Heat and Thermodynamics

Thermodynamics System

→ A definite quantity of matter bounded by a closed surface is called system. It is an assembly of large number of particles having certain pressure, volume and temperature.
The system may be mechanical device or a biological organism or specified quantity of materials such as air inside the tyre that is being inflated or it may be gas contained in a cylinder with a movable piston. The environment outside the system is known as its surrounding.
The thermodynamics system is one which interacts with its surrounding:

i) by transfer of heat in and out of the system.
ii) by performance of the work by or on the system.

* Open system

→ The system which exchange energy and matter with its surrounding is called open system.

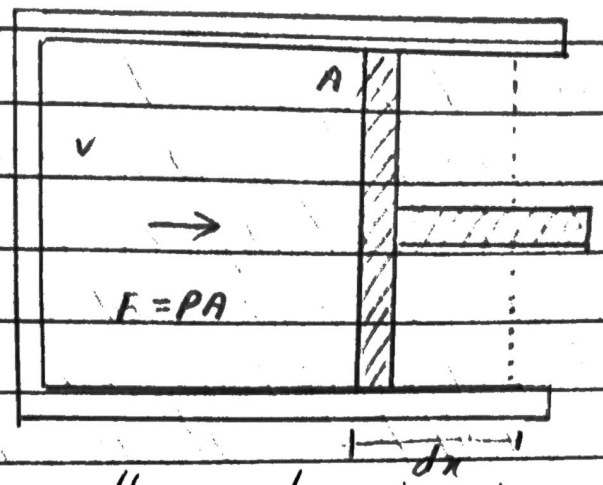
* Close system

→ The system which only exchange energy is called closed system

* Isolated system

→ The system which exchange nothing is called isolated system.

Work done by gas during expansion



Consider a gas in a cylinder fitted with movable piston which is frictionless as shown in the figure. If A be the cross sectional area of the piston and P be the pressure exerted by the gas then force exerted by the gas on the piston is given by $F = PA$

If the piston is displaced through a small distance dx against the constant pressure P then work done by gas is given by $dW = F dx$

$$= PA dx$$

So, work done by the gas is $dW = \cancel{F dx}\ P dV$ — ①

When the volume of gas increase from V_1 to V_2, the total workdone is obtained by integrating eqn ① from lim V_1 to V_2.

$$W = \int_{V_1}^{V_2} P dV$$

When final volume V_2 is greater than initial volume V_1, the work done by system during expansion is positive. On the otherhand, work done on the system during compression is negative

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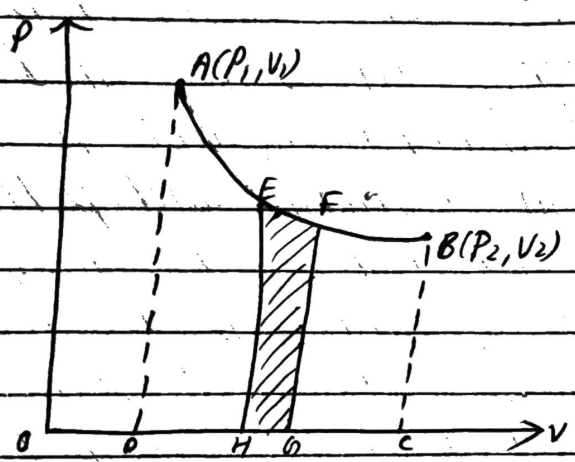
Indicator diagram

→ It is a graph between pressure and volume of a system undergoing the process of expansion and contraction. It is also called P-V diagram



Suppose a gas expands from initial state $A(P_1, V_1)$ to final state $B(P_2, V_2)$ as shown in figure.

Let E be the point on indicator diagram and P and V be the pressure and volume at that point. Let the volume increase by small amount dV to reach to the point F closed to the point E such that the pressure remains constant.

From the figure, $EH = FG = P$

$HG = dV$

Area of shaded strike $= EH \times HG$

$= PdV$

Work done in changing state from E to F

$dW = PdV$

Total workdone by the system (gas) during expansion from A to B = sum of area of all such strike

i.e W = Area of ABCDA

Thus, work done by the system is numerically equal to area under P-V diagram

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Internal energy of a gas

Matter consists of molecule which are in random motion. So, they have kinetic energy. The molecules of matter also attract one another due to intermolecular force of attraction. The energy possessed by the molecules due to their positions is called potential energy. The potential energy is against the intermolecular force. The sum of kinetic energy and potential energy of the molecules of the body or system is called its internal energy and it is denoted by 'U'. In an ideal gas, there is no intermolecular attraction and hence it doesnot posses potential energy. The internal energy of an ideal gas is wholly kinetic in nature and is a function of temperature only. The internal energy of ~~an~~ ~~ideal~~ real gas is partly made up of kinetic energy and partly of intermolecular potential energy and hence is a function of temperature and volume.

First law of thermodynamics

It states that if the quantity of heat supplied to a system is capable of doing work then quantity of heat ~~supplied~~ absorbed by the system is equal to the sum of increase in internal energy of the system and external workdone by it.

