

ELL-702: NON-LINEAR SYSTEMS

ASSIGNMENT – 3

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2023MEZ8467

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% Qn 1

%Defining transfer function
num = [1];
den = [1,6,11,6];

lim = 1000;
d = 0.1;
len = (2*lim/d)+1;
wr = linspace(-lim,lim,len);
Gr1 = zeros(size(wr)); % Real part of G(jw)
Gimag = zeros(size(wr)); % Imag part of G(jw)
Gimag_popov = zeros(size(wr)); % w*Imag(G(jw))
iter = 1;
w_zero_x = 0;
w_iter = 0;

img = 1i;

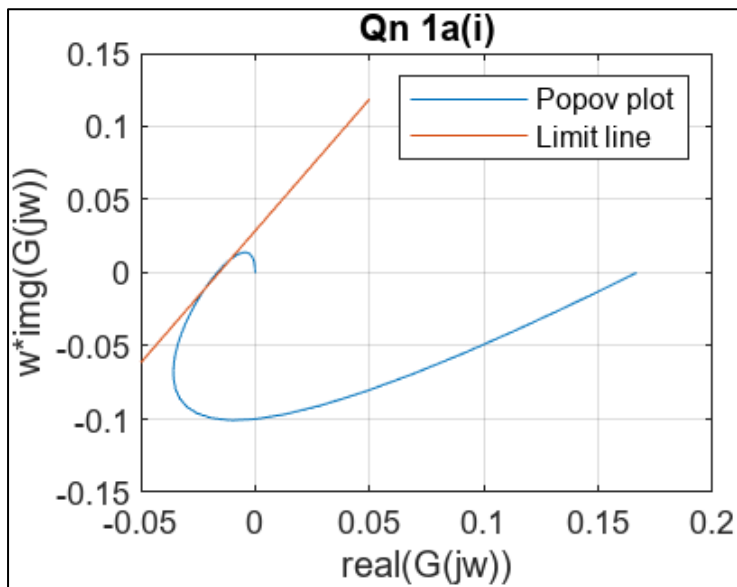
if size(num,2)>= size(den,2)
    disp("G(s) is not strictly proper!!");
else
    Gs = tf(num,den);
    for w = -lim:d:lim
        Gr1(iter) = real(evalfr(Gs,w*img));
        Gimag(iter) = imag(evalfr(Gs,w*img));
        % To identify the point of x-intercept
        if iter>1
            if Gimag(iter)*Gimag(iter-1)<=0
                w_zero_x = Gr1(iter);
                w_iter = iter;
            end
        end
        iter = iter+1;
    end
    Gimag_popov = wr.*Gimag;
```

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k = -(1/w_zero_x); % Sector limit(Based on intercept of curve)
%Slope of limit line
slope = ((Gimag(w_iter)*wr(w_iter))-(Gimag(w_iter-1)*wr(w_iter-1)))/(Gr1(w_iter)-Gr1(w_iter-1));
x_line = -0.05:0.01:0.05;
y_line = slope*(x_line-(-1/k));
plot(Gr1,Gimag_popov,x_line,y_line);
xlabel("real(G(jw))");
