Article

The SIOA Algorithm: A Bio-Inspired Approach for Efficient Optimization

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Abstract: The Sporulation-Inspired Optimization Algorithm (SIOA) is an innovative metaheuristic optimization method inspired by the biological mechanisms of microbial sporulation and dispersal. SIOA operates on a dynamic population of solutions ("microorganisms") and alternates between two main phases: sporulation, where new "spores" are generated through adaptive random perturbations combined with guided search towards the global best, and germination, in which these spores are evaluated and may replace the most similar and less effective individuals in the population. A distinctive feature of SIOA is its fully self-adaptive parameter control, as the dispersal radius and the probabilities of sporulation and germination are automatically adjusted in response to search progress. The algorithm also integrates a special "zero-reset" mechanism, enhancing its ability to detect global optima located near the origin. SIOA further incorporates a stochastic local search phase to refine solutions and accelerate convergence. Experimental results demonstrate that SIOA achieves high-quality solutions with a reduced number of function evaluations, especially in complex, multimodal, or high-dimensional problems. Overall, SIOA provides a robust and flexible optimization framework, suitable for a wide range of challenging optimization tasks.

Keywords: Optimization; SIOA Optimizer; Evolutionary Algorithms; Global Optimization; Evolutionary Techniques; Metaheuristics;

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1. Introduction

Mathematical Formulation of the Global Optimization Problem

Let $f : S \to \mathbb{R}$ be a real-valued function of n variables, where $S \subset \mathbb{R}^n$ is a compact subset. The global optimization problem is defined as the problem of finding

$$x^* = \arg\min_{x \in S} f(x), \tag{1}$$

where the feasible set S is given by the Cartesian product

$$S = \prod_{i=1}^{n} [a_i, b_i] \subseteq \mathbb{R}^n, \tag{2}$$

with the following conditions:

- $f \in C(S)$, i.e., f is a continuous function on S ($C(S)$ denotes the space of continuous functions on S),
- $[a_i, b_i] \subset \mathbb{R}$ are closed and bounded intervals for $i = 1, \ldots, n$,
- S is a compact and convex subset of the Euclidean space $\mathbb{R}^n$,

- $x^* \in S$ is the global minimizer of f over S.

Optimization represents a fundamental discipline in computational mathematics with widespread applications across scientific and industrial domains. Optimization techniques can be categorized into numerous classes based on their underlying principles and problem characteristics. Classical gradient-based methods include steepest descent [2,82], Newton's method [2,3], quasi-Newton methods [4–7], and Gauss-Newton approaches[2,8]. Stochastic optimization encompasses Monte Carlo methods [9,10], simulated annealing [11,12], stochastic tunneling [13], and parallel tempering [14]. Population-based methods feature genetic algorithms [15,16] differential evolution [17–19], biogeography-based optimization [20], and cultural algorithms [21]. Derivative-free techniques include pattern search [22], mesh adaptive direct search [15,23], and the Nelder-Mead simplex method [24]. Response surface methodologies incorporate kriging [25,26], radial basis functions[27,28], and polynomial chaos expansion [29]. Trust-region methods involve Bayesian optimization [30], sequential quadratic programming [31], and interpolation-based approaches [32,33]. Convex optimization techniques [34], span interior-point methods [35], subgradient methods [34,36], and cutting-plane algorithms [37]. Decomposition methods include Benders decomposition [38], Dantzig-Wolfe decomposition [39], and Lagrangian relaxation [40] Space-partitioning strategies comprise DIRECT algorithms [41], branch-and-bound methods [42], and interval analysis [43]. Machine learning-inspired optimization contains neural evolution [44], reinforcement learning [45], and deep Q-learning [46]. Socially-motivated algorithms feature particle swarm optimization [47], ant colony optimization [48], artificial bee colony [49], and firefly algorithms [50]. Physics-inspired methods include simulated crystallization [51], gravitational search [52], electromagnetism-like mechanisms [53], and charged system search [54]. Hybrid approaches combine neuro-fuzzy systems [55], memetic algorithms [56], and cultural evolution strategies [57] Biologically-inspired optimization encompasses genetic programming [58], artificial immune systems [59], bacterial foraging optimization [60], and invasive weed optimization[61]. Other notable methods include harmony search[62], teaching-learning-based optimization [63], water cycle algorithms [64], league championship algorithms [65,66], and imperialist competitive approaches [67]. Emerging directions involve quantum-inspired optimization [68], chemical reaction optimization [69], and social cognitive optimization [70].

Within the diverse landscape of metaheuristic optimization algorithms, the Sporulation-Inspired Optimization Algorithm (SIOA) introduces an innovative, biologically motivated approach inspired by natural mechanisms of microbial reproduction and dispersal. SIOA operates with a dynamic population of solutions, conceptualized as "microorganisms" that undergo processes of sporulation and germination. In this framework, each solution can generate "spores" via adaptive random perturbations, guided by the current best solution, with the intensity of dispersal dynamically regulated through self-adaptive parameters. A key distinguishing feature of SIOA is its two-phase search mechanism, combining the generation of spores (sporulation phase) with a germination process that evaluates and selectively integrates new solutions into the population using a similarity-based (crowding) replacement scheme, where each new spore replaces its most similar population member only if it achieves superior fitness. The algorithm incorporates an additional "zero-reset" mechanism, occasionally forcing solution components to zero, which helps to accelerate convergence towards global optima near the origin. The search is further enhanced by a stochastic, optional local search phase, which promotes the exploitation of promising regions in the solution space. One of SIOA's main strengths lies in its fully self-adaptive parameter control: not only the dispersal radius, but also the probabilities of sporulation and germination are automatically adjusted according to the algorithm's search progress. This adaptive strategy enables SIOA to balance exploration and exploitation effectively, enhancing its performance in a wide range of complex optimization tasks. Notably, in multimodal and high-dimensional problems, SIOA demonstrates a strong capability to avoid premature convergence and maintain population diversity, contributing to fast and robust convergence. Experimental results confirm that SIOA exhibits high efficiency and stability across a broad suite of benchmark functions, often outperforming established metaheuristics, especially in challenging optimization landscapes. The biological inspiration underpinning SIOA offers natural mechanisms for diversity maintenance and premature convergence avoidance, making it especially suitable for demanding applications where a balance between global exploration and focused exploitation is critical. This work thoroughly examines the theoretical underpinnings of SIOA, including its convergence properties, parameter sensitivity analysis, and practical implementation aspects. Furthermore, the potential for extensions and adaptations of the algorithm is explored, including constrained, multi-objective, and large-scale optimization scenarios. Overall, SIOA emerges as a powerful, modern, and flexible contribution to computational optimization methodology, with significant prospects for both research and real-world applications.

The rest of the paper is organized as follows:

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2. The SIOA Optimizer method

The following is the pseudocode of SIOA and the related analysis.

The SIOA algorithm 1 begins with an initialization phase, where an initial population of solutions (samples) of size NP is randomly generated within the specified bounds. For each solution, the fitness value is evaluated and stored in the fitness array, while the best solution (x_{best}) and its corresponding fitness (f_{best}) are also tracked. An empty list is initialized to collect spores that will be generated in each iteration.

During the main iteration loop, the algorithm executes three core operations in every cycle:

In the first phase (sporulation), each solution in the population has a probability (p_{spor}, which is self-adaptive) of generating a spore. The new spore is created by applying a combination of adaptive random perturbations and attraction towards the global best solution, with the strength of the perturbation determined by the current value of the adaptive dispersal radius (R). Additionally, with a certain probability, individual dimensions of the spore may be forcibly set to zero, especially when the best fitness value is near zero, enhancing the algorithm's ability to locate optima at or near the origin. All generated spores are ensured to remain within the problem boundaries.

In the second phase (germination), each spore has a probability (p_{germ}, also self-adaptive) to germinate. If so, its fitness is evaluated. The algorithm then uses a crowding (similarity-based) replacement strategy: the spore is compared against the most similar solution in the population (measured by Euclidean distance), and it replaces that solution only if its fitness is superior. If the spore achieves a new best fitness, the x_{best} and f_{best} are updated.

The third phase is optional local search, where each solution in the population has a probability (p_{loc}) of undergoing a specialized local search procedure. If the refined solution is better, it replaces the current one and updates the global best if necessary.

Throughout the process, all critical parameters including dispersal radius and the probabilities of sporulation and germination are dynamically self-adapted based on the search progress, specifically on improvements in the mean fitness of the population. This mechanism ensures that SIOA can automatically balance exploration and exploitation according to the evolving state of the search.

