Problem 3 Csci 104 Ioannis Tsotres

a) $t = \sqrt{n}$ $i=0$ $n=9$ $t=3$

```
for(int i=0; i<n; i++){
    n-=t;
}
```

i	0	1	2	3
n	9	6	3	0

$i=0 \rightarrow \sqrt{n}-1$

$$\sum_{i=0}^{\sqrt{n}-1} (\ )$$

```
for(int j=0; j<n; j++);
```

$$\sum_{j=0}^{n-1} \Theta(1) = \Theta(n-1) = \Theta(n)$$

$$\sum_{i=0}^{\sqrt{n}-1} \Theta(n) = \Theta(n\sqrt{n}) - \Theta(n)$$

$$\Theta(n\sqrt{n})$$

b)

```
for (int i=1; i<=n; i++){
    for(int K=1; K<=n; K++){
        if( A[K] == i ){
            for(int m=1; m<=n; m=m+m){
                //do something O(1)
            }
        }
    }
}
```

to check if

$$\sum_{i=1}^{n} \left(\sum_{K=1}^{n} \left[\Theta(1) + \sum_{x=1}^{\log_2 n} \Theta(1) \right] \right)$$

if every item is i

Stops

x	1	2	3	4	...	x	$\log_2 n$
m	2	4	8	16		2^x	n

$2^x = n$

$x = \log_2 n$

$$\sum_{x=1}^{\log_2 n} \Theta(1)$$

$$\sum_{i=1}^{n} \left(\sum_{K=1}^{n} \left[\Theta(1) + \sum_{x=1}^{\log_2 n} \Theta(1) \right] \right)$$

$$\sum_{K=1}^{n} \left[\Theta(1) + \Theta(\log_2 n) \right]$$

$$\sum_{i=1}^{n} \left[\Theta(n) + \Theta(n \log n) \right]$$

$$\Theta(n^2) + \Theta(n^2 \log n)$$

$$\boxed{\Theta(n^2 \log n)}$$

if $n <= 1$, $\Theta(1)$

c) if(n <= 1) return; $T(1) = \Theta(1)$

else {

f3 (A, n-2); $T(n-2) = \Theta(1) + 2T(n-4)$

f3 (A, n-2); $T(n-4) = \Theta(1) + 2 \cdot T(n-6)$

}

1st $T(n) = \Theta(1) + 2T(n-2) = \Theta(1) + 2T(n-2)$

2nd $\Theta(1) + \Theta(2) + 4 \cdot T(n-4) = \Theta(3) + 4T(n-4)$

3rd $\Theta(1) + \Theta(2) + \Theta(4) + 8 \cdot T(n-6)$

$\hookrightarrow = \Theta(7) + 8 \cdot T(n-6)$

kth $\Theta(2^k - 1) + 2^k \cdot T(n-2k)$

When do we stop? $n - 2k <= 1$

$n - 1 = 2k$ $\quad \frac{n-1}{2} = k$

$T(1) = \Theta(1)$, when $n = 1$

$T(n) = \Theta(2^{\frac{n-1}{2}} - 1) + 2^{\frac{n-1}{2}} \cdot \Theta(1)$

$\Theta(2^{\frac{n}{2}}) + \Theta(2^{\frac{n}{2}})$

$\Theta(2^{\frac{n}{2}+1})$

$T(n) = \Theta(2^{\frac{n}{2}})$

d)

for(int i=0; i<n; i++){ $\rightarrow \sum_{i=0}^{n-1} (\quad)$

if(i == size){ $\longrightarrow \Theta(1)$ + how many times if executes

int newsize = 4·size; $\rightarrow \Theta(1)$

int *b = new int[newsize]; $\rightarrow \Theta(1)$

for(int j=0; j<size; j++) b[j] = a[j];

$\hookrightarrow \sum_{j=0}^{i-1} \Theta(1);$

delete [] a;

a=b; $\rightarrow \Theta(1) + \Theta(1)$

size = newsize; $- \Theta(1)$

}

a[i] = i·i; $\Theta(1)$

when will the if execute?

k	0	1	2	3	...	kth
i	10	40	160	640	...	$10 \cdot 4^k$

$10, 10 \cdot 4^1, 10 \cdot 4^2, 10 \cdot 4^3$

kth

Stops

$i = n-1$

n

$10 \cdot 4^k = n-1$

$= n$

$4^k = \frac{n-1}{10}$

$k = \log_4 \frac{n-1}{10}$

$k = \log_4 \frac{n}{10} + 1$

$\sum_{k=0}^{\log_4 \frac{n-1}{10}}$

}

$i = 4^k \cdot 10$

$$k = \log_4 \frac{n}{10}$$

$$4^{\log_4 \frac{n}{10}} = \frac{n}{10} \cdot 10$$

$$n = 4^k \cdot 10$$

$$\sum_{i=0}^{n-1} \left[\Theta(2) + \sum_{k=0}^{\log_4 \frac{n}{10}} \left[\Theta(5) + \sum_{j=0}^{n-1} \Theta(1) \right] \right]$$

$$\Theta(2n) + \sum_{i=0}^{n-1} \left[\sum_{k=0}^{\log_4 \frac{n}{10}} \left[\Theta(5) + \sum_{j=0}^{n-1} \Theta(1) \right] \right]$$

$$\Theta[n] + \sum_{i=0}^{n-1} \left[\sum_{k=0}^{\log_4 \frac{n}{10}} \left[\Theta(5) + \sum_{j=0}^{4^k \cdot 10 - 1} \Theta(1) \right] \right]$$

$$\Theta(n) + \sum_{i=0}^{n-1} \left[\underbrace{\sum_{k=0}^{\log_4 \frac{n}{10}} \left[\Theta(5) + \Theta(4^k \cdot 10 - 1) \right]} \right]$$

$$\sum_{k=0}^{\log_4 \frac{n}{10}} \Theta(5) + \Theta(4^k \cdot 10)$$

$$\Theta(n) + \sum_{i=0}^{n-1} \left[5 \log_4 \frac{n}{10} + \Theta(n) \right]$$

$$\Theta(n) + \sum_{i=0}^{n-1} \Theta(\log n) + \Theta(n)$$

$$\Theta(n) + \Theta(n \log n) + \Theta(n^2)$$

$$\boxed{\Theta(n^2)}$$