MCL738 – Dynamics of Multibody Systems

Lab Assignment – 6

4-Bar Mech: Newton-Rapshon Method

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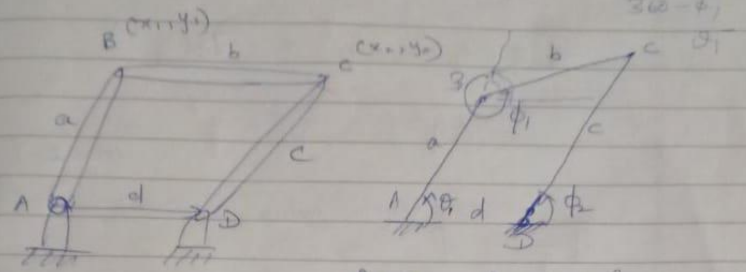
Qn 1)

MBD lab - 6



g.c = $\{\theta, \phi_1, \phi_2\}$

Breaking Joint C, we get

[Note: Since fixed link is shortest, it is a double crank mechanism.]

Constraint eqns:

(I) $a\cos\theta_1 + b\cos(\theta_1+\phi_1) = d + c\cos\phi_2$

(II) $a\sin\theta_1 + b\sin(\theta_1+\phi_1) = c\sin\phi_2$

DoF = 3 − 2 = 1

$\boxed{S + L < m + n \Rightarrow \text{Grashof mech}}$

Now once we fix $\theta_1 = \omega t$

We can figure out ϕ_1 & ϕ_2 from eqn (I) & (II)

Diff (1) & (2), we get $\dot\phi_1$ & $\dot\phi_2$ in terms of $\dot\theta_1$

Similarly double diff of (1) & (2) gives $\ddot\phi_1$ & $\ddot\phi_2$ in terms of $\ddot\theta_1$

Solve this using NR for multiple variables, multiple eqns.

In this case $\vec{u} = (\theta_1)$ $\vec{v} = (\phi_1, \phi_2)$

$\bar{q} = (\vec{u}, \vec{v})^T$

$\Phi_1(\bar{q}) = a\cos\theta_1 + b\cos(\theta_1+\phi_1) - d - c\cos\phi_2 = 0$

$\Phi_2(\bar{q}) = a\sin\theta_1 + b\sin(\theta_1+\phi_1) - c\sin\phi_2 = 0$

Using position analysis

$$\bar{\Phi}_{\bar{q}}(q^{i})\,\Delta\bar{q}^{i} = -\bar{\Phi}(\bar{q}^{i})$$

$$\Delta\bar{q}^{i} = \bar{q}^{i+1} - \bar{q}^{i}$$

$m = 2$ (ϕ_1, ϕ_2)

$n = 1$ (θ_1)

$\bar{\Phi}_{\bar{q}} \rightarrow m \times (n+m)$ matrix

passive variables

Active variables

$$\bar{\Phi}_{\bar{q}}(\bar{q}^{i}) = \begin{bmatrix} \partial\Phi_1/\partial\phi_1 & \partial\Phi_1/\partial\phi_2 & \partial\Phi_1/\partial\theta_1 \\ \partial\Phi_2/\partial\phi_1 & \partial\Phi_2/\partial\phi_2 & \partial\Phi/\partial\theta_1 \end{bmatrix}$$

$$\bar{q} = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \theta_1 \end{bmatrix}$$

In terms of passive joint variables alone

$$\bar{\Phi}_{\bar{u}}(\bar{u}^{i})\,\Delta\bar{u}^{i} = -\bar{\Phi}_{\bar{v}}(\bar{v}^{i})\,\Delta\bar{v}^{i} - \bar{\Phi}(\bar{q}^{i})$$

$$\bar{q}^{0} = \begin{Bmatrix} \bar{u}^{int} \\ \bar{v}^{init} \end{Bmatrix}$$

Assuming $\phi_1^0 = 20^\circ$, $\phi_2^0 = 10^\circ$

for $\theta_1^0 = 0^\circ$ //

$$\therefore \bar{q}^{0} = \begin{Bmatrix} 20^\circ \\ 10^\circ \\ 0^\circ \end{Bmatrix}$$

Now $\theta = \omega t$

Taking $\omega = 5^\circ/sec$

$\theta^i = \theta^{i-1} + \omega t$ (or)

Simulating with a time step of .5 secs

$\{\theta\} = \{0, 0.5, 1 \ldots \ldots 360\}_{(1\times 721)}$ $\quad \frac{360-0}{0.5} + 1 = 721$

Now onto velocity analysis

$$[\bar{\bar{\Phi}}_{\bar{u}}]\dot{\bar{u}} = -\bar{\bar{\Phi}}_{v}\dot{\bar{v}}$$

here $\bar{\bar{\Phi}}_{\bar{u}}$ & $\bar{\bar{\Phi}}_{v}$ are functions of $(\bar{u}, \bar{v})$