Algorithm 1 Peudocode of SIOA

Input:
- NP: Population size
- $Iter_{max}$: Maximum iterations
- p_{loc}: Local search rate
- $bounds$: Search space bounds
- $c1, c2$: Search coefficients

(Self-adaptive within the loop, initialized with default values:)
- R_{min}, R_{max}: Min/max dispersal radius
- p_{spor}: Initial sporulation probability
- p_{germ}: Initial germination probability

Output:
- x_{best}: Best solution found
- f_{best}: Corresponding fitness value

Initialization:
$dim \leftarrow$ Problem dimension
Initialize population $X = x_i | x_i\, U(bounds), i = 1, ..., NP$
Evaluate initial fitness $F = f_i = f(x_i) | i = 1, ..., NP$
$(x_{best}, f_{best}) \leftarrow argmin_{(x_i, f_i)} f_i$
// Set adaptive parameters:
$R \leftarrow R_{max}$
$ps_{ad} \leftarrow p_{spor}$
$pg_{ad} \leftarrow p_{germ}$
$meanP_{fitness} \leftarrow +\infty$
Main Optimization Loop:
for $iter$ = 1 to $Iter_{max}$ do
// Parameter self-adaptation
  $t \leftarrow \frac{iter/max}{iter}$
  $R \leftarrow R_{max} - t \cdot (R_{max} - R_{min})$
  $mean_{fitness} \leftarrow$ mean(F)
  $prog \leftarrow \frac{(bestprev - f_{best})}{(|bestprev| + \varepsilon)}, \varepsilon$ =1e-10
  if $prog$ >0.001 then
    $ps_{ad} \leftarrow$ clamp$(ps \cdot 0.98, 0.1, 1.0)$
    $pg_{ad} \leftarrow$ clamp$(pg \cdot 1.02, 0.1, 1.0)$
  else
    $ps_{ad} \leftarrow$ clamp$(ps_{ad} \cdot 1.02, 0.1, 1.0)$
    $pg_{ad} \leftarrow$ clamp$(pg_{ad} \cdot 0.98, 0.1, 1.0)$
  end if
  $meanP_{fitness} \leftarrow mean_{fitness}$
  // Sporulation phase
  $S \leftarrow \varnothing$
  for each x_i in X do
    Create vectror spore =[$spore_1, spore_2 ... spore_{dim}$]
    for d = 1 to dim do
      $spore_d \leftarrow X_{i,d} + U(-R, R) * c_1 + (x_{best,d}$ - $X_{i,d} + U(-R, R)) * c_2$
      // Special "reset to zero" rule
      if $U(0, 1) < 0.1$ and ($f_{best} \in (-3, 3)$ then
        $spore_d \leftarrow 0$
      end if
      $spore_d \leftarrow$ clamp($spore_d, blower_d, bupper_d$)
    end for
    $S \leftarrow S \cup spore$
  end for
  // Germination phase
  for each $spore$ in S do
    if $U(0, 1) < pg_{ad}$ then
      $f_{spore} \leftarrow f(spore)$
      $idx \leftarrow$ index of sample in X most similar to $spore$ (Euclidean distance)
      if $f_{spore} < f_{idx}$ then
        $x_{idx} \leftarrow spore$
        $f_{idx} \leftarrow f_{spore}$
      end if
      if $f_{spore} < f_{best}$ then
        $x_{best} \leftarrow spore$
        $f_{best} \leftarrow f_{spore}$
      end if
    end if
  end for
  // Local search (optional)
  for each x_i in X do
    if $U(0, 1) < p_{loc}$ then
      $(x_{ref}, f_{ref}) \leftarrow$ localSearch(x_i) [71]
      if $f_{ref} < f_i$ then
        $x_i \leftarrow x_{ref}$
        $f_i \leftarrow f_{ref}$
        if f_ref < f_$best$ then
          $x_{best} \leftarrow x_{ref}$
          $f_{best} \leftarrow f_{ref}$
        end if
      end if
    end if
  end for
  if termination criteria met then break: $\delta_{sim}^{(iter)} = \left| f_{sim,min}^{(iter)} - f_{sim,min}^{(iter-1)} \right|$ [72,73] or $Iter_{max}$ or Function evaluations (FEs)
end for
return (x_{best}, f_{best})

The use of similarity-based (crowding) replacement preserves population diversity and helps prevent premature convergence, while the special zero-reset rule increases the chance of discovering global optima at zero. The stochastic local search phase further enhances exploitation capability. Overall, the combination of these mechanisms creates a dynamic, self-adjusting system in which the algorithm continuously tunes its parameters and replacement strategies based on intermediate solution quality, thus maximizing its ability to efficiently explore complex, multimodal, and high-dimensional search spaces.

3. Experimental setup and benchmark results

The experimental framework is structured as follows: First, the benchmark functions used for performance evaluation are introduced, then a thorough examination of the experimental results is provided. A systematic parameter sensitivity analysis is conducted to validate the algorithm's robustness and optimization capabilities under different conditions. All experimental configurations are specified in Table 1.

Table 1. Parameters and settings

PARAMETER	VALUE	EXPLANATION
NP	100	Population for all methods
p_{spor}	$p_{spor} \in [0,1]$: adaptive, initial: 0.6	Sporulation propability for SIOA
p_{germ}	$p_{germ} \in [0,1]$: adaptive, initial: 0.9	Germination propability for SIOA
R_{min}	$R_{min} \in [0,1]$: adaptive, initial: 0.01	Smaller sporulation radius for SIOA
R_{max}	$R_{max} \in [0,1]$: adaptive, initial: 0.5	Larger sporulation radius for SIOA
c_1	0.6	Stochastic perturbation
c_2	0.4	Attraction toward the global best
$iter_{max}$	500	Maximum number of iterations for all methods
SR	Similarity of best fitness [72,73] or $iter_{max}$ or FEs	Stopping rule
N_s	12	Similarity $count_{max}$ for stopping rule
P_{loc}	0.005 (0.5%) etc.	Local search rate for all methods (optional)
C_{rate}	double, 0.1 (10%) (classic values)	Crossover for GA
M_{rate}	double, 0,05 (5%) (classic values)	Mutation for GA
cf_1, cf_2	1.193	Cognitive and Social coefficient for PSO
w	0.721	Inertia for PSO
$coef_1, coef_2$	1.494	Cognitive and Social coefficient for CLPSO
w	0.729	Inertia for CLPSO
F	0.8	Initial scaling factor for DE and SaDE
CR	0.9	Initial crossover rate for DE and SaDE
w	$w \in [0.5, 1]$ (random)	Inertia for PSO
NP_C	$Np = 4 + \lfloor 3 \cdot \log(\text{dimension}) \rfloor$	Population for CMA-ES

The computational experiments were conducted using a system equipped with an AMD Ryzen 5950X processor and 128GB of RAM, running Debian Linux. The testing framework involved 30 independent runs for each benchmark function, ensuring robust statistical analysis by initializing with fresh random values in every iteration. The experiments utilized a custom-developed tool implemented in ANSI C++ within the GLOBALOPTIMUS[80] platform, an open-source optimization library available at https://github.com/itsoulos/GLOBALOPTIMUS (last accessed: July 28, 2025). The algorithm's parameters, as detailed in Table 1, were carefully selected to balance exploration and exploitation effectively.

3.1. *Experiments with traditional methods and classical benchmark problems*

The evaluation of SIOA was first conducted on established benchmark function sets [74–76], in direct comparison with widely used traditional optimization methods, in order to assess its computational efficiency, convergence capability, and result stability under standard testing scenarios (Table 2)

Table 2. The benchmark functions used in the conducted experiments.

