Math 415 Midterm 1 Exam Check List SP 2018

Chapter 1 Section 1: Gaussian elimination, back substitution

- instill the process of Gaussian elimination and back substitution
- terms: pivot, inconsistent system, triangular system, singular system
- understand how elimination can break down (no row exchanges yet)

Chapter 1 Section 2: Column vectors, addition, scalar multiplication, the two geometric interpretations

- terms: vectors, vector addition, scalar multiplication, linear combos
- understanding vectors as "arrows" and the geometry of linear combos
- solving $2 \times 2 (3 \times 3)$ systems as the intersection of lines (planes)
- solving $2 \times 2 (3 \times 3)$ systems as a linear combo problem
- understanding geometrically how systems can be singular
- unique solution, no solution, infinitely many solutions

Chapter 1 Section 3: Matrices, addition, scalar and matrix multiplication, matrix form of a linear system

- terms: matrix, matrix addition, scalar multiplication, matrix multiplication
- introduce linear systems as the matrix equation $Ax = b$
- instill the associativity and non-commutativity of multiplication, distribution laws
- terms: identity matrix, elementary matrix (and role in row operations)

Chapter 1 Section 4: Elementary matrices, permutation matrices, LU factorization

- terms: triangular matrix, elementary matrix
- introduce the idea of an inverse via elementary matrices
- derive the triangular factorization of a matrix (when no row exchanges are needed)
- instill the importance of "one linear system = two triangular systems"
- terms: row exchanges, permutation matrices
- derive the general elimination principle $PA = LU$ for non-singular systems

Chapter 1 Section 5: Inverses, Gauss-Jordan elimination, transposes, symmetric matrices

- terms: inverse
- Gauss-Jordan elimination and computing inverses

- instilling the fact that "inverse exists = there is a full set of pivots = non-singular"
- terms: transpose, symmetric matrix

Chapter 2 Section 1: Vector spaces, subspaces, column space and null space of a matrix

- instill the idea of a vector space as a set of objects and addition and scalar multiplication behaving as for real numbers, but instead with lists of reals
- examples: column vectors, 2×3 matrices, polynomials of degree n or less
- terms: subspace, closure rules, column space, null space
- instill the importance of column and null spaces in studying linear systems

Chapter 2 Section 2: Finding the column space and null space, Echelon form, general factorization, pivot and free variables, superposition, rank, efficient solution methods

- terms: row echelon form, reduced row echelon form
- state the general factorization rule $PA = LU$
- terms: pivot variables, free variables, special solutions, complete solution
- intro the idea of dimension informally: dimension of the null space = number of free variables
- terms: superposition, complete solution, particular solution, null space solution
- drive home the idea that the reduced row echelon form $Rx = d$ of $Ax = b$ reveals all
- terms: span, rank
- relate the dimensions of column space and null space to rank r and m and n
- describing the column space in terms of pivot columns

Chapter 2 Section 3: Linear independence, basis, dimension, coordinates relative to a basis

- terms: linear independence and dependence
- instill how to check for independence
- terms: spanning set
- examples: column vectors, 2×3 matrices, symmetric matrices, polynomials
- terms: basis, minimal spanning set, maximal independent set
- recall now how to find bases for the column space and the null space
- uniqueness of representation in terms of a basis and "coordinates" of a vector relative to a basis

Chapter 2 Section 4: The four fundamental subspaces

- terms: column space, null space, row space, left null space, row rank, column rank
- give examples and find the dimensions of each subspace and what space they live in
- instill the geometric picture of A as a mapping