MATH 2210 - Applied Linear Algebra
Answers for Homework Questions

Austin Sec. 1.3

- #1 There is exactly one solution $(3, -1, 0)$.
- #2 (a) There is exactly one solution: $x = 400$, $y = 300$, $z = 550$, and $w = 160$.
- #3 Use Octave `rref` to find the following values in the right-most column: $T_1 = 14.86, T_2 = 20.50, T_3 = 26.19, T_4 13.95, T_5 = 20.93, T_6 = 29.28$.
- #4 Use Octave `rref` to find the following values in the right-most column: The first shift produces $x = 113$ pounds while the second shift produces $y = 65$ pounds. There are 30.8 pounds of sulfur.

Austin Sec. 1.2

- #1 (a) $\left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$, (b) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$, (c) $\left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & 2 \\ 0 & 1 & -2 & 0 & -3 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$
- #2 (a) Not reduced, inf many sol, (b) not reduced, no sols (c) not reduced, unique sol, (d) not reduced, inf many sol
- #5 a. Nothing, b. The leading entry of some row appears in the rightmost column of the augmented matrix, c. There is a column that is not the rightmost and that does not contain the leading entry of a row. Also, the rightmost column does not contain a leading entry. d. There are infinitely many solutions.
- #6 FTFFF

Austin Sec. 1.4

- #1 (a) Consistent, inf many sol, (b) Consistent, inf many sols, (c) Inconsistent, (d) Consistent, unique sol
- #3 (a) Cannot guarantee either, (b) Can guarantee inf many sols, (c) Pivot position in each column of coefficient matrix
- #4 FFFTF
- #6 (a) Consistent for $k = -6$ and never unique, inconsistent for $k \neq -6$, (b) Inconsistent $k = -2$ and $l \neq -3/2$. Inf may sols if $k = -2$ and $l = -3/2$. Unique sol if $k \neq -2$.
- #7 (a) $a = b = c = 0$, (b) $a = 1$ and $b = c = 0$, (c) $a = 9, b = 0$ and $c = -9$.
- #8 (a) There cannot be a pivot in the rightmost column, (b) There is a solution when all the variables are set to zero, (c) There are at least as many equations as unknowns.

Austin Sec. 2.1

- #3(a) $(5,-1)$, (b) $(c_1,c_2,c_3)=c_3(2/3,-2/3,1)$, (c) $(c_1,c_2)=(0,0)$, (d) $\mathbf{v}_3=-2/3\mathbf{v}_1+2/3\mathbf{v}_2$.
- #4 (a) $(120,105,1.0)$, (c) $(a,b)=(2,1)$, (d) Not possible.
- #5(a) $(c_1,c_2,c_3)=(5-2c_3,2-2c_3,c_3)$, (b) not possible, (c) $(c_1,c_2)=(2,2)$, (d) By part (c), $\mathbf{v}_3$ is a linear combination of other two vectors.
- #6 $k=10$.
- #7 TTTF

Austin Sec. 2.2

- #2 $2201, 135$
- #6 (a) $(-11,-2,13)$, (b) $(9,-1,2,0)+x_4(-2,1,-1,1)$ shifted line, (d) No, pivot in every row, (e) Can always be found for any given vector and the solution space will form a line in $\mathbb{R}^4$.
- #9 FTFTT
- #11 (a) $\mathbf{x}=x_3(-2,-1,1)$, (b) $\mathbf{x}=(1,-2,0)+x_3(-2,-1,1)$, (c) always have $\mathbf{x}=\mathbf{x}_h+\mathbf{x}_p$.
- #12 (b) $(1000,500)$, (c) $(708.5,291.5)$,
- #13 (d) $c_1=1000/7, c_2=1500/7$, (f) as $k\to\infty$, $(0.3)^k\to 0$ and $\mathbf{x}_{k+1}\to(5000/7,2000/7)$.

