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An Innovative Hybrid Approach to Global Optimization

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Abstract: Global optimization is critical in engineering, computer science, and various industrial applications, as it aims to find optimal solutions for complex problems. The development of efficient algorithms has emerged from the need for optimization, with each algorithm offering specific advantages and disadvantages. An effective approach to solving complex problems is the hybrid method, which combines different algorithms. This paper presents a hybrid global optimization method, calculating optimal solutions iteratively through vector operations. These operations are based on samples derived either from internal line searches or genetically modified samples in specific subsets of Euclidean space. Additionally, other relevant approaches are explored to enhance the method's efficiency.

Keywords: Optimization; Differential evolution; Genetic algorithm; Line search; Evolutionary techniques; Stochastic methods; Hybrid methods.

1. Introduction

The basic goal of global optimization is to find the global minimum by searching the appropriate range of the underlying objective problem. The global optimization method aims to find the global minimum of a continuous multidimensional function and is defined as

$$x^* = \arg\min_{x \in S} f(x) \tag{1}$$

where the set S is defined as follows:

$$S = [a_1, b_1] \times [a_2, b_2] \times \ldots [a_n, b_n]$$

Global Optimization refers to algorithms whose main objective is to find the global optimum of a problem. According to literature research there are a variety of real-world problems that can be formulated as global optimization problems, such as problems in mathematics [1–3], physics [4–6], chemistry [7–9], and medicine [10–12], biology [13,14], agriculture [15,16] and economics [17,18]. Optimization methods can be categorized into deterministic [19–21] and stochastic [22–24] based on how they approach solving the problem. The techniques used for deterministic are mainly interval methods [25,26]. In interval methods, the set S is divided into smaller regions that may contain the global minimum using certain criteria. On the other hand stochastic methods use randomness to explore the solution space. Recently, Sergeyev et al [27] proposed a comparison comparison between deterministic and stochastic methods.

A series of stochastic optimization methods are the so - called evolutionary methods, which attempt to mimic a series of natural processes. Such methods include the Genetic algorithms [28,29], the Differential Evolution method [31,32], Particle Swarm Optimization (PSO) methods [33–35], Ant Colony optimization methods [36,37], the Fish Swarm

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Algorithm [38], the Dolphin Swarm Algorithm [39], the Whale Optimization Algorithm (WOA) algorithm [40–42] etc. Also, due to the wide spread of parallel computing units, a variety of research papers related to evolutionary techniques has been appeared that use such processing units [43–45]. In the current work two evolutionary methods was incorporated in the final algorithm: Genetic Algorithms and the Differential Evolution method was combined in a hybrid optimization method.

Genetic algorithms were formulated by John Holland [46] and his team and the initially generate randomly candidate solutions to an optimization problem. These solutions altered iteratively through a series of operators that mimic natural processes, such as mutation, selection and crossover. Genetic algorithms have been used widely in areas such as networking [47], robotics [48,49], energy topics [50,51] etc. They can be combined with machine learning to solve complex problems, such as neural network training [52,53].

On the other hand, differential evolution (DE) is used in symmetric optimization problems [54,55] and in problems that are discontinuous and noisy and change over time. After studies, it was observed that differential evolution can be successfully combined with other techniques for machine learning applications, such as classification [56,57], feature selection [58,59], deep learning [60,61]etc.

Hybrid methods [62,63] in global optimization refer to techniques that combine multiple optimization strategies to solve complex problems. These methods aim to take advantage of different approaches to find the global optimum in a more efficient way, particularly when dealing with large-scale problems or strongly nonlinear optimization landscapes. A typical example of a hybrid method is the work of Shutao Li et al who propose a new hybrid PSO-BFGS strategy for the global optimization of multimodal functions [64]. To make the combination more efficient, they proposed an LDI to dynamically start the local search and a repositioning technique to maintain the particle diversity, which can effectively avoid the premature convergence problem. Another innovative hybrid method is the work of M. Andalib Sahnehsaraei et al where a hybrid algorithm using GA operators and PSO formula is proposed was presented through the use of efficient operators, for example, traditional and multiple crossovers, mutation and PSO formula [65].