NAME	FORMULA	DIMENSION
ACKLEY	$f(x) = -a\exp\left(-b\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - \exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(cx_i)\right) + a + \exp(1)\ a = 20.0$	**4**
BF1	$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1) - \frac{4}{10}\cos(4\pi x_2) + \frac{7}{10}$	**2**
BF2	$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1)\cos(4\pi x_2) + \frac{3}{10}$	**2**
BF3	$f(x) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1 + 4\pi x_2) + \frac{3}{10}$	**2**
BRANIN	$f(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right)^2 + 10\left(1 - \frac{1}{8\pi}\right)\cos(x_1) + 10$ $-5 \le x_1 \le 10,\ 0 \le x_2 \le 15$	**2**
CAMEL	$f(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4, \quad x \in [-5,5]^2$	**2**
DIFFERENT POWERS	$f(\mathbf{x}) = \sqrt{\sum_{i=1}^{n} \lvert x_i\rvert^{2+4\frac{i-1}{n-1}}}$	**10**
DIFFPOWER	$f(x) = \sum_{i=1}^{n} \lvert x_i - y_i\rvert^p\ p = 2,5,10$	**2,5,10**
DISCUS	$f(x) = 10^6 x_1^2 + \sum_{i=2}^{n} x_i^2$	**10**
EASOM	$f(x) = -\cos(x_1)\cos(x_2)\exp\left((x_2 - \pi)^2 - (x_1 - \pi)^2\right)$	**2**
ELP	$f(x) = \sum_{i=1}^{n} (10^6)^{\frac{i-1}{n-1}} x_i^2$	**10**
EQUAL MAXIMA	$f(x) = \sin^6(5\pi x) \cdot e^{-2\log(2)\cdot\left(\frac{x-0.1}{0.8}\right)^2}$	**10**
EXP	$f(x) = -\exp\left(-0.5\sum_{i=1}^{n} x_i^2\right), \quad -1 \le x_i \le 1$	**10**
GKLS[77]	$f(x) = \mathrm{Gkls}(x, n, w)\ w = 50, 100$	**n=2,3**
GOLDSTAIN	$f(x) = [1 + (x_1 + x_{2+1)})^2(19 - 14x_1 + 3x_1^2 14x_2 + 6x_1x_2 + 3x_2^2)]$ $[30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$	**2**
GRIEWANK ROSENBROCK	$f(\mathbf{x}) = \underbrace{\left(\frac{\lVert\mathbf{x}\rVert^2}{4000} - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1\right)}_{\text{Griewank}} \cdot \underbrace{\left(\frac{1}{10}\sum_{i=1}^{n-1}\left[100(x_{i+1} - x_i^2)^2 + (1 - x_i)^2\right]\right)}_{\text{Rosenbrock}}$	**10**
GRIEWANK2	$f(x) = 1 + \frac{1}{200}\sum_{i=1}^{2} x_i^2 - \prod_{i=1}^{2}\frac{\cos(x_i)}{\sqrt{(i)}}$	**2**
GRIEWANK10	$\mathrm{f}(x) = 1 + \frac{1}{200}\sum_{i=1}^{10} x_i^2 - \prod_{i=1}^{10}\frac{\cos(x_i)}{\sqrt{(i)}}$	**10**
HANSEN	$f(x) = \sum_{i=1}^{5} i\cos[(i-1)x_1 + i]\sum_{j=1}^{5} j\cos[(j+1)x_2 + j]$	**2**
HARTMAN3	$f(x) = -\sum_{i=1}^{4} c_i\exp\left(-\sum_{j=1}^{3} a_{ij}(x_j - p_{ij})^2\right)$	**3**
HARTAMN6	$f(x) = -\sum_{i=1}^{4} c_i\exp\left(-\sum_{j=1}^{6} a_{ij}(x_j - p_{ij})^2\right)$	**6**
POTENTIAL[78]	$V_{LJ}(r) = 4\epsilon\left[\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^6\right]$	**9,15,30**
RARSTIGIN2	$f(x) = x_1^2 + x_2^2 - \cos(18x_1) - \cos(18x_2)$	**2**
ROSENBROCK	$f(x) = \sum_{i=1}^{n-1}\left(100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2\right), \quad -30 \le x_i \le 30$	**4,8,16**
ROTATED ROSENBROCK	$f(\mathbf{x}) = \sum_{i=1}^{n-1}\left[100(z_{i+1} - z_i^2)^2 + (z_i - 1)^2\right],\ z = Rx$	**10**
SHEKEL5	$f(x) = -\sum_{i=1}^{5}\frac{1}{(x - a_i)(x - a_i)^T + c_i}$	**4**
SHEKEL7	$f(x) = -\sum_{i=1}^{7}\frac{1}{(x - a_i)(x - a_i)^T + c_i}$	**4**
SHEKEL10	$f(x) = -\sum_{i=1}^{10}\frac{1}{(x - a_i)(x - a_i)^T + c_i}$	**4**
SINUSOIDAL[79]	$f(x) = -(2.5\prod_{i=1}^{n}\sin(x_i - z) + \prod_{i=1}^{n}\sin(5(x_i - z))), \quad 0 \le x_i \le \pi$	**4,8,16**
STEP ELLIPSOIDAL	$f(\mathbf{x}) = \sum_{i=1}^{n}\lfloor x_i + 0.5\rfloor^2 + \alpha\sum_{i=1}^{n}\left(10^6 \cdot \frac{i-1}{n-1}\right)x_i^2,\ \alpha = 1$	**4**
TEST2N	$f(x) = \frac{1}{2}\sum_{i=1}^{n} x_i^4 - 16x_i^2 + 5x_i$	**4,5**
TEST30N	$\frac{1}{10}\sin^2(3\pi x_1)\sum_{i=2}^{n-1}\left((x_i - 1)^2(1 + \sin^2(3\pi x_{i+1}))\right) + (x_n - 1)^2(1 + \sin^2(2\pi x_n))$	**4,5**

The results presented in Table 3 were obtained using the parameter settings described in Table 1. An important observation is the consistency of the best solution across 12 consecutive runs, which demonstrates a high degree of stability and robustness in the optimization process. This stability was achieved with minimal reliance on local optimization, as the local search procedure was applied in only 0.5% of the cases. Such performance indicates that the algorithm's global search capabilities are sufficient to consistently identify optimal or near-optimal solutions without heavy dependence on local refinement methods.

Table 3. Comparison of function calls of SIOA method with others

FUNCTION	SIOA	GA	DE	PSO	ACO
ACKLEY	3028	3441	10694	5684(0.86)	3449
BF1	1204	2346	4963	2562	1558(0.4)
BF2	1177	2116	5139	2332	1523(0.96)
BF3	1144	2163	4730	2093	1410
BRANIN	950	1668	2022	1686	1054
CAMEL	1154	1835	3161	2029	1227
DIFFERENT POWERS10	2123	2507	3897	2608	2003
DIFFPOWER2	1590	1886	3239	2694	1740
DIFFPOWER5	3471	3770	5620	4472	3789
DIFFPOWER10	4407	3909	6546	5091	4582
DISCUS10	931	1640	2433	1658	1010
EASOM	776	1618	1784	1576	977
ELP10	1126	1771	2613	1867	1224
EQUAL MAXIMA10	2649	2212	4341	3401	2384
EXP10	1096	1764	2625	1795	1175
GKLS250	1202	1862	3427	1996	1245
GKLS350	1207	2038(0.86)	3637	2361	1550(0.86)
GOLDSTEIN	1161	1925	2621	1955	1249
GRIEWANK ROSENBROCK10	1684	2136	3743	2437	1843
GRIEWANK2	1061	2956(0.26)	4765(0.46)	1589(0.23)	839
GRIEWANK10	1899(0.6)	2936(0.2)	4582(0.5)	2209(0.36)	2444(0.33)
HANSEN	1486	2143(0.86)	3078	2964	1424(0.86)
HARTMAN3	1067	1744	2376	1760	1099
HARTMAN6	1129	1733(0.73)	2558	1917(0.7)	1222(0.93)
POTENTIAL3	1156	1754	2694	1875	1270
POTENTIAL5	1639	2106	3320	2424	1749
POTENTIAL10	3104(0.6)	3566(0.43)	5583(0.66)	4581(0.5)	3182(0.43)
RASTRIGIN2	933	2411(0.93)	4412	3017(0.96)	1661
ROSENBROCK4	1422	1783	2860	2069	1496
ROSENBROCK8	1558	2072	3962	2501	1751
ROSENBROCK16	1833	2506	4157	2781	2151
ROTATED ROSENBROCK10	1785	2237	3663	2675	1918(0.96)
SHEKEL5	1220	1770(0.66)	2884	1990(0.76)	1298(0.76)
SHEKEL7	1286	1812(0.83)	2890(0.96)	2080(0.83)	1351(0.83)
SHEKEL10	1345(0.9)	1867(0.66)	3625	2091(0.83)	1335
SINUSOIDAL4	1358	1938	3263	2213	1278
SINUSOIDAL8	1541(0.96)	1957	3241	2014	1459
SINUSOIDAL6	1814(0.53)	2319(0.76)	4209(0.7)	2680	1979(0.86)
STEP ELLIPSOIDAL4	994	1714(0.96)	2102	1960	1259
TEST2N4	1502(0.73)	2270(0.96)	3619	2153	1437(0.9)
TEST2N5	1338(0.5)	2185(0.66)	4556	2376(0.86)	1601(0.63)
TEST30N3	1142	1730	2381	1998	1116
TEST30N4	1261	1825	2408	2270	1167
TOTAL	**66,953(0.949)**	**93,941(0.901)**	**160,423(0.96)**	**106,484(0.928)**	**72,478(0.92)**

The comparative analysis of the results of Table 3 shows that the proposed SIOA method outperforms traditional GA, DE, PSO, and ACO methods across a wide range of benchmark functions, both in terms of the number of objective function evaluations and the success rate. In the vast majority of cases, SIOA achieves the minimum or one of the lowest evaluation counts, indicating high computational efficiency and faster convergence. The differences are particularly evident in multidimensional and multimodal problems, where traditional methods such as GA and DE require significantly more evaluations, often more than double or triple those of SIOA.

The success rate, which is 100% when not shown in parentheses, also presents a positive picture for SIOA. Its overall value reaches 94.9%, surpassing the corresponding rates of GA (90.1%) and PSO (92.8%) and coming very close to the best performances of DE (96%) and ACO (92%), but with considerably lower computational cost. In several challenging cases, such as the GRIEWANK and POTENTIAL functions, SIOA combines low evaluation requirements with competitive or even maximum success rates, demonstrating an ability to maintain a balance between exploration and exploitation.

The overall picture, as reflected in the last row of the table, confirms SIOA's general superiority, as it achieves the lowest total number of evaluations (66,953) compared to other methods, which range from about 72,478 (ACO) to 160,423 (DE). This high efficiency, combined with the stability of the results, suggests that the biologically inspired strategy of sporulation and germination, together with mechanisms for self-adaptation and diversity preservation, offers a clear advantage over classic evolutionary and swarm-based methods across a wide spectrum of optimization problems.

