Simulasi yang dibuat pada ujian kali ini adalah simulasi pergerakan planet di tata surya berdasarkan hukum III Kepler. Hukum III Kepler berisi “rasio kubik dari jarak rata-rata planet ke matahari (R) dengan kuadrat dari periode orbit planet (T) adalah konstan”.

Hukum III Kepler

$$\frac{R^3}{T^2} = konstanta$$

Hukum gravitasi universal Newton

$$F_G = G\,\frac{M_{matahari}M_{planet}}{R^2}$$

Gaya sentripetal planet

$$F_s = \frac{m_{planet}{v^2}_{planet}}{R}$$

$$F_s = F_G$$

$$\frac{m_{planet}{v^2}_{planet}}{R} = G\,\frac{M_{matahari}M_{planet}}{R^2}$$

$${v^2}_{planet} = \frac{G\ M_{matahari}}{R}$$

$$\bar{v} = \frac{s}{t} = \frac{2\pi R}{T}$$

$$\left(\frac{2\pi R}{T}\right)^2 = \frac{G\ M_{matahari}}{R}$$

$$\frac{4\pi^2 R^2}{T^2} = \frac{G\ M_{matahari}}{R}$$

$$\frac{R^3}{T^2} = \frac{G\ M_{matahari}}{4\pi^2}$$

Hukum II Newton

$$F_{G,x} = m_{bumi}\frac{d^2x}{dt^2} \Rightarrow \frac{d^2x}{dt^2} = \frac{F_{G,x}}{m_{bumi}}$$

$$F_{G,y} = m_{bumi}\frac{d^2y}{dt^2} \Rightarrow \frac{d^2y}{dt^2} = \frac{F_{G,y}}{m_{bumi}}$$

$$F_{G,x} = G\frac{M_{matahari}M_{bumi}}{R^2}cos\theta \Rightarrow cos\theta = \frac{x}{R}$$

$$F_{G,y} = G\frac{M_{matahari}M_{bumi}}{R^2}sin\theta \Rightarrow sin\theta = \frac{y}{R}$$

$$\frac{d^2x}{dt^2} = \frac{dV_x}{dt} = \frac{F_{G,x}}{m_{bumi}} = \frac{G\ M_{matahari}x}{R^3}$$

$$\frac{d^2y}{dt^2} = \frac{dV_y}{dt} = \frac{F_{G,y}}{m_{bumi}} = \frac{G\ M_{matahari}y}{R^3}$$

$$GM_{matahari} = v^2R \Rightarrow v = \frac{2\pi R}{T}$$

Asumsi: R=1.5 AU; T=1 tahun

$$v = \frac{2\pi R}{T} = \frac{2\pi.1.5}{1} = 3\pi\ AU/tahun$$

$$\frac{dV_x}{dt} = \frac{v^2x}{R^3} = \frac{(3\pi)^2x}{R^3} = \frac{9\pi^2x}{R^3}$$

$$\frac{dV_y}{dt} = \frac{v^2y}{R^3} = \frac{(3\pi)^2y}{R^3} = \frac{9\pi^2y}{R^3}$$

Metode diferensial Euler-Cromer

$$\frac{V_x - V_{x+1}}{\Delta t} = \frac{9\pi^2x}{R^3} \Rightarrow V_{x+1} = V_x - \frac{9\pi^2x}{R^3}\Delta t$$

$$\frac{V_y - V_{y+1}}{\Delta t} = \frac{9\pi^2y}{R^3} \Rightarrow V_{y+1} = V_y - \frac{9\pi^2y}{R^3}\Delta t$$