Report (Lab 5: DFT and FFT)

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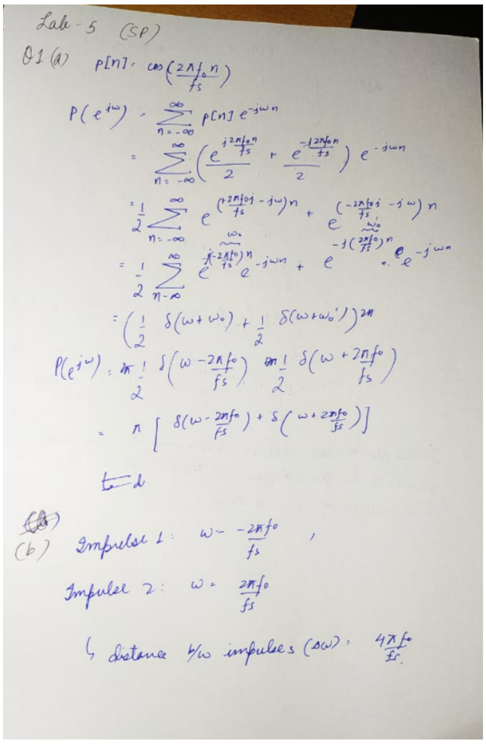
Question 1:

(a) and (b):



Lab - 5 (SP)

Q1 (a) $p[n] = \cos\left(\frac{2\pi f_0 n}{f_s}\right)$

$$P(e^{j\omega}) = \sum_{n=-\infty}^{\infty} p[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{e^{\frac{j2\pi f_0 n}{f_s}}}{2} + \frac{e^{\frac{-j2\pi f_0 n}{f_s}}}{2}\right) e^{-j\omega n}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} e^{\left(\frac{+2\pi f_0 j}{f_s} - j\omega\right)n} + e^{\left(\frac{-2\pi f_0 j}{f_s} - j\omega\right)n}$$

$$= \frac{1}{2}\sum_{n=-\infty}^{\infty} e^{j\left(\frac{2\pi f_0}{f_s}\right)n} e^{-j\omega n} + e^{-j\left(\frac{2\pi f_0}{f_s}\right)n} e^{-j\omega n}$$

$$= \left(\frac{1}{2}\delta(\omega + \omega_0) + \frac{1}{2}\delta(\omega + \omega_0')\right) 2\pi$$

$$P(e^{j\omega}) = \pi \frac{1}{2}\delta\left(\omega - \frac{2\pi f_0}{f_s}\right) + \pi\frac{1}{2}\delta\left(\omega + \frac{2\pi f_0}{f_s}\right)$$

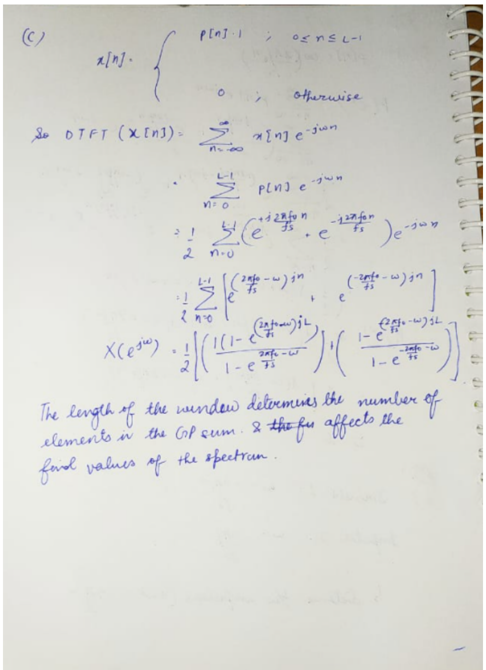
$$= \pi\left[\delta\left(\omega - \frac{2\pi f_0}{f_s}\right) + \delta\left(\omega + \frac{2\pi f_0}{f_s}\right)\right]$$

(b) Impulse 1: $\omega = -\frac{2\pi f_0}{f_s}$,

Impulse 2: $\omega = \frac{2\pi f_0}{f_s}$

distance b/w impulses ($\Delta\omega$): $\frac{4\pi f_0}{f_s}$

(c):



(c)

$$x[n] = \begin{cases} p[n]\cdot 1 & ; 0 \le n \le L-1 \\ 0 & ; \text{otherwise} \end{cases}$$

So $\text{DTFT}(x[n]) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

$$= \sum_{n=0}^{L-1} p[n]\, e^{-j\omega n}$$

$$= \frac{1}{2}\sum_{n=0}^{L-1}\left(e^{+j\frac{2\pi f_0 n}{f_s}} + e^{-j\frac{2\pi f_0 n}{f_s}}\right)e^{-j\omega n}$$

$$= \frac{1}{2}\sum_{n=0}^{L-1}\left[e^{\left(\frac{2\pi f_0}{f_s} - \omega\right)jn} + e^{\left(-\frac{2\pi f_0}{f_s} - \omega\right)jn}\right]$$

$$X(e^{j\omega}) = \frac{1}{2}\left[\left(\frac{1\left(1 - e^{\left(\frac{2\pi f_0}{f_s} - \omega\right)jL}\right)}{1 - e^{\frac{2\pi f_0}{f_s} - \omega}}\right) + \left(\frac{1 - e^{\left(\frac{2\pi f_0}{f_s} - \omega\right)jL}}{1 - e^{-\frac{2\pi f_0}{f_s} - \omega}}\right)\right]$$

The length of the window determines the number of elements in the GP sum. & ~~the~~ f_s affects the final values of the spectrum.

(d):

Yes the plots are consistent with the answer in part (c)

(e):

The DTFT of the signal gives us two impulses, as discussed in the previous parts, and as we increase the length of the DFT, we approach closer to the two impulses and the peaks in the plots become steeper, and start to look more like spiky impulses as we increase L.

(i):

Three strongest frequencies, obtained from the DFT:

(These are also obtained in the output of the program `q1_i.m`)

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Audio file 2:
6.1896
6.2273
6.0388
```

Question 2:

We can see that the outputs from both the ways match each other, thus verifying the way of computing convolution and circular convolution from DFT.

Basic idea:

$$x_1 * x_2 = IDFT(DFT(x_1).DFT(x_2))$$

Question 3:

We can identify the higher and lower frequencies from the spectrum by looking at the magnitude spectrum of the DFT, which for every k in the plot, gives us the contribution of the frequency $2\pi\frac{k}{N}$ in the original signal.

So we can judge the contribution of the frequencies / higher or lower frequencies by looking at the plot of DFT and identifying the lowest and highest peaks in the plot.