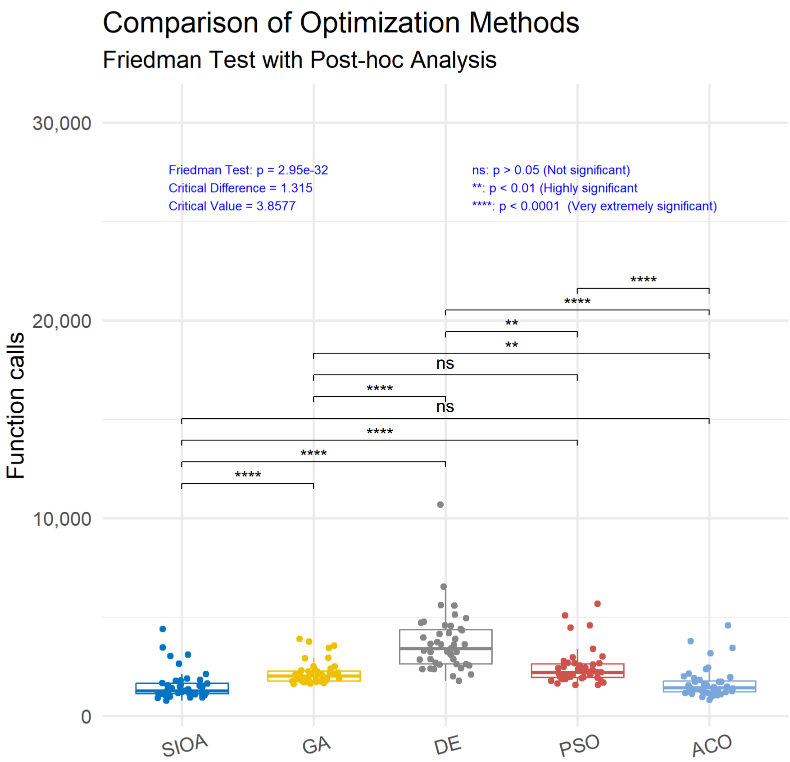

Figure 1. Statistical comparison of SIOA against other methods

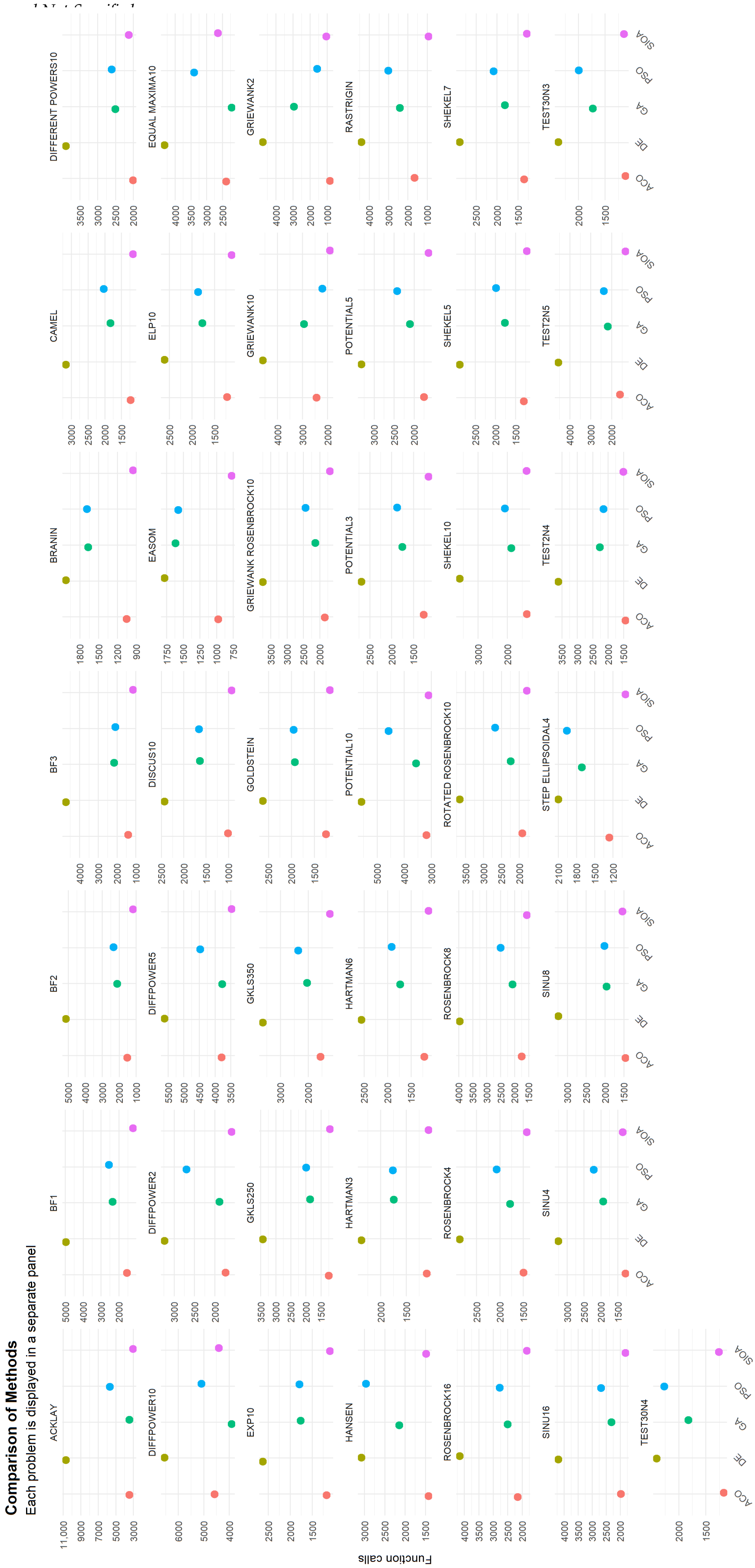

Figure 2. Performance of all methods on each problem

The analysis of the results (Friedman test [81]) presented in figure [friedman.png] and table [table.png] shows the performance comparison of the proposed SIOA optimization method against other established techniques. The values of the critical parameter p, which indicate the levels of statistical significance, reveal that SIOA demonstrates a very extremely significant superiority over GA, DE, and PSO, with p-values lower than 0.0001. In contrast, the comparison between SIOA and ACO did not show a statistically significant difference, as the p-value is greater than 0.05, indicating that the two methods exhibit a similar level of performance according to this statistical evaluation.

3.2. Experiments with advanced methods and real-world problems

Subsequently, SIOA was tested against more sophisticated algorithms on complex, large-scale problems derived from realistic application domains, aiming to evaluate its performance under increased complexity, constraint handling, and uncertainty.

Table 4. Real world problems CEC2011.