ylabel("w*img(G(jw))");
legend('Popov plot','Limit line')

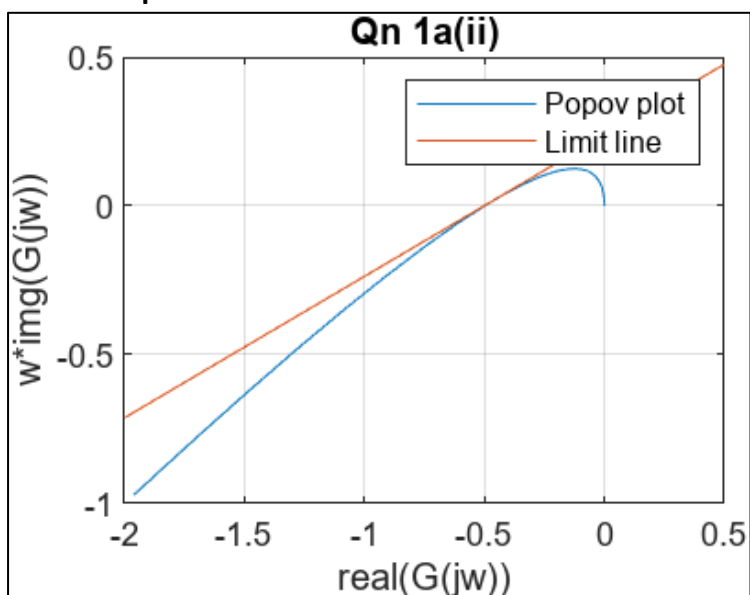
end

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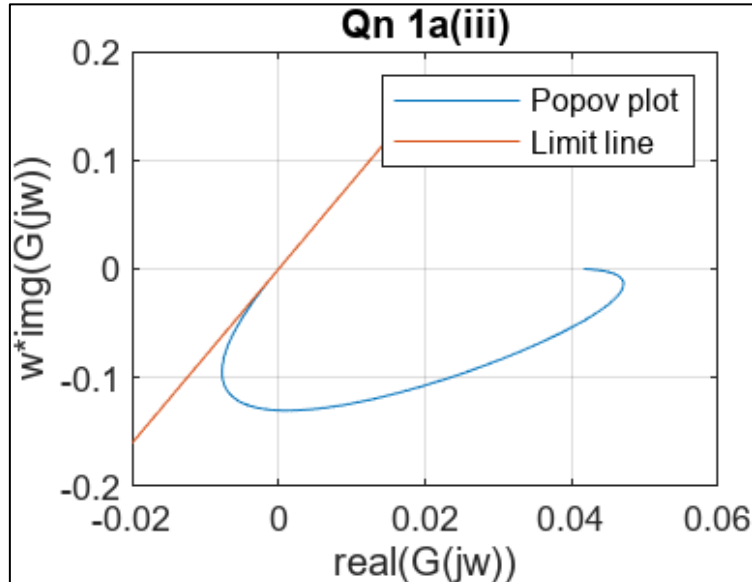
We get:

- $K = 63.4172$
- Slope = 1.8020



We get:

- $K = 1.9921$
- $\text{Slope} = 0.4751$



We get:

- $K = 10^6 \approx +(\text{infinity})$
- $\text{Slope} = 0.4751$

The Slope and intercept of the limit line was taken based on the Popov plot's intersection with x-axis. Then the sector limit was calculated using $(-1/x_{\text{intercept}})$.

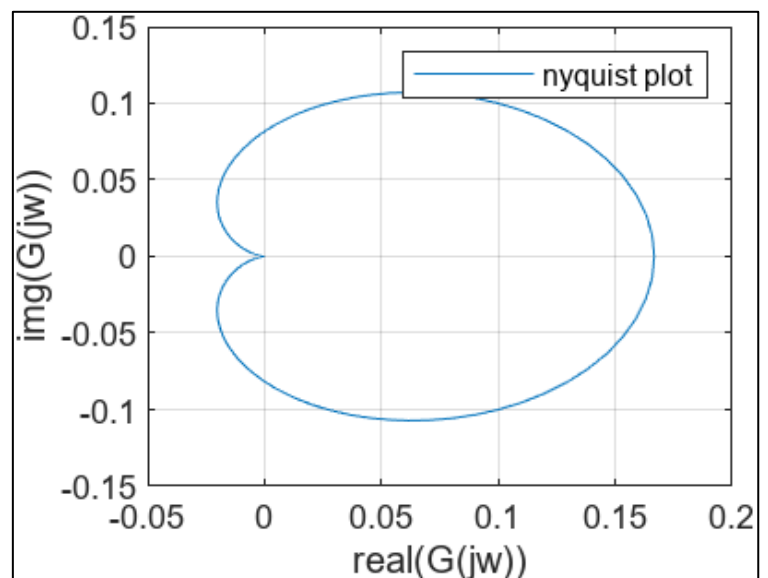
Qn 2)

Considering the transfer function from 1a(i):

We get the Nyquist plot of the system as

Since the system is Hurwitz(all the poles of $G(s)$ lies in the LH plane), We can consider the limit of non-linearity as (α, β) where $\alpha < 0$ and $\beta > 0$.

Then according to circular criterion, the Nyquist plot must lie in the interior of $\text{Disc}(\alpha, \beta)$.



If we consider the center of our disc to be the origin, then we will have our limits to be $(-\gamma_2, \gamma_2)$. And if we take γ_1 to be $\max(\text{real}(G(j\omega)))$, we can find γ_2 using $(\gamma_1 \gamma_2) < 1$.

From Nyquist plot we get γ_1 as 0.16667, thus we get γ_2 to be 5.99

Thus the limits for non-linearity for exponential stability will be $(-5.99, 5.99)$.