ELL-700: LINEAR SYSTEMS THEORY

ASSIGNMENT – 1

KALMAN DECOMPOSITION

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2023MEZ8467

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```
function [T,A_n,B_n,C_n] = Kalman_main(A,B,C)

n = size(A,1);
m = size(B,2);
p = size(C,1);

if(n<=0 || m<=0 || p<=0 || n<=max(m,p))
    disp("Invalid dimensions");
```

The function takes in three matrices (A,B,C) corresponding to the system parameters and gives out the transformation matrix and the transformed matrices (A_n,B_n,C_n) after Kalman decomposition.

'n' here is the total number of state variables, 'm' the total number of input variables and 'p' is the total number of output variables.

The "if" condition is used to check the consistency of the system equation, ensuring that the system is valid based on the dimensions of each input matrix.

```
else
    %Finding Controllability matrix
    iter = A*B;
    con = [B iter];
    for i = 1:n-2
        iter = A*iter;
        con = [con iter];
    end

    %Finding observability matrix
    iter2 = C*A;
    obs = [C;iter2];
    for j = 1:n-2
        iter2 = iter2*A;
        obs = [obs;iter2];
    end
```

Once the conditions on matrix dimensions are satisfied, we calculate the controllability and observability matrix. We can find the reachable subspace by finding the image of controllability matrix, and similarly find the unobservable subspace by finding the null-space of observability matrix.

In the first *for* loop, [B AB] is first initiated and A^2B, A^3B.....$A^{n-1}B$ are appended in each loop using *iter*.

In the second *for* loop for the observability matrix, [C;CA] is first initiated and CA^2,CA^3...CA^{n-1} are appended to each row in every loop using iter2.

Thus the controllability and the observability matrices are constructed using the *for* loops.

```
    reach = orth(con,1e-4);
    unobs = null(obs,1e-4);
    unreach = null(transpose(reach),1e-4);
    observ = null(transpose(unobs),1e-4);
```

Once the controllability and the observability matrices are constructed, we use them to find the reachable, unreachable, observable and unobservable subspaces from R^n.

orth function from MATLAB gives the orthonormal basis vectors of the Range space or Image of controllability matrix. A tolerance of 10^{-4} was used to round off all the values which are lesser than the tolerance to 0. This was done after encountering an error which removed some of the vectors from the intersection subspace due to very minor differences.

Unobservable subspace is calculated using the *null* function which gives the null-space of observability matrix in terms of orthonormal vectors.

Then the unreachable and observable subspaces can be determined by taking the complement of the reachable and unobservable subspaces. The relation between the null-space and complement is used to determine the complement subspace.

```
%Constructing intersection matrices
    S_cob = intersection(reach,unobs);
    S_co = intersection(reach,observ);
    S_cbob = intersection(unreach,unobs);
    S_cbo = intersection(unreach,observ);
```

Using the 4 subspaces determined in the previous code-block, the controllable but unobservable(S_cob), controllable and observable(S_co), uncontrollable and unobservable(S_cbob) and uncontrollable but observable(S_cbo) subspaces are determined.

```
function inter = intersection(x,y)
%INTERSECTION Summary of this function goes here
%   Detailed explanation goes here
kernel = null([x y],1e-4);
p1 = size(x,2);
p2 = size(y,2);

P_1 = kernel(1:p1,:);
P_2 = kernel(p1+1:size(kernel,1),:);
inter = x*P_1;
end
```

The intersection function is used to find the orthonormal basis of the intersection subspace of any two subspaces.

kernel stores the null-space of the union of basis vectors from ‘x’ and ‘y’ subspaces.

If kernel is an empty matrix, then it means that there are no common vectors in subspaces ‘x’ and ‘y’. If it is non-empty, then the first p1 rows of each column gives all the vectors which are common in both subspaces in terms of the coefficients of basis vectors from subspace ‘x’. Thus we can find the basis vectors of intersection of ‘x’ and ‘y’ subspace by multiplying ‘x’ to ‘P_1’ or ‘y’ to ‘P_2’.

```
  T = [S_cob S_co S_cbob S_cbo];  %Transformation matrix
    A_n = T\(A*T);
    B_n = T\B;
    C_n = C*T;
```

Once the 4 subspaces are determined, we can find the transformation matrix and the new system matrices are given by:

$$A_n = T^{-1}AT$$

$$B_n = T^{-1}B$$

$$C_n = CT$$