Complexity Study of Reasoning about Knowledge and Public Observations

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Joint work with
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How do we mathematically model certain scenarios?

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- Graphs, Flows, Linear Programs

$$\boxed{\text{Scenario}} \longrightarrow \boxed{\begin{array}{c}\text{Model}\\ \text{(Graph, LP)}\end{array}}$$

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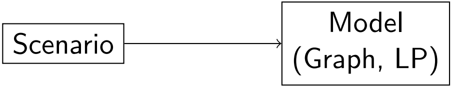

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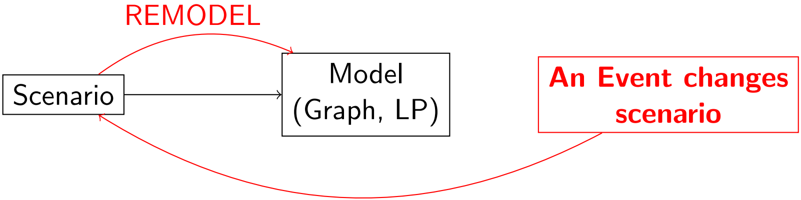

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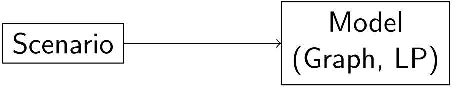

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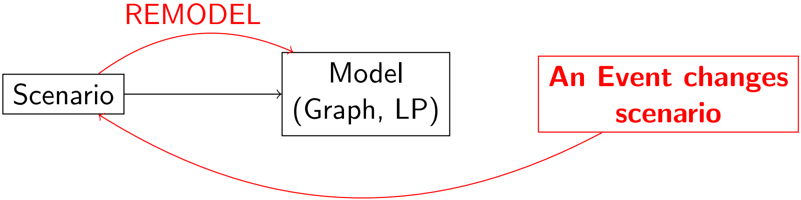

- Examples:

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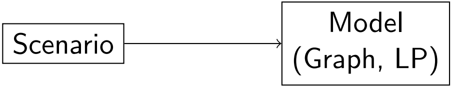

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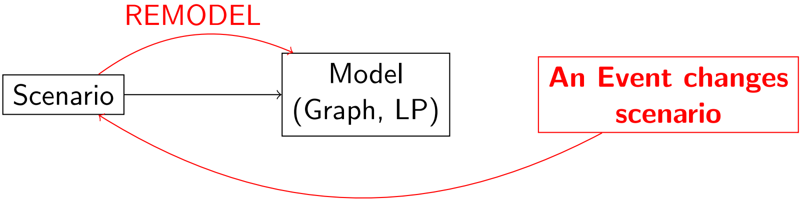

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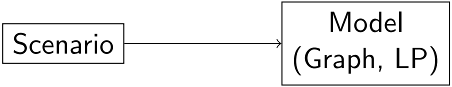

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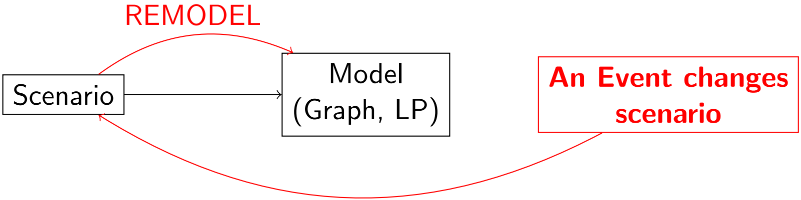

- Examples:
  - Supervisor Assignment Problem: Perfect Matching
  - Whether a certain town is reachable given a map: Reachability

Some More Examples

- Propositional Language (Valuation models):

$$\varphi_G \to \varphi_C$$

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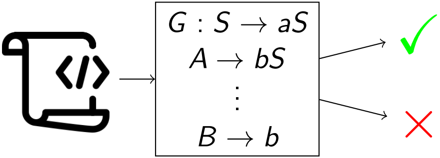

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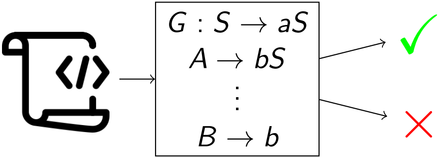

- Buchi Automata: Will an OS arrive at a deadlock EVENTUALLY?