Austin Sec. 2.2

- #5 (a) $\mathbf{x}=x_3(-2,1,1)$, (b) Choose two distinct vectors from part (a) to form columns of B.
- #7 (a) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&-3\end{bmatrix}$, (b) $\begin{bmatrix}0&0&1\\0&1&0\\1&0&0\end{bmatrix}$, (c) $\begin{bmatrix}1&0&0\\0&1&0\\3&0&1\end{bmatrix}$, (d) $\begin{bmatrix}1&0&0\\-2&1&0\\-1&2&1\end{bmatrix}$.
- #8 (a) $B=\begin{bmatrix}-1&-2\\-2&-3\end{bmatrix}$, (c) $I\mathbf{x}=\mathbf{x}$, (d) $\mathbf{x}=B\mathbf{b}=(-1,-4)$.

Austin Sec. 6.2

- #4 (b) F
- #5 (a) B^{-1}, (b) $A^TA+B^TA+A^TB+B^TB$, (c) AA^T+BA^T, (d) A^T+2I
- #6 (a) must be square, (b) Y, N, Y, Y

Austin Sec. 2.3

- #1(a) Yes, $\mathbb{R}^2$, (b) Yes, yes, no, plane in $\mathbb{R}^3$.
- #2 Yes, Yes, $\mathbf{0}$.
- #3(a) Need $\frac{1}{3}b_1+\frac{1}{2}b_2=0$, (b) All $\mathbf{b}\in\mathbb{R}^2$.

- #5 TTFTF
- #7(a) Yes, No, Yes, $n \geq 438$, No.
- #9 pivots on diagonal, span $\mathbb{R}^2$, always unique sol, trivial sol only.

Austin Sec. 2.4

- #1 (a) any set of more than three vectors in $\mathbb{R}^3$ must be dependent, (b) $\mathbf{v}_3 = 2\mathbf{v}_1 - \mathbf{v}_2$, (c) one example is $c_1 = -2, c_2 = 1, c_3 = 1, c_4 = 0$, (d) $\mathbf{x} = (-2, 1, 1, 0)$.
- #2 Linearly independent, span is $\mathbb{R}^3$ and other results follow from this by discussing pivot rows.
- #4 FTFTT
- #5 (a) $c_1 = 2, c_2 = -1, c_3 = 3, c_4 = 1$, (b) non-trivial solution to homogeneous equation, There are infinitely many solutions to the homogeneous equation $A\mathbf{x} = \mathbf{0}$, c. $v_4 = -2v_1 + v_2 - 3v_3$, d. One vector can be written as a linear combination of the others.
- #6 (a) $n \geq 27$, (b) $n \leq 27$, (c) $n = 27$, (d) consistent for all such vectors.
- #7 (a) $A : 5 \times 4$, pivot in all columns, (b) $A : 4 \times 4$, $A \sim I$, (c) $A : 4 \times 3$, not possible, (d) $A : 3 \times 5$, not possible, (e) $A : 4 \times 5$ must have free variable,
- #9 (a) $k = 3$, (b) $k \neq 3$.

Austin Sec. 3.1

- #3 b L^{-1} is always lower triangular because the only row operations needed to row reduce $L \sim I$ are replacements in which a multiple of one row is added to a row *underneath* it
- #5 (a) $A^{-1} = \begin{bmatrix} 0 & -1/3 & 0 \\ 1/2 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, (c) $A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1/2 \end{bmatrix}$.
- #6 (a) $A^{-1} = BA$, (b) $A^{-1} = BA^{99}$.
- #7 TFTFT.
- #8 (a) no, (b) no, (c) yes, (d) Span is $\mathbb{R}^n$, (e) yes, discuss inverse of B.
- #10 a. $B^2 = PA^2P^{-1}$, b. $B^{-1} = PA^{-1}P^{-1}$, c. $B^{-1} = PA^{-1}P^{-1}$, d. $C = (QP)A(QP)^{-1}$.
- #9 (c) $A = \begin{bmatrix} 3 & 2 & -1 \\ 6 & 6 & 4 \\ -12 & -7 & 3 \end{bmatrix}$.