PROBLEM	FORMULA	Dim	BOUNDS
Parameter Estimation for Frequency-Modulated Sound Waves	$\min_{x\in[-6.4,6.35]^6} f(x) = \frac{1}{N}\sum_{n=1}^{N} \left\lvert y(n;x) - y_{\text{target}}(n)\right\rvert^2$ $y(n;x) = x_0 \sin(x_1 n + x_2 \sin(x_3 n + x_4 \sin(x_5 n)))$	6	$x_i \in [-6.4, 6.35]$
Lennard-Jones Potential	$\min_{x\in\mathbb{R}^{3N-6}} f(x) = 4\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\left[\left(\frac{1}{r_{ij}}\right)^{12} - \left(\frac{1}{r_{ij}}\right)^{6}\right]$	30	$x_0 \in (0,0,0)$ $x_1, x_2 \in [0,4]$ $x_3 \in [0,\pi]$ x_{3k-3} x_{3k-2} $x_i \in [-b_k, +b_k]$
Bifunctional Catalyst Blend Optimal Control	$\frac{dx_1}{dt} = -k_1 x_1,\ \frac{dx_2}{dt} = k_1 x_1 - k_2 x_2 + k_3 x_2 + k_4 x_3,$ $\frac{dx_3}{dt} = k_2 x_2,\ \frac{dx_4}{dt} = -k_4 x_4 + k_5 x_5,$ $\frac{dx_5}{dt} = -k_3 x_2 + k_6 x_4 - k_5 x_5 + k_7 x_6 + k_8 x_7 + k_9 x_5 + k_{10} x_7$ $\frac{dx_6}{dt} = k_8 x_5 - k_7 x_6,\ \frac{dx_7}{dt} = k_9 x_5 - k_{10} x_7$ $k_i(u) = c_{i1} + c_{i2} u + c_{i3} u^2 + c_{i4} u^3$	1	$u \in [0.6, 0.9]$
Optimal Control of a Non-Linear Stirred Tank Reactor	$J(u) = \int_0^{0.72}\left[x_1(t)^2 + x_2(t)^2 + 0.1u^2\right]dt$ $\frac{dx_1}{dt} = -2x_1 + x_2 + 1.25u + 0.5\exp\left(\frac{x_1}{x_1+2}\right)$ $\frac{dx_2}{dt} = -x_2 + 0.5\exp\left(\frac{x_1}{x_1+2}\right)$ $x_1(0) = 0.9, \quad x_2(0) = 0.09,\ t \in [0, 0.72]$	1	$u \in [0,5]$
Tersoff Potential for model Si (B)	$\min_{\mathbf{x}\in\Omega} f(\mathbf{x}) = \sum_{i=1}^{N} E(\mathbf{x}_i)$ $E(\mathbf{x}_i) = \frac{1}{2}\sum_{j\neq i} f_c(r_{ij})\left[V_R(r_{ij}) - B_{ij} V_A(r_{ij})\right]$ where $r_{ij} = \lVert \mathbf{x}_i - \mathbf{x}_j \rVert$, $V_R(r) = A\exp(-\lambda_1 r)$ $V_A(r) = B\exp(-\lambda_2 r)$ $f_c(r)$: cutoff function with $f_c(r)$: angle parameter	30	$x_1 \in [0,4]$ $x_2 \in [0,4]$ $x_3 \in [0,\pi]$ $x_i \in \left[\frac{4(i-3)}{4}, 4\right]$
Tersoff Potential for model Si (C)	$\min_{\mathbf{x}} V(\mathbf{x}) = \sum_{i=1}^{N}\sum_{j>i}^{N} f_C(r_{ij})\left[a_{ij} f_R(r_{ij}) + b_{ij} f_A(r_{ij})\right]$ $f_C(r) = \begin{cases} 1, & r < R - D \\ \frac{1}{2} + \frac{1}{2}\cos\left(\frac{\pi(r-R+D)}{2D}\right), & \lvert r - R\rvert \le D \\ 0, & r > R + D \end{cases}$ $f_R(r) = A\exp(-\lambda_1 r)$ $f_A(r) = -B\exp(-\lambda_2 r)$ $b_{ij} = \left[1 + (\beta^n)\zeta_{ij}^n\right]^{-1/(2n)}$ $\sum_{k\neq i,j} f_C(r_{ik}) g(\theta_{ijk}) \exp\left[\lambda_3^3 (r_{ij} - r_{ik})^3\right]$	30	$x_1 \in [0,4]$ $x_2 \in [0,4]$ $x_3 \in [0,\pi]$ $x_i \in \left[\frac{4(i-3)}{4}, 4\right]$
Spread Spectrum Radar Polly phase Code Design	$\min_{x\in X} f(x) = \max\{\lvert\varphi_1(x)\rvert, \lvert\varphi_2(x)\rvert, \ldots, \lvert\varphi_m(x)\rvert\}$ $X = \{x \in \mathbb{R}^n \mid 0 \le x_j \le 2\pi,\ j = 1,\ldots,n\} m = 2n - 1$ $\varphi_j(x) = \begin{cases} \sum_{k=1}^{n-j}\cos(x_k - x_{k+j}) & \text{for } j = 1,\ldots,n-1 \\ n & \text{for } j = n \\ \varphi_{2n-j}(x) & \text{for } j = n+1,\ldots,2n-1 \end{cases}$ $\varphi_j(x) = \sum_{k=1}^{n-j}\cos(x_k - x_{k+j}), \quad j = 1,\ldots,n-1$ $\varphi_n(x) = n,\ \varphi_{n+\ell}(x) = \varphi_{n-\ell}(x), \quad \ell = 1,\ldots,n-1$	20	$x_j \in [0, 2\pi]$
Transmission Network Expansion Planning	$\min \sum_{l\in\Omega} c_l n_l + W_1 \sum_{l\in OL}\lvert f_l - \bar{f}_l\rvert + W_2 \sum_{l\in\Omega}\max(0, n_l - \bar{n}_l)$ $Sf = g - d$ $f_l = \gamma_l n_l \Delta\theta_l, \quad \forall l \in \Omega$ $\lvert f_l\rvert \le \bar{f}_l n_l, \quad \forall l \in \Omega$ $0 \le n_l \le \bar{n}_l, \quad n_l \in \mathbb{Z}, \quad \forall l \in \Omega$	7	$0 \le n_i \le \bar{n}_l$ $n_i \in \mathbb{Z}$
Electricity Transmission Pricing	$\min_x f(x) = \sum_{i=1}^{N_g}\left(\frac{C_i^{gen}}{P_i^{gen}} - R_i^{gen}\right)^2 + \sum_{j=1}^{N_d}\left(\frac{C_j^{load}}{P_j^{load}} - R_j^{load}\right)^2$ $\sum_j GD_{i,j} + \sum_j BT_{i,j} = P_i^{gen}, \quad \forall i$ $\sum_i GD_{i,j} + \sum_i BT_{i,j} = P_j^{load}, \quad \forall j$ $GD_{i,j}^{max} = \min(P_i^{gen} - BT_{i,j},\ P_j^{load} - BT_{i,j})$	126	$GD_{i,j} \in [0, GD_{i,j}^{max}]$
Circular Antenna Array Design	$\min_{r_1,\ldots,r_6,\varphi_1,\ldots,\varphi_6} f(\mathbf{x}) = \max_{\theta\in\Omega} AF(\mathbf{x},\theta)$ $AF(\mathbf{x},\theta) = \left\lvert\sum_{k=1}^{6}\exp\left(j\left[2\pi r_k\cos(\theta - \theta_k) + \varphi_k \frac{\pi}{180}\right]\right)\right\rvert$	12	$r_k \in [0.2, 1]$ $\varphi_k \in [-180, 180]$
Dynamic Economic Dispatch 1	$\min_{\mathbf{P}} f(\mathbf{P}) = \sum_{t=1}^{24}\sum_{i=1}^{5}\left(a_i P_{i,t}^2 + b_i P_{i,t} + c_i\right)$ $P_i^{\min} \le P_{i,t} \le P_i^{\max}, \quad \forall i = 1,\ldots,5,\ t = 1,\ldots,24$ $\sum_{i=1}^{5} P_{i,t} = D_t, \quad \forall t = 1,\ldots,24$ $P_{\min} = [10, 20, 30, 40, 50]$ $P_{\max} = [75, 125, 175, 250, 300]$	120	$P_i^{\min} \le P_{i,t} \le P_i^{\max}$
Dynamic Economic Dispatch 2	$\min_{\mathbf{P}} f(\mathbf{P}) = \sum_{t=1}^{24}\sum_{i=1}^{9}\left(a_i P_{i,t}^2 + b_i P_{i,t} + c_i\right)$ $P_i^{\min} \le P_{i,t} \le P_i^{\max}, \quad \forall i = 1,\ldots,5,\ t = 1,\ldots,24$ $\sum_{i=1}^{5} P_{i,t} = D_t, \quad \forall t = 1,\ldots,24$ $P_{\min} = [150, 135, 73, 60, 73, 57, 20, 47, 20]$ $P_{\max} = [470, 460, 340, 300, 243, 160, 130, 120, 80]$	216	$P_i^{\min} \le P_{i,t} \le P_i^{\max}$
Static Economic Load Dispatch (1,2,3,4,5)	$\min_{P_1,\ldots,P_{N_G}} F = \sum_{i=1}^{N_G} f_i(P_i)$ $f_i(P_i) = a_i P_i^2 + b_i P_i + c_i, \quad i = 1,2,\ldots,N_G$ $f_i(P_i) = a_i P_i^2 + b_i P_i + c_i + \lvert e_i \sin(f_i(P_i^{\min} - P_i))\rvert$ $P_i^{\min} \le P_i \le P_i^{\max}, \quad i = 1,2,\ldots,N_G$ $\sum_{i=1}^{N_G} P_i = P_D + P_L$ $P_L = \sum_{i=1}^{N_G}\sum_{j=1}^{N_G} P_i B_{ij} P_j + \sum_{i=1}^{N_G} B_{0i} P_i + B_{00}$ $P_i - P_i^0 \le UR_i \quad P_i^0 - P_i \le DR_i$	6 13 15 40 140	See Technical Report of CEC2011

The results shown in Table 5 were obtained using the parameter settings defined in Table 1. The termination criterion was set to 150,000 function evaluations, ensuring a uniform computational budget across all test cases. No local optimization procedures were applied during the runs, meaning that the reported outcomes reflect solely the global search capabilities of the algorithm without any refinement from local search techniques. This setup allows for an unbiased assessment of the method's performance under purely global exploration conditions.

Table 5. Algorithms' Comparison Based on Best and Mean after 1.5e+5 FEs

150000 Fes	CLPSO			SaDE			jDE			CMA-ES			SIOA		
Problem	**best**	**mean**	**st**	**best**	**mean**	**st**	**best**	**mean**	**st**	**best**	**mean**	**st**	**best**	**mean**	**st**
Parameter Estimation for Frequency-Modulated Sound Waves	0.1314837477	0.2124981688	0.030223102	0.1899428536	0.2025566839	0.009271896	0.116157541	0.146008756	0.035009569	0.18160915970	0.256863966	0.044727545	0.20618586	0.259930863	0.023021357
Lennard-Jones Potential	-13.43649135	-10.25073403	1.02903617	-24.86870825	-22.6693403	1.127265561	-29.98126575	-27.49258505	1.235083397	-28.42253189	-25.78783328	2.27119571	-28.51132554	-24.14612379	2.489334694
Bifunctional Catalyst Blend Optimal Control	-0.000286591	-0.000286591	1.157726295e-16	-0.000286591	-0.000286591	5.513684428e-20	-0.000286591	-0.000286591	5.513684428e-20	-0.000286591	-0.000286591	5.513684428e-20	-0.000286591	-0.000286591	9.17[illegible]681044e-11
Optimal Control of a Non-Linear Stirred Tank Reactor	0.3903767228	0.3903767228	0.00	0.3903767228	0.3903767228	0.00	0.390376723	0.390376723	0	0.3903767228	0.390376723	0	0.390376723	0.390376723	0
Tersoff Potential for model Si (B)	-28.23544117	-26.18834522	1.05654251	-3.107773136	25.4711091	16.7202543	-13.51157064	-3.983690794	6.666047747	-29.26244222	-27.5889735	1.040646284	-28.63594613	-27.11517851	1.084722973
Tersoff Potential for model Si (C)	-30.85200257	-28.87349048	0.988024149	-11.60719468	22.08963599	18.5809093	-18.76214649	-8.506037168	5.543190141	-33.19699356	-31.79270914	0.828194234	-33.50417851	-31.0138182	1.420690601
Spread Spectrum Radar Polly phase Code Design	1.085334991	1.343956153	0.148708837	1.536501579	2.150881715	0.198607499	1.525870558	1.812042166	0.171213339	0.01484822722	0.171988666	0.137892008	0.607180067	1.023498006	0.228610721
Transmission Network Expansion Planning	250.00	250.00	0.00	250.00	250.00	0.00	250.00	250.00	0.00	250.00	250.00	0.00	250.00	250.00	0
Electricity Transmission Pricing	13,775,010.10	13,775,395.07	222.9723613	23,481,009.86	30,034,934.81	3,264,767.4	13,774,627.84	14,020,953.78	276,142.5345	13,775,841.77	13,787,550.18	6136.744382	13,774,551.1	13,775,341.62	372.2433548
Circular Antenna Array Design	0.006933401045	0.05181551798	0.070674314	0.02142329927	0.03892428051	0.008183211	0.006820072	0.017657998	0.022383475	0.007204797576	0.008635655364	0.000917821	0.007425975	0.024989563	0.044360116
Dynamic Economic Dispatch 1	428,607,927.60	435,250,914.50	2,973,190.125	968,042,312.10	1,034,679,775.00	25667290.12	968,042,312.1	1,034,393,036	25,445,935.78	88,285.60	102,776.71	6688.08697	921,434,356.7	984,699,299.8	23,606,727.92
Dynamic Economic Dispatch 2	33,031,590.31	53,906,147.38	8,492,239.111	845,287,898.30	913,715,793.20	3,0667,287.54	340,091,475.3	397,471,715.1	37,259,947.14	502,699.42	477,720.15	193,951.4891	768,167,675.2	768,167,675.2	768,167,675.2
StaticEconomic Load Dispatch 1	6554.67	7668.33	1245.137667	16,877.92	101,588.39	81105.92078	6163.749006	6778.527028	3004.59066	6657.61	415,917.46	688,544.4983	6538.455462	877,097.0217	847,631.4535
Static Economic Load Dispatch 2	19,030.36	20,699.00	2922.047235	2,600,565.21	9,329,466.81	4,019,053.284	1,161,578.904	3,671,587.605	1,542,286.275	763,001.22	1,425,815.44	377,126.8219	24,026.88184	1,478,534.024	1,063,608.234
Static Economic Load Dispatch 3	470,192,288.30	470,294,703.20	57822.41621	478,069,615.30	541,898,763.00	20,128,777.18	471,058,115.8	471,963,142.3	529,633.389	470,023,232.30	470,023,232.30	1.848771369e-07	470,825,156.9	472,256,736.5	608,886.262
Static Economic Load Dispatch 4	884,980.56	1,423,887.36	285,794.4518	14,170,362.58	106,749,078.50	73,147,979.72	6,482,592.714	17,527,314.24	53,06,489.46	476,053.52	2,925,852.94	12,68,161.817	70,686.26733	580,122.834	340,707.3484
Static Economic	8,105,947,615	8,110,924,071.00	4,422,895.726	1.312720405e+10	13,543,754,650.00	213,865,059.7	8,453,090,778	8,459,337,082	2874,979.192	8,072,077,963	8,084,017,791	4,623,617.36	8,002,077,963	8,048,300,791	4,365,204.42

