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Adapt the parameters of RBF networks using Grammatical Evolution

Ioannis G. Tsoulos[1,†,‡,*], Alexandros Tzallas[2], Evangelos Karvounis[3]

[1] Department of Informatics and Telecommunications, University of Ioannina, Greece; itsoulos@uoi.gr
[2] Department of Informatics and Telecommunications, University of Ioannina, Greece; tzallas@uoi.gr
[3] Department of Informatics and Telecommunications, University of Ioannina, Greece; ekarvounis@uoi.gr
* Correspondence: itsoulos@uoi.gr;
† Current address: Department of Informatics and Telecommunications, University of Ioannina, Greece.
‡ These authors contributed equally to this work.

Abstract: Radial basis function networks are widely used in a multitude of applications in various scientific areas in both classification and data fitting problems. These networks deal with the above problems by adjusting their parameters with various optimization techniques. However, an important issue to address is finding a satisfactory interval of values for the network parameters before adjusting these parameters. This paper proposes a two-stage technique, where in the first stage, using Grammatical Evolution, rules are generated to create the optimal value interval of the network parameters. In the second stage of the technique, the parameters of the network are fine-tuned with some robust global optimization method, such as a genetic algorithm. The proposed technique was tested on a number of problems from the recent literature and found to reduce the classification or data fitting error by over 40% on most datasets. Furthermore, the method appears highly stable as increasing the number of network parameters does not significantly affect its performance.

Keywords: Neural networks; Genetic algorithms; Genetic programming; Grammatical evolution

1. Introduction

Many practical problems of the modern world can be thought of either as data fitting problems, as for example, problems that appears in physics [1,2], problems related to chemistry [3,4], economic problms [5,6], medicine problems [7,8], etc. A commonly used machine learning tool to tackle such problems, is the Radial Basis Function (RBF) network [9,10]. Usually, an RBF network is expressed using the following equation:

$$y(\overrightarrow{x}) = \sum_{i=1}^{k} w_i \phi\left(\left\| \overrightarrow{x} - \overrightarrow{c_i} \right\|\right) \quad (1)$$

where the symbols in the equation are defined as follows:

1. The element $\overrightarrow{x}$ stands for the input pattern from the dataset describing the problem. For the rest of this paper, the notation d will be used to represent the number of elements in $\overrightarrow{x}$.
2. The parameter k denotes the number of weights used to train the RBF network and the associated vector of weights is denoted as $\overrightarrow{w}$.
3. The vectors $\overrightarrow{c_i}$, $i = 1, .., k$ stand for the centers of the model.
4. The outcome of the equations $y(\overrightarrow{x})$ stands for the estimated value of the network for the input pattern $\overrightarrow{x}$.

The $\phi(x)$ function, in most cases represent the Gaussian function given by:

$$\phi(x) = \exp\left(-\frac{(x-c)^2}{\sigma^2}\right) \quad (2)$$

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The main advantages of RBF networks are:

1. They have a simpler structure than other machine learning models such as multilayer perceptron neural networks (MLPs)[11], since they have only one processing layer and therefore have faster training techniques and they have faster response times.
2. They can approximate any continuous function [12].

The RBF networks were used in many cases, such as problems from physics [13–16], solving differential equations [17–19], robotics [20,21], face recognition [22], digital communications [23,24], chemistry problems [25,26], economic problems [27–29], network security problems [30,31] etc. Also, recently a variety of papers have appeared proposing novel initialization techniques for the network parameters [32–34]. Also, Benoudjit et al [35] discuss the effect of kernel widths on RBF networks. Moreover, Neruda et al [36] presents a comparison of some learning methods for RBF networks. Additionally, a variety of pruning techniques [37–39] have been proposed to reduce the number of required parameters of the RBF networks. Due to the widespread usage of RBF networks but also because considerable computing time is often required for their effective training, in recent years a series of techniques have been proposed [40,41] for the exploitation of parallel computing units to adjust the parameters of neural networks.

In the same direction of research, other researchers propose to handle problems of classification or data fitting, techniques such as Support Vector Machines (SVM) [42,43], decision trees [44,45] etc. Also, Wang et al suggested an auto - encoder reduction method, applied on a series of large datasets[46]. This problem has also been tackled by various researchers during the past years, such as the work of Agarwal and Bhanot [47] proposed to adapt the RBF parameters, the usage of the ABC algorithm[48], the incorporation of the Firefly algorithm[49]. Furthermore, Gyamfi et al [50] recently proposed a differential RBF network that incorporates partial differential equations, aiming to make the network more robust in the presence of noise data. Also, Li et al [51] proposed a multivariate ensembles-based hierarchical linkage strategy (ME-HL) for system reliability evaluation of aeroengine cooling blades.

The parameters of the RBF network are modified in order to minimize the following loss - function, called training error of the network:

$$E(y(x,g)) = \sum_{i=1}^{m} \left(y(\overrightarrow{x}_i, \overrightarrow{g}) - t_i\right)^2 \tag{3}$$

Where the parameter m denotes the number of input patterns, the t_i represent the expected output for pattern $\overrightarrow{x}_i$. The vector $\overrightarrow{g}$ represents the parameter set of the RBF network.