It is also known as principle of conservation of energy. If a quantity of heat dQ is given to a system, dW is the workdone by it and dU is the increase in internal energy of the system, then from the principle of conservation of energy, we have

$$dQ = dU + dW \quad \text{——— (i)}$$

Since, $dW = PdV$

So, $dQ = dU + PdV$ ——— (ii)

Eqn (i) and (ii) are mathematical forms of first law of thermodynamics

Notes: i) If heat is given to system, then dQ is taken as positive and if heat is lost by the system, dQ is taken as negative.

ii) If work is done by the system, then dW is positive and if work is done on the system, dW is negative.

iii) Suppose, gas is contained in a cylinder fitted with moveable piston and the gas is heated with constant volume i.e $dV = 0$

then, $dQ = dV + 0$

$\therefore dQ = dV$

It means all heat is supplied to the system goes into increasing the internal energy of gas and hence temperature

iv) For a cyclic process, $dU = 0$ because the system is brought back to initial condition.

i.e $dQ = 0 + dW$ $[\because dU = 0]$

ie $dQ = dW$

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Specific heat of a gas

→ A gas can be heated at constant volume as well as at constant pressure. Different amount of heat is required when gas is heated at constant volume and at constant pressure.

Ⓐ Specific heat capacity

i) Specific heat capacity at constant volume (C_v):

→ It is the amount of heat required to raise the temperature of 1kg mass of gas through 1K at constant volume.

It's SI unit is $J\,kg^{-1}K^{-1}$

Heat supplied to a gas at constant volume is given by

$$(dQ)_v = mC_v dT$$

$$\text{or, } C_v = \frac{d(Q)_v}{m\,dT}$$

ii) Specific heat capacity at constant pressure (C_p)

It is the amount of heat required to raise the temp of 1kg mass of gas through 1K at constant pressure.

It's SI unit is $J\,kg^{-1}K^{-1}$

Heat supplied to a gas at constant pressure is given by

$$d(Q)_p = mC_p dT$$

$$C_p = \frac{(dQ)_p}{m\,dT}$$

where
$m \rightarrow$ mass of gas
$dT \rightarrow$ change in temperature

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(B) Molar heat capacity (Molar specific heat)

i) Molar heat capacity at constant volume (C_v)

→ It is the amount of heat required to raise the temperature of 1 mole of gas through 1K at constant volume. It's SI unit is $J\,mol^{-1}K^{-1}$.

We have,

$$C_v = \frac{d(Q)_v}{n\,dT} \quad \text{——} \; (1)$$

The total mass m and molar mass M of a gas is

$m = nM$ where $n \rightarrow$ no of mole

So, eqn (1) become

$$C_v = \frac{(dQ)_v}{nM\,dT}$$

$$M C_v = \frac{(dQ)_v}{n\,dT}$$

$$C_v = \frac{(dQ)_v}{n\,dT}$$

where, $C_v = MC_v$ is the molar ~~constant~~ heat capacity at constant volume

ii) Molar heat capacity at constant pressure (C_p)

→ It is the amount of heat required to raise the temperature of 1 mole of gas through 1K at constant pressure. It's SI unit is $J\,mol^{-1}K^{-1}$.

Specific heat capacity at constant pressure is

$$C_p = \frac{(dQ)_p}{m\,dT}$$

$$c_p = \frac{(dQ)_p}{nM\,dT} \quad [\because m = nM]$$

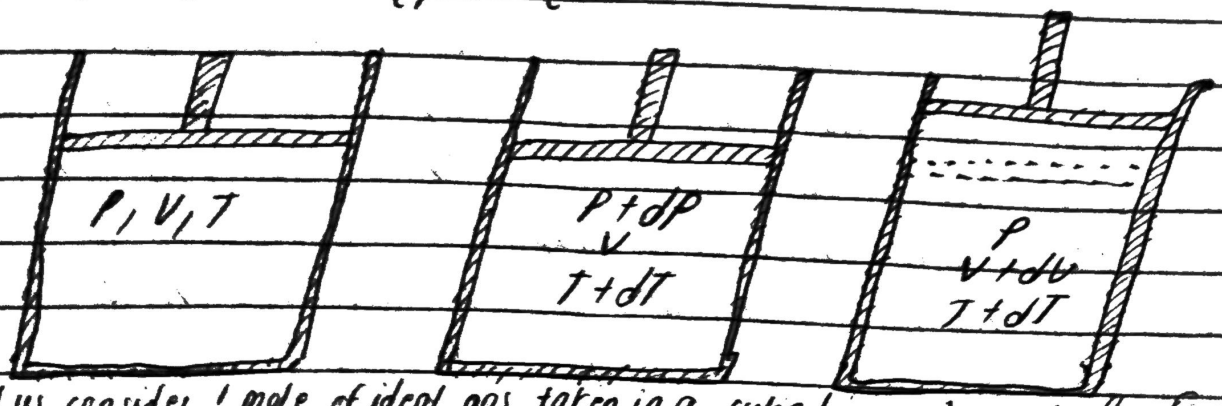
$$Mc_p = \frac{(dQ)_p}{n\,dT}$$

$$C_p = \frac{(dQ)_p}{n\,dT} \quad \text{where } C_p = Mc_p$$

C_p is greater than C_v

→ When a gas is heated with constant volume, gas does no work against external pressure and all the heat energy supplied to the system is used to increase temperature of gas ~~i.e.~~ that is internal energy. But if gas is heated at constant pressure, it's volume increase. So, heat energy is required not only to increase internal energy but also to do ~~ext~~ work against external pressure. Thus, more heat is required to raise the temperature of 1 mole of gas through 1K when it is heated at constant pressure than if it is heated at constant volume.

