CS173: Discrete Mathematical Structures, Spring 2008

First Midterm — February 14, 2008, 9:30am-10:45am

Name: Seong Hwang	Net ID: shwang30

Circle your discussion section.

Time	Th 12:00-12:50	Th 1:00-1:50	Th 2:00-2:50	Fri 10:00-10:50	Fri 11:00-11:50
Discussion Section	1A Aparna A-H	2A Tracy A-Kr	3A Mo A-Me	4A Dan M. A-Lin	5A Dan M. A-N
	1B Dan S. I-Z	2B Nana Ku-Z	3B Lucas Mi-Z	4B Yuzi Liu-Z	5B Dan S. O-Z

Instructions

1. This is a closed-everything exam. No notes or electronics of any kind are allowed.
2. Print your full name and your NetID in the boxes above.
3. Print your name at the top of every page.
4. Please write clearly and legibly. If we can't read your answer, we can't give you credit.
5. You have 75 minutes to complete the exam, plan accordingly. Do not spend too much time on any one of the problems. Problems do not necessarily appear in order of difficulty.
6. You may write "I Don't Know" on any problem to receive 25% of the total credit. However, you must ONLY write "I Don't Know". If you write anything else, it will be graded as though it is a solution.

#	0	1-6	7	8	9	10	11	12	13	Total
Max	5	30	8	12	10	10	15	15	15	120
Score	3	15	7	7.5	10	1	10	4	12	69.5

Course Questions

0. The easy problem:

   (a) **[1 point]** How many homeworks are you allowed to drop?

   2

   ✓

   (b) **[1 point]** What is the penalty for submitting late homework?

   Score of 0 ✓

   (c) **[1 point]** What percentage of your course grade is devoted to discussion section participation?

   ✓

   10%

   (d) **[2 points]** Circle the sources of information that you are *not* allowed to use while working on CS173 homework.

      i. The CS173 website.
      ii. Discussions with your friend Sarah who took CS173 last semester.
      iii. A textbook (other than the course text by Ensley and Crawley).
      iv. A brainstorming session with Josh, who is also taking CS173.
      v. A draft of the homework solutions that Josh produced after the brainstorming session.
      vi. Your section leader.
      vii. Wikipedia (a freely available online encyclopedia).

Multiple Choice (5 points each)

Indicate your answers by circling the correct one. **Each question has exactly one correct answer.** Read everything carefully.

1. What is the size of $\mathcal{P}(\mathcal{P}(\varnothing)) \times \mathcal{P}(\{1, 2, 3\})$? (Recall that we use $\mathcal{P}(A)$ to denote the powerset of a set A, and $\varnothing$ to denote the empty set.)

   (a) 64
   (b) 32
   (c) 16
   (d) 8
   (e) 6
   (f) 4
   (g) 2
   (h) 1
   (i) 0
   (j) None of the above.

2. *Read carefully.* Let $X = \{4, 8, 15, 16, 23, 42\}$. Which of the following is a partition of X?

   (a) $(\{4, 16\}, \{8, 15, 23\}, \{42\})$
   ✓ (b) $\{\{4, 16, 42, 4\}, \{15\}, \{8, 23\}\}$
   (c) $\{(4, 8, 16), (15, 23, 42)\}$
   (d) $\{4, 8, 15, 16, 23, 42\}$
   (e) None of the above.

3. Which of the following is the converse of the statement, "You are crazy if you are taking CS 173."

   ✓ (a) If you are crazy, then you are taking CS173.
   (b) If you are not taking CS 173, then you are not crazy.
   (c) If you are not crazy, then are you not taking CS 173.
   (d) You are taking CS173 and you are not crazy.
   (e) Butterflies are pretty.

4. Let A be the set of students and B be the set of classes that Albert takes. Consider the following predicates:

   - $P(x)$ means "x is stinky"
   - $Q(x)$ means "x has showered"
   - $R(x, y)$ means "x sits next to Albert in class y"

   Which of the following is equivalent to the statement "In every class Albert takes, some student that has not showered and is stinky sits next to Albert."