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  - **Event changes Knowledge**: A tells B it has 3, now for B: $\boxed{\begin{array}{l} A > 3 \\ B > 2 \\ C > 1 \end{array}}$

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  - Max $c^T x$
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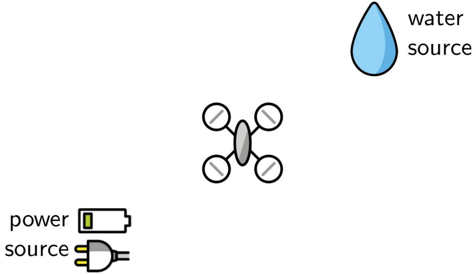
Motivation:A Farming Drone



- A water source on top right corner
- A power source on bottom right corner

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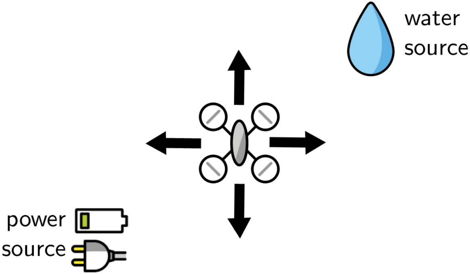
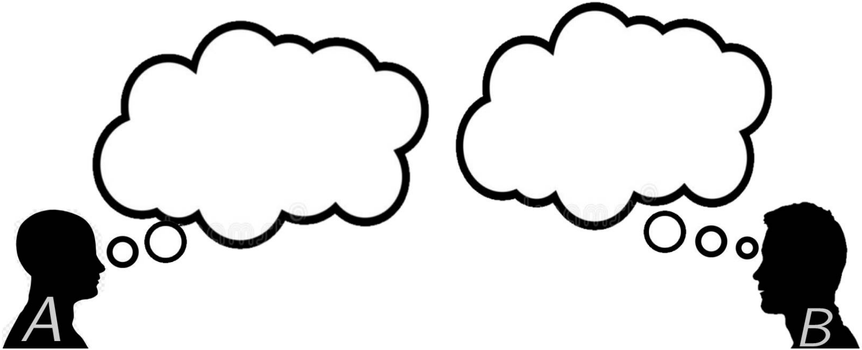


- Can move up, down, left or right
- Cannot move diagonally

Farming Drone: Agents and their expectations



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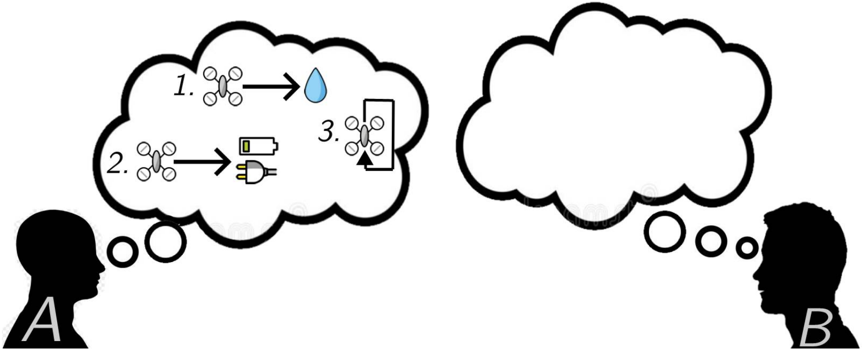

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3. Go patrolling in clockwise direction.

