Lab 4 – Discrete-time FT and LTI systems

Objectives: In this lab we will

- numerically compute DTFT of signals
- study a few discrete-time LTI systems

4.1 Discrete-time Fourier transform (DTFT)

The DTFT of any discrete-time signal x[n] is given as

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n]e^{-j\omega n}$$

(a) Write a **matlab function** of the form `X = DT_Fourier(x,N0,w)` that takes as inputs

- x, a discrete-time signal of finite duration (assume that the signal is zero elsewhere)
- N_0, location of the sample x[0] in the given input signal x, note that N_0 is a positive integer between 1 and length(x)
- ω, a vector of frequencies at which to compute the DTFT (though frequency is a continuous variable in DTFT we can evaluate it at only finite set of points)

The function should return X, a complex vector corresponding to the DTFT computed at the frequencies in ω. Write the function using **at most** one for-loop.

(b) Write a **matlab script** which calls the `DT_Fourier()` function. Set $\omega = -10:0.01:10$ for computing the DTFT. Your script should compute DTFT for each of the following discrete-time signals:

1. unit impulse $\delta[n]$
2. shifted unit impulse $\delta[n+3]$
3. rectangular pulse from -3 to 3
4. finite duration sinusoid $\sin(\omega_0 n)$ with $\omega_0 = \frac{\pi}{4}$ for -200 to 200

Appropriately choose inputs x and N_0 for each of the signals. For each signal, plot the DTFT spectrum (i.e. magnitude, phase, real, imaginary parts) in a 2x2 figure. Compare your plots with the analytical answers worked out in class.

(c) In the same script, compute DTFT for the signal $a^n u[n]$**.** Restrict your signal (n = 0 to 100) for finite computations. Set $\omega = -10:0.01:10$**.** In a 2x2 figure, plot two time domain signals (corresponding to $a = b$ and $a = -b$, values of b are given below) in the top panels and their DTFT magnitude spectrum in the bottom panels. Do this for b = 0.01, 0.5, 0.99 and note your observations as b changes.

4.2 Discrete-time filters

An order-M moving average filter is a discrete-time LTI system with input x[n] and output y[n] relation given by

$$y[n] = \frac{1}{M} \sum_{m=0}^{M-1} x[n-m]$$

(a) What is the impulse response h[n] of this LTI system?

(b) Look up the matlab convolution command `conv()`. You must implement the above filter using this command (do not write any for-loops)

(c) In a **matlab script** generate the sine wave $s[n] = 5\sin(\omega_0 n)$ and its noisy version given by $x[n] = s[n] + v[n]$. Let $\omega_0 = \frac{\pi}{200}$ and generate this for n = 0 to 1000. Here $v[n]$ is the Gaussian noise which you can generate using the command `randn()`. In one panel of a 2x2 figure plot $s[n]$ and $x[n]$ on top of each other.

(d) Filter the signal $x[n]$ with a moving average filter using the command `conv()` and set the shape parameter to 'full'. Do this for M = 5, 21, 51. For each M, plot the original signal $s[n]$ and the filtered signal $y[n]$ on top of each other in the remaining panels.

(e) What are your observations as M is changed? What are the trade-offs?

(f) Use your function `DT_Fourier()` to compute the DTFT of the noisy and the filtered signals for different M. Set $\omega = -10: 0.01: 10$. Plot the magnitude spectrum in a 2x2 figure. Note your observations.

(g) A simple digital differentiator has input output relation given by

$$y[n] = x[n] - x[n-1]$$

Repeat steps (a), (c), (d) & (f) for this filter and note your observations.

(h) In terms of frequency selectivity, what is the nature of all the above implemented filters?

4.3 Inverse DTFT (script)

(a) Write a matlab script which numerically computes inverse DTFT using the `int()` command. Recall that the inverse DTFT is given by the expression

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) e^{j\omega n}\, d\omega.$$

You must compute the inverse DTFT for the frequency domain rectangular wave which in the interval $[-\pi, \pi]$ is given by

$$X\left(e^{j\omega}\right) = \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| < \pi \end{cases}$$

Let $\omega_c = \frac{\pi}{16}$. Compute the signal $x[n]$ numerically for n = -100 to 100 and plot it as function of time n. Is $x[n]$ expected to be real or complex valued? Plot accordingly.

(b) Repeat above when $\omega_c = \frac{\pi}{8}$, $\omega_c = \frac{\pi}{4}$ and $\omega_c = \frac{\pi}{2}$ and compare your observations. What happens when $\omega_c = \pi$. Can you explain this observation using theory?

(c) Repeat part (a) when the DTFT is given by the band-pass signal of the form

$$X\left(e^{j\omega}\right) = \begin{cases} 1, & \omega_1 \le |\omega| \le \omega_2 \\ 0, & |\omega| < \omega_1 \ \ and\ \omega_2 < |\omega| < \pi \end{cases}$$

Let $\omega_1 = \frac{\pi}{8}$ and $\omega_2 = \frac{\pi}{4}$. Try another set of values for ω_1 and ω_2 and note your observations.