# MCL738 – Dynamics of Multibody Systems

# Lab Assignment – 7

# RK4 Problem and Euler Method Problem

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Qn 1) Euler Method Problem

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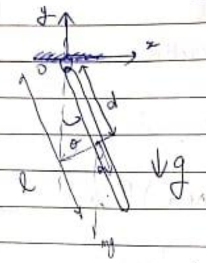


$\Delta t = h = 0.1$

$\theta(5) = ?$

$\dot{\theta}(5) = ?$

$\theta(0) = 3^\circ$

$\dot{\theta}(0) = 0^\circ/sec$

Now equations of motion will be

Euler's eqn abt pt O,

~~Since O is a pt of ...~~

Since O is a pt on the body which is fixed to the ground, we can directly apply euler's eqn abt that pt considering the MOI of the body abt that pt using $\parallel^{el}$ axis theorem.

$$I_o = I_{COM} + m d^2$$

here $d = \frac{l}{2}$ (COM at geometric center)

$$I_{COM} = \frac{ml^2}{12} = \frac{m(2d)^2}{12} = \frac{4md^2}{12} = \frac{md^2}{3}$$

$$I_o = \frac{md^2}{3} + md^2 = \frac{4}{3}md^2$$

Since it is a planar system, we only require $I_{33}$ (which is $I_o$ in this case).

$I_o \ddot{\theta} = \tau_{ext}$

$\tau_{ext} = -mg\sin\theta\, d$ (Since it is in $-\hat{3}$ direction)

$\Rightarrow I_o \ddot{\theta} = -mg\sin\theta\, d$

EOM :

$$I_o \ddot{\theta} + mg\sin\theta\, d = 0 \;//$$

We write

$\omega = \dot{\theta}$

$\dot{\omega} = \ddot{\theta}$

$\Rightarrow \quad \omega = \dot{\theta}$

$$\dot{\omega} = -\frac{mg\sin\theta\, d}{I_o} = -\frac{\cancel{m}g\sin\theta\cancel{d} \times 3}{4\cancel{m}d^{\cancel{2}}} = -\frac{3}{4}\frac{g}{d}\sin\theta$$

$\therefore \quad \dot{\theta} = \omega$

$$\dot{\omega} = \frac{-3}{4}\frac{g}{d}\sin\theta$$

In matrix form

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \omega \\ -\frac{3}{4}\frac{g}{d}\sin\theta \end{bmatrix}$$

$$Y^i = \begin{bmatrix} \theta^i \\ \omega^i \end{bmatrix}, \quad \dot{Y} = F(Y, t) = \begin{bmatrix} \omega \\ -\frac{3}{4}g\sin\theta \end{bmatrix}$$

$$Y^{i+1} = Y^i + hF(Y^i, t^i)$$

$$t^i = t^i + h$$

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If we linearize this

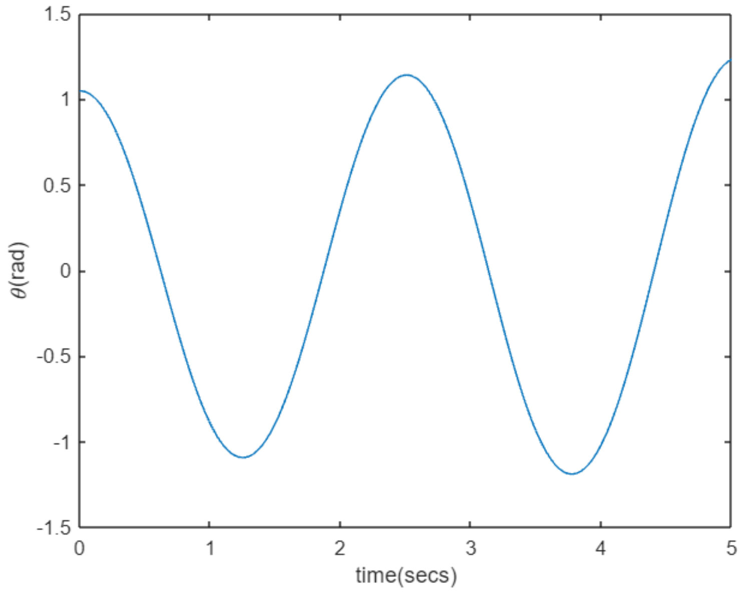
$$F(Y^i, t^i) = \begin{bmatrix} \omega \\ -\frac{g}{d}\left(\frac{3}{4}\right)\theta \end{bmatrix}$$

