Date- 20/Dec/2023

UNIT-I ⇒

- General Form of LPP -

max or min $(z) = ax + by$ -------- (i)

Subject to
$$\left.\begin{aligned} a_1x + b_1y &* c_1 \\ a_2x + b_2y &* c_2 \end{aligned}\right\} \text{--------- (ii)}$$
$$a_3x + b_3y * c_3$$
$$\vdots$$
$$x \geq 0 \text{ and } y \geq 0 \quad \text{------ (iii)}$$

where $* = \leq, \geq, <, >$

- Technical terms in LPP - In, eq^n (i) , the linear function (z) is called an objective function, it's value is optimized.

If $Z = ax + by$. then the variable x and y whose values are to be determined are called decision variables.
Since, the decision variable x and y whose represent number or amount of some items. Their values are non-negative.
Above given eq^n (iii) defines non-negative constraints.

Constants like a_1, b_1, a_2, b_2 in eq^n (ii) are called Distribution constant and constants c_1 and c_2 in eq^n (ii) are called required or ability.

In the objective function $Z = ax + by$, ~~constants~~ constraints a and b represents the profit or loss are the decision variables ~~or~~ x and y ~~repr~~ respectively.
A set of values of decision variable (x, y) this satisfies all linear inequalities is called a solution.
A solution which also satisfy the non-negative constraints is called a feasible solution.

A feasible solution which optimized the objective function is called an optimum solution or optimal solution.

Question

A manufacturing company makes two types of television set, one is black and white and other in colour. The company has resources to make at most 300 set a week. It Rs. 1800 to make a black and white set and Rs. 2700 to make a colour set. The company can spend not more than Rs. 648000 a week to make television set. If it makes profit of Rs. 510 per black & white and Rs. 675 per coloured set. How many set of each type should be produced so that the company has a maximum process? Formulate this problem as LPP given that the objective is to maximize the profit.

Solution – Let us suppose,
total no. of black & white and coloured Television set per week are x and y respectively

$\Rightarrow \quad x \geq 0$ and $y \geq 0$

Solution: Let us consider,
number of necklaces are x
and number of bracelets are y

Subjects according to their numbers and time taken for manufacturing

$$x+y \le 24$$
$$\frac{1}{2}x + y \le 16$$
$$x + 2y \le 32$$

maximum profit $Z = 100x + 300y$

LPP model for given problem is-
$Z = 100x + 300y$
Subjects are $x+y \le 24$
$\frac{x}{2} + y \le 16$ or $x + 2y \le 32$

(3) Some chemical company is producing two products A and B. The processing times are 3 and 4 hours per unit for A and operation 1 and 2 (two) respectively operations and 4 hrs and 5 hrs per unit for B and operation 1 and 2 respectively. The available time is 18 hours and 21 hrs operations 1 and 2 respectively. The product A can be sold for Rs. 3 profit per unit and B of Rs. 8 profit per unit. Formulate the LPP model.

max profit $Z = 20x + 10y$

Subjects are $1.5x + 3y \leq 42$
$3x + y \leq 24$

Exercise

(1) A man drives his motor cycle at the speed of 50 km hour. He has to spend Rs. 2 per km on petrol if he drives it at a faster speed of 80 km per hour. The petrol cost of increase ₹3 per km. He has atmost ₹120 to spend on petrol an one hour time. He wishes to find the maximum distance that he can travel. Express this problem as a LPP.

Solution: Let us consider, two speeds x and y.

According to question,

$$2x + 3y \leq 120 \quad \text{----(i)}$$

$$\frac{x}{50} + \frac{y}{80} \leq 1$$

$$80x + 50y \leq 4000$$

$$8x + 5y \leq 400 \quad \text{-----(ii)}$$

and

$$x \geq 0, \quad y \geq 0$$

maximum → $Z = x + y$.

(2) A small firm manufactures necklaces and bracelets. The total number of necklaces and bracelets that it can handle per day is at most 24. If it takes 1 hrs to make a bracelet and half an hour to make a necklaces. The maximum number of hours available per day 16. If the profit on necklace is 100 and bracelet is 300. Formulate the LPP model.

Given availability of black & white and coloured T.V. set at most 300.

$$x + y \le 300 \quad ----- (i)$$

$$1800x + 2700y \le 648000 \quad ------ (ii)$$

maximum profit $z = 510x + 675y$

max, $z = 510x + 675y$

Subject to, $x + y \le 300$

$1800x + 2700y \le 648000$

Que. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hrs. craftman time. in its making while a cricket bat takes 3 hrs of machine time and 1 hrs. of craftman time in a day. The factory has a availability of not more than 42 hrs of machine time and 24 hrs of craftman time. If the profit of the racket & on a bat is Rs.20 and ₹10 respectively. Formulate the LPP model and find the max profit.

Solution: Let us consider,
total number of tennis racket be x
and total number of cricket bat be y

then $x \ge 0$ and $y \ge 0$

Availability of machine time and craftmen time

$$1.5x + 3y \le 42 \quad --------- (i)$$

$$3x + y \le 24 \quad -------- (ii)$$

assembling in a day. The profit is rupees 50 each on type A and 60 each type should the be produced. This problem express in LPP.

Solution- Let us consider, number of dolls of type A be x.
and number of dolls of type B be y.

According to given question, the availibilities are:

$$\frac{5x}{60} + \frac{8y}{60} \leq 3 \text{ hour}$$

$$5x + 8y \leq 3 \times 60$$

$$5x + 8y \leq 180 \quad \text{-----(i)}$$

$$\frac{10x}{60} + \frac{8y}{60} \leq 4 \text{ hours}$$

$$10x + 8y \leq 4 \times 60$$

$$10x + 8y \leq 240 \quad \text{-------(ii)}$$

maximum profit $\rightarrow$ $Z = 50x + 60y$

(7) A producer has 30 and 17 units of labour and capital respectively which he can used to produce two types of goods x and y to produce 1 unit of x, 2 unit of labour and 3 units of capital are required similarly 3 units of labour and 1 unit

of According to given ~~possibilit~~ possible availability,

$$300x + 400y \geq 3400$$
$$80x + 50y \geq 640$$
and $$x + y \leq 10$$
$$x \geq 0 \; , \; y \geq 0$$

maximum ~~profit~~ $Z = 2\cancel{5}x + 22y$

(5) A young man drives his motorcycle at 25 km/hr, he has to spend 2 Rs. per km on petrol. If he drives it at a faster speed of 40 km/hr, the petrol cost increasing to 5 Rs. per km. He has 100 Rs to spend on petrol and wishes to find what is the maximum distance, he can travel within 1 hrs. Express this problem in LPP.

Solution- Let us consider, the distance ~~speed~~ be x and y

$$2x + 5y \leq 100$$
and $$\frac{x}{25} + \frac{y}{40} \leq 1$$
and $$x \geq 0 \; , \; y \geq 0$$
maximum profit
$$Z = x + y$$

(6) A company manufacturer 2 types dolls A and B. Type A requires 5 min each for cutting and 10 min each for assembling. Type B requires 8 min each for cutting and 8 min each for assembling. There are 3 hours available for cutting and 4 hrs

Solution – Let us consider, the number of products of A be x

and, the number of products of B be y.

According to questions, possible availability of product A and product B

$3x + 4y \leq 18$ ---- (i)

$4x + 5y \leq 21$ ----- (ii)

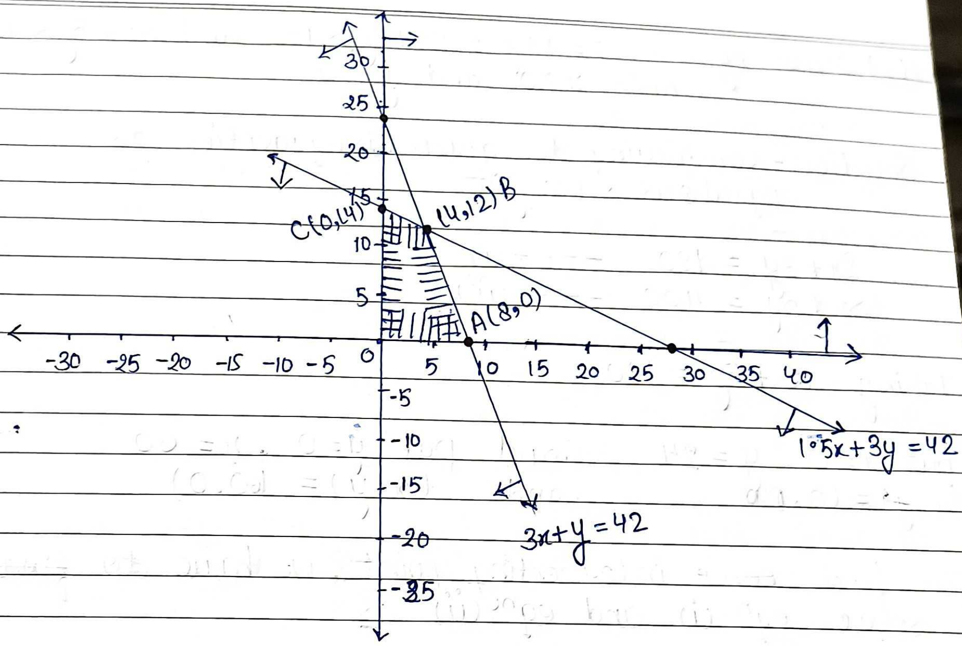
maximum profit, $Z = 3x + 8y$

and, $x \geq 0$, $y \geq 0$

(4) The postmaster of a local post office wishes to higher extra helper during Diwali Season because of a large increasing in the volume of maile handling and delivery. Because of the limited office space and the budgeting condition. The number of temporary helper must not exceed 10 according to past experience a man can handle 300 litre and 80 packages per day on the average and a women can handle 400 liters and 50 packages per day. The postmaster believe that the daily value of extra mail and packages will be no less than 3400 and 680 respectively. A man receives Rs. 25 a day and a women recives Rs. 22 a day per day. Formulate LPP model.

Solution – Let us consider, the number of male/man workers be x

and, the number of women workers be y



- The bounded region is OABC, given subjectives provides solutions of the objectives. Finite solution are provided by the graph.
This bounded region is defined by the corner point O(0,0), A(8,0), B(4,12), and C(0,14).

Now, we have to find the possible value of objective Z on these corner points.

Table-

Points	$Z = 20x + 10y$
O(0,0)	0
A(8,0)	160
B(4,12)	200
C(0,14)	140

Que-(2) Graphical solution-

We have, $Z = 20x + 10y$

$1.5x + 3y \le 42$ ----(i)

$3x + y \le 24$ -----(ii)

$x \ge 0$, $y \ge 0$

Now, $1.5x + 3y = 42$

put $x = 0$; $y = 14$ and put $y = 0$, $x = 28$

$(x, y) = (0, 14)$ and $(x, y) = (28, 0)$

$1.5x + 3y = 42$

$3x + y = 24$

$\Rightarrow$

$1.5x + 3y = 42$

$9x + 3y = 72$

$-7.5x = 30$

$\boxed{x = 4}$

$y = 12$

$(x, y) = (4, 12)$

and, $3x + y = 24$

put $x = 0$, $y = 24$ and $y = 0$, $x = 8$

$(x, y) = (0, 24)$ and $(8, 0)$

and $(x, y) = (4, 12)$

Here, coordinates of eq $1.5x + 3y = 42$ are $(0, 14)$, $(28, 0)$, $(4, 12)$. These are the ~~joinded~~ points.

coordinates of eq^n $3x + y = 24$ are $(0, 24)$, $(8, 0)$ and $(4, 12)$

Answer → Given objective Z has maximum value at point $(4, 12)$

⇒ A manufacturing company ~~ha~~ should make 4 numbers of tennis racket and 12 cricket bats to gain maximum profit per day.

Que. A dietician mixes two types of food in such a way that the between contents of a mixture contains atleast 8 unit of Vitamin A and 10 units of Vitamin C. Food X contains 2 unit of Vitamin A and 1 unit of Vitamin C. While Food Y contains 1 unit of Vitamin A and 2 unit of Vitamin C. 1 kg (unit) of Food X cost Rs. 5 whereas 1 kg (unit) of Food Y Rs. 7. Formulate the LPP.

Solution- Let, the total amount of ~~Vitamin A~~ Food X be x.
the total amount of ~~Vitamin B~~ Food Y be y.

According to question,

$$2x + y \geq 8$$

and $$x + 2y \geq 10$$

and $$z = 5x + 7y$$

Que- In a big motor & truck industry, there are two factories. In a first factory which basically assembles the parts 5 man days work on each truck while 2 man days hrs work on each motor. In the second factory which basically does the job of finishing, 3 man days are work ~~are~~ on each truck as well as on each motor. Due to limitation, of workers and machines, the first factory operates 180 ~~man~~ men days in a week while the second factory operates 135 man days in a week. If the manufacturer earns Rs. 3000 on each truck, Rs. 2000 on each motor. Express this problem in LPP.

(3) Motor cycle - $Z = x + y$; $2x + 3y \le 120$ and $8x + 5y \le 400$ and $x \ge 0$ and $y \ge 0$

Solution - Considering the given inequalities to equations, we get

$2x + 3y = 120$ ---- (i)
$8x + 5y = 400$ ---- (ii)

Taking $2x + 3y = 120$

put $x = 0$, $y = 24$ and put $y = 0$, $x = 60$
$(x, y) = (0, 24)$ and $(x, y) = (60, 0)$

To find ~~comp~~ intersecting point, we have to ~~find~~ solve eq^n (i) and eq^n (ii)

$2x + 3y = 120$ and $8x + 5y = 400$

multiplying 4 on both side of eq^n (i)

$$\begin{array}{r} 8x + 12y = 480 \\ -\ 8x + 5y = 400 \\ \hline 7y = 80 \end{array}$$

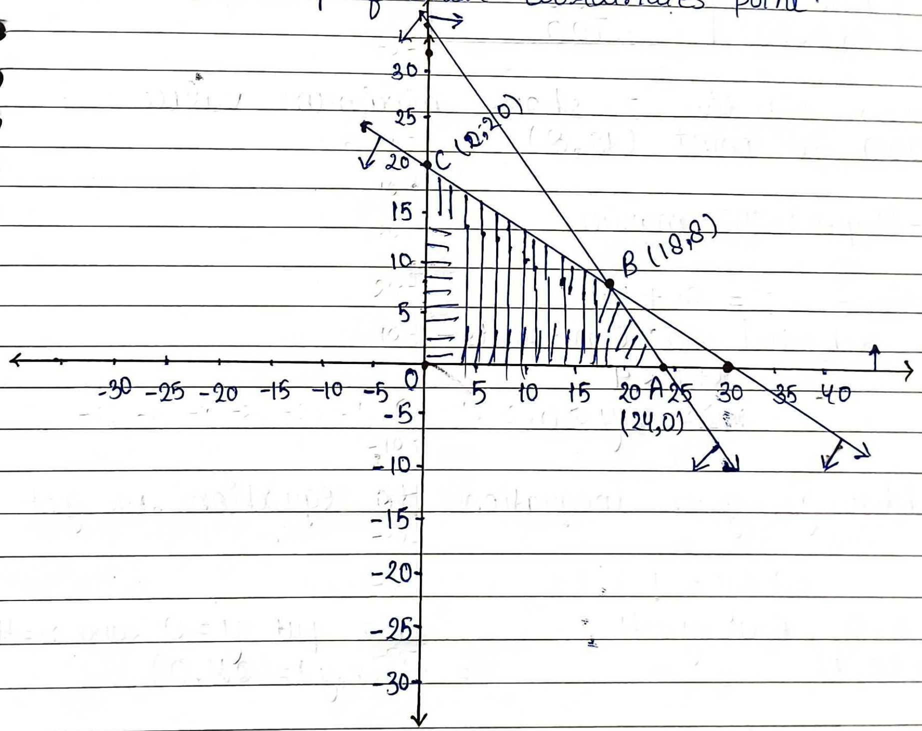
$y = \frac{80}{7}$ and $x = \frac{300}{7}$

$y = 11.42$ and $x = 42.86$

(i) coordinates of eqn $2x_1 + 3x_2 = 60$ are
$(0,20)$, $(30,0)$ and $(18,8)$

(ii) coordinates of eqn $4x_1 + 3x_2 = 96$ are
$(0,32)$, $(24,0)$ and $(18,8)$

We will draw a graph for given subjectives with the help of their coordinates point.



The bounded region or feasible region is OABC, This region is formed by the corner points O(0,0), A(24,0), B(18,8) and C(0,20)

Que - $Z = 40x_1 + 35x_2$

$2x_1 + 3x_2 \leq 60$

$4x_1 + 3x_2 \leq 96$

$x_1, x_2 \geq 0$

Solution - Considering the given inequalities to equations, we get

$2x_1 + 3x_2 = 60$ ------ (i)

$4x_1 + 3x_2 = 96$ ------ (ii)

Taking $2x_1 + 3x_2 = 60$

put $x_1 = 0$, then $x_2 = 20$ and, put $x_2 = 0$, then $x_1 = 30$

$(x_1, x_2) = (0, 20)$ and $(x_1, x_2) = (30, 0)$

To find intersecting point, we have to solve eq^n's (i) and (ii)

$$2x_1 + 3x_2 = 60$$
$$-4x_1 + 3x_2 = -96$$

$\Rightarrow \quad -2x_1 = -36 \quad \Rightarrow \quad \boxed{x_1 = 18}$

then $x_2 = 8$

$(x_1, x_2) = (18, 8)$

Taking eq^n $4x_1 + 3x_2 = 96$

put $x_1 = 0$, then $x_2 = 32$ and put $x_2 = 0$, then $x_1 = 24$

$(x_1, x_2) = (0, 32)$ and $(x_1, x_2) = (24, 0)$

intersecting point $(x_1, x_2) = (18, 8)$

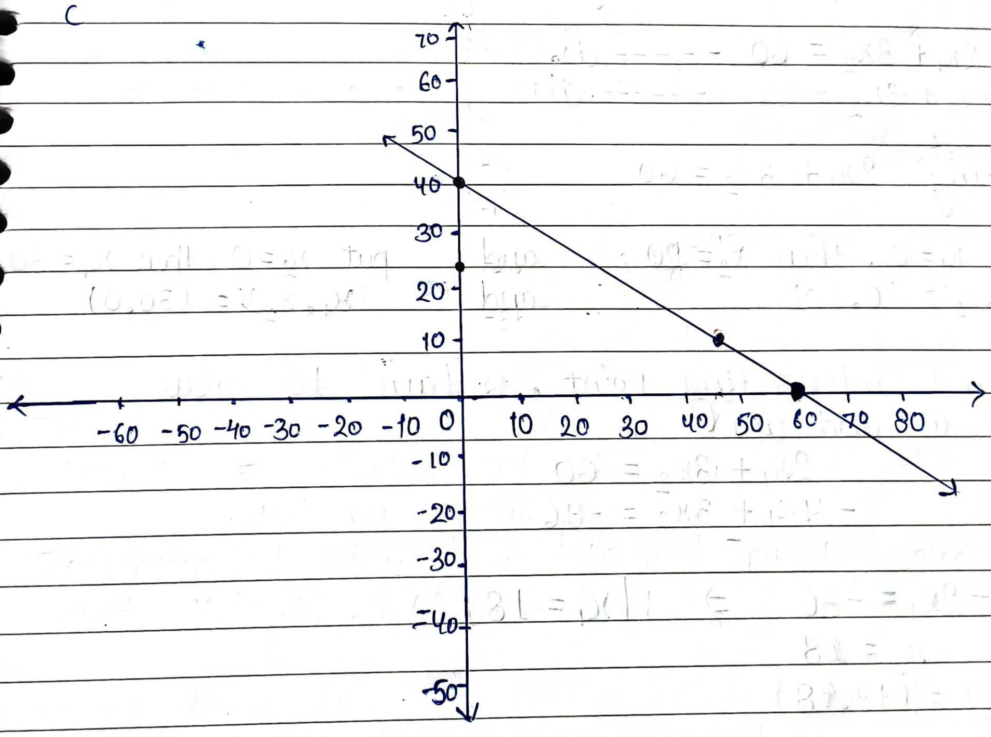
Taking eq^n $8x + 5y = 400$

put $x=0$, then $y = 80$ and put $y=0$, then $x = 50$

$(x,y) = (0,80)$ $(x,y) = (50,0)$

~~put~~ Intersecting points are $\left(\frac{300}{7}, \frac{80}{7}\right)$ or $(42.86, 11.42)$

