Lab Report 1

Sannidhya Gupta - `2021112012`

1.1: Finding Fourier Series coefficients

function present in fourierCoeff.m in the code folder

`(a)` → Script given in code folder (q1_a.m)

`(b)` → Script given in code folder (q1_b.m)

`(c)` → Script given in code folder (q1_c.m)

Here the Fourier coefficients have been computed for the expression: $2cos(10\pi t)$

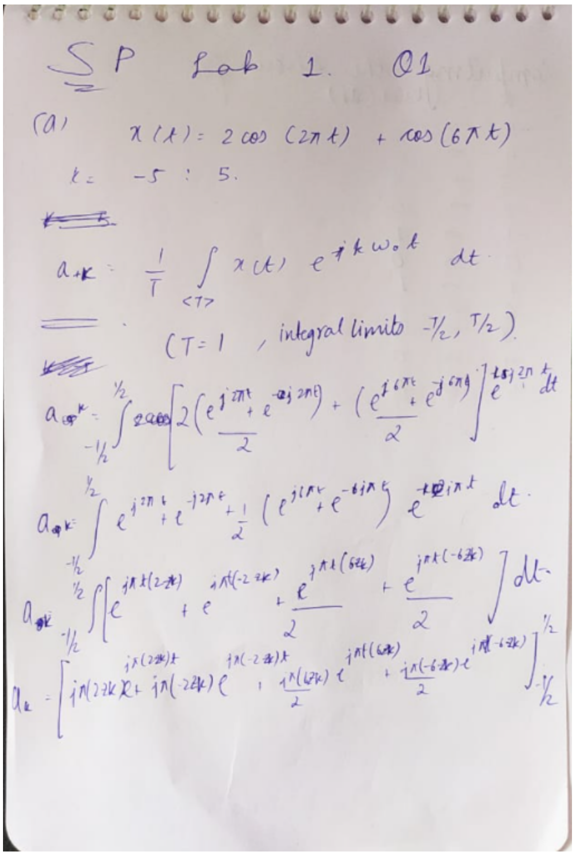
1.2: FS reconstruction and finite FS approximation error

function present in partialfouriersum.m in the code folder

`(a)`

Here, the first script and function are copied into the current script to generate the input signal's Fourier coefficients and then passed through the function "partialfouriersum" to reconstruct the original signal.

Verifying analytically:


SP Lab 1. Q1

(a) $x(t) = 2\cos(2\pi t) + \cos(6\pi t)$

$t = -5 : 5.$

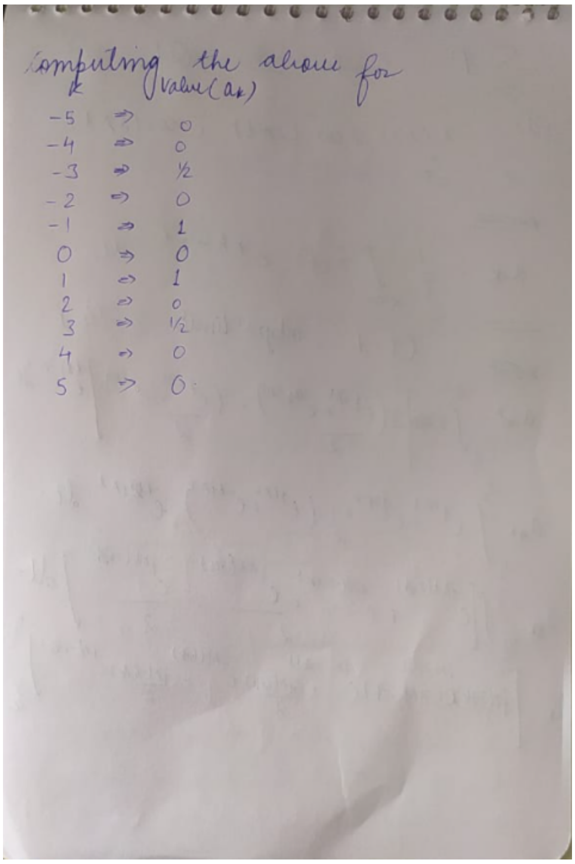
$$a_k = \frac{1}{T}\int_{<T>} x(t)\, e^{-jk\omega_0 t}\, dt$$

$(T = 1$, integral limits $-T/2, T/2)$

$$a_k = \int_{-1/2}^{1/2} \left[2\left(\frac{e^{j2\pi t} + e^{-j2\pi t}}{2}\right) + \left(\frac{e^{j6\pi t} + e^{-j6\pi t}}{2}\right) \right] e^{-j2\pi k t}\, dt$$

$$a_k = \int_{-1/2}^{1/2} \left[e^{j2\pi t} + e^{-j2\pi t} + \frac{1}{2}\left(e^{j6\pi t} + e^{-j6\pi t}\right) \right] e^{-j2\pi k t}\, dt$$

$$a_k = \int_{-1/2}^{1/2} \left[e^{j\pi t(2-2k)} + e^{j\pi t(-2-2k)} + \frac{e^{j\pi t(6-2k)}}{2} + \frac{e^{j\pi t(-6-2k)}}{2} \right] dt$$