Sub the results from position analysis

$$\dot{\bar{v}} = \omega = \dot{\theta}_1$$

Finally accn analysis

$$\left(\bar{\bar{\Phi}}_{\bar{q}}\dot{\bar{q}}\right)_{\bar{q}} \Longrightarrow \begin{array}{l} \cdot\underbrace{\left(\frac{\partial\bar{\Phi}_1}{\partial\phi_1}\dot{\phi}_1 + \frac{\partial\bar{\Phi}_1}{\partial\phi_2}\dot{\phi}_2 + \frac{\partial\bar{\Phi}_1}{\partial\theta_1}\dot{\theta}_1\right)}_{\psi_1} \\ \cdot\underbrace{\left(\frac{\partial\bar{\Phi}_2}{\partial\phi_1}\dot{\phi}_1 + \frac{\partial\bar{\Phi}_2}{\partial\phi_2}\dot{\phi}_2 + \frac{\partial\bar{\Phi}_2}{\partial\theta_1}\dot{\theta}_1\right)}_{\psi_2} \end{array}$$

$$\downarrow$$

$$\begin{bmatrix} \frac{\partial\psi_1}{\partial\phi_1} & \frac{\partial\psi_1}{\partial\phi_2} & \frac{\partial\psi_1}{\partial\theta_1} \\ \frac{\partial\psi_2}{\partial\phi_1} & \frac{\partial\psi_2}{\partial\phi_2} & \frac{\partial\psi_2}{\partial\theta_1} \end{bmatrix}$$

Finally we get

$$\bar{\bar{\Phi}}_{\bar{u}}\ddot{\bar{u}} = -\bar{\bar{\Phi}}_{\bar{v}}\ddot{\bar{v}} - \left(\bar{\bar{\Phi}}_{\bar{q}}\dot{\bar{q}}\right)_{\bar{q}}\dot{\bar{q}}$$

In this case

$$\ddot{\bar{v}} = \ddot{\theta}_1 = 0$$

To get $\left(\bar{\Phi}_q \dot{\bar{q}}\right)_{\bar{q}}$

We can diff $\bar{\Phi}_1$ & $\bar{\Phi}_2$ (both constr eqns) with time and then diff both eqns partially with $\bar{q}$

Sub $\dot{\bar{q}}$ and $\bar{q}$ obtained in position & velocity analysis

- Diff constraint eqn once

(i) $-a\dot{\theta}_1 \sin\theta_1 - b\sin(\theta_1+\phi_1)(\dot{\theta}_1+\dot{\phi}_1) = -c\dot{\phi}_2 \sin\phi_2$

(ii) $a\dot{\theta}_1\cos\theta_1 + b(\dot{\theta}_1+\dot{\phi}_1)\cos(\theta_1+\phi_1) = c\dot{\phi}_2\cos\phi_2$

Diff it again

(i) $+a\ddot{\theta}_1\sin\theta_1 + a(\dot{\theta}_1)^2\cos\theta_1 + b(\ddot{\theta}_1+\ddot{\phi}_1)\sin(\theta_1+\phi_1)$
$+b(\dot{\theta}_1+\dot{\phi}_1)^2\cos(\theta_1+\phi_1) = +c\ddot{\phi}_2\sin\phi_2$
$+c(\dot{\phi}_2)^2\cos\phi_2$

(ii) $a\ddot{\theta}_1\cos\theta_1 - a(\dot{\theta}_1)^2\sin\theta_1 + b(\ddot{\theta}_1+\ddot{\phi}_1)\cos(\theta_1+\phi_1) - b(\dot{\theta}_1+\dot{\phi}_1)^2\sin(\theta_1+\phi_1)$
$= c\ddot{\phi}_2\cos\phi_2 - c(\dot{\phi}_2)^2\sin\phi_2$

$\Rightarrow$ (i) $\ddot{\theta}_1\left(a\sin\theta_1 + b\sin(\theta_1+\phi_1)\right) + \ddot{\phi}_1\left(b\sin(\theta_1+\phi_1)\right) - \ddot{\phi}_2\left(c\sin\phi_2\right)$
$= -a\cos\theta_1(\dot{\theta}_1)^2 + c\cos\phi_2(\dot{\phi}_2)^2 - b\cos(\theta_1+\phi_1)(\dot{\theta}_1+\dot{\phi}_1)^2$

(ii) $\ddot{\theta}_1\left(a\cos\theta_1 + b\cos(\theta_1+\phi_1)\right) + \ddot{\phi}_1\left(b\cos(\theta_1+\phi_1)\right) - \ddot{\phi}_2\left(c\cos\phi_2\right)$
$= a\sin\theta_1(\dot{\theta}_1)^2 - c\sin\phi_2(\dot{\phi}_2)^2 + b(\dot{\theta}_1+\dot{\phi}_1)^2\sin(\theta_1$

(i) $\ddot{\theta}_1 [a\sin\theta + b\sin(\theta_1+\phi_1)] + \ddot{\phi}_1 [b\sin(\theta_1+\phi_1)] - \ddot{\phi}_2 [c\sin\phi_2]$
$= -a\cos\theta_1 (\dot{\theta}_1)^2 + c\cos\phi_2 (\dot{\phi}_2)^2 - b\cos(\theta_1+\phi_1)(\dot{\theta}_1+\dot{\phi}_1)$