C



$\begin{array}{r} 240 \\ -36 \\ \hline 204 \end{array}$

intersecting point of eqn $4x_1 + 8x_2 = 36$ and $3x_1 = 180$

$3x_1 = 180$

$\boxed{x_1 = 60}$

$4 \times 60 + 6x_2 = 36$

$240 + 6x_2 = 36$

$6x_2 = 36 - 240$

$6x_2 = -204$

$x_2 = -34$

$(x_1, x_2) = (60, -34)$

intersecting point of eqn $4x_1 + 6x_2 = 36$ and $5x_2 = 200$

$5x_2 = 200 \Rightarrow \boxed{x_2 = 40}$

$4x_1 + 6 \times 40 = 36$

$4x_1 + 240 = 36$

$4x_1 = -204$

$x_1 = -51$

$(x_1, x_2) = (-51, 40)$

Taking $3x_1 = 180$

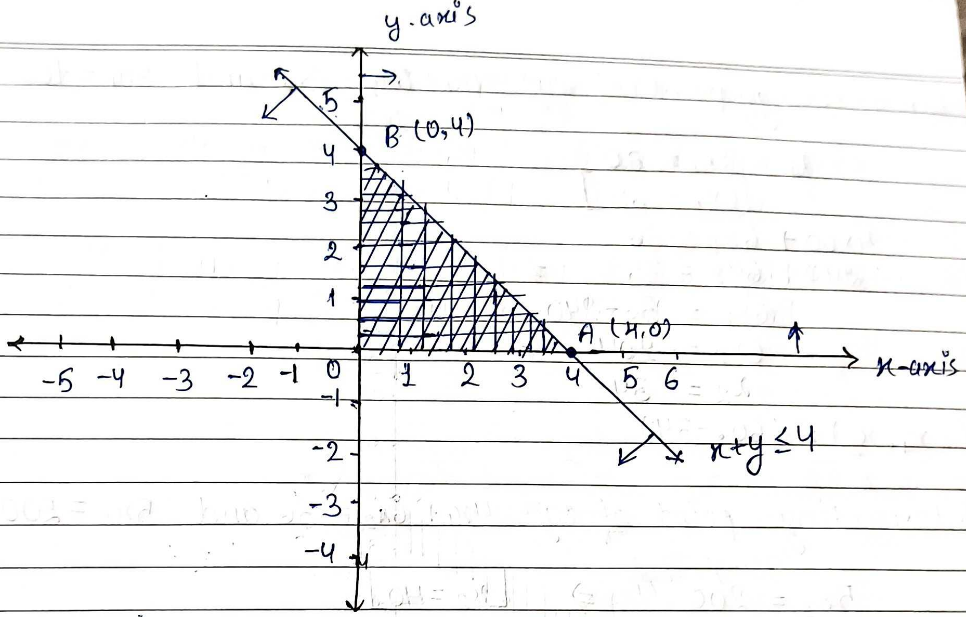
$x_1 = 60$, $(x_1, x_2) = (60, 0)$

and intersecting point $(x_1, x_2) = (60, -34)$

Taking $5x_2 = 200$

$x_2 = 40$

and intersecting point $(x_1, x_2) = (-51, 40)$



Que - Using Graphical method -

$$\max \quad Z = 15x_1 + 18x_2$$
$$4x_1 + 6x_2 \geq 36$$
$$3x_1 \leq 180$$
$$5x_2 \leq 200 \quad , \quad x_1 ; x_2 \geq 0$$

Solution - Considering given inequations in equation, we got

$4x_1 + 6x_2 = 36$ ------ (i)

$3x_1 = 180$ ------ (ii)

$5x_2 = 200$ ------ (iii)

Taking, $4x_1 + 6x_2 = 36$

put $x_1 = 0$, then $x_2 = 6$ and put $x_2 = 0$, then $x_1 = 9$

$(x_1, x_2) = (0, 6)$ and $(x_1, x_2) = (9, 0)$

Following table shows the possible (optimal) value of objective z on the corner points of OABC -

point	$Z = 40x_1 + 35x_2$
O (0,0)	0
A (24,0)	960
B (18,8)	1000
C (0,20)	700

Given objective z shows maximum value 1000 at point (18,8)

Que- (Paper) The maxim

Solution- $Z = 3x + 4y$

subjected to constraits -

$x + y \le 4$

$x \ge 0, y \ge 0$

Considering, given inequations to equations, we get

$x + y = 4$

put $x = 0$, then $y = 4$ and put $y = 0$ and $x = 4$

$(x, y) = (0, 4)$ $(x, y) = (4, 0)$

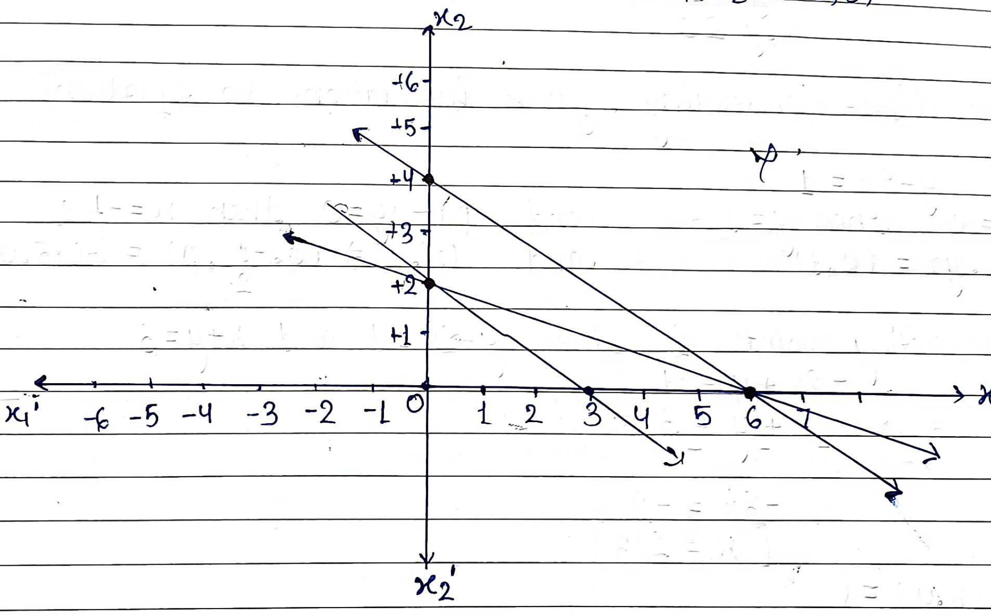
points	$Z = 3x + 4y$
(0,4)	16 → maximum optimal value.
(4,0)	12

Taking $\quad 4x_1 + 6x_2 = 24$

$2x_1 + 3x_2 = 12$

put $x_1 = 0$ and $x_2 = 4$ and put $x_2 = 0$ and $x_1 = 6$

$(x_1, x_2) = (0, 4)$ and $(x_1, x_2) = (6, 0)$



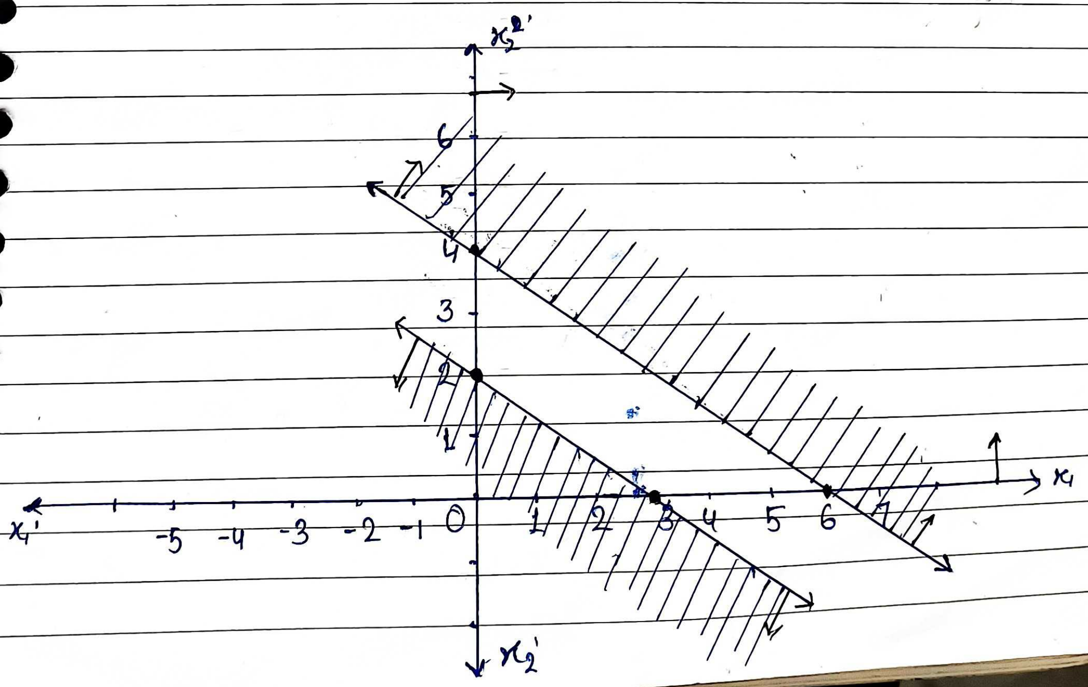


table →

points (ABCDE)	$Z = 15x_1 + 18x_2$
A(0, 6)	108
B(9, 0)	135
C(60, 0)	900
D(60, 40)	1620
E(0, 40)	720

max optimal value is at point (60, 40) · (1620)

Que- max -, $Z = 4x_1 + 3x_2$

$2x_1 + 3x_2 \leq 6$

$4x_1 + 6x_2 \geq 24$

$x_1, x_2 \geq 0$

Solution: Considering inequalities in equations, we get

$2x_1 + 3x_2 = 6$ ------(i)

$4x_1 + 6x_2 = 24$ --------(ii)

Taking $2x_1 + 3x_2 = 6$

put $x_1 = 0$ then $x_2 = 2$ and put $x_2 = 0$ then $x_1 = 3$

$(x_1, x_2) = (0, 2)$ and $(x_1, x_2) = (3, 0)$

intersecting point of eq^n $2x_1 + 3x_2 = 6$ and $4x_1 + 6x_2 = 24$

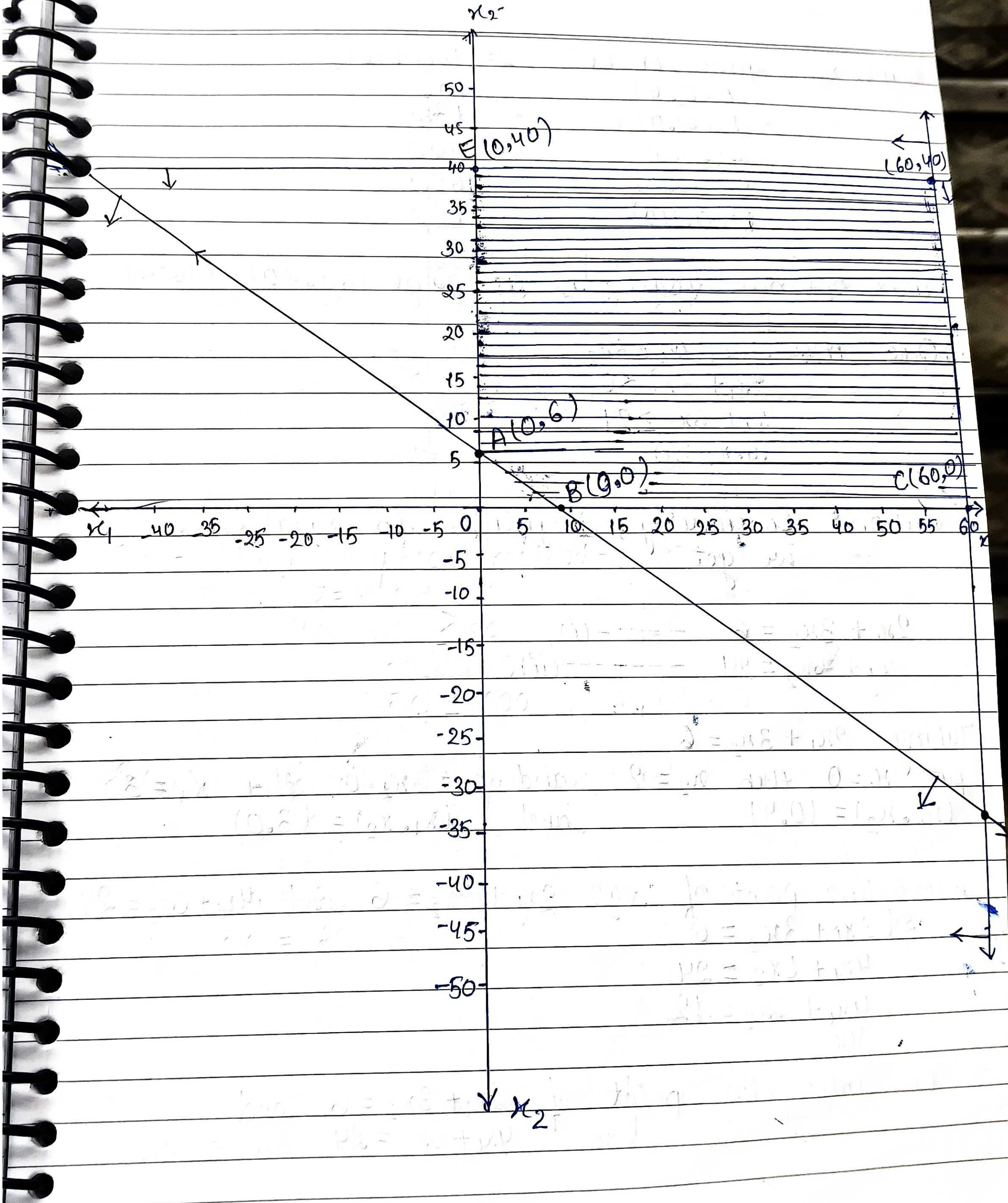
$2 \times [2x_1 + 3x_2 = 6]$

$4x_1 + 6x_2 = 24$

$4x_1 + 6x_2 = 12$

$4x$

⇒ No Intersecting point of $2x_1 + 3x_2 = 6$ and $4x_1 + 6x_2 = 24$



x_2
E (0,40)
(60,40)
A (0,6)
B (9,0)
C (60,0)
x_1
x_2

		C_j	4	6	2	0	0	
B	C_B	x_B	x_1	x_2	x_3	S_1	S_2	min ratio
x_1	4	1	1	0	-1	4/3	-1/3	
x_2	6	2	0	1	2	-1/3	1/3	
		Z_j	4	6	8	10/3	2/3	
		$C_j - Z_j$	0	0	-6	-10/3	-2/3	

$\Rightarrow x_1 = 1,\ x_2 = 2,\ x_3 = 0$

max $Z = 4x_1 + 6x_2 + 2x_3$

$Z = 4\times1 + 6\times2 + 2\times0$

$\boxed{Z = 16}$

Que- $Z = 10x_1 + 5x_2$

$4x_1 + 5x_2 \le 100$

$5x_1 + 2x_2 \le 80$

$x_1, x_2 \ge 0$

Solution- $Z = 10x_1 + 5x_2 + 0S_1 + 0\cdot S_2$

LPP in standard form,

and $4x_1 + 5x_2 + S_1 + 0\cdot S_2 = 100$

$5x_1 + 2x_2 + 0\cdot S_1 + S_2 = 80$

$x_1, x_2, S_1, S_2 \ge 0$

Table 1

		C_j	10	5	0	0	
B	C_B	X_B	x_1	x_2	S_1	S_2	minimum ratio (x_B/x_i)
S_1	0	100	4	5	1	0	25
S_2	0	80	5	2	0	1	16 ← Key row
		Z_j	0	0	0	0	
		$C_j - Z_j$	10	5	0	0	

↑ Key column

5 is a key element.

Simplex Method

Que $Z = 4x_1 + 6x_2 + 2x_3$

$x_1 + x_2 + x_3 \leq 3$

$x_1 + 4x_2 + 7x_3 \leq 9$

$x_1, x_2, x_3 \geq 0$

Solution- $Z = 4x_1 + 6x_1 + 2x_3 + 0 \cdot S_1 + 0 s_2$

C_j = constant and coefficient of Z

and $x_1 + x_2 + x_3 + S_1 + 0 \cdot S_2 = 3$

$x_1 + 4x_2 + 7x_3 + 0 \cdot S_1 + S_2 = 9$

$x_1, x_2, x_3, S_1, S_2 \geq 0$

Table-1

$Z_j = x_{s_1} + x_{s_2}$

		C_j	4	6	2	0	0	
B	C_B	x_B	x_1	x_2	x_3	S_1	S_2	Min R (x_B/x_i)
S_1	0	3	1	1	1	1	0	$3/1 = 3$
S_2	0	9	1	4	7	0	1	$9/4 = 2.25$ ← Key row
		Z_j	0	0	0	0	0	
		$C_j - Z_j$	4	6	2	0	0	

Key element Key column

Table-2

		C_j	4	6	2	0	0	min.
B	C_B	x_B	x_1	x_2	x_3	S_1	S_2	R (x_B/x_i)
S_1	0	3/4	3/4	0	-3/4	1	-1/9	1 ← Key row
x_2	6	9/4	1/4	1	-7/4	0	1/4	9
		Z_j	3/2	6	21/2	0	3/2	
		$C_j - Z_j$	5/2	0	-17/2	0	-3/2	

Key column

Key element $= \frac{3}{4}$

Que- max - $Z = x+y$

$$y - 2x \le 1$$
$$x \le 2$$
$$x + y \le 3$$
$$x, y \ge 0$$

Solution- Considering, given inequations to equations.

$y - 2x = 1$

$x = 0$ then $y = 1$ and put $y = 0$ then $x = -1/2$

$(x, y) = (0, 1)$ and $(x, y) \Rightarrow \left(-\frac{1}{2}, 0\right) = (-0.5, 0)$

intersecting point of lines $y - 2x = 1$ and $x + y = 3$

$$-2x + y = 1$$
$$x + y = 3$$
$$- \quad - \quad -$$
$$-3x = -2$$
$$[x = 2/3]$$

$(x, y) = ($

Table-1

		C_j	6	8	0	0		
B	C_B	x_B	x_1	x_2	S_1	S_2	min-ratio.	(x_B/x_i)
S_1	0	300	30	20	1	0	15	
S_2	0	110	5	10	0	1	11	← Key row
		Z_j	0	0	0	0		
		C_j-Z_j	6	8	0	0		

Key element ↑ Key column

Table-2

		C_j	6	8	0	0		
B	C_B	x_B	x_1	x_2	S_1	S_2	min ratio	
S_1	0	80	20	0	1	-2	$80/20 = 4$	← Key row
x_2	8	11	1/2	1	0	1/10	$11/(1/2) = 22$	
		Z_j	4	8	0	4/5		
		C_j-Z_j	2	0	0	-4/5		

↑ Key column

20 is a key element

Table-3

		C_j	6	8	0	0	
B	C_B	x_B	x_1	x_2	S_1	S_2	min ratio
x_1	6	4	1	0	1/20	-1/10	
x_2	8	9	0	1	-1/40	3/20	
		Z_j	6	8	1/10	18/10	
		C_j-Z_j	0	0	-1/10	-18/10	

From table 3, we get, $(C_j - Z_j)$ all elements are zeroes and negative.