Austin Sec. 3.4

- #3 $\det(A) = \det(PBP^{-1}) = \det(P)\det(B)\det(P^{-1}) = \det(B) = 30$
- #4 (a) $\det(A) = k - 16$ (b) $k \neq 16$

- #5 FFTTF
- #6 (a)-64, (b) -8, (c) -10, (d) 2, (e) $-2/5$
- #7 (a) discuss value of $\det(AB) = \det(A)\det(B)$ for A, B invertible, (b) discuss value of $\det(AB) = \det(A)\det(B)$ for AB invertible.
- #8 Determinant zero for all. Discuss invertibility in all cases.
- #9 (a) $\det(A) = 2x - y - 2z = 0$, (b) If we replace the third column by either $\mathbf{v}_1$ or $\mathbf{v}_2$, we obtain a matrix whose columns are linearly dependent.
- #11 $\det(A) = -1$, $\det(B) = 1$, $\det(C) = abcd$

Austin Sec. 2.5

- #1 (a) $T : \mathbb{R}^3 \to \mathbb{R}^3$, $S : \mathbb{R}^3 \to \mathbb{R}^2$, (b) $(6, -1, -6)$, (c) $(-6, 0)$, (d) $(7, -1)$, (e) BA.
- #3 (a) $\mathbf{x} = x_3(2, -1, 1)$ (b) No vectors satisfy the equation (c) $\mathbf{x} = (-2, -2, 0) + x_3(2, -1, 1)$
- #4 (a) Use linearity of T; $2\mathbf{v}_1 + \mathbf{v}_2 + 2\mathbf{v}_3$, (b) $\mathbf{v}_2$, (c) $\mathbf{x} = x_3(1, 1, 1)$.
- #6 FTFT.
- #7 (a) Use $\mathbf{x}, \mathbf{y}$ notation from part (b) here; $T((1, 0, 0)) = T(\mathbf{e}_1) = (20, 15)$, $T((0, 1, 0)) = T(\mathbf{e}_2) = (30, 5)$, $T((0, 0, 1)) = T(\mathbf{e}_3) = (0, 40)$, (b) Use results from part (a): $A = [T(\mathbf{e}_1) \;\; T(\mathbf{e}_2) \;\; T(\mathbf{e}_3)]$, (c) Use $\mathbf{y}, \mathbf{z}$ notation from part (d); $S((1, 0)) = S(\mathbf{e}_1) = (5, 8)$, $S((0, 1)) = S(\mathbf{e}_2) = (6, 10)$, (d) Use results from part (d); $B = [S(\mathbf{e}_1) \;\; S(\mathbf{e}_2)]$, (e) Compute $S(T(\mathbf{x})) = (11700, 19100)$, (f) $(S \circ T)(\mathbf{x}) = (BA)\mathbf{x}$.
- #8 (a) use linearity, $T(\mathbf{e}_1) = (1, -2)$, (b) use linearity $T(\mathbf{e}_2) = (2, 0)$ and $A = [T(\mathbf{e}_1) \;\; T(\mathbf{e}_2)]$, (c) $(-6, -8)$.
- #9 (a) $\mathbf{x}_1 = (1.4, 3.1)$, (b) Solve $A\mathbf{x} = \mathbf{x}_1$; $\mathbf{x} = (0.8, 1.2)$, (c) Use Octave to calculate $A^5\mathbf{x}_0 = (6.8, 15.9)$, where $\mathbf{x}_0 = (1, 2)$, (d) Use Octave to calculate $A^{12}\mathbf{x}_0 = (116.8, 272.5)$, where $\mathbf{x}_0 = (1, 2)$.
- #10 (a) $1000T(\mathbf{e}_1) = (950, 50)$, (b) $1000T(\mathbf{e}_2) = (500, 500)$, (c) use linearity to find $A = [T(\mathbf{e}_1) \;\; T(\mathbf{e}_2)]$, (d) $(1665, 135)$, (e) 1778, (f) Use Octave to calculate $A^7\mathbf{x}_0 \approx 1636$, (g) Goes to ≈ 1636.36.