(ii) $\ddot{\theta}_1 [a\cos\theta_1 + b\cos(\theta_1+\phi_1)] + \ddot{\phi}_1 [b\cos(\theta_1+\phi_1)] - \ddot{\phi}_2 [c\cos\phi_2]$
$= a\sin\theta_1 (\dot{\theta}_1)^2 - c\sin\phi_2 (\dot{\phi}_2)^2 +$
$b(\dot{\theta}_1+\dot{\phi}_1)^2 \sin(\theta_1+\phi_1)$

$\Rightarrow$ $\cancel{(a\sin\theta + b\sin(\theta_1+\phi_1))}$

$$\Rightarrow \begin{bmatrix} b\sin(\theta_1+\phi_1) & -c\sin\phi_2 \\ b\cos(\theta_1+\phi_1) & -c\cos\phi_2 \end{bmatrix} \begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \end{bmatrix} = \begin{bmatrix} -[a\sin\theta_1 + b\sin(\theta_1+\phi_1)] \\ -[a\cos\theta_1 + b\cos(\theta_1+\phi_1)] \end{bmatrix} \ddot{\theta}_1$$

$$+ \begin{bmatrix} (-a\cos\theta_1\,\dot{\theta}_1) & (c\cos\phi_2\,\dot{\phi}_2) & (-b\cos(\theta_1+\phi_1)(\dot{\theta}_1+\dot{\phi}_1)) \\ (a\sin\theta_1\,\dot{\theta}_1) & (-c\sin\phi_2\,\dot{\phi}_2) & (b\sin(\theta_1+\phi_1)(\dot{\theta}_1+\dot{\phi}_1)) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_1+\dot{\theta}_1 \end{bmatrix}$$

Since $\dot{\theta}_1 = \text{const} = K$; $\ddot{\theta}_1 = 0$

$\therefore$

$$\begin{bmatrix} b\sin(\theta_1+\phi_1) & -c\sin\phi_2 \\ b\cos(\theta_1+\phi_1) & -c\cos\phi_2 \end{bmatrix} \begin{bmatrix} \ddot{\phi}_1 \\ \ddot{\phi}_2 \end{bmatrix} = \cancel{[(-ac\theta_1\,\dot{\theta}_1)\ (cc\phi_2\,\dot{\phi}_2)\ (-bc(\theta_1+\phi_1)}$$

$$= \begin{bmatrix} (-ac\theta_1\,\dot{\theta}_1) & (cc\phi_2\,\dot{\phi}_2) & (-bc(\theta_1+\phi_1)(\dot{\theta}_1+\dot{\phi}_1)) \\ (as\theta_1\,\dot{\theta}_1) & (-cs\phi_2\,\dot{\phi}_2) & (bs(\theta_1+\phi_1)(\dot{\theta}_1+\dot{\phi}_1)) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\phi}_2 \\ \dot{\phi}_1+\dot{\theta}_1 \end{bmatrix}$$

$$\vec{P}_x' = a\cos\theta_1\,\hat{i} + a\sin\theta_1\,\hat{j}$$

$$\vec{V}_x' = -a\dot{\theta}_1\sin\theta_1\,\hat{i} + a\dot{\theta}_1\cos\theta_1\,\hat{j}$$

$$|\vec{V}_x'| = a\dot{\theta}_1$$

$$\vec{a}_x' = -a\ddot{\theta}_1\sin\theta_1\,\hat{i} - a(\dot{\theta}_1)^2\cos\theta_1\,\hat{i} + a\ddot{\theta}_1\cos\theta_1\,\hat{j} - a(\dot{\theta}_1)^2\sin\theta_1\,\hat{j}$$

