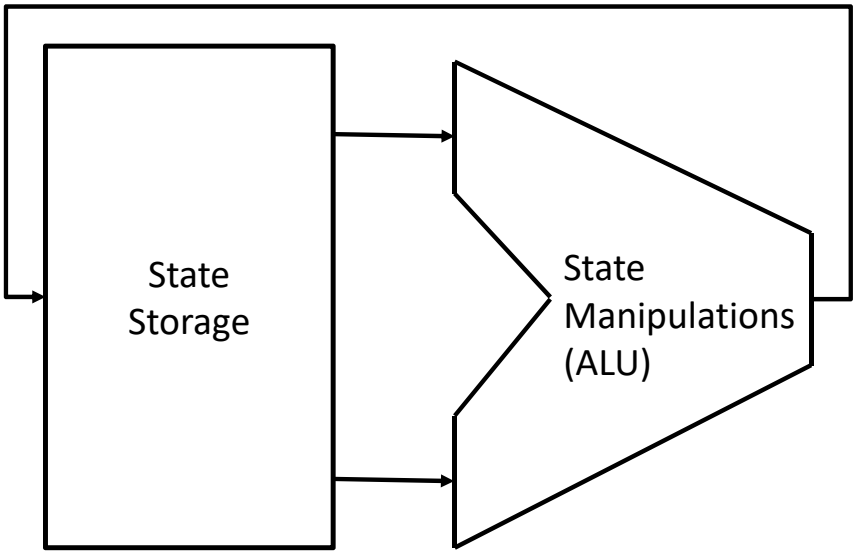
Finite State Machines

State – the central concept of computing

How do we generate control signals for circuits? Finite State Machines

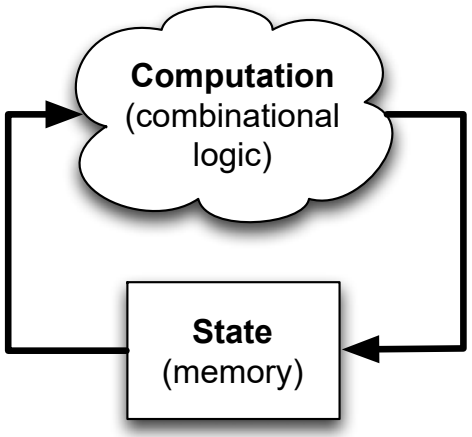


Today's lecture

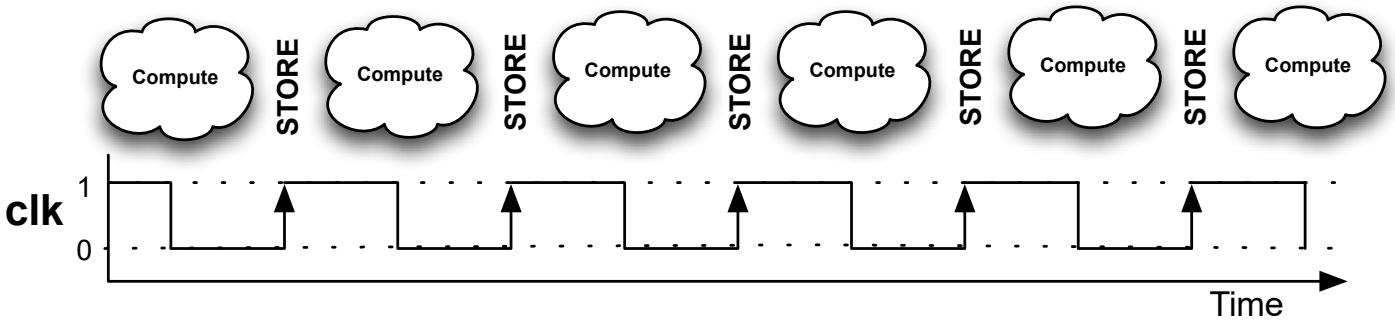
- Goal: Build a sequential circuit from a state diagram
  - Step 0: Problem specification
  - Step 1: Build the state diagram
  - Setp 2: Build the state table
  - Step 3: Build the sequential circuit using D flip-flops
- Timing diagram
- Another example: Sequence recognizer

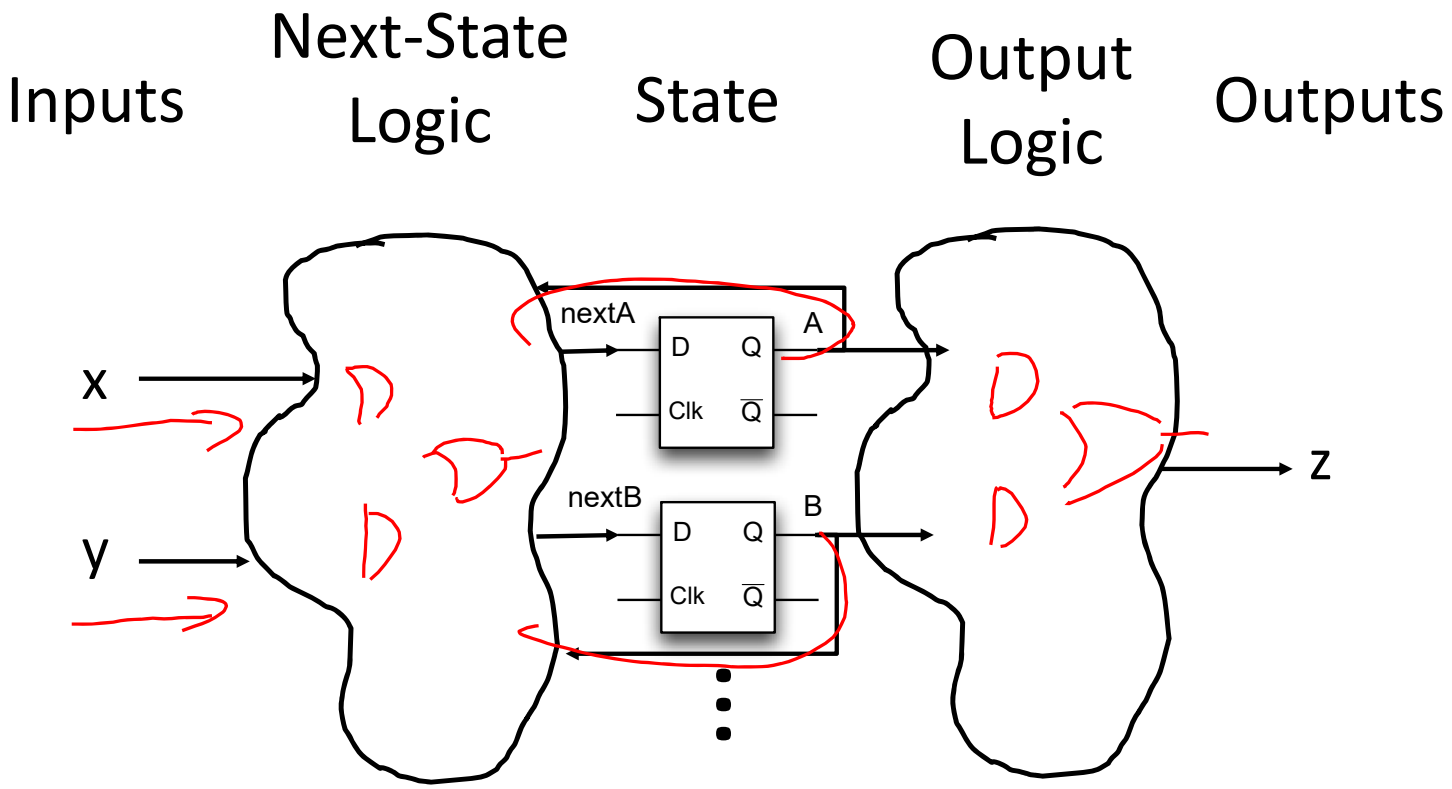
If a combinational logic circuit is an implementation of a Boolean function, then a sequential logic circuit can be considered an implementation of a finite state machine.

Synchronous Design, reminder



- Alternate between computation and updating state.




