An improved Controlled Random Search method

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Abstract

A modified version of common global optimization method named Controlled Random Search is presented here. The method aims to locate the global minimum of multidimensional functions inside a rectangular hyperbox. The new method modifies the original algorithm by incorporating a new sampling method, a new termination rule and the periodical application of local search optimization algorithm to the points sampled. The new version is compared against the original one on a series of test functions from the relevant literature and the results are reported.

1 Introduction

Global optimization[1] is considered a problem of high complexity with many applications. The problem is defined as the location of the global minimum of a multi - dimensional function $f(x)$:

$$x^* = \arg\min_{x \in S} f(x) \tag{1}$$

Where $S \subset R^n$ is formulated as:

$$S = [a_1, b_1] \otimes [a_2, b_2] \otimes \ldots [a_n, b_n] \tag{2}$$

This problem is applicable in a wide range of areas such as physics [2, 3], chemistry [6, 7], medicine [4, 5], economics[8] etc. In modern theory there are two different categories of global optimization methods: the stochastic methods and the deterministic methods. The first category contains the vast majority of methods such as Simulated Annealing methods [9, 10, 11], Genetic Algorithms [12, 13, 14], Tabu Search methods [15], Particle Swarm Optimization [16, 17, 18] etc. A common method that also belongs to stochastic methods is the Controlled Random Search (CRS) method [19], which is a population based algorithm.

This method initially creates a set with randomly selected points and repeatedly replaces the worst point in that set with a randomly generated point and this process can continue until some convergence criterion is met. The CRS method has been used intensively in many problems such as geophysics problems [20, 21], optimal shape design problems [22], the animal diet problem [23], the heat transfer problem [24] etc.

This CRS method has been thoroughly analyzed by many researchers in the field such as the work A of Ali and Storey where two new variants of the CRS method are proposed [25]. These variants have proposed alternative techniques for the selection of the initial sample set and usage of local search methods. Also, Pillo *et al* [26] suggested a hybrid CRS method where the base algorithm is combined with a Newton - type unconstrained minimization algorithm [27] to enhance the efficiency of the method in various test problems. Another work is of Kaelo and Ali , in which they suggested [28] some modifications on the method and especially in the new point generation step. Also, Filho and Albuquerque have suggested [29] the usage of a distribution strategy to accelerate the Controlled Random Search method. The current work proposed three major modifications in the CRS method: a new point replacement strategy, a stochastic termination rule and a periodical application of some local search method. The first modification is used to better explore the domain range of the function. The second modification is made in order to achieve a better termination of the method without wasting valuable computational time. The third modification is used in order to speed up the method by applying a small amount of steps of a local search method.

The rest of this article is organized as follows: in Section 2 the major steps of the CRS method as well as the proposed modifications are outlined, in section 3 the results from the application of the proposed method on a series of test functions are provided and finally in section 4 some conclusions are presented.

2 Method Description

The Controlled Random Search has a series of steps that are described in Algorithm 1. The changes proposed by the new method focus on three points:

1. The creation of a test point (**New_Point** step) is performed with a new technique described in subsection 2.1.

2. In the **Min_Max** step the stochastic termination rule described in subsection 2.2 is used. The aim of this rule is to terminate the method when, with some certainty, no lower minimums are to be found.

3. Apply a few steps of a local search procedure after **New_Point** step in the $\tilde{z}$ point. This procedure is used to bring the test points closer to the corresponding minimums. This speeds up the process of searching for new minima, although it obviously leads to an increase in function calls

Algorithm 1 The Controlled Random Search algorithm.

Initialization Step:

1. **Set** the value for the parameter N. A commonly used value for that is $N = 25n$.
2. **Set** a small positive value for ϵ.
3. **Create** the set $T = \{z_1, z_2, ..., z_N\}$, by randomly sampling N points from S.

Min_Max Step:

1. **Calculate** the points $z_{\text{min}} = \text{argmin} f(z)$ and $z_{\text{max}} = \text{argmax} f(z)$ and their function values
$$f_{\text{max}} = \max_{z \in T} f(z)$$
and
$$f_{\text{min}} = \min_{z \in T} f(z)$$
2. **If** $|f_{\text{max}} - f_{\text{min}}| < \epsilon$, **then goto Local_Search** Step.