Relation between C_p and C_v



Let us consider 1 mole of ideal gas taken in a cylinder as shown in the figure.

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Let P, V and T be the pressure, volume and temp of gas respectively. Let the gas be heated at constant volume keeping the piston fixed so that temperature of gas increase by dt but no work is done. From first law of thermodynamics we have, $dQ = dU + dW$

i.e. $dQ = dU$ $[\because dW = 0]$

The amount of heat supplied to the system at constant volume is $(dQ)_v = dU = nC_v dT$

or, $dU = 1 \times C_v dT$ $[\because n = 1$, for 1 mole$]$

or, $dU = C_v dT$ —— (I)

where C_v is molar heat capacity at constant volume.

Again,

When the gas is heated at constant pressure so that its temperature increase by same amount dT.

In this case, supplied heat is used to increase internal energy and to do external work. The amount of heat supplied to the system at constant pressure is given by $(dQ)_p = 1 \times C_p dT$

$= C_p dT$ where molar heat capacity at constant pressure.

Again from first law of thermodynamics, we have

$(dQ)_p = dU + PdV$

i.e. $C_p dT = dU + PdV$ —— (II)

From (I) and (II) we get

$C_p dT = C_v dT + PdV$ —— (III)

~~For~~ For 1 mole of gas, ideal gas equation is

$PV = RT$

Differentiating above eqn, we get

$$d(PV) = d(RT)$$

$$PdV + VdP = RdT$$

For constant pressure $dP = 0$

$$\therefore PdV = RdT \quad \text{——— (iv)}$$

From eqn (iii) and (iv), we get

$$C_P dT = C_V dT + RdT$$

$$C_P = C_V + R$$

$$C_P - C_V = R \quad \text{——— (v)}$$

This is a relation between molar heat capacities for a gas

Dividing on both side on eqn (v) by molar mass of the gas

$$\frac{C_P}{M} - \frac{C_V}{M} = \frac{R}{M}$$

$$c_P - c_V = r \quad \text{where } r = \frac{R}{M} \text{ is gas constant of molar mass}$$

This is the relation of specific heat capacity of gas.

Thermodynamics process

A thermodynamic process takes place when some change occurs in a system the state of thermodynamic system. The change is due to the change in thermodynamic parameters which are pressure, volume and temperature. Some important thermodynamic process are as follows:

(1) Isothermal process

The process in which pressure and volume of a system change without any change in temperature is called isothermal process. In such a process, there is a free exchange of heat between system and surrounding. During isothermal process, the walls of the cylinder are perfectly conducting so that free exchange of heat between gas and its surrounding make the temperature constant. Also, the process of compression or expansion in this process is slow.

Eg: melting of ice at its melting point, vapourization of water at BP.

If a gas expands isothermally from initial state $A(P_1, V_1)$ to final state $B(P_2, V_2)$ then during isothermal process we have $P_1V_1 = P_2V_2 = \text{constant}$

This is the equation of isothermal process for an ideal gas

Application of 1st law of thermodynamic to isothermal process

From first law of thermodynamic, we have

$$dQ = d(U) + d(W) \quad \text{——— (1)}$$

In an isothermal process, there will be no change in the internal energy of the gas i.e $dU = 0$

So, eqn (1) becomes,

$$dQ = dW$$

Hence, in isothermal process, the heat supplied to the system is equal to work done by the system.

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Workdone by ideal gas during isothermal process

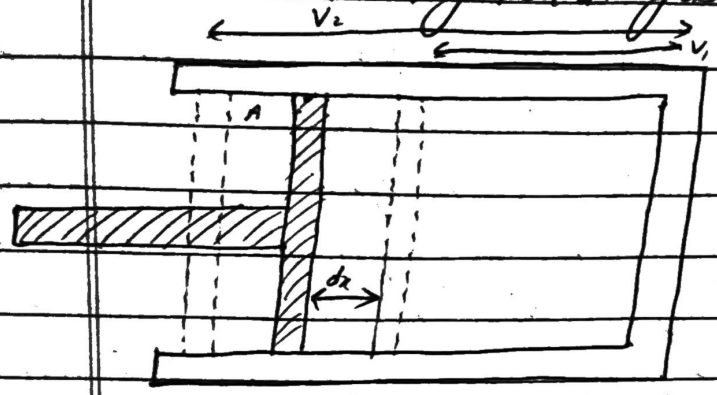

fig a) Workdone during isothermal process

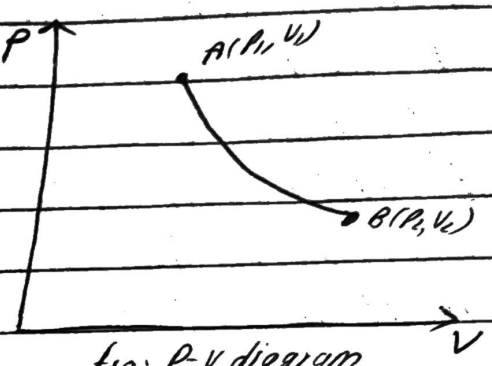

fig: P-V diagram for isothermal process

Consider 1 mole of an ideal gas enclosed in a cylinder having perfectly conducting walls fitted with movable piston which is frictionless. When a piston moves through a small distance dx, then small workdone dW is given by $dW = F dx$