Tangential accn of point B $= a\ddot{\theta}_1$

$P_x^B = a\cos\theta_1$

$P_y^B = a\sin\theta_1$

Similarly

$|v^2| = c\dot{\phi}_2$

$P_x^C = d + c\cos\phi_1$

$P_y^C = c\cos\phi_2$

Tangential accn of point C $= c\ddot{\phi}_2$

Centripetal accn of point B $= (\dot{\theta}_1)^2 a$

Centripetal accn of point C $= (\dot{\phi}_2)^2 c$

MATLAB Code:

```
syms theta_1 phi_1 phi_2;

a = 2.1; b = 1.6; c = 2.6; d = 0.9;

const1 = (a*cos(theta_1)) + (b*cos(theta_1+phi_1)) - d - (c*cos(phi_2));
const2 = (a*sin(theta_1)) + (b*sin(theta_1+phi_1)) - (c*sin(phi_2));

cons = [const1;const2];

jacU = [diff(const1,phi_1) diff(const1,phi_2);diff(const2,phi_1)
diff(const2,phi_2)];
jacV = [diff(const1,theta_1);diff(const2,theta_1)];
q_init_u = [0.2;0.4];

omega = 1; %rad/sec
time = transpose(0:0.05:10);
phi_1_vals = zeros(size(time));
phi_2_vals = zeros(size(time));
q_ud = zeros(2,1);
q_udd = zeros(2,1);
phi_1d_vals = zeros(size(time));
phi_2d_vals = zeros(size(time));

%Acceleration analysis
jacU_acc = [b*sin(theta_1+phi_1),-(c*sin(phi_2));b*cos(theta_1+phi_1),-
(c*cos(phi_2))];
jac_qd = [(-a*cos(theta_1)),(c*cos(phi_2)),(-
b*cos(theta_1+phi_1));(a*sin(theta_1)),(-
c*sin(phi_2)),(b*sin(theta_1+phi_1))];

phi_1dd_vals = zeros(size(time));
phi_2dd_vals = zeros(size(time));

for t = 1:size(time)
    th = omega*time(t);
    [phi1,phi2] = Pose_NR(q_init_u,jacU,jacV,cons,th,phi_1,phi_2,theta_1);
    q_init_u(1) = phi1; q_init_u(2) = phi2;
    phi_1_vals(t) = phi1;
    phi_2_vals(t) = phi2;
    q_ud = -
((vpa(subs(jacU,[phi_1,phi_2,theta_1],[phi1,phi2,th])))\(vpa(subs(jacV,[phi_1
,phi_2,theta_1],[phi1,phi2,th]))))*omega;
    phi_1d_vals(t) = q_ud(1);
    phi_2d_vals(t) = q_ud(2);
    q_udd =
((vpa(subs(jacU_acc,[phi_1,phi_2,theta_1],[phi1,phi2,th])))\(vpa(subs(jac_qd,
```

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[phi_1,phi_2,theta_1],[phi1,phi2,th]))))*[omega*omega;q_ud(2)*q_ud(2);(omega+
q_ud(1))*(omega+q_ud(1))];
     phi_1dd_vals(t) = q_udd(1);
     phi_2dd_vals(t) = q_udd(2);

end
```

```
 %function for Newton-Raphson Iteration of multiple equations
function [phi1,phi2] = Pose_NR(qinit,jacU,jacV,cons,thet,phi_1,phi_2,theta_1)

 qiter = qinit;
 tol = 10^(-6);
 i = 1;

 while true

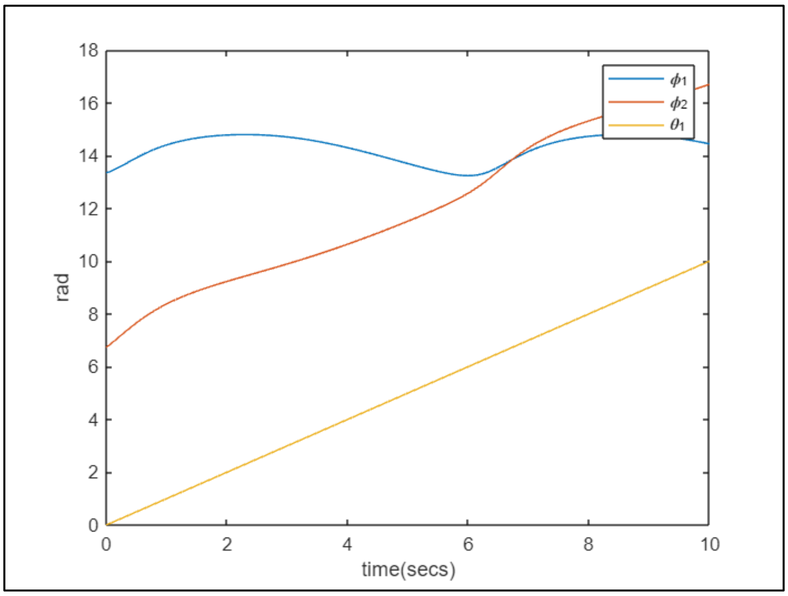
     del_qi = -
(vpa(subs(jacU,[phi_1,phi_2,theta_1],[qiter(1),qiter(2),thet])))\(vpa(subs(co
ns,[phi_1,phi_2,theta_1],[qiter(1),qiter(2),thet])));