Inputs
Next-State Logic
State
Output Logic
Outputs
x
y
nextA
nextB
D
Q
Clk
Q
A
B
z

Step 0: Problem Specification

- We have a candy machine that dispenses candies that cost 15-cents
  - Accepts
    - nickels (5-cents)
    - dimes (10-cents)
  - Dispenses a candy if the balance is ≥ 15-cents
  - When the customer overpays
    - the machine does not return change, but
    - keeps the balance for future transactions

Step 1: Build the State Diagram

- Inputs

  d = 1, iff dime is inserted

  n = 1, iff nickel " "

- Ouputs

  c = 1, iff candy should be dispensed

d n	meaning
0 0	no dime or nickel inserted
0 1	nickel "
1 0	dime "
1 1	not possible

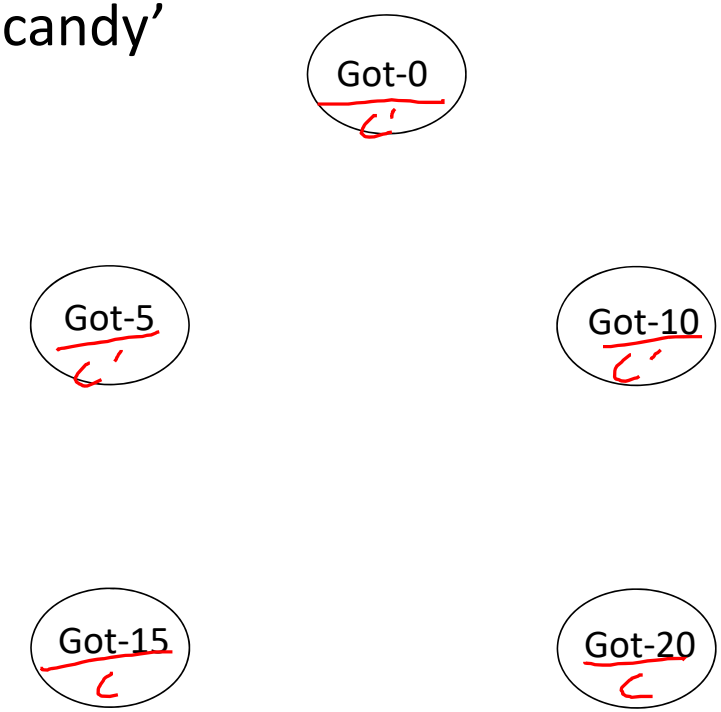
c	
0	no candy
1	candy

- State identification

  0¢, 5¢, 10¢, 15¢, 20¢

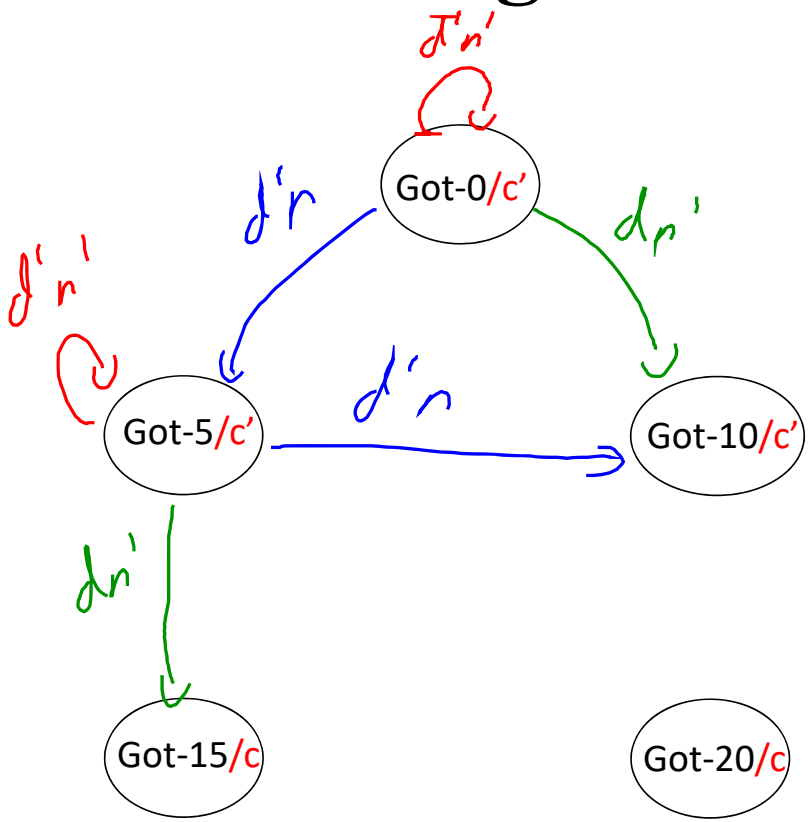
Step 1: Build the State Diagram

Outputs: candy or candy’



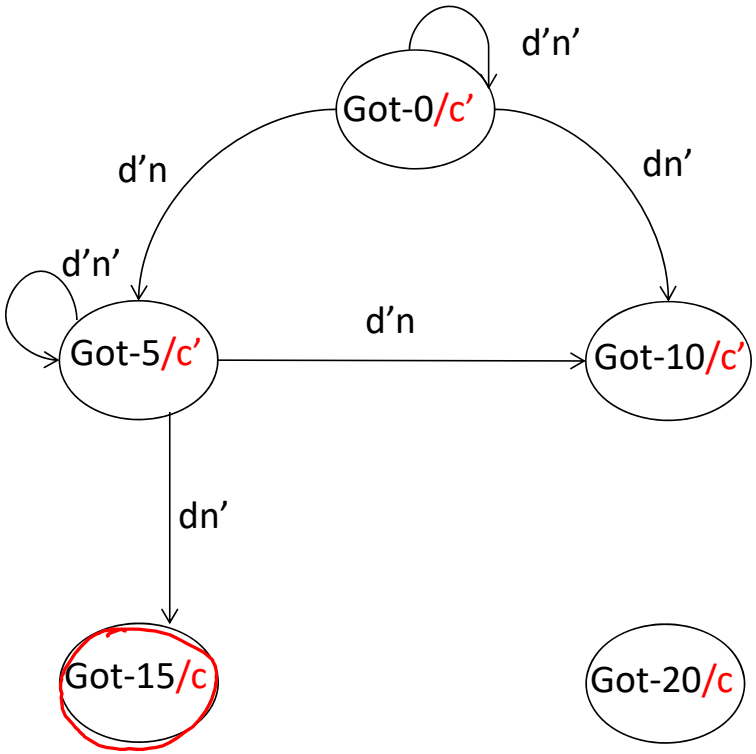
a) candy
b) candy'

Step 1: Build the State Diagram



Inputs: d'n', d'n, dn'

Step 1: Build the State Diagram



What are the transitions for state Got-15?