The current work produces through a series of steps trial solutions using the genetic operators of the Genetic Algorithm as well as solutions identified by a line search procedure. In the current work the Armijo line search is a method is used. This method is incorporated to estimate an appropriate step when updating the trial points and it was originated in the work of Armijo[66]. The solutions produced in the previous step are used to formulate new trial solutions using a process derived from Differential Evolution.

The remainder of this paper is divided into the following sections: in section 2, the proposed method is described, in section 3 the experimental results and statistical comparisons are presented, and finally in section 4 some conclusions and guidelines for future improvements are discussed.

2. The overall algorithm

The proposed method combines some aspects from different optimization algorithms and the main steps are subsequently:

1. **Initialization step.**
   (a) **Set** the population size $N \geq 4$.
   (b) **Set** n the dimension of the benchmark function.
   (c) **Initialize** the samples $x_i, i = 1, \ldots, N$ using uniform distribution.
2. **Calculation step.**
   (a) **For** $i = 1 \ldots \mathrm{N}$ **do**
      i. **Obtain** sample x_i.

ii. **Find nearest sample** c_ι **from** x_ι**:**

$$d(x,c) = \sqrt{\sum_{i=1}^{n}(x_i - c_i)^2} \tag{2}$$

where $d(x,c)$ is the Euclidean Distance.

iii. **Set** direction vectors: $p_1 = -\nabla f(x_i)$ end $p_2 = -\nabla f(c_i)$

iv. **Set** initial step size for Armijo $a = a_0$

v. **Compute** with line search Armijo the sample:

- **Find** new points using line search minLS(x,c): $x_i^{new} = x_i + ap_1$ and $c_i^{new} = c_i + ap_2$
- **Adjust** step size a until Armijo condition is met:

$$f(x_i^{new}, {c_i}^{new}) \leq f(x_i, c_i) + c_1 a \nabla f(x_i, c_i)^T (p_1, p_2) \tag{3}$$

vi. **Make** sample-child with crossover with random number $g_k \in [0.0, 1.0]$:

$$\text{child}\left(x, x^{best}\right) = g_k x_k + (1 - g_k) x_k^{best} \tag{4}$$

vii. **For** $j = 1, \ldots, n$ **do**

- **Set** trial vector:

$$y_j = x_j + F \times \left(\text{minLS}(x_\iota, c_\iota)_j - \text{child}(x_i, c_i)_j\right) \tag{5}$$

where F is the so - called Differential Weight of Differential Evolution algorithm.

- **If** $y_j \notin [a_j, b_j]$**,** then $y_j = x_{i,j}$

viii. **EndFor**

- **Set** $r \in [0,1]$ a random number. If $r \leq p_m$ then $x_i = \text{LS}(x_i)$, where LS(x) is a local search procedure like the BFGS procedure[67].
- **If** $f(y) \leq f(x)$ then $x = y$, $x^{best} = y$.

(b) **EndFor**

3. **Check the termination rule stated in** [68], which means that the method checks the difference between the current optimal solution $f_{min}^{(t)}$ and the previous $f_{min}^{(t-1)}$ one. The algorithm terminates when the following:

$$\left| f_{\text{min}}^{(k)} - f_{\text{min}}^{(k-1)} \right| \leq \epsilon \tag{6}$$

holds for N_t iterations. The value ϵ is a small positive value. In the conducted experiments the value $\epsilon = 10^{-5}$ was used. If the termination rule of equation 6 does not hold, then the algorithm continues from Step 2.

4. **Return** the sample x^{best} in the population with the lower function value $f\left(x^{best}\right)$.

Initially, the algorithm initializes the trial solutions as well as the necessary parameters. At every iteration the method constructs trial solutions using the genetic algorithms operations as well as the Armijo line search method and these solutions are combined using a process from Differential Evolution, using the following steps:

- Identification of the nearest point c_ι for each sample x_ι.
- Calculation of a sample minLS(x,c) through Armijo line search, between the sample x_i and the samplec_i.
- Generation of the sample using the crossover process of the Genetic Algorithm, between the sample x_ι and the best samplex_i^{best} .
- Computation of the trial point y_i using a process derived from Differential Evolution.

