ELL 801: Nonlinear Control

HW #1

Due on January 19, 2024

1. **(Output feedback)** Design an output feedback controller to stabilize the origin of

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2 \\ \dot{x}_2 &= x_1 - x_2 - x_1 x_3 + u \\ \dot{x}_3 &= x_1 + x_1 x_2 - 2x_3 \\ y &= x_1\end{aligned}$$

2. **(Integral control)** A simplified model of the low-frequency motion of a ship is

$$\tau\ddot{\psi} + \dot{\psi} = ku$$

where ψ is the heading angle of the ship and u is the rudder angle (for us, the control input). Design a state feedback integral controller so that ψ tracks a desired angle $\psi_r = \pi/2$. Use Matlab/Mathematica/any other software of your choice to illustrate the robustness of your design against $\pm 10\%$ perturbations in the values of the parameters $\tau = k = 1$.

3. **(Relative degree)** Consider the system

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2 - x_3 \\ \dot{x}_2 &= -x_1 x_3 - x_2 + u \\ \dot{x}_3 &= -x_1 + u \\ y &= x_3\end{aligned}$$

Answer the following questions:

(a) Is the system input-output linearizable?

(b) If the answer to the previous question is positive, transform it into normal form, specifying the region over which the transformation is valid.

(c) What is the zero dynamics? Is the system minimum phase?