Austin Sec. 2.6

- #1 Use the notation from pre-lecture video: R_θ rotation matrix for counter clockwise rotation by angle θ; H_θ reflection matrix about line through the origin that makes an angle θ relative to x-axis. (a) R_π, (b) see video, (c) $H_{-\pi/4}$, (d) $R_{\pi/3}$, (e) product $H_{\pi/4}R_{\pi/3}$,
- #2 a. This is the same as a counterclockwise 90° rotation, b. This is the same as a clockwise 90° rotation, c. This is the same as a 180° rotation, d. This is the same as a reflection in the horizontal axis.
- #4 (a) $R_{\pi/2}^{-1} = R_{-\pi/2}$, (b) $H_{\pi/4}^{-1} = H_{\pi/4}$, (c) $A^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$, (d) $A^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 1/3 \end{pmatrix}$,

- #5 Use the notation for homogeneous coordinates transformation matrices in Activity 2.6.4; (a) $a = 1, b = 0, c = -1, d = 0, e = 1, f = -2$, (b) $a = 0, b = -1, c = 0, d = 1, e = 0, f = 0$, (c) $a = 1, b = 0, c = 1, d = 0, e = 1, f = 2$, (d) take product of three matrices $A_3 A_2 A_1$,

- #7 (a) stretches vectors by factor r, (b) CCW rotation by angle θ, (e) CCW rotation by angle θ followed by stretch factor r, (f) do the transformation in part (e) twice in sequence using different angles θ and ϕ with stretches r and s. Stretches vector by rs and rotates it by $\theta + \phi$.

Austin Sec. 3.1

- #2 Use the notation from pre-lecture video: R_θ rotation matrix for counter clockwise rotation by angle θ; (a) $A = R_{\pi/4}$, $A^{-1} = R_{-\pi/4}$, (b) $A = A^{-1} = R_\pi$, (c) Construct matrices that define reflections.

Austin Sec. 3.4

- #2 $\det A = 1$, $\det B = -1$.

Austin Sec. 3.2

- #1 (a) The vectors are linearly independent and span $\mathbb{R}^2$, (b) The grid on the figure indicates that 1. $\mathbf{x} = (-6, 3)$, 2. $\mathbf{x} = (-2, -4)$, 3. $\mathbf{x} = (6, 3)$, (c) The grid on the figure indicates that 1. $\{\mathbf{x}\}_\mathcal{B} = (0, -1)$, 2. $\{\mathbf{x}\}_\mathcal{B} = (1, 2)$, 3. $\{\mathbf{x}\}_\mathcal{B} = (-2, -1)$, (d) $\{\mathbf{x}\}_\mathcal{B} = (20, 50)$.

- #2 (a) put each set as columns of a matrix, then row reduce to examine pivot structure, (b) $\mathbf{x}_\mathcal{B} = (12/5, 13/5)$, $\mathbf{x}_\mathcal{C} = (13, 21)$, (c) $\mathbf{x} = (-2, 16)$, $\mathbf{x}_\mathcal{C} = (-20, -38)$, (d) $\mathbf{x}_\mathcal{C} = (-37/5, -3/5)$, $\mathbf{x} = (-8, -13)$, (e) Compute $P_\mathcal{B}^{-1} P_\mathcal{C}$.

- #3 (a) put vectors as columns of matrix and show definition of basis is satisfied, (b) matrix-vector product $C_\mathcal{B}\mathbf{x}_\mathcal{B}$, (c) Solve system $C_\mathcal{B}\mathbf{x}_\mathcal{B} = \mathbf{x}$ for $\mathbf{x}_\mathcal{B}$, (d) $\mathbf{x} = (23, -11, -2, 9)$, (e) $\mathbf{x}_\mathcal{B} = (3, 4, 1, -3)$.

- #4 No; more than three vectors in the set, so cannot be linearly independent in $\mathbb{R}^3$, (b) put vectors into a matrix and take pivot columns as a basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$, (c) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4, \mathbf{v}_6\}$.

- #6 TTTTF.

- #7 (a) Yes, put into matrix and discuss pivots (b) Yes, since A invertible this implies $A \sim I$, (c) Yes, since columns of A are linearly independent, all columns have pivots. Row operations do not change pivot structure, (d) Yes. Discuss pivots of matrix.

