Math 415 Midterm 3 Exam Check List SP 2018

Chapter 3 Section 5: Function spaces, Fourier series and orthogonal polynomials

- motivate the integral inner product for functions
- prove orthogonality of the Fourier trig functions, then normalize
- derive Fourier series and the Fourier coefficients
- apply Gram-Schmidt to basis $1, t, t^2$ in P_2
- derive best approximation of a general function with a quadratic function

Chapter 4 Section 1: Definition and properties of the determinant

- describe the three properties defining the determinant
- describe seven additional properties that derive from the first three (product rule, transpose rule, inverse rule, singularity rule, etc.)

Chapter 4 Section 2: Formulas for the determinant, cofactor expansion

- derive the product of pivots rule
- derive the permutation form (optional)
- derive the cofactor expansion form of the determinant (important for next section)

Chapter 4 Section 3: Applications of determinants

- cofactor matrix formula for the inverse of a matrix
- Cramer's rule
- volume of a box in terms of determinants

Chapter 5 Section 1: Eigenvalues and eigenvectors and computation examples

- motivate the eigenvalue problem for A using const coefficient differential systems
- present in detail a 2×2 example and its full solution
- discuss what the eigenvalue problem looks like for diagonal, triangular and projection matrices
- discuss the sum and product rules for eigenvalues

Chapter 5 Section 2: Diagonalization of a matrix

- present the diagonalization results for a square matrix and a justification
- point out that not all matrices are diagonalizable, but those with distinct eigenvalues are

- discuss the issue of possibly not enough eigenvectors when eigenvalues are repeated
- discuss the fact that complex eigenvalues can arise even with real matrices
- discuss how to find powers of matrices using diagonalization
- discuss how the eigenvalues of a matrix and its transpose are related

Chapter 5 Section 3: Complex numbers and complex eigenvalue problems

- terms: i, complex number, real part, imaginary part, modulus, conjugate
- define addition, multiplication, division of complex numbers
- define conjugate and outline the properties of conjugates
- do an eigenvalue analysis with complex eigenvalues and eigenvectors
- prove theorem that eigenvalues and eigenvectors of a real matrix appear in conjugate pairs
- terms: complex plane, rectangular and polar forms of a complex number
- Euler's identity