A common method of calculating the parameters in these neural networks uses a technique to calculate the centers of the functions $\phi(x)$ and then the weight vector $\overrightarrow{w}$ is calculated as a solution of a linear system of equations. Typically, the method used to calculate the centers is the well - known k-means method [52]. In many cases, this way of estimating the parameters of the neural network leads to over-fitting of the model so that it cannot generalize satisfactorily to unknown data. Furthermore, since there is no range of values for the parameters, there is the possibility that they will take extremely large or extremely small values, with the result that any generalizability of the model is lost. This work suggests a two phase method to minimize the error of equation (3). During the first phase, an attempt is made to bound the parameter values to intervals at which the training error is likely to be significantly reduced. The identification of the most promising intervals for the parameters is performed using a technique that utilizes Grammatical Evolution[53], that collects information from the training data. During the second phase, the parameters of the RBF network can be trained within the optimal range found in the first phase using some global optimization method [54,55]. In the proposed approach, the widely used method of genetic algorithm [56–58] was used for the second phase of the process. The main contributions of the suggested approach are:

1. The first phase procedure seeks to identify a range of values for the network parameters while also reducing the error of the network on the training data set.
2. The rules Grammatical Evolution uses in the first phase are simple and can be generalized to any data set for data classification or fitting.
3. The determination of the value interval is done in such a way that it is faster and more efficient to train the parameters of the neural network with some optimization method during the second phase of the method.
4. After identifying a promising value interval from the first phase, any global optimization method can be used on that value interval to effectively minimize the network training error.

The rest of this paper is divided in the following sections: in section 2 the proposed method is fully described, in section 3 the datasets used in the experiments are listed as well as the experimental results and finally in section 4 some conclusions are provided.

2. Method description

This section starts with a detailed description of the Grammatical Evolution technique and the grammar that will be used to generate partition rules for the parameter set of RBFs. Subsequently, the first phase of the proposed methodology will be extensively analyzed and then the second phase, where a Genetic Algorithm will be applied to the outcome of the first phase.

2.1. Grammatical Evolution

Grammatical evolution is a special case of genetic algorithm. Genetic Algorithms, suggested by John Holland [59] are inspired by biology and the algorithm starts by creating an initial population of the so -called chromosomes that stand for potential solutions to the objective problem. These chromosomes are gradually altered using the genetic operators of selection, crossover and mutation[60]. The chromosomes in the Grammatical Evolution represent production rules of any given BNF (Backus–Naur form) grammar[61]. Grammatical Evolution has been used successfully in a variety of cases, such as function approximation[62,63], solution of trigonometric equations [64], automatic music composition of music [65], neural network construction [66,67], creating numeric constraints[68], video games [69,70], estimation of energy demand[71], combinatorial optimization [72], cryptography [73] etc. The BNF grammar can be used to describe the syntax of programming languages and usually it is defined as $G = (N, T, S, P)$ where

- N is a set that defines the non - terminal symbols of the grammar. Each non - terminal symbol is associated with a set of production rules. The application of these production rules produces series of terminal symbols.
- T stands for the set of terminal symbols.
- S is the start symbol that initiates the production rules with the property $S \in N$.
- P defines the set of production rules. These are rules are following the following notations: $A \to a$ or $A \to aB,\ A, B \in N,\ a \in T$.

The algorithm begins using the symbol S and gradually creates terminal symbols with the assistance of the production rules. The production rules are selected through the following procedure:

- Read the next element V from the chromosome that is being processed.
- Get the rule: Rule = V mod R, where R is the total number of production rules for the current non – terminal symbol.

The BNF grammar used in this work is presented in Algorithm 1. The symbols enclosed in <> denote the non-terminal symbols. The numbers in parentheses in the right part of the grammar stand for the sequence numbers of each production rule. Every RBF network with k weights is constructed by the following series of parameters:

1. A series of vectors $\overrightarrow{c_i},\ i = 1, \ldots, k$ that stand for the centers of the model.
2. For every Gaussian unit an additional parameter σ_i is required.

3. The output weight vector $\overrightarrow{w}$.

The number n is the total number of parameters of the problem. In the case of this paper, it is the total number of parameters of the RBF network. For the current work, the number n can be computed using the following formula:

$$n = (d + 2) \times k \tag{4}$$

The number n in the corresponding grammar is computed as follows:

1. For every center $\overrightarrow{c_i}$, $i = 1, .., k$ there are d variables. Hence, the total number of parameters required by the centers is $d \times k$.
2. Every Gaussian unit requires an additional parameter: σ_i, $i = 1, .., k$, which means k more parameters.
3. The weight vector $\overrightarrow{w}$ used in the output has k parameters.

As an example of production, consider the chromosome $x = [9, 8, 6, 4, 15, 9, 16, 23, 8]$ and $d = 2$, $k = 2$, $n = 8$. The steps to produce the final program $p_{\text{test}} = (x7, 0, 1), (x1, 1, 0)$ are outlined in Table 1. Every partition program consists of a series of partition rules. Each partition rule contains three elements:

1. The variable for which its original interval will be partitioned, for example x_7.
2. An integer number with values 0 and 1 at the left end of the value interval. If this value is 1, then the left end of the corresponding variable's value field will be divided by two, otherwise no change will be made.
3. An integer number with values 0 and 1 at the right end of the range of values of the variable. If this value is 1, then the right end of the corresponding variable's value field will be divided by two, otherwise no change will be made.

Hence, for the example program p_{test} the two partition rules will divide the right end of the variable x_7 and the left end of the variable x_1.

Algorithm 1 The BNF grammar used in the current work, to produce intervals for the RBF parameters. By using this grammar in the first phase of the proposed procedure, the optimal interval of values for the parameters of the neural network will be identified.

```
S::=<expr>   (0)
<expr> ::=  (<xlist> , <digit>,<digit>)  (0)
           |<expr>,<expr>                (1)
<xlist>::=x1    (0)
           | x2 (1)
           .........
           | xn (n)
<digit>  ::= 0 (0)
           | 1 (1)
```

Table 1. Steps to produce a valid expression from the BNF grammar.