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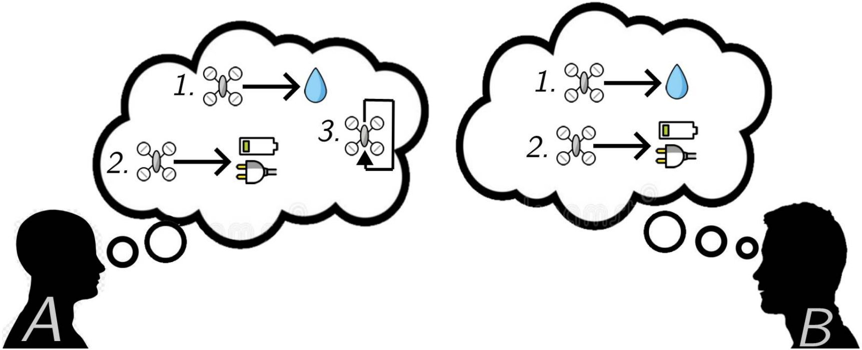

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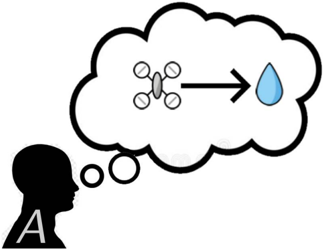

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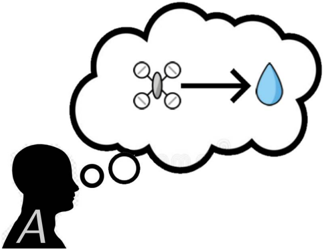

- What is the minimal number of  moves that A has to **observe** to know its goal?
- Does there exist a sequence of  moves such that by observing it, B would know its goal but A would not?

Modelling Knowledge: Epistemic Model

- Epistemic model (W, R, V).

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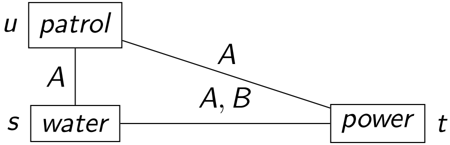

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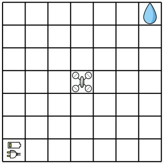

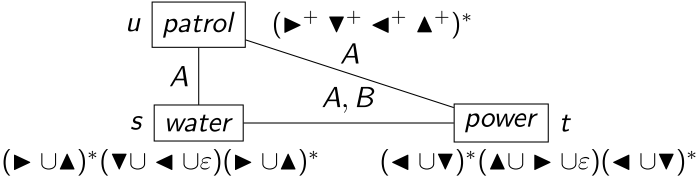

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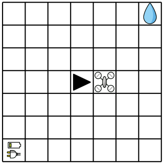

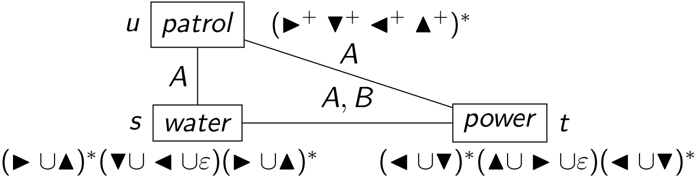

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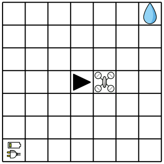

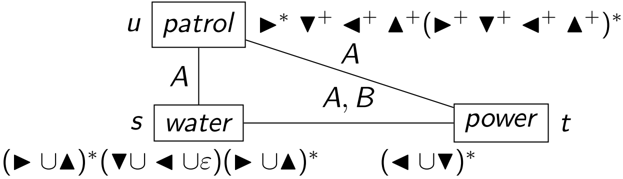

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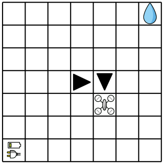

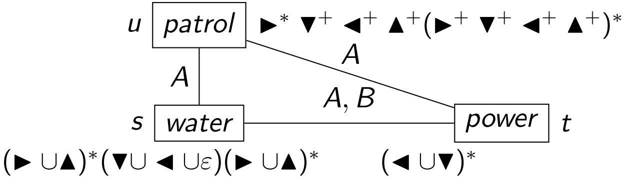

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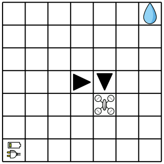

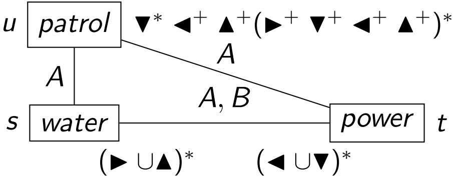

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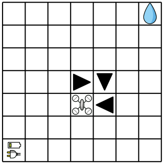

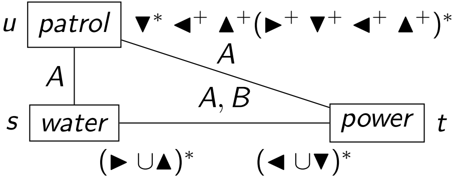

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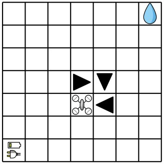

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Public Observation Logic: Syntax[1]

The language of POL:

$$\varphi \quad ::= \quad \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi \mid \hat{K}_i\varphi \mid [\pi]\varphi \mid \langle\pi\rangle\varphi$$

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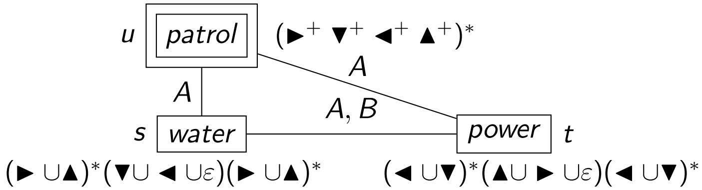

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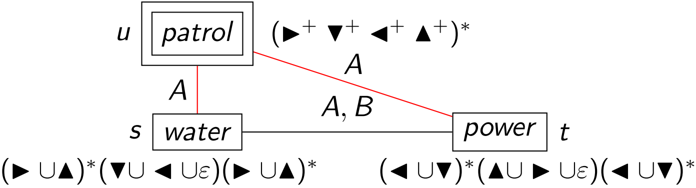

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Public Observation Logic: Syntax[1]

The language of POL:

$$\varphi \quad ::= \quad \top \mid p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi \mid \hat{K}_i\varphi \mid [\pi]\varphi \mid \langle\pi\rangle\varphi$$

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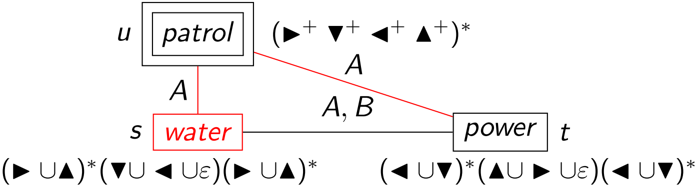

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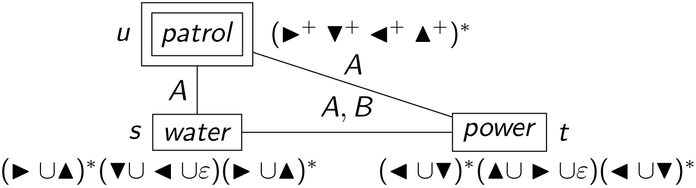

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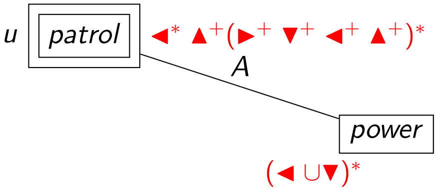

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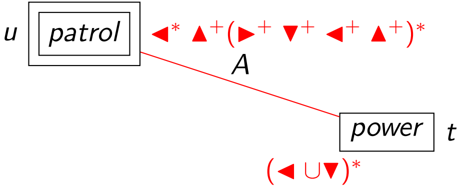

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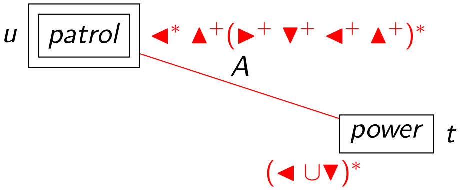

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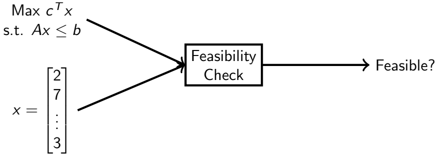
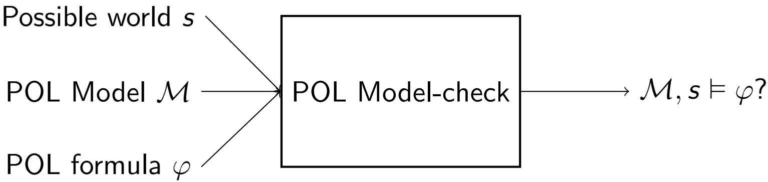
    - 

    : Check CNF Assignment

Recall: Checking Solution vs Finding a Solution

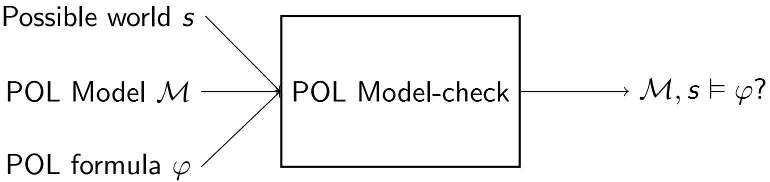
- Recall the questions using Linear Programs:
  - Max $c^T x$ s.t. $Ax \leq b$ and $x = \begin{bmatrix} 2 \\ 7 \\ \vdots \\ 3 \end{bmatrix}$ $\rightarrow$ $\boxed{\text{Feasibility Check}}$ $\rightarrow$ Feasible? : Check CNF Assignment
  - Max $c^T x$ s.t. $Ax \leq b$ $\rightarrow$ $\boxed{\text{Feasible Solution Finding}}$ $\rightarrow$ $\exists$? a feasible solution : CNF-SAT

The Model Checking Question



Is POL Model-checking **decidable**?

The Model Checking Question



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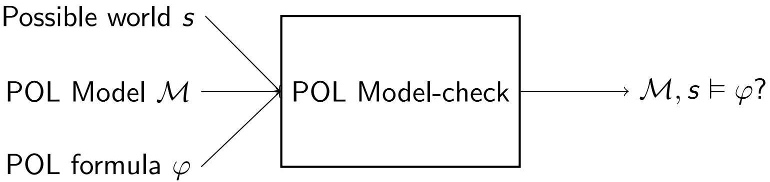

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Theorem: POL Model Checking Complexity [IJCAI'22]

The model-checking problem of POL is PSPACE-Complete.

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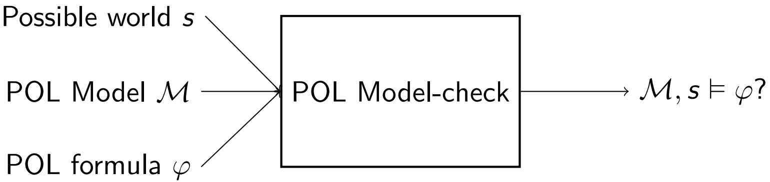

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It's too hard, isn't it?
Enter: **Fragments**

POL Model-checking

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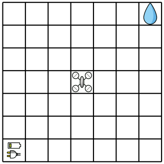

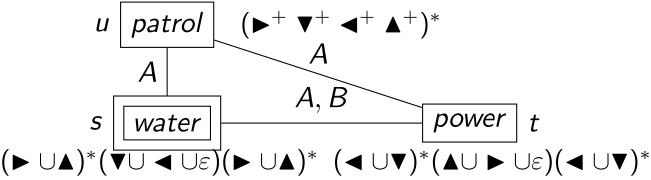

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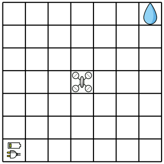

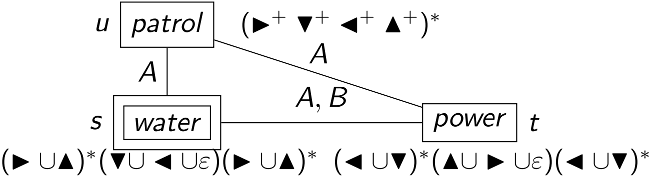

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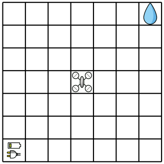

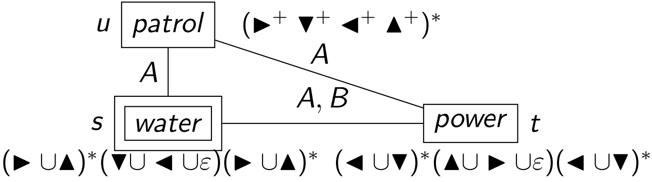

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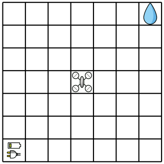

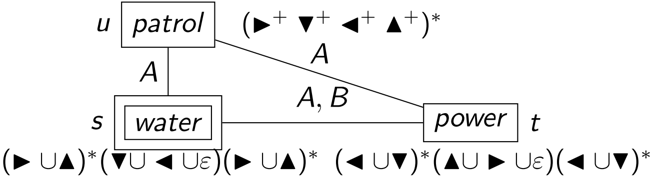

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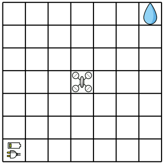

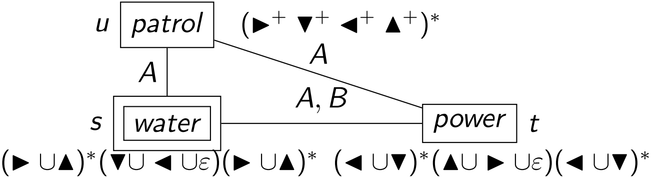

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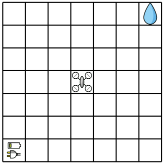

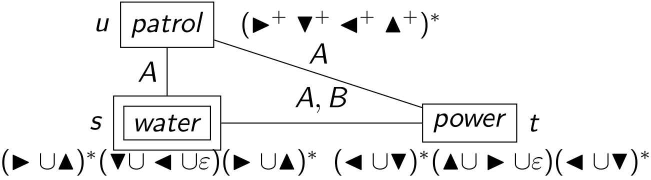

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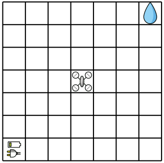
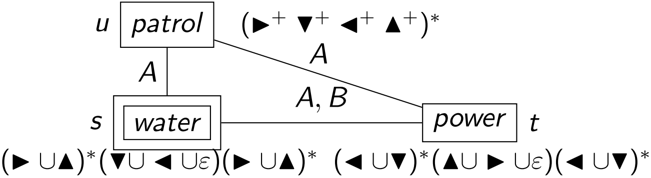
POL Model-checking





- **Verification of a plan,** Word **Fragment**: only **word** in π.

$$\mathcal{M}, s \models \langle \blacktriangleright\blacktriangleright\blacktriangleright \rangle K_A water$$

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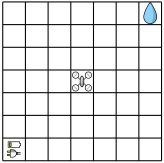

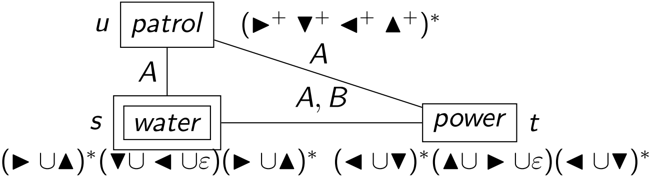

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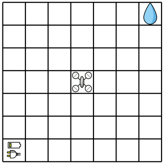
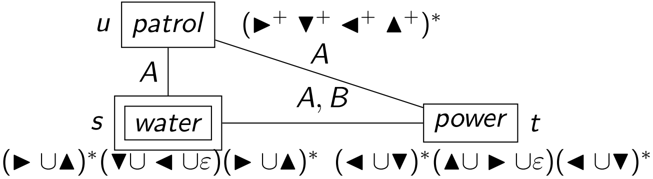
Can A know the goal is after observing **three** $\blacktriangleright$ **moves** (a plan)?

POL Model-checking





- Star-Free **Fragment**: no Kleene Star ($*$) in π.

$$\mathcal{M}, s \models [(\blacktriangleright \cup \blacktriangledown \cup \blacktriangleleft \cup \blacktriangle)^2] \neg K_A water \wedge \langle (\blacktriangleright \cup \blacktriangledown \cup \blacktriangleleft \cup \blacktriangle)^3 \rangle K_A water$$

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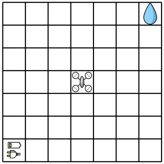

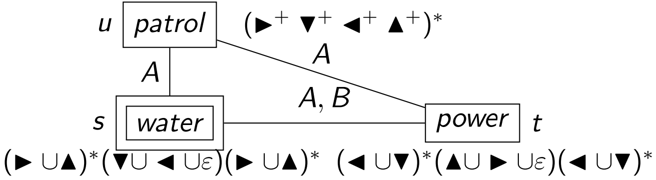

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- A cannot **know** about the goal until the length of the sequence of moves is at least 3.

Are these more efficient fragments?

Star-Free fragment	$[aab+b]K_ip$, $\cancel{[aab^*]K_ip}$	PSPACE-Hard ($TQBF$)
Existential fragment	$\langle aab^*\rangle K_ip$, $\cancel{[aab^*]K_ip}$	PSPACE-Hard (Intersection Non-Emptiness Problem)
Star-Free$-$Existential fragment	$\langle aab+b\rangle \hat{K}_ip$, $\cancel{\langle aab^*\rangle K_ip}$	NP-Complete (3-SAT)
Word fragment	$[aab]K_ip$, $\cancel{[aab+b]K_ip}$	P

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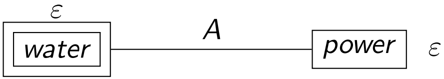

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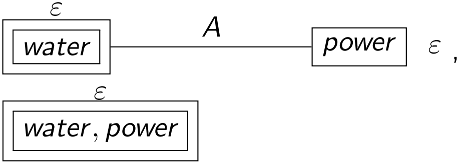

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If NO MODEL then φ is a property

Else it is not

POL Satisfiability Results

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Open: Lower Bound

POL Fragment Results (KR'23)[1]

Star-Free Multi-agent fragment	$[aab + b]K_i p$, $\cancel{[aab^*]K_i p}$	NEXPTIME-Complete (Tiling Problem)
Star-Free Single-agent fragment	$[aab]K_i p \vee K_i q$, $\cancel{[aab^* + b]K_i p \vee K_j q}$	PSPACE-Hard (TQBF)
Word Multi-agent fragment	$[aab]K_i p$, $\cancel{[aab + b]K_i p}$	PSPACE-Complete (PAL Reduction)
Word Single-agent fragment	$[aab]K_i p \vee K_i q$, $\cancel{[aab + b]K_i p \vee K_j q}$	NP-Complete (PAL Reduction)

[1] Chakraborty S.; Ghosh A.; Ghosh S.; and Schwarzentruber F. 2023. On simple expectations and observations of intelligent agents: A complexity study., KR 2023, Rhodes, Greece, September 2-8, 2023, 136–145.

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- Consider the formula $\langle \pi^{\star} \rangle \psi$.