   (a) $\exists y \in B, \forall x \in A, R(x, y) \vee P(x)$

   (b) $\exists y \in B, \forall x \in A, R(y, x) \vee P(x)$

   (c) $\forall y \in B, \exists x \in A$, if $R(x, y)$ then $\neg Q(x)$

   (d) $\exists y \in B, \forall x \in A, R(x, y) \vee P(x) \vee \neg Q(x)$

   (e) $\exists y \in B, \forall x \in A, R(y, x) \vee P(x) \vee \neg Q(x)$

   (f) $\forall y \in B, \exists x \in A, R(y, x) \wedge P(x) \wedge \neg Q(x)$

   (g) $\forall y \in B, \exists x \in A, R(x, y) \wedge P(x) \wedge \neg Q(x)$

   (h) None of the above.

5. Which of the following is not a *member* of the set $\mathcal{P}(\{4, 5\}) \times \{1, 2, 3\} \times \{\pi, e\}$?

   (a) $(4, \{1, \pi\})$ ✓

   (b) $(\emptyset, 2, \pi)$

   (c) $(\{4, 5\}, 2, e)$

   (d) $(\{4\}, 3, \pi)$

   (e) None of the above.

   ∅ 4 5 4 5

6. Let $A(x)$ be the predicate "x is honest", and let $B(x)$ be the predicate "x is a politician". Let S be the set of people in the world. Which of the following is equivalent to the *negation* of the statement: $\forall x \in S$, if $\neg A(x)$ then $B(x)$?

   (a) All honest politicians are people.

   (b) Some dishonest person is not a politician.

   (c) Some honest person is not a politician.

   (d) Every politician is honest.

   (e) No politician is honest.

   (f) There is an honest politician.

   (g) There is a dishonest politician.

   (h) None of the above.

   $\exists x \in S$ st Ax & $\neg Bx$

Short Answer

7/8 7 5/12

Write your solutions in the space provided.

7. [8 points] Fill in the blanks.

   (a) Every proof by induction is an argument that there can be no min) counter example (2 words).

   (b) A proof by induction consists of three parts: the introduction (e.g. "The proof is by induction on n."), the Base case (2 words), and the inductive step (2 words).

   (c) The inductive hypothesis (2 words) allows us to assume that theorem holds for all smaller inputs than the one we are presently considering.

8. [12 points] Express the following sets as simply as you can. **Use explicit lists for finite sets.** For infinite sets, use set builder notation (both "form description" and "property description" answers are acceptable).

   Let $A = \{n \in \mathbb{Z} : n \text{ is even}\}$, $B = \{n \in \{2, 3, 4, \ldots\} : n \text{ is prime}\}$, $C = \{n^2 : n \in \mathbb{Z}\}$, and $D = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

   (a) $B \cap C$

   disjointed (there's no $B \cap C$ since n^2 can't be a prime number)

   3

   (b) $C - A$

   $\{n^2 : n \in \mathbb{Z} \text{ and } n \text{ is odd}\}$

   3

   (c) $(B \cap D) \times (A \cap C \cap D)$ o too

   $\{2, 3, 5, 7\} \times \{4\}$

   1.5/3 $\{(2,4), (3,4), (5,4), (7,4)\}$

   (d) $\mathcal{P}(D \cap C \cap A) \cup \mathcal{P}((D \cap C) - A)$

   $P\{4\}$ $\{4, 9\} - A$

   $(\emptyset, 4)$ $P\{9\}$ $(\emptyset, 9)$

   0/3 $\{\emptyset, 4, 9, (\emptyset, 4), (\emptyset, 9)\}$

9. **[10 points]** Count the number of integers in $\{0, 1, 2, \ldots, 500\}$ which are divisible by 4 or 5.

$\frac{500}{5} + 1 = 101$ $\frac{500}{20} + 1 = 26$

$\frac{500}{4} + 1 = 126$

$101 + 126 - 26 = \boxed{201}$ ✓

10. **[10 points]** Let $n \geq 1$ be an integer and let $U = \{1, 2, \ldots, n\}$. Prove or disprove:

(a) $\forall (x, y) \in U \times U, \quad ((x - y) \in U) \vee ((y - x) \in U)$

0/5

Since $\forall (x,y) \in U \times U$, x and y can be any number from 1 to n.
$(x-y) \in U$ and $(y-x) \in U$ show that (any number from 1 to n) $\in U$.
thus, it holds.

why does this prove it?