Expression	Chromosome	Operation
	9,8,6,4,15,9,16,23,8	9 mod 2=1
<expr>,<expr>	8,6,4,15,9,16,23,8	8 mod 2=0
(<xlist>,<digit>,<digit>),<expr>	6,4,15,9,16,23,8	6 mod 8=6
(x7,<digit>,<digit>),<expr>	4,15,9,16,23,8	4 mod 2=0
(x7,0,<digit>),<expr>	15,9,16,23,8	15 mod 2=1
(x7,0,1),<expr>	9,16,23,8	9 mod 2 =1
(x7,0,1),(<xlist>,<digit>,<digit>)	16,23,8	16 mod 8=0
(x7,0,1),(x1,<digit>,<digit>)	23,8	23 mod 2=1
(x7,0,1),(x1,1,<digit>)	8	8 mod 2=0
(x7,0,1),(x1,1,0)		

2.2. The first phase of the proposed algorithm

The purpose of this phase is to initialize the bounds of the RBF model and discover a promising interval for the corresponding values. For this initialization, the K-Means algorithm [52] technique is used, which is also used for the traditional RBF network training technique. A description of this algorithm in a series of steps is shown in Algorithm 2.

Algorithm 2 The K-Means algorithm.

1. **Repeat**
   (a) Set $S_j = \{\}$, $j = 1..k$
   (b) **For** every pattern x_i, $i = 1, ..., m$ **do**
      i. **Set** $j^* = \min_{i=1}^{k} \{D(x_i, c_j)\}$.
      ii. **Set** $S_{j^*} = S_{j^*} \cup \{x_i\}$.
   (c) **EndFor**
   (d) **For** every center c_j, $j = 1..k$ **do**
      i. **Set** as M_j the number of points in S_j
      ii. **Compute** c_j as

$$c_j = \frac{1}{M_j} \sum_{i=1}^{M_j} x_i$$

   (e) **EndFor**
2. **Calculate** the quantities s_j as

$$\sigma_j^2 = \frac{\sum_{i=1}^{M_j} (x_i - c_j)^2}{M_j}$$

3. **Stop** the algorithm, if centers c_j do not change anymore.

Having calculated the centers c_i and the corresponding variances σ_i, the algorithm continues to compute the vectors $\overrightarrow{L}$, $\overrightarrow{R}$ with dimension n, that will be used as the initial bounds of the parameters. The above vectors are calculated through the procedure of the algorithm 3.

Algorithm 3 Algorithm to locate the vectors $\overrightarrow{L}$, $\overrightarrow{R}$

1. **Set** m=0
2. **Set** $F > 1$, the scaling factor.
3. **Set** $B > 0$, the initial upper bound for the weight vector $\overrightarrow{w}$.
4. **For** $i = 1..k$ **do**
   (a) **For** $j = 1..d$ **do**
      i. **Set** L_m=$-F \times c_{ij}$, R_m=$F \times c_{ij}$
      ii. **Set** $m = m + 1$
   (b) **EndFor**
   (c) **Set** $L_m = -F \times \sigma_i$, $R_m = F \times \sigma_i$
   (d) **Set** $m = m + 1$
5. **EndFor**
6. **For** $j = 1, ..., k$ **do**
   (a) **Set** $L_m = -B$, $R_m = B$
   (b) **Set** $m = m + 1$
7. **EndFor**

The bounds for the first $(d + 1) \times k$ variables of any given RBF network is computed as a multiple of the quantity F with the values calculated by the K-Means algorithm. The value B can be utilized to initialize the bounds for the weight $\overrightarrow{w}$. Afterwards, the following genetic algorithm is executed to locate the most promising vectors $\overrightarrow{L}$, $\overrightarrow{R}$ for the RBF parameters:

1. **Define** as N_c the number of chromosomes that will participate in the the Grammatical Evolution procedure.
2. **Define** as k the number of processing nodes of the used RBF model.
3. **Define** as N_g the number of allowed generations.
4. **Define** as p_s the used selection rate, with $p_s \leq 1$.
5. **Define** as p_m the used mutation rate, with $p_m \leq 1$.
6. **Define** as $N_s$as the total number of RBF networks that will be created randomly in every fitness calculation.
7. **Initialize** N_c chromosomes as sets of random numbers.
8. **Set** $f^* = [\infty, \infty]$, the fitness of the best chromosome. The fitness function f_g of any provided chromosome g is considered as an interval $f_g = \left[f_{g,\mathbf{low}}, f_{g,\mathbf{upper}}\right]$
9. **Set** iter=0.
10. **For** $i = 1, \ldots, N_c$ **do**
    (a) **Produce** the partition program p_i using the grammar of Figure 1 for the chromosome i.
    (b) **Produce** the bounds $\left[\vec{L_{p_i}}, \overrightarrow{R_{p_i}}\right]$ for the partition program p_i.
    (c) **Set** $E_{\min} = \infty$, $E_{\max} = -\infty$
    (d) **For** $j = 1, \ldots, N_S$ **do**
       i. **Create** randomly a set of parameters $\overrightarrow{g_j} \in \left[\vec{L_{p_i}}, \overrightarrow{R_{p_i}}\right]$
       ii. **Calculate** the error $E_{\overrightarrow{g_j}} = \sum_{k=1}^{M} \left(y(\overrightarrow{x_k}, \overrightarrow{g_j}) - t_k\right)^2$
       iii. **If** $E_{\overrightarrow{g_j}} \leq E_{\min}$ **then** $E_{\min} = E_{\overrightarrow{g_j}}$
       iv. **If** $E_{\overrightarrow{g_j}} \geq E_{\max}$ **then** $E_{\max} = E_{\overrightarrow{g_j}}$
    (e) **EndFor**
    (f) **Set** the fitness$f_i = [E_{\min}, E_{\max}]$
11. **EndFor**