$= PA\,dx$ where A is cross sectional area of piston

$= P\,dV$ —— (i)

When the gas expands from initial state $A(P_1, V_1)$ to final state $B(P_2, V_2)$ then total amount of workdone is given by

$$W = \int_{V_1}^{V_2} P\,dV \quad \text{—— (ii)}$$

For 1 mole of ideal gas, we have

$$PV = RT$$

$$\text{or, } P = \frac{RT}{V} \quad \text{—— (iii)}$$

Substituting the value of P in eqn (ii) we get

$$W = \int_{V_1}^{V_2} \frac{RT}{V}\, dV$$

$$= RT \int_{V_1}^{V_2} \frac{1}{V}\, dV$$

$$= RT \left[\log_e V \right]_{V_1}^{V_2}$$

$$= RT \left[\log_e V_2 - \log_e V_1 \right]$$

$$W = RT \log_e \left(\frac{V_2}{V_1} \right) \quad \text{——— (iv)}$$

For isothermal process, we have

$$P_1 V_1 = P_2 V_2$$

$$\frac{P_1}{P_2} = \frac{V_2}{V_1} \quad \text{——— (v)}$$

From (iv) and (v), we get

$$W = RT \log_e \left(\frac{P_1}{P_2} \right) \quad \text{——— (vi)}$$

Eqn (iv) and (vi) are the expression for workdone in an isothermal process for 1 mole of an ideal gas.

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② Adiabatic process

A thermodynamic process in which pressure, volume and temperature change but there is no exchange of heat between the system and surrounding is called adiabatic process.
In adiabatic process, no heat is allowed either to enter the system or to leave it. If a gas is suddenly compressed, external work done on it increases its internal energy and hence its temp rises or increase. Similarly if the gas suddenly expands, it does work against the surrounding in the expense of its internal energy and hence its temperature falls.
Thus, there is no exchange of heat between gas and its surrounding.

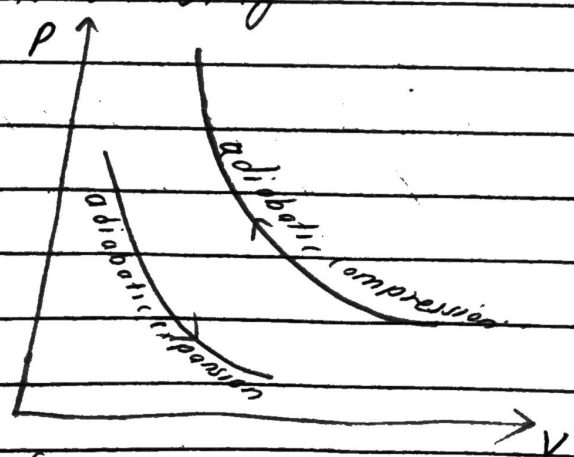

fig: P-V diagram during adiabatic process

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Application of 1st law of thermodynamics to adiabatic process

From 1st law of thermodynamic, we have,

$$dQ = dU + dW \quad \text{—— (1)}$$

In adiabatic process, no heat enters or leaves out of the system i.e $dQ = 0$

∴ eqn (1) becomes

$$0 = dU + dW$$

$$dU = -dW$$

When a gas expand adiabatically, dW is positive then $dU = -(+dW) = -dW$ i.e internal energy of the system decreases. On the other hand, when the gas contracts adiabatically, dW is negative then $dU = -(-dW) = +dW$ i.e internal energy of the system increases.

Equation of adiabatic process

Consider 1 mole of an ideal gas contained in a cylinder having perfectly insulating wall. Let the gas be compressed adiabatically, so that piston moves inward through a distance dx.

Let A be cross sectional area of piston.

∴ Workdone during adiabatic process is

$$dW = F\,dx$$
$$= PA\,dx$$
$$= P\,dV$$

Internal energy due to the rise in temperature dT is $dU = 1 \times C_v dT$ —— (ii)

where C_v is molar heat capacity at constant volume.

Also,

From 1st law of thermodynamics

$$dQ = dU + dW$$

For adiabic process, $dQ = 0$

$$\therefore 0 = dU + dW \quad \text{—— (iii)}$$

From eqn (i), (ii) and (iii), we have

$$C_v dT = PdV = 0 \quad \text{—— (iv)}$$

For 1 mole gas, we have

$$PV = RT$$

Differenting this eqn, we get

$$d(PV) = d(RT)$$

$$PdV + VdP = RdT$$

$$dT = \frac{PdV + VdP}{R}$$

Substituting the value of dT in eqn (iv)

$$C_v\left(\frac{PdV + VdP}{R}\right) + PdV = 0$$

or, $C_v PdV + C_v VdP + RPdV = 0$

or, $C_v PdV + C_v VdP + (C_p - C_v)PdV = 0$

or, $\cancel{C_v PdV} + C_v VdP + C_p PdV - \cancel{C_v PdV} = 0$

or, $C_v VdP + C_p PdV = 0$

Dividing both sides by $C_v PV$, we get

$$\frac{C_v V dP}{C_v PV} + \frac{C_p P dV}{C_v PV} = 0$$

$$\frac{dP}{P} + \gamma \frac{dV}{V} = 0 \quad \text{where } \gamma = \frac{C_p}{C_v}$$