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- **Solution:** $\pi^\star$ is a regular expression $\longrightarrow$ NFA/DFA (**FINITE**)

Decidability: Model-Checking

- Consider the formula $\langle \pi^\star \rangle \psi$.
- As per semantics, given a $\mathcal{M}, s$, we search for a $w \in \mathcal{L}(\pi^\star)$ such that...
- s survives in $\mathcal{M}|_w$ and...
- Recursively check $\mathcal{M}|_w, s \vDash \psi$.
- **Problem:** How far in $\mathcal{L}(\pi^\star)$ to search since the language will have infinite words?
- **Solution:** $\pi^\star$ is a regular expression $\longrightarrow$ NFA/DFA (**FINITE**)
- Given a model $\mathcal{M}$, bound the number of **unique** $\mathcal{M}|_w$ over any $w \in \Sigma^\star$.

Decidability of POL^{-}: Tableau Term

- **Survival Term**

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  - $(\sigma \quad w \quad \checkmark)$: World σ survives after updated by w.

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- **Formula Term**

Decidability of POL$^-$: Tableau Term

- **Survival Term**
  - $(\sigma \quad w \quad \checkmark)$: World σ survives after updated by w.

- **Formula Term**
  - $(\sigma \quad w \quad \varphi)$: φ holds in World σ after updating by w

Decidability of POL^{-}: Some Tableau Rules

Propositional Rules

Clash rule
$$\frac{(\sigma \quad w \quad p), \quad (\sigma \quad w \quad \neg p)}{\perp}$$

Diamond and Box Rules

Diamond Project
$$\frac{(\sigma \quad w \quad \langle a \rangle \psi)}{(\sigma \quad wa \quad \checkmark), (\sigma \quad wa \quad \psi)}$$

Box Project
$$\frac{(\sigma \quad w \quad [\pi]\psi), \quad (\sigma \quad wa \quad \checkmark)}{(\sigma \quad wa \quad [\pi \backslash a]\psi)}$$

Survival Rules

Constant Valuation Up
$$\frac{(\sigma \quad w \quad p)}{(\sigma \quad \epsilon \quad p)} \qquad \frac{(\sigma \quad w \quad \neg p)}{(\sigma \quad \epsilon \quad \neg p)}$$

Survival Chain
$$\frac{(\sigma \quad wa \quad \checkmark)}{(\sigma \quad w \quad \checkmark)}$$

Decidability: Satisfiability

Again consider the formula: $\langle \pi^\star \rangle \psi$.

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This implies either $\mathcal{M}, s \vDash \psi$ (Inductively checked), where $\epsilon \in \mathcal{L}(\pi^\star)$ is the witness word.

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OR $\mathcal{M}, s \vDash \langle \pi \rangle \langle \pi^\star \rangle \psi$

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OR $\mathcal{M}, s \vDash \langle \pi \rangle \langle \pi^\star \rangle \psi$

The input formula becomes subformula of a bigger formula $=>$ problem in proving inductively.

Decidability: Satisfiability

Again consider the formula: $\langle \pi^\star \rangle \psi$.

Say there is a model $\mathcal{M}, s \models \langle \pi^\star \rangle \psi$

This implies either $\mathcal{M}, s \models \psi$ (Inductively checked), where $\epsilon \in \mathcal{L}(\pi^\star)$ is the witness word.

OR $\mathcal{M}, s \models \langle \pi \rangle \langle \pi^\star \rangle \psi$

The input formula becomes subformula of a bigger formula $=>$ problem in proving inductively.

Approach: Look towards Automaton structures

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Concluding...

- Model-Checking and Satisfiability problem for POL (Full language ongoing)
- Complete Axiomatic System for extension of POL : Epistemic Protocol Logic[1]
- Programs can be interpreted more efficiently in CFL.
  How about CFG instead of regular?

[1] H van Ditmarsch, S Ghosh, R Verbrugge, and Y Wang. Hidden protocols: Modifying our expectations in an evolving world. Artificial Intelligence, 208:18–40, 2014