Austin Sec. 3.5

- #1 (a) 6, 4, (b) $\dim(\mathrm{Nul}(A)) = \dim(\mathrm{Col}(A)) = 3$, (c) $\{\mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_5\}$, (d) $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$, where $\mathbf{u} = (1, 0, 0, 0, 0, 0)$, $\mathbf{v} = (0, 2, -1, 1, 0, 0)$, $\mathbf{w} = (0, -3, -4, 0, 1, 1)$

- #2 YNNYN

- #3 FTFFT

- #6 (a) No; discuss free variables, (b) $\dim\text{Col}(A) = 3$, (c) $\dim\text{Col}(A) = 2$, (d) $\dim\text{Col}(A) = 1$, (e) $\dim\text{Col}(A) = 0$meaning A is the zero matrix

- #7 This is a VERY GOOD question! Know $A = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{a}_3 \quad \mathbf{a}_4]$. Find $\mathbf{a}_3, \mathbf{a}_4$ from basis for Null space. Remember that solutions from Null space give linear dependence relationships between the columns of a matrix. Hence *one* option is $A = \begin{bmatrix} 2 & -1 & -7 & 8 \\ 3 & 2 & -7 & -2 \\ -1 & 4 & 7 & -18 \end{bmatrix}$.

- #8 (a) $\{\mathbf{0}\}$, (b) $\mathbb{R}^8$

- #9 (a) $\{\mathbf{0}\}$, (b) $\mathbb{R}^3$

Austin Sec. 4.1

- #1 (a) $\lambda = 3, -2$, (b) $\mathbf{x} = -3\mathbf{v}_1 + 2\mathbf{v}_2$, (c) $A\mathbf{x} = -9\mathbf{v}_1 - 4\mathbf{v}_2$, (d) $A^2\mathbf{x} = -27\mathbf{v}_1 + 8\mathbf{v}_2$, (e) $A^{-1}\mathbf{x} = -\mathbf{v}_1 - \mathbf{v}_2 = -\mathbf{x}$.

- #3 (a) First, prove that $\lambda = 0$ leads to non-trivial solutions, then prove that non-trivial solutions lead to $\lambda = 0$. Remember that e-vectors are non-zero vectors, by definition! (b) Explain both directions separately using same strategy is above, (c) Assume A is invertible, and apply inverse to the definition of an evector, (d) Assume evector exists and apply A to the definition, (e) λ_1^5, λ_2^5, but same eigenvectors.

- #4 See results of #3 to sketch correct vectors.

- #6 $\mathbf{x} = 2\mathbf{v}_1 - \mathbf{v}_2$, $A^4\mathbf{x} = 32\mathbf{v}_1 - 8\mathbf{v}_2 = (145, -130)$. $A(1,0) = (1,2)$, $A(0,1) = (2,-2)$. Think of A as a linear transformation and use previous result to get A.

- #7 TFTFF

- #10 (a) $\lambda_1 = 1.4, \lambda_2 = 0.9$, (b) $\mathbf{x}_0 = 10\mathbf{v}_1 + 14\mathbf{v}_2$, (d) $\mathbf{x}_k = 10(1.4)^k\mathbf{v}_1 + 14(0.9)^k\mathbf{v}_2$, (e) $P_k/Q_k \to 1/3$

Austin Sec. 4.2

- #1 (a) $\lambda = 2$ multiplicity 2, (b) $\lambda = 3, 4, -6$, all multiplicity 1, (c) $\lambda = -2$ multiplicity 2, (d) $\lambda = 3, -2$, all multiplicity 1

- #2 (a) $\lambda = 2$, $\mathbf{v}_1 = (1,2)$ defective matrix so no basis, (b) basis must exist by Proposition 4.2.14, (c) $\mathbf{v}_1 = (1,0)$, $\mathbf{v}_2 = (0,1)$, (d) basis must exist by Proposition 4.2.14