New_Point Step:

1. **Select** randomly the reduced set $\tilde{T} = \{z_{T_1}, z_{T_2}, .., z_{T_{n+1}}\}$ from T.
2. **Compute** the centroid G:
$$G = \frac{1}{n} \sum_{i=1}^{n} z_{T_i}$$
3. **Compute** a trial point $\tilde{z} = 2G - z_{T_{n+1}}$.
4. **If** $\tilde{z} \notin S$ **or** $f(\tilde{z}) \geq f_{\text{max}}$ **then** goto New_Point step.

Update Step:

1. $T = T \cup \{\tilde{z}\} - \{z_{\text{max}}\}$.
2. **Goto Min_Max** Step.

Local_Search Step:

1. $z^* = \text{localSearch}(z)$.
2. Return the point z^* as the discovered global minimum.

Algorithm 2 Proposed method to compute the trial point.

1. **Compute** the centroid G:

$$G = \frac{1}{n}\sum_{i=1}^{n} z_{T_i}$$

2. Set $G = G + \frac{1}{n} z_{\text{min}}$
3. **Compute** a trial point $\tilde{z} = G - \frac{1}{n} z_{T_{n+1}}$.

2.1 A new method for trial points

The proposed technique to compute the trial point $\tilde{z}$ is shown in Algorithm 2. According to this, the calculation of the test point $\tilde{z}$ does not contain product with high values as in the basic algorithm, so that the test point is not too far from the centroid. This technique avoids vector jumps from the centroid, where it has great gravity in the calculation for starting the local optimization.

2.2 A new stopping rule

It is quite common in the optimization techniques to use a predefined number of maximum iterations as the stopping rule of the method. Even though this termination rule is easy to implement sometime could require an excessive number of functions calls before termination and a more sophisticated termination rule is needed. The termination rule proposed here is inspired from [30]. At every iteration k the variance $\sigma^{(k)}$ of the quantity f_{min} is calculated. If the optimization technique did not manage to find a new estimation of the global minimum for some iterations, then it is highly possible that the global minimum is already located and hence the algorithm should terminate. The termination rule is defined as follows, terminate when

$$\sigma^{(k)} \le \frac{\sigma^{\left(k_{\textbf{last}}\right)}}{2} \tag{3}$$

The term $k_{\textbf{last}}$ represents the last iteration where a new global minimum was located.

3 Experiments

3.1 Test functions

The modified version of the CRS was tested against the traditional CRS on series of benchmark functions from the relevant literature [31, 32]. The following functions were used:

- **Bf1** function. The function Bohachevsky 1 is given by the equation

$$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1) - \frac{4}{10}\cos(4\pi x_2) + \frac{7}{10}$$

with $x \in [-100, 100]^2$.

- **Bf2** function. The function Bohachevsky 2 is given by the equation

$$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1)\cos(4\pi x_2) + \frac{3}{10}$$

  with $x \in [-50, 50]^2$.

- **Branin** function. The function is defined by $f(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$ with $-5 \le x_1 \le 10,\ 0 \le x_2 \le 15$. The value of global minimum is 0.397887.with $x \in [-10, 10]^2$.

- **CM** function. The Cosine Mixture function is given by the equation

$$f(x) = \sum_{i=1}^{n} x_i^2 - \frac{1}{10}\sum_{i=1}^{n}\cos(5\pi x_i)$$

  with $x \in [-1, 1]^n$. The value of the global minimum is -0.4 and in our experiments we have used $n = 4$.

- **Camel** function. The function is given by

$$f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4, \quad x \in [-5, 5]^2$$

- **Easom** function. The function is given by the equation

$$f(x) = -\cos(x_1)\cos(x_2)\exp\left((x_2 - \pi)^2 - (x_1 - \pi)^2\right)$$

  with $x \in [-100, 100]^2$.

- **Exponential** function. The function is given by

$$f(x) = -\exp\left(-0.5\sum_{i=1}^{n} x_i^2\right), \quad -1 \le x_i \le 1$$

  The global minimum is located at $x^* = (0, 0, ..., 0)$ with value -1. In our experiments we used this function with $n = 2, 4, 8, 16, 32, 64, 100$ and the corresponding functions are denoted by the labels EXP2, EXP4, EXP8, EXP16, EXP32, EXP64, EXP100.

