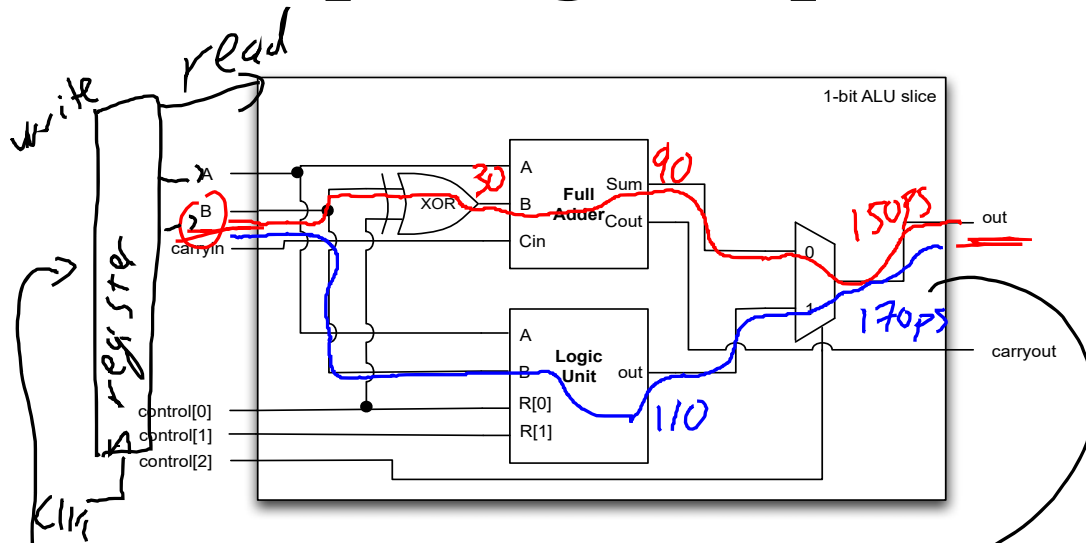


# Computing components from ALU1



What is the worst case propagation delay from B to out?  
(consider both arithmetic and logic operations)

- A: 120ps
- B: 150ps
- C: 160ps
- D: 170ps
- E: 190ps

XOR gate	In	Out	Delay
	A,B	out	<u>30ps</u>

Full Adder	In	Out	Delay
	A,B	Sum	<u>60ps</u>
	Cin	Sum	30ps
	A,B	Cout	90ps
	Cin	Cout	60ps

Logic Unit	In	Out	Delay
	A,B	out	110ps
	R	out	10ps

2-to-1 Multiplexor	In	Out	Delay
	A,B	out	<u>60ps</u>
	<del>A,B</del> S	out	80ps

2 pages for handout

# **More Finite State Machines: Multiplication as an Example**

Lab 4

Exam 1

# Learning Objectives

- Design a Finite State Machine that controls a datapath
- Use Register Transfer Language to describe how state is modified each clock cycle

# **Step 1: Understand your problem**

**We can perform binary multiplication using the same algorithm as decimal multiplication**

$$\begin{array}{r}
 1101 \quad (13) \\
 \times 1010 \quad (10) \\
 \hline
 1110000 \\
 110100 \\
 000000 \\
 + 110100 \\
 \hline
 1000010 \quad (130)
 \end{array}$$

a) Add 0000

b) Add 1101

c) Add 1010

**Multiplying two unsigned N-bit integers produces at most a 2N-bit integer**

$$\begin{array}{r} \phantom{0000} 1111 \quad (15) \\ \times \phantom{000} 1111 \quad (15) \\ \hline \phantom{0000} 1111 \\ \phantom{000} 1111 \\ \phantom{00} 1111 \\ + 1111 \\ \hline 11100001 \quad (225) \end{array}$$

$$\begin{array}{r} 23 \\ \times 2 \\ \hline 46 \end{array}$$

**Multiplication can be described as a series of shift and add operations**

$$\begin{array}{r}
 \overbrace{0000}^{2N} 1101 \quad (13) \quad \underline{A} \\
 \times 1010 \quad (10) \\
 \hline
 0000 \quad = 0 \\
 \\
 \underline{00011010} \quad A \ll 1 \\
 0000 \\
 + \underline{01101000} \quad A \ll 3 \\
 \hline
 10000010 \quad (130) \quad \underbrace{P = (A \ll 1) + (A \ll 3)}
 \end{array}$$