3. Experiments

Settings and benchmark functions

To ensure the reliability of the experimental results, the experiments were repeated 30 times using different seeds for the random number generator. The experiments were conducted on an an AMD Ryzen 5950X processor with 128 GB of RAM and the used operating system was Debian Linux. The software as well as the objective function was written entirely in ANSI-C++ with the assistance of the the open - source optimization environment OPTIMUS, that is available from http://www.github.com/itsoulos/GlobalOptimus (accessed on 17 September 2024). The test functions used in the experiments are presented in Table 1.

Table 1. The benchmark functions used in the conducted experiments.

NAME	FORMULA	DIMENSION
BF1	$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1) - \frac{4}{10}\cos(4\pi x_2) + \frac{7}{10}$	2
BF2	$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1)\cos(4\pi x_2) + \frac{3}{10}$	2
BF3	$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1)\cos(4\pi x_2) + \frac{3}{10}$	2
BRANIN	$f(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$	2
CAMEL	$f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4, \quad x \in [-5,5]^2$	2
Easom	$f(x) = -\cos(x_1)\cos(x_2)\exp\left((x_2-\pi)^2 - (x_1-\pi)^2\right)$	2
ELP	$f(x) = \sum_{i=1}^{n}\left(10^6\right)^{\frac{i-1}{n-1}}x_i^2$	$n = 10, 20, 30$
Exp	$f(x) = -\exp\left(-0.5\sum_{i=1}^{n}x_i^2\right), \quad -1 \leq x_i \leq 1$	$n = 4, 8, 16, 32$
Gkls[69]	$f(x) = \mathrm{Gkls}(x, n, w)$	$n = 2, 3$ $w = 50, 100$
Griewank2	$f(x) = 1 + \frac{1}{200}\sum_{i=1}^{2}x_i^2 - \prod_{i=1}^{2}\frac{\cos(x_i)}{\sqrt{(i)}}$	2
Griewank10	$\mathrm{f}(x) = 1 + \frac{1}{200}\sum_{i=1}^{10}x_i^2 - \prod_{i=1}^{10}\frac{\cos(x_i)}{\sqrt{(i)}}$	10
Hansen	$f(x) = \sum_{i=1}^{5} i\cos[(i-1)x_1 + i]\sum_{j=1}^{5} j\cos[(j+1)x_2 + j]$	2
Hartman3	$f(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{3} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	3
Hartman6	$f(x) = -\sum_{i=1}^{4} c_i \exp\left(-\sum_{j=1}^{6} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	6
Potential[70]	$V_{LJ}(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]$	$n = 9, 15, 21, 30$
Rastrigin	$f(x) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2)$	2
Rosenbrcok	$f(x) = \sum_{i=1}^{n-1}\left(100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right), \quad -30 \leq x_i \leq 30$	$n = 4, 8, 16$
Shekel5	$f(x) = -\sum_{i=1}^{5}\frac{1}{(x-a_i)(x-a_i)^T + c_i}$	4
Shekel7	$f(x) = -\sum_{i=1}^{7}\frac{1}{(x-a_i)(x-a_i)^T + c_i}$	4
Shekel10	$f(x) = -\sum_{i=1}^{10}\frac{1}{(x-a_i)(x-a_i)^T + c_i}$	4
Sinusoidal[71]	$f(x) = -(2.5\prod_{i=1}^{n}\sin(x_i - z) + \prod_{i=1}^{n}\sin(5(x_i - z))), \quad 0 \leq x_i \leq \pi$	$n = 4, 8$
Test2N	$f(x) = \frac{1}{2}\sum_{i=1}^{n}x_i^4 - 16x_i^2 + 5x_i$	$n = 4, 5, 7$
Test30N	$\frac{1}{10}\sin^2(3\pi x_1)\sum_{i=2}^{n-1}\left((x_i-1)^2(1+\sin^2(3\pi x_{i+1}))\right) + (x_n-1)^2(1+\sin^2(2\pi x_n))$	$n = 3, 4$

The functions used in the conducted experiments have been proposed by various researchers [72,73] in the relevant literature. For a more accurate comparison of the methods, efforts were made to maintain certain parameter values at equal or similar levels. The values for the parameters of the algorithm are presented in Table2, along with some explanation of each parameter.