- #3 FTTFT

- #4 (a) $\lambda = -6, 6, -10$, (c) Yes, Proposition 4.2.14, (d) $\lambda = 0, -2, 2$; A not invertible, Yes by Prop. 4.2.14, (e) (i) Sub λ^2 into equation, (ii) sub λ^{-1} into equation

- #7 (a) $\lambda_1 = 1.1, \mathbf{v}_1 = (1,1)$, $\lambda_2 = 0.8, \mathbf{v}_2 = (2,1)$, (b) $\mathbf{x} = \mathbf{v}_1 + 5\mathbf{v}_2$, (c) $\mathbf{x}_k = (1.1)^k\mathbf{v}_1 + 5(0.8)^k\mathbf{v}_2$, (d) $\mathbf{x}_k \to (1.1)^k\mathbf{v}_1$

- #8 (a) $\lambda_1 = 1, \mathbf{v}_1 = (1,2)$, $\lambda_2 = 0.1, \mathbf{v}_2 = (1,-1)$, (b) $\mathbf{x} = \frac{1}{3}\mathbf{v}_1 - \frac{1}{3}\mathbf{v}_2$, (c) $\mathbf{x}_k = \frac{1}{3}\mathbf{v}_1 - \frac{1}{3}(0.1)^k\mathbf{v}_2$, (d) $\mathbf{x}_k \to \frac{1}{3}\mathbf{v}_1$

Austin Sec. 4.3

- #1 (a) $\lambda = -3, 2$ yes, (b) $\lambda = -2, -2$ no, (c) $\lambda = 1 \pm 2i$ no, (d) $\lambda = -2, 1, 4$ yes, (e) $\lambda = -1, -1, 5$ yes,
- #2 (a) no, (b) no, (c) yes $\lambda = 1 - 2i$.
- #3 FTTTF
- #4 (a) Yes, by prop. 4.2.11, (b) no, not diagonalizable in general, (c) yes (d) only identity, (e) $4I$
- #7 (a) Projection onto x-axis, (b) Projection onto $\mathbf{v} = (1, 2)$ eigenvector, (c) $A^k = A$ for all k, (d) no, $\lambda = 0$
- #8 (a) $\lambda = 3 \pm i$, (b) use formula given to find C, (c) $\mathbf{v} = (1 - i, i)$, (d) $\mathbf{v}_1 = (1, 1)$, $\mathbf{v}_2 = (0, 1)$.

Austin Sec. 6.1

- #2 (a) $\mathbf{u} \cdot \mathbf{u} = 9$, $\mathbf{u} \cdot \mathbf{v} = 1$, $\mathbf{u} \cdot \mathbf{w} = -6$ (b) 3, -5, -15, 1, (c) $k = 30$
- #3 (a) 2, (b) $\cos\theta = 2/(\sqrt{5})$ so $\theta \approx 26.6$ degrees, (c) $t = -2$
- #4 (a) $\mathbf{v} \cdot \mathbf{v} = 3^2(\mathbf{w} \cdot \mathbf{w})$, (b) $|\mathbf{w}| = |3||\mathbf{v}|$, (c) $\mathbf{v} \cdot \mathbf{v} = s^2(\mathbf{w} \cdot \mathbf{w})$, $|\mathbf{v}| = |s||\mathbf{w}|$, (d) $s = 1/\sqrt{17}$
- #6 (a) $2x + 4z = 0$, (b) Solve system $2x + 4z = 0$, $-x + 2y - 4z = 0$, (c) $\mathbf{x} = (x, y, z) = z(-2, 1, 1)$. Represents the line of intersection of the two planes in part (b), (d) $\mathbf{x} = (x, y, z) = z(-2, 0, 1) + y(0, 1, 0)$. Represents the plane from part (a).
- #7 (a) Yes, use dot product distributive properties, (b) must be the zero vector
- #8 (a) discuss linear independence of orthogonal vectors, (c) $c_1 = 2$, $c_2 = -1$, $c_3 = 2$
- #10 (a) 13, (b) $x^2 + 4xy + y^2$, (c) λ.