$\Rightarrow \quad x_1 = 4$ and $x_2 = 9$

$Z_1 = 6 \times 4 + 8 \times 9$

$Z_1 = 24 + 72$

$[Z_1 = 96]$ ans

		C_j	10	5	0	0	min
B	C_B	X_B	x_1	x_2	S_1	S_2	ratio
x_2	5	180/17	0	1	5/17	−4/17	
x_1	10	200/17	1	0	−2/17	25/85	
		Z_j	10	5	5/17	30/17	
		$C_j - Z_j$	0	0	−5/17	−30/17	

$$X_1 = \frac{200}{17} \qquad X_2 = \frac{180}{17}$$

$$Z = 10x_1 + 5x_2$$

$$= 10 \times \frac{200}{17} + 5 \times \frac{180}{17} = \frac{2000}{17} + \frac{900}{17}$$

$$Z = ₹\frac{2900}{17}$$ Ans

Que- Use simplex method to solve the following LPP problem-

max $Z = 6x_1 + 8x_2$

subject to $30x_1 + 20x_2 \le 300$

$5x_1 + 10x_2 \le 110$

$x_1, x_2 \ge 0$

Solution: LPP in standard form

$$Z = 6x_1 + 8x_2 + 0 \cdot S_1 + 0 \cdot S_2$$

$$30x_1 + 20x_2 + S_1 + 0 \cdot S_2 = 300$$

$$5x_1 + 10x_2 + 0 \cdot S_1 + S_2 = 110$$

$$x_1, x_2, S_1, S_2 \ge 0$$

Table 2

		C_j	10	5	0	0	min.
B	C_B	x_B	x_1	x_2	S_1	S_2	ratio (x_B/x_i)
S_1	0	36	0	17/5	1	-4/5	
x_2	5	16	1	2/5	0	1/5	
		C_j	5	2	0	1	
		$C_j - Z_j$	5	3	0	-1	

↑ Key column

$-\frac{36 \times 4}{5} + \frac{16}{5}$

$= \frac{-144 + 16}{5}$

Table 2

		C_j	10	5	0	0	min	
B	C_B	x_B	x_1	x_2	S_1	S_2	ratio (x_B/x_i)	
S_1	0	36	0	17/5	1	-4/5	180/17	← Key row
x_1	10	16	1	2/5	0	1/5	40	←
		Z_j	10	4	0	2	-128/5	
		$C_j - Z_j$	0	1	0	-2		

↑ Key column

$\frac{5}{1} \quad \frac{8}{5}$

100	4	5	1	0
64	4	$\frac{8}{5}$	0	$\frac{4}{5}$
36	0	$\frac{17}{5}$	1	$\frac{-4}{5}$

Table-3

		C_j	10	5	0	0	min
B	C_B	x_B	x_1	x_2	S_1	S_2	ratio (x_B/x_i)
x_1	10	180/17	0	1	5/17	-4/17	
x_2	5	200/17	1	0	-2/17	9/85	
		Z_j	5	10	40/17	-31/17	
		$C_j - Z_j$	50	-5	-40/17	+31/17	

↑

Table-2

		c_j	7	6	0	0	min ratio
B	C_B	x_B	x_1	x_2	S_1	S_2	
S_1	0	1	0	1/2	1	-1/2	2
x_1	7	3	1	1/2	0	1/2	6
		Z_j	7	7/2	0	7/2	
		$C_j - Z_j$	0	5/2	0	-7/2	

← Key row

$\frac{1}{2}$ is a key element

↑ Key column

Table-3

		c_j	7	6	0	0	min ratio
B	C_B	x_B	x_1	x_2	S_1	S_2	
x_2	6	2	0	1	2	-1	
x_1	7	2	1	0	-1	1	
		Z_j	7	6	5	1	
		$C_j - Z_j$	0	0	-5	-1	

$\Rightarrow$ $x_1 = 2$ and $x_2 = 2$

max, $Z = 7x_1 + 6x_2$

$Z = 7 \times 2 + 6 \times 2$

$Z = 14 + 12$

$[Z = 26]$ Ans

Table-3

B	C_B	C_j / x_B	8 / x_1	10 / x_2	7 / x_3	0 / S_1	0 / S_2	min ratio
S_1	0	51/5	2/5	0	7/5	1	-3/5	
x_1	8	8/5	1/5	1	14/5	0	1/5	
		Z_j	8/5	8	8/5	0	8/5	
		$C_j - Z_j$	32/5	2	27/5	0	-8/5	

↑

Que- maximise $Z = 7x_1 + 6x_2$

sub $x_1 + x_2 \leq 4$

$2x_1 + x_2 \leq 6$

$x_1, x_2 \geq 0$

Solution- $Z = 7x_1 + 6x_2 + 0 \cdot S_1 + 0 \cdot S_2$

and $x_1 + x_2 + S_1 + 0 \cdot S_2 = 4$

$2x_1 + x_2 + 0 \cdot S_1 + S_2 = 6$

$x_1, x_2, S_1, S_2 \geq 0$

Table-1

B	C_B	C_j / x_B	7 / x_1	6 / x_2	0 / S_1	0 / S_2	min ratio
S_1	0	4	1	1	1	0	4
S_2	0	6	2	1	0	1	3
		Z_j	0	0	0	0	
		$C_j - Z_j$	7	6	0	0	

← Key row

↑ Key column

2 is a Key element

Que. Use simplex method to solve the following L.P.P problem.

Solution - $Z = 8x_1 + 10x_2 + 7x_3$

Subject to, $x_1 + 3x_2 + 2x_3 \leq 15$

$x_1 + 5x_2 + x_3 \leq 8.$

$x_1, x_2, x_3 \geq 0$

Given LPP in standard form

$Z = 8x_1 + 10x_2 + 7x_3 + 0 \cdot S_1 + 0 \cdot S_2 + \cancel{0 \cdot S_3}$

and, $x_1 + 3x_2 + 2x_3 + S_1 + 0 \cdot S_2 + \cancel{0 \cdot S_3} = 15$

$x_1 + 5x_2 + x_3 + 0 \cdot S_1 + S_2 + \cancel{0 \cdot S_3} = 8$

$x_1, x_2, x_2, S_1, S_2 \geq 0.$

Table-1

		c_j	8	10	7	0	0	min
B	C_B	x_B	x_1	x_2	x_3	S_1	S_2	ratio.
S_1	0	15	1	3	2	1	0	$15/3 = 5$
S_2	0	8	1	5	1	0	1	$8/5 = 1.6$
		z_j	0	0	0	0	0	
		$c_j - z_j$	8	10	7	0	0	

(x_B/x_i)

← Key row

↑ Key column

5 is a key element

Table-2

		c_j	8	10	7	0	0	min
B	C_B	x_B	x_1	x_2	x_3	S_1	S_2	ratio
S_1	0	$51/5$	$2/5$	0	$7/5$	1	$-3/5$	$51/2 = 25.5$
x_2	10	$8/5$	$1/5$	1	$1/5$	0	$1/5$	8
		z_j	2	10	2	0	2	
		$c_j - z_j$	6	0	5	0	-2	

← Key row

↑ Key column

Table-1

		C_j	30	40	0	0	0	min
B	C_B	x_B	x	y	S_1	S_2	S_3	ratio
S_1	0	10	2	1	1	0	0	10
S_2	0	7	1	1	0	1	0	7
S_3	0	12	1	2	0	0	1	6
		Z_j	0	0	0	0	0	
		$C_j - Z_j$	30	40	0	0	0	

← Key row

2 is a key-element

↑ Key column

Table-2

		C_j	30	40	0	0	0	min
B	C_B	x_B	x	y	S_1	S_2	S_3	ratio
S_1	0	4	3/2	0	1	0	-1/2	$8/3 = 2.66$
S_2	0	1	1/2	0	0	1	-1/2	2
y	40	6	1/2	1	0	0	1/2	12
		Z_j	20	40	0	0	20	
		$C_j - Z_j$	10	0	0	0	-20	

← Key row

$\frac{1}{2}$ is a key element

↑ Key column

Table-3

		C_j	30	40	0	0	0	min
B	C_B	x_B	x	y	S_1	S_2	S_3	ratio
S_1	0	1	0	0	1	-3	1	
x	30	2	1	0	0	2	-1	
y	40	5	0	1	0	-1	1	
		Z_j	30	40	0	20	10	
		$C_j - Z_j$	0	0	0	-20	-10	

$\Rightarrow \quad x = 2$ and $y = 5$

$P \Rightarrow 30x + 40y = 60 + 200 = \underline{260}$

Table-3

		c_j	40	60	0	0	0	min ratio
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	
x_1	40	3	1	0	1/2	0	-1/4	
S_2	0	27	0	0	-13/6	1	3/4	
x_2	60	9	0	1	-1/6	0	1/4	
		z_j	40	60	10	0	5	
		c_j-z_j	0	0	-10	0	-5	

From above table-3 we conclude that all the elements of $c_j - z_j$ are negative and zeroes.

$\Rightarrow \quad x_1 = 3 \quad , x_2 = 9$ and $S_2 = 27$

$Z = 40 \times 3 + 60 \times 9 + 0 \times 27 + 0$
$Z = 120 + 540 + 0 + 0$
$Z = \underline{660}$ Ans

Que. maximise $P = 30x + 40y$
Sub to. $2x + y \leq 10$
$x + y \leq 7$
$x + 2y \leq 12$ and $x, y \geq 0$

Solution- Given LPP in standard form
$P = 30x + 40y + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$

and $2x + y + S_1 + 0 \cdot S_2 + 0 \cdot S_3 = 10$
$x + y + 0 \cdot S_1 + S_2 + 0 \cdot S_3 = 7$
$x + 2y + 0S_1 + 0S_2 + S_3 = 12$

$x, y, S_1, S_2, S_3 \geq 0$

Date
30/01/24

Que - max. $z = 40x_1 + 60x_2$
Sub to. $3x_1 + 3x_2 \le 36$
$5x_1 + 2x_2 \le 60$
$2x_1 + 6x_2 \le 60$
$x_1, x_2 \ge 0$

Solution - Given LPP in standard form
$Z = 40x_1 + 60x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$
and $3x_1 + 3x_2 + S_1 + 0 \cdot S_2 + 0 \cdot S_2 = 36$
$5x_1 + 2x_2 + 0 \cdot S_1 + S_2 + 0 \cdot S_2 = 60$
$2x_1 + 6x_2 + 0 \cdot S_1 + 0 \cdot S_2 + S_3 = 60$

$x_1, x_2, S_1, S_2, S_3 \ge 0$

Table-1

		C_j	40	60	0	0	0	min
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	ratio
S_1	0	36	3	3	1	0	0	12
S_2	0	60	5	2	0	1	0	30
S_3	0	60	2	6	0	0	1	10 ← Key row
		Z_j	0	0	0	0	0	
		$C_j - Z_j$	40	60	0	0	0	

↑ Key column

6 is a key element

Table-2

		C_j	40	60	0	0	0	min
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	ratio
S_1	0	6	2	0	1	0	$-1/2$	3 ← Key row
S_2	0	40	$13/3$	0	0	1	$-1/3$	$120/13$
x_2	60	10	$1/3$	1	0	0	$1/6$	30
		Z_j	20	60	0	0	10	
		$C_j - Z_j$	20	0	0	0	-10	

↑ Key column

2 is a Key element

Date
1/Feb/2024

(1) max $z = 21x_1 + 15x_2$
$-x_1 - 2x_2 \geq -6$
$4x_1 + 3x_2 \leq 12$

Solution- Given LPP in standard form
max $z = 20x_1 + 10 \cdot x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3 - MA_1 - MA_2$ ~~$+ 0A_1 + 0A_2$~~

and $x_1 + 2x_2 + S_1 + 0 \cdot S_2 + 0 \cdot S_3 + 0 \cdot A_1 + 0 \cdot A_2 = 40$
$3x_1 + x_2 + 0 \cdot S_1 - S_2 + 0 \cdot S_3 + A_1 + 0 \cdot A_2 = 30$
$4x_1 + 3x_2 + 0 \cdot S_1 + 0 \cdot S_2 - S_3 + 0 \cdot A_1 + A_2 = 60$

Table-1

		C_j							
B	CB	x_B	x_1	x_2	S_1	S_2	S_3	A_1	A_2
S_1	0	40							
A_1	-M	30							
A_2	-M								

Table-3

		C_j	50	60	0	0	0	min
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	ratio
S_1	0							
x_2	60	320	1	1	0	$-4/5$	$3/5$	
x_1	50	63	1	0	0	$7/5$	$-4/5$	
		Z_j						
		$C_j - Z_j$						

Table-2.

		C_j	50	60	0	0	0	min
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	ratio
S_1	0	184	$10/7$	0	1	0	$-1/7$	128.8
S_2	0	45	$5/7$	0	0	1	$-4/7$	63
x_2	60	116	$4/7$	1	0	0	$1/7$	203
		Z_j	$240/7$	60	0	0	$60/7$	
		$C_j - Z_j$	$110/7$	0	0	0	$-60/7$	

← Key row

Key column

$\frac{5}{7}$ is a key-element

380
740

Table-3

		C_j	50	60	0	0	0	min	
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	ratio	
S_1	0	94	0	0	1	-2[illegible]	1		
x_1	50	63	1	0	0	$7/5$	$-4/5$		
x_2	60	80	0	1	1	$-4/5$	$3/5$		
		Z_j	50	60	60	[illegible]	$\cancel{140/7}$		4
		$C_j - Z_j$	0	0	-60	-[illegible]	$\cancel{-140/7}$		4
						-26			

~~∃ $S_1 = 94$, $x_1 = 63$ and $x_2 = 80$~~

Que- max $Z = 50x_1 + 60x_2$

Sub to $2x_1 + x_2 \le 300$

$3x_1 + 4x_2 \le 509$

$4x_1 + 7x_2 \le 812$

$x_1, x_2 \ge 0$

Solution- Given LPP in standard form

$Z = 50x_1 + 60x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$

~~$3x_1 + 4x_2 + S_1 + 0 \cdot S_2 + 0 \cdot S_3 = 300$~~

and, $2x_1 + x_2 + S_1 + 0 \cdot S_2 + 0 \cdot S_3 = 300$

$3x_1 + 4x_2 + 0 \cdot S_1 + S_2 + 0 \cdot S_3 = 509$

$4x_1 + 7x_2 + 0 \cdot S_1 + 0 \cdot S_2 + S_3 = 812$

Table. 1

		C_j	50	60	0	0	0	min
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	ratio
S_1	0	300	2	1	1	0	0	300
S_2	0	509	3	4	0	1	0	127.25
S_3	0	812	4	7	0	0	1	~~203~~ 116
		Z_j	0	0	0	0	0	
		$C_j - Z_j$	50	60	0	0	0	

← ~~Key row~~

← Key row

4 is a key element

↑ Key column

Table-2

		C_j	50	60	0	0	0	min
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	ratio
S_1	0	691/4	5/4	0	1	-1/4	0	691/4 = 138.2
x_2	60	509/4	3/4	1	0	1/4	0	169.7
S_3	0	-315/4	-5/4	0	0	-7/4	1	63
		Z_j	45	60	0	15	0	
		$C_j - Z_j$	5	0	0	0	0	

← Key row

$\frac{-5}{4}$ is a key element.

↑ Key column

$\Rightarrow \quad S_2 = \frac{5}{2} \quad , \quad x_2 = \frac{3}{2} \quad , \quad x_1 = 0$

max. $Z = x_1 + 5x_2$

$Z = 0 + 5 \times \frac{3}{2}$

$Z \Rightarrow \frac{15}{2} = 7.5$

Que- min $z = 3x_1 + 8x_2$

$x_1 + x_2 = 200 \; ; \; x_1 \le 80$ and $x_2 \ge 60$.

Solution- Given LPP in standard form

max $z' = -3x_1 - 8x_2 + 0 \cdot S_1 + 0 \cdot S_2 - M \cdot A_1 - MA_2$

and $x_1 + x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot A_1 + 0 \cdot A_2 = 200$

$x_1 + 0 \cdot x_2 + S_1 + 0 \cdot S_2 + 0 \cdot A_1 + 0 \cdot A_2 = 80$

$0 \cdot x_1 + x_2 + 0 \cdot S_1 - S_2 + 0 \cdot A_1 + A_2 = 60$

Table-1

		c_j	-3	-8	0	0	$-M$	$-M$	min
x_B	C_B	x_B	x_1	x_2	S_1	S_2	A_1	A_2	ratio.
A_1	$-M$	200	1	1	0	0	1	0	
S_1	0	80	1	0	1	0	0	0	
A_2	$-M$	60	0	1	0	-1	0	1	
		z_j	$-M$	$-2M$	0	M	$-M$	$-M$	
		$c_j - z_j$	$-3+M$	$-8+2M$	0	$-M$	0	0	

Table-1

B	C_B	C_j / x_B	1 / x_1	5 / x_2	0 / S_1	0 / S_2	−M / A_1	min ratio
S_1	0	6	4	4	1	0	0	6/4 = 1.5
A_1	−M	2	1	3	0	−1	1	0.6 ← Key row
		Z_j	−M	−3M	0	M	−M	
		$C_j - Z_j$	1+M	5+3M	0	−M	0	

3 is a key element   Key column

Table-2

B	C_B	C_j / x_B	1 / x_1	5 / x_2	0 / S_1	0 / S_2	−M / A_1	min ratio
S_1	0	10/3	8/3	0	1	4/3	−4/3	+10/4 = +2.5 ← Key row
x_2	5	2/3	1/3	1	0	−1/3	1/3	−2
		Z_j	5/3	5	0	−5/3	5/3	
		$C_j - Z_j$	−2/3	0	0	5/3	$\frac{-3M-5}{3}$	

$-\frac{1}{3}$ is a key element

Key column

Table-3

B	C_B	C_j / x_B	1 / x_1	5 / x_2	0 / S_1	0 / S_2	−M / A_1
S_1	0	6	4	4	1	0	0
S_2	0	−2	−1	−3	0	1	−1
		Z_j					
		$C_j - Z_j$					

Table-4

B	C_B	C_j / x_B	1 / x_1	5 / x_2	0 / S_1	0 / S_2	−M / A_1	min ratio
S_2	0	5/2	2	0	3/4	1	−1	
x_2	5	3/2	1	1	1/4	0	0	
		Z_j	5	5	5/4	0	0	
		$C_j - Z_j$	−4	0	−5/4	0	−M	

Que- max $Z = x_1 + 5x_2$

Sub. to. $4x_1 + 4x_2 \leq 6$

$x_1 + 3x_2 \geq 2$

Solution - $Z = x_1 + 5x_2 + 0 \cdot S_1 + 0 \cdot S_2 - MA_1$

$4x_1 + 4x_2 + S_1 + 0 \cdot S_2 + 0 \cdot A_1 = 6$

$x_1 + 3x_2 - S_2 + A_1 + 0 \cdot S_1 = 2$

Table - 1

Que.

Ware / fac.	W_1	W_2	W_3	W_4	Supply
F_1	48	60	56	58	140
f_2	45	55	53	60	260
f_3	50	65	60	62	360
f_4	52	64	55	61	220
Require.	200	320	250	210	980 / 980

Total supply = 140 + 260 + 360 + 220
Total supply = 980

Total requirement = 200 + 320 + 250 + 210
Total requirement = 980

Ware / f	W_1	W_2	W_3	W_4	Supply
f_1	48 $\boxed{140}$	60	56	58	~~140~~ 0
f_2	45 $\boxed{60}$	55 $\boxed{200}$	53	60	~~260~~ ~~200~~ 0
f_3	50	65 $\boxed{120}$	60 $\boxed{240}$	62	~~360~~ ~~240~~ 0
f_4	52	64	55 $\boxed{10}$	61 $\boxed{210}$	~~220~~ 210
Requ	~~200~~	~~320~~	~~250~~	210	980 / 980
	0	~~120~~	~~240~~		
		0	0		

Minimum cost = $48 \times 140 + 45 \times 60 + 55 \times 200 + 65 \times 120$
$+ 60 \times 240 + 55 \times 10 + 61 \times 210$
$= 55980$

Ware / f	W_1	W_2	W_3	W_4	Supply
f_1	48 $\boxed{140}$				
f_2	45 $\boxed{60}$	55 $\boxed{200}$			
f_3		65 $\boxed{120}$	60 $\boxed{240}$	6	
f_4			55 $\boxed{10}$	61 $\boxed{210}$	

Q.2)

ware / fa.	W_1	W_2	W_3	W_4	factory's capa	
f_1	19 $\boxed{5}$	30 $\boxed{2}$	50	10	~~7~~ ~~2~~ 0	0
f_2	70	30 $\boxed{6}$	40 $\boxed{3}$	60	~~9~~ ~~3~~	0
f_3	40	8	70 $\boxed{4}$	20 $\boxed{14}$	~~18~~	0
Requi	~~5~~	~~8~~	~~7~~	~~14~~	Total 34	
	0	~~6~~ 0	~~4~~ 0	0		

Total capacity ⇒ $7+9+18 = 34$
Total requirement ⇒ $5+8+7+14 = 34$

Total capacity = Total requirement
Given T.P. is balanced.

