



# Report: Signal Processing Lab 2

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## Question 1:

### ROC, Causality, and Stability (function + script)

Here I have created the function file q1\_a.m which houses the function that calculates the required parameters: [N,ROC,C,S]

We call the function with the given inputs in the script and the outputs are received in the format:

```
Output:
for p = 3
    N:
    2

    ROC:
    0    3
    3    Inf

    C:
    0
```

```

1

S:
1
0

for p = 0.1
N:
2

ROC:
      0      0.1000
0.1000      Inf

C:
0
1

S:
0
1

for p = 0
N:
1

ROC:
0      Inf

C:
1

S:
1

for p = 0      0.5
N:
2

ROC:
      0      0.5000
0.5000      Inf

C:
0
1

S:
0
1

for p = 2      -0.5
N:
3

ROC:
      0      0.5000
0.5000      2.0000

```

```

2.0000      Inf

C:
0
0
1

S:
0
1
0

for p = 0.5      -0.5
N:
2

ROC:
      0      0.5000
0.5000      Inf

C:
0
1

S:
0
1

for p = 2  2  2
N:
2

ROC:
0      2
2      Inf

C:
0
1

S:
1
0

for p = 0  1  2
N:
3

ROC:
0      1
1      2
2      Inf

C:
0
0
1

```

```

S:
0
0
0

for p = -0.5+0i          0+1i
N:
3

ROC:
    0    0.5000
0.5000    1.0000
1.0000    Inf

C:
0
0
1

S:
0
0
0

for p = 2  2  2
N:
2

ROC:
0    2
2    Inf

C:
0
1

S:
1
0

for p = 0  1  2
N:
3

ROC:
0    1
1    2
2    Inf

C:
0
0
1

S:
0
0
0

```

```

for p = -0.5+0i      0+1i
N:
3

ROC:
    0    0.5000
0.5000    1.0000
1.0000    Inf

C:
0
0
1

S:
0
0
0

for p = 0+0i    0+1i    0-1i
N:
2

ROC:
0    1
1    Inf

C:
0
1

S:
0
0

for p = 0.5+0i      -0.5+0i      2+1i
2-1i
N:
3

ROC:
    0    0.5000
0.5000    2.2361
2.2361    Inf

C:
0
0
1

S:
0
1
0

for p = 1+1i    1+2i    1+3i    2+1i
N:

```

```

4

ROC:
    0      1.4142
1.4142    2.2361
2.2361    3.1623
3.1623      Inf

C:
0
0
0
1

S:
1
0
0
0

```

## Question 2:

$$H(z) = \frac{1}{1 + pz^{-1}}$$

We use the function `zplane`, `freqz`, `impz` to calculate the poles, zeroes, frequency/magnitude responses, impulse response for the given system.

The given system represents an FIR filter as the impulse response when plotted can be observed to be of finite duration.

(c) When  $p$  is brought down from 0.8 to 0.1, we can see that the number of points in the impulse response reduces.

(e)

$$H(z) = \frac{z - p^{-1}}{z - p}$$

This also represents an FIR system as in the plot it is observed that the impulse response is of finite duration

## Question 3:

$$H(z) = \frac{z^2 - (2\cos\theta)z + 1}{z^2 - (2r\cos\theta)z + r^2}, \quad r \in (0, 1), \theta \in [0, \pi]$$

## Poles and Zeroes of the function:

Q3

$$H(z) = \frac{z^2 - (2\cos\theta)z + 1}{z^2 - (2r\cos\theta)z + r^2}$$

↓ Poles

$$z = \frac{-(-2r\cos\theta) \pm \sqrt{4r^2\cos^2\theta - 4r^2}}{2}$$

$$= \frac{2r\cos\theta \pm \sqrt{4r^2\cos^2\theta - 4r^2}}{2} = \boxed{r\cos\theta \pm i/r\sin\theta}$$

Poles

Radius of ROC circle

$$= \sqrt{r^2\cos^2\theta + r^2\sin^2\theta}$$

$$= r\sqrt{\cos^2\theta + \sin^2\theta} = r$$

And since  $r \in (0, 1)$ , our ROC's will be like  $[0, r)$ ,  $(r, \infty)$ .

Both stable & causal.  $\leftarrow$  And this will contain 1. Since this extends to infinity too.

↓ Zeros

$$z = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$

$$= \boxed{\cos\theta \pm i\sin\theta}$$

Zeros

Yes the system can be causal and stable simultaneously, for ROC  $(r, \infty)$

As  $r < 1$  this interval contains 1 which makes it stable, and extends to infinity which makes it causal at the same time.

## Question 4:

The frequency response and magnitude response have been plotted by using the freqz function and poles of the function have been calculated by using the zplane function.