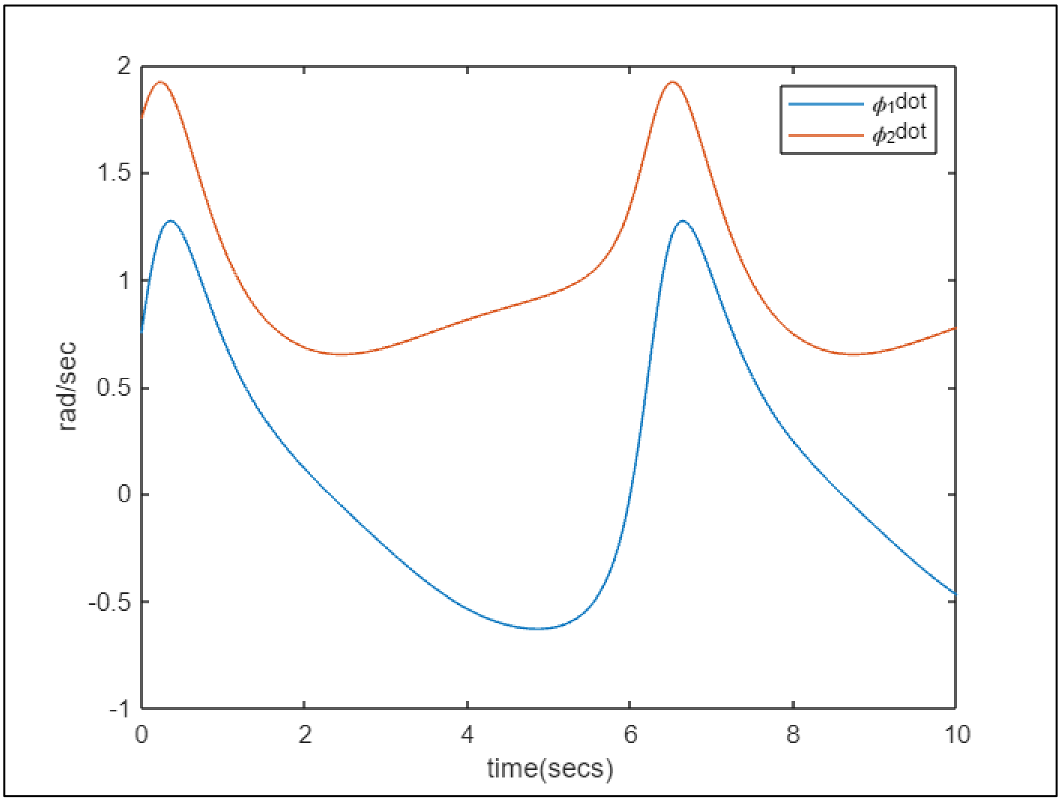
     if all(abs(del_qi)<tol)
         break;
     end

     i = i+1;
     qiter = qiter + del_qi;
 end
 phi1 = qiter(1);
 phi2 = qiter(2);

