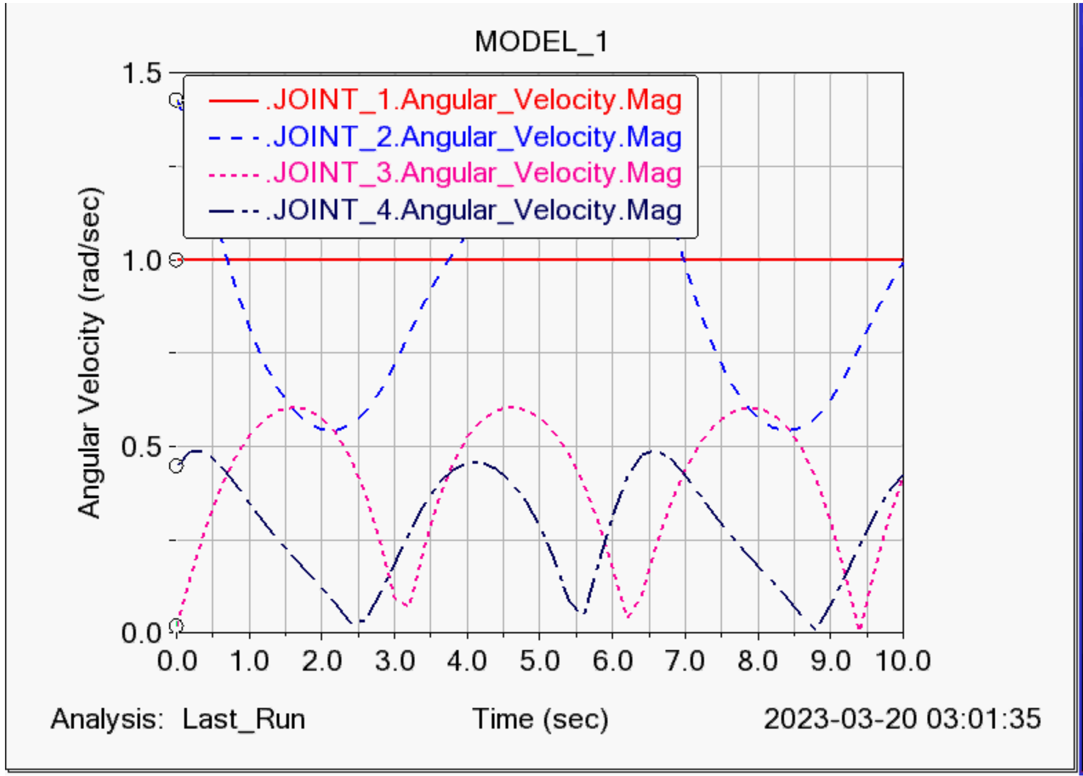
- When the Crank is at 45deg:

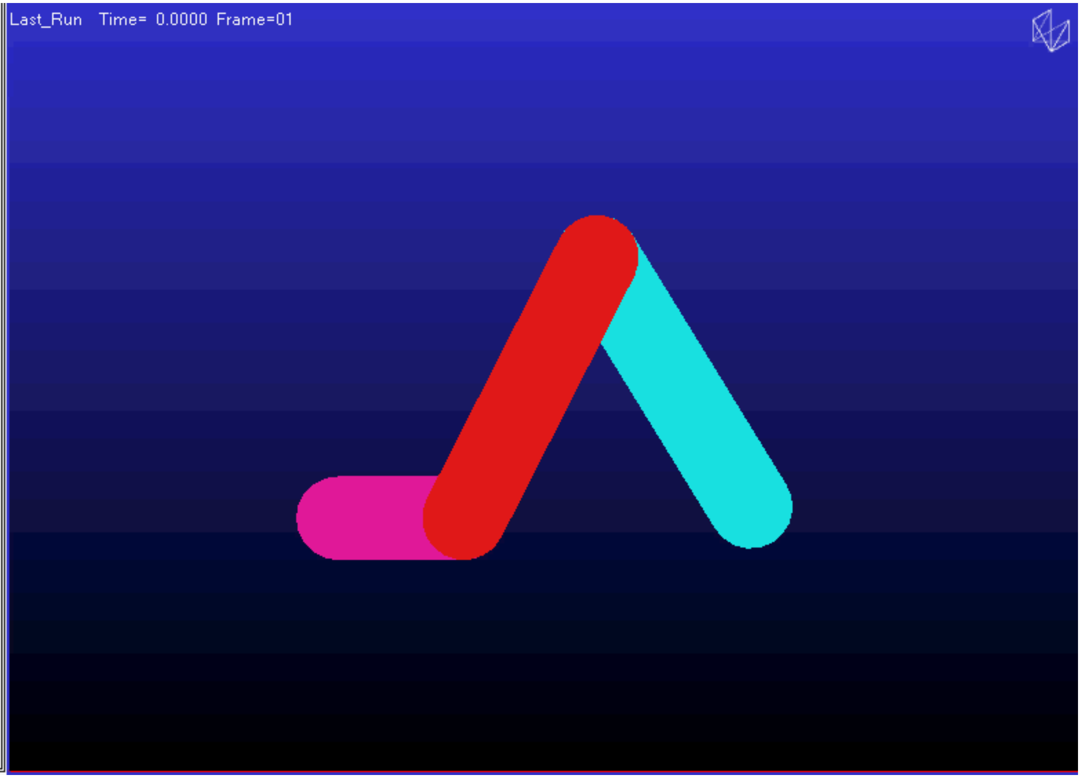




Four bar Using ADAMS View

Crank=0.06m,coupler=0.14m,output link=0.16,base=0.2m

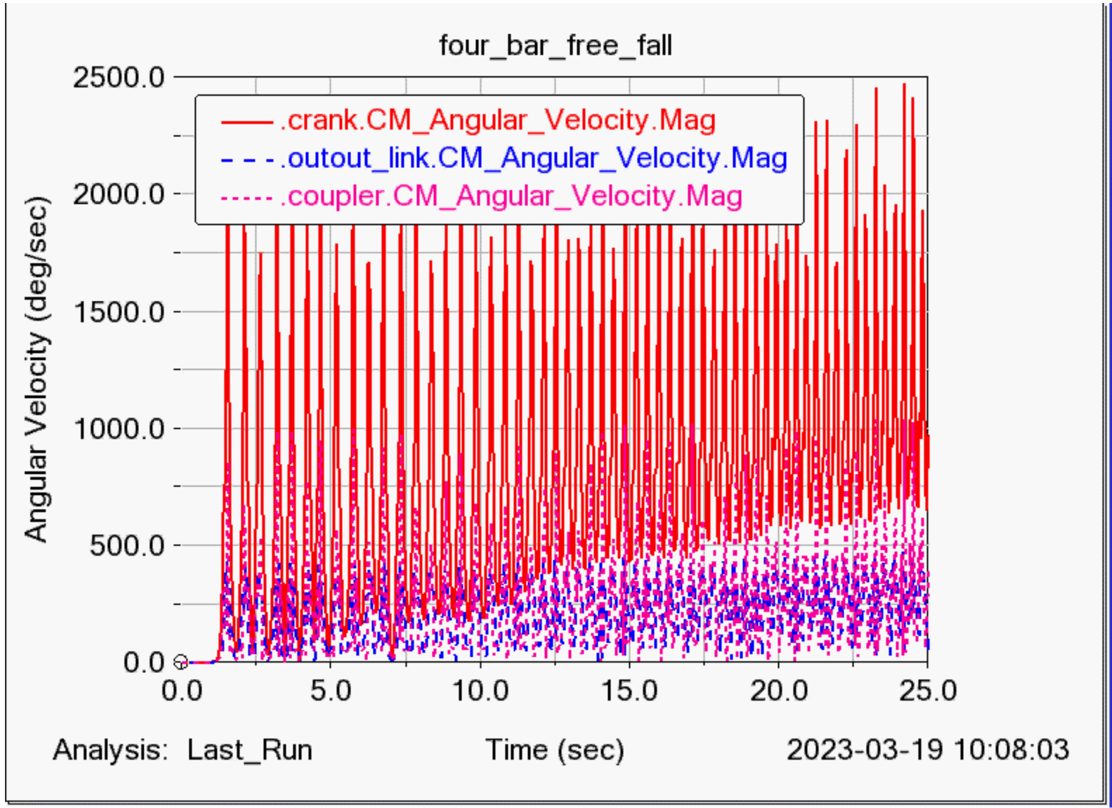


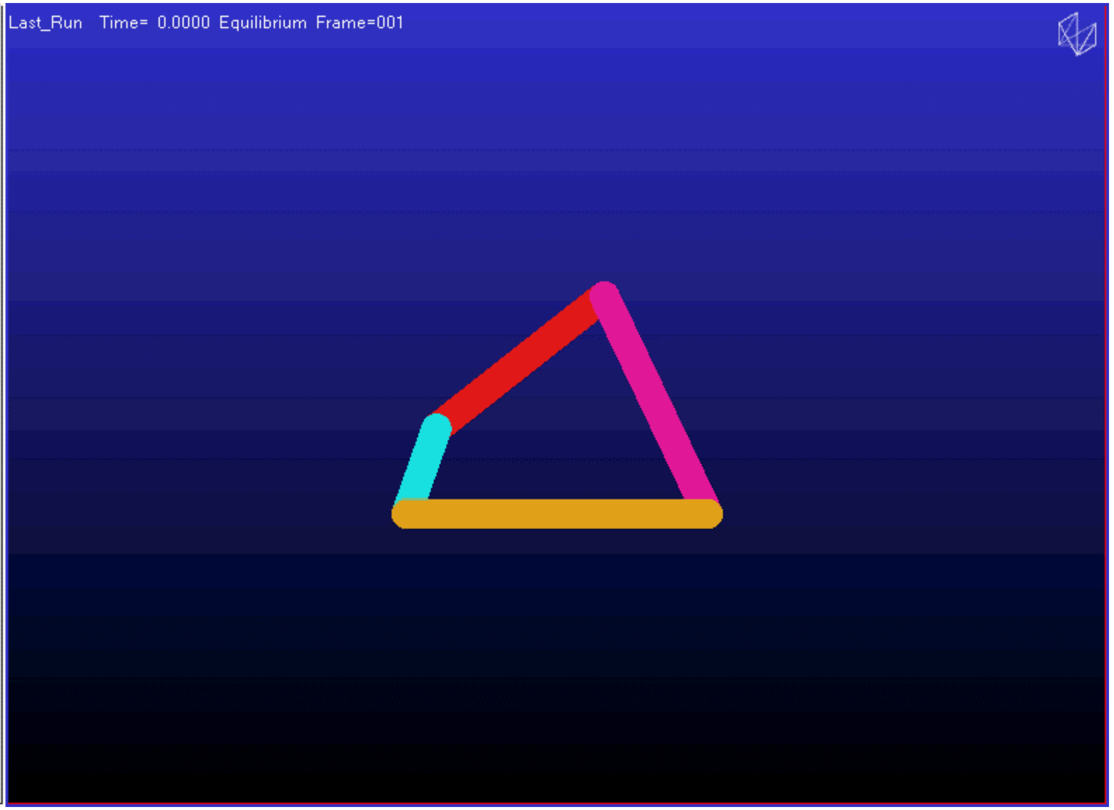


Vid.1:constant angular velocity at crank base (1 rad/sec)

Four bar Using ADAMS View

Crank=0.06m,coupler=0.14m,output link=0.16,base=0.2m





Vid.2: free fall under gravity without any torque at crank-base joint