A: d'n': Got0; d'n: Got20; dn': Got20

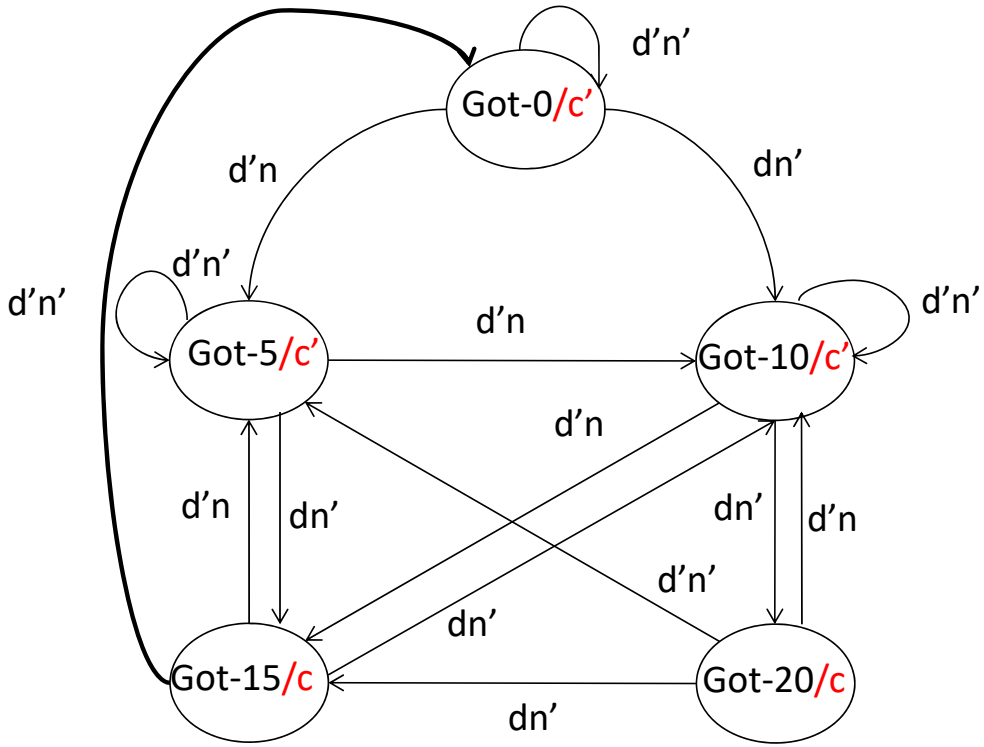
B: d'n': Got0; d'n: Got0; dn': Got0

C: d'n': Got0; d'n: Got5; dn': Got10

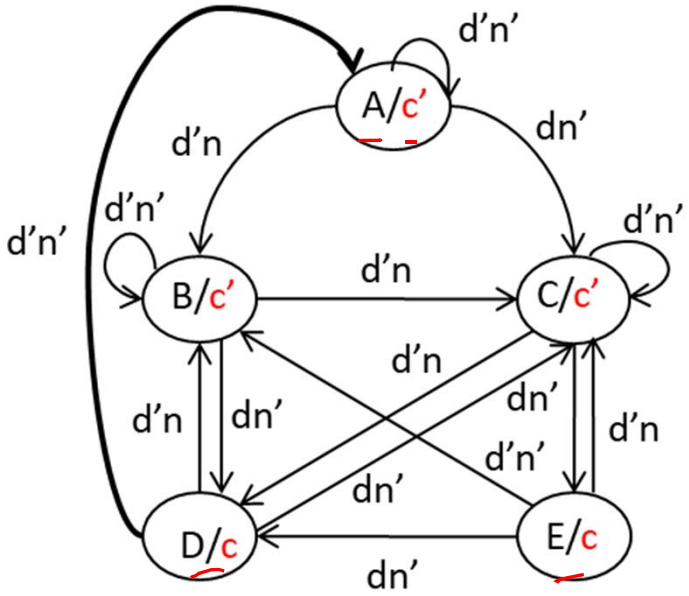
D: d'n': Got15; d'n: Got5; dn': Got10

Inputs: d'n', d'n, dn'

Final State Diagram



Step 2: Build a State Table

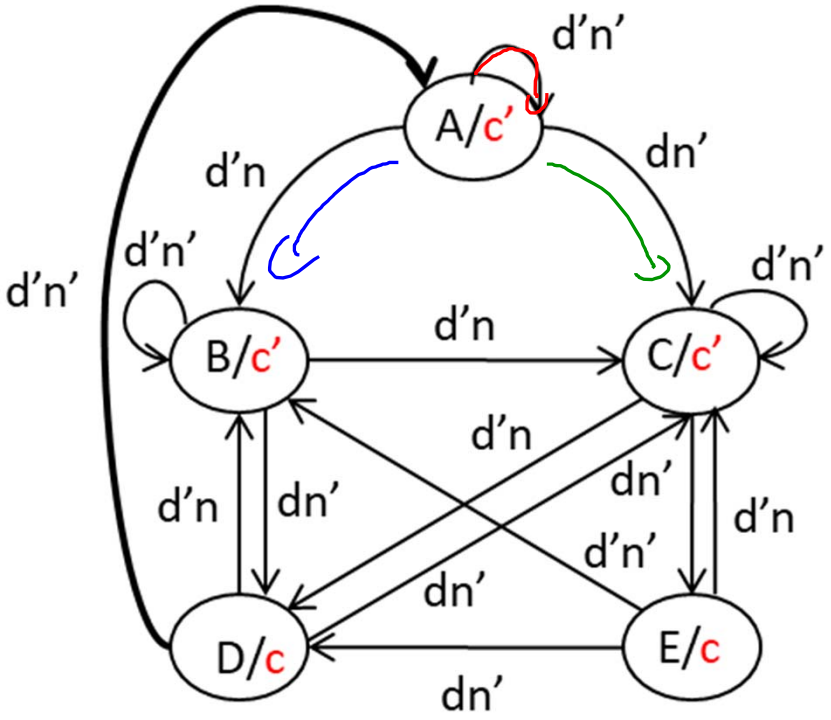


Current State	Output **candy**
A	0
B	0
C	0
D	1
E	1

candy = D + E

In a 'Moore machine', the outputs are a function only of the current state.

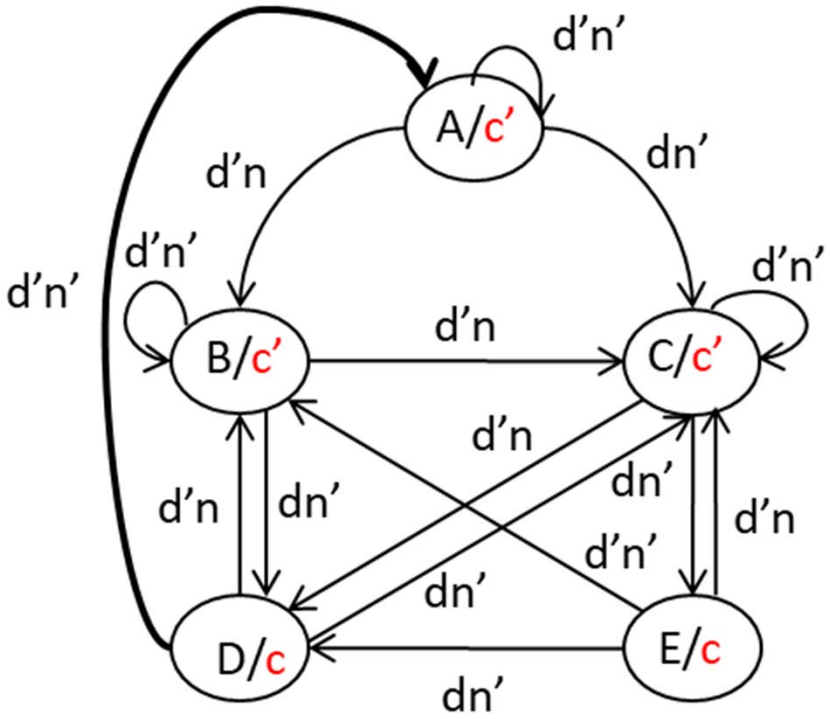
Step 2: Build a Next-State Table



Current State	Input	Next State
A	d'n'	A
A	d'n	B
A	dn'	C
B	d'n'	
B	d'n	
B	dn'	
C	d'n'	
C	d'n	
C	dn'	
D	d'n'	
D	d'n	
D	dn'	
E	d'n'	
E	d'n	
E	dn'	