Table 6. Detailed Ranking of Algorithms Based on Best and Mean after 1.5e+5 FEs

Problem	**CLPSO best**	**CLPSO Mean**	**SaDE best**	**SaDE Mean**	**jDE Best**	**jDE Mean**	**CMA-ES best**	**CMA-ES Mean**	**EO Best**	**EO Mean**
Parameter Estimation for Frequency-Modulated Sound Waves	2	3	4	2	1	1	3	4	5	5
Lennard-Jones Potential	5	5	4	4	1	1	3	2	2	3
BifunctionalCatalyst Blend Optimal Control	1	1	1	1	1	1	1	1	1	1
Optimal Control of a Non-Linear Stirred Tank Reactor	1	1	1	1	1	1	1	1	1	1
Tersoff Potential for model Si (B)	3	3	5	5	4	4	1	1	2	2
Tersoff Potential for model Si (C)	3	3	5	5	4	4	2	1	1	2
Spread Spectrum Radar Polly phaseCode Design	3	3	5	5	4	4	1	1	2	2
Transmission Network Expansion Planning	1	1	1	1	1	1	1	1	1	1
Electricity Transmission Pricing	3	1	5	5	2	4	4	3	1	2
Circular Antenna Array Design	2	5	5	4	1	2	3	1	4	3
Dynamic Economic Dispatch 1	2	2	5	5	4	4	1	1	3	3
Dynamic Economic Dispatch 2	2	2	5	5	3	3	1	1	4	4
Static Economic Load Dispatch 1	3	2	5	5	1	1	4	3	2	4
Static Economic Load Dispatch 2	1	1	5	5	4	4	3	2	2	3
Static Economic Load Dispatch 3	2	2	5	5	4	3	1	1	3	4
Static Economic Load Dispatch 4	3	2	5	5	4	4	2	3	1	1
Static Economic Load Dispatch 5	3	3	5	5	4	4	2	2	1	1
TOTAL	**40**	**40**	**71**	**68**	**44**	**46**	**34**	**29**	**36**	**42**

Table 7. Comparison of Algorithms and Final Ranking

Method	**Best**	**Mean**	**Overall**	**Average**	**Rang**
CMA-ES	34	29	63	1.75	1
EO	36	42	78	2.16	2
CLPSO	40	40	80	2.22	3
jDE	44	46	90	2.5	4
SaDE	71	68	139	3.86	5

The comparative analysis of the optimization methods, based on both best and mean performance after 150,000 function evaluations, reveals clear distinctions in their overall effectiveness. CMA-ES achieved the highest ranking, excelling in both peak and consistent performance, followed by EO and CLPSO, which demonstrated strong competitiveness. SIOA ranked closely behind these top methods, showing notable strengths in complex, high-dimensional, and multimodal problems, where its adaptive sporulation and germination mechanisms effectively balanced exploration and exploitation. In certain cases, such as the Tersoff Potential and Static Economic Load Dispatch problems, SIOA's results approached those of CMA-ES, highlighting its capacity to rival advanced evolutionary strategies. However, its slightly higher variance in some problem instances, particularly in less multimodal landscapes, reduced its mean performance score, preventing it from achieving the top overall rank. Despite this, SIOA emerges as a modern and competitive algorithm with strong potential for further improvement, especially through integration with specialized local search schemes aimed at enhancing stability and precision.

3.3. Exploration and exploitation

In this study, the trade-off between exploration and exploitation is assessed using a specific set of quantitative indicators: Initial Population Diversity (IPD), Final Population Diversity (FPD), Average Exploration Ratio (AER), Median Exploration Ratio (MER), and Average Balance Index (ABI). These metrics, although fundamentally grounded in population diversity measurements, are designed to capture both the temporal evolution of exploration by monitoring diversity changes over the course of the optimization and the degree of exploitation through the level of convergence in the final population. While these indicators provide a structured way to examine algorithmic behavior, further investigation employing more direct analysis tools, such as attraction basin mapping or tracking the clustering of solutions around local or global optima, could yield deeper insights into the search dynamics. Such approaches are considered a promising avenue for extending the current work.

The metrics reported in Tables 8 quantify and track the interplay between exploration and exploitation throughout the execution of the SIOA algorithm. Their computation relies on diversity measurements at different stages of the optimization process and on how these values evolve over iterations.

The IPD quantifies the diversity present at the very start of the optimization and is obtained by computing the mean Euclidean distance between all pairs of individuals in the initial population:

$$IPD = \frac{2}{NP(NP-1)} \sum_{i=1}^{NP-1} \sum_{j=i+1}^{NP} d(x_i, x_j) \tag{3}$$

where $d(x_i, x_j)$ is the Euclidean distance between solutions $x_i$and $x_j$end NP denotes the population size.

The FPD is computed using the same formulation, but applied to the final set of solutions after the algorithm completes.

The AER reflects the average level of exploration across all iterations and is defined as:

$$AER = \frac{1}{G}\sum_{g=1}^{iter_{max}} \frac{IPD_g}{IPD_1} \tag{4}$$

where $iter_{max}$is the total number of iterations, IPD_grepresents the diversity at iteration g, and IPD_1 is the initial diversity value.

The MER is the median value of the exploration ratios recorded over all generations:

$$MER = \text{median}\left(\frac{IPD_g}{IPD_1}\right), \quad \text{for } g = 1, \ldots, iter_{max} \tag{5}$$

The ABI serves as a composite measure of the exploration–exploitation balance. It is typically calculated as a weighted function of AER and FPD (or other exploitation-related indicators):

$$ABI = \frac{AER}{AER+\epsilon} \cdot \left(1 - \frac{FPD}{IPD}\right) \tag{6}$$

where ϵ is a small constant introduced to avoid division by zero. An ABI value close to 0.5 generally indicates a well-balanced interplay between exploration and exploitation.