- **GoldStein & Price**

$$
\begin{aligned}
f(x) &= [1+(x_1+x_2+1)^2 \\
&\quad (19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)] \times \\
&\quad [30+(2x_1-3x_2)^2 \\
&\quad (18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]
\end{aligned}
$$

The function has 4 local minima in the range $[-2,2]^2$ and global minimum $f^*=3.0$.

- **Griewank2** function. The function is given by

$$f(x) = 1 + \frac{1}{200}\sum_{i=1}^{2} x_i^2 - \prod_{i=1}^{2}\frac{\cos(x_i)}{\sqrt{(i)}}, \quad x \in [-100,100]^2$$

  The global minimum is located at the $x^* = (0,0,...,0)$ with value 0.

- **Gkls** function. $f(x) = \mathrm{Gkls}(x,n,w)$, is a function with w local minima, described in [33] with $x \in [-1,1]^n$ and n a positive integer between 2 and 100. The value of the global minimum is -1 and in our experiments we have used $n=2,3$ and $w=50$. The corresponding functions are denoted by the labels GKLS250 and GKLS350.

- **Guilin Hills** function. $f(x) = 3 + \sum_{i=1}^{n}\left(c_i\frac{x_i+9}{x_i+10}\sin\left(\frac{\pi}{1-x_i+\frac{1}{2k_i}}\right)\right)$, $x \in [0,1]^n$, $c_i > 0$ and k_i being positive integers. The function has $\prod_{i=1}^{n} k_i$ local minima. In our experiments we have used $n=5,10$ and we set the values of k_i so that the number of minima was 50. The produced functions are entitled GUILIN550 and GUILIN1050 in the result tables.

- **Hansen** function. $f(x) = \sum_{i=1}^{5} i\cos\left[(i-1)x_1+i\right]\sum_{j=1}^{5} j\cos\left[(j+1)x_2+j\right]$, $x \in [-10,10]^2$. The global minimum of the function is -176.541793.

- **Hartman 3** function. The function is given by

$$f(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} a_{ij}\left(x_j-p_{ij}\right)^2\right)$$

  with $x \in [0,1]^3$ and $a = \begin{pmatrix} 3 & 10 & 30 \\ 0.1 & 10 & 35 \\ 3 & 10 & 30 \\ 0.1 & 10 & 35 \end{pmatrix}$, $c = \begin{pmatrix} 1 \\ 1.2 \\ 3 \\ 3.2 \end{pmatrix}$ and

$$p = \begin{pmatrix} 0.3689 & 0.117 & 0.2673 \\ 0.4699 & 0.4387 & 0.747 \\ 0.1091 & 0.8732 & 0.5547 \\ 0.03815 & 0.5743 & 0.8828 \end{pmatrix}$$

- **Hartman 6** function.

$$f(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} a_{ij}\left(x_j - p_{ij}\right)^2\right)$$

with $x \in [0,1]^6$ and $a = \begin{pmatrix} 10 & 3 & 17 & 3.5 & 1.7 & 8 \\ 0.05 & 10 & 17 & 0.1 & 8 & 14 \\ 3 & 3.5 & 1.7 & 10 & 17 & 8 \\ 17 & 8 & 0.05 & 10 & 0.1 & 14 \end{pmatrix}$, $c = \begin{pmatrix} 1 \\ 1.2 \\ 3 \\ 3.2 \end{pmatrix}$

and

$$p = \begin{pmatrix} 0.1312 & 0.1696 & 0.5569 & 0.0124 & 0.8283 & 0.5886 \\ 0.2329 & 0.4135 & 0.8307 & 0.3736 & 0.1004 & 0.9991 \\ 0.2348 & 0.1451 & 0.3522 & 0.2883 & 0.3047 & 0.6650 \\ 0.4047 & 0.8828 & 0.8732 & 0.5743 & 0.1091 & 0.0381 \end{pmatrix}$$

- **Rastrigin** function. The function is given by

$$f(x) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2), \quad x \in [-1,1]^2$$

- ***Rosenbrock** function*

  This function is given by

$$f(x) = \sum_{i=1}^{n-1}\left(100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right), \quad -30 \le x_i \le 30.$$

  The global minimum is located at the $x^* = (0, 0, ..., 0)$ with $f(x^*) = 0$. In our experiments we used this function with $n = 20$.

- **Shekel 7** function.

$$f(x) = -\sum_{i=1}^{7} \frac{1}{(x - a_i)(x - a_i)^T + c_i}$$

with $x \in [0,10]^4$ and $a = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 \\ 3 & 7 & 3 & 7 \\ 2 & 9 & 2 & 9 \\ 5 & 3 & 5 & 3 \end{pmatrix}$, $c = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4 \\ 0.6 \\ 0.3 \end{pmatrix}$.

- **Shekel 5** function.

$$f(x) = -\sum_{i=1}^{5} \frac{1}{(x - a_i)(x - a_i)^T + c_i}$$

with $x \in [0,10]^4$ and $a = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 \\ 3 & 7 & 3 & 7 \end{pmatrix}$, $c = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4 \end{pmatrix}$.

- **Shekel 10** function.

$$f(x) = -\sum_{i=1}^{10} \frac{1}{(x-a_i)(x-a_i)^T + c_i}$$

with $x \in [0,10]^4$ and $a = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 \\ 8 & 8 & 8 & 8 \\ 6 & 6 & 6 & 6 \\ 3 & 7 & 3 & 7 \\ 2 & 9 & 2 & 9 \\ 5 & 5 & 3 & 3 \\ 8 & 1 & 8 & 1 \\ 6 & 2 & 6 & 2 \\ 7 & 3.6 & 7 & 3.6 \end{pmatrix}$, $c = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.2 \\ 0.4 \\ 0.4 \\ 0.6 \\ 0.3 \\ 0.7 \\ 0.5 \\ 0.6 \end{pmatrix}$.

- **Sinusoidal** function. The function is given by

$$f(x) = -\left(2.5\prod_{i=1}^{n}\sin\left(x_i - z\right) + \prod_{i=1}^{n}\sin\left(5\left(x_i - z\right)\right)\right), \quad 0 \le x_i \le \pi.$$

  The global minimum is located at $x^* = (2.09435, 2.09435, ..., 2.09435)$ with $f\left(x^*\right) = -3.5$. In our experiments we used $n = 4, 8, 16, 32$ and $z = \frac{\pi}{6}$ and the corresponding functions are denoted by the labels SINU4, SINU8, SINU16, SINU32.

- **Test2N** function. This function is given by the equation

$$f(x) = \frac{1}{2}\sum_{i=1}^{n} x_i^4 - 16x_i^2 + 5x_i, \quad x_i \in [-5, 5].$$

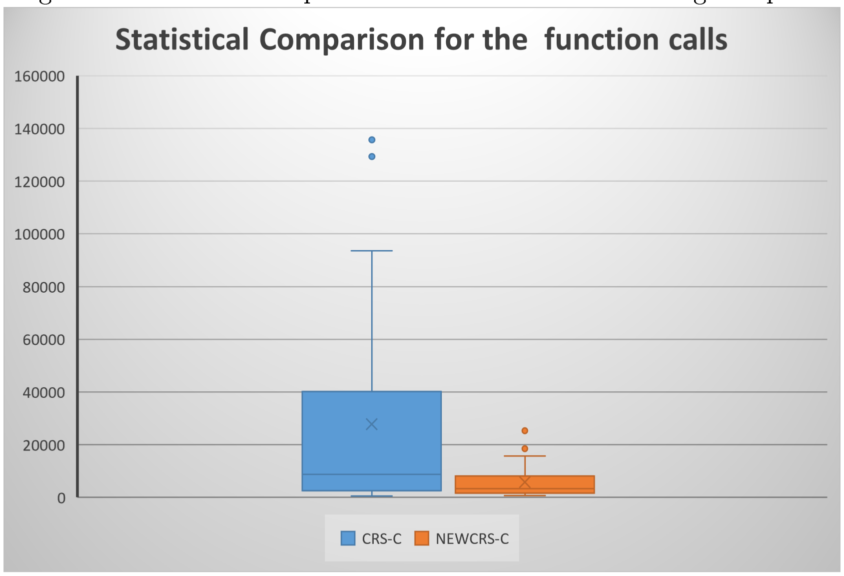
  The function has 2^n in the specified range and in our experiments we used $n = 4, 5, 6, 7$. The corresponding values of global minimum is -156.664663 for $n = 4$, -195.830829 for $n = 5$, -234.996994 for $n = 6$ and -274.163160 for $n = 7$.

- **Test30N** function. This function is given by

$$f(x) = \frac{1}{10}\sin^2\left(3\pi x_1\right)\sum_{i=2}^{n-1}\left(\left(x_i - 1\right)^2\left(1 + \sin^2\left(3\pi x_{i+1}\right)\right)\right) + \left(x_n - 1\right)^2\left(1 + \sin^2\left(2\pi x_n\right)\right)$$

  with $x \in [-10, 10]$. The function has 30^n local minima in the specified range and we used $n = 3, 4$ in our experiments. The value of global minimum for this function is 0.0

Figure 1: Statistical comparison for the function calls using box plots.



3.2 Results

In the experiments two different values were measured: the rejection rate in the **New_Point** step and the average number of function calls required. In the first case we measure the percentage of points rejected during the **New_Point** step, i.e. points created that are outside the domain range of the function. All the experiments were conducted 30 times and different seed for the random number generator was used each time. The local search method that used in the experiments and denoted as localsearch(x), was a BFGS variant due to Powell[34]. The experimental results are listed in Table 1. The column FUNCTION stands for the name of the objective function, the column CRS-R represents the rejection rate for the CRS method while the column NEWCRS-R represents the same measure for the proposed method. Similar, the column CRS-C represents the average function calls for the CRS method and the column NEWCRS-C stands for the average function calls of the proposed method. Also a statistical comparison between the CRS and the proposed method is shown in Figure 1.

The proposed method almost annihilates the rejection rate in every test function. This is an evidence that the new mechanism proposed here to create new point is more accurate than the traditional one. Also, the proposed method requires lower number of function calls than the CRS method as one can deduce from the relevant columns and the statistical comparison.

Table 1: Experimenting with rejection rates.

FUNCTION	CRS-R	CRS-C	NEWCRS-R	NEWCRS-C
BF1	1.37%	2523	0.00%	1689
BF2	1.33%	2506	0.17%	1569
BRANIN	16.00%	2014	9.13%	851
CAMEL	1.67%	2235	0.20%	1487
EASOM	51.03%	591	11.43%	635
EXP2	3.03%	1290	0.70%	644
EXP4	2.67%	4688	0.00%	1302
EXP8	2.77%	16453	0.00%	2601
EXP16	4.00%	47400	0.00%	5207
EXP32	7.70%	93520	0.00%	10414
EXP64	18.80%	135638	0.00%	13602
EXP100	38.53%	129327	0.00%	14506
GKLS250	3.87%	1784	0.27%	1684
GKLS350	6.43%	3881	0.03%	2088
GOLDSTEIN	3.60%	2154	0.70%	1829
GRIEWANK2	1.20%	2503	0.03%	2742
GUILIN550	8.33%	9129	0.00%	25333
GUILIN1050	9.63%	30806	0.00	10561
HANSEN	47.60%	2643	4.03%	1736
HARTMAN3	9.97%	3009	6.13%	1331
HARTMAN6	13.37%	13615	0.00%	6091
RASTRIGIN	9.17%	2130	1.33%	2986
ROSENBROCK	0.00%	59024	0.00%	15719
SHEKEL5	4.73%	8974	0.00%	2967
SHEKEL7	3.70%	8606	0.00%	3236
SHEKEL10	2.73%	9264	0.00%	3479
SINU4	3.90%	6525	0.00%	2889
SINU8	5.10%	21561	0.00%	4946
SINU16	8.43%	62194	0.00%	9539
SINU32	14.40%	135986	0.00%	18456
TEST2N4	24.57%	10198	0.00%	3756
TEST2N5	34.17%	20850	0.00%	4806
TEST2N6	42.50%	43290	0.00%	6075
TEST2N7	50.37%	92658	0.00%	7005
TEST30N3	24.10%	4011	0.00%	5691
TEST30N4	27.30%	7432	0.00%	8579
TOTAL	**13.67%**	1000412	**0.86%**	208031

4 Conclusions

Three important modifications were proposed in the current work for the CRS method. The first modification has to do with the new test point generation process, which seems to be more accurate than the original one. The new method almost every time creates points that are within the domain range of the function. The second change adds a new termination rule based on stochastic observations. The third proposed modification applies a few steps of a local search procedure to every trial point created by the algorithm. Judging by the results, it seems that the proposed changes have two important effects. The first is that the success of the algorithm in creating valid test points is significantly improved. The second is the large reduction in the number of function calls required to find the global minimum.

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