A	B	C	D	E
A	B	B	B	C
B	C	C	D	D
C	D	E	A	E

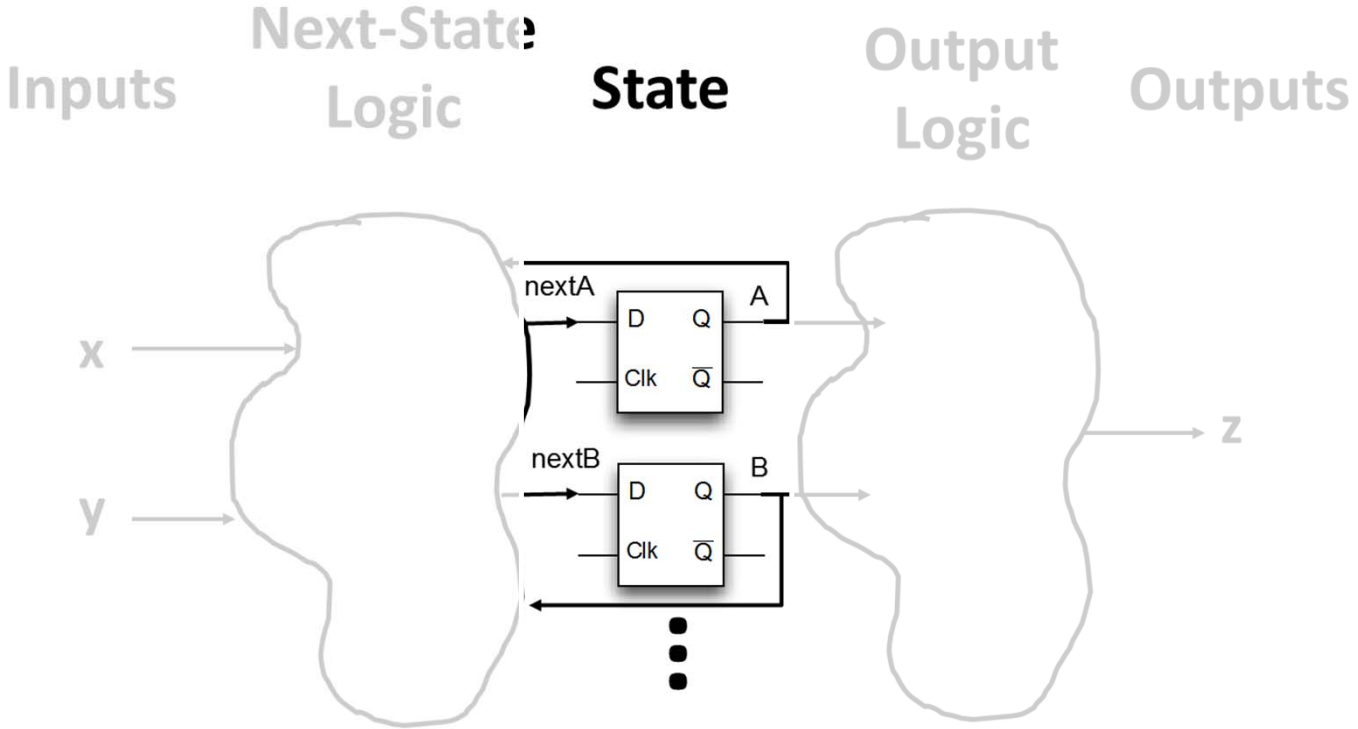
Step 2: Build a Next-State Table



Why do we need sequential logic to build this circuit?

Current State	Input	Next State
A	d'n'	A
A	d'n	B
A	dn'	C
B	d'n'	B
B	d'n	C
B	dn'	D
C	d'n'	C
C	d'n	D
C	dn'	E
D	d'n'	A
D	d'n	B
D	dn'	C
E	d'n'	B
E	d'n	C
E	dn'	D

Step 3: Build Sequential Circuit



Step 3: Build Sequential Circuit (State)

Current State	Input	Next State
A	d'n'	A
A	d'n	B
A	dn'	C
B	d'n'	B
B	d'n	C
B	dn'	D
C	d'n'	C
C	d'n	D
C	dn'	E
D	d'n'	A
D	d'n	B
D	dn'	C
E	d'n'	B
E	d'n	C
E	dn'	D

State Encoding:

5 states: How many bits?

a) 2

2^3 b) 3 minimal (hard)

c) 4

d) 5 one hot (easy)

e) 6

Step 3: Build Sequential Circuit (State)

Current State	Input	Next State
A	d'n'	A
A	d'n	B
A	dn'	C
B	d'n'	B
B	d'n	C
B	dn'	D
C	d'n'	C
C	d'n	D
C	dn'	E
D	d'n'	A
D	d'n	B
D	dn'	C
E	d'n'	B
E	d'n	C
E	dn'	D