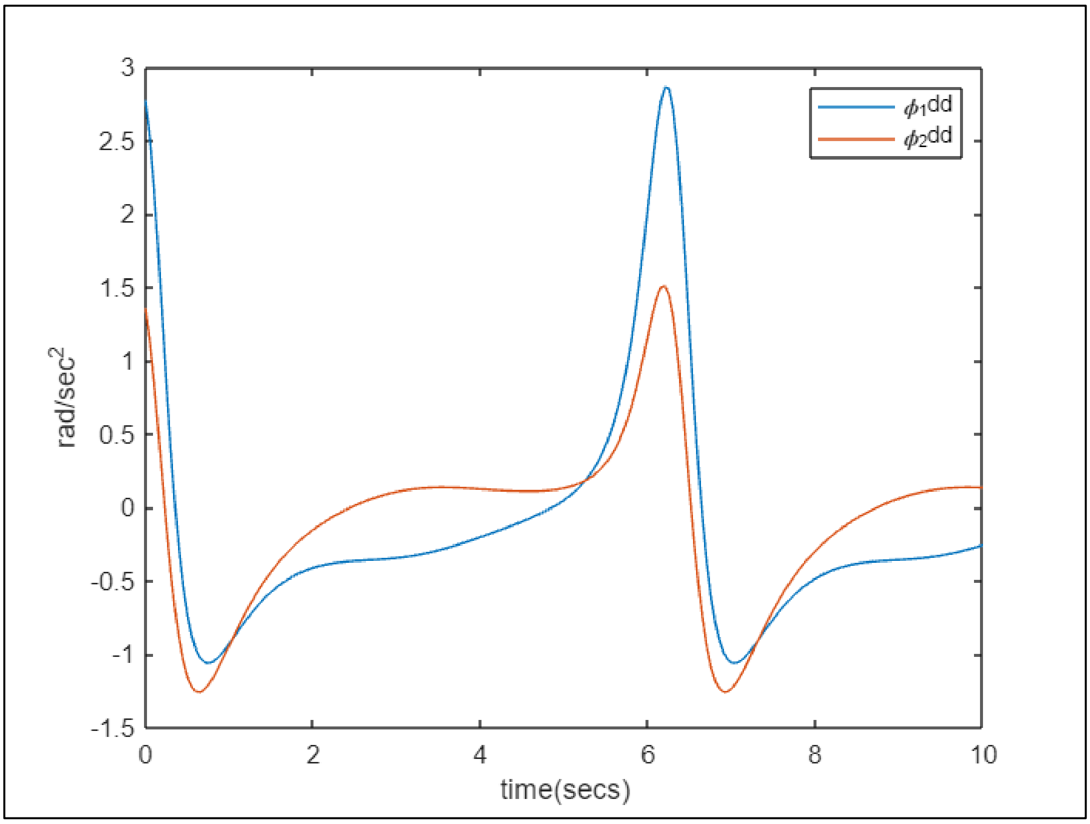
 end
```

MATLAB Output:



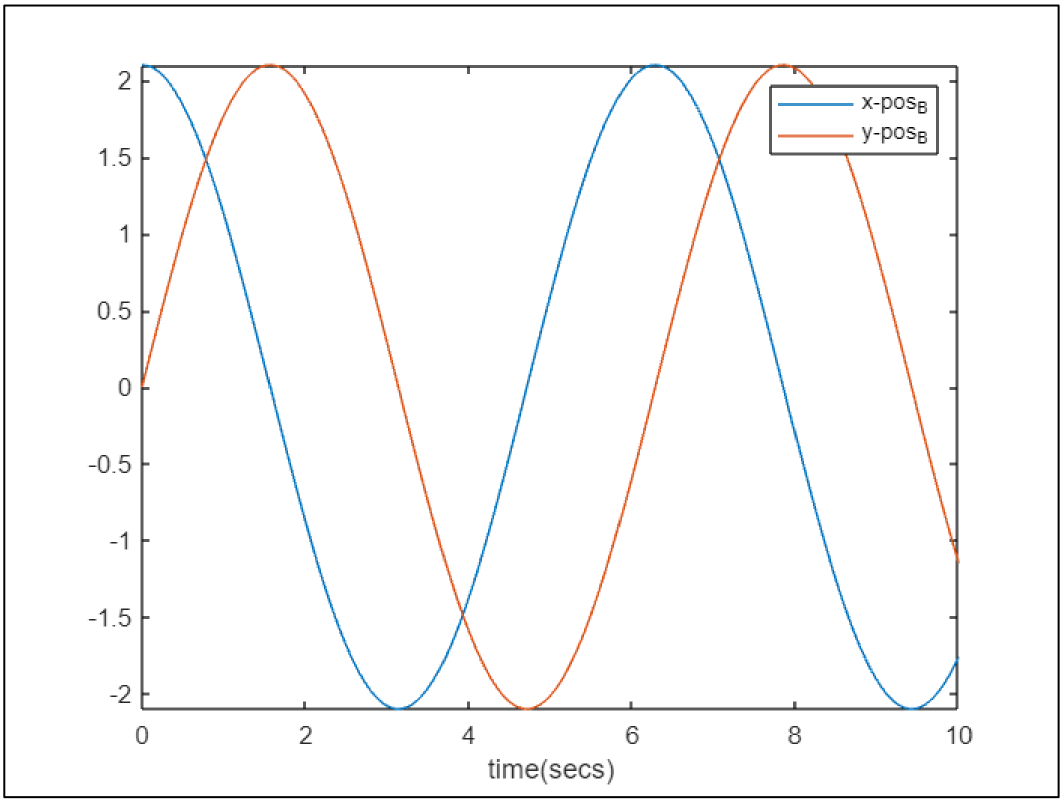


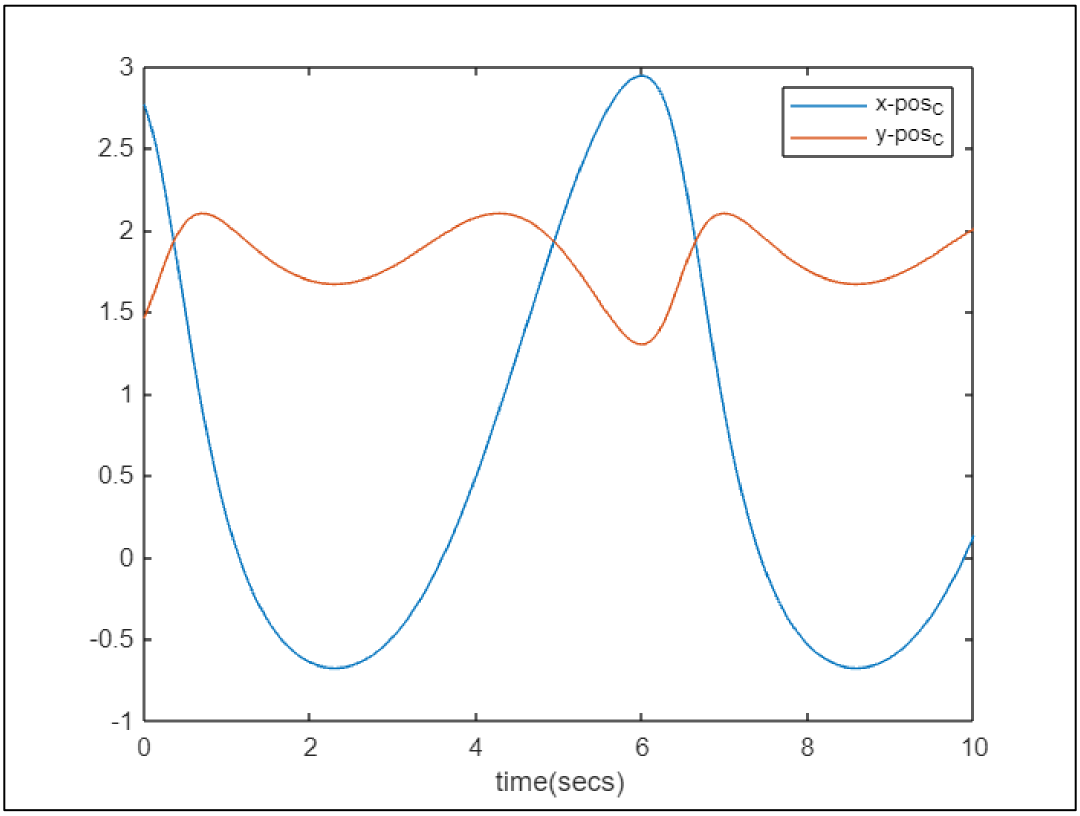
2
1.5
1
0.5
0
-0.5
-1
0
2
4
6
8
10
rad/sec
time(secs)
$\phi_1$dot
$\phi_2$dot