(b) $\forall A \in \mathcal{P}(U), \exists B \in \mathcal{P}(U), \quad (A \cup B = U) \wedge (A \cap B = \emptyset)$

$P(U)$ consists of every possible subset of U. Thus, $\forall A \in P(U)$ states that $A \subseteq U$. Same goes for $\exists B \in P(U)$ that assumes $B \subseteq U$. Thus, since $\forall A \in P(U)$ and $\exists B \in P(U)$, $A \cup B \subseteq U$.

1/5

~~The~~ other way works too such that since $\forall A \in P(U)$, $U \subseteq A$ not true. thus as long as $\exists B \in P(U)$ $U \subseteq A \cup B$.

However, $(A \cap B = \emptyset)$ has to be $\emptyset \subseteq A \cap B$ such that $\emptyset$ is a subset of $A \cap B$ because $\emptyset$ is not the only subset of $A \cap B$.

?

Long Answer (15 points each)

$\frac{10}{15}$

Write your solutions in the space provided. (If you run out of room, you may continue your answer on another page, but please tell us where to look!)

11. Prove that for each integer $n \geq 0$, the following identity holds: $\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$.

$\sum_{k=0}^{n} 2^k = 2^{n+1} - 1$ for $n \geq 0$, $\forall n \in \mathbb{Z}$

Base case: when $n=0$, $\sum_{k=0}^{0} 2^k = 2^1 - 1$

$2^0 = 2 - 1$

$1 = 1$ So the identity holds.

I.S.: Let there be an integer $1 \leq m \leq n$ for $\forall m \in \mathbb{Z}$. what does this mean?

$\sum_{k=0}^{m} 2^k = \left(\sum_{k=0}^{m-1} 2^k\right) + 2^m$

Where is your induction hypothesis?

By induction hypothesis,

$\sum_{k=0}^{m} 2^m = \left(2^{(m-1)+1} - 1\right) + 2^m$

$= 2^m - 1 + 2^m$

$= 2(2^m) - 1$

$\sum_{k=0}^{m} 2^m = 2^{m+1} - 1$

Thus, by inductive steps, the identity holds

12. Let S be a subset of $\{1, 2, \ldots, 3n\}$ which contains $2n + 1$ numbers. Show that S contains 3 consecutive integers.

(* n has to be greater than 0
$n > 0$)

$\{1, 2, \ldots, 3n\}$ ($2n+1$ numbers) has 3 consecutive integers.

Base case: when $n = 1$, $S = \{1, 2, 3\}$, so it has 3 consecutive integers 1, 2, 3.

4

IS: Let $1 < m \leq n$ for $\forall m \in \mathbb{Z}$.

Let there be $2m+1$ numbers.

This means that no matter what m is, it is greater than 1 (by I.H.) This means that the set will always contain the base case where $m = 1$.

This proves that, by inductive step, subset S will always have 3 consecutive integers (as long as $n > 0$, $\forall n \in \mathbb{Z}$)

no! I can make an S which does not contain $\{1, 2, 3\}$!

13. Prove by induction that any positive integer can be written as a sum of *distinct* powers of 2. 'Distinct' means that each power of 2 appears at most once in the sum. For example:

~~10~~ 12

$$4 = 2^2 \qquad 17 = 2^4 + 2^0 \qquad 23 = 2^4 + 2^2 + 2^1 + 2^0 \qquad 173 = 2^7 + 2^5 + 2^3 + 2^2 + 2^0$$

In other words, prove that any positive integer can be written in binary!

$\forall k \in \mathbb{Z}$ where $\mathbb{Z}$ is positive

Base case: Let $k = 2^n$ where $n \geq 0$, $\forall n \in \mathbb{Z}$.

Let $k = 1$

$1 = 2^0$ so it holds ($n=0$)

Let $k=2$

$2 = 2^1$ so it holds ($n=1$)

Let $k=3$

$3 = 2^1 + 2^0$ so it holds.

Let $k=4$

$4 = 2^2$ so it holds ($n=2$)

Induction step: Let L be a positive integer when $4 < L \leq k$.

Why $\leq k$? what is k?

