4. (CP) Apply the following world population figures to estimate the 1980 population, using the cubic curve through four data points. Compare with the 1980 estimate of 4452584592.

```
import numpy as np

def newton_divided_difference(x, y):
    n = len(y)
    pyramid = np.zeros([n, n]) # Create a square 2D array for the pyramid
    pyramid[::,0] = y # first column is y

    for j in range(1,n):
        for i in range(n-j):
            # create pyramid by updating other columns
            pyramid[i][j] = (pyramid[i+1][j-1] - pyramid[i][j-1]) /
(x[i+j] - x[i])

    return pyramid

def newton_polynomial(x_data, y_data, x):
    pyramid = newton_divided_difference(x_data, y_data)
    n = len(x_data) - 1
    p = pyramid[0][n]

    for k in range(1,n+1):
        p = pyramid[0][n-k] + (x - x_data[n-k])*p

    return p

# Given data points
years = np.array([1960, 1970, 1990, 2000])
populations = np.array([3039585530, 3707475887, 5281653820, 6079603571])

# Estimate the 1980 population
estimated_1980_population = round(newton_polynomial(years, populations,
1980))

print(f"Estimated 1980 population using Newton Divided Differences Method
: {estimated_1980_population}")
```

```
print("Estimated 1980 population data: ", 4452584592)
print(f"Difference : {abs(estimated_1980_population - 4452584592)}")
```

```
Estimated 1980 population using Newton Divided Differences Method : 4472888288
Estimated 1980 population data:  4452584592
Difference : 20303696
```

1.

a) Lagrange Interpolation

$$L_1(x) = \frac{(x-2)(x-4)}{(0-2)(0-4)} = \frac{(x-2)(x-4)}{8}$$

$$L_2(x) = \frac{x(x-4)}{(2-0)(2-4)} = \frac{-x(x-4)}{4}$$

$$L_3(x) = \frac{x(x-2)}{(4-0)(4-2)} = \frac{x(x-2)}{8}$$

$$P_2(x) = y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x) = -\frac{(x-2)(x-4)}{4} - \frac{x(x-4)}{4} + \frac{x(x-2)}{2} = -\frac{x^2-6x+8}{4} - \frac{x^2-4x}{4} + \frac{x^2-2x}{2} = \frac{1}{4}(-x^2+6x-8-x^2+4x+2x^2-4x)$$

$$= \frac{1}{4}(6x-8) = \frac{3}{2}x - 2$$

b) Newton's divided differences

0	-2		
		3/2	
2	1		0
		3/2	
4	4		

$\Rightarrow P_2(x) = -2 + \frac{3}{2}(x-0) + 0(x-0)(x-2) = \frac{3}{2}x - 2$

2.

-2	8			
		-2		
0	4		0	
		-2		0
1	2		0	
		-2		
3	-2			

$\rightarrow P(x) = 8 - 2(x+2) + 0.(x+2)(x-0) + 0.(x+2)(x-0).(x-1)$

$= 8 - 2x - 4 = -2x + 4$

- Degree 0: None
- Degree 1: $P(x) = -2x + 4$
- Degree 2: None
- Degree 3: None
- Degree 4: Infinitely many $P_4(x) = -2x + 4 + c(x+2)x(x-1)(x-3)$

3.

1	0						
		0					
2	0		0				
		0		0			
3	0		0		0		
		0		0		0	
4	0		0		0		0
		0		0		1/12	
5	0		0		5/12		
		0		5/3			
6	0		5				
		10					
7	10						

$$P_6(x) = \frac{1}{72}(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)$$