Numerical Methods (1/2020)
Midterm Examination Part B

1. [10 points] An area of a triangle can be found using the Heron's Formula for the area of a triangle (since Babylonian time). Write a Python program using this formula along with the Heron's formula for finding square root to solve for the area of the triangle whose 3 sides are 18, 24, and 30.

   Heron's method for finding the area of a triangle with a, b, c as the lengths of the sides of the triangle where p is $(a+b+c)/2$

   $$\text{area of a triangle} = \sqrt{p(p-a)(p-b)(p-c)}$$

   Heron's method for finding the square root of a number N.

   $$\sqrt{N} = \lim_{i\to\infty} x_i$$

   where

   $$x_{i+1} = \frac{1}{2}\left(x_i + \frac{N}{x_i}\right)$$

2. [15 points] Write a python program to convert a number in IEEE-754 32 bits to base 10. The input are a set of 3 numbers in 32-bit IEEE-754 form: sign, exponent and fraction, and the output is a number of decimal. Try with the following inputs:

   Sign bit: `1`
   Exponent: `0111 1100`$_2$ `(124`$_{10}$`)`
   Mantissa: `1011 1000 0000 0000 0000 000`$_2$ `(6029312`$_{10}$`)`

   You can check your answer at
   https://www.h-schmidt.net/FloatConverter/IEEE754.html

3. [10 points] We know that $\tan(\pi/6) = 1/\sqrt{3}$. Use the following Taylor series expansion for arctan to evaluate the value of π for up to 6 significant digits.

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \cdots$$

$$= \sum_{n=1}^{\infty} (-1)^{(n-1)} \frac{x^{2n-1}}{2n-1} \overset{\text{or}}{=} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$$

QUESTION: IS $y = \arctan(x)$ EVEN, ODD, OR NEITHER?

$x \in [-1, 1]$

4. [20 points] For function $f(x) = \ln(2x) - 0.5x + 1$,
   a. Plot $f(x)$
   b. Solve for x where $f(x) = 0$ using interval search, Newton-Raphson, and false position during interval x = [0,1] and x = [6,8]
   c. Solve for x where f(x) is maximum using equal interval search and parabolic interpolation using the initial interval x = [0.5,3] or initial points 0.5, 1 and 3.

5. [10 points] Given the following dot_product between 2 matrices function. Debug the function and verify it by performing the dot product between the following 2 matrices.

```
def dot_product(A,B):
    (nrow_A,ncol_A) = A.shape
    (nrow_B,ncol_B) = B.shape
    if (ncol_B == nrow_A):
        C = np.zeros((nrow_A,ncol_B))
        (nrow_C,ncol_C) = C.shape
        for i in range(0,nrow_C):
            for j in range(0,ncol_C):
                product_sum = 0
                for k in range(0, ncol_B):
                    product_sum +=  B[i,k] * A[k,j]
                C[i,j] = product_sum
            # end for j
        # end for l
    else:
        print('Error')
        return
    # end if (ncol_A == nrow_B)
    return C
# end function
```

6. [15 points] Write a Python program that solve the following system of linear equations using Gauss's Elimination with pivoting. Make sure that the rows are ordered such that the first row has the highest value of the coefficient of x_1 and the last row has the lowest value of the coefficient of x_1. Your answer must consist of 3 functions: pivoting, forward elimination, and back-substitution.

$$
\begin{aligned}
-1x_1 + 2x_2 + 5x_3 + x_4 &= -4 \\
-3x_1 + 3x_2 - x_3 + 4x_4 &= 20 \\
4x_1 + x_2 + 2x_3 - 3x_4 &= -16 \\
5x_1 + 4x_2 + 3x_3 - x_4 &= -10
\end{aligned}
$$

7. [20 points] A factory manufactures a product which is being sold at the price of 35 baht per piece. The cost of producing this product consists of machine setup cost for 200 baht, material cost of 10 baht per product, and dealer cost of 0.15 baht per square of product. The revenue R(x) is 35x and the cost C(x) is $200+10x+0.15x^2$ where x is the number of products produced. Given that profit is R(x) – C(x). Write a Python program to find the values of x at the breakeven point using Bisection method, and the maximum profit using Golden Section search method.

8. [Extra credits][25 points] Somchai would like to buy a house valued at 3 million baht. He plans to loan money from a bank which doesn't need a down payment. The bank wants Somchai to pay up the principal and the loan interest in 20 years. In the first 3 years, the bank uses 1.5% annual interest rate. After the first 3 years, the bank will use 5% annual interest rate. How much Somchai have to pay per month? Assume that the bank uses compound interest rate. Proof your results.