Computing the above for

k		Value(a_k)
-5	⇒	0
-4	⇒	0
-3	⇒	½
-2	⇒	0
-1	⇒	1
0	⇒	0
1	⇒	1
2	⇒	0
3	⇒	½
4	⇒	0
5	⇒	0

`(b)` → Script given in code folder (q2_b.m)

The script plots the original signal and the output of the reconstructed wave from `(a)` in the same plot.

`(c)` → Script given in code folder (q2_c.m)

Obtained output:

```
MAE = 1.3323e-15
RMS error = 1.7827e-08
```

`(d)` → Script given in code folder (q2_d.m)

Obtained output: (for square wave):

N = 1 MAE = 0.5 RMS Error = 0.41399
N = 2 MAE = 0.5 RMS Error = 0.41399
N = 3 MAE = 0.5 RMS Error = 0.33028
N = 4 MAE = 0.5 RMS Error = 0.33028
N = 5 MAE = 0.5 RMS Error = 0.2853
N = 6 MAE = 0.5 RMS Error = 0.2853
N = 7 MAE = 0.5 RMS Error = 0.25733
N = 8 MAE = 0.5 RMS Error = 0.25733
N = 9 MAE = 0.5 RMS Error = 0.23644
N = 10 MAE = 0.5 RMS Error = 0.23644
N = 11 MAE = 0.5 RMS Error = 0.22272
N = 12 MAE = 0.5 RMS Error = 0.22272
N = 13 MAE = 0.5 RMS Error = 0.20876
N = 14 MAE = 0.5 RMS Error = 0.20876
N = 15 MAE = 0.5 RMS Error = 0.19646
N = 16 MAE = 0.5 RMS Error = 0.19646
N = 17 MAE = 0.5 RMS Error = 0.19128
N = 18 MAE = 0.5 RMS Error = 0.19128
N = 19 MAE = 0.5 RMS Error = 0.18491
N = 20 MAE = 0.5 RMS Error = 0.18491
N = 21 MAE = 0.5 RMS Error = 0.17886
N = 22 MAE = 0.5 RMS Error = 0.17886
N = 23 MAE = 0.5 RMS Error = 0.17312
N = 24 MAE = 0.5 RMS Error = 0.17312
N = 25 MAE = 0.5 RMS Error = 0.16375
N = 26 MAE = 0.5 RMS Error = 0.16375
N = 27 MAE = 0.5 RMS Error = 0.15999
N = 28 MAE = 0.5 RMS Error = 0.15999
N = 29 MAE = 0.5 RMS Error = 0.1547
N = 30 MAE = 0.5 RMS Error = 0.1547
N = 31 MAE = 0.5 RMS Error = 0.15107
N = 32 MAE = 0.5 RMS Error = 0.15107
N = 33 MAE = 0.5 RMS Error = 0.1508
N = 34 MAE = 0.5 RMS Error = 0.1508
N = 35 MAE = 0.5 RMS Error = 0.14901
N = 36 MAE = 0.5 RMS Error = 0.14901
N = 37 MAE = 0.5 RMS Error = 0.14662

Observations:

It is observed that, when the value of N increases, the maximum absolute error remains the same, however, the RMS error (which gives more of an idea for deviated the reconstruction is from the original) gets reduced.

So, in order to do a more precise reconstruction, we can increase the number of fourier coefficients we take into consideration for reconstruction of the signal.