Table 2. Parameters of optimization methods settings

PARAMETER	VALUE	EXPLANATION
N	200	Number of samples for all methods
N_k	200	Maximum number of iterations for all methods
SR	$\left\|f_{\min}^{(k)} - f_{\min}^{(k-1)}\right\|$	Best fitness: Stopping rule for all methods
N_t	12	Similarity max count for all methods
F	0.8	Differential weight for Differential Evolution
CR	0.9	Crossover Probability for Differential Evolution
C_1, C_2	0.5	Parameters of PSO
G_c	0.1 (10%)	Crossover rate for Genetic Algorithm
G_m	0.05 (5%)	Mutation rate for Genetic Algorithm

Experimental results

In Table 3, the average of objective function calls for each method is presented. All experiments were executed 30 times with different initialization for the random generator each time and the average number of function calls was depicted. The following notation was used in this table:

1. The column FUNCTION represents the name of the used objective problem.
2. The column DE represents the average function calls for the Differential Evolution optimization technique.
3. the column IPSO stands for the application of the Improved PSO algorithm as suggested by Charilogis and Tsoulos [74] to the objective problems.
4. The column GENETIC represents the application of the Genetic Algorithm to the objective problem.
5. At the end of the table, the total sum of calls for each method is listed.
6. The number in parentheses indicates the percentage of executions in which the global optimum was successfully found. The absence of this number signifies that the global minimum was computed in every independent run with 100% success.

Table 3. Comparison of average function calls of proposed method against others

FUNCTION	DE	IPSO	GENETIC	PROPOSED
BF1	8268	4113	4007	3951
BF2	7913	3747	3793	3382
BF3	6327	3305	3479	2736
BRANIN	4101	2522	2376	1622
CAMEL	5609	2908	2869	2027
EASOM	2978	1998	1958	969
ELP10	6288	4397	3131	2820
ELP20	10794	6883	6160	5337
ELP30	14172	9438	9576	7070
EXP4	5166	3177	2946	2370
EXP16	6498	3477	3250	2654
EXP32	7606	3728	3561	2652
GKLS250	3834	2495	2280	1115
GKLS350	3919	2658	2612	945(93)
GOLDSTEIN	6781	3856	3687	2676
GRIEWANK2	7429(96)	3168	4500(96)	2453(50)
GRIEWANK10	18490	7942	6409	6351
HANSEN	4185	2892	3209	2525
HARTMAN3	5190	3103	2751	1945
HARTMAN6	5968	3688	3219	2832
POTENTIAL3	6218	5154	4351	3537
POTENTIAL5	9119	10128	7704	6735
POTENTIAL6	10509(76)	11780(46)	10177(70)	7706(76)
POTENTIAL10	12721(96)	16550(86)	13357	11517
RASTRIGIN	6216	3539	4106	2125(86)
ROSENBROCK4	8452	5858	3679	4442
ROSENBROCK8	11530	7843	5269	6726
ROSENBROCK16	17432	11450	8509	7310
SHEKEL5	6662	3886	3325	3722
SHEKEL7	6967	4009	3360	3817
SHEKEL10	6757	3985	2488	3317
SINU4	5953	3409	2990	2192
SINU8	6973	3995	3441	3171
SINU16	6979	4680	4320	5250
TEST2N4	6396	3390	3330	2235
TEST2N5	6271	3604	4000(96)	2530(93)
TEST2N7	7074	4020(96)	4775(73)	2939(50)
TEST30N3	6178	4018	3210	2728
TEST30N4	7006	4504	3678	3593
TOTAL SUM	296929	195297	171842	144024

At the end of every global optimization procedure a BFGS variant of Powell [75] is utilized to improve the discovered solution and identify with certainty a local minimum. Additionally, Figure 1 presents a statistical comparison between the used optimization methods and the data are provided from Table 3.