**We determine whether to add each iteration by one-bit from the multiplicand**

$$\begin{array}{r} 1101 \quad (13) \\ \times \quad \underline{1010} \quad (10) \\ \hline 0000 \\ 1101 \\ 0000 \\ + \underline{1101} \\ \hline 10000010 \quad (130) \end{array}$$



**We can use bitshifting and bitmasks to extract one bit from the multiplicand for control**

$$\begin{array}{r}
 1101 \quad (13) \\
 \times 1010 \quad (10) \\
 \hline
 0000 \\
 1101 \\
 0000
 \end{array}$$

Handwritten notes and calculations:

- $B$
- $B \& 1$
- $B \gg 1$
- $B \& 1$
- $B \gg 1$
- $B \& 1$

Handwritten calculations for the right side:

- $1010$
- $\underline{0001}$
- $0000$
- $0101$
- $\underline{0001}$
- $0001$
- $0010$
- $\underline{0001}$
- $0000$

**You try another example**

$$\begin{array}{r} 1001 \quad (9) \\ \times 0011 \quad (3) \\ \hline \end{array}$$

# You try another example

$$\begin{array}{r} 1001 \quad (9) \\ \times 0011 \quad (3) \\ \hline 1001 \\ 1001 \\ 0000 \\ +0000 \\ \hline 00011011 \quad (27) \end{array}$$

0001  
0000

# Write some C code that can perform the shift-add multiply algorithm

```

      1101 (13)
x   1010 (10)
-----
    0000
   1101
  0000
+ 1101
-----
10000010 (130)
  
```

**A**

**B**

*unsigned - 32-bit*

*16-bit*

```

int shift_add_multiply(unsigned short X,
unsigned short Y) {
    unsigned int A = (unsigned int) X;
    unsigned short B = Y;
    unsigned int P = 0;
  
```

return P;

}

**P**

```

i n t  s h i f t _ a d d _ m u l t i p l y ( u n s i g n e d  s h o r t  X ,  u n s i g n e d  s h o r t  Y )  {
    u n s i g n e d  i n t  A  =  ( u n s i g n e d  i n t )  X ;
    u n s i g n e d  s h o r t  B  =  Y ;
    u n s i g n e d  i n t  P  =  0 ;

    w h i l e  ( B  ! =  0 )  {
        i f  ( ( B  &  1 )  ==  1 )  {
            P = P + A;
        }
        B  =  B  >>  1 ;
        A  =  A  <<  1 ;
    }
    r e t u r n  P ;
}

