Section: G Group: 01 Trimester: Spring 25 Date of Submission:________

Experiment No. 04

Name of the Experiment: Determination of the spring constant and effective mass of a given spiral spring.

Theory:

If a spring is clamped vertically at the end P, and loaded with a mass m_0 at the other end A, then the period of vibration of the spring along a vertical line is given by,

$$T = 2\pi\sqrt{\frac{m' + m_0}{k}} = 2\pi\sqrt{\frac{M}{k}} \quad \ldots\ \ldots\ \ldots\ (1)$$

Where, m' is a constant called the effective mass of the spring and k, the spring constant i.e. the ratio between the added force and the corresponding stretch of the spring.

The contribution of the mass of the spring to the effective mass of the vibrating system can be shown as follows: Consider the kinetic energy of a spring and its load undergoing simple harmonic motion. At the instant under consideration, let the load m_0 be moving with velocity v_0 as shown in the figure.

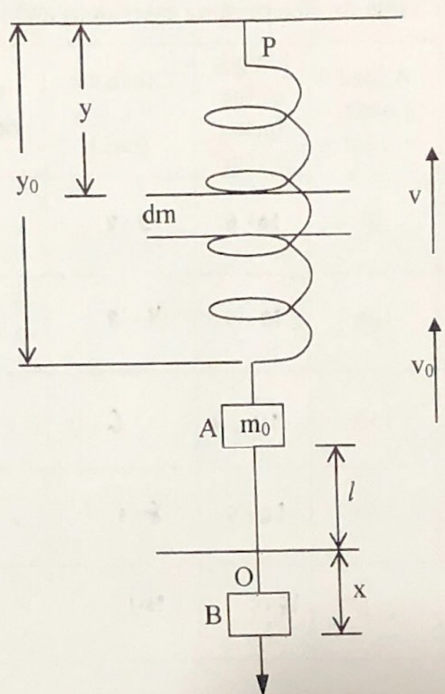


Fig. 01

At the same instant, an element dm of the mass m of the spring will also be moving up but with a velocity v which is smaller than v_0. It is evident that the ratio between v and v_0 is just the ratio between y and y_0. Hence, $\frac{v}{y} = \frac{v_0}{y_0}$ i.e. $v = \frac{v_0}{y_0}y$. The kinetic energy of the spring alone will be $\int \frac{v^2}{2}dm$. But dm maybe written as $\frac{m}{y_0}dy$, where, m is the mass of the spring.

Thus the integral equals to $\frac{1}{2}\left(\frac{m}{3}\right)v_0^2$. The total kinetic energy of the system will then be $\frac{1}{2}\left(m_0 + \frac{m}{3}\right)v_0^2$ and the total mass of the system is therefore, $M = \left(m_0 + \frac{m}{3}\right)$

Hence, effective mass, $m' = \frac{m}{3}$. The applied force $m_0 g$ is proportional to the extension l within the elastic limit.

$$\text{Therefore, } mg = kl \text{ or, } l = \frac{g}{k} m$$

$$k = \frac{g}{l/m} = \frac{g}{\text{slope of l vs m graph}} = \frac{981}{0.04} = 24252 \text{ dynecm}^{-1}$$

Apparatus:

- A spiral spring
- Convenient masses with hanging arrangement
- Clamp or a hook attached to a rigid framework of heavy metal rods
- Weighing balance
- Stopwatch and scale

Experimental Data:

(A) Initial length of the Spring, L_0 = 8.4 cm

(B) Table for determining extensions and time periods:

No. of Obs.	Added Loads, m_0 (gm.)	Length of the Spring, L (cm)	Extension, $l = L - L_0$ (cm.)	No. of vibrations	Total time (sec.)	Mean time (sec.)	Period, T (sec.)	T^2 (sec.^{2})
1	50	10.6	2.2	10	5.19 5.21	5.20	0.52	0.270
2	100	12.7	4.3	10	5.92 5.85	5.89	0.589	0.347
3	150	14.4	6	10	6.41 6.39	6.40	0.64	0.41
4	200	16.5	8.1	10	7.13 7.03	7.08	0.708	0.501
5	250	18.5	10.1	10	7.80 7.69	7.75	0.775	0.601

Calculation:

(A) Effective mass, m' (from graph) = $(110 - 100) = 10$ gm

(B) Effective mass, m' (from calculation or theoretical effective mass) = $\frac{m}{3} = \frac{30}{3} = 10$ gm

(C) Difference (%) = $\frac{\text{Experimental Result} \sim \text{Theoretical Result}}{\text{Theoretical Result}} \times 100\% = \frac{10-10}{10} \times 100\% = 0\%$

(D) Accuracy = 100% - % Difference = $100\% - 0\% = 100\%$

(E) Spring Constant $k = \frac{g}{\text{slope of } l \text{ vs } m_0 \text{ graph}}$

$= \frac{981}{0.04}$

$= 24252$ dyne cm^{-1}

Slope = slope of l vs $m_0 = l/m_0$

$= \frac{4}{100} = 0.04 \text{ cmg}^{-1}$

Results:

(A) Value of the Effective Mass, $m' =$ 10 gm

(B) Value of the Spring Constant, $k =$ 24252 dyne cm^{-1}

Discussions:

Q: What do you understand by the term *Spring Constant*?

Spring constant is basically a factor that decides the stretching of a spring. It depends on the material and construction of the spring. Spring constant is denoted by k. $k = \frac{F}{x}$

Q: What is the *Effective Mass* of a spring?

In an ideal spring linear density is not same as the mass because every particle does not move the same distance. Linear density is 1/3 of mass of the spring. That is called effective mass.

Q: Infer the extension of your spring when a load of 300 gm is used from the load ~ extension graph.

The extension of the spring when a 300gm load is applied is 12cm.

Q: Does the time period of oscillation depends on the displacement from the equilibrium position? Explain

we know that, $T = 2\pi\sqrt{l/g}$ $\quad T \propto \sqrt{l}$

here, l is displacement and it is propotional to period. So, we can say that period of oscillation does depend on displacement.

Q: What happens to the time period if you keep increasing the load?

We know, $T = 2\pi\sqrt{\frac{m}{k}}$

So, if we increase the load period will increase.

Q: What type of motion does the spring-block system have? Write the differential equation for such motion. Write the solution of the differential equation.

The spring block system has simple harmonic motion. Differential equation —

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$ma = -kx$$

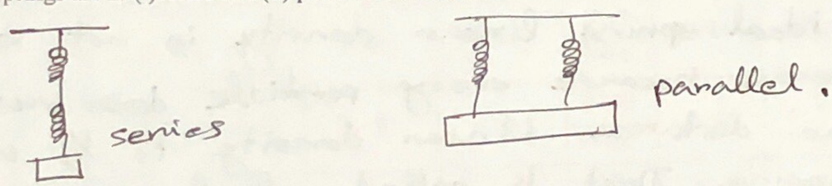
$$\Rightarrow \frac{d^2x}{dt^2} = -kx$$

$$\Rightarrow \frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$v = \frac{dx}{dt} \quad A\omega\cos(\omega t + \delta)$$

$$a = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \delta)$$

Q: Suppose two spring with the same spring constant k. Draw the diagram, when these two springs are in (i)series and (ii) parallel combination.



