Probability HW 5 Ioannis Tsotras CSCI 104

1. $|S| = \frac{7!}{3!4!}$ ways or $7 \cdot 5 = 35$

E_1 = choosing ~~I~~nnovative, then analytical

E_2 = choosing Empathetic & Innovative

$E_1 \rightarrow$ 3 Cases — 2 choices —

$\overline{\#}\ \overline{I}\ \overline{A}$ or $\overline{I}\ \overline{A}\ \overline{\#}$ or $\overline{I}\ \overline{\#}\ \overline{A}$

↓ how many times

5 times + 5 times + 5 times

~~15 times~~ 35 times $|E_1| = 15$

$|E_2|$ → order doesn't matter

How many cases?

$\overline{E}\ \overline{I}\ \overline{\#}$ or $\overline{I}\ \overline{E}\ \overline{\#}$

$\overline{\#}\ \overline{E}\ \overline{I}$ or $\overline{\#}\ \overline{I}\ \overline{E}$

$\overline{E}\ \overline{\#}\ \overline{I}$ or $\overline{I}\ \overline{\#}\ \overline{E}$

$\binom{5}{4} * \binom{2}{1} * \binom{1}{0}$

$\frac{5!}{4! * 1!} * \frac{2!}{1!\,1!} = 10$

$\frac{10+15}{35} = \frac{25}{35} = \boxed{5/7}$

$n=10\ k=4$

1.2 Bernoulli Trials $p=5/7 \quad q=2/7$

$$\binom{10}{4} p^4 q^6 = \boxed{\binom{10}{4}\left(\frac{5}{7}\right)^4 \cdot \left(\frac{2}{7}\right)^6}$$

2. $|S| = 2^4 = 16$

2.1 A = 2 consecutive heads B = 1st or last is tails

HHTT HTHH
HHTH TTHH $|A|=8$
HHHT
HHHH
THHT
THHH

$P(A)=\frac{8}{16}$ $P(B)=\frac{12}{16}$

$2^3+2^3-2^2=12$

HHH
THHT
TTHT
THTT
TTTT

2^2 duplicates

$P(A\cap B)=P(A)\cdot P(B)$ $\frac{1}{2}\cdot\frac{3}{4}=3/8$

$P(A\cap B)$

THHT
THHH HHHT
HHTT
TTHH

$=\frac{5}{16}$

$\frac{5}{16} \neq \frac{3}{8}$

They aren't independent

2.2 X = # of pairs of heads in 4 consec. flips

TTTT
TTTH THTH } X=0 = 8/16
TTHT HTHT } 0
THTT
HTTT HTTH

HHTT
THHT } X=1 = 3/16
TTHH

HHHT } X=2 = 2/16
THHH

HHHH } X=3 = 1/16

Expected Value of X

E(X=0) = 0 * 8/16 = 0
E(X=1) = 1 * 3/16 = 3/16
E(X=2) = 2 * 2/16 = 4/16
E(X=3) = 3 * 1/16 = 3/16

$E(X) = \frac{5}{8}$

$\frac{3}{16} + \frac{4}{16} + \frac{3}{16} = \frac{10}{16}$

$\frac{5}{8}$

3. Expected # of moves

X = # of moves

|S| = 8

NASTY as start

NASTY is word

NASTY → done!

1 move

NASTY is word → $X = 1$ → $\frac{1}{8}$

HASTY is word

NASTY → HASTY → done!

2 moves

NASTY → HASTY, NASTY → BOARD, NASTY → HOARD, NASTY → MATHS → $X = 2$ → $\frac{4}{8}$

BOARD is word

NASTY → BOARD (2 moves)

NASTY → HOARD → BOARD (3 moves)

NASTY → HOARD → BOARD, NASTY → BOARD → HOARD, NASTY → HASTY → MATHS → $X = 3$ → $\frac{3}{8}$

HOARD is word

NASTY → BOARD → HOARD (3)

NASTY → HOARD (2)

$1 \cdot \frac{1}{8} + 2 \cdot \frac{4}{8} + 3 \cdot \frac{3}{8} =$

$\frac{1}{8} + \frac{6}{8} + \frac{8}{8} =$

$1\frac{1}{8}$ 1 1/8 of a move

MATHS is word

NASTY → MATHS (2)

NASTY → HASTY → MATHS (3)

HASTY as start

HASTY is word

HASTY → 1 move

$|S| = 5$

HASTY is word $\{X=1 = \frac{1}{5}$

NASTY is word

HASTY → NASTY = 2 moves

NASTY
BOARD
HOARD
MATHS
$\}X=2 = \frac{4}{5}$

BOARD is word

HASTY → BOARD = 2 moves

HASTY → HOARD → 1

HOARD is word

HASTY → HOARD = 2 moves

$1 \cdot \frac{1}{5} + 2 \cdot \frac{4}{5} = \frac{9}{5}$

$1\frac{4}{5}$

MATHS

HASTY → MATHS = 2 moves

1 4/5 expected move for HASTY

1 7/8 expected moves for NASTY

S = matches suspect

$|S| = 100{,}000$ I = innocent

4.

4.1 $P(I \mid S) = \frac{P(I \cap S)}{P(S)}$

$P(S) = \frac{100}{100{,}000} = \frac{1}{1000}$

$P(I \cap S) = \frac{99}{100000} \cdot \frac{99}{100000} \cdot \frac{1000}{1} = \frac{99}{100} = 99\%$

99% suspect is innocent if he matches the description

4.2 $\frac{P(I \cap S)}{P(I)}$ $P(I) = \frac{99999}{100{,}000}$

$\frac{\frac{99}{100000}}{\frac{99999}{100000}} = \frac{99}{99999} = .09900099\%$

Suspect innocent if matches description if they are innocent

4.3 1 - Prob they wouldn't match

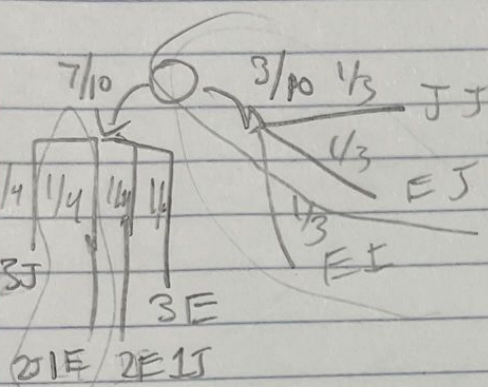
$\left[1 - \frac{\binom{99100}{1000}}{\binom{100{,}000}{1000}}\right]$ ← at least prob one would match?

5.

3/10 to have 2 Planets → 1/3 chance to have 2J

7/10 to have 3 → 1/4 chance to have a Earth

.6% to not detect it



$$p(E|2J) = \frac{p(2J \cap E)}{p(2J)}$$

chance we don't detect but it has there

$$p(E|2J) \quad \frac{\frac{3}{10} * \frac{1}{4} * .6}{\frac{7}{10} * \frac{1}{4} * .8 + \frac{1}{3} * \frac{3}{10}}$$

$p(E|2J) = 58.3\%$ chance it has an Earth planet.