N = 38 MAE = 0.5 RMS Error = 0.14662
N = 39 MAE = 0.5 RMS Error = 0.14571
N = 40 MAE = 0.5 RMS Error = 0.14571
N = 41 MAE = 0.5 RMS Error = 0.14458
N = 42 MAE = 0.5 RMS Error = 0.14458
N = 43 MAE = 0.5 RMS Error = 0.14448
N = 44 MAE = 0.5 RMS Error = 0.14448
N = 45 MAE = 0.5 RMS Error = 0.14533
N = 46 MAE = 0.5 RMS Error = 0.14533
N = 47 MAE = 0.5 RMS Error = 0.14795
N = 48 MAE = 0.5 RMS Error = 0.14795
N = 49 MAE = 0.5 RMS Error = 0.15987
N = 50 MAE = 0.5 RMS Error = 0.15987
N = 51 MAE = 0.5 RMS Error = 0.14748
N = 52 MAE = 0.5 RMS Error = 0.14748
N = 53 MAE = 0.5 RMS Error = 0.1435
N = 54 MAE = 0.5 RMS Error = 0.1435
N = 55 MAE = 0.5 RMS Error = 0.1408
N = 56 MAE = 0.5 RMS Error = 0.1408
N = 57 MAE = 0.5 RMS Error = 0.13863
N = 58 MAE = 0.5 RMS Error = 0.13863
N = 59 MAE = 0.5 RMS Error = 0.13708
N = 60 MAE = 0.5 RMS Error = 0.13708
N = 61 MAE = 0.5 RMS Error = 0.13511
N = 62 MAE = 0.5 RMS Error = 0.13511
N = 63 MAE = 0.5 RMS Error = 0.13307
N = 64 MAE = 0.5 RMS Error = 0.13307
N = 65 MAE = 0.5 RMS Error = 0.13248
N = 66 MAE = 0.5 RMS Error = 0.13248
N = 67 MAE = 0.5 RMS Error = 0.13147
N = 68 MAE = 0.5 RMS Error = 0.13147
N = 69 MAE = 0.5 RMS Error = 0.13056
N = 70 MAE = 0.5 RMS Error = 0.13056
N = 71 MAE = 0.5 RMS Error = 0.12901
N = 72 MAE = 0.5 RMS Error = 0.12901
N = 73 MAE = 0.5 RMS Error = 0.12714
N = 74 MAE = 0.5 RMS Error = 0.12714
N = 75 MAE = 0.5 RMS Error = 0.12477

N = 76 MAE = 0.5 RMS Error = 0.12477

N = 77 MAE = 0.5 RMS Error = 0.12364

N = 78 MAE = 0.5 RMS Error = 0.12364

N = 79 MAE = 0.5 RMS Error = 0.1243

N = 80 MAE = 0.5 RMS Error = 0.1243

N = 81 MAE = 0.5 RMS Error = 0.12409

N = 82 MAE = 0.5 RMS Error = 0.12409

N = 83 MAE = 0.5 RMS Error = 0.12436

N = 84 MAE = 0.5 RMS Error = 0.12436

N = 85 MAE = 0.5 RMS Error = 0.12399

N = 86 MAE = 0.5 RMS Error = 0.12399

N = 87 MAE = 0.5 RMS Error = 0.12318

N = 88 MAE = 0.5 RMS Error = 0.12318

N = 89 MAE = 0.5 RMS Error = 0.12386

N = 90 MAE = 0.5 RMS Error = 0.12386

N = 91 MAE = 0.5 RMS Error = 0.12357

N = 92 MAE = 0.5 RMS Error = 0.12357

N = 93 MAE = 0.5 RMS Error = 0.12413

N = 94 MAE = 0.5 RMS Error = 0.12413

N = 95 MAE = 0.5 RMS Error = 0.12504

N = 96 MAE = 0.5 RMS Error = 0.12504

N = 97 MAE = 0.5 RMS Error = 0.1268

N = 98 MAE = 0.5 RMS Error = 0.1268

N = 99 MAE = 0.5 RMS Error = 0.13391

N = 100 MAE = 0.5 RMS Error = 0.13391

1.3: Gibbs phenomenon - revisit square wave

`(a)`

Here, we calculate the FS coefficients by using the fourierCoeff function from part `1.1: (a)` and send the required wave as input. Here, the construction of the wave by using `piecewise()` takes up a lot of processing power, so instead of writing the expression for the wave, we just take the wave to be = 1 at all times, and perform the integration in the FS coefficient calculation from $[-T_1, T_1]$

`(b)` → **Script given in code folder (q3_b.m)**

The script plots the scaled FS coefficients of the periodic square wave in `(a)`. Here the script can be modified in order to obtain graphs for various values of T.

`(c)` → **Script given in code folder (q3_c.m)**

Here FS reconstruction is done by using the function partialfouriersum from part `1.2: (a)` and the script here can be modified in order to plot the reconstruction for various values of N.
We can see that on increasing the value of N or considering a wider range of FS coefficients, we get a more accurate and precise reconstruction, with less amount of deviation from the original periodic wave.

1.4: Fourier series – more examples and symmetry properties

`(a)` → **Script given in code folder (q4_a.m)**

Here the wave $x_1(t)$ has been constructed by using the piecewise function, and FS coefficients have been calculated and plotted by using fourierCoeff function from part `1.1: (a)`

`(b)` → **Script given in code folder (q4_b.m)**

Here the wave $x_2(t)$ has been constructed by using the piecewise function, and FS coefficients have been calculated and plotted by using fourierCoeff function from part `1.1: (a)`

In both the above parts, N (a parameter for fourierCoeff) has been taken to be equal to 20.

`(c)`

The FS coefficients are symmetric → $a_k = a_{-k}$

The Fourier series coefficients seem to be decreasing in $x_1(t)$ as k increases
The Fourier series coefficients seem to be increasing in $x_2(t)$ as k increases

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