Show that the equivalent spring constant

(i) $k_{series} = \frac{k}{2}$

Spring 1, $F = k\Delta x_1$

Spring 2, $F = k\Delta x_2$

$$\Delta x_1 + \Delta x_2 = F\left(\frac{1}{k} + \frac{1}{k}\right)$$

$$\Rightarrow \frac{1}{k_{series}} = \frac{2}{k}$$

$$\therefore k_{series} = \frac{k}{2}$$

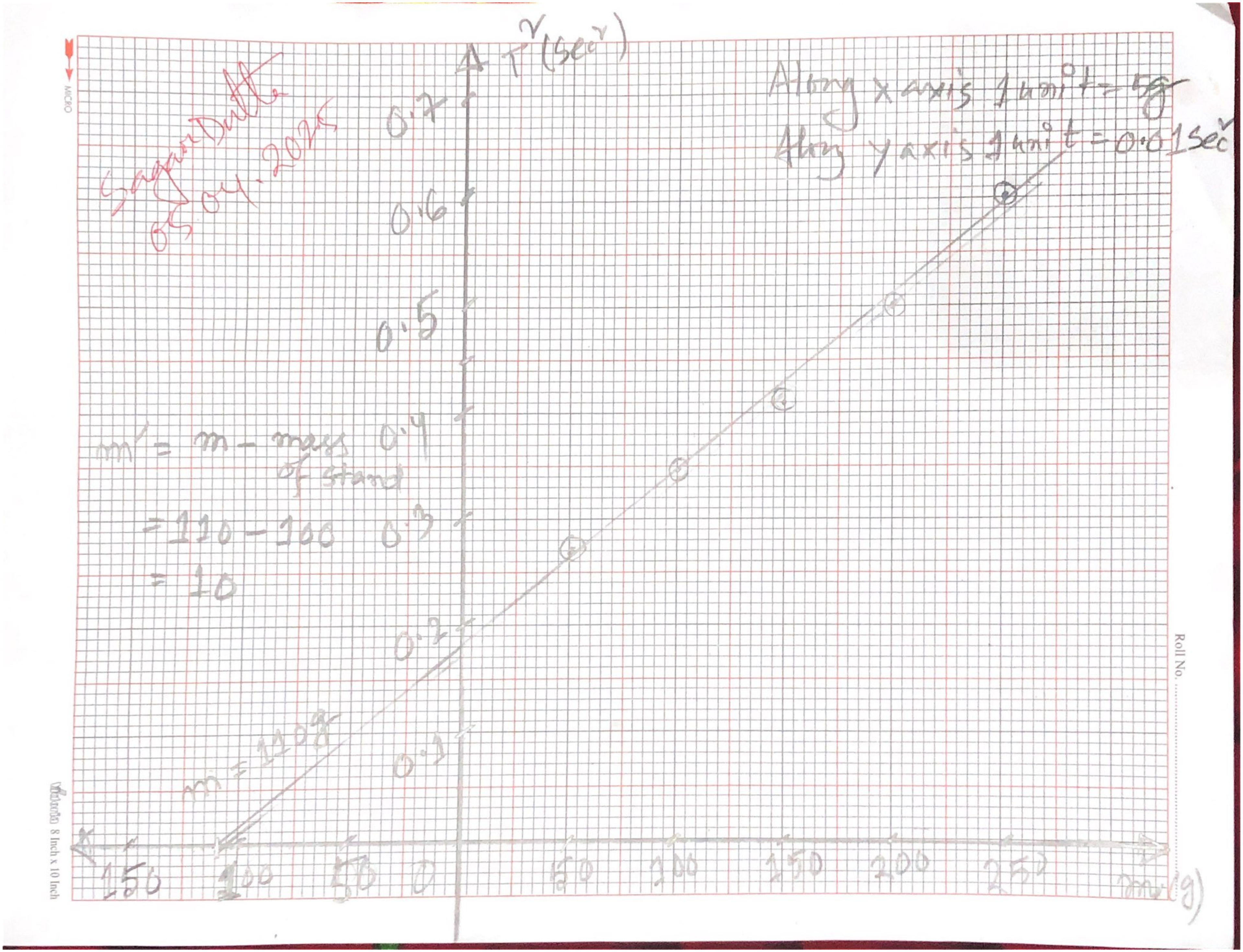
(ii) $k_{parallel} = 2k$

S_1, $F = k\Delta x_1$ $\quad$ S_2, $F = k\Delta x_2$

$$k_{par} = \frac{F}{\Delta x_1} + \frac{F}{\Delta x_2}$$

$$= \frac{F}{\Delta x} + \frac{F}{\Delta x} = 2\frac{F}{\Delta x}$$

$$\therefore k_{par} = 2k$$



Sagor Dutta
05.04.2025
T^2 (sec²)
Along x axis 1 unit = 5g
Along y axis 1 unit = 0.01 sec²
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$m' = m -$ mass of stand
$= 110 - 100$
$= 10$
m = 110g
150
100
50
0
50
100
150
200
250
m (g)
Roll No.
8 Inch x 10 Inch
MICRO

Roll No. ....

Along x axis 1 unit = 5g

Along y axis 1 unit = 0.2 cm

$\Delta m_0 = 100g$

$\Delta l = 4cm$

$$\text{slope of } l \text{ vs } m_0 = \frac{\Delta l}{\Delta m_0} = \frac{4}{100} = 0.04\ cm\,g^{-1}$$

$$K = \frac{g}{\text{slope of } l \text{ vs } m_0} = \frac{981}{0.04}$$

$$= 24525\ dyne/cm$$