Austin Sec. 6.2

- #1 (a) $\dim(W) = \dim(W^\perp) = 2$ (b) $\{\mathbf{v}_1, \mathbf{v}_2\}$, where $\mathbf{v}_1 = (-2, -3, 1, 0)$, $\mathbf{v}_2 = (-1, 1, 0, 1)$
- #2 (a) $\text{rank}(A) = 2$, basis $\text{Col}(A) = \{\mathbf{a}_1, \mathbf{a_2}\}$ (b) $(\dim(\text{Col}A))^\perp = 1$, basis $(5, 3, 1)$
- #3 (a) $\dim W = 3$, $\dim W^\perp = 1$, (b) $\{(3, 1, 0, 0), (-5, 0, 1, 0), (2, 0, 0, 1)\}$, (c) $\{(-1, 3, 5, 2)\}$, (d) $\{(A, B, C, D)\}$
- #4 TFTTF
- #7 (a) Use $\det(A) = \det(A^T)$ and consider $\det((A - \lambda I)^T)$, (b) A, A^T have same characteristic polynomial, so same solutions to $\det(A - \lambda I) = 0$ (c) Let $Q = P^{-1}$, then $A^T = Q^T D (Q^T)^{-1}$.
- #8 (a) Use dot prod properties for $|\mathbf{v} + \mathbf{w}|^2$, (b) orthogonal vectors.
- #9 (a) $\mathbf{x} \cdot (A\mathbf{y}) = (A^T\mathbf{y}) \cdot \mathbf{x}$, (c) use part (a) to get $(\lambda_1 - \lambda_2)(\mathbf{v}_1 \cdot \mathbf{v}_2) = 0$.

- #10 (a) $\mathbb{R}^{15}$, (b) $(\text{Row}(A))^\perp = \text{Nul}(A)$, (c) All three rows of A are basis for Row(A), $(-2, 1, 0, 0)$ basis Nul(A).

Austin Sec. 6.3

- #1 (a) Orthogonal vectors are independent, (b) $\hat{\mathbf{b}} = (2/3)\mathbf{w}_1 - (1/6)\mathbf{w}_2 = (1/2, 1, 1/2)$, (c) $\mathbf{u}_1 = \mathbf{w}_1/\sqrt{3}$, $\mathbf{u}_2 = \mathbf{w}_2/\sqrt{6}$, (d) $P = QQ^T = \frac{1}{6}\begin{pmatrix} 3 & 0 & 3 \\ 0 & 6 & 0 \\ 3 & 0 & 3 \end{pmatrix}$

- #2 (a) Orthogonal set of three vectors in $\mathbb{R}^3$, (b) Diagonal due to orthogonality, diagonal elements are the squares of the lengths of each vector, (c) $\mathbf{b} = -2\mathbf{w}_1 + 3\mathbf{w}_2 + 2\mathbf{w}_3$, (d) normalize vectors to form basis, (e) $Q^{-1} = Q^T$.

- #3 (a) $\hat{\mathbf{b}} = -2\mathbf{w}_1 + \mathbf{w}_2 = (-1, -2, 1, 3)$, (b) $\mathbf{b}^\perp = \mathbf{b} - \hat{\mathbf{b}} = (3, 1, -7, 4)$, (c) Basis $\{(-1, 1, 1, 0), (-1, 2, 0, 1)\} = \{\mathbf{a}_1, \mathbf{a}_2\}$, $\mathbf{b}^\perp = -7\mathbf{a}_1 + 4\mathbf{a}_2$.

- #5 (a) $\frac{2}{9}\mathbf{w}, -\frac{1}{9}\mathbf{w}, \frac{2}{9}\mathbf{w}$, (b) $P = [\frac{2}{9}\mathbf{w} \;\; -\frac{1}{9}\mathbf{w} \;\; \frac{2}{9}\mathbf{w}]$, (c) $P = [T(\mathbf{e}_1) \;\; T(\mathbf{e}_2) \;\; T(\mathbf{e}_3)]$, columns found in part (a), (d) $L = \text{Col}(P)$ since P has rank 1.