Integrating on both sides

$$\int \frac{dP}{P} + \gamma \int \frac{dV}{V} = \text{constant}$$

or, $\log_e P + \gamma \log_e V = \text{constant}$

or, $\log_e P + \log_e V^\gamma = \text{constant}$

or, $\log_e (PV^\gamma) = \text{constant}$

$$\Rightarrow PV^\gamma = \text{constant}$$

This is the relation of adiabatic process relating pressure and volume of the gas.

Relation between temperature and volume

We have,

$$PV = RT \quad (\text{for 1 mole of ideal gas})$$

$$P = \frac{RT}{V}$$

Substituting the value of P in eqn (v), we get

$$\frac{RT}{V} \times V^\gamma = \text{constant}$$

$$RTV^{-1} \times V^\gamma = \text{constant}$$

$$RTV^{\gamma-1} = \text{constant}$$

$$TV^{\gamma-1} = \text{constant}.$$

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Relation between pressure and temperature.

For 1 mole of ideal gas, we have

$$PV = RT$$

$$V = \frac{RT}{P}$$

So, eqn ① becomes

$$P\left(\frac{RT}{P}\right)^{\gamma} = \text{constant}$$

$$P\frac{R^{\gamma}T^{\gamma}}{P^{\gamma}} = \text{constant}$$

$$P^{1-\gamma}T^{\gamma} = \text{constant}$$

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V/m

Workdone during an adiabatic process

Consider 1 mole of an ideal gas enclosed in a perfectly insulating walls of the cylinder fitted with frictionless piston. Let the gas expands adiabatically from the initial volume V_1 to final volume V_2 as shown in PV diagram. Then, workdone by the gas in this process is given by

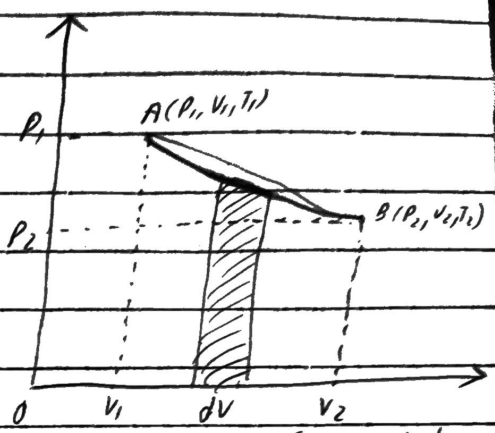

fig: PV diagram for workdone in adiabatic process

$$W_b = \int_{V_1}^{V_2} P\,dV \quad \text{——— (I)}$$

The adiabatic equation is

$$PV^{\gamma} = \text{constant} = k \text{ (say)} \quad \text{——— (II)}$$

where $\gamma = \frac{C_p}{C_v}$

$$\text{or, } P = \frac{k}{V^{\gamma}} \quad \text{——— (III)}$$

From eqn (I) and (III), we get

$$W = \int_{V_1}^{V_2} \frac{k}{V^{\gamma}}\,dV$$

$$= k\int_{V_1}^{V_2} V^{-\gamma}\,dV$$

$$= k\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}\left[V^{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{K}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right)$$

$$= \frac{1}{1-\gamma}\left(KV_2^{1-\gamma} - KV_1^{1-\gamma}\right)$$

But we have,

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} = K$$

So eqn (iv) become,

$$W = \frac{1}{1-\gamma}\left(P_2V_2^{\gamma}V_2^{1-\gamma} - P_1V_1^{\gamma}V_1^{1-\gamma}\right)$$

$$W = \frac{1}{1-\gamma}\left(P_2V_2 - P_1V_1\right)$$

For an ideal gas, we have

$$PV = RT \text{ (for 1 mole)}$$

So eqn (v) become

$$W = \frac{1}{1-\gamma}\left(RT_2 - RT_1\right)$$

$$\therefore W = \frac{R}{1-\gamma}\left(T_2 - T_1\right)$$

This is the required expression for workdone in an adiabatic process and it shows that workdone depends upon on only initial and final temperature.

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3) Isochoric process

The thermodynamic process that takes place at constant volume is called isochoric process. In this process the working substance is kept in a non expanding chamber. Since there is no change in volume, no workdone is done in this process. So, from 1st law of thermodynamics, we have

$$dQ = dU + dW$$

$$dQ = dU \quad [dW = 0 \text{ for isochoric process}]$$

Thus, whole of the heat supplied is used to increase the internal energy of the system.

The heat supplied in this process is given by

$$dQ = 1 \times C_v dT \quad [\because \text{For 1 mole of gas}]$$

$$\therefore dU = C_v dT$$

Not in syllabus

4) Isobaric process

The thermodynamic process that takes place at constant pressure is called isobaric process. In this process, the working substances is kept in expanding chamber.

The amount of heat dQ given to the system is partly used in increasing the temperature of working substance and partly used in doing external work.