12. **Perform** the procedure of selection: Initially, the chromosomes of the population are sorted according to their fitness values. Since the fitness values are intervals, the L^* operator is defined as:

$$L^*(f_a, f_b) = \begin{cases} \text{TRUE}, & a_1 < b_1, \text{OR } (a_1 = b_1 \text{ AND } a_2 < b_2) \\ \text{FALSE}, & \text{OTHERWISE} \end{cases} \quad (5)$$

As a consuquence, the fitness value f_a is considered smaller than f_b if $L^*(f_a, f_b) =$ TRUE. The first $(1 - p_s) \times N_c$ chromosomes with smaller fitness values are copied without changes to the next generation of the algorithm. The rest of chromosomes are replaced by chromosomes created in the crossover procedure.

13. **Perform** the crossover procedure. The crossover procedure will create new $p_s \times N_c$ chromosomes. For every pair of created offsprings two parents (z, w) are selected from the curent population using the tournament selection. These parent will produce the offsprings $\tilde{z}$ and $\tilde{w}$ using the one - point crossover, shown in Figure 1.
14. **Perform** the mutation procedure. In this process a random number $r \in [0, 1]$ is drawn for every element of each chromosome. The corresponding element is changed randomly if $r \leq p_m$.
15. **Set** iter=iter+1
16. **If** iter$\leq N_g$ goto step 10.

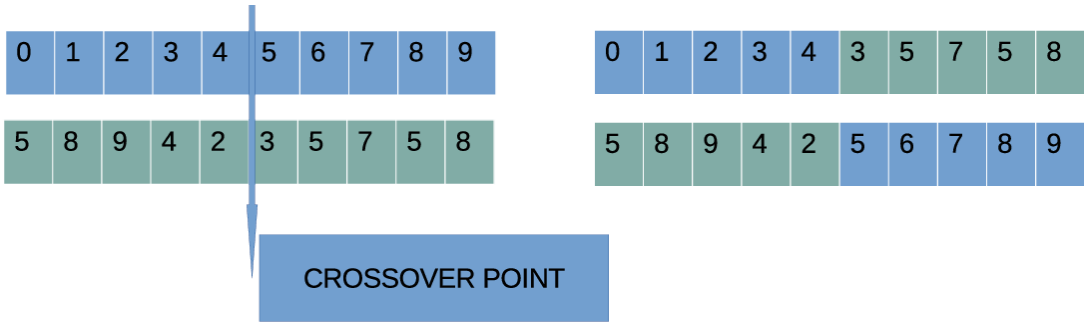

Figure 1. An example of the one point crossover procedure, as used in the Grammatical Evolution.

2.3. The second phase of the proposed algorithm

The second phase utilizes a genetic algorithm, to optimize the parameters of the RBF network. The optimization of the parameters uses as bounds the best interval produced in the first phase of the method. The layout of each chromosome is shown in Figure 2.

Figure 2. The layout of chromosomes in the second phase of the proposed algorithm.

c_{11}	c_{12}	...	c_{1d}	σ_1	c_{21}	c_{22}	...	c_{2d}	σ_2	...	c_{k1}	c_{k2}	...	c_{kd}	σ_k	w_1	w_2	...	w_k

1. **Initialization Step**
   (a) **Define** as N_c as the number of chromosomes.
   (b) **Define** as N_g the total number of generations.
   (c) **Define** as k the number of processing nodes of the used RBF model.
   (d) **Define** as $S = [L_{\text{best}}, R_{\text{best}}]$ the best located interval of the first stage of the algorithm, of subsection 2.2.
   (e) **Produce** N_C random chromosomes in S.
   (f) **Define** as p_s the used selection rate, with $p_s \leq 1$.
   (g) **Define** as p_m the used mutation rate, with $p_m \leq 1$.
   (h) **Set** iter=0.
2. **Fitness calculation Step**

(a) **For** $i = 1, \ldots, N_g$ **do**

i. **Compute** the fitness f_i of each chromosome g_i as $f_i = \sum_{j=1}^{m} \left(y(\overrightarrow{x_j}, \overrightarrow{g_i}) - t_j \right)^2$

(b) **EndFor**

3. **Genetic operations step**

(a) **Selection procedure.** Firstly, sort the chromosomes according to their fitness values. The first $(1 - p_s) \times N_c$ chromosomes having the lowest fitness values are copied without changes to the next generation. The rest of chromosomes are replaced by chromosomes created in the crossover procedure.

(b) **Crossover procedure**: For every pair of created offsprings two parents (z, w) are selected from the curent population using the tournament selection. These parent will produce the offsprings $\tilde{z}$ and $\tilde{w}$ according to the equations:

$$\begin{aligned} \tilde{z}_i &= a_i z_i + (1 - a_i) w_i \\ \tilde{w}_i &= a_i w_i + (1 - a_i) z_i \end{aligned} \tag{6}$$

The value a_i is a random number, where $a_i \in [-0.5, 1.5]$ [74].

(c) **Perform** the mutation procedure. In this process a random number $r \in [0, 1]$ is drawn for every element of each chromosome. The corresponding element is changed randomly if $r \leq p_m$.

4. **Termination Check Step**

(a) **Set** $iter = iter + 1$

(b) **If** iter$\leq N_g$ goto step 2.