W / f	W_1	W_2	W_3	W_4
f_1	19 $\boxed{5}$	30 $\boxed{2}$		
f_2		30 $\boxed{6}$	40 $\boxed{3}$	
f_3			70 $\boxed{4}$	20 $\boxed{14}$

Minimum cost $= 19\times5 + 30\times2 + 30\times6 + 40\times3 + 70\times4 + 20\times14$
=
=

ware / fac	W_1	W_2	W_3	W_4	Supply	
f_1	48 $\boxed{140}$	60	56	58 ~~58~~	~~140~~	0
f_2	~~60~~ 45 $\boxed{60}$	55 $\boxed{200}$	53	60	~~260~~	~~200~~ 0
f_3	50	65 $\boxed{120}$	60	62	~~360~~	240
f_4	52	64	55	61	220	
Requ	~~200~~	~~320~~	250	210	980 / 980	
	0	~~120~~ 0				

Transportation

Transportation problem is a special kind off linear programming in which goods a transported from a set of source to a set of destination subject to the supply and demand of the source and destination respectively such that the total cost of transportation is minimized. It is also sometimes called as Hichcock problem.

* Types of transportation problem -

(i) Balanced Transportation Problem - When bath supply and demand are equal then the problem is set to be a balanced transportation problem.

(ii) Unbalanced Transportation Problem - When the supply and demands are not equal then it is set of to be an unbalanced transportation problem. In this type of problem either a dummy row or a dummy column according to be the requirement to make it a balance problem, then it can be solved similar to the balanced problem.

(3) VOGEL'S APPROXIMATION METHOD (VAM) -

Que.

	W_1	W_2	W_3	W_4	Supply
F_1	48	60	56	58	140
F_2	45	55	53	60	260
F_3	50	65	60	62	360
F_4	52	64	55	61	220
	200	320	250	210	980 / 980

Solution - Total supply ⇒ 140 + 260 + 360 + 220 = 980

Total demand ⇒ 200 + 320 + 250 + 210 = 980

	W_1	W_2	W_3	W_4	Supply	P_1	P_2	P_3	P_4	P_5	P_6
F_1	48	60 [60]	56 [30]	58 [50]	~~140~~ ~~80~~ 50	8	2	2	2	2	58
F_2	45	55 [260]	53	60	~~260~~ 0	8	2	2	-	-	-
F_3	50 [200]	65	60	62 [160]	~~360~~ ~~160~~ 0	10	2	2	2	2	62
F_4	52	64	55 [220]	61	~~220~~ 0	3	6	-	-	-	-
Demand	~~200~~ 0	~~320~~ ~~60~~ 0	~~250~~ ~~30~~ 0	~~210~~ ~~50~~ 0							P_7

P_1	3	5	2	2
P_2	-	5	2	2
P_3	-	5	3	2
P_4	-	5	4	4
P_5	-	-	4	4
P_6	-	-	-	4

	W_1	W_2	W_3	W_4
F_1		60 [60]	56 [30]	58 [50]
F_2		55 [260]		
F_3	50 [200]			62 [160]
F_4			55 [220]	~~8~~

(2) Least Cost Method -

Que.

To / Source	D	E	F	Supply
A	5	8	4	50
B	6	6	3	40
C	3	9	6	60
Demand	20	95	35	150 / 150

Solution -

(3)

Dest / Sources	D_1	D_2	D_3	D_4	Supply
S_1	21	16	15	3	11
S_2	17	18	14	23	13
S_3	32	27	18	41	19
Demand	6	10	12	15	43 / 43

Solution – Total supply ⇒ $11 + 13 + 19 = 43$

Total demand ⇒ $6 + 10 + 12 + 15 = 43$

Total supply = Total demand

Given T.P. is balanced.

D / S	D_1	D_2	D_3	D_4	Supply	
S_1	21 [6]	16 [5]	15	3	~~11~~ ~~0~~ ~~5~~	0
S_2	17	18 [5]	14 [8]	23	~~13~~ ~~8~~	0
S_3	32	27	18 [4]	41 [15]	~~19~~ ~~15~~	
Demand	~~6~~ 0	~~10~~ ~~5~~ 0	~~12~~ ~~4~~ 0	~~15~~ 0	43 / 43	

D / S	D_1	D_2	D_3	~~Sup~~ D_4
S_1	21 [6]	16 [5]		
S_2	–	18 [5]	14 [8]	
S_3	–	–	18 [4]	41 [15]

Minimum cost $= 21\times6 + 16\times5 + 18\times5 + 14\times8 + 18\times4 + 41\times15$

Minimum cost $= 126 + 80 + 90 + 112 + 72 + 615$

Minimum cost $= 1095$

By Least cost method

	I	II	III	IV	Supply	
A	7 $\boxed{6}$	13 $\boxed{6}$	5 $\boxed{17}$	5 $\boxed{5}$	34 ~~28~~	11 6 0
B	5 $\boxed{15}$	5	7	6	~~15~~ 0	- - -
C	8	6	6	5 $\boxed{12}$	~~12~~ 0	- - -
D	6	11 $\boxed{19}$	6	4	~~19~~ 0	- - -
	~~21~~	~~25~~	~~17~~	~~17~~		
	~~6~~	~~6~~	0	~~5~~		
	0	0		0		

	I	II	III	IV
A	7 $\boxed{6}$	3 $\boxed{6}$	5 $\boxed{17}$	5 $\boxed{5}$
B	5 $\boxed{15}$			
C		1 $\boxed{19}$		5 $\boxed{12}$
D				

Minimum cost $= 7\times6 + 5\times15 + 3\times6 + 1\times19 + 5\times17 + 5\times5$
$+ 5\times12$

$= 42 + 75 + 18 + 19 + 85 + 25 + 60$

Minimum cost $= 324$.

By VAM

	I	II	III	IV	Supply	P_1	P_2	P_3	P_4
A	~~7~~ $\boxed{6}$	3 $\boxed{6}$	5 $\boxed{17}$	5 $\boxed{5}$	34 ~~28~~ 11	2	2	2	2
B	5 $\boxed{15}$	5	7	6	~~15~~ 0	1	1	1	1
C	8	6	6	5 $\boxed{12}$	~~12~~ 0	1	1	1	3
D	6	1 $\boxed{19}$	6	4	~~19~~ 0	3	-	-	-
Demand	~~21~~	~~25~~	~~17~~	~~17~~					
	~~15~~ 0	~~6~~ 0	0	~~5~~ 0					
P_1	1	1	1	1					
P_2	2	2	1	1					
P_3	2	-	1	1					
P_4	2	-	-	1					
P_5	2	-	-	1					

P_5	P_6
2	7
1	5
-	-
-	-

(1)

	I	II	III	IV	Supply
A	7	3	6	5	34
B	5	5	7	6	15
C	8	6	6	5	12
D	6	1	6	4	19
Demand	21	25	17	17	

Solution- Total supply $\Rightarrow 34 + 15 + 12 + 19 = 80$
Total demand $\Rightarrow 21 + 25 + 17 + 17 = 80$

Total supply = Total demand

By North-West Corner Method-

	I	II	III	IV	Supply
A	7 $\boxed{21}$	3 $\boxed{13}$	5	5	~~34~~ ~~13~~ 0
B	5	5 $\boxed{12}$	7 $\boxed{3}$	6	~~15~~ ~~3~~ 0
C	8	6	6 $\boxed{12}$	5	~~12~~ 0
D	6	1	6 $\boxed{2}$	4 $\boxed{17}$	~~19~~ 17
Demand	~~21~~	~~25~~	~~17~~	~~17~~	
	0	~~12~~	~~14~~	0	
		0	~~12~~		
			0		

~~Minimum~~

	I	II	III	IV
A	7 $\boxed{21}$	3 $\boxed{13}$		
B		5 $\boxed{12}$	7 $\boxed{3}$	
C			6 $\boxed{12}$	
D			6 $\boxed{2}$	4 $\boxed{17}$

Minimum cost $= 7\times21 + 3\times13 + 5\times12 + 7\times3 + 6\times12 + 6\times2 + 4\times17$

$\Rightarrow 147 + 39 + 60 + 21 + 72 + 12 + 68 = 419$

$\frac{55}{\times 20}$

minimum cost $= 50 \times 200 + 60 \times 60 + 55 \times 260 + 56 \times 30$
$58 \times 50 \quad + 55 \times 220 + 62 \times 160$
$= 10000 + 3600 + 14300 + 1680 + 2900$
$+ 12100 + \cancel{37200}\ 9920$
$= 54500$ Rs. /– Ans

Minimum cost $= 5\times300 + 20\times100 + 15\times200 + 10\times400$
$+ 0\times300$
$= 1500 + 2000 + 3000 + 4000$
$= \underline{10500}$ Rs/-

Origin	A	B	C	D
X	30 $\boxed{300}$	20 $\boxed{300}$	10 $\boxed{200}$	
Y	5		25 $\boxed{200}$	

minimum cost = 30×300 + 20×300 + 2000 + 5000
= 9000 + 6000 + 2000 + 5000
minimum cost = 22000 Rs/-

	A	B	C	D	Avai	
X	30	20 $\boxed{100}$	10 $\boxed{400}$	0 $\boxed{300}$	~~800~~ ~~500~~	100
Y	5 $\boxed{300}$	15 $\boxed{200}$	25	0	~~500~~ ~~200~~	0
Dem	~~300~~ 0	~~300~~ ~~100~~ 0	~~400~~ 0	~~300~~ 0		

	A	B	C	D
X		20 $\boxed{100}$	10 $\boxed{400}$	0 $\boxed{300}$
Y	5 $\boxed{300}$	15 $\boxed{200}$		

Minimum cost = 5×300 + 20×100 + 15×200 + 10×400
= 1500 + 2000 + 3000 + 4000
= 10500 Rs/-

	A	B	C	D	Avail	P_1	P_2	P_3
X	30	20 $\boxed{100}$	10 $\boxed{400}$	0 $\boxed{300}$	~~800~~ ~~400~~ 100	10	10	20
Y	5 $\boxed{300}$	15 $\boxed{200}$	25	0	~~500~~ ~~200~~	5	15	15
Demo.	~~300~~ 0	~~300~~ 0	~~400~~ 0.	~~300~~ 0.				
P_1	25	5	15	0				
P_2	-	5	15	0				
P_3	-	5	-	0				

	I	II	III	IV
A	7 $\boxed{6}$	3 $\boxed{6}$	5 $\boxed{17}$	5 $\boxed{15}$
B	5 $\boxed{15}$			
C				5 $\boxed{12}$
D		1 $\boxed{19}$		

Minimum cost $= 7\times6 + 5\times15 + 3\times6 + 1\times19 + 5\times17$
$+ 5\times5 + 5\times12$
$= 42 + 75 + 18 + 19 + 85 + 25 + 60$
$= 324$ Rs/-

→ Unbalanced Transportation Problem ←

When Supply > Demand ⇒ Add Dummy Column
Supply > Demand ⇒ Add Dummy Row

Que -

Origin	A	B	C	Availability
X	30	20	10	800
Y	5	15	25	500
Demand	300	300	400	

Total availability = 800 + 500 = 1300
Total demand = 300 + 300 + 400 = 1000

Here, A > D
Therefore add a dummy column to a given table

Orig.	A	B	C	D	Availability
X	30 $\boxed{300}$	20 $\boxed{300}$	10 $\boxed{200}$	0	~~800~~ ~~500~~ ~~200~~ 0
Y	5	15	25 $\boxed{200}$	0 $\boxed{300}$	~~500~~ ~~300~~
Dem	~~300~~ 0	~~300~~	~~400~~ ~~200~~ 0	~~300~~ 0	

(2)

Source / Supply	D_1	D_2	D_3	D_4	Supply
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34 / 34

Total supply $\Rightarrow 7+9+8 = 34$
Total demand $\Rightarrow 5+8+7+4 = 34$

By VAM method.

Source	D_1	D_2	D_3	D_4	Supply	P_1	P_2	P_3	P_4	P_5	P_6
S_1	19 $\boxed{5}$	30	50	10 $\boxed{2}$	$\cancel{7}$ $\cancel{2}$ $\cancel{0}$	9	9	40	40	–	–
S_2	70	30	40 $\boxed{7}$	60 $\boxed{2}$	$\cancel{9}$ 7	10	20	20	20	20	40
S_3	40	8 $\boxed{8}$	70	20 $\boxed{10}$	$\cancel{18}$ $\cancel{10}$ $\cancel{0}$	12	20	50	50	–	–
Dem	$\cancel{5}$ 0	$\cancel{8}$ 0	$\cancel{7}$ 0	$\cancel{14}$ $\cancel{4}$ 2 0							

	D_1	D_2	D_3	D_4
P_1	21	22	10	10
P_2	21	–	10	10
P_3	–	–	10	10
P_4	–	–	10	50
P_5	–	–	10	60

P_6 – – 10 –

By Least-cost method-

Source	D_1	D_2	D_3	D_4	Supply
S_1	21	16	15	3 $\boxed{11}$	~~11~~ 0
S_2	17 $\boxed{1}$	18	14 $\boxed{12}$	23	~~13~~ ~~1~~ 0
S_3	32 $\boxed{5}$	27 $\boxed{10}$	18	41 $\boxed{4}$	~~19~~ ~~9~~ ~~4~~ 0
Demand	~~6~~ ~~5~~ 0	~~10~~ 0	~~12~~ 0	~~15~~ ~~4~~ 0	43 / 43

Source	D_1	D_2	D_3	D_4
S_1				3 $\boxed{11}$
S_2	17 $\boxed{1}$		14 $\boxed{12}$	
S_3	32 $\boxed{5}$	27 $\boxed{10}$		41 $\boxed{4}$

Minimum cost $= 17\times1 + 32\times5 + 27\times10 + 14\times12 + 3\times11 + 41\times4$

$= 17 + 160 + 270 + 168 + 33 + 164$

$= 812$ Ans

VAM method-

Source	D_1	D_2	D_3	D_4	Supply	P_2
S_1	21	16	15	3	11	12
S_2	17	18	14	23	13	3
S_3	32	27	18	41	19	9
Demand	6	10	12	43	43 / 43	4
P_1	11	6				

Exercise -

(1)

Source	D_1	D_2	D_3	D_4	Supply
S_1	21	16	15	3	11
S_2	17	18	14	23	13
S_3	32	27	18	41	19
Demand	6	10	12	15	43 / 43

Solution - Total supply = 43
Total demand = 43

Total Supply = Total demand

North-West corner -

Source	D_1	D_2	D_3	D_4	Supply
S_1	21 $\boxed{6}$	16 $\boxed{5}$	15	3	~~11~~ ~~5~~ 0
S_2	17	18 $\boxed{5}$	14 $\boxed{8}$	23	~~13~~ ~~8~~ 0
S_3	32	27	18 $\boxed{4}$	41 $\boxed{15}$	~~19~~ ~~15~~ 0
Demand	~~6~~ 0	~~10~~ ~~5~~ 0	~~12~~ ~~4~~ 0	~~15~~ 0	43 / 43

Source	D_1	D_2	D_3	D_4
S_1	21 $\boxed{6}$	16 $\boxed{5}$		
S_2		18 $\boxed{5}$	14 $\boxed{8}$	
S_3			18 $\boxed{4}$	41 $\boxed{15}$

Minimum cost $= 21\times6 + 16\times5 + 18\times5 + 14\times8 + 18\times4 + 41\times15$
$= 126 + 80 + 90 + 112 + 72 + 615$
= Rs. 1095 /-

DEGENERACY PROBLEM

(1)

	A	B	C	Supply
S_1	8	7	3	60
S_2	3	8	9	70
S_3	11	3	5	80
Demand	50	80	80	

	A	B	C	Supply	P_1	P_2	P_3	P_4
S_1	8	7	3 $\boxed{60}$	~~60~~ 0	-4	4	-	-
S_2	3 $\boxed{50}$	8	9 $\boxed{20}$	~~70~~ ~~20~~ 0	5	1	1	9
S_3	11	3 $\boxed{80}$	5	~~80~~ 0	2	2	2	-
Dem.	~~50~~ 0	~~80~~ 20	~~80~~ 20					

P_1	5	4	2
P_2	-	4	2
P_3	-	5	4
P_4	-	-	9

	A	B	C	Supply
S_1	8	7	3 $\boxed{60}$	60
S_2	3 $\boxed{50}$	8	9 $\boxed{20}$	70
S_3	11	3 $\boxed{80}$	5 $\boxed{\alpha}$	80
Demand	50	80	80	

No. of allocations = 4

and No. of allocations

$= m+n-1$

$= 3+3-1$

$\Rightarrow 6-1 = 5$

This is a degeneracy problem

Note : In Non-degeneracy, number of allocations $= m+n-1$

- In degeneracy, number of allocations $\neq m+n-1$

$\Delta_{33} \Rightarrow c_{33} - (u_3' + v_3) = 70 - (20 - 20) = 70$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 $\boxed{5}$	30 $\boxed{32}$	50 $\boxed{42}$	10 $\boxed{2}$	7	$u_1 = 0$
S_2	70 $\boxed{19}$	30 $\boxed{\cancel{2}}$ $\boxed{2}$	40 $\boxed{7}$	60 $\boxed{18}$	9	$u_2 = 32$
S_3	40 $\boxed{11}$	8 $\boxed{6}$	70 $\boxed{52}$	20 $\boxed{12}$	18	$u_3 = 10$
Demand	5	8	7	19		
v_j	$v_1 = 19$	$v_2 = -2$	$v_3 = 8$	$v_4 = 10$		

$c_{ij} = u_i + v_j$

$c_{11} \Rightarrow u_1 + v_1 = 19$; $v_1 = 19$

$c_{14} \Rightarrow u_1 + v_4 = 10$; $v_4 = 10$

$c_{22} \Rightarrow u_2 + v_2 = 30$; $u_2 = 32$

$c_{23} \Rightarrow u_2 + v_3 = 40$; $v_3 = 8$

$c_{32} \Rightarrow u_3 + v_2 = 8$; $v_2 = -2$

$c_{34} \Rightarrow u_3 + v_4 = 20$; $v_3 = 10$

$$\left.\begin{aligned}
\Delta_{12} &= c_{12} - (u_1 + v_2) = 32 \\
\Delta_{13} &= c_{13} - (u_1 + v_3) = 42 \\
\Delta_{21} &= c_{21} - (u_2 - v_1) = 19 \\
\Delta_{24} &= c_{24} - (u_2 - v_4) = 18 \\
\Delta_{31} &= c_{31} - (u_3 + v_1) = 11 \\
\Delta_{33} &= c_{33} - (u_3 + v_3) = 52
\end{aligned}\right\} \Delta_{ij} > 0$$

Optimal solution $= 19 \times 5 + 10 \times 2 + 30 \times 2 + 40 \times 7 + 8 \times 6 + 20 \times 1$