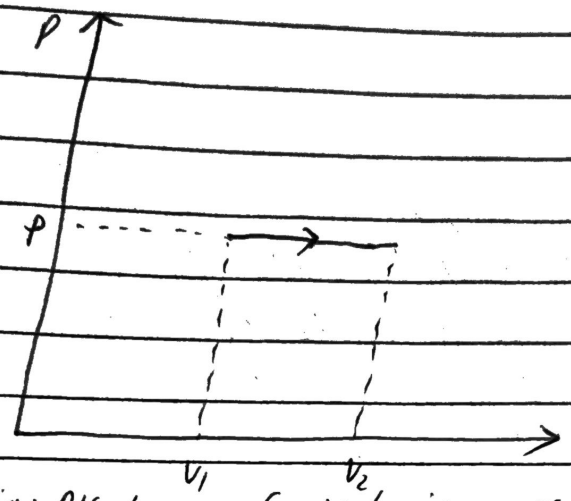

fig: PV diagram for isobaric process

Reversible process

The process which can be retraced in the opposite order (reversed order) so that it passes exactly through same state as in the direct process is called reversible process

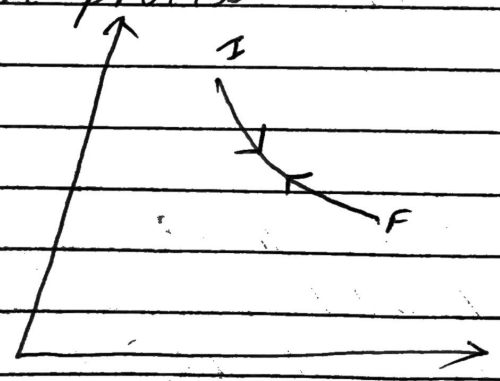

fig: PV diagram for reversible process

Examples: All the mechanical processes taking place under the action of conservative force

Irreversible Process

The process which can't be retraced in the opposite order by reversing the controlling factors is called irreversible process. Eg: All the process in nature.

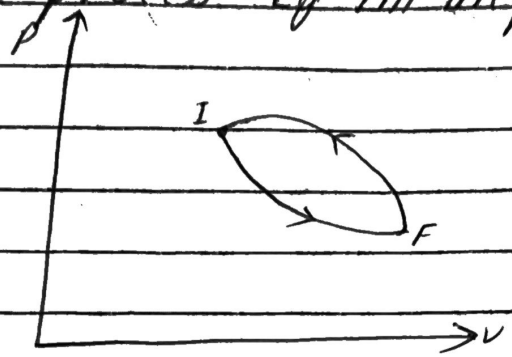

fig: PV diagram for irreversible process

Formulae

① $dQ = nC_v dT$

② $dQ = nC_p dT$

③ $dQ = dU + dW$

④ Ideal gas equation :- $PV = nRT$

⑤ Eqn of isothermal process :- $P_1V_1 = P_2V_2$

⑥ Work done during isothermal process :-

i) $W = nRT\log_e\left(\frac{V_2}{V_1}\right)$

ii) $W = nRT\log_e\left(\frac{P_1}{P_2}\right)$

⑦ Equation for adiabatic process

i) $PV^\gamma = \text{constant}$

i.e $P_1V_1^\gamma = P_2V_2^\gamma$

ii) $TV^{\gamma-1} = \text{constant}$

i.e $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

iii) $P^{1-\gamma} T^{\gamma} = \text{constant}$

$P_1^{1-\gamma} T_1^{\gamma} = P_2^{1-\gamma} T_2^{\gamma}$

(8) Workdone during adiabatic process

$$W = \frac{nR}{1-\gamma}(T_2 - T_1)$$

(9) $C_p - C_v = R$

(10) $C_p - C_v = \gamma$

(11) $\frac{C_p}{C_v} = \gamma$

(12) $PV = mRT$

(13) $n = \frac{m}{M}$

(14) At constant pressure

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Numericals

(1) A certain volume of dry air at NTP is allowed to expand five times of its original volume under adiabatic condition. Calculate final pressure and temperature.

Here,

At NTP, for dry air

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Pressure $(P_1) = 760$ mm of Hg

$= \frac{76}{100} \times 13600 \times 10$

$= 1.01 \times 10^5 \, Nm^{-2}$

Temperature $(T_1) = 273K$

let initial volume $(V_1) = V$ (say

final volume $(V_2) = 5V$

final pressure $(P_2) = ?$

final temperature $(T_2) = ?$

Ratio of molar capacity $(\gamma) = 1.4$

For adiabatic expansion, we have

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

$1.01 \times 10^5 \times V^{\gamma} = P_2 \times (5V)^{\gamma}$

$P_2 = \frac{1.01 \times 10^5 \times V^{\gamma}}{5^{\gamma} \times V^{\gamma}}$

$= \frac{1.01 \times 10^5}{5^{1.4}}$

$= 1.06 \times 10^4 \, N/m^2$

Again, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$T_2 = \frac{T_1 \times V^{\gamma-1}}{5^{\gamma-1} V^{\gamma-1}}$

$= \frac{373}{5^{1.4-1}}$

$= 196 K$

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2) A certain volume of dry air at NTP is allowed to expand four time of its original volume under i) isothermal conditions ii) adiabatic condition. Calculate final pressure and temperature in each case ($\gamma = 1.4$)

Here,

Initial temperature (T_1) = 273K

Initial pressure (P_1) = $1.01 \times 10^5 \, Nm^{-2}$

Initial volume (V_1) = V (let)

$\gamma = 1.4$

Final pressure (P_2) = ?