Table 8. Balance between exploration and exploitation of the SIOA method in each benchmark function after 1.5e+5 FEs

PROBLEM	BEST	MEAN	SD	IPD	FDP	AER	MER	ABI
Parameter Estimation for Frequency-Modulated Sound Waves	0.20618586	0.259930863	0.023021357	8.5901	4.16015	4.16015	0	0.49979
Lennard-Jones Potential	-28.51132554	-24.14612379	2.489334694	13.91823	4.466	0.00021	0	0.49967
Bifunctional Catalyst Blend Optimal Control	-0.000286591	-0.000286591	9.177681044e-11	0.0743	0.08161	0.00007	0	0.50005
Optimal Control of a Non-Linear Stirred Tank Reactor	0.390376723	0.390376723	0	49184124.11	0.00019	17134746.47	0	0.49745
Tersoff Potential for model Si (B)	-28.63594613	-27.11517851	1.084722973	5.52126	1.71041	0.0002	0	0.49968
Tersoff Potential for model Si (C)	-33.50417851	-31.0138182	1.420690601	5.52126	2.74916	0.00017	0	0.49971
Spread Spectrum Radar Polly phase Code Design	0.607180067	1.023498006	0.228610721	8.06994	5.64058	0.00008	0	0.49988
Transmission Network Expansion Planning	250.00	250.00	0	0.96619	0.92498	0.00001	0	0.5
Electricity Transmission Pricing	13774551.1	13775341.62	372.2433548	6.50993	0.06077	0.00845	0.00078	0.49851
Circular Antenna Array Design	0.007425975	0.024989563	0.044360116	245.62332	26.21386	0.00052	0	0.49938
Dynamic Economic Dispatch 1	921434356.7	984699299.8	23606727.92	530.86265	0.06496	0.54586	0.0025	0.49827
Dynamic Economic Dispatch 2	768167675.2	768167675.2	768167675.2	890.76948	0.08559	0.68792	0.00216	0.49823
Static Economic Load Dispatch 1	6538.455462	877097.0217	847631.4535	141.01729	149.57653	0.00001	0	0.50002
Static Economic Load Dispatch 2	24026.88184	1478534.024	1063608.234	238.24613	207.89492	0.00008	0	0.49982
Static Economic Load Dispatch 3	470825156.9	472256736.5	608886.262	218.59546	25.75625	0.00082	0	0.49941
Static Economic Load Dispatch 4	70686.26733	580122.834	340707.3484	410.06721	3.95079	0.01195	0	0.49942
Static Economic Load Dispatch 15	1.241487588e+10	1.284582323e+10	197599745.4	750.05361	0.06971	0.71341	0.00243	0.49825

3.4. Parameters Sensitivity

By adopting the parameter sensitivity examination framework proposed by Lee et al. [82], this study provides a solid foundation for understanding how optimization algorithms react to changes in their configuration and sustain their reliability across varying conditions.

Table 9. Sensitivity analysis of the method parameters for the Potential problem (Dimension 10)

Potential 10	Value	Mean	Min	Max	Iters	Main range
c1	0.1	-16.36153	-23.37956	-11.01324	150	2.42989
	0.3	-15.99563	-20.23252	-10.85775	150	
	0.5	-15.31793	-21.19271	-10.81508	150	
	0.7	-14.64094	-20.05168	-10.78772	150	
	0.9	-13.93164	-19.82466	-10.51742	150	
c2	0.1	-15.64469	-18.92493	-12.61004	150	3.03774
	0.3	-16.10363	-23.37956	-10.99769	150	
	0.5	-15.43998	-20.58774	-11.0502	150	
	0.7	-14.64094	-16.71041	-10.51742	150	
	0.9	-13.93164	-20.23252	-11.37413	150	

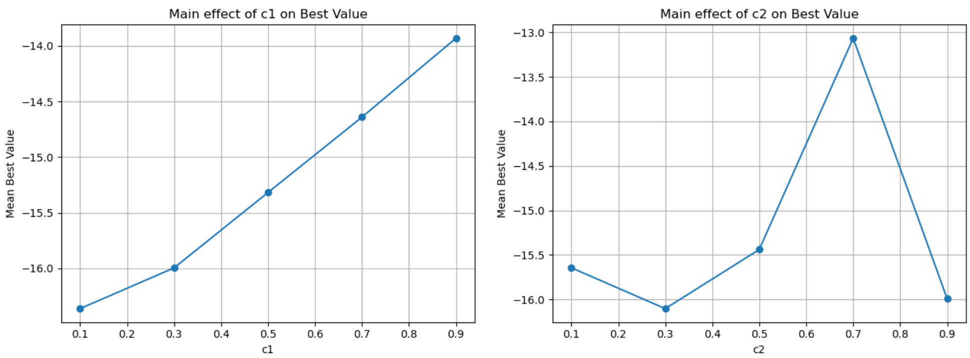

Figure 3. Graphical representation of c1 and c2 for the Potential problem

Table 10. Sensitivity analysis of the method parameters for the Rastrigin (Dimension 4)

Rastrigin 4	Value	Mean	Min	Max	Iters	Main range
c1	0.1	2.13634	0	11.10018	150	0.88378
	0.3	1.32079	0	8.30785	150	
	0.5	1.52523	0	8.1495	150	
	0.7	1.25256	0	7.10786	150	
	0.9	1.74395	0	6.95643	150	
c2	0.1	3.25941	0	11.10018	150	2.44349
	0.3	1.51291	0	6.55699	150	
	0.5	0.81592	0	5.19549	150	
	0.7	1.34782	0	5.19957	150	
	0.9	1.04281	0	6.60519	150	

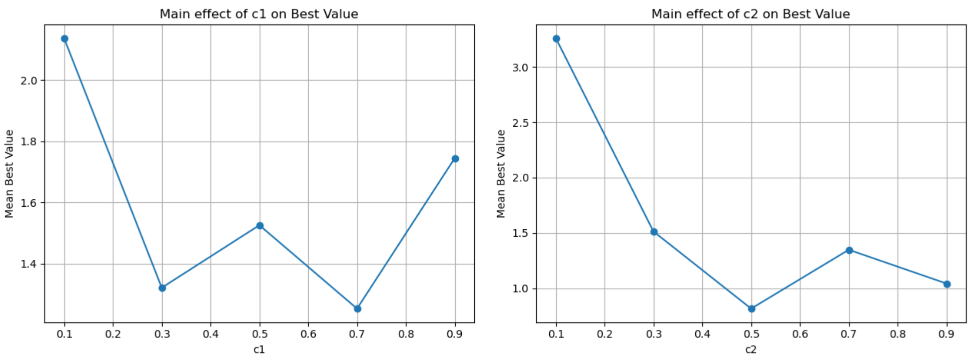

Figure 4. Graphical representation of c1 and c2 for the Rastrigin problem

Table 11. Sensitivity analysis of the method parameters for the Test2n problem (Dimension 4)

Test2n 4	Value	Mean	Min	Max	Iters	Main range
c1	0.1	-146.95432	-156.66451	-128.37343	150	2.81625
	0.3	-146.19977	-156.66454	-128.38355	150	
	0.5	-146.38681	-156.66442	-114.25247	150	
	0.7	-147.60809	-156.6641	-114.25223	150	
	0.9	-149.01602	-156.66437	-114.24072	150	
c2	0.1	-152.40955	-156.66454	-128.39005	150	8.94298
	0.3	-149.87331	-156.66447	-128.38459	150	
	0.5	-146.48201	-156.66451	-114.25223	150	
	0.7	-143.46657	-156.66437	-128.37376	150	
	0.9	-143.93359	-156.664	-114.24072	150	

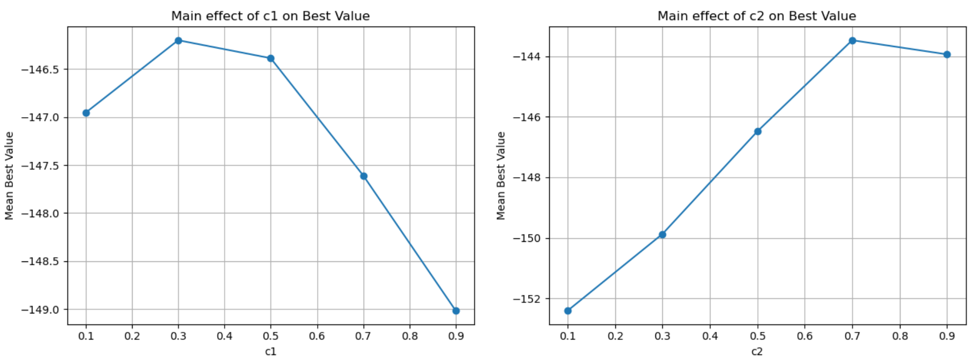

Figure 5. Graphical representation of c1 and c2 for the Test2n problem

Table 12. Sensitivity analysis of the method parameters for the Rosenbrock problem (Dimension 4)

Rosenbrock 4	Value	Mean	Min	Max	Iters	Main range
c1	0.1	35.1061	0	1354.34838	150	30.29039
	0.3	14.66004	0	1038.60285	150	
	0.5	13.3723	0	593.67884	150	
	0.7	9.95311	0	524.37725	150	
	0.9	4.81572	0	314.03933	150	
c2	0.1	18.00132	0	1354.34838	150	20.91708
	0.3	4.43389	0	235.09807	150	
	0.5	11.78364	0	593.67884	150	
	0.7	18.33745	0	400.1598	150	
	0.9	25.35097	0	1244.51484	150	

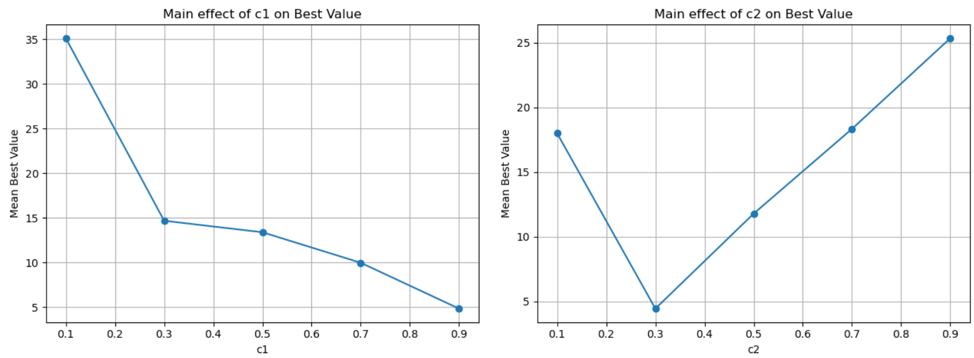

Figure 6. Graphical representation of c1 and c2 for the Rosenbrock problem

In Potential problem (Table 9 and Figure 3), the mean best value improves as c_1 decreases: the Mean Best moves from −13.93 (c_1=0.9) toward −16.36 (c_1=0.1), with a main effect range of 2.43. This indicates that for this high-dimensional, strongly multimodal potential, excessive stochastic dispersion (high c_1) "blurs" exploitation of promising areas, whereas mild dispersion supports steady improvement. The impact of c_2 is stronger (range 3.04) and non-monotonic: moderate values around 0.3 yield the best mean performance (−16.10), while very low or very high values degrade results. Therefore, in potential a clear preference emerges for a "moderate" pull toward the best solution ($c_2 \approx 0.3$) combined with a low stochastic perturbation (small c_1).