The steps of the current algorithm are also outlined graphically in Figure 3 using a flowchart.

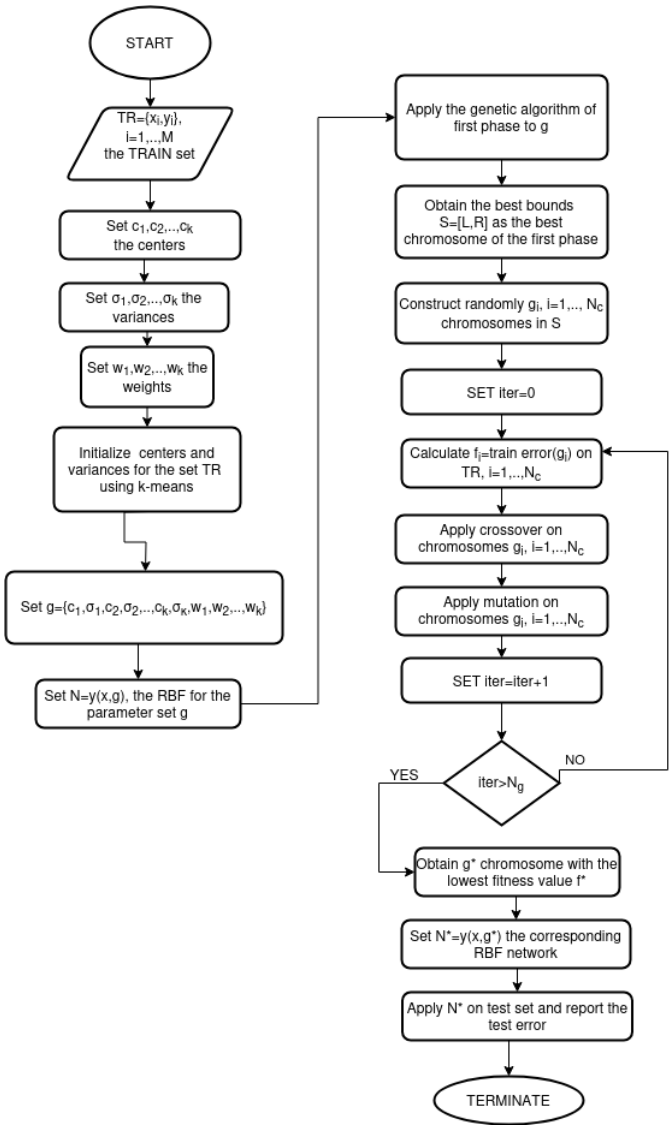

Figure 3. The flowchart of the proposed algorithm.

3. Experiments

3.1. Experimental datasets

The proposed method was tested on a wide set of classification and regression problems found in the relevant literature. The method was compared against some other well -known machine learning models. The following databases were used to obtain the datasets:

1. The UCI dataset repository, https://archive.ics.uci.edu/ml/index.php(accessed on 9 September 2023)
2. The Keel repository, https://sci2s.ugr.es/keel/datasets.php(accessed on 9 September 2023)[75].
3. The Statlib URL http://lib.stat.cmu.edu/datasets/(accessed on 9 September 2023).

The classification datasets are listed in Table 2 and the regression datasets are listed in Table 3.

Table 2. The classification datasets used in the experiments. The column DATASET denotes the number of the dataset, the column CLASSES stands for the number of classes in each dataset and the column REFERENCE points to the bibliography where the use of the particular data set is presented.

DATASET	CLASSES	REFERENCE
APPENDICITIS	2	[76]
AUSTRALIAN	2	[77]
BALANCE	3	[78]
CLEVELAND	5	[79,80]
DERMATOLOGY	6	[81]
HAYES ROTH	3	[82]
HEART	2	[83]
HOUSEVOTES	2	[84]
IONOSPHERE	2	[85,86]
LIVERDISORDER	2	[87]
MAMMOGRAPHIC	2	[88]
PARKINSONS	2	[89]
PIMA	2	[90]
POPFAILURES	2	[91]
SPIRAL	2	[92]
REGIONS2	5	[93]
SAHEART	2	[94]
SEGMENT	7	[95]
WDBC	2	[96]
WINE	3	[97,98]
Z_F_S	3	[99]
ZO_NF_S	3	[99]
ZONF_S	2	[99]
ZOO	7	[100]

Table 3. The regression datasets used in the experiments. The column DATASET denotes the number of the dataset and the column REFERENCE points to the bibliography or URL (KEEL or STATLIB) where the use of the particular data set is presented.

DATASET	REFERENCE
ABALONE	[101]
AIRFOIL	[102]
BASEBALL	STATLIB
BK	[103]
BL	STATLIB
CONCRETE	[104]
DEE	KEEL
DIABETES	KEEL
FA	STATLIB
HOUSING	[105]
MB	[106]
MORTGAGE	KEEL
NT	[107]
PY	[108]
QUAKE	[109]
TREASURY	KEEL
WANKARA	KEEL

3.2. Experimental results

The RBF model used in the experiments was implemented in ANSI C++ with the assistance of the open source Armadillo library [110]. The optimization methods used were also freely available from the OPTIMUS software, downloaded from https://github.com/itsoulos/OPTIMUS/(accessed on 9 September 2023). The proposed method was validated using the 10 - fold validation technique for all datasets. Also, all the experiments were executed 30 times with different seed number for the random generator each time. In the conducted experiments, the drand48() random function of the C - programming language was employed. For the classification datasets the average classification error was measured and and the average mean test error for the regression datasets. The experiments were conducted on an AMD Ryzen 5950X with 128GB of RAM, running the Debian Linux operating system. All the values for the parameters of the used algorithms are shown in Table 4. The results obtained for the classification datasets are shown in Table 5 and for the regression datasets are listed in Table 6.