Optimal Solution $= 743$

.	D_1	D_2	D_3	D_4
S_1	19 $\boxed{5}$		~~40~~	10 $\boxed{12}$
S_2		8 $\boxed{8}$	40 $\boxed{7}$	60 $\boxed{2}$
S_3				20 $\boxed{10}$

	D_1	D_2	D_3	D_4	Supply	u_i
S_1	19 $\boxed{5}$	30 $\boxed{32}$	50	10 $\boxed{2}$	7	$u_1 = 10$
S_2	70 $\boxed{1}$	30 $x=2$	40 $\boxed{7}$	60 $\boxed{2}$ =0	9	$u_2 = 60$
S_3	40 $\boxed{11}$	8 $\boxed{8}$ =6	70	20 $\boxed{10}$ =12	8	$u_3 = 20$
Dem	5	8	7	14		
v_j	$v_1 = 9$	$v_2 = -12$	$v_3 = -20$	$v_4 = 0$		

$C_{ij} = u_i + v_j$

$C_{11} = u_1 + v_1 = 19$; $v_1 = 9$

$C_{14} = u_1 + v_4 = 10$; $u_1 = 10$

$C_{23} = u_2 + v_3 = 40$; $v_3 = -20$

$C_{24} = u_2 + v_4 = 60$; $u_2 = 60$

$C_{32} = u_3 + v_2 = 8$; $v_2 = -12$

$C_{34} = u_3 + v_4 = 20$; $u_3 = 20$

Now, $\Delta_{ij} = C_{ij} - (u_i + v_j)$

$\Delta_{12} \Rightarrow C_{12} - (u_1 + v_2) = 30 - (10 - 12)$

$\Delta_{12} = 32$

$\Delta_{13} \Rightarrow C_{13} - (u_1 + v_3) = 50 - (10 - 20) = 60$

$\Delta_{21} \Rightarrow C_{21} - (u_2 + v_1) = 70 - (60 + 9) = 1$

$\Delta_{22} \Rightarrow C_{22} - (u_2 + v_1) = 30 - (60 - 12) = -18$

$\Delta_{31} \Rightarrow C_{31} - (u_3 + v_1) = 40 - (20 + 9) = 11$

P6

–

40

–

(2)

	A	B	C	D	E
F	10	5	13	15	16
G	3	9	18	13	6
H	10	7	2	2	2
I	7	11	9	7	12
J	7	9	10	4	12

No. of rows = 5
No. of columns = 5

Step-1 Row reduction

	A	B	C	D	E
F	5	0	8	10	11
G	0	6	15	10	3
H	8	5	0	0	0
I	0	4	2	0	5
J	3	5	6	0	8

Step-2 Column reduction

	A	B	C	D	E
F	6	$\boxed{0}$	8	10	11
G	$\boxed{0}$	6	15	10	3
H	8	5	$\boxed{0}$	0*	0*
I	0*	4	2	$\boxed{0}$	5
J	3	5	6	0*	8

Step-3 Optimal solution does not exist

Let $x = 2$

	A	B	C	D	E
F	7	$\boxed{0}$	8	12	11
G	$\boxed{0}$	4	13	10	1
H	10	5	0*	2	$\boxed{0}$
I	0*	2	$\boxed{0}$	0*	3
J	3	3	4	$\boxed{0}$	6

F → B
G → A
H → E
I → C
J → D

Minimum cost = 5 + 3 + 2 + 9 + 4
= Rs. 23 /- Answer

∴ Solution - Number of rows = 4
Number of columns = 4

Here, no. of rows = no. of columns
given assignment problem is balanced.

Step-1 Row Reduction

	P	Q	R	S
A	3	9	0	8
B	3	1	6	0
C	1	34	3	0
D	4	0	2	[illegible]

Step-2 Column-Reduction

	P	Q	R	S
A	2	9	0	8
B	2	1	6	0
C	0	4	3	0
D	3	0	2	1

Step-3

	P	Q	R	S
A	2	9	$\boxed{0}$	8
B	2	1	6	$\boxed{0}$
C	$\boxed{0}$	4	3	0×
D	3	$\boxed{0}$	2	1

Number of assigment = 4
Number of rows = 4

Minimum cost = 8 + 6 + 13 + 10
= 37 Rs. Ans

ASSIGNMENT

Assignment method is a scheduling method used for assigning individual jobs to individual processing component on a 1 to 1 basis, with the goal of minimizing the total cost of time for accomplishing all the jobs.

* Types of assignment problem-

(1) Balanced- Assignment Problem- An assignment problem is said to be balanced if the number of person are equal to the number of jobs.

(2) Unbalanced - Assignment Problem - An assignment problem is said to be unbalanced if the number of person are not equal to the number of jobs.

Exercise

(1) Find the optimal solution for assignment problem with profit cost price.

Area \ S.M.	P	Q	R	S
A	11	17	8	16
B	9	7	12	6
C	13	16	15	12
D	14	10	12	11

5)

	A	B	C	D	E
F	2	9	2	7	1
G	6	8	7	6	1
H	4	6	5	3	1
I	4	2	7	3	1
J	5	3	9	5	1

Step-1. Row-Reduction

	A	B	C	D	E
F	1	8	1	6	0
G	5	7	6	5	0
H	3	5	4	2	0
I	3	1	6	2	0
J	4	2	8	4	0

Step-2 Column Reduction

	A	B	C	D	E
F	$\boxed{0}$	~~8~~ 7	0×	4	0× ✓
G	4	6	5	~~4~~ 3	$\boxed{0}$ ✓
H	2	4	3	$\boxed{0}$	×0
I	2	$\boxed{0}$	5	0×	×0
J	3	1	7	2	0 ✓

Step-3

	A	B	C	D	E
F	$\boxed{0}$	7	0×	4	0×
G	4	6	5	3	$\boxed{0}$ ✓
H	2	4	3	$\boxed{0}$	×0 ✓
I	2	$\boxed{0}$	5	×0	×0 ✓
J	3	1	7	2	×0 ✓

Step-4. Let minimum value, $x = 2$

	A	B	C	D	E
F	0	2	0	6	2
G	2	6	3	3	0
H	0	4	1	0	0
I	0	0	3	0	0
J	1	1	5	2	0

(4)

	1	2	3	4	5
I	12	8	7	15	4
II	7	9	17	14	10
III	9	6	12	6	7
IV	7	6	14	6	10
V	9	6	12	10	6

Step-1- Row Reduction

	1	2	3	4	5
I	8	4	3	11	0
II	0	2	10	7	3
III	3	0	6	0	1
IV	1	0	8	0	4
V	3	0	6	4	0

Step-2 - Column Reduction

	1	2	3	4	5
I	8	4	0	11	0
II	0	2	7	7	3
III	3	0	3	0	1
IV	1	0	5	0	4
V	3	0	3	4	0

Step-3

	1	2	3	4	5
I	8	4	$\boxed{0}$	11	0x
II	$\boxed{0}$	2	7	7	3
III	3	$\boxed{0}$x	3	$\boxed{0}$	1
IV	1	$\boxed{0}$x	5	0x	4
V	3	0x	3	4	$\boxed{0}$

When we allocate III job to IV (4) and IV job to 2

I → 3
II → 1
III → 4
IV → 2
V → 5

Minimum cost = 7+7+6+6+6
= ~~8~~ Rs. 32 /- Answer

(3)

	A	B	C	D	E
F	8	4	2	6	1
G	0	9	5	5	4
H	3	8	9	2	6
I	4	3	1	0	3
J	9	5	8	9	5

Step-1. Row reduction

	A	B	C	D	E
F	7	3	1	5	0
G	0	9	5	5	4
H	1	6	7	0	4
I	4	3	1	0	3
J	4	0	3	4	0

Step-2 Column Reduction

	A	B	C	D	E
F	7	3	0	5	0
G	0	9	4	5	4
H	1	6	6	0	4
I	4	3	0	0	3
J	4	0	2	4	0

Step-3.

	A	B	C	D	E
F	7	3	0	5	$\boxed{0}$
G	$\boxed{0}$	9	4	5	4
H	1	6	6	$\boxed{0}$	4
I	4	3	$\boxed{0}$	$0^{\times}$	3
J	4	$\boxed{0}$	2	4	0

Number of allocation
= Number of rows

F → E
G → A
H → D
I → C
J → B

Minimum cost = 1 + 0 + 2 + 1 + 5
= 10 Rs. /-   Answer

1 → D
2 → B
3 → C
4 → A
5 → E

Minimum cost = 28 + 24 + 33 + 25 + 39
=

Que.

	A	B	C	D
1	1	4	6	3
2	9	7	10	9
3	4	5	11	7
4	8	7	8	5

Solution - No. of rows = 4
No. of columns = 4
Given assignment problem is balanced

Step-1 Row reduction

	A	B	C	D
1	0	3	5	2
2	2	0	3	2
3	0	1	7	3
4	3	2	3	0

Step-2 Column reduction

	A	B	C	D
1	$\boxed{0}$	3	2	2
2	2	$\boxed{0}$	0×	2
3	$\boxed{0}$×	1	~~3~~4	3
4	3	2	$\boxed{0}$	$\boxed{0}$×

Step-3

	A	B	C	D
1	$\boxed{0}$	3	2	2 ✓
2	2	$\boxed{0}$	0×	2
3	0×	1	4	3 ✓
4	3	2	$\boxed{0}$	0×

Step-4 Optimal solⁿ does not exist
Let, minimum cost
$x = 1$

Que-

	A	B	C	D
1	7	5	8	4
2	5	6	7	4
3	8	7	9	8

Step-3-

	A	B	C	D	E
1	2	6	8	$\boxed{0}$	6
2	19	$\boxed{0}$	2	0×	9
3	11	0×	$\boxed{0}$	1	0×
4	$\boxed{0}$	10	11	11	5
5	0×	30	8	5	4

Step-4-

	A	B	C	D	E
1	3	6	8	$\boxed{0}$	6
2	20	$\boxed{0}$	2	0×	9
3	12	0×	$\boxed{0}$	1	0×
4	$\boxed{0}$	9	10	10	4
5	0×	29	7	4	3

Step-4-

	A	B	C	D	E
1	6	6	8	$\boxed{0}$	6
2	23	$\boxed{0}$	2	0×	9
3	15	0×	$\boxed{0}$	1	0×
4	$\boxed{0}$	6	7	7	1
5	0×	26	4	1	$\boxed{0}$

No. of allocated zeroes = No. of Rows

Date -

Que-

	A	B	C	D	E
1	30	37	40	28	40
2	40	24	27	21	36
3	40	32	33	30	35
4	25	38	40	36	36
5	29	62	41	34	39

Solution- No. of rows = 5
No. of columns = 5.

Given problem is balanced.

Step-1 Row reduction

	A	B	C	D	E
1	2	9	12	0	12
2	19	3	6	0	15
3	10	2	3	0	5
4	0	13	15	11	11
5	0	33	12	5	10

Step-2 column Reduction

	A	B	C	D	E	
1	2	7	9	$\boxed{0}$	7	✓
2	19	1	3	0×	10	✓
~~3~~	~~10~~	~~$\boxed{0}$~~	~~0×~~	~~0×~~	~~0×~~	✓
4	$\boxed{0}$	11	12	11	6	✓
5	0×	31	9	5	5	✓

No. of ele. 3
min. ... = 5
3 ≠ 5

6 + X = 1

Step-3

	A	B	C	D	E
1	2	7	9	$\boxed{0}$	7
2	19	1	3	0×	10
3	10	$\boxed{0}$	0×	0×	0×
4	$\boxed{0}$	11	12	11	6
5	0×	31	9	5	5

Step

line.

row - unmu.

Col - mu.

Date- 4/03/2024

UNIT- V ⇒ JOB SEQUENCIES

(Problem of n-jobs and 2-machines)

Que- Suppose that there are five jobs and 2 machines. Determine the sequence and ideal time.

Solution :

Jobs	A	B	C	D	E
Prin.	5	1	9	3	10
bind	2	6	7	8	4

Optimal sequence of jobs - Printing | B | D | C | E | A | Binding

Jobs	Printing		Binding	
	in	out	in	out
B	0	1	1	7
D	1	4	7	15
C	4	13	15	22
E	13	23	23	27
A	23	28	28	30

Total ellapse time = 30 hrs.

Idle Time for printing = 30-28 = 2 hrs.

Idle Time for binding = 1+1+1 = 3 hrs.

Step-3 Assign (0)

	A	B	C	D
1	$\boxed{0}$	6	10	14
2	0×	5	9	11
3	0×	5	9	12
4	0×	$\boxed{0}$	0×	0×

Step-4 Optimal solⁿ does not exist

Let, minimum cost = 5

	A	B	C	D
1	0	2	6	
2	0	1	5	
3	0	1	5	
4	5	0	0	0

Table - Step-4

	A	B	C	D
1	$\boxed{0}$	1	5	9
2	0×	$\boxed{0}$	4	6
3	0×	0×	4	0
4	5	0×	$\boxed{0}$	0×

Step-5

	A	B	C	D
1	$\boxed{0}$	1	1	5
2	0×	$\boxed{0}$	0	2
3	0×	0×	0	3
4	9	8	0	0

Here, the allocation possibility is more than one.

⇒ 1→A   minimum cost = 18 + 13 + 19 + 0
2→B   minimum cost = 50
3→C
4→D

and 1→A   minimum cost = 18 + 17 + 15 + 0
2→B   minimum cost = 50
3→C
4→D

Step-4

	A	B	C	D
1	$\boxed{0}$	2	1	1
2	3	0×	$\boxed{0}$	2
3	0×	$\boxed{0}$	3	2
4	4	2	×0	$\boxed{0}$

Here, no. of assigned zeroes = no. of rows

A 1 → A
2 → C
3 → B
4 → D

minimum cost = 1 + 10 + 5 + 5
= Rs 21/-

Que.

	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	10	15	19	22

⇒

	A	B	C	D
1	18	24	28	32
2	8	13	17	19
3	18	15	19	22
4	0	0	0	0

Solution - No. of rows = 3
No. of columns = 4
Here, No. of rows ≠ No. of colums

Step-1 - Row reduction

	A	B	C	D
1	0	6	10	14
2	0	5	9	11
3	0	5	9	12
4	0	0	0	0

Step-2- Column reduction

	A	B	C	D	
1	$\boxed{0}$	6	10	14	✓
2	×0	5	9	11	✓
3	×0	5	9	12	✓
4	×0	$\boxed{0}$	0 ×	0 ×	

Que.

Job	A	B	C	D	E	F
M_1	5	3	2	10	12	6
M_2	3	2	5	11	10	7

Solution – Optimal solution of job sequence –

M_1

C	F	D	E	A	B

M_2

Jobs	M_1 in	M_1 out	M_2 in	M_2 out
A/C	0	2	2	7
~~B~~/F	2	8	8	15
~~C~~/D	8	18	18	29
~~D~~/E	18	30	30	40
~~E~~/A	30	35	40	43
~~F~~/B	35	38	43	45

Total ellapse time = 45

Idle time for M_1 = 45-38
= 7

Idle time for M_2 = 2+1+3
+1
= 7

Que.

Jobs	1	2	3	4	5	6	7
M_1	4	13	16	7	11	12	10
M_2	9	11	11	7	13	2	4

Optimal Solution of job sequence –

M_1

1	4	5	3	2	7	6

M_2

Jobs	M_1 in	M_1 out	M_2 in	M_2 out
1	0	4	4	13
4	4	11	13	20
5	11	22	22	35
3	22	38	38	49
2	38	51	51	62
7	51	61	62	66
6	61	73	73	75

Total ellapse time = 75

Idle time for M_1 = 2

Idle time for M_2 = 4+2+3
+2+7
= 18

Que -

Jobs	J_1	J_2	J_3	J_4	J_5
Mach. A	5	1	9	3	10
Mach. B	2	6	7	8	4

Solution - Optimal sequences of jobs -

A → | J_2 | J_4 | J_3 | J_5 | J_1 | ← B

Jobs	Machine A		Machine B	
	in	out	in	out
J_2	0	1	1	7
J_4	1	4	7	15
J_3	4	13	15	22
J_5	13	23	23	27
J_1	23	28	28	30

Total ellapse time = 30

Idle time of Machine A = 30 − 28 = 2

Idle time of Machine B = 1 + 1 + 1 = 3

Que.

Job/Mach	A	B	C	D	E
M_1	5	1	9	3	10
M_2	2	6	7	8	4

Solution - Optimal Sequence of jobs -

M_1 → | B | D | C | E | A | ← M_2

Jobs	M_1		M_2	
	in	out	in	out
B	0	1	1	7
D	1	4	7	15
C	4	13	15	22
E	13	23	23	27
A	23	28	28	30

Que.

Jobs	A(1)	B(2)	C(3)	D(4)	E(5)
Machine A	6	2	3	4	11
Machine B	3	7	10	9	15

Optimal Sequence of jobs

Mac A				Mac B
B(2)	3	4	5	1

Jobs
2
3
4
5
1

Jobs	Machine A		Machine B	
~~in~~	in	out	in	out
2	0	2	2	9
3	2	5	9	19
4	5	9		
5				
1				

Que

Jobs	1	2	3	4	5
Machine A	6	2	10	4	11
Machine B	3	7	8	9	6

Optimal Sequence of Jobs -

A				B
2	4	3	5	1

Jobs	Machine A		Machine B	
	in	out	in	out
2	0	2	2	9
4	2	6	9	18
3	6	16	18	26
5	16	27	27	32
1	27	33	33	36

Total ellapse time = 36

Idle time for machine A = 36 - 33 = 3.
Idle time for machine B = 2 + 1 + 1 = 4

Date - 11/March/2024

UNIT-4 ⇒ REPLACEMENT THEORY

★ Replacement Theory - The replacement theory in operational research is used in the decision making process of replacing a used equipments with a substitute, mostly a new equipments of better usage. The replacement might be necessary due to failure or breakdown of particular equipments.

The replacement theory is used in the cases like existing item have ~~out lived~~. out lived or it may not be economical any more to continue with them, or items might have been destroyed either by accident or otherwise. The above discussed situation can be solved mathematically and categorized on some basis called as replacement model.

Replacement Model - (a) Value of money does not change with time.
(b) Value of money changes with time.

→ Replacement of equipment that fails suddenly -
(a) Individual Replacement Policy.
(b) Group Replacement Policy.

Policy of the first Replacement Model

If the running or ~~m~~ ~~replo~~ maintenance cost of a machine for the next year is more than the average annual cost of the selected year, the

Find (i) decide the optimum sequence of processing of different product in order to minimise the total manufacturing time of product.
(ii) The total minimum ellapse time
(iii) Idle for time for machine A and machine B.
(iv) Name and discuss the scheduling model used.

Solution – Optimal Sequence of jobs –

(i) MA

VIII	III	IV	VII	VI	V	II	I

MB

(ii)

Product	MA in	MA out	MB in	MB out
VIII	0	10	10	30
III	10	25	30	80
IV	25	405	80	115
VII	45	110	115	165
VI	230	230	230	270
V	230	310	310	346
II	310	355	355	385
I	355	3805	385	405

Total ellapse time = 405 minutes

(iii) Idle f time for machine A ⇒ 405 − 3805 = 20

Idle time for machine B ⇒ 10 + 65 + 40 + 90 = 120

(iv) The scheduling model used is (n−2) job sequencing model.

Que.

Jobs	A	B	C	D	E	F	G	H	I
M_1	4	7	6	11	8	10	9	7	6
M_2	8	10	9	6	5	11	5	10	3

Optimal sequence of jobs -

M_1 M_2

A	C	I	B	H	F	/	D	~~D~~G	E

Jobs	M_1 in	M_1 out	M_2 in	M_2 out
A	0	4	4	12
C	4	10	12	21
I	10	16	21	34
B	16	23	34	44
H	23	30	44	54
F	30	40	54	65
D	40	51	65	71
G	51	60	71	76
E	60	68	76	81

Total ellapse time = 81 hrs

Idle time for M_1 = 81 - 68
= 13 hrs

Idle time for M_2 = 4 hrs

Que. In a machine shop, 8 different product are being manufacturer. Each requiring time on 2 machines A and B as given below:-

product	Time (in min) on machi. (A)	Time (in min) mach. (B)
I	30	20
II	45	30
III	15	50
IV	20	35
V	80	36
VI	120	40
VII	65	50
VIII	10	20

9	6000	23300	15000	38300	4255.56

Capital cost ⇒ 13000 + 3600 = 16600 Rs.