```

0011

0001

0000

## **Step 2: Understand the datapath**

# Identify needed state components from variables (what state is stored?) X

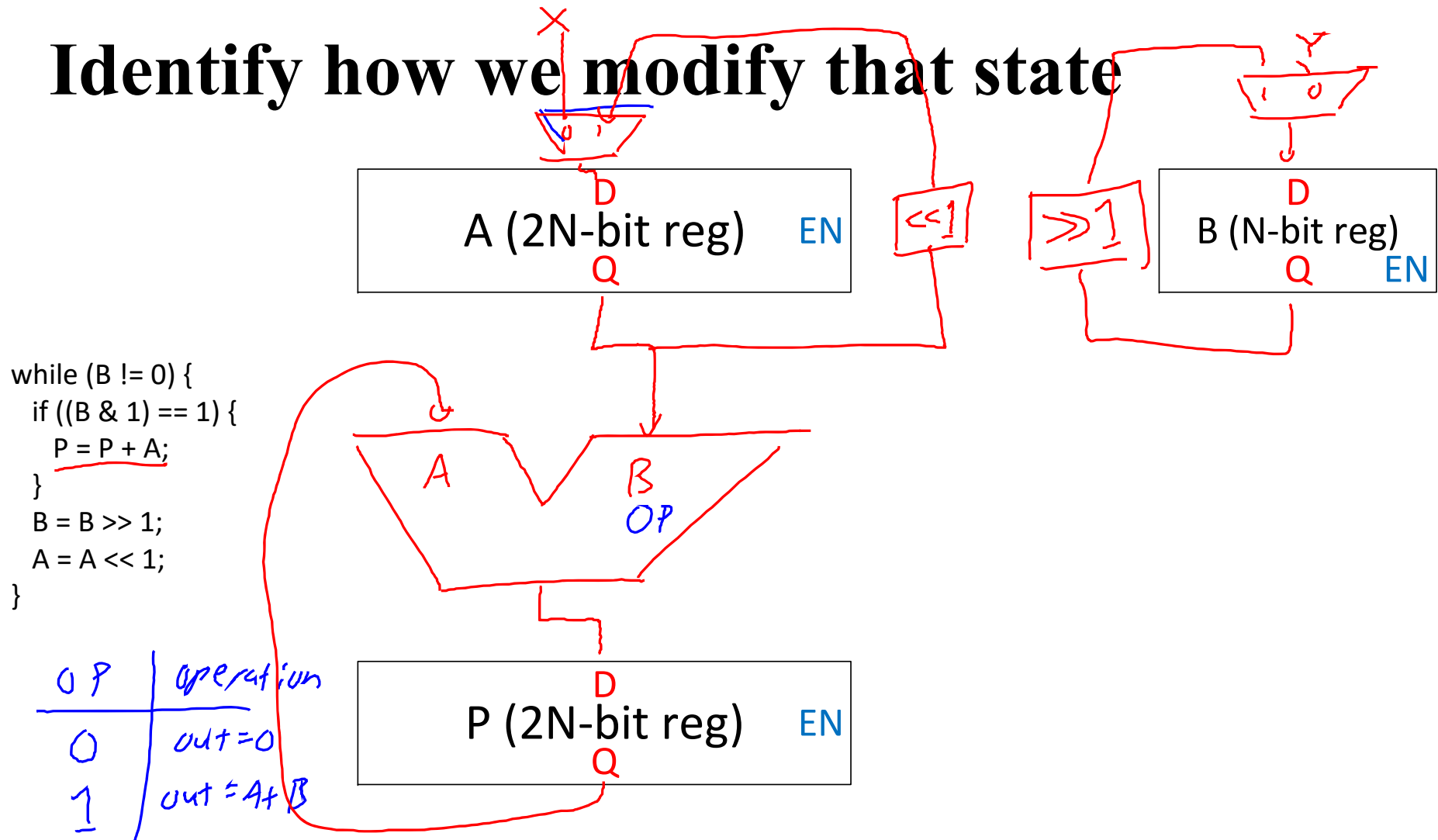
```
int shift_add_multiply(...) {  
    unsigned int A = (unsigned int) X;  
    unsigned short B = Y;  
    unsigned int P = 0;  
  
    return P;  
}
```

reg A (2N-bits)

reg B (n-bits)

reg P (2N-bits)