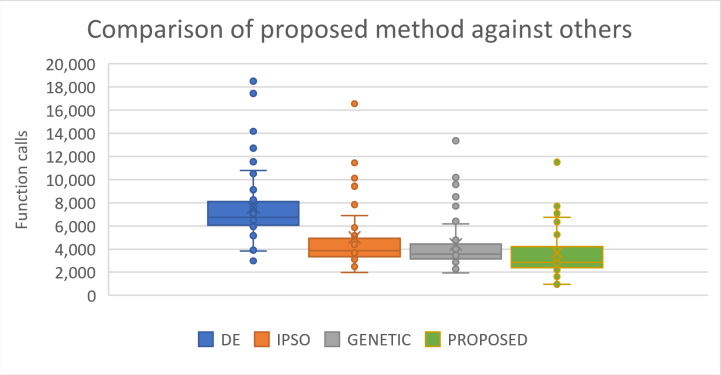


Figure 1. Comparison of function calls of proposed method against others

The proposed method seems to significantly reduce the required number of functional calls compared to other techniques, and in many functions this reduction can exceed 50%. This trend is also confirmed by Figure 2, where a statistical comparison is presented between universal optimization methods used in the experiments.

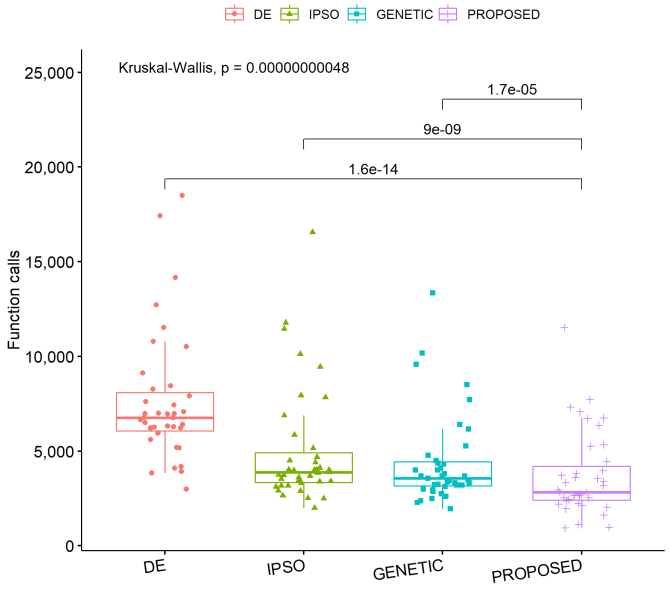


Figure 2. Statistical comparison of proposed method against others

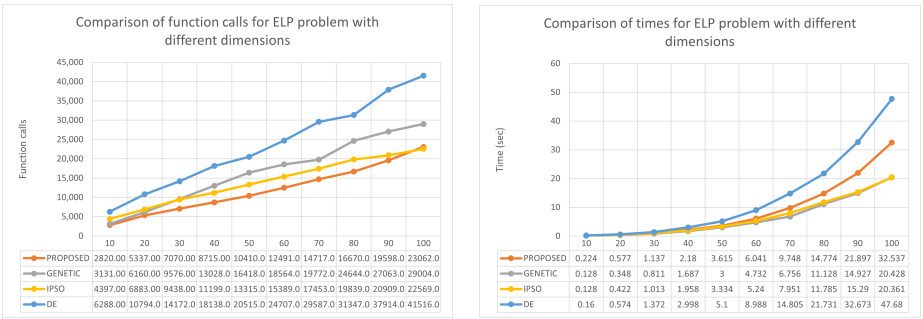
From this figure, it is evident that the proposed method demonstrates significantly better performance compared to DE, with a substantial reduction in cost (144,024 versus 296,929). The t-test confirmed the statistically significant difference (p-value < 0.05). Compared to IPSO, the proposed method also outperforms, with lower values in almost all functions (144,024 versus 195,297). The Kruskal-Wallis test further confirmed the statistical significance of this difference (p-value < 0.05). Regarding GENETIC, the proposed method proves to be more efficient, with a lower total cost (144,024 versus 171,842), and the t-test indicates a statistically significant superiority.

In conclusion, the proposed method is more efficient compared to DE, IPSO, and GENETIC. The differences in objective function costs are statistically significant, as confirmed by both parametric (t-test) and non-parametric (Kruskal-Wallis) analyses, demonstrating its superiority in solving the specific functions.

