1. $- c_A -- \wedge ---$

3 cases

$$\underline{a-z} \quad \frac{+}{s} \frac{s}{t} \quad \text{or} \quad \frac{+}{s} \quad \underline{a-z} \quad \frac{s}{t} \quad \text{or} \quad \frac{+}{s} \frac{s}{t} \quad a-z$$

$$\binom{26}{1} \cdot \binom{2}{1} \quad + \quad \binom{26}{1} \cdot \binom{2}{1} \quad + \quad \binom{26}{1} \binom{2}{1}$$

$$52 \quad + \quad 52 \quad + \quad 52 = 156$$

Overlap

tts	+ts	tst
tst	tss	tss
sts	sst	sts
sst	stt	st+

tts → 2	tss → 2	
tst → 2	stt → 2	$156 - 6 \text{ duplicates} = 150$
sst → 2		
sts → 2		150 unique strings

2. Alice, Bob, Charlie

3,4,5,6   2-6   2-6
(↓)   (↓)   (↓)

where they are on different floors

$$\text{A} \downarrow \quad \text{B} \downarrow \quad \text{C} \downarrow$$
$$\rightarrow 4 \times 4 \times 3 = 48$$

where they are all on the same floor $\Rightarrow 4 \cdot \overset{3,4,5,6}{\underset{}{1}} \rightarrow 4$

only two on the same floor $_{3,4,5,6}$ $\rightarrow$ the three one

$\rightarrow$ Alice & Bob $= 4 \times \underset{\text{same}}{\underset{\text{bor}}{1}} \times 4 = 16$

$\rightarrow$ Alice & Charlie $= 4 \times 4 \times 1 = 16$

$\rightarrow$ Bob & Charlie = Alice doesn't go on 3

     $\rightarrow$ if floor 3: $4 \cdot 1 \cdot 1 = 4$

     $\rightarrow$ if any other floor : $3 \cdot 4 \wedge 1 = 12$

48 + 4 + 16 + 16 + 4 + 12

48 + 4 + 48 = 100

100 different events

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3. 21 Total Pokemon, 7 water, 5 fire, 9 other

3.1 only 1 fire & 1 water type or all 3 one other

$$\frac{7!}{1!6!} \cdot \frac{5!}{1!4!} \cdot \frac{9!}{1! \cdot 8!} \quad + \quad \frac{9!}{3!6!} \quad \checkmark \text{ all 3 one other}$$

Choose 2 matters

315 + 84 = **399 combinations**

1 fire
1 other

$399 + \frac{7!}{116!} \cdot \frac{9!}{7!2!} + \frac{5!}{214!} \cdot \frac{9!}{7!2!} = \boxed{831 \text{ combos}}$

3.2: $\dfrac{(10+2)!}{10! \, 2!} = \dfrac{12!}{2! \, 10!} = \dfrac{132}{2!}$ water   fire   other
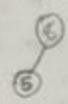
66 different combinations

4. $[\_,\_,\_,\_]$ Indist over Ind 1st.
only 2 copies

$(5,1,0,0), (4,2,0,0), (3,3,0,0)$ $= 3$

$(4,1,1,0), (3,2,1,0), (2,2,2,0)$ $\overline{only\ 3}+$
$= 3$

$(3,1,1,1), (2,2,1,1)$ 4 copies $= 2 = 8$

8 total ways

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5.   3 nodes → ⓪ - 12   x>y

ways to form
Bst of 2 nodes

$\Delta * \frac{2!}{1!\,1!} \cdot \frac{1}{1} = 2$ ways
↓
make
a 1 node   → 2 places to put it

BST of 3 Nodes:
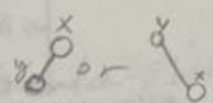
$\underset{\substack{\downarrow \\ make \\ a\ 2\ node \\ BST}}{2} * \underset{\substack{\downarrow \\ 3 \\ roots \\ choose \\ from}}{\frac{3!}{2!\,2!}} - \underset{\substack{\downarrow \\ duplicate \\ structure}}{1} = 5$
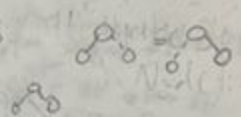
BST OF 4 Nodes

$\underset{\substack{\downarrow \\ make \\ a\ BST \\ of\ 3 \\ \downarrow \\ not \\ counting}}{5} * \underset{\substack{\downarrow \\ 4\ places \\ to\ add \\ a\ new \\ node}}{\frac{4!}{2!\,3!}} - 6\ duplicates = 14\ trees$
structures
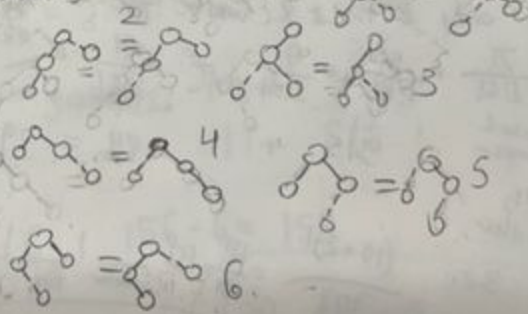
$14 = 2 * (5+2)$
$2 * 7 = 14$

4 Nodes = 2(2 Nodes + 3 Nodes)

5 Node BST $= 2*(4 \text{ Nodes} + 3 \text{ Nodes})$

$\quad\quad\quad\quad 2*(14 + 5)$

$\quad\quad\quad\quad = 2 * 19$

$\quad\quad\quad\quad 38$ ways to make 5-Node BST

BST of 13 Nodes $= 2(12 \text{ Nodes} + 11 \text{ Nodes})$

12-Node BST $= 2(11N + 10N)$

11 Node BSTs $= 2(10N + 9N)$

10 N BSTs $= 2(9N + 8N)$

9 N BSTs $= 2(8N + 7N)$

8 N BSTs $= 2(7N + 6N)$

7 N BSTs $= 2(6N + 5N \text{ BSTs})$

6 N BSTs $= 2(5N + 4N \text{ BSTs})$

5N BSTs $= 2(4N + 3N)$ BSTs

4 N BSTs $= 2(3N + 2 \text{ BSTs})$

3 N BSTs $= 2(2N + 1N)$ BSTs

2 N BSTs $= 2(1N + 0N)$ BSTs

$\quad\quad$ ↳ for this → there is only one valid BST of ∞

$2N = 1$

$1N = 1$

$\quad\quad\quad\quad 1 + 1 = (2) 2 = 4$

$\quad\quad 3N = 4$

$4N = (4+1)*2 \to 10$

$5N = (10+4)2 = 28$ BSTs

$6N = 2(38) = 76$ BSTs

$7N = 2(76 + 28) = 208$ BSTs

$8N = 2(208 + 76) = 598$ BSTs

$9N = 2(598 + 208) = 1612$ BSTs

$\boxed{90064 * 111}$

$\quad$ ↳ different BSTs

$10N = 2(1612 + 598) = (2210)2$

$\quad\quad\quad 4420$ BSTs

$11N = (1612 + 4420)2 = 12064$ BSTs

$12N = (12064 + 4420)*2 = 32968$

$13N = (12064 + 32968)*2 = 90064$

$90064 * 111$

$\quad\quad$ ↳ 11 to

$\quad\quad$ 1-base from