# Identify how we modify that state

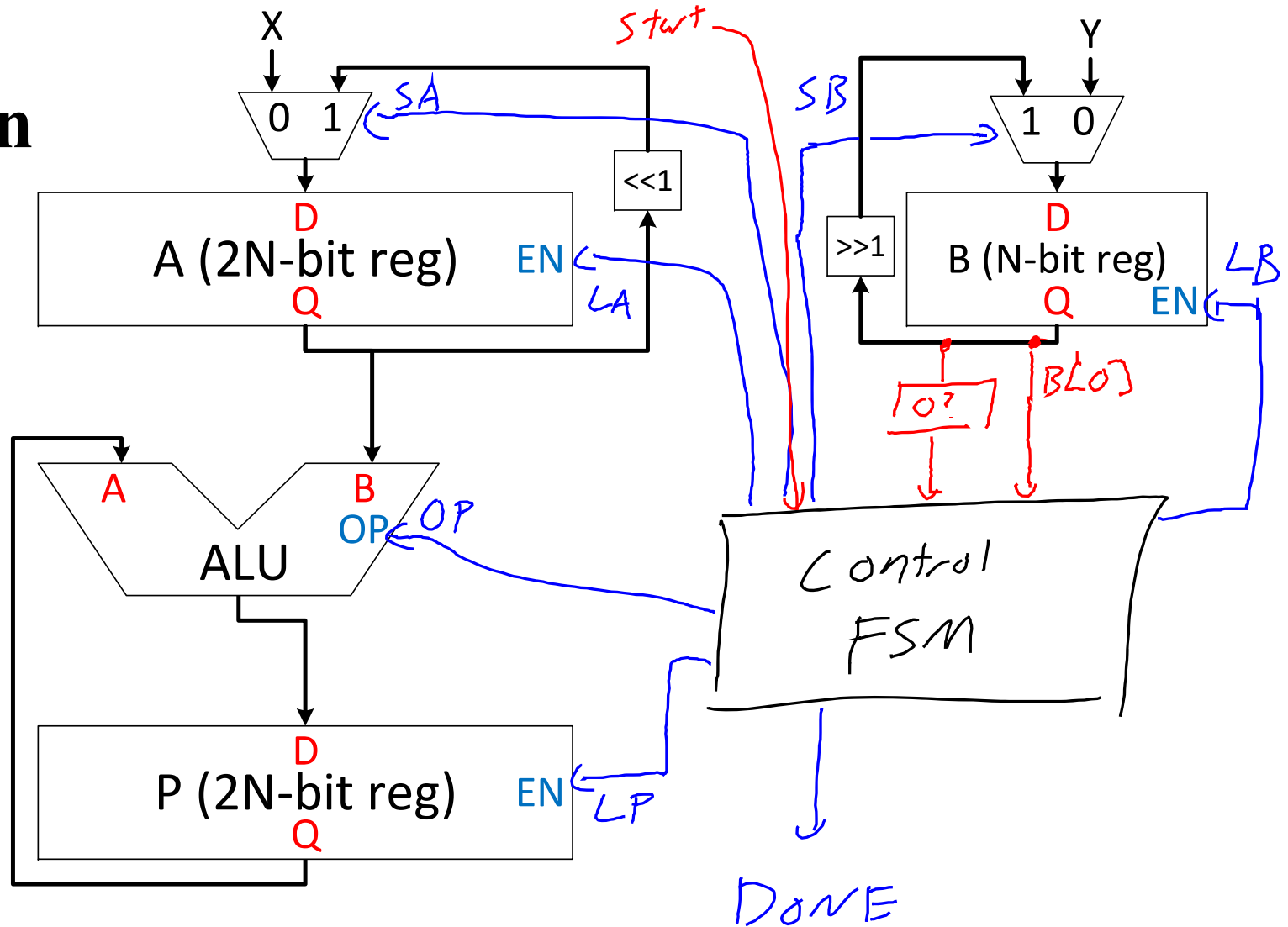





# Define specification for control FSM

```

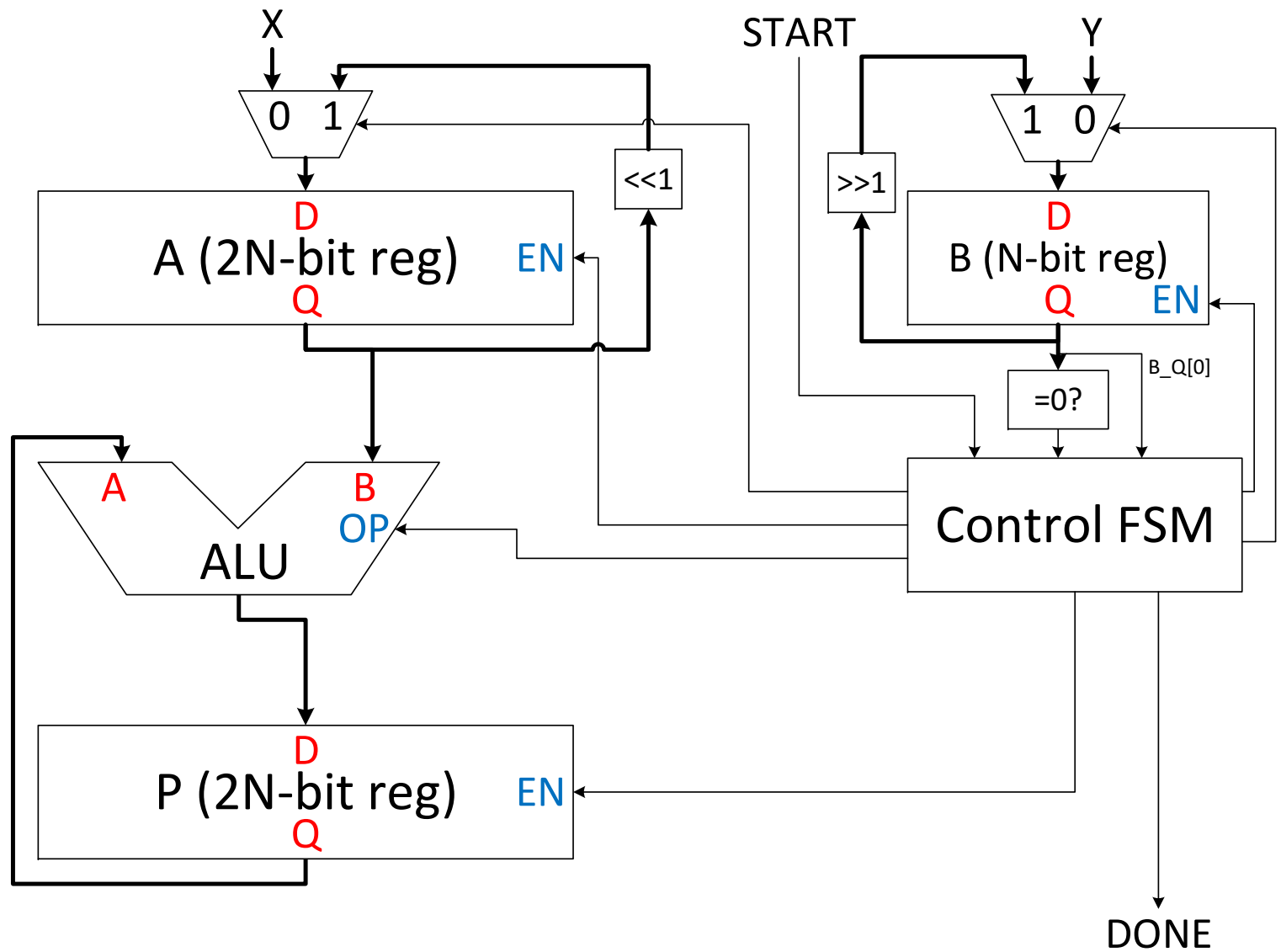
while (B != 0) {
  if ((B & 1) == 1) {
    P = P + A;
  }
  B = B >> 1;
  A = A << 1;
}
  
```



# Control FSM has 3 inputs and 7 outputs

- 
- B is zero (Z)
  - B\_Q[0] is 1 (B[0])
  - START
  - Load A (LA)
  - Shift A (SA)
  - Load B (LB)
  - Shift B (SB)
  - Add when 1/Zero when 0 (OP)
  - Load P (LP)
  - DONE

# Control and Datapath



**Step 3: Design your FSM  
(don't forget timing!)**

```

int shift_add_multiply(...) {
    unsigned int A = (unsigned int) X;
    unsigned short B = Y;
    unsigned int P = 0;

```

Init/Load  


---

 $A = X, B = Y,$   
 $P = 0$

COMPARE  


---

```

    while (B != 0) {
        if ((B & 1) == 1) {
            P = P + A;
        }
        B = B >> 1;
        A = A << 1;
    }
    return P;
}