One hot encoding

ABCDE

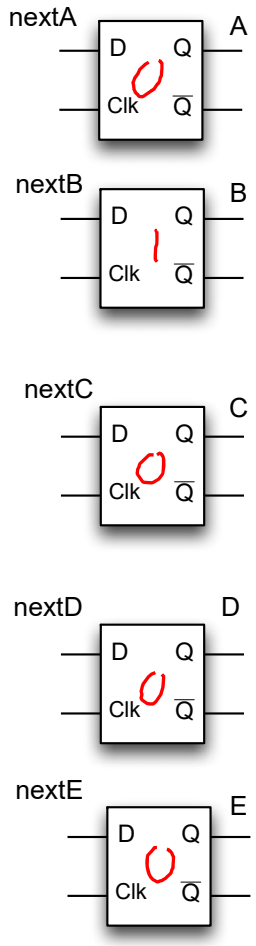
State A = 10000

State B = 01000

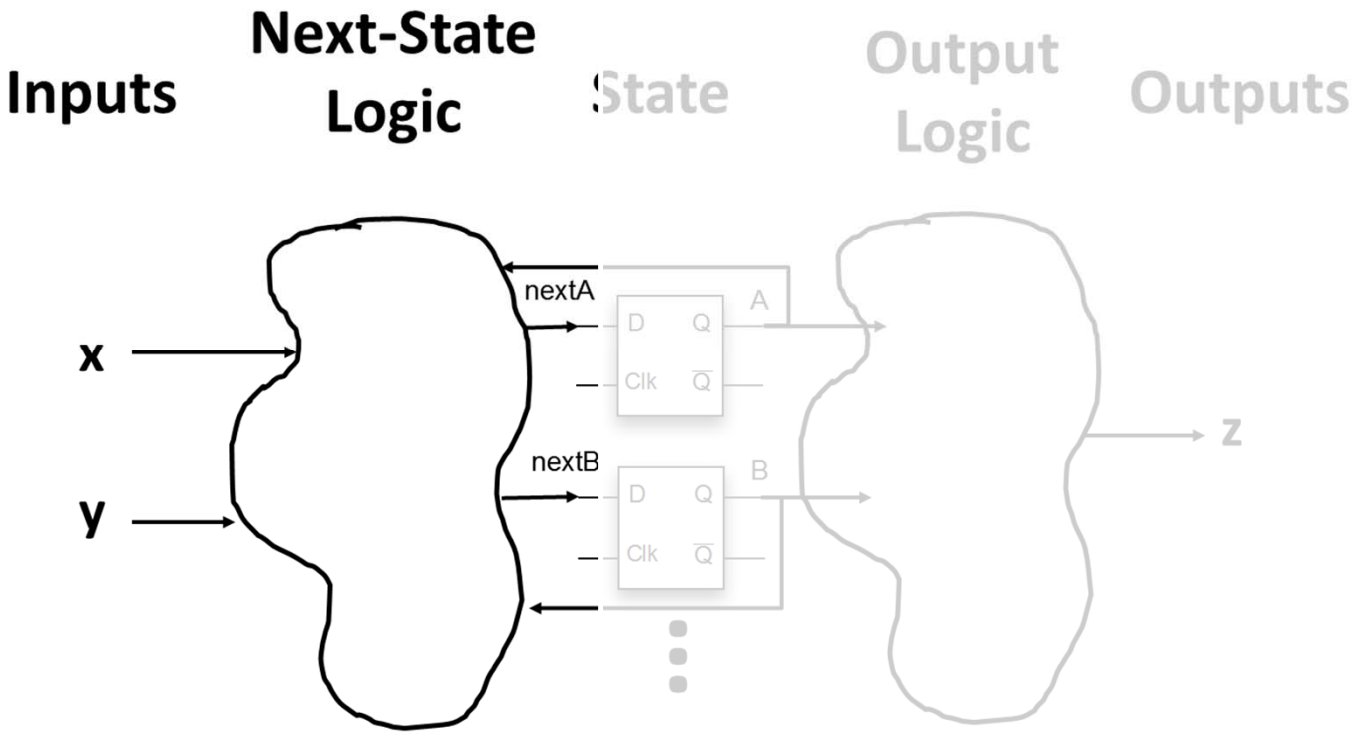
State C = 00100

State D = 00010

State E = 00001

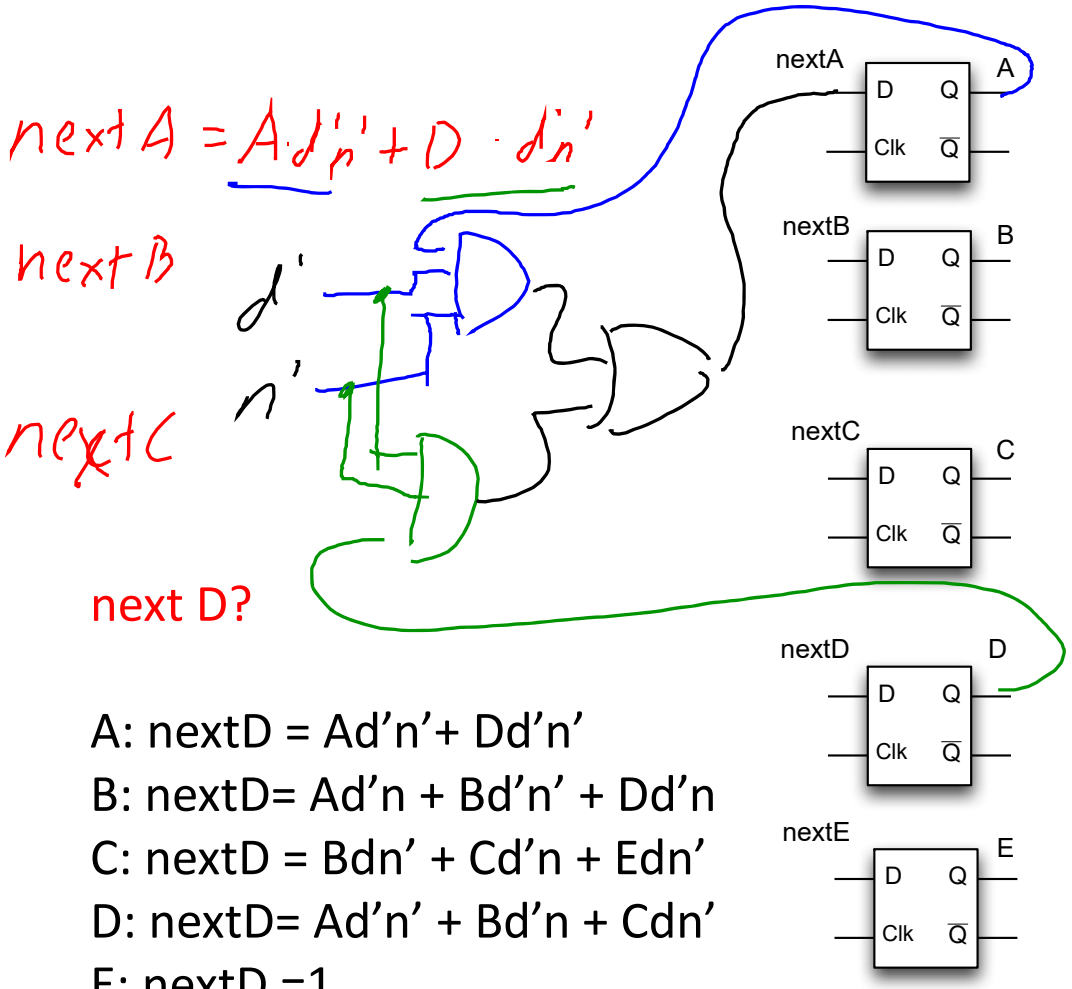


Step 3: Build Sequential Circuit



Step 3: Build Sequential Circuit (Next-State Logic)

Current State	Input	Next State
A	d'n'	A
A	d'n	B
A	dn'	C
B	d'n'	B
B	d'n	C
B	dn'	D
C	d'n'	C
C	d'n	D
C	dn'	E
D	d'n'	A
D	d'n	B
D	dn'	C
E	d'n'	B
E	d'n	C
E	dn'	D



A: nextD = Ad'n'+ Dd'n'

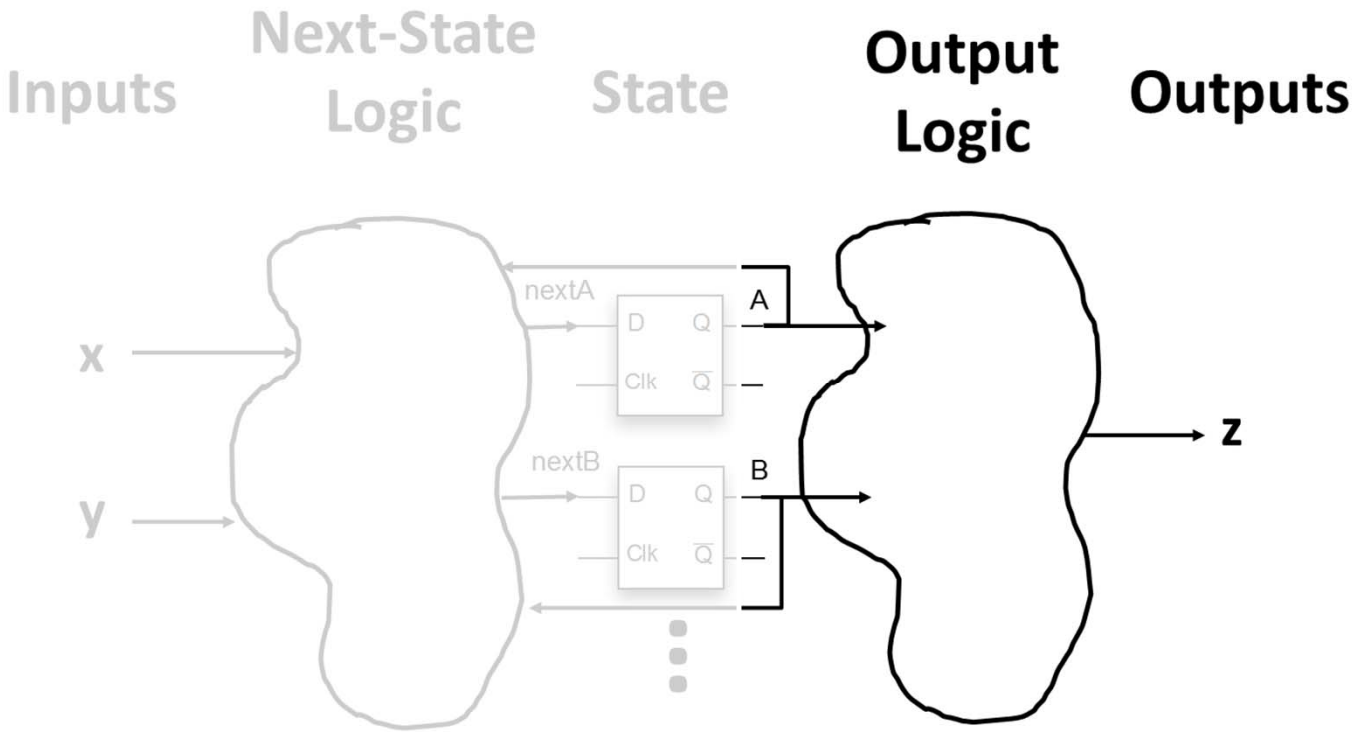
B: nextD= Ad'n + Bd'n' + Dd'n

C: nextD = Bdn' + Cd'n + Edn'

D: nextD= Ad'n' + Bd'n + Cdn'

E: nextD =1

Step 3: Build Sequential Circuit



Step 3: Build Sequential Circuit (output logic)

Current State	Input	Next State	Output
A	d'n'	A	0
A	d'n	B	0
A	dn'	C	0
B	d'n'	B	0
B	d'n	C	0
B	dn'	D	0
C	d'n'	C	0
C	d'n	D	0
C	dn'	E	0
D	d'n'	A	1
D	d'n	B	1
D	dn'	C	1
E	d'n'	B	1
E	d'n	C	1
E	dn'	D	1

nextA = Ad'n'+ Dd'n'

nextB = Ad'n + Bd'n' + Dd'n + Edn'

nextC = And' + Bd'n + Cd'n' + Ddn' + Ed'n

nextD = Bdn' + Cd'n + Edn'

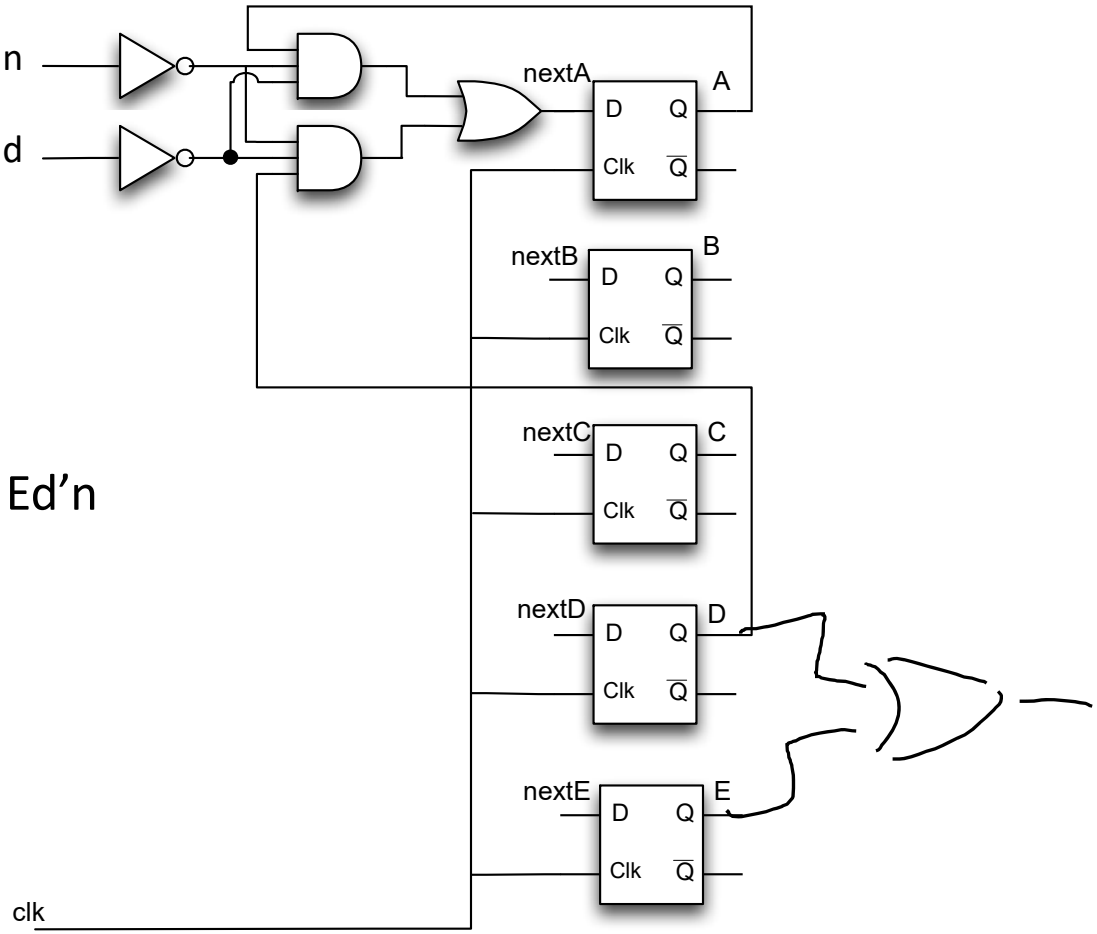
nextE = Cdn'