The following applies to the results tables:

1. The column RPROP represents an artificial neural network [111,112]. This neural network has 10 processing nodes and was trained using the Rprop method [113].
2. The column denoted as ADAM stands for an artificial neural network with 10 hidden nodes trained by the Adam optimizer [114,115].
3. The column NEAT (NeuroEvolution of Augmenting Topologies) [116] denotes the application of the NEAT method for neural network training.
4. The RBF-KMEANS column denotes the original two - phase training method for RBF networks. During the first phase the centers and variances are estimated through the K-Means algorithm and during the second phase the output weights are calculated by solving a linear system of equations.
5. The column GENRBF stands for the RBF training method introduced in [117].
6. The column PROPOSED represents the results obtained by the proposed method.
7. For both tables, an additional line was added under the title AVERAGE. This line indicates the average classification or regression error for all datasets.

Table 4. The values used for the experimental parameters.

PARAMETER	VALUE
N_c	200
N_g	100
N_s	50
F	10.0
B	100.0
k	10
p_s	0.90
p_m	0.05

Table 5. Experimental results for the classification datasets. The first column denotes the name of the dataset. Every number in cells denote average classification error as calculated on the test set.

DATASET	RPROP	ADAM	NEAT	RBF-KMEANS	GENRBF	PROPOSED
Appendicitis	16.30%	16.50%	17.20%	12.23%	16.83%	15.77%
Australian	36.12%	35.65%	31.98%	34.89%	41.79%	22.40%
Balance	8.81%	7.87%	23.14%	33.42%	38.02%	15.62%
Cleveland	61.41%	67.55%	53.44%	67.10%	67.47%	50.37%
Dermatology	15.12%	26.14%	32.43%	62.34%	61.46%	35.73%
Hayes Roth	37.46%	59.70%	50.15%	64.36%	63.46%	35.33%
Heart	30.51%	38.53%	39.27%	31.20%	28.44%	15.91%
HouseVotes	6.04%	7.48%	10.89%	6.13%	11.99%	3.33%
Ionosphere	13.65%	16.64%	19.67%	16.22%	19.83%	9.30%
Liverdisorder	40.26%	41.53%	30.67%	30.84%	36.97%	28.44%
Mammographic	18.46%	46.25%	22.85%	21.38%	30.41%	17.72%
Parkinsons	22.28%	24.06%	18.56%	17.41%	33.81%	14.53%
Pima	34.27%	34.85%	34.51%	25.78%	27.83%	23.33%
Popfailures	4.81%	5.18%	7.05%	7.04%	7.08%	4.68%
Regions2	27.53%	29.85%	33.23%	38.29%	39.98%	25.18%
Saheart	34.90%	34.04%	34.51%	32.19%	33.90%	29.46%
Segment	52.14%	49.75%	66.72%	59.68%	54.25%	49.22%
Spiral	46.59%	48.90%	50.22%	44.87%	50.02%	23.58%
Wdbc	21.57%	35.35%	12.88%	7.27%	8.82%	5.20%
Wine	30.73%	29.40%	25.43%	31.41%	31.47%	5.63%
Z_F_S	29.28%	47.81%	38.41%	13.16%	23.37%	3.90%
ZO_NF_S	6.43%	47.43%	43.75%	9.02%	22.18%	3.99%
ZONF_S	27.27%	11.99%	5.44%	4.03%	17.41%	1.67%
ZOO	15.47%	14.13%	20.27%	21.93%	33.50%	9.33%
AVERAGE	**26.56%**	**32.36%**	**30.11%**	**28.84%**	**33.35%**	**18.73%**

Table 6. Experimental results for the regression datasets. The first column is the name of the regression dataset. Also, the numbers in cells denote average regression error on the test set.

DATASET	RPROP	ADAM	NEAT	RBF-KMEANS	GENRBF	PROPOSED
ABALONE	4.55	4.30	9.88	7.37	9.98	5.16
AIRFOIL	0.002	0.005	0.067	0.27	0.121	0.004
BASEBALL	92.05	77.90	100.39	93.02	98.91	81.26
BK	1.60	0.03	0.15	0.02	0.023	0.025
BL	4.38	0.28	0.05	0.013	0.005	0.0004
CONCRETE	0.009	0.078	0.081	0.011	0.015	0.006
DEE	0.608	0.630	1.512	0.17	0.25	0.16
DIABETES	1.11	3.03	4.25	0.49	2.92	1.74
HOUSING	74.38	80.20	56.49	57.68	95.69	21.11
FA	0.14	0.11	0.19	0.015	0.15	0.033
MB	0.55	0.06	0.061	2.16	0.41	0.19
MORTGAGE	9.19	9.24	14.11	1.45	1.92	0.014
NT	0.04	0.12	0.33	8.14	0.02	0.007
PY	0.039	0.09	0.075	0.012	0.029	0.019
QUAKE	0.041	0.06	0.298	0.07	0.79	0.034
TREASURY	10.88	11.16	15.52	2.02	1.89	0.098
WANKARA	0.0003	0.02	0.005	0.001	0.002	0.003
AVERAGE	**11.71**	**11.02**	**11.97**	**10.17**	**12.54**	**6.46**

On average, the proposed technique appears to be 30-40% more accurate than the immediate best. In many cases, this percentage exceeds 70%. Moreover, in the vast majority

of problems, the proposed technique significantly outperforms the next best available method in terms of test error. In order to validate the results, an additional experiment was executed on the classification datasets, where the number of nodes increases from 5 to 20 and the results are graphically outlined in Figure 4. From this experiment, one can draw two conclusions: firstly, the proposed technique has a significant advantage over the others to a large extent in terms of average classification error, and secondly, the proposed method is shown to be robust and not significantly dependent on the increase of processing nodes , since 5–10 processing nodes are enough to achieve low classification errors.