```

Shift  


---

 $A = A \ll 1,$   
 $B = B \gg 1$

Shift-add  


---

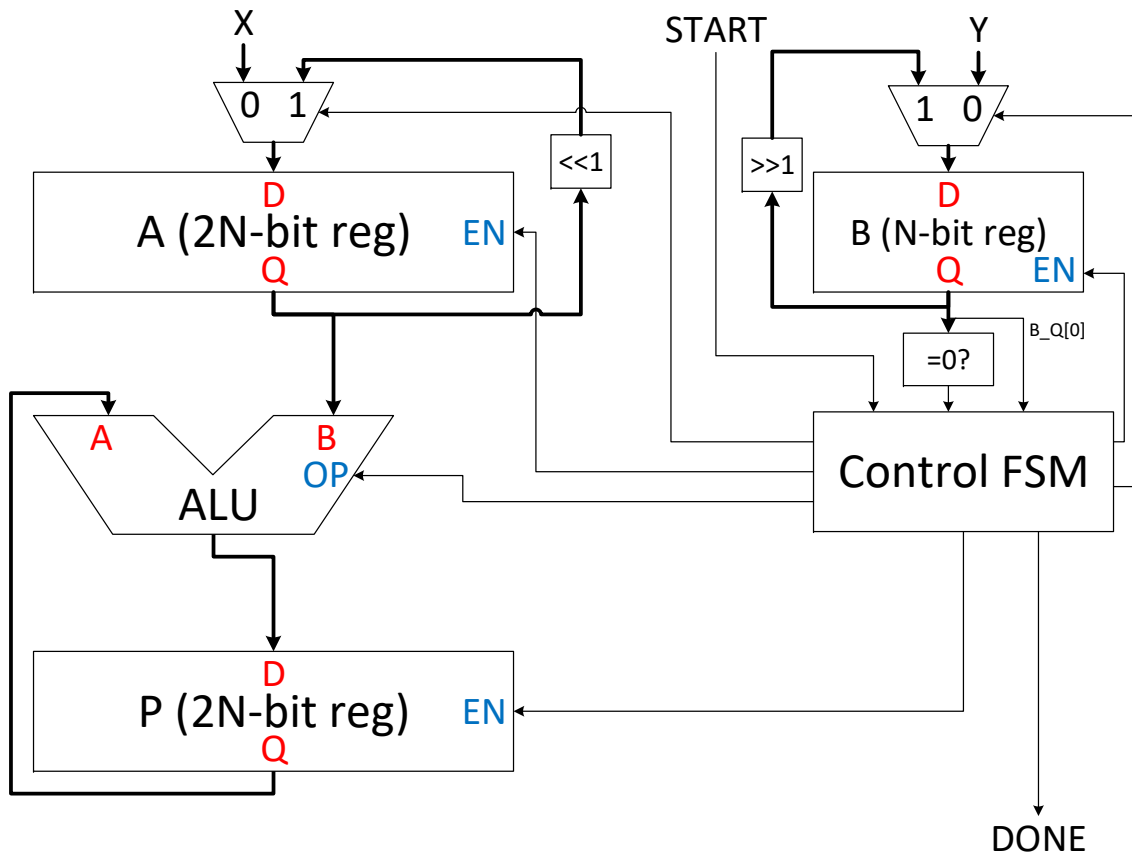
 $A = A \ll 1, B = B \gg 1$   
 $P = P + A$