~~Suppose $L=5$.~~

should not be here

This L consists of $2^2 + 2^0$ where 2^2 is the greatest power of 2 that's also less than L.

So the gap is 1, when $L - 2^m = \text{gap}$ ($\forall m \in \mathbb{Z}$)

(where 2^m is the greatest power of 2 that's less than L)

~~Suppose $L = 7$~~

Right idea, but a proof cannot have ambiguous language...

Repeating what I just did, $L = 2^m + $ 'gap' when 2^m is the greatest power of 2 that's less than or equal to L.

This means that the 'gap' is $L - 2^m$ where gap $< 2^m$ because if the 'gap' is at least 2^m, than $L \geq 2^m + 2^m = 2^{m+1}$ which can be expressed with a single power of 2.

By induction hypothesis, the 'gap', which is $< 2^m$, can be expressed by previous numbers that are still powers of 2

Thus, By induction steps, the statement holds.

(scratch paper)

CS173: Discrete Mathematical Structures, Spring 2008

Second Midterm — April 1, 2008, 9:30am-10:45am

Name: Seong Hwang	**Net ID:** shwang30

Circle your discussion section.

Time	Th 12:00-12:50	Th 1:00-1:50	Th 2:00-2:50	Fri 10:00-10:50	Fri 11:00-11:50
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	1B Dan S. I-Z	2B Nana Ku-Z	3B Lucas Mi-Z	4B Yuzi Liu-Z	5B Dan S. O-Z

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#	1	2	3	4	5	6	7	8	9	Total
Max	10	10	10	10	10	15	10	15	10	100
Score	8	2	10	1.5	10	14	10	15	8	78.5

1. (10 points) Let $f(n) = 2n\log_2(2n)$ and let $g(n) = n\log_2 n$. Prove that $f(n) = \Theta(g(n))$.

1.) $f(n) = O(g(n))$

6

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{2n\log_2 2n}{n\log_2 n} \approx 2 < \infty \quad \therefore f(n) = O(g(n))$$

by L'hopital

show

2.) $f(n) = \Omega(g(n))$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \frac{2n\log_2 2n}{n\log_2 n} \approx 2 > 0 \quad \therefore f(n) = \Omega(g(n))$$

by L'hopital

show

This shows that f(n) and g(n) have a constant relation, 2, so $f(n) = \Theta(g(n))$

2. (10 points) Find a recurrence for $f(n)$, the number of bitstrings of length n that do not have three consecutive ones. For example, $f(3) = 7$ because out of the 8 bitstrings of length 3, only one has three consecutive ones.

$$\{000, 001, 010, 011, 100, 101, 110, \cancel{111}\}$$

You do not have to solve the recurrence - just give the recurrence and explain your answer.

$f(0) = 0$ X

$f(1) = 2$ ✓ $\{0,1\}$

$f(2) = 4$ ✓ $\{00,01,10,11\}$

2

$$f(n) = \underbrace{2^n}_{\text{total \# of strings at } n} - 2(\overbrace{2^{n-1}}^{\text{total \# of strings at } n-1} - \underbrace{f(n-1)}_{\text{total \# of non 111 strings at } n-1}) \quad n \geq 3$$

$2^{n-1} - f(n-1)$ → total # of 111's at n−1

This reccurrence first finds the total # of strings at n. then it subtracts the # of 111's from n−1, and we multiply it by 2 because # of 111 at n is always twice as many as the previous. This gives the total # of non 111 strings at n.

3. (5 points per part, 10 points total) Give the general form for the following recurrences. Note that you do not have to solve for constants. For example, the general form for the recurrence $R(n) = 3R(n-1) + 5$ would be $R(n) = c_1 3^n + c_2$, where c_1 and c_2 are constants.

(a) $A(n) = -3A(n-1) + 4 \cdot 5^n$

$x = -3$

$A(n) = c_1(-3)^n + c_2(5)^n$ ✓

(b) $B(n) = B(n-1) + 2B(n-2) + 2^n(n-2)$

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, x = -1$

$(c_3 n + c_4) 2^n$ but since $x = 2$

$(c_3 n^2 + c_4 n + c_5) 2^n$

$B(n) = c_1(2)^n + c_2(-1)^n + c_3 n^2 2^n + c_4 n 2^n + c_5 2^n$ ✓ ✓ ✓ ✓ ✓

Great!