Since we put $\sin\theta \approx \theta$

**MATLAB Code**:

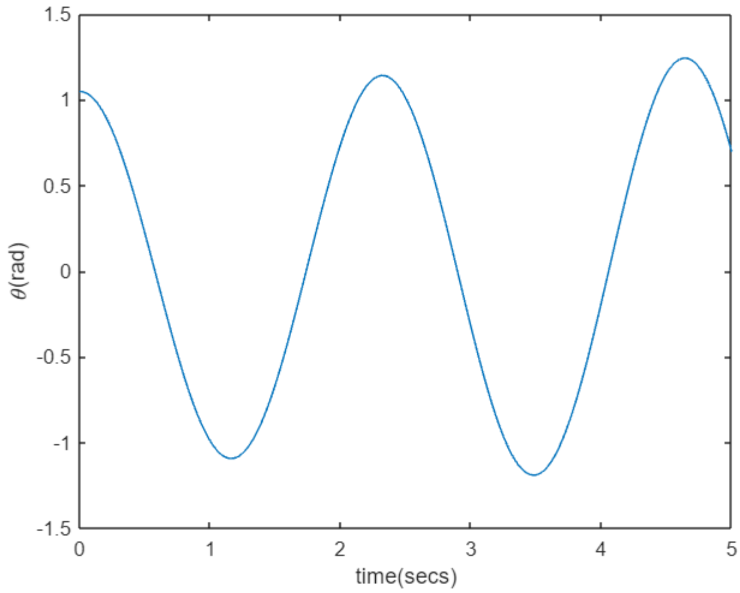
```
%Constants

m = 1; %Kg
l = 2;
ratio = 0.5;
d = ratio*l;
g = 9.8;
h = 0.01; %Time step


%Iteration - 1 without Linearization
syms theta omega;

init_t = 0;
final_t = 5;
time = init_t:h:final_t;

q_init = [60*pi/180;0];
q_iters = zeros([2 size(time,2)]);
q_iters(:,1) = q_init;

F = [omega;(-3/4)*g*sin(theta)/d];
```

```
i = 1;

digits(10);
for t = init_t:h:final_t-h
    q_iters(:,i+1) = q_iters(:,i) +
(h*vpa(subs(F,[theta,omega],transpose(q_iters(:,i)))));
    i = i+1;
end
```




```
%Iteration-2 after linearizing

syms theta omega;

init_t = 0;
final_t = 5;
time = init_t:h:final_t;

q_init = [60*pi/180;0];
q_iters = zeros([2 size(time,2)]);
q_iters(:,1) = q_init;

F = [omega;(-3/4)*g*(theta)/d];
i = 1;

digits(10);
for t = init_t:h:final_t-h
    q_iters(:,i+1) = q_iters(:,i) +
(h*vpa(subs(F,[theta,omega],transpose(q_iters(:,i)))));
    i = i+1;
end
```

```
plot(time,q_iters(1,:))
```



Qn 2) RK4 Problem



Assignment :-

Using RK4 method evaluate the soln of

$$\frac{d^2y}{dt^2} - y(1-y)\frac{dy}{dt} + y = 0$$

where $y(0) = 1$

$\dot{y}(0) = 1$

upto $t = 5$ with step size of 0.1

Ans:- $h = 0.1$, $t_{end} = 5$ secs $y(0) = 1$ ; $\dot{y}(0) = 1$

taking (1) $\dot{y} = \frac{dy}{dt} = v$

$$\frac{dv}{dt} = y(1-y)v - y \qquad Y(t) = \begin{bmatrix} y \\ v \end{bmatrix}$$

$$\begin{bmatrix} \dot{y} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} v \\ y(1-y)v - y \end{bmatrix}$$

$$\therefore \quad \dot{Y}(t) = F(Y,t) = \begin{bmatrix} v \\ y(1-y)v - y \end{bmatrix}$$

$K_1 = hF(Y^i, t^i)$

$K_2 = hF(t^i + \frac{h}{2}, Y^i + \frac{K_1}{2})$

$K_3 = hF(t^i + \frac{h}{2}, Y^i + \frac{K_2}{2})$

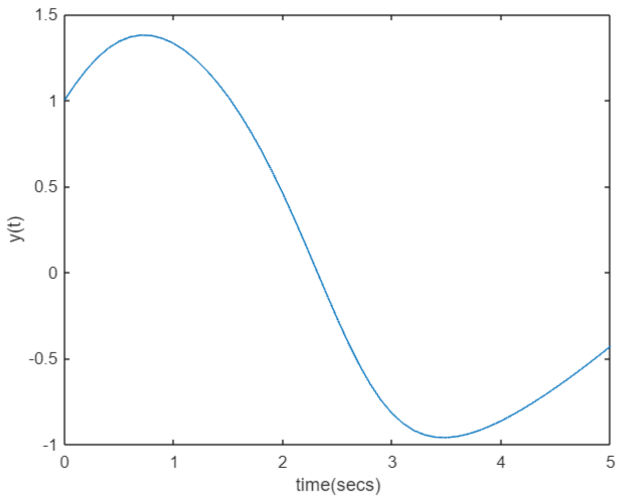
$K_4 = hF(t^i + h, Y^i + K_3)$

$$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$Y^{i+1} = Y^i + K \quad ; \quad t^{i+1} = t^i + h$$

**MATLAB Code**:

```
syms y v;
F = [v;(y*(1-y)*v)-y];
Y = [y;v];
h = 0.1;
init_t = 0;
final_t = 5;
time = init_t:h:final_t;

q_init = [1;1];
q_iters = zeros([2 size(time,2)]);
q_iters(:,1) = q_init;

i = 1;
K1 = zeros(2,1);
K2 = zeros(2,1);
K3 = zeros(2,1);
K4 = zeros(2,1);
K = zeros(2,1);
y_iter = zeros(2,1);
digits(10);
for t = init_t:h:final_t-h
    y_iter = (double(subs(Y,[y,v],transpose(q_iters(:,i)))));
    K1 = h*((double(subs(F,[y,v],transpose(y_iter)))));
    K2 = h*((double(subs(F,[y,v],transpose(y_iter + (K1/2))))));
    K3 = h*((double(subs(F,[y,v],transpose(y_iter + (K2/2))))));
    K4 = h*((double(subs(F,[y,v],transpose(y_iter + (K3))))));
    K = (K1 + (2*K2) + (2*K3) + (K4))/6;
    q_iters(:,i+1) = q_iters(:,i) + K;
    i = i+1;
end

plot((time),q_iters(1,:));
```



```
plot((time),q_iters(2,:));
```