9 Tests of Hypotheses for a Single Sample

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LEARNING OBJECTIVES

After careful study of this chapter, you should be able to do the following:

1. Structure engineering decision-making problems as hypothesis tests
2. Test hypotheses on the mean of a normal distribution using either a *Z*-test or a *t*-test procedure
3. Test hypotheses on the variance or standard deviation of a normal distribution
4. Test hypotheses on a population proportion
5. Use the *P*-value approach for making decisions in hypotheses tests
6. Compute power, type II error probability, and make sample size selection decisions for tests on means, variances, and proportions
7. Explain and use the relationship between confidence intervals and hypothesis tests
8. Use the chi-square goodness of fit test to check distributional assumptions
9. Use contingency table tests

9-1 Hypothesis Testing

9-1.1 Statistical Hypotheses

Definition

A **statistical hypothesis** is a statement about the parameters of one or more populations.

9-1 Hypothesis Testing

9-1.1 Statistical Hypotheses

For example, suppose that we are interested in the burning rate of a solid propellant used to power aircrew escape systems.

- Now burning rate is a random variable that can be described by a probability distribution.
- Suppose that our interest focuses on the **mean** burning rate (a parameter of this distribution).
- Specifically, we are interested in deciding whether or not the mean burning rate is 50 centimeters per second.

9-1 Hypothesis Testing

9-1.1 Statistical Hypotheses

Two-sided Alternative Hypothesis

H_0: $\mu = 50$ centimeters per second   null hypothesis

H_1: $\mu \neq 50$ centimeters per second   alternative hypothesis

One-sided Alternative Hypotheses

H_0: $\mu = 50$ centimeters per second

H_1: $\mu < 50$ centimeters per second

or

H_0: $\mu = 50$ centimeters per second

H_1: $\mu > 50$ centimeters per second

9-1 Hypothesis Testing

9-1.1 Statistical Hypotheses

Test of a Hypothesis

• A procedure leading to a decision about a particular hypothesis

• Hypothesis-testing procedures rely on using the information in a **random sample from the population of interest**.

• If this information is *consistent* with the hypothesis, then we will conclude that the hypothesis is **true**; if this information is *inconsistent* with the hypothesis, we will conclude that the hypothesis is **false**.

9-1 Hypothesis Testing

9-1.2 Tests of Statistical Hypotheses

$$H_0\colon \mu = 50 \text{ centimeters per second}$$
$$H_1\colon \mu \neq 50 \text{ centimeters per second}$$

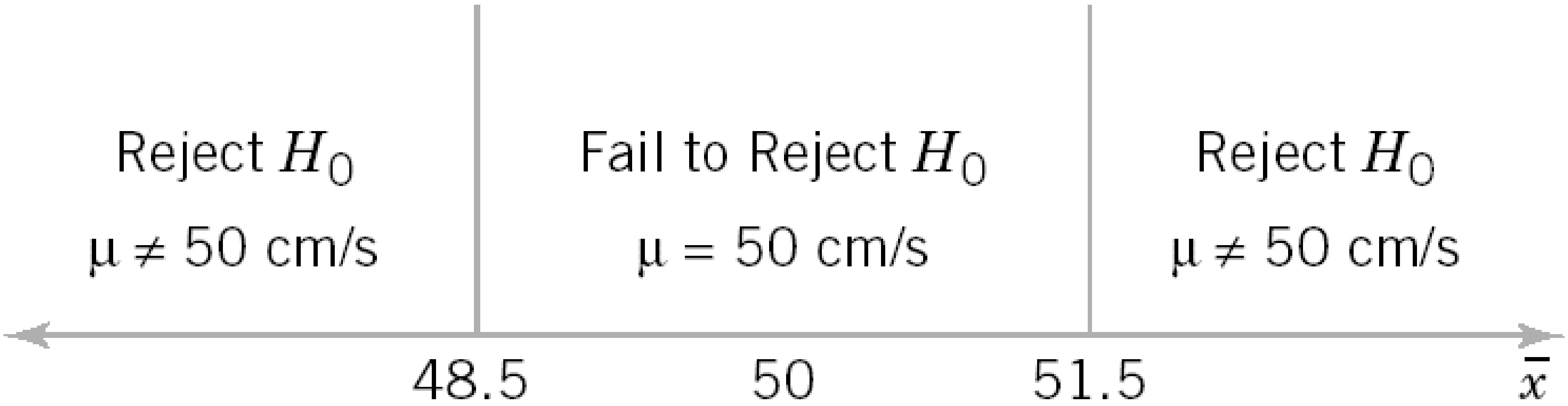


Figure 9-1 Decision criteria for testing $H_0{:}\mu = 50$ centimeters per second versus $H_1{:}\mu \neq 50$ centimeters per second.