Final temperature (T_2) = ?

i) When air expanded isothermally, to 4 time its volume

$V_2 = 4V$

Then, $P_2 V_2 = P_1 V_1$ [isothermal condition]

$$P_2 = \frac{P_1 V_1}{V_2} = \frac{1.01 \times 10^5 \times V}{4V} = 25250 \, N/m^2$$

Again, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ or, $T_2 = \frac{V_2 \times T_1}{V_1} = \frac{4V \times 273}{V} = 1092K$

ii) When air is expanded adiabatically to 4 time it volume

$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$

or, $P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$

$= 1.01 \times 10^5 \times \left(\frac{1}{4}\right)^{1.4}$

$= 1.01 \times 10^5 \times 0.1436$

$= 14502.3 \, N/m^2$

Again,

$T_1 V_1^{\gamma - 1} = T_2 V_2^{\gamma - 1}$

$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma - 1}$

$= 273 \left(\frac{V}{4V}\right)^{1.4 - 1}$

$= 157 K$

3) Air is compressed adiabatically half of its volume. Calculate the change in temperature

Here,

let Initial volume $(V_1) = V$ (say)

Final volume $(V_2) = \frac{V}{2}$

let T_1 and T_2 be initial and final temperature.

For adiabatic process,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_1 V^{\gamma-1} = T_2 \left(\frac{V}{2}\right)^{\gamma-1}$$

$$T_1 V^{1.4-1} = \frac{T_2 V^{1.4-1}}{2^{1.4-1}}$$

$$T_1 \times 2^{0.4} = T_2$$

$$\frac{T_2}{T_1} = 2^{0.4} = 1.32$$

$$\frac{T_2}{T_1} - 1 = 1.32 - 1$$ [subtracting 1 on both side]

$$\frac{T_2 - T_1}{T_1} = 0.32$$

$$\left(\frac{T_2 - T_1}{T_1}\right) \% = 32\%$$

Hence, temperature is changed by 32%

4) An ideal gas initially at 4 atm and 300K is permitted to expand adiabatically twice its initial volume. Find the final pressure and temperature if the gas is i) monoatomic ($\gamma = \frac{5}{3}$) ii) diatomic with $C_v = \frac{5R}{2}$

Here,

Initial pressure (P_1) = 4 atm

Final pressure (P_2) = ?

Initial temperature (T_1) = 300K

Final temperature (T_2) = ?

Initial volume (V_1) = V

Final volume (V_2) = 2V

i) For mono-atomic gas,

$$\gamma = \frac{5}{3} = 1.67$$

We have,

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_2 = \frac{4}{2^{1.67}} = 1.257 \text{ atm}$$

Also,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300 \times \left(\frac{1}{2}\right)^{0.67}$$

$$= 188.55K$$

ii) For diatomic $\gamma = 7/5 = 1.4$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$= 4 \times \left(\frac{V}{2V}\right)^{1.4}$$

$$= 4 \times \left(\frac{1}{2}\right)^{1.4}$$

$$= 1.52 \text{ atm}$$

Also,

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300 \times \left(\frac{1}{2}\right)^{0.4}$$

$$= 227.36 K$$

5) A gas in a cylinder initially at a temperature of 10°C and one atm is to be compressed adiabatically to 1/8 of its volume. Find final temperature (Take $\gamma = 1.4$)

Here,

Initial Temp (T_1) = 10°C = 273+10 = 283K

Initial pressure (P_1) = 1atm = 1.01×10^5 N/m²

Final Temp (T_2) = ?

$\gamma = 1.4$

Initial volume (V_1) = V

Final volume (V_2) = $\frac{V}{8}$

We have

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

or, $$283\, V^{\gamma-1} = T_2 \left(\frac{V}{8}\right)^{\gamma-1}$$

or, $$283\, V^{1.4-1} = T_2 \left(\frac{V}{8}\right)^{1.4-1}$$

$$\therefore T_2 = 283 \times 8^{1.4-1}$$

$$T_2 = 650\,K$$

6) Five moles of an ideal gas are kept at constant temperature of 53°C while the pressure of the gas is increased from 1 atm to 3 atm. Calculate work done by gas.

Here,

No of mole $(n) = 5$

Temperature $(T) = 53°C = 273 + 53 = 326\,K$

Initial pressure $(P_1) = 1\,atm = 1 \times 10^5\ N/m^2$

Final pressure $(P_2) = 3\,atm = 3 \times 10^5\ N/m^2$

Work done $(W) = ?$

We know work done in isothermal pressure

$$W = nRT \log\left(\frac{V_2}{V_1}\right)$$

$$W = nRT \log\left(\frac{P_1}{P_2}\right)$$

$$= 5 \times 8.31 \times 326 \log\left(\frac{1}{3}\right)$$

$$= -14881\,J$$

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31 If the ratio of specific heat capacities of a gas is 1.4 and its ~~at~~ density at STP is 0.09 kg/m^3, calculate the value of specific heat capacities at constant pressure and at constant volume.