Output: Candy = D + E

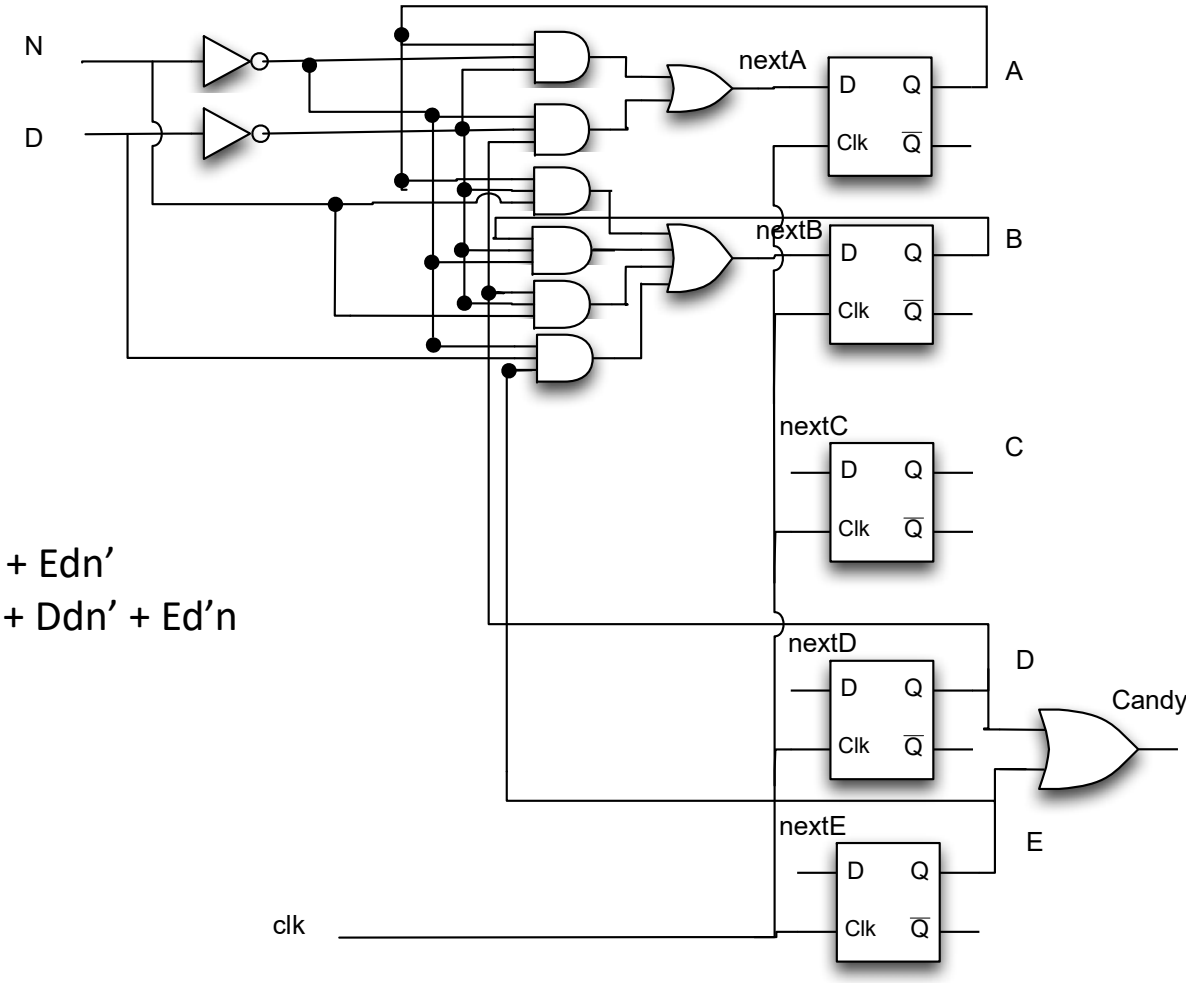
Step 3:Build Sequential Circuit (output logic)

nextA = Ad'n'+ Dd'n'
nextB = Ad'n + Bd'n' + Dd'n + Edn'
nextC = And' + Bd'n + Cd'n' + Ddn' + Ed'n
nextD = Bdn' + Cd'n + Edn'
nextE = Cdn'

Output: c = D + E



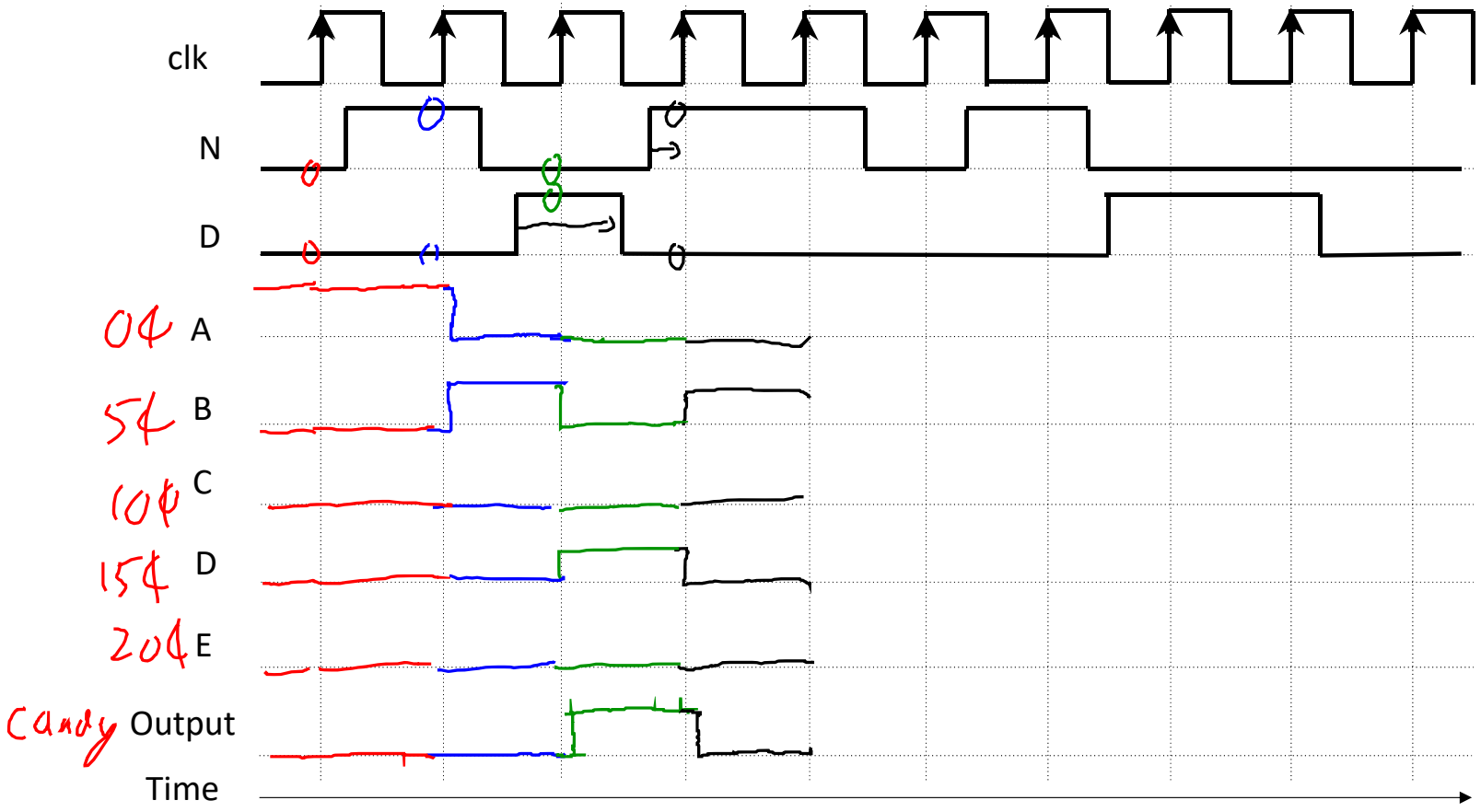
Step 3: Build Sequential Circuit ("finished")