An additional experiment was executed for the High Elliptic function, which is defined as:

$$f(x) = \sum_{i=1}^{n} \left(10^6\right)^{\frac{i-1}{n-1}} x_i^2$$

where the parameter n defines the dimension of the function. In this experiment the proposed method was applied to this function as the dimension n changes from 10 to 100. The Figure 3a presents the performance of the proposed method for this function in terms of function calls across various dimensions. It is evident that as the dimensionality increases, the number of function calls required to solve the problem also rises. Specifically, for a dimension of 10, the algorithm requires 2,820 function calls, whereas for a dimension of 100, it demands 23,062 function calls, which is approximately eight times higher. This increase in function calls as the problem's dimensionality grows suggests that the complexity of the ELP problem scales significantly with dimensionality. The relationship between dimensions and function calls appears to be nonlinear, indicating that higher dimensions introduce additional computational challenges. For instance, moving from 10 to 20 dimensions more than doubles the required function calls, from 2,820 to 5,337, while the increase between 90 and 100 dimensions, although significant, shows a slightly smaller relative increase (19,598 to 23,062). The Figure 3b shows the response times of the ELP problem across various dimensions, from 10 to 100, measured in seconds. As the dimensionality increases, the execution time also rises significantly. For instance, with 10 dimensions, the execution time is only 0.224 seconds, while for 100 dimensions, it escalates to 32.537 seconds, marking a more than 145-fold increase. This substantial growth in execution time as the problem's dimensionality increases highlights the complexity and computational demands of higher-dimensional problems. The relationship between dimensions and time appears to be nonlinear, as seen in the sharp rise in time as dimensions grow. For example, moving from 10 to 20 dimensions results in more than doubling the time, from 0.224 to 0.577 seconds, while increasing from 90 to 100 dimensions causes a larger jump, from 21.897 to 32.537 seconds.



(a) ELP: Function calls **(b)** ELP: Times

Figure 3. Different variations of the ELP problem

In each iteration of the algorithm, a trial point is calculated through vector operations, similar to the process of optimization using differential evolution. The main difference of the proposed method compared to differential evolution lies in the fact that the samples for calculating the trial point are selected from nearby regions of the initial distribution, rather than being chosen randomly. However, the performance of the two methods differs in their ability to find optimal solutions, as shown in Figure4. The extraneous samples of Figure 4 have been removed.

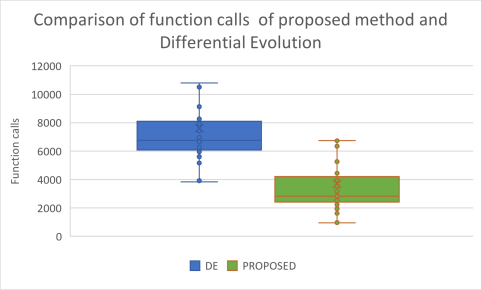


Figure 4. Comparison of proposed method against DE

4. Conclusions

The proposed optimization method demonstrates significantly better performance compared to the other methods (DE, IPSO, GENETIC) in terms of objective function calls, with fewer calls indicating better efficiency. This suggests that the proposed method is more efficient, achieving optimal solutions with fewer objective function evaluations, which is crucial in problems where each call is computationally expensive. Statistical tests, including both the t-test and Kruskal-Wallis, confirm that the differences in the number of calls between the proposed method and the others are statistically significant (p-value < 0.05). This indicates that the proposed method not only uses fewer resources but also achieves reliable results with improved efficiency. In summary, the proposed method clearly excels in efficiency compared to DE, IPSO, and GENETIC, significantly reducing the number of objective function calls and optimizing computational cost. Potential improvements to the algorithm could involve identifying samples that contribute more effectively toward finding the optimal solution. Additionally, since this is a novel optimization method, exploring alternative termination criteria or different initial sample distributions may lead to enhanced performance.

Author Contributions: V.C., I.G.T. and V.S. conceived the idea and methodology and supervised the technical part regarding the software. V.C. conducted the experiments, employing several datasets, and provided the comparative experiments. I.G.T. performed the statistical analysis. V.S. and all other authors prepared the manuscript. All authors have read and agreed to the published version of the manuscript.

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