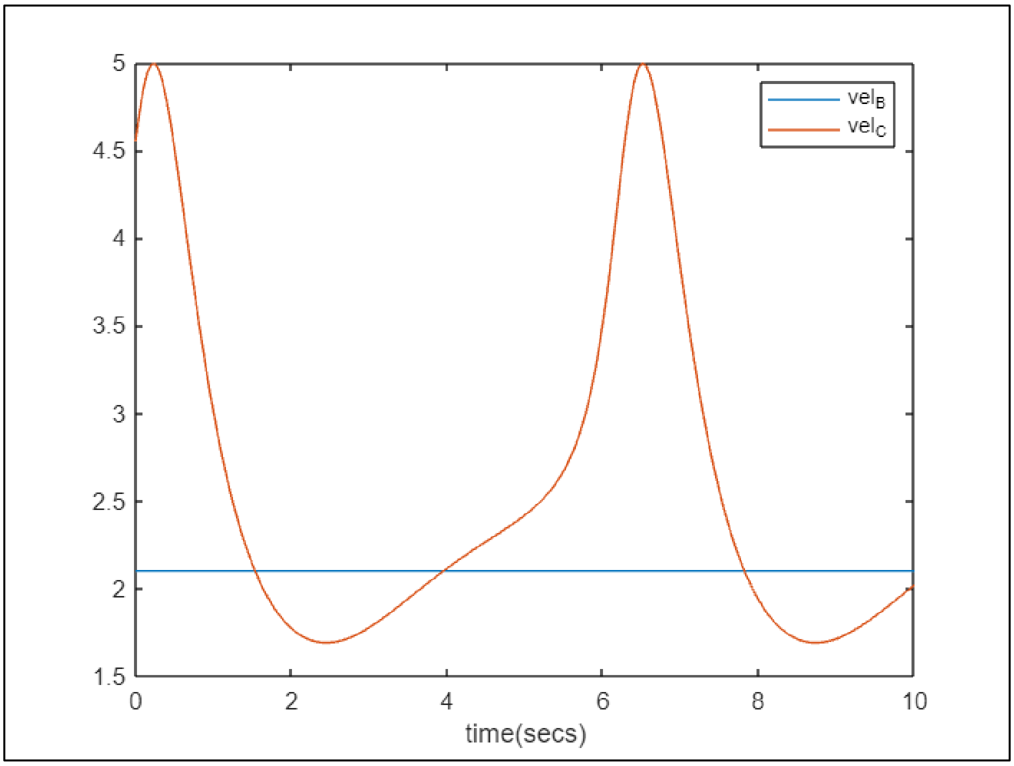


3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10
rad/sec^2
time(secs)
$\phi_1$dd
$\phi_2$dd

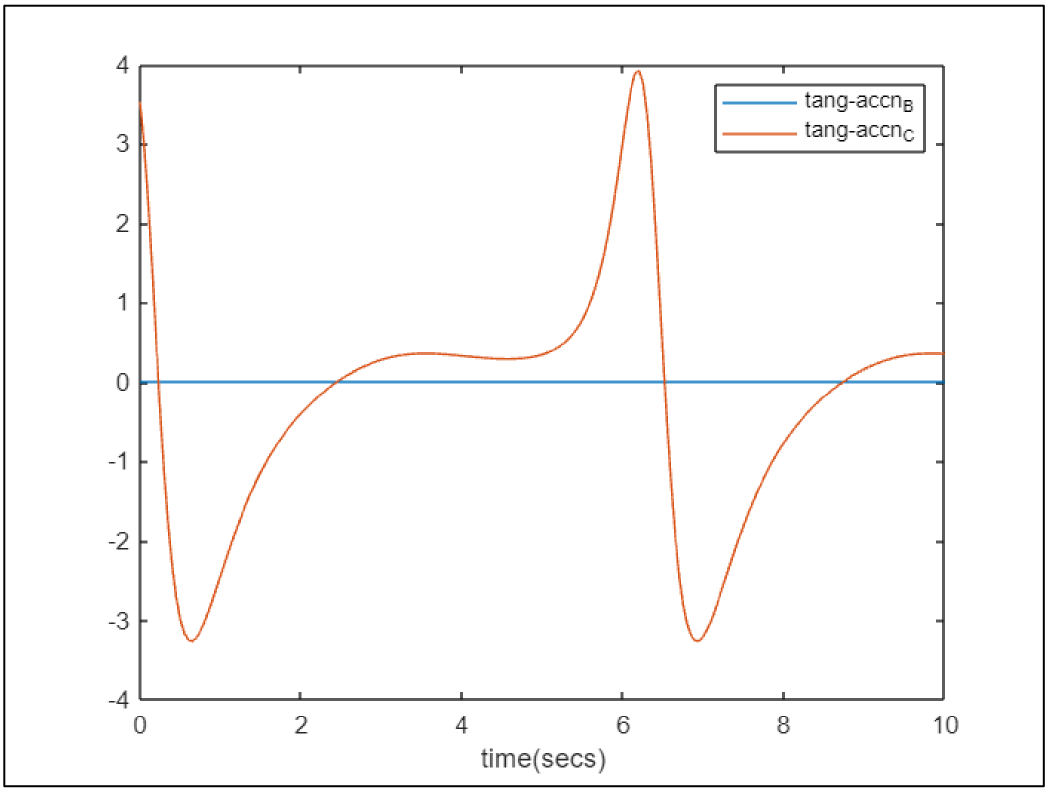
Position, Velocity and Acceleration of points B and C:



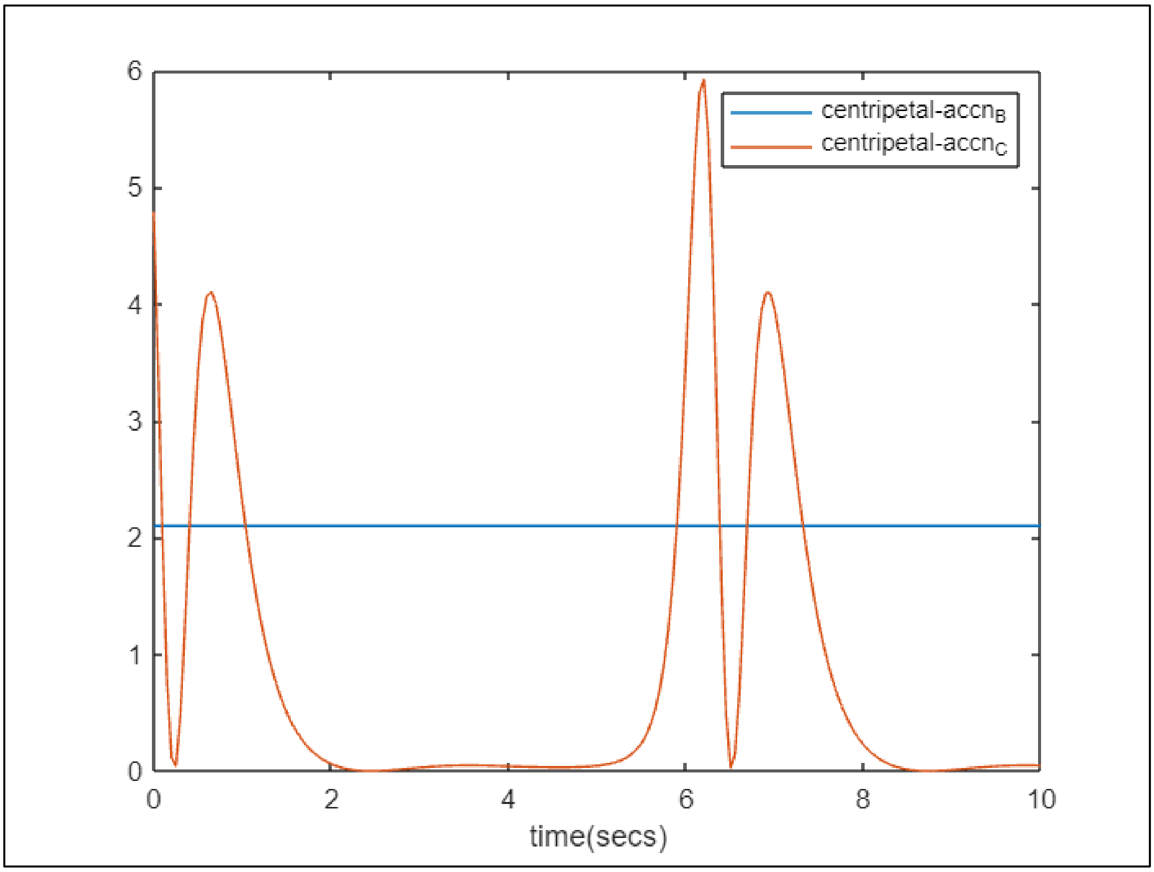




5
4.5
4
3.5
3
2.5
2
1.5
0
2
4
6
8
10
time(secs)
velB
velC



4
3
2
1
0
-1
-2
-3
-4
0
2
4
6
8
10
time(secs)
tang-accnB
tang-accnC



Verification of Solution by plotting change in norm of Constraints with time: