Homogeneous coordinates and matrix representation

To perform sequence of transformation such as translation followed by rotation and scaling, we need a sequence of process

① Translate the coordinates.
② Rotate the translated coordinates. ~~and then~~
③ scale the rotated coordinates and complete the composite transformation

To convert a 2X2 matrix into 3X3 & 4X4 matrix in this way we can represent the points by three number instead of 2 number, which is call homogeneous co-ordinates system.

In this system, we represent all the transformation equation in matrix multiplication.

Old(x, y)

$h = 2$

$xh = x \cdot h$

$xh \quad 1 \times 2 = 2$

$yh = y \cdot h$

$yh = 2 \times 2$

$yh = 4$

new (x', y')

$P(x, y) = (1, 2)$

$x' = xh / h$

$y' = yh / h$

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Example - We have a I co-ordinate P(2,3) convert into homogeneous co-ordinates here $h = 2$, what is the new co-ordinates

Solution - old coordinates

$$(x, y) = (2,3)$$

$$h = 2$$

$$x'h = x \cdot h$$

$$x'h = 2 \times 2$$

$$x'h = 4$$

$$y'h = y \cdot h$$

$$y'h = 3 \times 2$$

$$y'h = 6$$

$$h = 2$$

new coordinates = $(4, 6, 2)$ 3D

Again old coordinates -

$$x' = x = \frac{x'h}{h}$$

$$x = \frac{4}{2}$$

$$x = 2$$

$$y' = y = \frac{y'h}{h}$$

$$y = \frac{6}{2}$$

$$y = 3$$

$$(x, y) = (2,3)$$

Rotation – Ratates an object arrounds specify points or origin

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scaling – In scaling resizes an object x and y axis

$$\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Successive and composite 2D Transformation

A sequence of transformation can be combined in to single transformation called Composition. we can perform get it by sequence of 3 transformation-

(1) Translation
(2) Rotation
(3) Reverse Translation.

$$\left[S' = T_v \, S(s_x, s_y) \, T_{-v} \right]$$

$$\begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scaling -

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Homogeneous coordinates of scaling.
2D to 3D

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Matrix Representation of 2D Transformation

In computer graphics, 2D transformation is represented using 3x3 transformation matrix allows far transformation, rotation and scaling.

Translation - In translation moves object in its y direction

Matrix representation 3x3

$$\begin{vmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{vmatrix}$$

Point the homogenous co-ordinates using translation, rotation, scaling

Translation -

$$x' = x + tx$$
$$y' = y + ty$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} tx \\ ty \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$

We have to convert 2X2 matrix in 3X3 matrix in homogeneous coordinates of translation

$$\begin{bmatrix} x' \\ y' \\ hr \end{bmatrix} = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

here, $hr = 1$

Rotation -

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cdot\cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

homogeneous co-ordinates of rotation
2D → 3D

The window to viewport - Real word things we have to see in screen of mobiles, ~~see~~ laptop our computers this is called a window to viewport. we see ~~in~~ things in large size on real word but in our screen we see the thing in small size. ~~A window~~.

• Clipping - What we have to see and which devicte we have to see the picture.

World co-ordinate system - Real image like pen our other image

Screen co-ordinate System - The space in which the image will display screen.

Window - What image we want to see.
Viewport - What think we want to see.

* [You have given window co-ordinates and view port co-ordinates if not mention in the Question then we assume four normalised coordinates are (0,0) (1,1) four normalised device]

$$\begin{bmatrix} 2+0+0 & 0+0+0 & -2+0+1 \\ 0+0+0 & 0+2+0 & 0+(-2)+1 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

Now we have to perform scaling which coordinates are given

our

Now we will fine the resultant

$$P' = S'(x,y)\,P(x,y)$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Homogeneous representation

$$\begin{bmatrix} 0+0-1 & 4+0-1 & 4+0-1 & 0+0-1 \\ 0+0-1 & 0+0-1 & 0+4-1 & 0+4-1 \\ 0+0+1 & 0+0+1 & 0+0+1 & 0+0+1 \end{bmatrix}$$

$$\begin{array}{c} \begin{bmatrix} -1 & 3 & 3 & -1 \\ -1 & -1 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \\ \begin{matrix} A' & \;B' & \;C' & \;D' \end{matrix} \end{array}$$

$A'(-1,-1)$ $B'(3,-1)$ $C'(3,3)$ $D'(-1,3)$

Question- A square with vertices (0,0) (2,0) (2,2) (0,2) is scaled 2 units in x direction as well as y direction about the fixed point which is center of square (1,1) find the co-ordinates of vertices of new sequare (0,0)

Solution- $S' = T_v \, S(S_x, S_y) \cdot T_{-v}$

$$\begin{bmatrix} 1 & 0 & x_{vr} \\ 0 & 1 & y_{vr} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_{vr} \\ 0 & 1 & -y_{vr} \\ 0 & 0 & 1 \end{bmatrix}$$

fixed point is always be (1, 1) & that is ~~(xvr)~~ (x_{vr}, y_{vr})
(1, 1)

Scaling factor is $S_x = 2$ and $S_y = 2$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2+0+0 & 0+0+0 & 0+0+1 \\ 0+0+0 & 0+2+0 & 0+0+1 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

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Numerical (window to viewport)

Find the normalization transformation that makes of window whose lower left corner is add (1,1) and upper left corner is add (3,5) on to a viewport that is entire normalized device screen

$$N = \begin{bmatrix} 1 & 0 & V_{xmin} \\ 0 & 1 & V_{ymin} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -W_{xmin} \\ 0 & 1 & -W_{ymin} \\ 0 & 0 & 1 \end{bmatrix}$$

$\downarrow$

Normalization.

Viewport (0,0) (1,1)

V_{xmin} V_{ymin} V_{xmax} V_{ymax}.

(1,1) (3,5) (0,0) (1,1)

$W_{xmin} = 1$

$W_{ymin} = 1$

$W_{xmax} = 3$

$W_{ymax} = 5$

$V_{xmin} = 0$

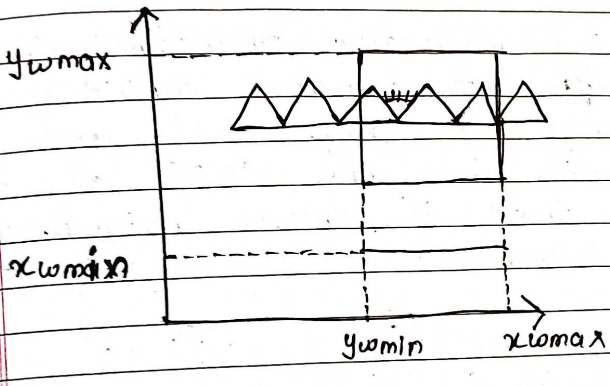
$V_{ymin} = 0$

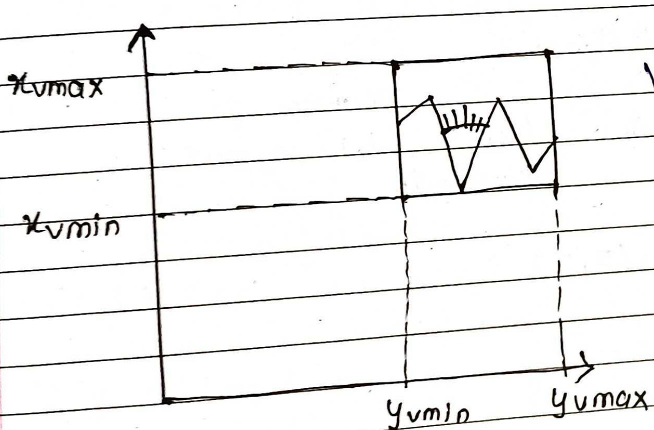
$V_{xmax} = 1$

$V_{ymax} = 1$

$$S_x = \frac{\text{Viewport's } x \text{ difference}}{\text{Window's } x \text{ difference.}}$$

$$S_x = \frac{V_{xmax} - V_{xmin}}{W_{xmax} - W_{xmin}}$$



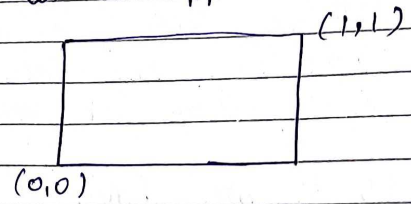


Viewport

Device co-ordinate

$Vx_{mn} = 0$ & $Vy_{mn} = 0$.
$Vx_{max_1} = 1$ & $Vy_{min} = 1$

(0,0) is a lower left corner
(1,1) is a upper left corner.



We assume normalized co-ordinates (0,0) (1,1)

Graphic package allows a user to specify which part of a define image is to be display and where the parts of an image is display on displayed device using a concept Knocus ~~Kpan~~ as clipping.

Window - Selecting portion of a drawing is called windowing and rectangle area which is selected called window

Viewport - rectangle of a interface window that defines where the image accutually appear called viewport.

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$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$T_x = 3$, $T_y = 3$, $T_z = 2$.
$x = 5$, $y = 6$, $z = 7$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 7 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 5+0+0+3 \\ 0+6+0+3 \\ 0+0+7+2 \\ 0+0+0+1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 9 \\ 9 \\ 1 \end{bmatrix}$$

$x' = 8$, $y' = 9$, $z' = 9$

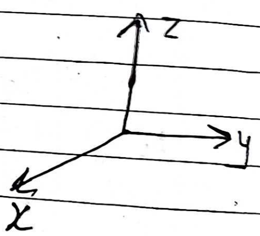
Introduction to 3D Transformation Matrix

In this 3D transformation we have 3 Co-ordinates A, B, C. in

In this 3D transformation process of manipulating the view of 3D object with respect to its original position by modifying its physical attributes through various methods of transformation like translation, scaling, rotation, reflextion, shearing.

Translation -

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$\boxed{P' = T \cdot P}$

Homogeneous representation

Example - (x, y, z) direction (5, 6, 7) and co-ordinate of points are (5, 6, 7) and 3 units in x direction and 3 unit of y direction and 2 units of z direction. Find the new co-ordinates.

$$Sy = \frac{\text{Viewport's } y \text{ difference}}{\text{Window's } y \text{ difference}}$$

$$Sy = \frac{Vymax - Vymin}{Wymax - Wymin}$$

$$Sx = \frac{1-0}{3-1} \qquad Sy = \frac{1-0}{5-1}$$

$$= \frac{1}{2} \qquad\qquad = \frac{1}{4}$$

$$N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1/2+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1/4+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1/2+0+0 & 0+0+0 & -1/2+0+0 \\ 0+0+0 & 0+1/4+0 & 0+(-1/4)+0 \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$N = \begin{bmatrix} 1/2 & 0 & -1/2 \\ 0 & 1/4 & -1/4 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation - Rotating an object with respect of an angle. Let co-ordinates of $P(x, y, z)$ ~~rotation angle~~.

rotation angle $= \theta$

new co-ordinates ~~that~~ are $P'(x', y', z')$

Three possible types of rotation

x axis   y axis   z axis

X axis

rotation with respect to x - axis

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

rotation with respect to y axis -

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$X' = Z\sin\theta + X\cos\theta$

$Y' = Y$

$z' = Z\cos\theta - x\sin\theta$

Reflection — (xy, yz, zx plane)

(a) reflection in xy plane

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(b) reflection in yz plane

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(c) reflection is zx plane

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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Homogeneous representation of scaling.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Example- A(0,3,3), B(3,3,6), C(3,0,1)
2 Units in x direction, 3 Units in y direction and 3 units in z direction. find the new co-ordinates.

Solution- A (0,3,3)

$A' = x' = x \times S_x$
$y' = y \times S_y$
$z' = z \times S_z$

$x' = 0 \times 2 = 0$
$y' = 3 \times 3 = 9$
$z' = 3 \times 3 = 9$

$B' = x' = x \times S_x = 3 \times 2 = 6$
$y' = y \times S_y = 3 \times 3 = 9$
$z' = z \times S_y = 6 \times 3 = 18$

$C' = x' = x \times S_x = 3 \times 2 = 6$
$y' = y \times S_y = 0 \times 3 = 0$
$z' = z \times S_z = 1 \times 3 = 3$

Numerical :-

four a given matrix first apply rotation of 45° about y axis followed by rotation of 45° about x axis determine the resultant matrix.

Solution - given matrix

$$\begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 0 & 3 & 6 & 1 \end{bmatrix}$$

There are two rotations ~~are~~ rx and ry

$\theta = 45°$

$$R_y = \begin{bmatrix} \cos 45° & 0 & \sin 45° & 0 \\ 0 & 1 & 0 & 0 \\ -\sin 45° & 0 & \cos 45° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 45° & -\sin 45° & 0 \\ 0 & \sin 45° & \cos 45° & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now we have to perform to composite transformation.

Shearing in y - direction (axis) -

$x' = x + Sh_x \cdot y$
$y' = y$
$z' = x + Sh_z \cdot y$

Matrix representation -

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & Sh_x & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & Sh_z & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Shearing in z - axis -

$x' = x + Sh_x \cdot z$
$y' = y + Sh_y \cdot z$
$z' = z$

Matrix Representation -

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Sh_x & 0 \\ 0 & 1 & Sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

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rotation with respect to z axis.

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$x' = x\cos\theta - y\sin\theta$
$y' = x\sin\theta + y\cos\theta$
$z' = z$

Sharing — It is used to object in 3D plane either in x, y or in z direction. It changes the shape of object.

Difference types of shearing transformation —

Shearing in x - direction
Shearing in y - direction
Shearing in z - direction

(a) Shearing in x direction —

$x' = x$
$y' = y + x \cdot shy$
$z' = z + x \cdot shy$

Matrix representation —

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ Shy & 1 & 0 & 0 \\ Shz & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$A \times B \neq B \times A$

$$R = R_x \circ R_y$$

$$R = \begin{array}{cc} R_y & R_x \\ \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$R = \begin{bmatrix} \cancel{1/\sqrt{2}+0+0+0} & 1/\sqrt{2} \cdot 1 \\ \cancel{0+0+0+0} & \\ \cancel{-1/\sqrt{2}\cdot} & \\ & \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1/\sqrt{2}+0+0+0 & 0+0+0+0 & 1/\sqrt{2}+0+0+0 & 0+0+0+0 \\ 0+0+1/2+0 & 0+1/\sqrt{2}+0+0 & 0+0+(-1/2)+0 & 0+0+0+0 \\ 0+0+(-1/\sqrt{2})+0 & 0+1/\sqrt{2}+0+0 & 0+0+1/2+0 & 0+0+0+0 \\ 0+0+0+0 & 0+0+0+0 & 0+0+0+0 & 0+0+0+1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 1/2 & 1/\sqrt{2} & -1/2 & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R = R_x \cdot R_y$

$$R = \begin{array}{cccc} 0 & 0 & 0 & 1 \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} & 0 \end{array}$$
$$\begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 1 & 0 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1/\sqrt{2}+0+0+0 & 0+0+1/2+0 & 0+0+1/2+0 \\ 0+0+0+0 & 0+1/\sqrt{2}+0+0 & 0-1/\sqrt{2}+0+0 \\ -1/\sqrt{2}+0+0+0 & 0+0+1/2+0 & 0+0+1/2+0 \\ 0+0+0+0 & 0+0+0+0 & 0+0+0+0 \end{bmatrix}$$

$$\begin{bmatrix} 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+0 \\ 0+0+0+1 \end{bmatrix}$$

$$R = \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ \cancel{0}\ 1/2 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 1/2 & 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

the corner are points of interest on the object

Edges – Edges are the line sequence segment connecting pairs of vertices they form the boundries of polygon and are defining the shape of object

Faces – faces are the polygon formed by connecting three or more vertices with edges.
triangle (four sided polygon)

A mesh is a collection of vertices, edges and the faces that describe the shape of 3D object
A. vertex is a single point. An edge is a straight line segment connecting two vertices.

Quadratic & Super quadratic Surface –

Quadratic surface is in computer graphics refers to the surfaces that are defined by quadratic equations involves second – degree terms
Example – Example of Quadratic surface which includes a shapes like Cones, spheare, paraboloids, Hyperbeloids, ellipsoids

UNIT-04

Curves & Surface

Polygon Surfaces — Polygon is a representation of surface. it is primitive which is closed in nature. It is formed using a collection of lines it is called as many sided figure. these lines combined to form polygon are called sides or edges figure. It is used to represent the 3D Object Example- Triangle, rectange, Hexagon, pentagon.

Polygon - Meshes In polygon meshes are a common way to represent 3D surfaces in computer graphics. In one mesh include verticees, edges and faces this is based on some points

Vertices — These are a points in 3D's space and represented by their coordinates (x, y, z) vertices define

R X Resultant Matrix.

$$= \begin{bmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} & 0 \\ 1/2 & 1/\sqrt{2} & -1/2 & 0 \\ -1/\sqrt{2} & 1/2 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 3 & 0 & 0 \\ 4 & 0 & 1 & 0 \\ 0 & 3 & 6 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2/\sqrt{2} + 0 + 1/\sqrt{2} + 0 \\ 1 + 0 - 1/2 + 0 \\ -2 \\ \end{bmatrix}$$

Super quadratic Surfaces –
Super quadratic surfaces are the mathematical models frequentily used In computer Graphics to repuresent 3 daimentional shape.

They extends 3D diamentional shape
They extend the concept of basic geometric like sphere, cylinder

Super quadratic are define by parameters that controls their shapes such as roundness, bending.

Componants –

Roundness – In roundness determine sharpness of corner & edges

Bending – In bending control how much bends the surface ~~for. depends~~ along each axis.

- Size parameter – These controls the size of super quadratic along each axis

- Scale factor – Scale the super quadrate along each axis independent

~~Orientation parameter~~

Paraboloid — A paraboloid is a surface that extends in a one direction while parabola shape in a perpendicular direction.

Eq^n — $z^2 = ax^2 + by^2$ [illegible] constant

Hyperboloid — Hyperbolic surface consist of two separate pieces or a sheet.

Eq^n — $x^2 + y^2 - z^2 = \pm 1$

or

$z^2 - x^2 - y^2 = \pm 1$

Cone — Geometric surfaces play a [illegible] role in the [illegible] [illegible] and [illegible] [illegible] [illegible]

Cone — Cone is a 3D object circular base.

Eq^n — $z^2 = ax^2 + by^2$

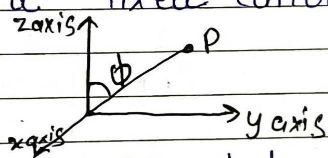
$z^2 = x^2 + y^2$ is representing the right circular cone along with z axis

Each of these shapes can be describe mathematically using quadratic. They are essential for create in realistic An visually 3D scenes in the application such as animation and rendering, common quadratic surfaces.

~~(i) Sphere -~~

Common Quadratic surfaces -

Sphere - A sphiere is a three daimentional object with all equal distance from a fixed center point

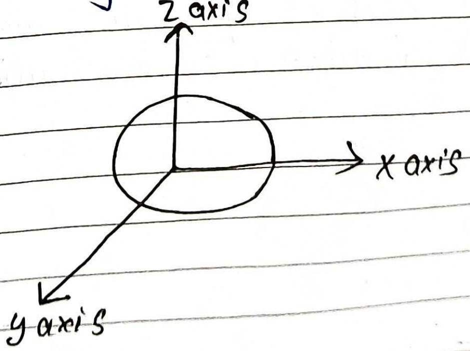


$Eq^n = [x^2 + y^2 + z^2 = w^2]$

Ellipsoid - A electron is a 3D shape resembling a complex sphiere

eqn :- $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Where a, b, c represented semi major axis along x, y, z axis respectively.



Orientation Angle

(i) Rotation Angle — Its specify the rotation arround x, y & z axis. Super quadratic surfaces is a flexible way to represent the object in computer graphics with relative ~~in~~ & small sec of parameter, making render realistics seems. super quadratic is defines by parameter that control there sheeks and sizes

Spline ~~cause~~ curve & Representation of spline curve —
In this process we used mathematical representation four which is easy to ~~with~~ build interface. that will allow a user to design & control the shape of complex curves and surfaces. This curve mathematically discribe with in a piece of cubic polynomial function.

Control point -

tion Angle

Angle — Its specify the
rotation arround
axis . Super quadratic
is a flexible way to
the object in computer
ith relative ~~in~~ & small
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Control point —

Orientatio

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Control point –

Approximate Spline

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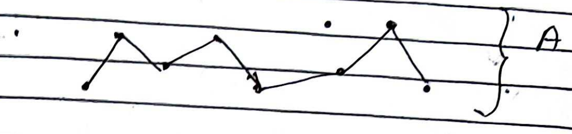
iii) Control points are also given there but our control curve make with all of the control point.



In C.G., this spline is mathematically represented using a series of Control point and basic function to ~~enter~~ Interpolate or Approximate Curve. One common representation B-spline curve, which is defined by a set of control points, a degree.

B-spline curve by introducing weights to control each the each control points. These representation allows for flexible and appreciate efficient manipulation of curve in C.G. Application.

- **Components** - (a) B-spline Curve
  (b) Hermite Curve
  (c) Beizer Curve

(a) **B-spline Curve** - These are the defined by a set of control points, degree. They provides the local control and commonly used in animation.

that controls their shapes and sizes.

→ Spline Curves & Representation of Spline Curve ←

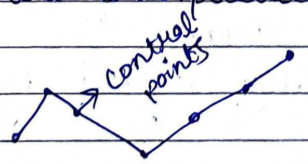
In this process we use mathematical representation for switch which is easy build an interface that allows a user to design and control the shape curves and surface.

This curve mathematically that describes within a piece of cubic polynomial function.

* **Control Point** - By giving a sets of coordinates positions, called the control points which indicates the general shape of the curve. These control points are fixed with piece curve continuous.
There are two types of curve/spline -

(1) Interpolation Spline
(2) Approximate Spline

(1) **Interpolation Spline** - When we use the points and make the curve passes through all the control points then resulting curve are interpolate set control points



Orientation Angle

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Control point –

Date- 22/April/2024

UNIT 7 COMPUTER ANIMATION

Computer Animation in C.G. refers to the process of creating of moving images using computer technology. It involves generating sequence of images, computer animation enabling the creation of visually and effects in movies, video games use of animation performs cartoons, video games, films and advertisements.
~~Animation~~

* **Components of Computer Animation-**

(1) Modelling - Modelling is a process of creating 3-D models of objects or environment using especialized software. Model defines the shape and structure.

(2) Rendering - Rendering is a process of generating 2-D image from 3-D images. This involves calculating scene, apply Shading, output of the image.

(3) Lighting - Lighting is essential for setting the mood and atmosphere of a scene. It involves placing lights within 3-D environment and adjusting their properties to achieve Lighting effects.

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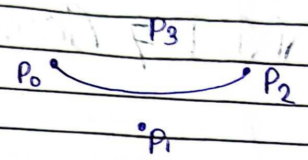
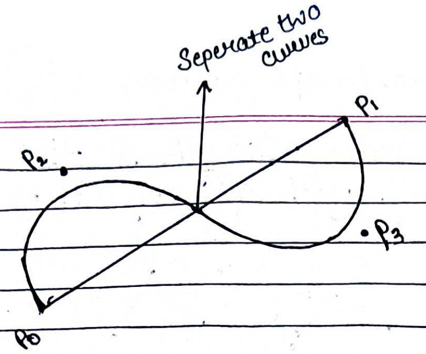
- 4-point curve -



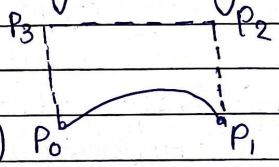
Degree = 4-1
= 3
(Cubic Curve)



(b) **Hermite Curves** – These are defined by specifying not only all the control points. They are useful for creating curves with ~~specific~~ tangent direction at each points. Tangents is a mathematical concept which curve or a function slope or a direction is described.

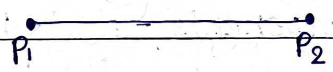
(c) **Beizer Curve** – In beizer curve, points are not always always on the curve.

Two-point curve.



Control points (P_0, P_1, P_2, P_3)

- Two-point curves –

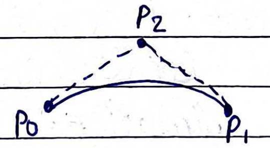


To find Degree $= n-1$

In 2-point curve, $n=2$

Degree $=1$ (Linear Curve)

- 3-point curves –



Degree $= (n-1)$
$= (3-1)$

Quadratic Curve

Degree $= 2$

Scan conversion - (1)

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* **Key frame System** - Key frame is a important concept used for editing animation or videos. This can represent specific point like. Key frames can define the specific moments mark. This moment describes movement or change in transform. Animators allows specific moment control desired animation effects achieve. Key frame concept used in animation to define specific points where important changes occur. These key frame serve as reference points that helps to determine state of object, scene at a particular moment during animation sequence.

(a) Defining key frames - Animators first identify key movements or passes within animation sequence.
These key frames represent significant changes in position, rotation, scale.

* Component of Morphing -

(1) Key frames - Process begin with which are starting and ending points of transformation. Key frames contain images or shapes.

(2) Rendering - Once intermediate frames are calculated, they are rendered to create final animation. This involves generating pixel for each frame based.

Morphing can be used for various purposes -

(a) Special effects
(b) Image Manipulation

(a) Special Effects - Creating shape shifting transformation for objects in moves, to show video games.

(b) Image Manipulation - Purposes to correct in perfection in photographs.

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* Application of Computer Animation-

(1) Entertaiment Industry - Animation is used in movies, t.v. shows and video games to bring creating experiences for audience.

(2) Education and Training - Teaching concepts, procedures and processes in fields like medicines and engineering.

(3) Digital art and Design - Animation in software allows a artist and designers to create effects, motion graphics; ~~web~~

(4) Web Design and User Interface - Animation is used to enhance user experience on web sites and applications.

* Morphing - In this we convert one image into another image or a shape. Morphing in C.G. refers to the process of smoothly transforming one image into another and is commonly used in animation and visual effects to create between different state ~~and~~ or forms.