- #6 (a) $(-3, 2, 1)$, (b) $\mathbf{b}^\perp = \mathbf{b} - \hat{\mathbf{b}} = (6, -4, -2)$, (c) $\mathbf{z} = \hat{\mathbf{b}} = 2\mathbf{v}_1 - \mathbf{v}_2$

- #7 TTFTT

- #8 (b) $|Q\mathbf{x}|^2 = |\mathbf{x}|^2$, (c) consider $|Q\mathbf{x}| = |\lambda\mathbf{x}|$.

- #9 (a) Consider $QQ^T = I$ and take determinant, (b) The four columns of Q form a basis for the column space $Col(QQ^T)$, (c) $\mathbf{b}^\perp = \mathbf{b} - \hat{\mathbf{b}}$

Austin Sec. 6.4

- #1 (a) $\{\mathbf{w}_1, \mathbf{w}_2\} = \{(1, 1, 1), (2, -1, -1)\}$ (b) $\{\mathbf{u}_1, \mathbf{u}_2\} = \{\frac{1}{\sqrt{3}}\mathbf{w}_1, \frac{1}{\sqrt{6}}\mathbf{w}_2\}$, (c) $P = QQ^T$, where $Q = [\mathbf{u}_1 \;\; \mathbf{u}_2]$, (d) $\hat{\mathbf{b}} = P\mathbf{b} = (3, 1, 1)$

- #2 $Q = [\mathbf{u}_1 \;\; \mathbf{u}_2]$, where $\mathbf{u}_1 = (2/3, -1/3, 2/3)$, $\mathbf{u}_2 = (1/\sqrt{5}, 2/\sqrt{5}, 0)$, $R = \begin{pmatrix} 6 & 6 \\ 0 & 15/\sqrt{5} \end{pmatrix}$

- #3 (a) $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\} = \{(2/\sqrt{12}, -2/\sqrt{12}, 2/\sqrt{12}), (-2/\sqrt{8}, -2/\sqrt{8}, 0), (2/\sqrt{24}, -2/\sqrt{24}, -4/\sqrt{24})\}$, (b) $Q = [\mathbf{u}_1 \;\; \mathbf{u}_2 \;\; \mathbf{u}_3]$, $R = Q^TA = \begin{pmatrix} 12/\sqrt{12} & 6/\sqrt{12} & -6/\sqrt{12} \\ 0 & 8/\sqrt{8} & -4/\sqrt{8} \\ 0 & 0 & 24/\sqrt{24} \end{pmatrix}$, (c) 1. $\mathbf{y} = Q^T\mathbf{b}$, 2.$R$ is upper triangular, 3. $\mathbf{x} = (-2, 1, -2)$

- #5 (a) Gram-Schmidt fails as $\mathbf{w} = \mathbf{0}$, (b) columns of A are linearly dependent

- #7 (a) 7×7 orthogonal projection matrix onto Col(A), (b) 4×4 identity, (c) 4×4 upper triangular, (d) already orthogonal (but not necessarily orthonormal)

Austin Sec. 6.5

- #1 (a) $\{\mathbf{w}_1, \mathbf{w}_2\} = \{(1, 2, -1), (0, 1, 2)\}$, (b) $(2, 1, -8)$, (c) $(-1, -3)$

- #2 (a) $\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} b \\ m \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}$, (b) $\left(\begin{array}{cc|c} 4 & 10 & 5 \\ 10 & 30 & 14 \end{array}\right)$, (c) $\hat{\mathbf{x}} = (1/2, 3/10)$, (d) 31/20 (e) skip.
- #3 (a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix}$, (b) Use Octave to find QR, $\hat{\mathbf{x}} = (7/4, -19/20, 1/4)$, (c) 1.488., (d) skip
- #4 (a) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \\ 1 & 2 & 2 \\ 1 & 3 & 2 \\ 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix} = \begin{pmatrix} 4.2 \\ 3.3 \\ 5.9 \\ 5.1 \\ 7.5 \\ 6.3 \end{pmatrix}$, (b) Use Octave to find normal equations and solve, $\hat{\mathbf{x}} = (2.94545, 2.00227, -0.854545)$
- #5 T skip FTF