Depriciation cost ⇒ 16600 − 1600 = ₹15000

Minimum Average Annual Cost = ₹ 3916.67

⇒ 4000 > 3916.67

∵ Maintenance cost is greater than minimum average annual cost.

Thus, we can replace the machine at the end of the selected 6^{th} year.

(3) A Simplex Engineering Company has a machine whose purchase prize is ₹180000. The expected maintenance cost and resale price in different year are as given below–

Year	1	2	3	4	5	6	7
Mainte. cost	1000	1200	1600	2400	3000	3900	5000
Resale cost	75000	72000	70000	65000	58000	50000	45000

After what time interval in your opinion should the machine be replaced.

Minimum Average Annual Cost = ₹ 15750

⇒ Selected Year is 6^{th} year.

16000 > 15750

∵ A. Maintenance Cost is greater than minimum average annual cost.

Thus, we can replace the machine at the end of the selected 6^{th} year.

(2) A firm is using a machine whose purchase price is ₹13000. The installation charges amount to ₹3600 and the machine has a scrap value of only ₹1600. The maintenance cost in various year is given in the following year-

Year	1	2	3	4	5	6	7	8	9
Maintenance cost ₹	250	750	1000	1500	2100	2900	4000	4800	6000

The firm wants to determine after how many years should be machine replaced on economic consideration, assuming that the machine replacement can be done only at the ends year.

Year (1)	main. cost (₹) 2	Cummu main cost (₹) 3	Deprecia cost (C-S) 4	Total cost 5 = (3+4)	Avg. annual mainte cost → 6 = 5/N
	250	250	15000	15250	15250
	750	1000	16000	16000	8000
	1000	2000	15000	17000	5666.67
	1500	3500	15000	18500	4625
	2100	5600	15000	20600	4120
	2900	8500	15000	23500	3916.67
	4000	12500	15000	27500	3928.57
	4800	17300	15000	32300	4037.5
			15000		

replacement of the machine at end of the selected year.

* Some related ~~to~~ terms to replacement value

(i) C (Capital Cost)
(ii) S (Scrap cost) of a machine (Resale value)
(iii) N (Number of years the machine could be used)
(iv) F(t) (~~Maind~~ Maintenance cost or running cost)
(v) A(n) (Average Annual Cost)

(vi) Total cost = Capital Cost − Scrap Value + Maintenance Cost

or

Total cost = Depreciation Cost + Maintenance Cost

Depreciation cost = Capital Cost − Scrap Value

Que. The cost of the machine is 61000 Rs. and its scrap value is ₹ 1000. The maintenance cost found from the past experiences are as follows–

Year	1	2	3	4	5	6	7	8
Maintenance	1000	2500	4000	6000	9000	12000	16000	20000

Year	Maint cost	Comm. main cost	Depreci. cost (C−S)	Total cost 5=3+4	Avg. Annual cost (6)=5/N
1	1000	1000	60000	61000	61000
2	2500	3500	60000	63500	31750
3	4000	7500	60000	67500	22500
4	6000	13500	60000	73500	18375
5	9000	22500	60000	82500	16500
6	12000	34500	60000	94500	15750
7	16000	50500	60000	110500	15785.71
8	20000	70500	60000	130500	16312.5

(6) Given cost price ₹12000 and scrap value is ₹200.
Given running cost in Rupees-

Year	1	2	3	4	5	6	7	8
Running cost	200	500	800	1200	1800	2500	3200	4000

Year (1)	running cost (2)	Cummu run cost (3)	Depreciation 4 = C-S	Total cost 5 = 4+3	Average annual cost 5 = 5/8
1	200	200	11800	12000	12000
2	500	700	11800	12500	6250
3	800	1500	11800	13300	4433.3
4	1200	2700	11800	14500	3625
5	1800	4500	11800	16300	3260
6	2500	7000	11800	18800	3133.3
7	3200	10200	11800	22000	3142.8
8	4000	14200	11800	26000	3250

⇒ Selected year is 3^{rd} year.
⇒ $6200 > 5200$
Maintenance cost is greater than the average annual maintenance cost.

Thus, we can replace a machine at ~~sele~~ the end of the selected third (3^{rd}) year.

(5) A taxi owner estimate from his past record that the cost per year for operating a taxi whose purchase price when new is ₹ 60000/- These are given below -

Year	1	2	3	4	5
Mainte cost Operating	10000	12000	15000	18000	20000

After five years the operating cost is ₹6000K where where K = 6, 7, 8, 9, 10 age in year. If the resale value decreased by 10% of purchase price each year. What is the optimum replacement policy?

Year Main-t Cummu

Year (1)	Main. Cost (2) (₹)	Cummu. Main Cost (₹) (3)	Deprecio. cost (4) 4 = C−S Scrap value (S)	Depre. (₹) 5	Total Cost (₹) 5 = 3+4	Ave. Annual mainte. cost 6 = 5/N
1	1000	1000	75000	5000	6000	6000
2	1200	2200	72000	8000	10200	5100
3	1600	3800	70000	10000	13800	4600 ←
4	2400	6200	65000	15000	21200	5300
5	3000	9200	58000	22000	31200	6240
6	3900	13100	50000	30000	43100	7183.33
7	5000	18100	45000	35000	53100	7585.71

Que-(8) Machine of a cost ₹9000. Annual Operating cost are ₹200 for first year and increased by ₹2000 every year. Determine the best age at which to replace the machine. (5 years)

Solution –

Year	1	2	3	4	5
Maint. (₹)	200	2200	4200	6200	8200

Year (1)	Maint cost (₹) (2)	Cummu Main cost (₹) (3)	Depreciation cost (4) 4 = C−S	Total cost (₹) 5 = 3+4	Ave. annual main. cost 6 = 5/N
1	200	200	9000	9200	9200
2	2200	2400	9000	11400	5700
3	4200	6600	9000	15600	5200 ←
4	6200	12800	9000	21800	5450
5	8200	21000	9000	30000	6000

$x + 3y = 900$

Taking $1.5x + y = 750$

put $x = 0$ then $y = 750$

coordinates are $(0, 750)$

~~put~~ put $y = 0$ then $x = 500$

coordinates are $(500, 0)$

intersecting point of $1.5x + y = 750$ and $x + 3y = 900$

coordinates are $\left(\frac{2700}{7}, \frac{1200}{7}\right)$

Taking $x + 3y = 900$

put $x = 0$ then $y = 300$

coordinates are $(0, 300)$

put $y = 0$ then $x = 900$

coordinates are $(900, 0)$

intersecting points are $\left(\frac{2700}{7}, \frac{1200}{7}\right)$ or $(385.7, 171.42)$

intersecting point of $x = 450$ and $1.5x + y = 750$

intersecting point of $y = 250$ and $x + 3y = 900$

(8) A manufacturing company produced two product A and B. Each product undergoes two operations are machine M_1 and M_2. The time require to perform these operations with available capacity of machine M_1 and M_2 given below-
The market survey has predicted that not more than 450 unit of product A and not more than 250 unit of product B can sold in the given quarter. The company wants to maximise profit. The unit profits ofor product A and B are Rs. 20 and Rs. 40.

Machine	Production time A	Production time B	Available capacity (hrs)
M_1	1.5 hrs	1 hrs.	750
M_2	1 hrs	3 hrs	900
Profit	20₹	40₹	

Solution- Formulation of given LPP.
Let, number of product A = x
number of product B = y

According to given problem;

$$x \leq 450$$
$$y \leq 250$$
$$1.5x + y \leq 750$$
$$x + 3y \leq 900$$
$$x, y \geq 0$$

maximum profit $Z = 20x + 40y$

Given LPP after replacing inequalities

$$x = 450$$
$$y = 250$$
$$1.5x + y = 750$$

255000
349000
———
349000

Selected year is 6^{th} year.

(7) cost price = ₹17500 and scrap value = ₹500

Year	1	2	3	4	5	6	7	8
Main.	200	300	350	1200	1800	2400	3300	4500

Year (1)	main (2)	Cummu. (3)	Depreciation (4) = C−S	Total cost 5 = 4+3	Average annual cost 6 = 5/8
1	200	200	17000	17200	17200
2	300	500	17000	17500	8750
3	350	850	17000	17850	5950
4	1200	2050	17000	19050	4762.5
5	1800	3850	17000	20850	4170
6	2400	6250	17000	23250	3875
7	3300	9550	17000	26550	3792.8
8	4500	14050	17000	31050	3881.2

Here, minimum average annual cost is ₹ 3792.8

3792.8 < 4500 (Maintenance cost

⇒ Selected year is 7^{th} year

Thus, machine can be replaced at the end of the 7^{th} year.

* 2-person zero sum Game - If the algebraic sum of gains and losses of all the players is zero (0) in a game. Then such game is called zero-sum game.
A game with two players, one person's gain is the loss of other, is called two-person zero-sum game.

* Pay-off Matrix - The gains resulting from a 2-person zero sum game can be represented in the matrix form is called pay-off matrix.

* Finite Game - If the no. of strategies of the players are finite then a game is said to be finite game.

* Infinite Game - If at least one of the players has infinite number of strategies, the game is said to be infinite game.

- If the best strategy for each player is to play one particular strategy through out the game, it is pure-strategy game.
- If the optimal plan for each player is to choose different strategy at different situation the game is called a mixed-strategy game.
- Maximum value of row minima value of a pay-off matrix is called as maximin value. It is denoted by $\underline{V}$.
- The minima value of column maximum value of a pay-off matrix is called as minimax value. and it is denoted by $\overline{V}$.

Date-1/April/2024

UNIT-III → GAME THEORY

The aim of the theory is to analysis the different situation each player has to face and different situation he has to choose according to those of the opponent.
The term 'game' represent a conflict between two or more parties such as chess, cricket, etc.

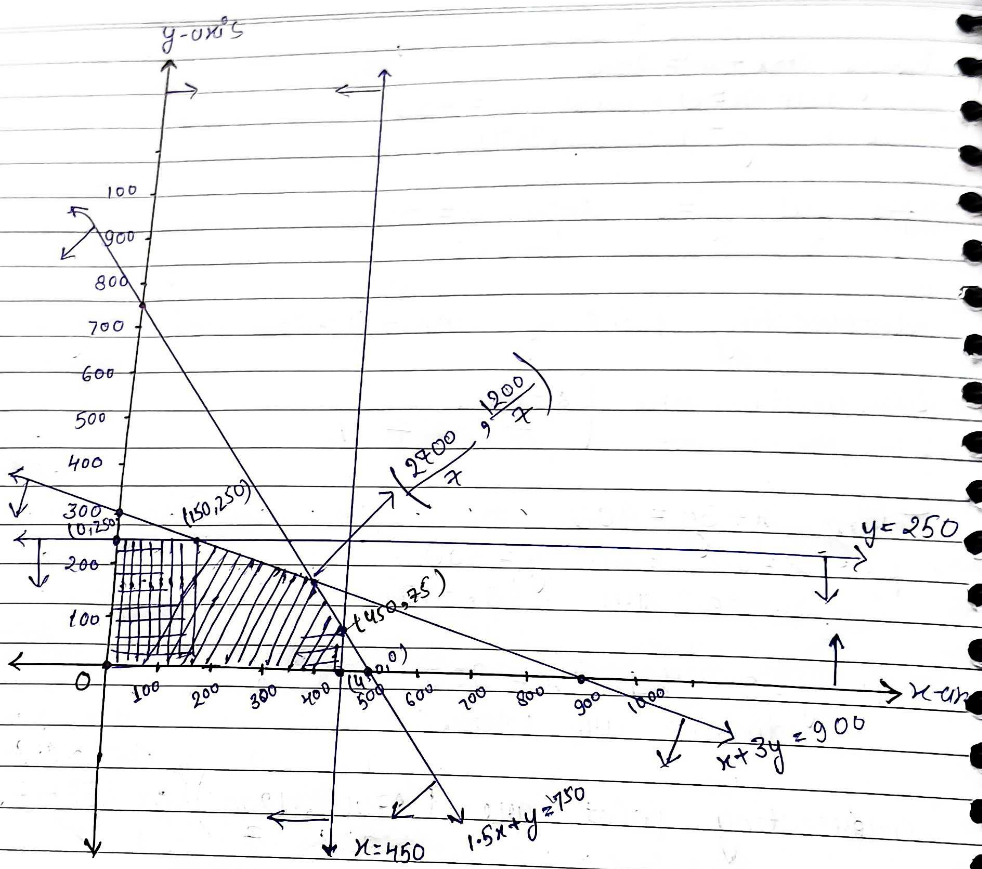
The application of 'Game Theory' is not limited to game in ordinary sense of it but also includes in Economics, Social Behaviour, etc.
In theory of games, we introduce the term players, strategy, pay-off, pay-off matrix, saddle point and without saddle point.

* Game - A competative situation is called a Game.

* Player - The number of competitors in game are called players.

* Pay-off - ~~Pay~~ Every play is associated with an outcome, known as the pay-off.

- Each player attempt to maximise his gains or minimise his loss.
- A game involving 'n' players is called n-person Game.



Coordinates	Z = 20x + 40y	
(0,0)	0	
(0,250)	10000	
(150,250)	13000	
$\left(\frac{2700}{7}, \frac{1200}{7}\right)$	14571.42 ←	maximum
(450,75)	12000	
(450,0)	9000	

$p \leq 7$
$q \geq 7$

(4) Determine the range of p and q that will make the pay-off element a_{22} and the saddle point of the game with the following game-

	B_1	B_2	B_3	max(min)
A_1	2	4	5	2
A_2	10	7	q	7 ←
A_3	4	p	8	4
min(max)	10	7	8	

Saddle point (A_2, B_2)

According to given matrix
~~$q \leq 7$~~ $q = 7$
and ~~$p \leq 7$~~

(5)

Player B

		B_1	B_2	max(min)
Player A	A_1	5	1	1
	A_2	3	4	3 ←
	min(max)	5	4 ↑	

$\Rightarrow$ $\underline{V} \neq \overline{V}$ (No saddle point)

Here, $a_{11} = 5$, $a_{12} = 1$, $a_{21} = 3$ and $a_{22} = 4$

value of game (V) $= \dfrac{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}{(a_{11} + a_{22}) - (a_{21} + a_{12})}$

$\Rightarrow \dfrac{20 - 3}{9 - 4} = \dfrac{17}{5} = 3.4$

(2) Solve the following games -

		Player B			maxmin
		B_1	B_2	B_3	
Player A	A_1	$\boxed{1}$	3	$\boxed{1}$	1 ←
	A_2	0	-4	~~3~~ 3	-4
	A_3	$\boxed{1}$	$\boxed{5}$	-1	-1
	minimax	1 ↑	5	1 ↑	

⇒ maxmin = minimax ⇒ $\underline{V} = \overline{V} = 1$

Saddle Point = (A_1, B_1) and (A_1, B_3)

Value of game = 1

optimal strategy of player A = A_1.
Optimal strategy of player B = B_1, B_3

(3) For what value of λ, ________ the game with the following matrix is strictly determinable.

		B_1	B_2	B_3	maxmin
Player A	A_1	λ	6	2	
	A_2	-1	λ	-7	
	A_3	-2	4	λ	
	minimax				

Here, on putting value of λ -1,

* **Saddle point** – A saddle point of a pay-off matrix is that position in the pay off matrix where maximum of row minima is equal to the minimum of column maxima.

- When the maxmin equal to minimax, the value of the game corresponding to pure strategies are called **optimum strategies**.
- The value of game is denoted by v.

* **Fair Game** – $\underline{v} = \overline{v} = v = 0$

* **Strictly Determinable Game** – $\underline{v} = \overline{v} = v$

When maxmin ≠ minimax, then pure strategy fails and this condition is of mixed strategy.

Questions

(1)

Player B

		B_1	B_2	B_3	max (min)
Player A	A_1	20	15	22	15
	A_2	$\boxed{35}$	$\boxed{45}$	$\boxed{40}$	35 ←
	A_3	18	20	25	18
min (max)		35	45	40	

Date - 5/April/2024

Graphical Method (2xn) or (nx2) order

(1) Player A

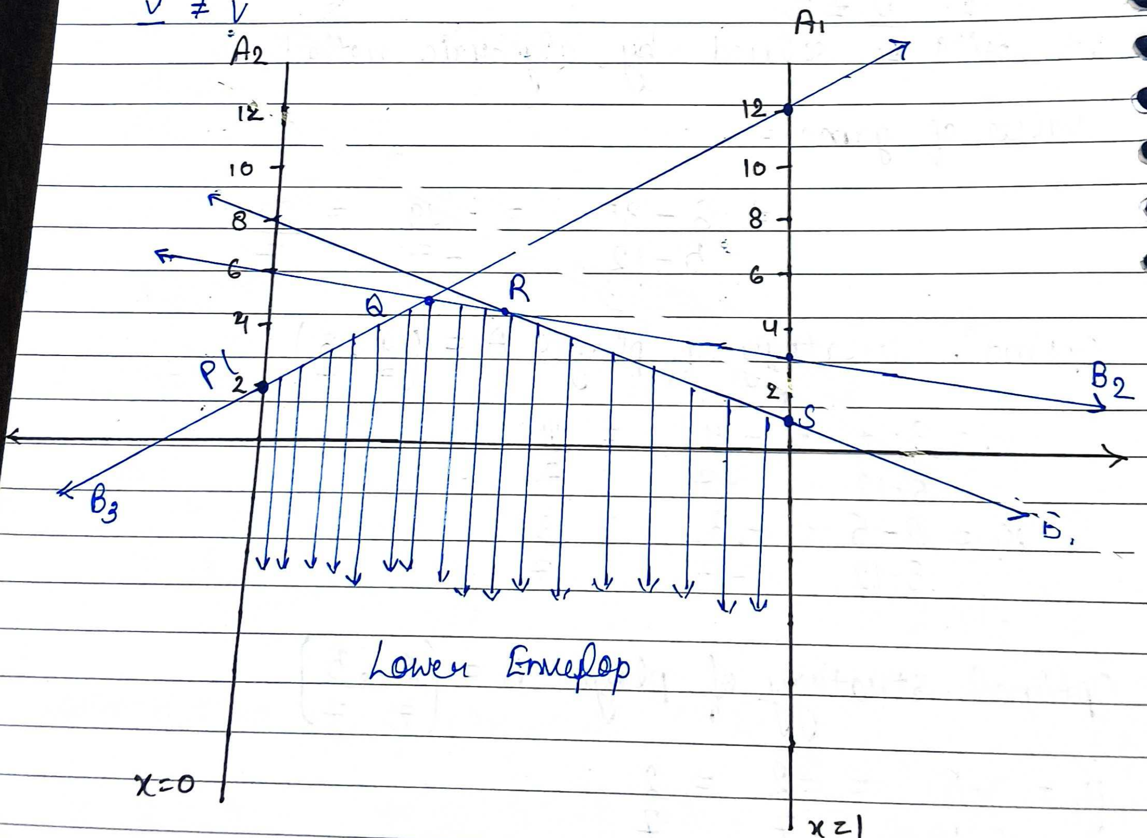
$$\begin{array}{c c} & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 1 & 3 & 12 \\ 8 & 6 & 2 \end{bmatrix} \end{array}$$

When 2 Rows are given

Solution

	B_1	B_2	B_3	max(min)
A_1	1	3	12	1
A_2	8	6	2	2 ✓
min(max)	8	6 ↑	12	

$v \neq \bar{v}$



max(min) = Q

Q is the point of intersection b/w B_2 and B_3

$$y_2 = \frac{5-3}{9-4} = \frac{2}{5} = 0.4$$

Solve the game with 2x2 pay-off matrix-

$$\begin{array}{c|cc|c} & B_1 & B_2 & \\ A_1 & 2 & 5 & 2 \\ A_2 & 7 & 3 & 3 \leftarrow \\ \hline & 7 & 5 & \\ & \uparrow & \uparrow & \end{array}$$

Solution- Here $a_{11} = 2$, $a_{12} = 5$, $a_{21} = 7$, $a_{22} = 3$

$\therefore \; \underline{v} \neq \overline{v}$

It will be solved by algebraic method

Value of game =

$$\Rightarrow \frac{6-35}{5-12} = \frac{-29}{-7} = \frac{29}{7}$$

Optimal strategy of player $A = \left(\frac{4}{7}, \frac{3}{7}\right)$

$$x_1 = \frac{3-7}{5-12} = \frac{-4}{-7} = \frac{4}{7}$$

$$x_2 = \frac{2-5}{5-12} = \frac{-3}{-7} = \frac{3}{7}$$

Optimal strategy of player $B = \left(\frac{2}{7}, \frac{5}{7}\right)$

$$y_1 = \frac{3-5}{5-12} = \frac{-2}{-7} = \frac{2}{7}$$

$$y_2 = \frac{2-7}{5-12} = \frac{-5}{-7} = \frac{5}{7}$$

* Algebraic Method (Analytical Method) –
Used in mixed strategy game when pay-off matrix is of (2X2) order.

