

## Lab 5 – DFT and FFT

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**Objectives:** In this lab we will use DFT to process signals

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### 5.1 DFT for frequency analysis of CT signals

Reading prerequisite: Section 7.4 of the Proakis textbook

Consider the signal  $p[n] = \cos(2\pi f_0 n / f_s)$ . We want to estimate frequency  $f_0$  of  $p[n]$  from finite number of its samples.

(a) [DTFT] What is the Fourier transform  $P(e^{j\omega})$  of  $p[n]$ ?

(b) From (a), what is the relation between the location of the impulses?

Consider a finite number of samples of  $p[n]$  which can be obtained by the windowing operation  $x[n] = p[n] \times w[n]$  where  $w[n]$  is the rectangular window function which is 1 for  $0 \leq n \leq L - 1$  and 0 otherwise.

(c) [DTFT] What will be the Fourier transform  $X(e^{j\omega})$  of  $x[n]$ ?

Hint: multiplication property of DTFT. What is the effect of the window on spectrum? (focus on magnitude spectrum for simplicity)

Any practical signal processing will involve a window function  $w[n]$  of some kind to get finite length sequences. We now consider  $x[n]$  as an L-length sequence and study it using DFT.

Write a matlab script for the following tasks.

(d) Let  $f_0 = 12 \text{ Hz}$  and  $f_s = 64 \text{ Hz}$ . Generate samples  $x[n]$  for  $L = 16$ . Compute its DFT  $X[k]$  of length-N using the `fft` command and plot magnitude of DFT for  $N = mL$  where  $m = \{1, 2, 4, 8\}$ . Plot all of them in a single 2x2 figure. Are your plots consistent with the answer in part (c)? (read about spectral leakage from Proakis 7.4).

Note that `fft` command automatically performs required zero-padding to compute length-N DFT (if signal length  $L < N$ ). For  $m = \{1, 2\}$  use `stem`, for  $m = \{4, 8\}$  use `plot`.

- (e) Now repeat above for changing  $L = \{16, 32, 64, 128\}$  and fixed  $N = 8L$ . Plot magnitude spectrum in a single 2x2 figure. Comment on spectrum shape as  $L$  changes? What can you conclude about the length of the signal ( $L$ ) and the frequency resolution?
- (f) Repeat (d) when  $f_0 = 11 \text{ Hz}$ .
- (g) Repeat (d) when  $w[n]$  is an  $L$ -length Hanning window. Use the matlab command `hann` to get this window. Comment on changes in main-lobe width and spectral leakage compared to the figure in part (d).
- (h) From the plots of DFT magnitude spectrum, how would you estimate  $f_0$  given  $f_s$ ? Use this method to estimate  $f_0$  in part (d), (f) and (g). Does  $N$  affect your answers?
- (i) Load one of the audio files shared in the repository, you should load the file number given by `roll_number` (modulo 10). Use the analysis above to find the three strongest frequencies (in Hz) present in the audio signal. You are encouraged to process other files as well.

## 5.2 Direct and DFT based convolutions

We can compute time-domain convolutions using various methods. In this matlab script we will compute linear convolution and circular convolution of a pair of signals using two methods for each.

- (a) Generate two finite length sequences as follows. The sequence  $x_1[n]$  is a random Gaussian sequence of length 10 and  $x_2[n]$  is first 10 samples of the signal  $\delta[n - 3]$  starting from  $n = 0$ .
- (b) Perform linear and circular convolutions of  $x_1[n]$  and  $x_2[n]$  directly using the commands `cconv` and `conv`, respectively (read up MATLAB documentation of these commands). Make sure each result is of the expected length.

- (c) Perform linear and circular convolutions of  $x_1[n]$  and  $x_2[n]$  using the DFT method. You must perform these convolutions using DFT and inverse DFT of the appropriate signals. Use the matlab commands `fft` and `ifft` to compute the DFT and inverse DFT of the signals. Make sure each result is of the expected length.
- (d) Plot the four outputs in a single 2x2 figure and verify that the two methods (direct and DFT based) give same answer for linear convolution. Similarly for circular convolution.

### 5.3 DFT of some signals

In this task we will compute DFT of some standard signals using the built-in `fft` command. In a matlab script compute DFT for various N-length sequences given as follows:

- a) `[ones(L,1); zeros(N-L,1)]`, for fixed  $L = 4$  repeat for  $N = 4, 16, 64$
- b)  $\sin(\omega_0 n)$ , for  $\omega_0 = 3\pi/10$  and  $N = 20$
- c)  $\cos(\omega_0 n)$ , for  $\omega_0 = 3\pi/10$  and  $N = 20$
- d)  $\sin(\omega_0(n - 1))$ ,  $\omega_0 = 3\pi/10$  and  $N = 20$
- e)  $(0.8)^n$  for  $N = 20$
- f)  $(-0.8)^n$  for  $N = 20$

In the same 3x1 figure, plot the sequence and the magnitude and phase spectrum of its DFT. Can you identify the low frequencies and high frequencies from the spectrum?