Report - Lab 3

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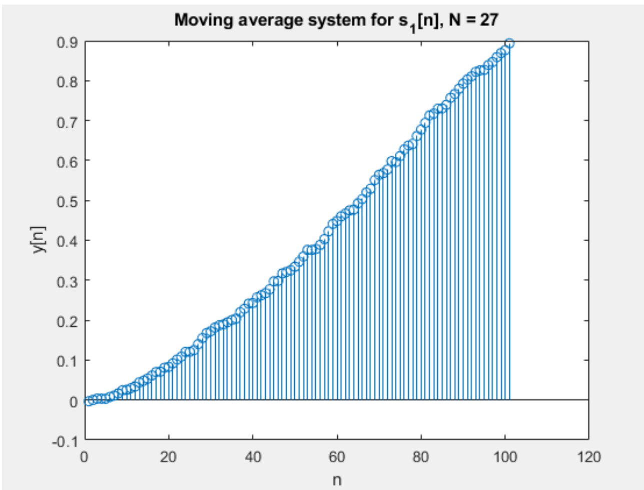
Question 3.1:

We have constructed the system by using sliding window average method

We experiment for different values of N. The signal, as it is looks like a ramp function with some noise added into it which causes disturbance in the waveform. So we need to choose a value of N which denoises the signal and we obtain the ramp signal back.

After testing for various values of N, I arrived to the conclusion that N=27 results in the best output, or an output that is closest to a ramp function



Question 3.1.1:

The system that is constructed by using the convolution $h[n] * x[n]$, and when we compare it with the original system, we can see that we get some extra unnecessary values in the system obtain through convolution.

Pros of convolution is that it does not require building or constructing complex algorithms for implementation of the system, if we assume that convolution is an operator which can be performed easily, this can be considered as a con for the original method too. Again the cons of the same are that it may result in extra values that are not meant to present in the output.
The pros of the original way of the implementing the system is that it gives only the required output, and does not result in unnecessary extra values.

Question 3.2:

Upsampling is basically zooming into the signal, which results in empty points in the signal that need to filled. The filling of these empty spaces after upsampling is called interpolation. In this question zeroth order, and first order interpolation have been implemented.

Zeroth order interpolation just copies previous values to current value, whereas first order implementation uses an approach where it sets an empty point value based on its adjacent non empty values, which results in a more smoother waveform.

Question 3.3:

We observe that the message signal on multiplying with the carrier wave, gets its amplitude changed at various points, which is known as Amplitude Modulation or AM.

Q3

(b)

$$y[n] = (\cos \omega_0 n)\, x[n] \quad \sim \quad y = (\cos\omega_0 n) X$$

$$h[n] = \cos(\omega_0 n)$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]\, z^{-n}$$

$$H(z) = \sum_{n=-\infty}^{\infty} \frac{1}{2}\left(e^{i\omega_0 n} + e^{-i\omega_0 n}\right) z^{-n}$$

$$= \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(\frac{e^{i\omega_0}}{z}\right)^n + \left(\frac{1}{e^{i\omega_0} z}\right)^n$$

On calculating: $H(z) = \dfrac{z(z - \cos(x))}{z^2 - 2z\cos x + 1}$

(where $x = 20000$).

Zeroes: $z = 0$, $z = \cos(x) = \cos(20000) = 0.813$

Poles: $z^2 - 2(0.813)z + 1$

$\boxed{\text{Inside circle of } r = 0.949}$

$z_1 = 0.813 + 0.491i \qquad z_2 = 0.813 - 0.491i$

RoC: ~~|z| ... ROC ...~~ $ROC < |z| \rightarrow \boxed{ROC < 0.949}$

Question 3.4:

Q4

$$H(z) = \frac{z^2 - 2\cos\theta\, z + 1}{z^2 - 2r\cos\theta\, z + r^2}$$

$$= \frac{z^2 - 2\left(\frac{e^{i\theta}+e^{-i\theta}}{2}\right) z + \left(e^{i\theta}\cdot e^{-i\theta}\right)}{z^2 - 2\left(\frac{re^{i\theta}+re^{-i\theta}}{2}\right) z + \left(re^{i\theta}\cdot re^{-i\theta}\right)}$$

$$= \frac{(z - e^{i\theta})(z - e^{-i\theta})}{(z - re^{i\theta})(z - re^{-i\theta})}$$

(a) Zeroes: roots of numerator polynomial

↳ $z = e^{i\theta}$, $z = e^{-i\theta}$

(b) Poles: Roots of denominator polynomial

↳ $z = re^{i\theta}$, $z = re^{-i\theta}$

⇒ When θ & r is changed, zeroes just rotates around on the unit circle on complex plane. Whereas, poles, not only rotate but the radius of circle that they lie on changes with change in r.