Formula – value of game $(v) = \dfrac{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

Optimal strategy of player A = (x_1, x_2)

where, $x_1 = \dfrac{a_{22}-a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

$x_2 = \dfrac{a_{11}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

Optimal strategy of player B = (y_1, y_2).

where, $y_1 = \dfrac{a_{22}-a_{12}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

$y_2 = \dfrac{a_{11}-a_{21}}{(a_{11}+a_{22})-(a_{12}+a_{21})}$

Optimal strategy of player A = $(0.5, 0.8)$ or $\left(\frac{1}{5}, \frac{4}{5}\right)$

$x_1 = \dfrac{4-3}{9-4} = \dfrac{1}{5} = 0.5$

$x_2 = \dfrac{5-1}{9-4} = \dfrac{4}{5} = 0.8$

Optimal strategy of player B = $\left(\frac{3}{5}, \frac{2}{5}\right)$ or $(0.6, 0.4)$

$y_1 = \dfrac{4-1}{9-4} = \dfrac{3}{5} = 0.6$

value of game $(v) = \dfrac{a_{11} \cdot a_{22} - a_{21} \cdot a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$

$v = \dfrac{6-3}{5+4} = \dfrac{3}{9} = \dfrac{1}{3}$

Optimal strategy of player A = $\left(\dfrac{5}{9}, \dfrac{4}{9}\right)$

$x_1 = \dfrac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{21}+a_{12})}$

$x_1 = \dfrac{2+3}{5+4} = \dfrac{5}{9}$

$x_2 = \dfrac{a_{11} - a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})} = \dfrac{3+1}{5+4} = \dfrac{4}{9}$

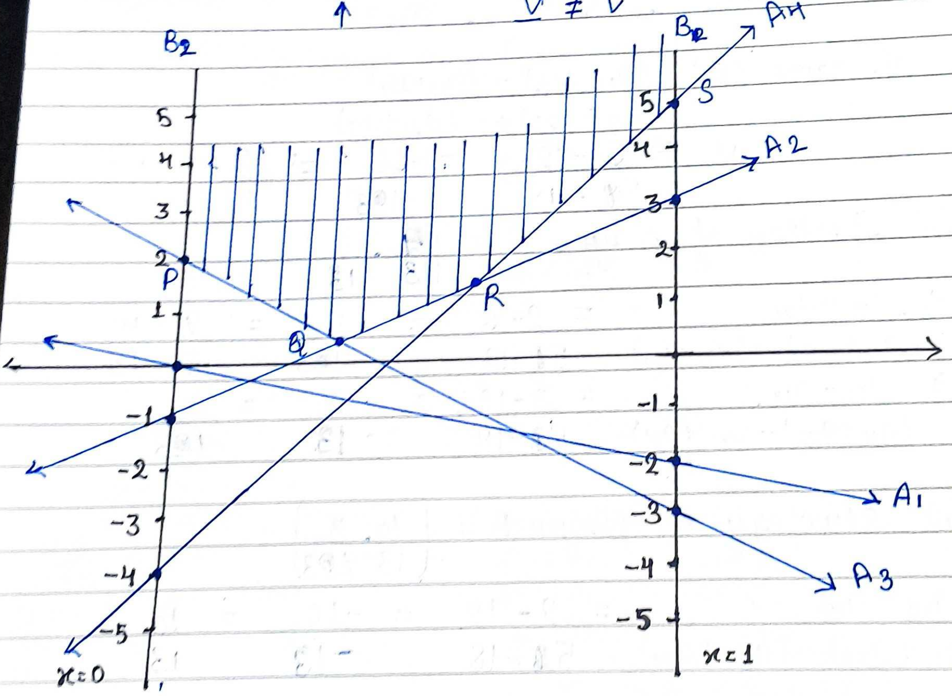
Optimal Strategy of player B = $\left(\dfrac{1}{3}, \dfrac{2}{3}\right)$

$y_1 = \dfrac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})} = \dfrac{2+1}{5+4} = \dfrac{3}{9} = \dfrac{1}{3}$

$y_2 = \dfrac{a_{11} - a_{21}}{(a_{11}+a_{22}) - (a_{21}+a_{12})} = \dfrac{3+3}{5+4} = \dfrac{6}{9} = \dfrac{2}{3}$

$$\begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} & \text{max (min)} \\ (2) \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} -2 & 0 \\ 3 & -1 \\ -3 & 2 \\ 5 & -4 \end{bmatrix} & \begin{array}{c} -2 \\ -1 \checkmark \\ -3 \\ -4 \end{array} \\ \text{min(max)} & \begin{array}{cc} 5 & 2 \\ & \uparrow \end{array} & \end{array}$$

$\underline{v} \neq \overline{v}$



min (max) = Q

Q is the intersecting point b/w A_2 and A_3

New matrix is

$$\begin{array}{cc} & \begin{array}{cc} B_1 & B_2 \end{array} \\ \begin{array}{c} A_2 \\ A_3 \end{array} & \begin{bmatrix} 3 & -1 \\ -3 & 2 \end{bmatrix} \end{array}$$

Here, $a_{11} = 3$, $a_{12} = -1$, $a_{21} = -3$ and $a_{22} = 2$

Then, new matrix will be

$$\begin{array}{c|cc|c} & B_2 & B_3 & \min(\min) \\ \hline A_1 & 3 & 12 & 3 \\ A_2 & 6 & 2 & 2 \\ \hline \min(m) & 6 & 12 & \end{array}$$

No saddle point are there in the matrix

$a_{11} = 3$, $a_{12} = 12$, $a_{21} = 6$, $a_{22} = 2$

value of game $(v) = \dfrac{(a_{11} \cdot a_{22}) - (a_{12} \cdot a_{21})}{(a_{11} + a_{22}) - (a_{21} + a_{12})}$

$v \Rightarrow \dfrac{6 - 72}{5 - 18} = \dfrac{-66}{-13}$ $\dfrac{33}{5}$

Optimal strategy of player A = $\left(\dfrac{4}{13}, \dfrac{9}{1}\right)$

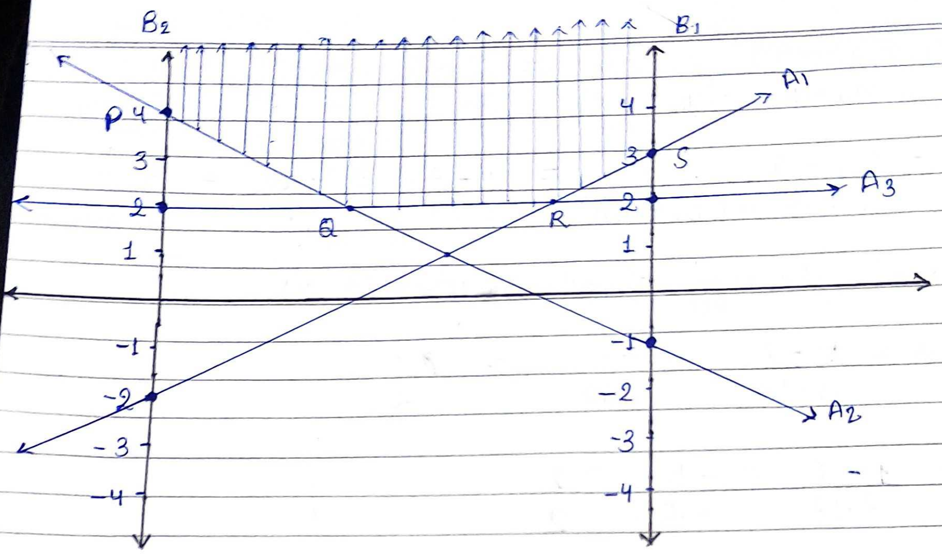
$x_1 \Rightarrow \dfrac{a_{22} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \dfrac{2 - 6}{5 - 18} = \dfrac{-4}{-13} = \dfrac{4}{13}$

$x_2 \Rightarrow \dfrac{a_{11} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \dfrac{3 - 12}{5 - 18} = \dfrac{-9}{-13} = \dfrac{9}{13}$

Optimal strategy of player B = $\left(\dfrac{10}{13}, \dfrac{3}{13}\right)$

$y_1 \Rightarrow \dfrac{a_{22} - a_{12}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \dfrac{2 - 12}{5 - 18} = \dfrac{-10}{-13} = \dfrac{10}{13}$

$y_2 \Rightarrow \dfrac{a_{11} - a_{21}}{(a_{11} + a_{22}) - (a_{12} + a_{21})} = \dfrac{3 - 6}{5 - 18} = \dfrac{-3}{-13} = \dfrac{3}{13}$



min (max) =

$$x_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{21}+a_{12})} = \frac{2-6}{3-13} = \frac{-4}{-10} = \frac{2}{5}$$

$$x_2 = \frac{a_{11} - a_{12}}{(a_{11}+a_{22}) - (a_{21}+a_{12})} = \frac{1-7}{3-13} = \frac{-6}{-10} = \frac{3}{5}$$

Optimal strategy of player B = (2, 2)

$$y_1 = \frac{2-7}{3-13} = \frac{-5}{-10} = \frac{1}{2}$$

$$y_2 = \frac{1-6}{3-13} = \frac{-5}{-10} = \frac{1}{2}$$

(2)

$$\begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \\ \hline A_1 & 3 & -2 & 4 & 5 \\ A_2 & -1 & 4 & 2 & 5 \\ A_3 & 2 & 2 & 6 & 10 \end{array}$$

$$\begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & 3 & -2 & 4 \\ A_2 & -1 & 4 & 2 \\ A_3 & 2 & 2 & 6 \\ \hline \text{Sum-} & 4 & 4 & 12 \end{array}$$

$$\begin{array}{c|cc|c} & B_1 & B_2 & \\ \hline A_1 & 3 & -2 & 1 \\ A_2 & -1 & 4 & 3 \\ A_3 & 2 & 2 & 4 \end{array}_{3\times 2}$$

or

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{max(min)} \\ \hline A_1 & 3 & -2 & -2 \\ A_2 & -1 & 4 & -1 \\ A_3 & 2 & 2 & 2 \leftarrow \\ \hline \text{min(max)} & 3 & 4 & \\ & \uparrow & & \end{array}$$

$\underline{v} \neq \overline{v}$

Date - 6/April/2024

→ Dominance Property ← (Matrix)

(1)

$$\begin{array}{c|c} & \text{Player B} \\ & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \text{Player A} \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} 1 & 7 & 2 \\ 6 & 2 & 7 \\ 5 & 1 & 6 \end{bmatrix} \end{array}$$

~~Cocd~~ Cancelling rows -

$$\begin{array}{cccc|c} & B_1 & B_2 & B_3 & \\ A_1 & 1 & 7 & 2 & 10 \\ A_2 & 6 & 2 & 7 & 15 \\ \cancel{A_3} & \cancel{5} & \cancel{1} & \cancel{6} & \cancel{12} \end{array}$$

Cancelling columns -

$$\begin{array}{cccc} & B_1 & B_2 & B_3 \\ A_1 & 1 & 7 & \cancel{2} \\ A_2 & 6 & 2 & \cancel{7} \\ & 7 & 9 & \cancel{9} \end{array}$$

New matrix -

$$\begin{array}{ccc|c} & B_1 & B_2 & \text{max(min)} \\ A_1 & 1 & 7 & 1 \\ A_2 & 6 & 2 & 2 \leftarrow \\ \text{min(max)} & 6 & 7 & \\ & \uparrow & \cancel{\uparrow} & \end{array} \quad (2\times 2)$$

$\underline{V} \neq \overline{V}$

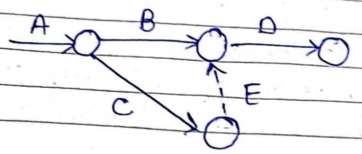
No saddle points are found

Here, $a_{11} = 1$, $a_{12} = 7$, $a_{21} = 6$, $a_{22} = 2$

$$\text{value of game} = \frac{1\times 2 - 7\times 6}{(1+2) - (7+6)} = \frac{2-42}{3-13} = \frac{-40}{-10} = 4$$

Optimal strategy of player A $= \left(\frac{2}{5}, \frac{3}{5}\right)$

★ Dummy activities-



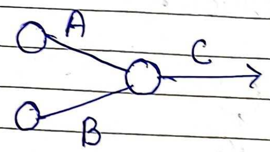
Here E is a dummy activity.

★ Event - The beginning and end point of an activity is called event.

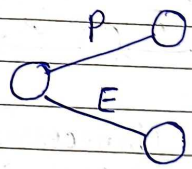
It has three type-

(i) Merge ~~ve~~ event
(ii) Burst event
(iii) Merge & Burst event

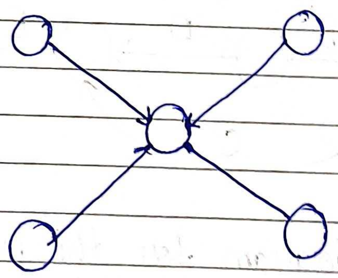
(i) Merge event -



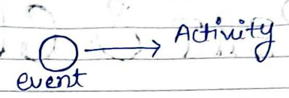
(ii) Burst event -



(iii) Merge & Burst Event



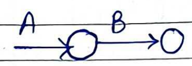
Sequencing - Maintain the relationship.



★ Activity - Any individual operations or jobs or task are called activities. There are four types of activity-

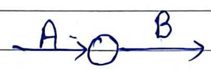
(i) Pre-decessor activity
(ii) Successor activity
(iii) Concurrent activity
(iv) Dummy activity

(i) Pre-decessor activity -



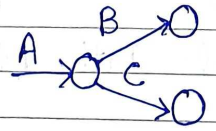
For a completion of event first we take a activity and for a completion of other event we will take B activity. Thus A is a pre-decessor activity of B.

(ii) Successor activity -



Which activity is running present time, called Successor activity. Here B is a successor activity.

(iii) Concurrent Activity -



When two activities are come from a event, these are called concurrent activity. Here B and C concurrent activity.

Date - 10/April/2024

UNIT - PERT & CPM

* Project Management - A project defines a combination of inter related activities which must be executed in a certain order. Before the entire task can be completed.

* Process of Project Management -

(1) Planning - The planning phases is started by splitting the total project into small project. (Analysis)

(2) Scheduling - Objective of the scheduling phases is to prepare a time chart showing the start and finish time for each activity as well as its relationship to other activities.

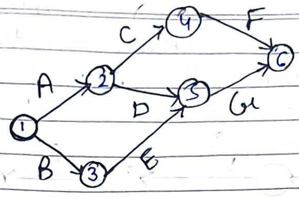
(3) Controlling - Controlling phases is the follow up of the planning and scheduling phases.

*Technique of Project Management -

(1) PERT - Project Evoluation Review Techniques

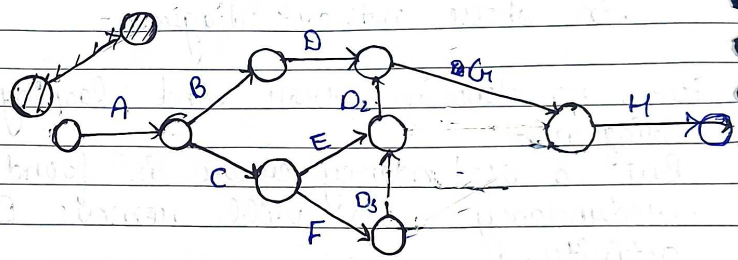
(2) CPM - Critical Path Method

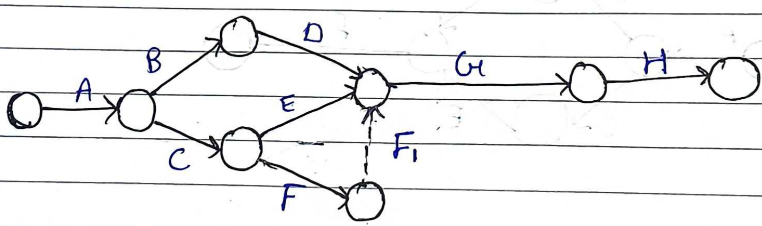
* Network Diagram - It is the graphical representation of logical and sequencially connected arrows and nodes representing activities and events of the project.



Que.

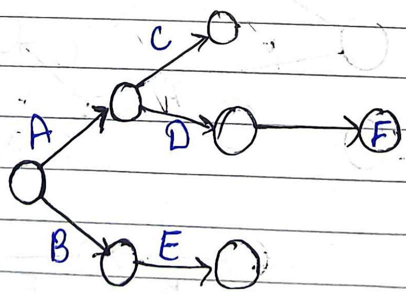
Acti.	Prede.
A	–
B	A
C	A
D	B
E	C
F	C
G	D, E, F
H	G





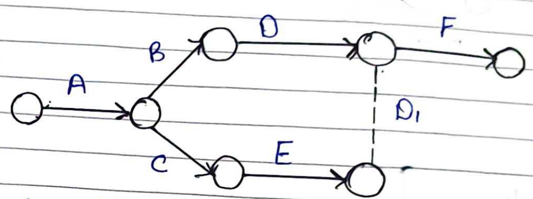
Que.

Act.	Pre
A	-
B	-
C	A
D	A
E	B
F	D
G	B, C



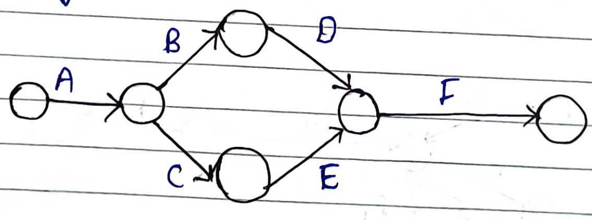
Activity	Pred. R
A	–
B	A
C	A
D	B
E	C
F	D, E

First we will draw a duplicate network diagram with the help of the given table-



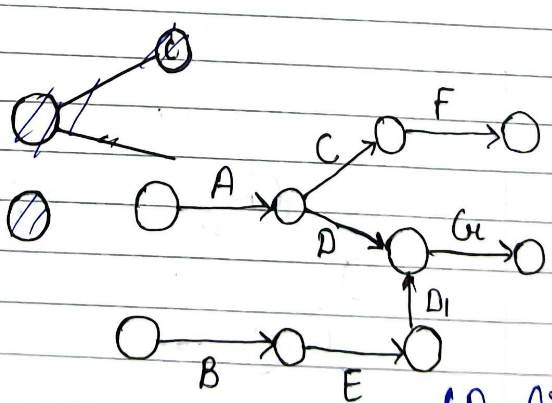
Now, we will check if there are possible errors in above network diagram-

Here, no dangling error and looping error in a diagram.
But a redundancy error is found, to remove redundancy, we will remove D_1 Dummy activity.



Que.