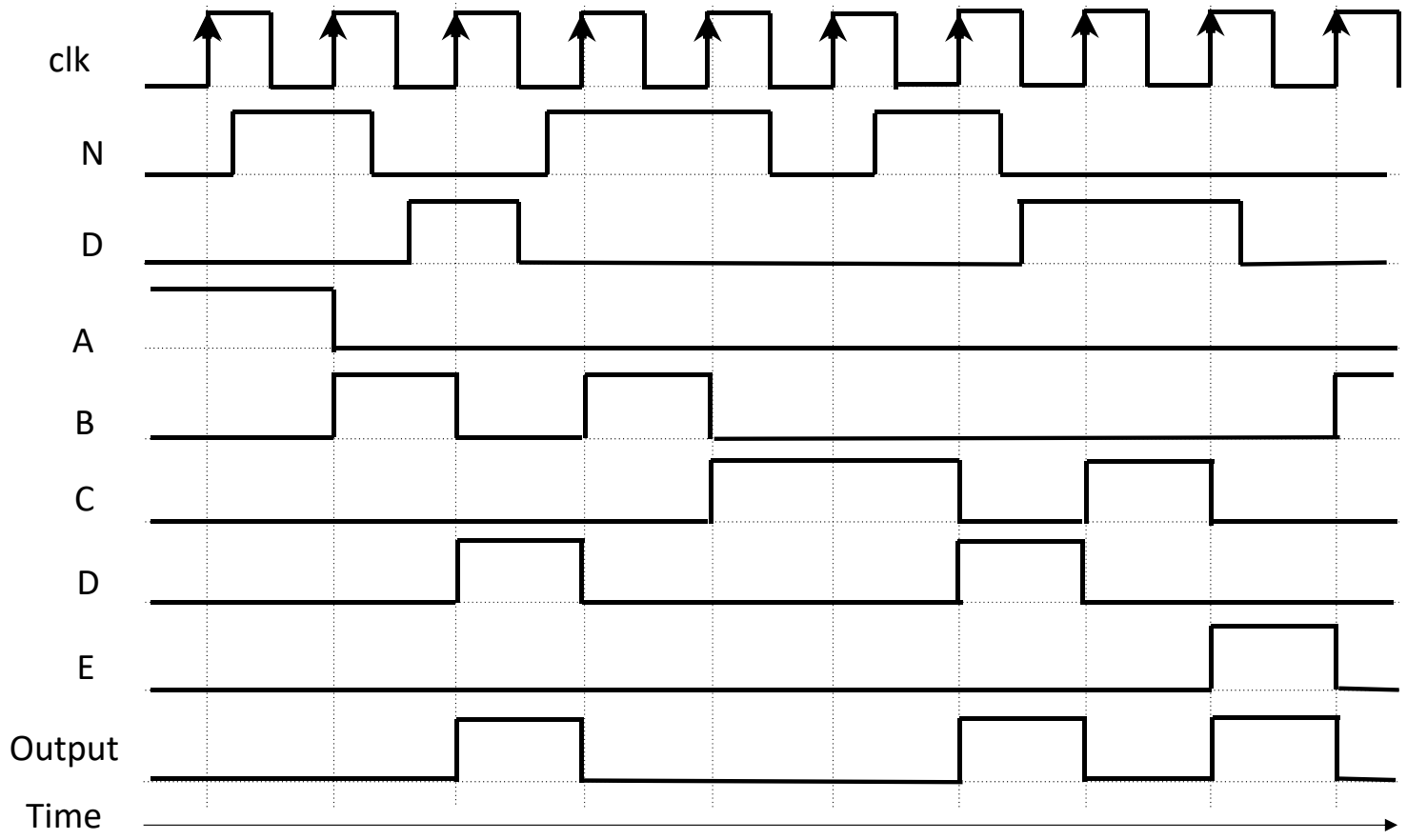
nextA = Ad'n'+ Dd'n'
nextB = Ad'n + Bd'n' + Dd'n + Edn'
nextC = And' + Bd'n + Cd'n' + Ddn' + Ed'n
nextD = Bdn' + Cd'n + Edn'
nextE = Cdn'

Output = Candy = D + E

Timing Diagram (update state on clock edges)



Timing Diagram



Another example: Sequence recognizer

- A sequence recognizer is a special kind of sequential circuit that looks for a special bit pattern in some input.
- The recognizer circuit has one input, X.
- There is one output, Z, which is 1 when the desired pattern is found.

- Our example will detect the bit pattern "1001":

Inputs:	1 1 1 0 0 1 1 0 1 0 0 1 0 0 1 1 0 …
Outputs:	0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0 …

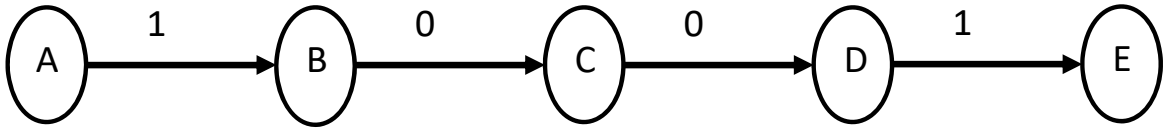
Here, one input and one output bit appear every clock cycle.

- This requires a sequential circuit because the circuit has to "remember" the inputs from previous clock cycles, in order to determine whether or not a match was found.

State diagram

State	Meaning
A	None of the desired pattern (1001) has been input yet.
B	We've already seen the first bit (1) of the desired pattern.
C	We've already seen the first two bits (10) of the desired pattern.
D	We've already seen the first three bits (100) of the desired pattern.
E	We've seen the pattern (1001)

State diagram

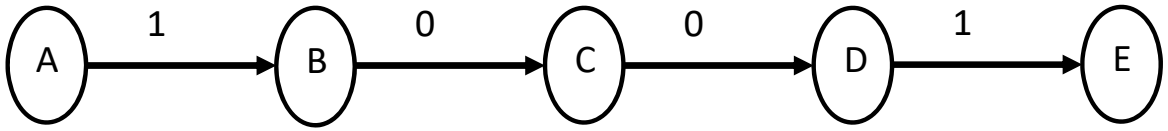


State	Meaning
A	None of the desired pattern (1001) has been input yet.
B	We've already seen the first bit (1) of the desired pattern.
C	We've already seen the first two bits (10) of the desired pattern.
D	We've already seen the first three bits (100) of the desired pattern.
E	We've seen the pattern (1001)

State diagram

i-clicker

- We need *two* outgoing arrows for each node, to account for the possibilities of X=0 and X=1.