9-1 Hypothesis Testing

9-1.2 Tests of Statistical Hypotheses

Definitions

Rejecting the null hypothesis H_0 when it is true is defined as a **type I error.**

Failing to reject the null hypothesis when it is false is defined as a **type II error.**

9-1 Hypothesis Testing

9-1.2 Tests of Statistical Hypotheses

Table 9-1 Decisions in Hypothesis Testing

Decision	H_0 Is True	H_0 Is False
Fail to reject H_0	no error	type II error
Reject H_0	type I error	no error

$$\alpha = P(\text{type I error}) = P(\text{reject } H_0 \text{ when } H_0 \text{ is true})$$

Sometimes the type I error probability is called the **significance level**, or the **α-error**, or the **size** of the test.

9-1 Hypothesis Testing

9-1.2 Tests of Statistical Hypotheses

$$\alpha = P(\overline{X} < 48.5 \text{ when } \mu = 50) + P(\overline{X} > 51.5 \text{ when } \mu = 50)$$

The z-values that correspond to the critical values 48.5 and 51.5 are

$$z_1 = \frac{48.5 - 50}{0.79} = -1.90 \quad \text{and} \quad z_2 = \frac{51.5 - 50}{0.79} = 1.90$$

Therefore

$$\alpha = P(Z < -1.90) + P(Z > 1.90) = 0.028717 + 0.028717 = 0.057434$$

9-1 Hypothesis Testing

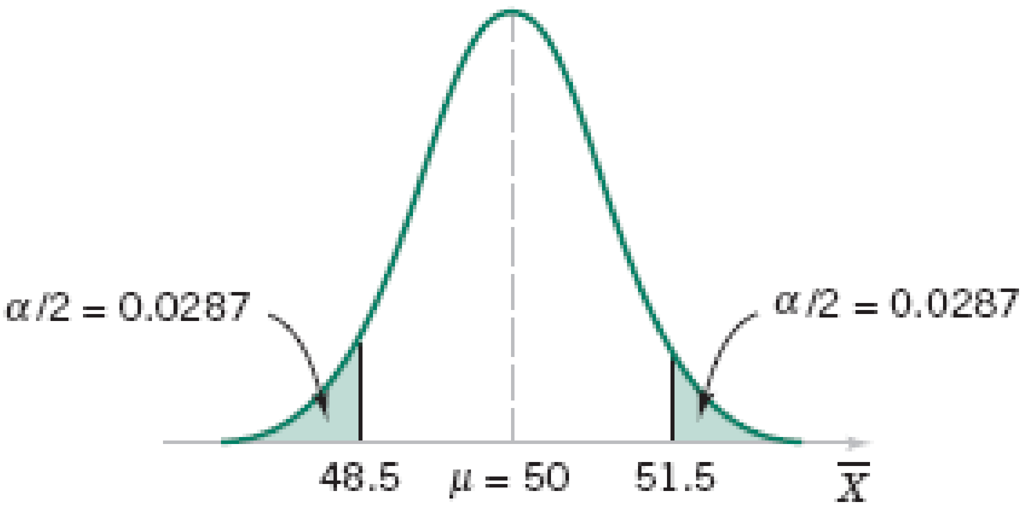


Figure 9-2 The critical region for H_0: $\mu = 50$ versus H_1: $\mu \neq 50$ and $n = 10$.

$$\alpha = P(\text{type I error}) = P(\text{reject } H_0 \text{ when } H_0 \text{ is true}) \qquad (9\text{-}3)$$