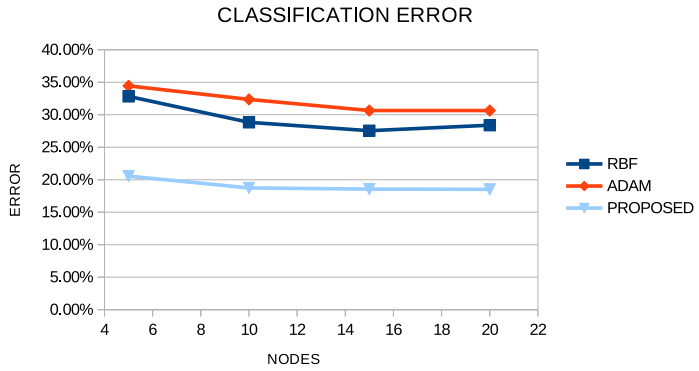

Figure 4. Average classification error for all classification datasets. The number of nodes increases from 5 to 20 and three models were used: the ADAM optimizer to optimize a neural network, the original RBF training method of two phases and the proposed method.

However, the proposed technique consists of two stages and in each of them a genetic algorithm should be executed. This means that it is significantly slower in computing time compared to the rest of the techniques and, of course, it needs more computing resources. This is graphically shown in Figure 5, where the average execution time for the method ADAM and the proposed method is shown for the classification datasets, when the number of processing nodes increases from 5 to 20. As expected, the proposed technique requires significantly more time than a traditional neural network training technique such as ADAM, since it consists of two sequential genetic algorithms.

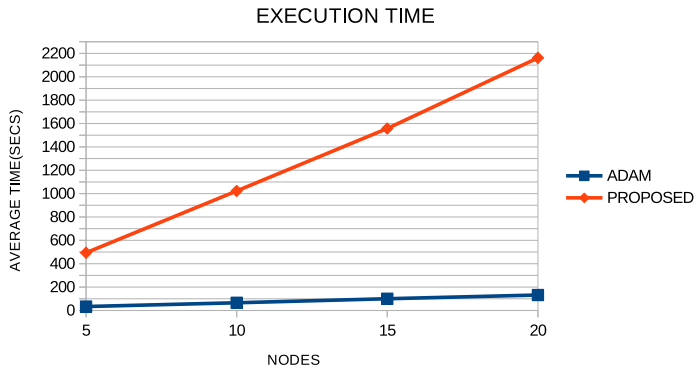

Figure 5. Average execution time for the ADAM method used to train a neural network and the proposed technique.

Of course, since we are talking about Genetic Algorithms, the training time required could be significantly reduced by using parallel techniques that take advantage of modern parallel computing structures such as the MPI interface [118] or the OpenMP library [119]. The superiority of the proposed technique is also reinforced by the statistical tests carried out on the experimental results and presented in figure 6.

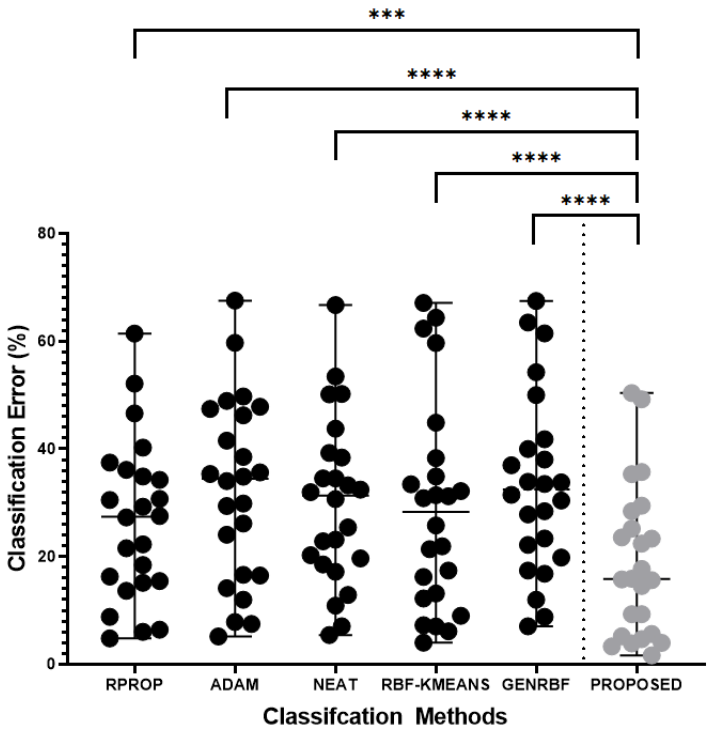

Figure 6. Scatter plot representation and the two-sample paired (Wilcoxon) signed-rank test results of the comparison for each of the five (5) classification methods (RPROP, ADAM, NEAT, RBF-KMEANS, and GENRBF) with the PROPOSED method regarding the classification error in twenty-four (24) different public available classification datasets. The stars only intend to flag significance levels for the two most used groups. A p-value of less than 0.001 is flagged with three stars (**). A p-value of less than 0.0001 is flagged with four stars (***).