Pt	Product
A	–
B	–
C	A
D	A
E	B
F	C
G	D, E



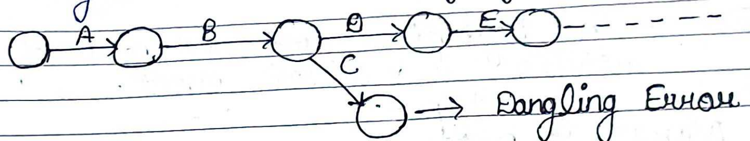
(Duplicate Network Diagram)

* Starting and Ending point should be unique.
* There can not be two activities going to a event ~~but not~~ starting from a same event.

* Common Error in Drawing Network –

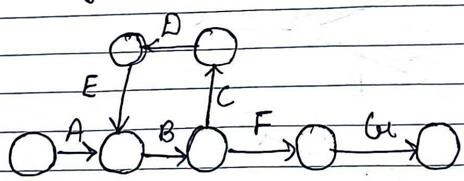
(1) Dangling Errors – To disconnect the activity before the completion of all activities in a network diagram is called Dangling Error.

Example



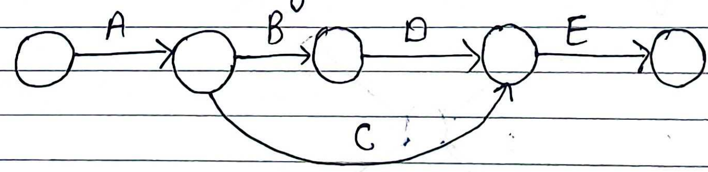
(2) Looping or Cyclic Errors – Drawing a endless loop in a network is known as looping errors or Cyclic Error.

Example,



(3) Redundancy Error – Unnecessary inserting a dummy activity in a network diagram is known as Redundancy Error.

Example,



Que. Draw the network diagram for the project whose activities and their precedence relationship as below –

Note 2: For finding Latest event of time, we will use Backward Pass Communication.

(1) For ending event of time $E = L$

(2) $(L_f)_{ij} = L_j$
(3) $(L_s)_{ij} = (L_f)_{ij} - D_{ij}$
(4) $(L_s)_{ij} = L_j - D_{ij}$

* **Total float** – Total float represents the maximum time within which an event can be delay without effecting the project.

$$(T_f)_{ij} = (L_s)_{ij} - (E_s)_{ij}$$
$$= (L_j - E_i) - D_{ij}$$

Free float $= (E_j - E_i) - D_{ij} = (F_f)_{ij}$

Free float = Total float $(T_f)_{ij}$ – Slack of Head event

Independent float = Free float – Tail slack

Independent float $\leq$ Free float $\leq$ Total float

Note- for critical event, slack time is zero.
Critical activities, ~~st~~ total float is zero.

Slak Time $= L_i - E_j$

Note 2: For finding Latest event of time, we will use Backward Pass Communication.

(1) For ending event of time $E = L$

(2) $(L_f)_{ij} = L_j$

(3) $(L_s)_{ij} = (L_f)_{ij} - D_{ij}$

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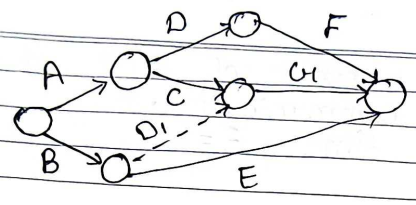
* **Total float** – Total float represents the maximum time within which an event can be delay without effecting the project.

$$(T_f)_{ij} = (L_s)_{ij} - (E_s)_{ij}$$
$$= (L_j - E_i) - D_{ij}$$



(Network Diagram)

Date - 18/April/2024

→← CPM (Critical Path Method) →←

(1) (i,j) = Activity (i,j), with tail event 'i' and head event 'j'. $(i) \xrightarrow{A} (j)$

(2) E_i = Earliest occurrence time of event 'i'.

(3) L_j = Latest allowable occurrence time of event 'j'.

(4) D_{ij} = Estimated completion time of activity

(5) $(E_s)_{ij}$ = Earliest starting time of activity ij.

(6) $(E_f)_{ij}$ = Earliest finishing time of activity ij.

(7) $(L_s)_{ij}$ = Latest starting time of activity ij.

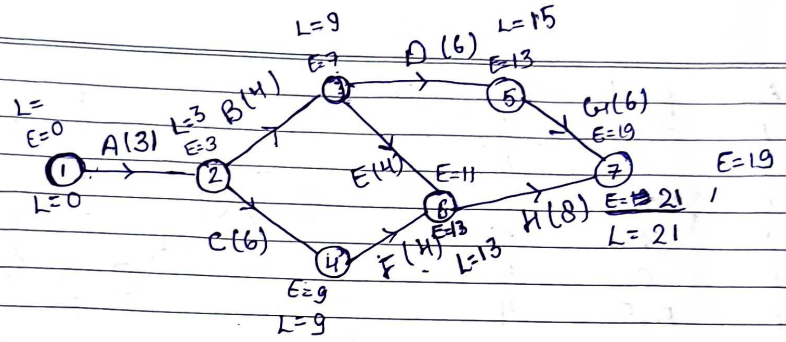
(8) $(L_f)_{ij}$ = Latest finishing time of activity ij.

Note - For finding earliest event of time we will use Forward Pass Computation.

Formulae - (1) $(E_s)_{ij} = E_i$

(2) $(E_f)_{ij} = (E_s)_{ij} + D_{ij}$

(3) $(E_f)_{ij} = E_i + D_{ij}$

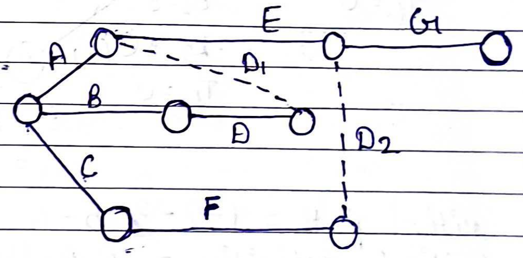


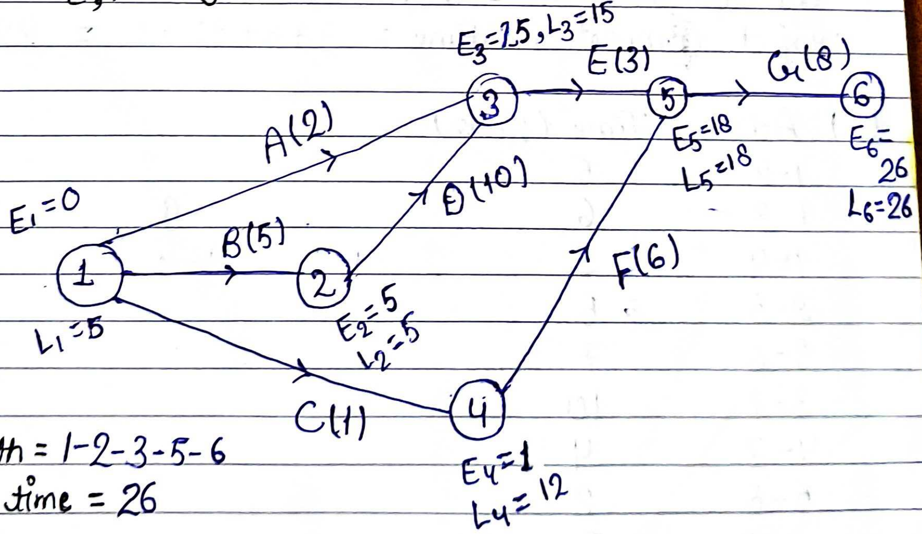
Critical Path = 1-2-4-6-7

Minimum time = 3+6+4+8 = 21 Days

(3)

Act.	pre.	Time
A	–	2
B	–	5
C	–	1
D	B	10
E	A,D	3
F	C	6
G	E,F	8

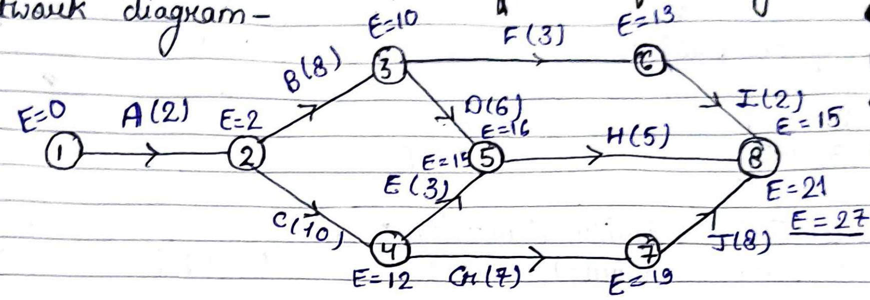




Critical path = 1-2-3-5-6
Minimum time = 26

Date - 22 | April | 2024

(1) Find out the critical path of the following network diagram -

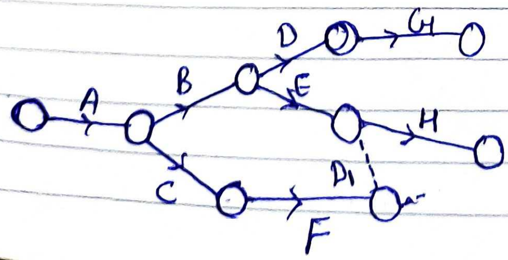


Critical Path = 1-2-4-7-8

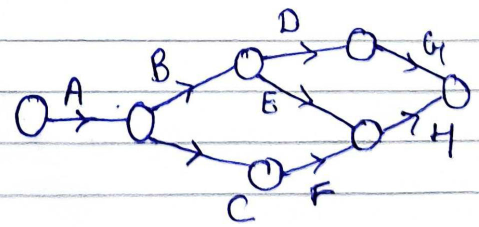
Minimum time = 2+10+7+8 = 27

(2)

Activity	Prede. Act	Time (Days)
A	-	3
B	A	4
C	A	6
D	B	6
E	B	4
F	C	4
G	D	6
H	E, F	8



Final N.D



Que. ______, is (the latest time by which an activity can be finished without delaying the completion of the project.

(1) LST (2) EST
(3) LRFI (4) EFT

Que. EST + activity duration =
(1) EFT (True) (2) LST (3) LFT (4) None of th

* Critical Event - The event with 0 slack time is called critical event.

* Critical Activity - Since the difference between the latest start time and earliest start time of an activity is usually called float. The activities with zero total float is called critical activity.

* Critical Path - The sequence of critical activities in a network is called a critical path. The critical path is the longest path from the starting event to ending event and define the minimum time required to complete the event / process.

Date- 26/April/2024

* PERT- It stands for Program Project Evaluation and Review Technique.
PERT involve uncertain into the project - completion time. It is an Event Oriented Network.

Note - CPM is an activity oriented activity.

(1) Optimistic Time (t_o) - This is the minimal possible time in which an activity can be completed most ideale completion.

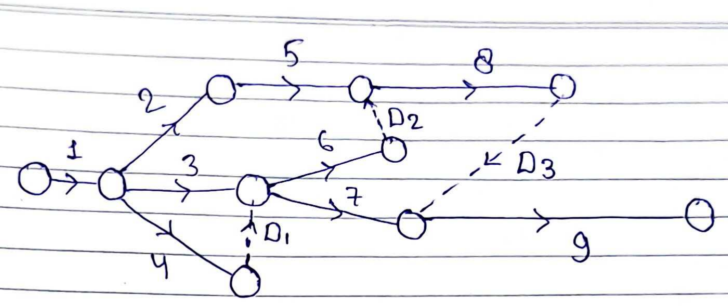
(2) Pessimistic Time (t_p) - This is the maximum time require to complete activity to under the words possible condition.

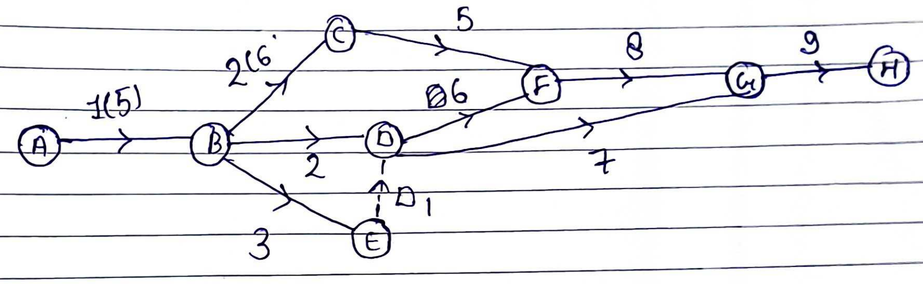
(3) Most likely time (t_m) - This is the time to require to complete the activity under natural working condition.

Exercise

(1) The time estimates (in weeks) for the activities of PERT. Network data is given below-

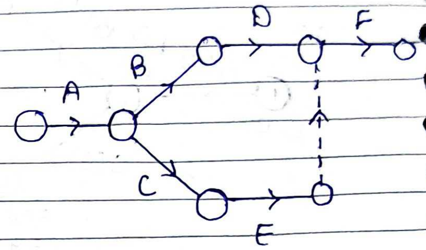
(a) Draw the project network.
(b) Determine the expected project length.
(c) Calculate the SD (Standard Deviation) and variable of project length.
(d) If the project due date is 19 weeks what is the probability of not meeting the due data.

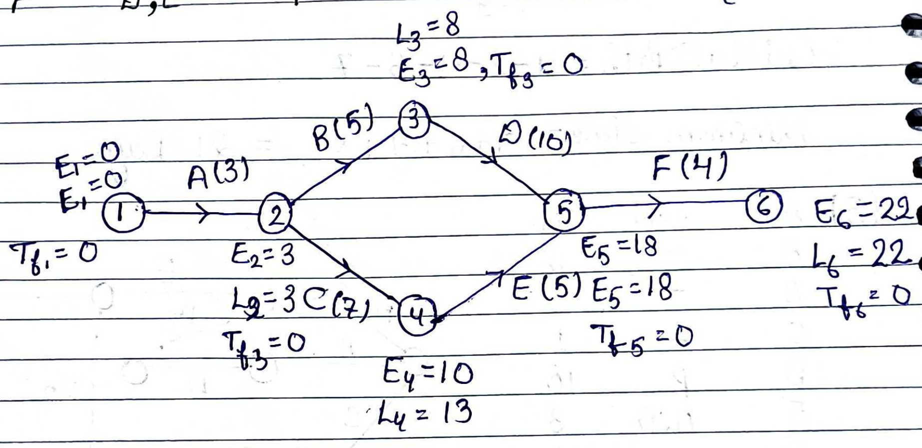

1
2
3
4
5
6
7
8
9
D1
D2
D3


A
B
C
D
E
F
G
H
1(5)
2(6
2
3
5
6
7
8
9
D1

(4)

Act.	prede.	Time (Weeks)
A	–	3
B	A	5
C	A	7
D	B	10
E	C	5
F	D, E	4





Critical path = 1-2-3-5-6
Critical activities = A-B-D-F
project completion time = 3+5+10+4 = 22 weeks

(5)

Act.	Time (Weeks)		
1-2	5		
1-3	6	7-9	6
1-4	3	8-9	4
2-5	5		
3-6	7		
3-7	10		
4-7	4 H		
5-8	2		
6-8	5		

Date -

Objective

(1) What is the ~~starting~~ earliest start time rule?

(a) It compares the activity start time for an activity sucessor

(b) It compares the activity end time for an activity predecessor.

(c) It direct when a project can start

(d) None of the above.

→ Completion event ~~of~~ in CPM N/D. → Collecter, event, node.

(2) Activities A, B and C are immediate predecessor of Y activity if the earliest finishing time for three act. 12, 15, 10, then what will E.S.T for y.

(a) 10 (b) 15 (c) 12 (d) Can't be determ.

(3) Activity P, Q and R instantly follow activity M and their Current Starting time 12, 18, 10 what is the L.F.T for activity M?

(a) 11 (b) 10 (b) 18 (d) Can't be deter-

(c) $Z = \frac{x-u}{6} = \frac{20-17}{3} = \frac{3}{3} = 1$

$P(Z_1) = 0.8413$

(d) $Z = \frac{x-u}{6} = \frac{19-17}{3} = \frac{2}{3} = 0.66$

$Z = 0.67$

$P(Z_{0.67}) = 0.7486$

$\overline{P} = 1-P = 1-0.7486 = 0.2514$

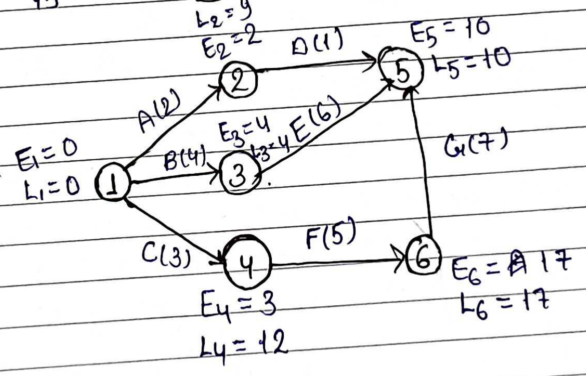
(e) The probability that the project will be completed on Schedule if the schedule completion time is 20 weeks.

(f) what should be the scheduled completion to 90%.?

Soln -

AC	to	tm	tp	expected time $t_e = \frac{t_o + }{6}$	variation $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$
1-2	1	1	7	2	1
[1-3]	1	4	7	4	1
1-4	2	2	8	3	~~0~~1
2-5	1	1	1	1	~~0~~
[3-5]	2	5	14	6	4
4-6	2	5	8	5	1
[5-6]	3	6	15	7	4

(a) Project Network -



(b) Expected Project Length = 4 + 6 + 7 = 17 weeks

(c) S.D. = σ^2 = 1 + 4 + 4 = 9

Variation = $\sqrt{\sigma^2} = \sigma = \sqrt{9} = 3$

$12y^6$

Questions

(1) $\vec{u} = 12x^6 \cdot y^6 \hat{i} + 3x^3y^3 z\hat{j} + 3x^2yz^2\hat{k}$

$$\text{div.}\vec{u} = \left(\hat{i}\cdot\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\cdot\frac{\partial}{\partial z}\right)\cdot\left(12x^6y^6\hat{i} + 3x^3y^3z\hat{j} + 3x^2yz^2\hat{k}\right)$$

$$= \frac{\partial}{\partial x}(12x^6y^6) + \frac{\partial}{\partial y}(3x^3y^3z) + \frac{\partial}{\partial z}(3x^2yz^2)$$

$$= 72x^5y^6 + 9x^3y^2z + 6x^2yz$$

(2) $\vec{u} = x^2yz\hat{i} + (3x+2y)z\hat{j} + 21z^2x\hat{k}$

$$\text{curl.}\vec{u} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2yz & 3xz+2yz & 21z^2x \end{vmatrix}$$

$$= \hat{i}(0-0) - \hat{j}(21z^2 - x^2y) + \hat{k}(3z + x^2z)$$

$$= \hat{i}(3x+2y) - (21z^2 - x^2y)\hat{j} + (3z - x^2z)\hat{k}$$

(3) $xy\frac{dy}{dx} = 1 + x + y + xy$

$$\frac{dy}{dx} = \frac{1}{xy} + \frac{1}{y} + \frac{1}{x} + 1$$

$$xy\frac{dy}{dx} = (1+x) + y(1+x) = (1+x)(1+y)$$

$$\frac{y\,dy}{1+y} = \left(\frac{1+x}{x}\right)dx$$

$xy\frac{dy}{dx} = 1 + x + y(1+x)$

$xy\frac{dy}{dx} = (1+x)\cdot(1+y) \Rightarrow \int \frac{y}{1+y}\, dy = \int \frac{1+x}{x}\cdot dx.$

$\int \frac{1-1+y}{1+y}\cdot dy = \int \frac{1}{x}dx + \int x\, dx$

$\int \frac{1+y}{1+y}dy - \int \frac{1}{1+y}\, dy = \int \frac{1}{x}dx + \int x\, dx$

$\int 1\cdot dy - \int \frac{1}{1+y}\, dy = \log x + \frac{x^2}{2} + c$

$\left[y - \log(1+y) = \log x + \frac{x^2}{2} + c \right]$