Here,

Ratio of specific heat capacities $(\gamma) = 1.4$

Density of gas at STP $(\rho) = 0.09\ kg\,m^{-3}$

Specific heat capacity of gas at constant pressure $(r) = ?$

Specific heat capacity of gas at constant volume $((v) = ?$

We have ideal gas equation

$$PV = mrT$$

$$\text{or, } r = \frac{PV}{mT} = \frac{P}{\frac{m}{V}T} = \frac{P}{\rho T}$$

$$= \frac{1.01 \times 10^5}{0.09 \times 273}$$

$$= 4110.7\ J\,kg^{-1}K^{-1}$$

Now,

$$\frac{C_p}{C_v} = 1.40$$

$$\text{or, } C_p = 1.4\,C_v$$

But, $C_p - C_v = r$

$$\text{or, } 1.4\,C_v - C_v = r$$

$$\text{or, } 0.4 \times C_v = 4110.7$$

$$C_v = 10276.76$$

$$= 1.03 \times 10^4\ J\,kg^{-1}K^{-1}$$

$$C_p = 1.4\,C_v = 1.4 \times 1.03 \times 10^4 = 1.44 \times 10^4\ J\,kg^{-1}K^{-1}$$

8) An ideal gas is slowly compressed at constant temp of 50°C to one half of its original volume. In this process, 80 cal of heat was given. How much work was done and what was the change in the internal energy of the gas?

Here

Constant temperature $(T) = 50°C$
$= 50 + 273$
$= 323 K$

Initial volume $(V_1) = V$ (say)

Final volume $(V_2) = \frac{V}{2}$

Heat given $(Q) = 80$ cal
$= 80 \times 4.2 J$
$= 336 J$

Number of mole of gas $(n) = 1$

$\therefore$ Work done is given by

$$W = RT \log_e \frac{V_2}{V_1}$$

$$= 8.3 \times 323 \times \log_e \frac{V}{2V}$$

$$= -807.03 J$$

Change in internal energy $= 336 + 807.63$
$= 1143.03 J$

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9) The density of the gas is $1.775\ kg\,m^{-3}$ at $27°C$ and $10^5\ Nm^{-2}$ pressure. If the specific heat capacity at constant pressure is $846\ Jkg^{-1}K^{-1}$. Find the ratio of specific heat capacity at constant pressure to that at constant volume.

Here,

Density of gas $(\rho) = 1.775\ kg/m^3$

Temperature of gas $(T) = 27°C = 27+273 = 300K$

Pressure $(P) = 10^5\ N/m^2$

Specific heat capacity at constant pressure $(C_p) = 846\ Jkg^{-1}K^{-1}$

$\gamma = ?$

We have,

$$PV = nRT$$

$$PV = m\gamma T$$

$$\gamma = \frac{PV}{mT}$$

$$= \frac{P}{\rho T}$$

$$= \frac{10^5}{1.775 \times 300}$$

$$= 187.8\ Jkg^{-1}K^{-1}$$

Again, $C_p - C_v = \gamma$

$$846 - C_v = 187.8$$

$$C_v = 846 - 187.8$$

$$= 658.2\ J/kg\,K$$

Also, $\gamma = \frac{C_p}{C_v} = \frac{846}{658.2} = 1.29$

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10 In a certain chemical process, a lab technician supplies of heat to a system. At the same time, 73J of work are done on the system by its surroundings. What is the increase in the internal energy of the system?

Here,

Amount of heat supplied $(dQ) = 254J$

Workdone by surrounding $(dW) = -73J$

Increase in internal energy, $dV = ?$

From 1st law of thermodynamics, we have

$$dQ = dV + dW$$

$$254 = dV - 73$$

$$dV = 254 + 73 = 327J$$

Increase in internal energy $= 327J$

11 Two moles of an ideal gas are heated at constant pressure from $T = 27°C$ to $107°C$. Calculate the work done by the gas.

Here,

Number of moles $(n) = 2$

Initial temperature $(T_1) = 27°C = 300K$

Final temperature $(T_2) = 107 = 380K$

Work done by gas $(dW) = ?$

We have

$$dW = PdV = P(V_2 - V_1)$$

$$= PV_2 - PV_1$$

$$= nRT_2 - nRT_1 \qquad [\because PV = nRT]$$

$= nR(T_2 - T_1)$

$= 2 \times 8.314(380-300)$

$= 1330 J$

$\therefore$ Amount of workdone $(dW) = 1330 J$

11. A gas is suddenly compressed to $1/4^{th}$ of its original volume. Calculate the rise in temperature. The original temperature being $27°C$ and $\gamma = 1.5$

Here,

let the initial volume $(V_1) = V$

Final volume $(V_2) = V/4$

Initial temperature $(T_1) = 27+273$

$= 300K$

let $T_2 K$ be final temperature after compression

Since the change is adiabatic

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300 \times \left(\frac{V}{V/4}\right)^{1.5-1}$$

$$= 300 \times (4)^{1/2}$$

$$= 600K$$

$$= 600 - 273 = 327°C$$

$\therefore$ Rise in temperature $= 327 - 27°C$

$= 300°C$