In addition, an additional set of experiments was performed on the classification data in which the critical parameter F took the values 3, 5 and 10. The aim of this set of experiments was to establish the sensitivity of the proposed technique to changes in its parameters. The experimental results are presented in the table 7 and a statistical test on the results is presented in figure 7. The results and the statistics test indicate that there is no significant difference in the efficiency of the method for different values of the critical parameter F.

Table 7. Experimental results with the proposed method and using different values for the parameter F on the classification datasets.

DATASET	$F = 3$	$F = 5$	$F = 10$
Appendicitis	15.57%	16.60%	15.77%
Australian	24.29%	23.94%	22.40%
Balance	17.22%	15.39%	15.62%
Cleveland	52.09%	51.65%	50.37%
Dermatology	37.23%	36.81%	35.73%
Hayes Roth	35.72%	32.31%	35.33%
Heart	16.32%	15.54%	15.91%
HouseVotes	4.35%	3.90%	3.33%
Ionosphere	12.50%	11.44%	9.30%
Liverdisorder	28.08%	28.19%	28.44%
Mammographic	17.49%	17.15%	17.72%
Parkinsons	16.25%	15.17%	14.53%
Pima	23.29%	23.97%	23.33%
Popfailures	5.31%	5.86%	4.68%
Regions2	25.97%	26.29%	25.18%
Saheart	28.52%	28.59%	29.46%
Segment	44.95%	48.77%	49.22%
Spiral	15.49%	18.19%	23.58%
Wdbc	5.43%	5.01%	5.20%
Wine	7.59%	8.39%	5.63%
Z_F_S	4.37%	4.26%	3.90%
ZO_NF_S	3.79%	4.21%	3.99%
ZONF_S	2.34%	2.26%	1.67%
ZOO	11.90%	10.50%	9.33%
AVERAGE	**19.03%**	**18.93%**	**18.73%**

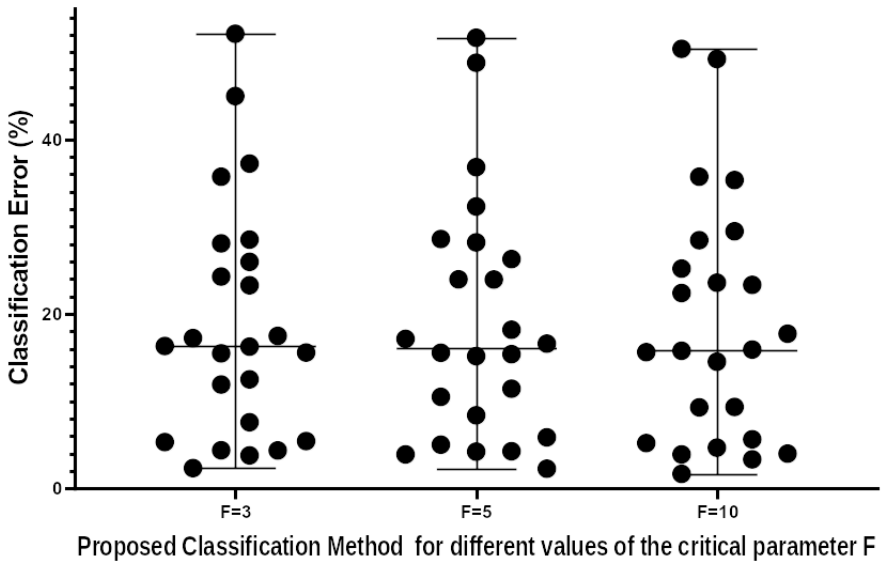

Figure 7. A Friedman test was conducted to determine whether different values of the critical parameter F had a difference or not in the classification error of the proposed method in twenty-four (24) other publicly available classification datasets. The analysis results for three different values of the critical parameter F (F=3, F=5, F=10) indicated no significant difference.

4. Conclusions

In the current work, an innovative two-stage technique was proposed for efficient training of RBF artificial neural networks. In the first stage of the application, using Grammatical Evolution, the interval of values of the neural network parameters is partitioned, so as to find a promising range that may contain low values of the training error. In the second stage, the neural network parameters are trained within the best range of values found in the first stage. The training of the parameters of the second phase is carried out using a Genetic Algorithm. The proposed method was applied on a wide series of well -known datasets from the relevant literature and was tested against a series of machine learning models. The new training technique was compared with the traditional method of training RBF networks but also with other training techniques of machine learning models and from the experimental results its superiority is evident in percentages that exceed 40%. However, since the proposed technique consists of two genetic algorithms executed sequentially, the execution time required is longer compared to other techniques especially for datasets with many patterns. An immediate solution to reduce the execution time of the method would be the use of parallel computing techniques, since genetic algorithms can by nature be directly parallelized.

Future improvements to the proposed method may include:

1. Application of the proposed method to other types of artificial neural networks.
2. Use of intelligent learning techniques in place of the K-Means technique to initialize the neural network parameters.
3. Using techniques to dynamically determine the number of necessary parameters of the neural network. For the time being, the number of parameters is considered constant, but this has the consequence of observing over-training phenomena in various data sets.
4. Implementation of crossover and mutation techniques that focus more on the existing interval construction technique for the model parameters.
5. Use of efficient termination techniques for Genetic Algorithms, for the most efficient termination of techniques without wasting computing time on unnecessary iterations.
6. Incorporation of parallel programming techniques to speed up the method.

Author Contributions: I.G.T., A.T. and E.K. conceived the idea and methodology and supervised the technical part regarding the software. I.G.T. executed the experiments, employing several datasets. A.T. performed the statistical analysis and all authors prepared the manuscript. All authors have read and agreed to the published version of the manuscript.

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