In Rastrigin problem (Table 10 and Figure 4), the behavior differs: c_1 has a relatively small main effect (0.88), and the best mean value occurs around $c1$=0.7 (Mean Best≈1.25), with similar performance at c_1=0.3. In contrast, c_2 is more decisive (range 2.44), with the optimal zone around 0.5 (Mean Best≈0.82). The Rastrigin function, with its pronounced symmetric multimodality, benefits from a stronger attraction mechanism toward the best (moderate c_2), which helps "lock in" low-value basins, while a moderate c_1 maintains enough exploration without destabilizing convergence. It is notable that the minima are often 0.00, indicating that all combinations can reach the global minimum, but mean values differentiate reliability and stability.

In Test2n problem (Table 11 and Figure 5), the picture is even clearer in favor of low c_2: the main effect of c_2 is very high (8.94), and the best mean performance appears at c_2=0.1 (Mean Best≈−152.41). Increasing c_2 toward 0.7–0.9 significantly worsens mean performance, although the minima remain near −156.664 for all settings. This shows that excessive attraction toward the best induces premature convergence into local basins and increases performance variability. c_1 has a moderate impact (2.82), with a trend suggesting that larger values (e.g., 0.9) may slightly improve mean performance, likely by helping to escape narrow polynomial valleys. Overall, in Test2n4, the guidance is clear: keep c_2 low and allow c_1 to be medium-to-high to maintain consistent solution quality.

In Rosenbrock4 problem (Table 12 and Figure 6), c_1 has the largest overall effect across all cases (range 30.29), with a dramatic improvement in mean performance as it increases from 0.1 to 0.9 (Mean Best from ~35.11 to ~4.82). The Rosenbrock function's narrow curved valley and anisotropy explain why stronger stochastic perturbation helps maintain mobility along the valley and avoid "dead zones" in step progression. c_2 shows a U-shaped trend: the best mean performance occurs at 0.3 (Mean Best≈4.43), while very low or very high c_2 increases the risk of large outliers, as seen in maximum values that can spike dramatically. Thus, in [rosenbrock4], a high c_1 is recommended to keep search activity within the valley, and a moderate $c_2 \approx 0.3$ helps avoid both over-pulling, which can distort the valley geometry, and overly loose guidance, which delays convergence.

Synthesizing these findings, a consistent tuning pattern emerges: in highly multimodal landscapes with many symmetric basins such as Rastrigin, a moderate c_2 around 0.5 and a moderate c_1 around 0.3–0.7 minimize mean values and stabilize convergence. In "parabolic" or polynomial landscapes like Test2n, a low c_2 and medium-to-high c_1 improve stability and mean performance, preventing premature convergence. In narrow-valley problems like Rosenbrock, strong c_1 and moderate $c_2 \approx 0.3$ appear to be the most robust choice. Finally, for dense multimodal potentials like Potential, the optimal zone tends toward low c_1 and moderate $c_2 \approx 0.3$, balancing small, targeted jumps with steady, controlled attraction toward the best.

In practical terms, the ranges that reappear as "safe defaults" are c_2 in the moderate range of 0.3–0.5, and c_1 adapted to landscape morphology: low for Potential-type landscapes, moderate for Rastrigin, high for Rosenbrock, and medium-to-high for polynomial Test2n landscapes. The min/max values per setting highlight the tendency for extreme deviations when c_2 is too high or too low especially in Rosenbrock reinforcing that the "high c_1 – moderate c_2" combination is often the most resilient operating point when the goal is high mean performance rather than isolated best cases.

3.5. Analysis of Computational Cost and Complexity of the SIOA Algorithm

Figure 7 illustrates the complexity of the proposed method, showing the number of objective function calls and the execution time (in seconds) for problem dimensions ranging from 20 to 260. The experimental settings follow the parameter values specified in Table 1, with the termination criterion based on the homogeneity of the best value. In addition, a limited local optimization procedure is applied at a rate of only 0.5%, enhancing the exploitation of promising regions in the search space without significantly affecting the overall global exploration strategy.

More specifically, in the ELLIPSOIDAL problem, the execution time increases gradually from 0.111 seconds at dimension 20 to 41.714 seconds at dimension 260, while the corresponding objective function calls range from 1,398 to 6,457. Similarly, for the ROSENBROCK problem, the execution time rises from 0.144 seconds at dimension 20 to 40.873 seconds at dimension 260, with the number of calls increasing from 2,83 to 9200. The results indicate that both execution time and the number of calls grow as the problem dimensionality increases, with ROSENBROCK generally requiring greater computational effort in higher dimensions compared to ELLIPSOIDAL. This observation highlights the sensitivity of the method's complexity to the nature of the problem, while also confirming its ability to scale efficiently across a wide range of search space sizes.

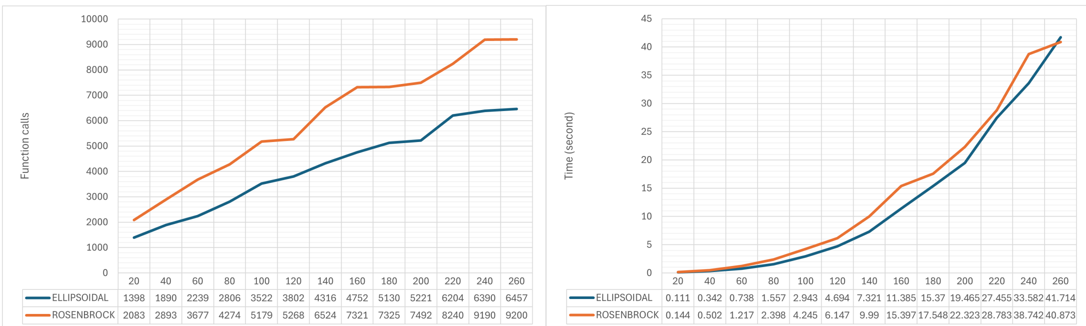

Figure 7. Computational performance (Calls anad Time) of the proposed method on ELLIPSOIDAL and ROSENBROCK across dimensions 20-260

4. Conclusions

Based on the experiments conducted, SIOA proves to be a mature, competitive, and efficient metaheuristic. In classical benchmark problems, it consistently outperforms GA, DE, PSO, and ACO in terms of required objective function calls, while maintaining a high success rate; the overall evaluation footprint is significantly lower than that of traditional methods, translating into faster convergence for a given computational budget. This performance profile supports the view that the biologically inspired "sporulation–germination" mechanism, combined with self-adaptive parameter control and similarity-based replacement, provides a tangible advantage across a wide range of problem types.

The method also demonstrates notable stability: with the parameter settings of Table 1, the best result was reproduced uniformly in 12 consecutive runs, while local optimization was used minimally (only 0.5%), indicating that SIOA's global search is sufficient to locate optimal or near-optimal solutions without relying heavily on exploitation. The algorithm's core components stochastic perturbation around an adaptive radius, attraction toward the global best, the "zero-reset" rule when the optimum lies near the origin, and replacement through crowding collectively explain both the maintenance of diversity and the ability to avoid premature convergence.

In more demanding, realistic scenarios with a uniform budget of 150,000 function evaluations and no local optimization, SIOA remains highly competitive against advanced techniques. Although CMA-ES achieved the top overall rank, SIOA came very close, with results in certain cases (e.g., Tersoff Potential and Static Economic Load Dispatch) approaching the best of the leading competitors. A slightly higher variance in some less multimodal landscapes limited the mean performance, highlighting a margin for improvement in stability without undermining the overall strength of the method.

The scalability analysis shows that both runtime and function evaluations increase with problem dimension and landscape ruggedness, with problems such as Rosenbrock generally requiring more computational effort than smoother ellipsoidal forms an observation consistent with the expected behavior of metaheuristics in difficult, poorly scaled valleys. In all cases, SIOA maintains an economical evaluation profile compared to competing approaches, a feature of direct value in costly simulations.

Overall, the method is realistically ready for application: fast in terms of evaluations, stable without relying on intensive local search, and sufficiently flexible to dynamically adapt critical parameters as the search progresses. At the same time, clear opportunities for further improvement remain. Realistic next steps include integrating more specialized, problem-sensitive local optimizers to reduce variance and improve final accuracy, as well as extending SIOA to constrained, multi-objective, and large-scale problems, where the combination of self-adaptation, crowding, and "zero-reset" may yield even greater benefits. Equally promising are explorations of hybrid versions augmented with surrogate modeling for expensive problems, further parallelization and GPU/multi-threaded implementations, the use of restart strategies and dynamic similarity thresholds, and the development of fully parameter-free versions with stronger theoretical convergence guarantees. The indicated extensions to constrained, multi-objective, and large-scale applications, along with reinforcement via dedicated local search schemes, underscore SIOA's realistic potential as a modern foundation for further research and practical deployment.

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