4. (10 points) Let R_1 and R_2 be relations on a set A. Prove or disprove: if R_1 and R_2 are transitive, then $R_1 \cap R_2$ is transitive.

$R_1 \cap R_2$ includes R_1 and R_2 by definition.

Since R_1 and R_2 are transitive, $\forall a,b,c \in A$ that $-\frac{1}{2}$

$(a,b) \in R_1, (b,c) R_1$, then $(a,c) R_1$, and $(a,b) \in R_2, (b,c) R_2$, then

$(a,c) \in R_2$. Since $R_1 \cap R_2$, it has to have

both R_1 and R_2 that are transitive. Since

R_1 and R_2 are transitive, $\forall a,b,c \in A$ that $(a,b) \in R_1 \cap R_2, (b,c) \in R_1 \cap R_2$

then $(a,c) \in R_1 \cap R_2$. Thus, $R_1 \cap R_2$ is transitive.

This makes no sense

why?

-8

5. Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ be given by $f(m,n) = mn$.

(a) (5 points) Is f one-to-one? Prove your answer.

Not one-to-one

$f(2,1) = 2 \cdot 1 = 2$

$f(1,2) = 1 \cdot 2 = 2$

$(2,1) \neq (1,2)$

(b) (5 points) Is f onto? Prove your answer.

Yes.

When either m or n is 1,

$f(1,n) = 1 \cdot n = n \to \mathbb{Z}$

$f(m,1) = m \cdot 1 = m \to \mathbb{Z}$

Thus, any $\mathbb{Z}$ can be expressed.

6. (5 points per part, 15 points total) For any finite set S of numbers, let "$\max S$" denote the largest element in S and "$\min S$" denote the smallest element in S. For each of the following relations on $A = \mathcal{P}(\{1, 2, \ldots, 10\})$, decide if the relation is reflexive, irreflexive, transitive, antisymmetric, or symmetric. Each can satisfy more than one of these properties; circle all that apply. You do not need to prove your answers are correct.

(a) $R_1 = \{(S,T) \in A \times A : \max S \leq \max T\}$.
reflexive ✓ irreflexive ✓ transitive ✓ antisymmetric X symmetric ✓

$\max S \leq \min T \leq \max T \leq \min U$
$\min T \leq \max T$

(b) $R_2 = \{(S,T) \in A \times A : \max S \leq \min T\}$.
reflexive ✓ irreflexive ✓ transitive ✓ antisymmetric ✓ symmetric ✓

(c) $R_3 = \{(S,T) \in A \times A : S \subseteq T \text{ or } T \subseteq S\}$.
reflexive ✓ irreflexive ✓ transitive ✓ antisymmetric ✓ symmetric ✓

$\max S \leq \min T$
$\max T \leq \min S$

$\frac{14}{15}$ Good!

7. (10 points) Let $T(n) = 6T(\frac{n}{6}) + n^2$, with $T(1) = 1$. Solve the recurrence $T(n)$ asymptotically. Show your work. You may use any theorems or methods presented in class. If you use the guess and check method, then you must prove that your answer is correct. If you use other methods from class, then you do not need to prove your answer is correct.

$a = 6$

$b = 6$ $f(n) = n^2$

$a \cdot f(\frac{n}{b}) = c \cdot f(n)$

$6(\frac{n}{6})^2 = \frac{1}{6} n^2$

$c < 1$

✓

$T(n) = \Theta(n^2)$ by Master's theorem

8. (15 points) Let $T(n) = \sqrt{n}T(\sqrt{n})+n$, with $T(2) = 1$. Solve the recurrence $T(n)$ asymptotically. Show your work. You may use any theorems or methods presented in class. If you use the guess and check method, then you must prove that your answer is correct. If you use other methods from class, then you do not need to prove your answer is correct.

level	input	#node	work/node	work/level
0	n	1	n	n
1	$n^{1/2}$	$n^{1/2}$	$n^{1/2}$	$n^{1/2}\cdot n^{1/2}=n$
2	$n^{1/4}$	$n^{1/4}\cdot n^{1/2}=n^{3/4}$	$n^{1/4}$	$n^{1/4}\cdot n^{3/4}=n$
⋮				
i	$n^{1/2^i}$	$\frac{n}{n^{1/2^i}}$	$n^{1/2^i}$	n
		+4	+7	

