Discussion Forum

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Type-2 Fuzzy Logic: Challenges and Misconceptions

This material is based on contributions by a number of people including Janet Aisbett, Humberto Bustince, Oscar Castillo, Jonathan M. Garibaldi, Hani Hagras, Vladik Kreinovich, Jerry M. Mendel, Frank Rhee, Terry Rickard, Woei Wan Tan and Christian Wagner including notably discussions at a Panel Session at the 2011 IEEE Symposium on Advances in Type-2 Fuzzy Logic Systems organized by Garibaldi, Hagras and Rhee. This paper focuses on the challenges of theoretical underpinning including notation, learning type-2 sets and systems and broadening the areas where type-2 system are applied. The misconceptions we discuss are around computational performance of type-2 systems and notion that type-2 systems are intended to replace type-1 fuzzy systems.

1. Introduction

Type-1 fuzzy sets [30] are the basis of a proven methodology capable of modelling uncertainty, vagueness and imprecision. There have been a host of successful real-world applications of type-1 fuzzy logic particularly in consumer products and control applications. As far back as 1975, Zadeh [29] highlighted the problems of trying to model linguistic uncertainties with type-1 fuzzy sets, proposing type-2 fuzzy sets as a model capable of capturing linguistic uncertainties. A type-2 fuzzy set can be viewed in a variety of ways using a variety of representations. Perhaps the simplest conceptual model is to think of type-2 fuzzy sets as type-1 fuzzy sets where the degree of belonging of a given element is measured using a type-1 fuzzy set. This means there is some uncertainty about the degree of membership of the element in the fuzzy set. Figure 1 depicts the three main types of membership function in fuzzy logic with the membership grades at a point *p*. Figure 1(a) depicted a type-1 fuzzy set. Note that this figure includes a third dimension which is implicit in the definition of a type-1 fuzzy set: it shows that each membership grade is certain and is therefore modelled as a crisp singleton. Figure 1(b) depicts a type-2 interval fuzzy set where the membership grade of a given element is modelled by a crisp interval. Figure 1(c) depicts a type-2 general fuzzy set where the membership grade is modelled by a fuzzy set whose support is in the interval [0, 1]. Type-2 fuzzy sets and systems are more conceptually and computationally complex than type-1 systems and have only received significant attention in the last 15 years. Prior to this, most work was focused on creating the theoretical underpinning of type-2 sets and systems. Early work included defining the logical connectives for general type-2 fuzzy sets [25, 24, 7] and defining type-2 interval fuzzy sets, at that time called interval-valued or IV fuzzy sets [8, 26, 15]. One of the important pieces of research increased interest in the area of type-2 fuzzy sets was the definition of type-reduction and the subsequent KM iterative procedure for type-reduction (Karnik, Mendel and Liang [14]). This work allowed researchers to implement complete type-2 fuzzy logic systems and

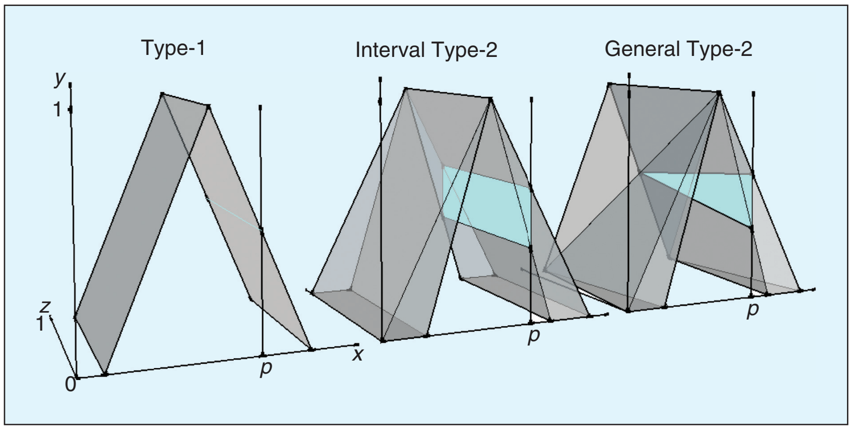

FIGURE 1 An example of the three types of fuzzy sets. The same input *p* is applied to each fuzzy set. (a) Type-1 fuzzy set. (b) Interval type-2 fuzzy set. (c) General type-2 fuzzy set. (Figure reproduced from [12].)

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use type-2 interval systems in a wide range of applications. In recent years the field has seen significant growth in applied and theoretical research with the first type-2 fuzzy logic symposium taking place in April 2011 as part of the 2011 Symposium Series on Computational Intelligence.

This recent growth in interest has not been without its problems. Along with a succession of high quality award winning papers there has been some less rigorous work done, which has attracted negative attention. There have been a number of public and private discussions about the field of type-2 fuzzy logic and it's role in computational intelligence. This article presents some of the reflections on this debate from the view of people working the field of type-2 fuzzy logic. We have separated the issues into challenges now facing the field and misconceptions people have about the research going on in type-2.

2. Challenges

2.1. Theoretical Underpinning

The theoretical underpinnings for type-2 fuzzy logic are well established. However, many researchers are exploring new ideas and in particular looking to learn from type-1 fuzzy logic theory. From our perspective there are two main theoretical issues—performance and defining and understanding uncertainty.

2.1.1. Notation

Some researchers across the fuzzy community have raised concerns about the notation employed in the papers published on type-2 fuzzy logic. The following definition appears in [22] A type-2 fuzzy set, denoted $\tilde{A}$, is characterized by a type-2 membership function $\mu_{\tilde{A}}(x,u)$, where $x \in X$ and $u \in J_x \subseteq [0,1]$, i.e.,

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) \mid \forall x \in X, \forall u \in J_x \subseteq [0,1]\} \tag{1}$$

in which $0 \le \mu_{\tilde{A}}(x,u) \le 1$, X is the domain of the fuzzy set and J_x is the domain of the secondary membership function at x. $\tilde{A}$ can also be expressed as

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u)/(x,u) \quad J_x \subseteq [0,1], \tag{2}$$

where $\iint$ denotes union over all admissible x and u. For discrete universes of discourse $\int$ is replaced by $\sum$. It is felt that this notation is "old fashioned" and doesn't use modern mathematical notation. In [1] there is a discussion about notation and many other researchers employ similar, yet different, modern notations. In our view agreement on a single notation is unlikely but this is also true in type-1.

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2.1.2. Computational Performance

Type-1 fuzzy logic based systems are extremely straightforward to implement and perform well on personal computers and lower end machines. Type-2 fuzzy systems require a lot more understanding, but importantly are too slow often for real time applications, particularly if we wish to use generalized type-2 fuzzy sets rather than interval valued fuzzy sets. The reason for these speed issues is primarily in defuzzification. However there is much new research in this area that helps (e.g. [9, 6, 17]). Nevertheless this is a problem. When comparing type-2 fuzzy systems applications with type-1 there is no comparing "like with like." Typically researchers develop a type-1 system (that may or may not be optimized) and then blur the membership functions and then optimize the system in some sense. So, a challenge is to provide a theoretical framework that allows us to compare type-1 and type-2 solutions and help us to decide whether to use a type-1 approach. Because type-1 systems are easier to understand and build there has to be some potential improvement to warrant the extra "work" involved. Since general type-2 fuzzy logic is computationally intensive one possible solution would be to have a hardware implementation. Some work has already been done using FPGAs [18, 19, 20] but more work is needed.

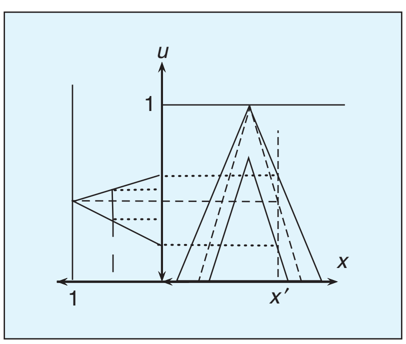

FIGURE 2 2D representation of the Type-2 fuzzy set.

2.1.3. Understanding of Uncertainty

There are a number of issues here. The term "uncertainty" is widely used usually in a way that is not formally defined. There is an inherent "we know what we mean by uncertainty" underpinning much of the written work in the field of fuzzy logic Researchers talk about levels and orders of uncertainty. The challenge is to understand the whole notion of uncertainty *from a fuzzy perspective* and place it in the context of type-1 and type-2 fuzzy logic. In engineering applications we often use the term uncertainty to refer to noise and imprecision which are quite different to the kinds of uncertainty which occur in computing with words problems. This use of uncertainty as a blanket term which can be used to cover areas where we have a lack of understanding which is not helpful.

Another aspect is that of measures of uncertainty in the output of a type-2 fuzzy system. After inferencing we defuzzify a type-2 fuzzy set to a crisp number often through type-reduction (although there are other methods emerging that miss out that step). But there is no accepted way of attaching some confidence to the solution. We throw away a lot of information in

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defuzzification (by the way this is also true in type-1 fuzzy systems).

2.2. Learning

The properties of a fuzzy system arise from a complex combination of a number of contributing components including the rules, the shape of the fuzzy sets, the positions of the fuzzy sets and the operators used in the logical operations. We may loosely call this set of facts which describe a fuzzy systems the system parameters. The parameters used in any given system will typically be arrived at in one of two ways, expert knowledge is captured and recorded in the systems operation or the parameters are learned in someway from a data set. A key research challenge in type-2 fuzzy systems, particularly in general type-2, is understanding what the parameter set for a type-2 fuzzy system should be and how those parameters should be arrived at. This is perhaps not an issues limited to type-2 fuzzy logic, however the additional parameters and the semantics of these parameters make the problem more interesting and more challenging.

2.2.1. Membership Function Form

In type-1 fuzzy systems a set of membership function forms are commonly used: Gaussian, triangular, trapezoidal, shoulder, and Gaussian shoulder. These give the membership functions clear form from a concise, easily manipulated sets of parameters. For example if a Gaussian function is used to define a fuzzy set it can be totally defined by two parameters: centre and width, completely defining the fuzzy set over a continuous domain. For general type-2 fuzzy system we have a new situation, a membership function defined over three dimensions. So we face a new question: what form should a three dimensional fuzzy membership function have?

For a long time the type-2 community essentially ignored this issues, believing it had been addressed by the concept of blurring a type-1 membership. A paragraph from Mendel and John's seminal paper [22] describes this approach eloquently:

> "Imagine blurring the type-1 membership function depicted in Fig. 3(a) by shifting the points on the triangle either to the left or to the right and not necessarily by the same amounts, as in Fig. 3(b). Then, at a specific value of x, say x', there no longer is a single value for the membership function (u'); instead, the membership function takes on values wherever the vertical line intersects the blur. Those values need not all be weighted the same; hence, we can assign an amplitude distribution to all of those points. Doing this for all $x \in X$, we create a three-dimensional membership functional type-2 membership function that characterizes a type-2 fuzzy set."

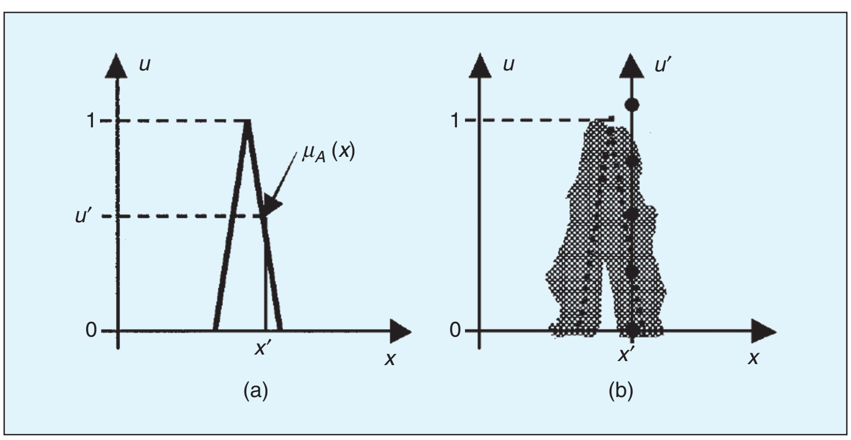

FIGURE 3 (a) Type-1 membership function and (b) blurred type-1 membership function, including discretization at $x = x'$. (Figure reproduced from [22].)

A careful rereading of this excerpt will demonstrate that the notion of blurring is used to illustrate and communicate the form of a general type-2 membership function and less so an advocated methodology for constructing a type-2 membership function. In some applications blurring may work well [5], however there is no guarantee of success and in certain applications areas this approach will fail. When the blurring approach is used the secondary membership functions will take on a specific form, depending on which direction the blurring is performed. If blurring is up and down, this will create a different result to blurring left to right.

This gives another way of viewing general type-2 membership functions; type-1 membership functions where the membership grades are themselves fuzzy sets. These secondary membership functions can take one of the typical membership function forms, or perhaps more interestingly they can take on custom form arrived at by data. In engineering applications it may be possible to measure the natural variation inherent in a given sensor and to build this into the fuzzy set model [28]. The benefits of doing so are not yet clear.

The discussion presented in this Section is predicated on the notion that this third-dimension, as it has become to be known, is crucial to system performance. Whilst such observations have been made [5], a piece of research using the alpha-plane representation has reached a different conclusion. In [23], Mendel et al investigated the effect the form of secondary membership function has on the defuzzified value of a type-2 fuzzy sets modelled using the alpha-planes representation. Extreme forms were used for the secondary membership functions, from left shoulder, to triangular to right shoulder. These extreme variations resulted in the value of the overall centroid varying by less than 1%. More research is need to understand this result in the context of a working system and compare the result to a similar experiment which does not make use of the alpha-plane representation.

Given the variety of forms and representations available to construct type-2 fuzzy sets an intuitive and appealing approach is learning the membership functions from data.

2.2.2. Learning Algorithms

Given the variety of forms and representations available to construct type-2 fuzzy sets an intuitive and appealing approach is learning the membership functions from data. Robert John's early work looked how an ANFIS like approach might be applied to general type-2 fuzzy systems. Mendel et al showed how to compute derivatives for interval type-2 fuzzy systems and how to perform supervised learning for such systems using gradient descent type algorithms [16, 21]. Hani Hagras et al [27, 11] have used evolutionary computation techniques to successfully tune interval type-2 fuzzy systems in a range of engineering applications. Almaraashi *et al* are conducting ongoing work [3, 2] into the use of simulated annealing for tuning type-2 fuzzy systems.

In any work which is learning type-2 sets it is necessary to define a set of parameters which define the form of the fuzzy set. Learning algorithms tune these parameterized type-2 fuzzy systems, as happens in the majority of tuned type-1 systems. The choice of parameters used to define a type-2 set restricts the form the membership function can take and restricts the form each secondary membership function can take. There are three approaches which can be taken to parameterize a general type-2 fuzzy set:

- ❏ **No slicing.** The membership function is defined by a set of parameters across three dimensions. In practice this is often done with a curried function, a function which it self returns a function i.e., $\mu_{\tilde{A}} : X \rightarrow [0,1] \rightarrow [0,1]$.
- ❏ **Vertical slicing.** The membership function is defined in two parts; a principal membership function and a secondary membership function for each point on the principal membership function. The principal membership function is a type-1 function and make use of standard type-1 parameterization. Each secondary membership function is also a type-1 function and makes use of standard type-1 parameterization.
- ❏ **Alpha-plane/Z-slicing.** The membership function is sliced into a number of interval type-2 fuzzy sets. Each of these interval type-2 fuzzy sets can be defined using appropriate parameters.

The research challenge for learning type-2 systems is to investigate which learning algorithms tune the sets efficiently for a given problem. Another important question is which parameter model used to define a type-2 fuzzy set gives the right level of flexibility without raising the level of computational complexity of learning prohibitively high. Information measures for type-2 fuzzy sets are in their infancy [13, 31] but are likely to inform research in this area.

2.3. Applications

2.3.1. Application Areas

Type-2 fuzzy system have shown improved performance over type-1 systems in engineering applications where high levels of uncertainty are present [10, 5]. This improved performance is not guaranteed. There has been a great deal of debate about the performance of type-2 systems, largely centered around engineering applications. This debate is in danger of missing a key objective for type-2 fuzzy systems, effectively modelling of linguistic uncertainties. The type-2 community have demonstrated in a range of applications that it is possible to construct a type-2 systems which gives improved performance over a type-1 system. Perhaps a worthy research questions is: does an application exist which a type-2 fuzzy system can do which a type-1 system cannot? In our opinion whilst this killer application does not exist then the debate of the usefulness of type-2 will continue.

The majority of the applications of type-2 reported in the literature are in areas where type-1 has previously been applied. The ambition of researchers working on type-2 applications should be to identify areas where type-2 systems haven't worked and investigate these areas. Areas where there are high levels of uncertainty, particularly linguistic uncertainty, would appear to be a good place to begin. These areas include medical decision making, scheduling and timetabling and complex geographical information systems.

2.3.2. Performance Comparisons

Claims made about applications of type-2 fuzzy systems need be substantiated by hard evidence. In an increasing number of papers in the field, rigorous qualitative analysis is being used to support claims being made about type-2 systems. Clearly a thorough analysis of experimental results is the best way of supporting claims made in a piece of research, and vital when performing comparisons. However, the other aspect of any experiment is the way in which the experiment is setup to ensure like is being compared with like.

When comparing a type-1 fuzzy system with a type-2 fuzzy system the question of system equality is a complex one. The simplistic answer is to use systems with the same number of sets and rules [4, 5]. When this approach has been taken the type-2 system often gives a better performance over the type-1 system, but in this case are the two system equivalent? If we consider the complexity of such systems then the systems are clearly not equivalent. There are a number ways of measuring system complexity, for example the number of terms, the number of design parameters or the level of computational complexity. Garibaldi advocates an investigation into an information measure analogous to Akaike's Information Criterion (AIC). AIC provides a way of measuring the fitness of a statistical model by essentially measuring the information lost when fitting the statistical model. This allows for fair comparisons of models of different complexities using appropriate qualitative techniques. Such a methodology would be of great benefit when comparing fuzzy systems of different types.

3. Misconceptions

3.1. Performance

The first misconception is that type-2 fuzzy logic systems/controllers will always outperform their type-1 counterparts. This is not necessarily true and the type-2 community have sometimes made claims where there is clearly little or no difference. As discussed above great care needs to be taken when comparing type-1 and type-2. Indeed we would argue that this is probably a waste of time for a given application. The evidence is that in a very noisy environment or where there are linguistic uncertainties type-2 fuzzy logic *may* assist in providing good solutions. In a way it is still early days for the type-2 applications as only in the recent few years have we had the algorithms to use generalized type-2 fuzzy systems. Applications of type-1 fuzzy logic have been around for decades.

A related misconception is that real time type-2 fuzzy systems are not possible. Recent advances using alpha planes and z slices may well allow for real time and indeed if we deploy interval valued type-2 fuzzy systems then "fast enough" systems can be deployed.

3.2. The Role of Type-1

The second misconception is that type-2 fuzzy logic will become the norm and replace type-1 fuzzy systems. This is not our view. As reiterated a number of times here we see that type-1 fuzzy logic will continue to be important and type-2 should be used if necessary. This is an open research question.

4. Summary

In this paper we have tried to collate some thoughts on challenges and misconceptions of type-2 fuzzy systems. There are many challenges for new researchers in the field and many interesting avenues to pursue. When considering when to use type-2 the chief message is that it is not type-1 vs type-2 but, to use an English phrase, "Horses for Courses." That is we can use type-1 or type-2. Experience to date shows that type-2 can help in certain cases. The type-2 applications field is still very new and we have much with which to look forward.

References

[1] J. Aisbett, J. T. Rickard, and D. Morgenthaler, "Type-2 fuzzy sets as functions on spaces," *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 4, pp. 841–844, 2010.

[2] M. Almaraashi and R. John, "Tuning of type-2 fuzzy systems by simulated annealing to predict time series," in *Proc. Int. Conf. Computational Intelligence and Intelligent Systems*, London, July 2011.

[3] M. Almaraashi, R. John, S. Coupland, and A. Hopgood, "Time series forecasting using a tsk fuzzy system tuned with simulated annealing," in *Proc FUZZ-IEEE*, Barcelona, Spain, July 2010, pp. 1–6.

[4] S. Coupland, M. Gongora, R. John, and K. Wills, "A comparative study of fuzzy logic controllers for autonomous robots," in *Proc. IPMU*, Paris, France, 2006, pp. 1332–1339.

[5] S. Coupland and R. John, "Geometric type-1 and type-2 fuzzy logic systems," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 1, pp. 3–15, Feb. 2007.

[6] S. Coupland and R. John, "A fast geometric method for defuzzification of type-2 fuzzy sets," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 4, pp. 929–941, 2008.

[7] D. Dubois and H. Prade, "A class of fuzzy measures based on triangular-norms," *Int. J. General Syst.*, vol. 8, pp. 43–61, 1982.

[8] M. B. Gorzalczany, "A method of inference in approximate reasoning based on interval-valued fuzzy sets," *Fuzzy Sets Syst.*, vol. 21, pp. 1–17, 1987.

[9] S. Greenfield, F. Chiclana, S. Coupland, and R. John, "The collapsing method of defuzzification for discretised interval type-2 fuzzy sets," *Inform. Sci.*, vol. 179, no. 13, pp. 2055–2069, June 2009.

[10] H. Hagras, "Type-2 FLCs: A new generation of fuzzy controllers," *IEEE Comput. Intell. Mag.*, vol. 2, no. 1, pp. 30–43, Feb. 2007.

[11] H. Hagras, F. Doctor, A. Lopez, and V. Callaghan, "Evolving type-2 fuzzy agents for ambient intelligent environments," in *Proc. Int. Symp. Evolving Fuzzy Systems*, 2006, pp. 208–213.

[12] H. Hagras and C. Wagner, "Introduction to interval type-2 fuzzy logic controllers—Towards better uncertainty handling in real world applications," *IEEE Syst., Man Cybern. eNewsletter*, no. 27, June 2009.

[13] H. Hamrawi and S. Coupland, "Non-specificity measures for type-2 fuzzy sets," in *Proc. FUZZ-IEEE*, Jeju Island, South Korea, 2009, pp. 732–737.

[14] N. N. Karnik, J. M. Mendel, and Q. Liang, "Type-2 fuzzy logic systems," *IEEE Trans. Fuzzy Syst.*, vol. 7, pp. 643–658, 1999.

[15] G. J. Klir and T. A. Folger. *Fuzzy Sets, Uncertainty, and Information*. Englewood Cliffs, NJ: Prentice-Hall, 1988.

[16] Q. Liang and J. M. Mendel, "Interval type-2 fuzzy logic systems: Theory and design," *IEEE Trans. Fuzzy Syst.*, vol. 8, pp. 535–549, 2000.

[17] F. Liu, "An efficient centroid type-reduction strategy for general type-2 fuzzy logic system," *Inform. Sci.*, vol. 178, no. 9, pp. 2224–2236, 2008.

[18] C. Lynch, H. Hagras, and V. Callaghan, "Embedded type-2 FLC for real-time speed control of marine and traction diesel engines," in *Proc. FUZZ-IEE*, Reno, AZ, May 2005, pp. 347–352.

[19] C. Lynch, H. Hagras, and V. Callaghan, "Parallel type-2 fuzzy logic co-processors for engine management," in *Proc. FUZZ-IEEE*, London, 2007, pp. 907–912.

[20] M. Melgarejo and C. Pena-Reyes, "Hardware architecture and FPGA implementation of a type-2 fuzzy system," in *Proc. GLSVSLI*, Boston, MA, Apr. 2004, pp. 458–461.

[21] J. M. Mendel, "Computing derivatives in interval type-2 fuzzy logic systems," *IEEE Trans. Fuzzy Syst.*, no. 1, 2004.

[22] J. M. Mendel and R. I. John, "Type-2 fuzzy sets made simple," *IEEE Trans. Fuzzy Syst.*, vol. 10, no. 2, pp. 117–127, 2002.

[23] J. M. Mendel, F. Liu, and D. Zhai, "Alpha-plane representation for type-2 fuzzy sets: Theory and applications," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 5, pp. 1189–1207, 2009.

[24] M. Mizumoto and K. Tanaka, "Fuzzy sets of type 2 under algebraic product and algebraic sum," *Fuzzy Sets Syst.*, vol. 5, pp. 277–290, 1981.

[25] M. Mizumoto and H.-J. Zimmerman, "Some properties of fuzzy set of type-2," *Inform. Contr.*, vol. 31, pp. 312–340, 1976.

[26] D. G. Schwarz, "The case for an interval-based representation of linguistic truth," *Fuzzy Sets Syst.*, vol. 17, pp. 153–165, 1985.

[27] C. Wagner and H. Hagras, "Evolving type-2 fuzzy logic controllers for autonomous mobile robots," in *Proc. Analysis and Design of Intelligent Systems Using Soft Computing Techniques*. Berlin: Springer-Verlag, 2007, pp. 16–25.

[28] C. Wagner and H. Hagras, "Novel methods for the design of general type-2 fuzzy sets based on device characteristics and linguistic labels surveys," in *Proc. IFSA*, July 2009, pp. 537–543.

[29] L. Zadeh, "The concept of a linguistic variable and its application to approximate reasoning," *Inform. Sci.*, vol. 8, pp. 199–249, 1975.

[30] L. A. Zadeh, "Fuzzy sets," *Inform. Contr.*, vol. 8, pp. 338–353, 1965.

[31] D. Zhai and J. M. Mendel, "Uncertainty measures for general type-2 fuzzy sets," *Inform. Sci.*, vol. 181, pp. 503–518, Feb. 2011.