Done  


---

 $DONE = 1$

# Register transfers show source state elements, operations, and destination state elements



$A = X$

$B = Y$

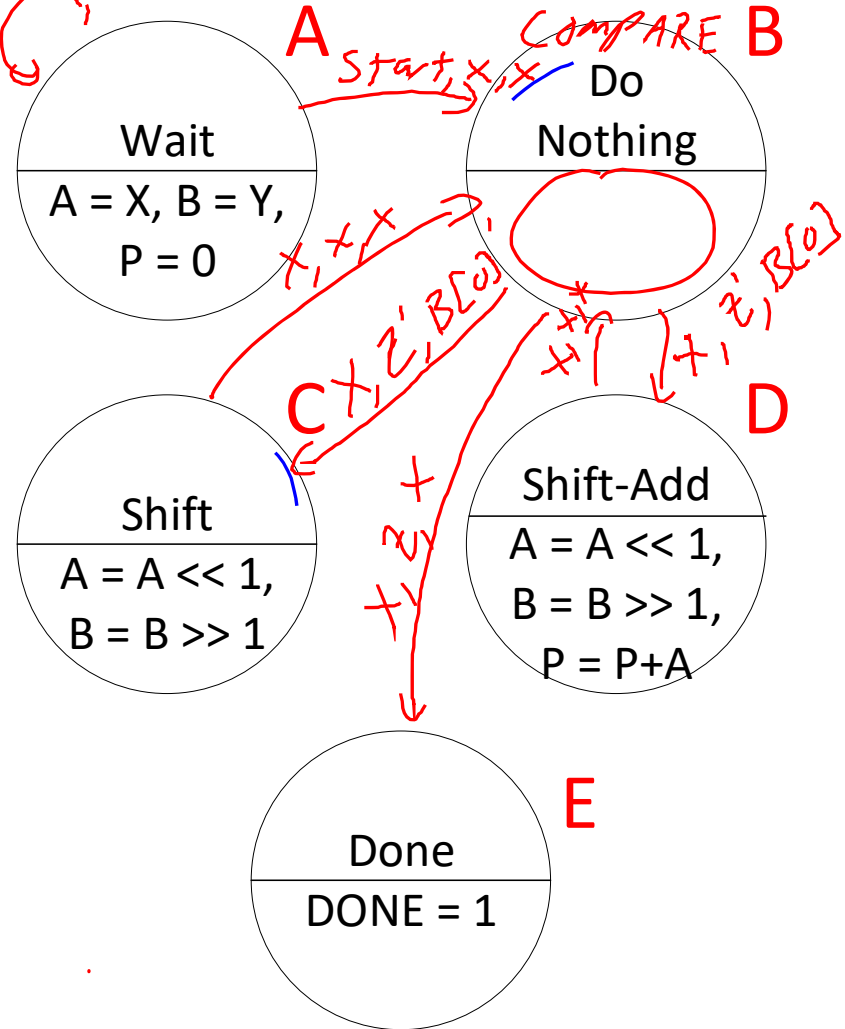
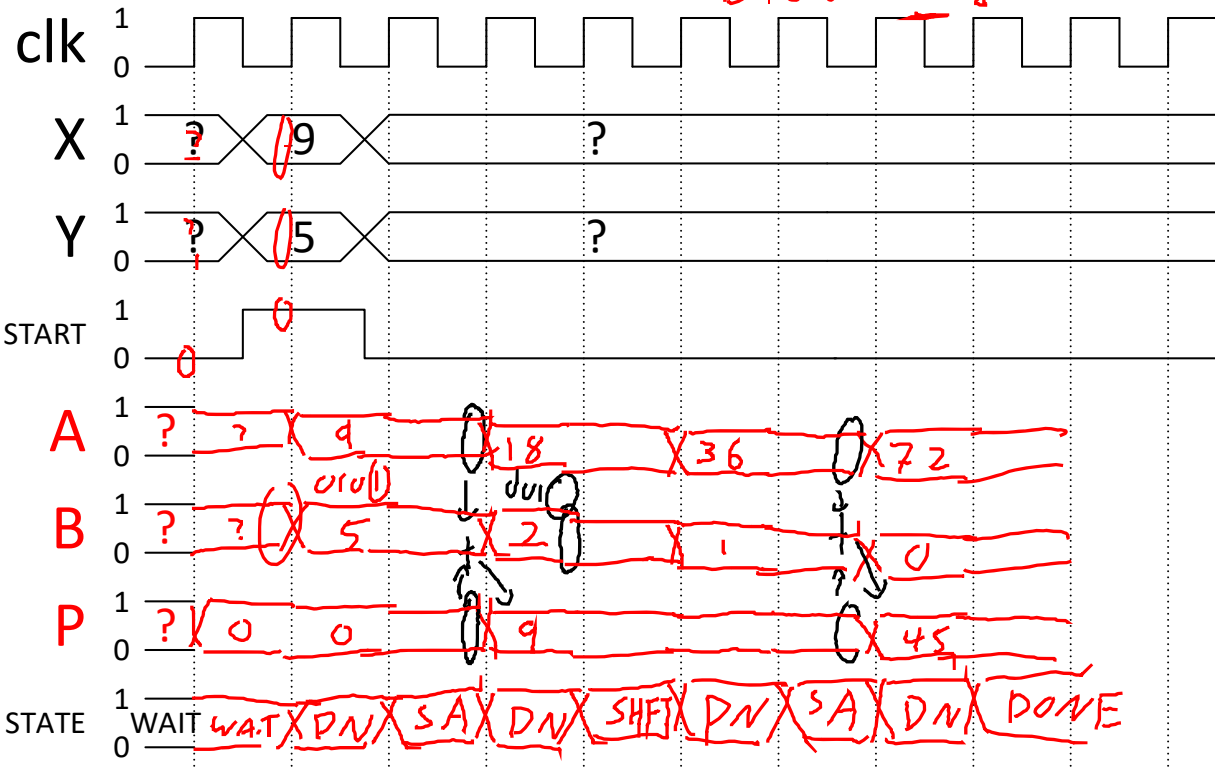
$P = 0$

$A = A \ll 1$

$B = B \gg 1$

$P = P + A$

Start, Z, BC03 A: 0101001000 Start, X, X  
 B: 00000000



# Translate register transfers into control signals