$n^{\frac{1}{2^d}} = 2$

$\log_2 n^{\frac{1}{2^d}} = 1$

$\frac{1}{2^d}\log_2 n = 1$

$\log_2 n = 2^d$

$\log_2 \log_2 n = d$

$T(n) = dn$

$T(n) = O(n \log_2\log_2 n)$

also, it is shown that you need at least $n\log_2\log_2 n$ amount of work, so

$T(n) = \Omega(n\log_2\log_2 n)$

Thus, $T(n) = \Theta(n\log_2\log_2 n)$

9. (10 points) Let A be a finite set, and $f : A \to A$ be a one-to-one function. Prove that f is also onto.

8

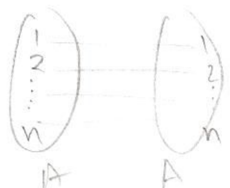
Since A is a finite set, $|A|$ is not ∞.

Since f is one to one, every element of A in domain has only 1 element of A in codomain.

Since f is going from A to A, which have the same cardinalities, it is obvious that every element in codomain A has to be linked to an element in domain A because f is one to one.

ick?

No - use pigeonhole!



Thus, f is onto.

Another fact about f:

$|A| \leq |B|$ assures it is one to one, and $|A| \geq |B|$ assures that it is onto. Since our case is $|A| = |A|$, it is both one to one and onto, which also tells it's a bijection, which further implies f is also onto.

NO this is converse of what we showed!!

(scratch paper)

CS173 Cheat Sheet (Spring 2008)

Set Theory Notation		
empty set	$\varnothing$	$\{\,\}$
subset	$A \subseteq B$	$\forall x\colon x \in A \to x \in B$
proper subset	$A \subset B$	$A \subseteq B \land \exists y \in B\colon y \notin A$
superset	$A \supseteq B$	$B \subseteq A$
proper superset	$A \supset B$	$B \subset A$
set equality	$A = B$	$A \subseteq B \land B \subseteq A$
union	$A \cup B$	$\{x \mid x \in A \lor x \in B\}$
intersection	$A \cap B$	$\{x \mid x \in A \land x \in B\}$
difference	$A - B$	$\{x \mid x \in A \land x \notin B\} = A \cap \overline{B}$
symmetric difference	$A \Delta B$	$\{x \mid x \in A \leftrightarrow x \notin B\}$
complement	$\overline{A}$	$\{x \mid x \notin A\} = U - A$
Cartesian product	$A \times B$	$\{(a,b) \mid a \in A \land b \in B\}$
power set	$\mathcal{P}(A)$	$\{B \mid B \subseteq A\}$
cardinality	$\lvert A \rvert$	# of elements (if finite)

Binary relation $R \subseteq A \times A$	
relation notation	a and b are related $\iff (a,b) \in R$
inverse R^{-1}	$\{(b,a) \in A \times A \mid (a,b) \in R\}$
reflexive	$\forall a \in A,\ (a,a) \in R$
symmetric	$\forall a,b \in A$, if $(a,b) \in R$ then $(b,a) \in R$
antisymmetric	$\forall a,b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a = b$
transitive	$\forall a,b,c \in A$, if $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$

Equivalence relation $\sim$

An equivalence relation is a binary relation which is reflexive, symmetric, and transitive

Partial order $\preceq$

A partial order, or poset, is a binary relation which is reflexive, antisymmetric, and transitive.

Function $f\colon A \to B$

A function f from A to B associates each element $a \in A$ to exactly one element $b \in B$.

Notation	$b = f(a)$ if b is associated to a
one-to-one (or injective)	$\forall a_1, a_2 \in A$, if $a_1 \neq a_2$ then $f(a_1) \neq f(a_2)$
onto (or surjective)	$\forall b \in B, \exists a \in A$ such that $f(a) = b$
bijection	one-to-one *and* onto
inverse $f^{-1}\colon B \to A$	$\{(b,a) \mid b = f(a)\}$ (if f is a bijection)