Inputs: 1 1 1 0 0 1 1 0 1 0 0 1 0 0 1 1 0 …

Outputs: 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0 …

Transitions for E:

A: x=0-> A; x=1 -> B

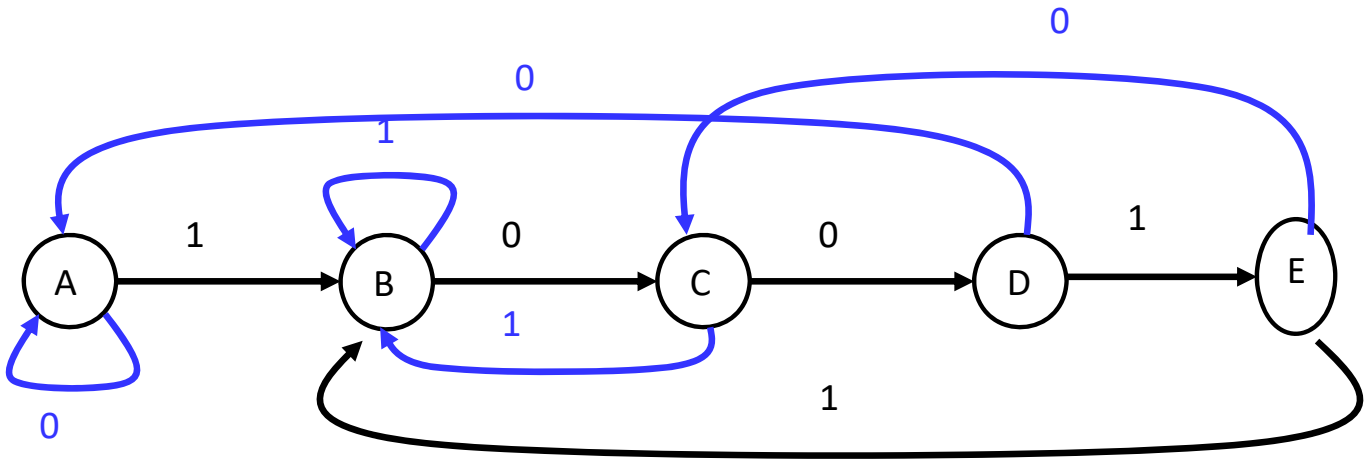
B: x=0-> C; x=1 -> B

C: x=0-> B; x=1 -> A

D: x=0-> D; x=1 -> B

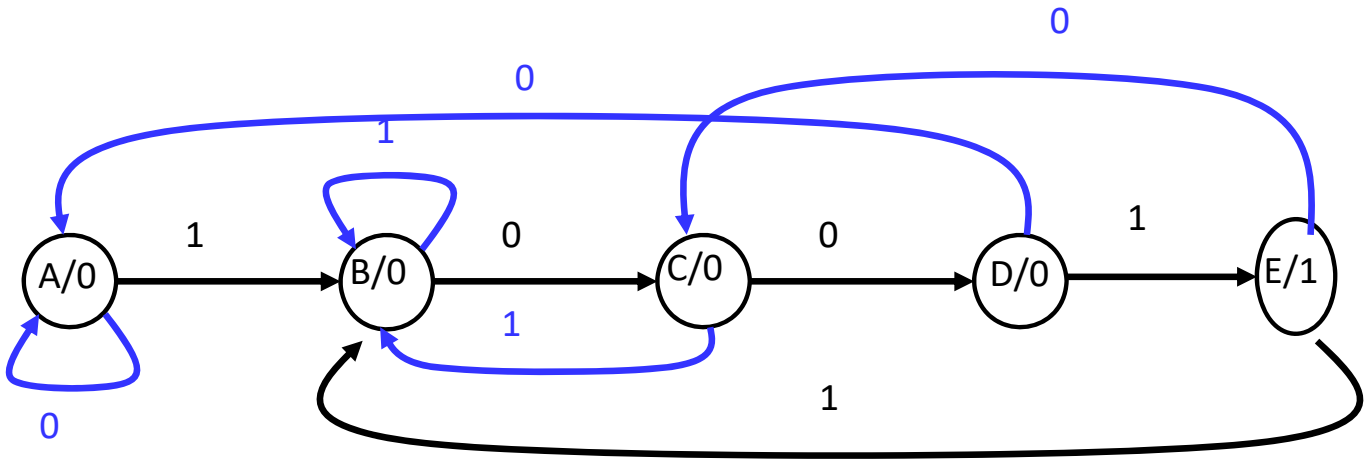
State diagram

- We need *two* outgoing arrows for each node, to account for the possibilities of X=0 and X=1.

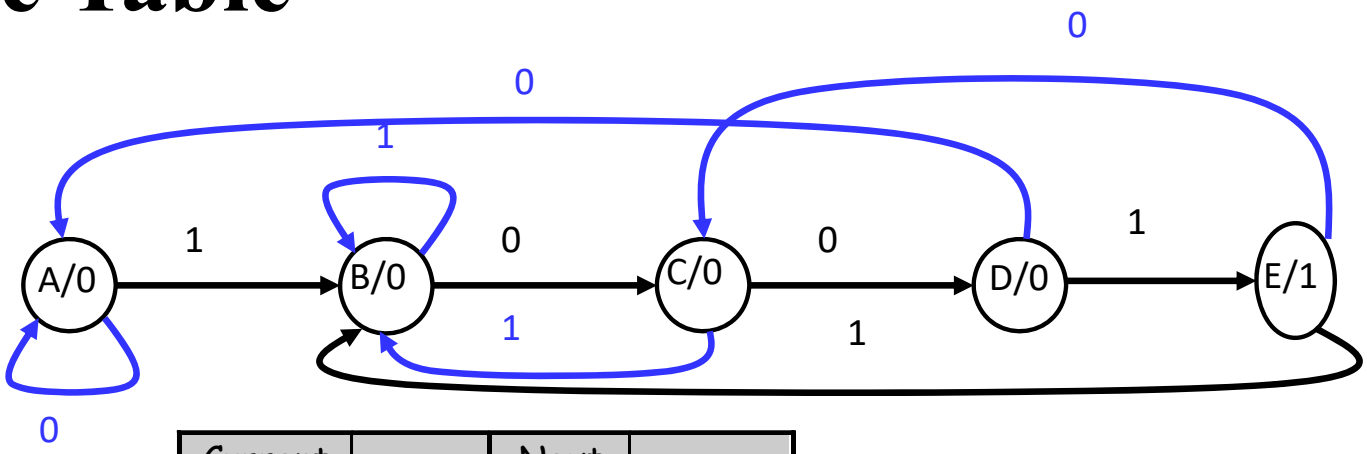


State diagram

- Need to determine the output

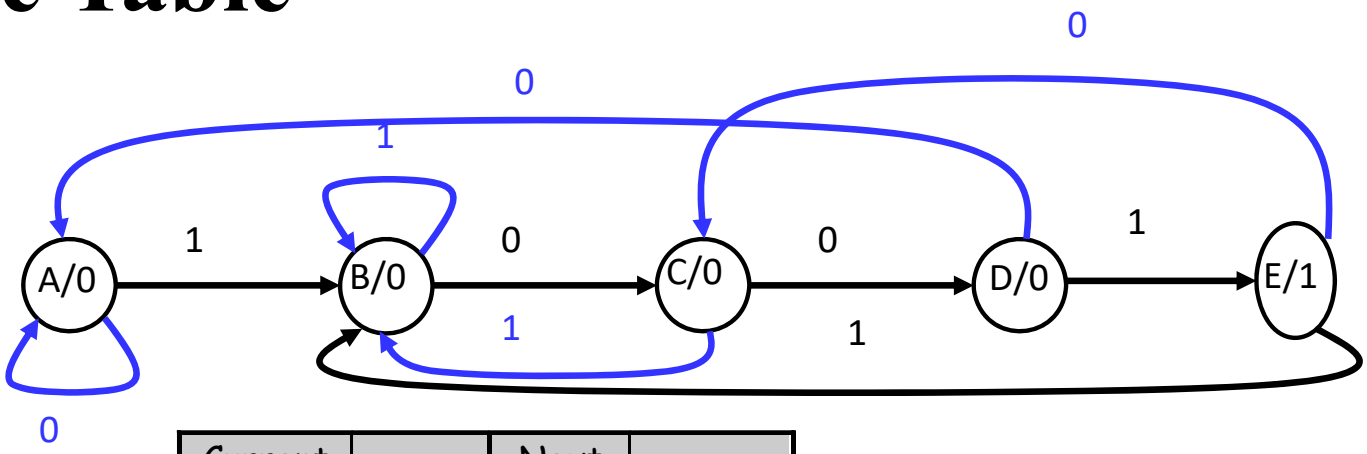


State Table



Current State	Input	Next State	Output
A	0	A	0
A	1	B	0
B	0	C	0
B	1	B	0
C	0	D	0
C	1	B	0
D	0	A	0
D	1	E	1
E	0	C	0
E	1	B	0

State Table



Current State	Input	Next State	Output
A	0	A	0
A	1	B	0
B	0	C	0
B	1	B	0
C	0	D	0
C	1	B	0
D	0	A	0
D	1	E	1
E	0	C	0
E	1	B	0

Anext = Ax' + Dx'
Bnext = Ax + Bx + Cx + Ex
Cnext = Bx' + Ex'
Dnext = Cx'
Enext = Dx

Output = E