9-1 Hypothesis Testing

$$\beta = P(\text{type II error}) = P(\text{fail to reject } H_0 \text{ when } H_0 \text{ is false}) \tag{9-4}$$

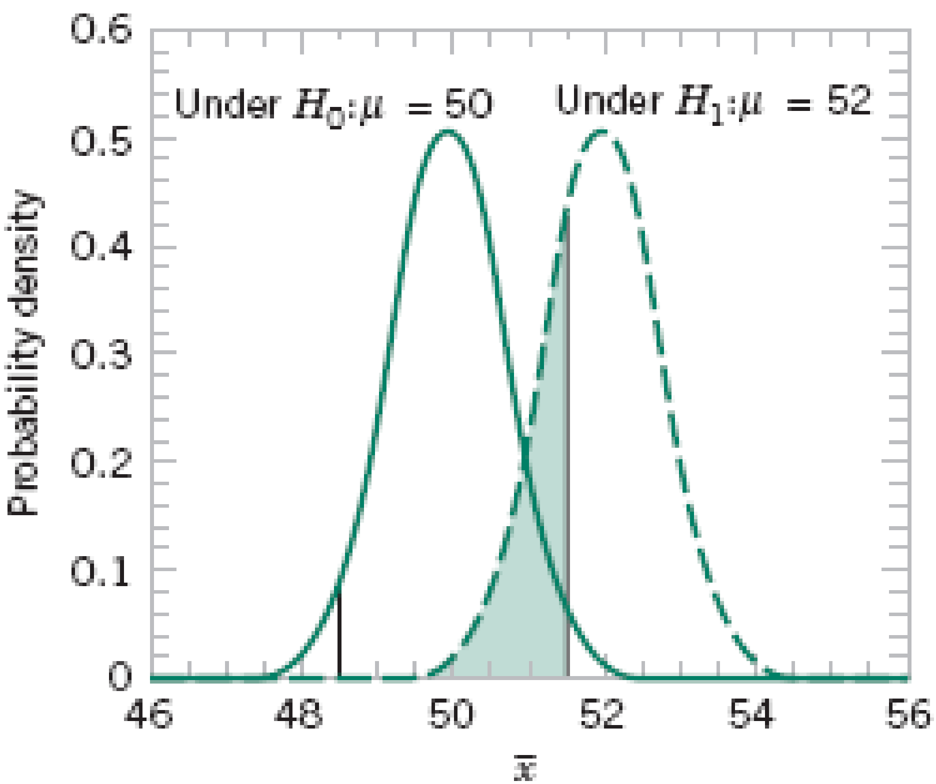


Figure 9-3 The probability of type II error when $\mu = 52$ and $n = 10$.

9-1 Hypothesis Testing

Definition

The **power** of a statistical test is the probability of rejecting the null hypothesis H_0 when the alternative hypothesis is true.

- The power is computed as 1 - β, and power can be interpreted as *the probability of correctly rejecting a false null hypothesis.* We often compare statistical tests by comparing their power properties.

- For example, consider the propellant burning rate problem when we are testing $H_0 : \mu = 50$ centimeters per second against $H_1 : \mu$ not equal 50 centimeters per second . Suppose that the true value of the mean is $\mu = 52$. When $n = 10$, we found that $\beta = 0.2643$, so the power of this test is $1 - \beta = 1 - 0.2643 = 0.7357$ when $\mu = 52$.

9-1 Hypothesis Testing

9-1.3 One-Sided and Two-Sided Hypotheses

Two-Sided Test:

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0$$

One-Sided Tests:

$$H_0: \mu = \mu_0 \qquad H_1: \mu > \mu_0$$

or

$$H_0: \mu = \mu_0 \qquad H_1: \mu < \mu_0$$

9-1 Hypothesis Testing

Example 9-1

Consider the propellant burning rate problem. Suppose that if the burning rate is less than 50 centimeters per second, we wish to show this with a strong conclusion. The hypotheses should be stated as

$$H_0\colon \mu = 50 \text{ centimeters per second}$$
$$H_1\colon \mu < 50 \text{ centimeters per second}$$

Here the critical region lies in the lower tail of the distribution of $\overline{X}$. Since the rejection of H_0 is always a strong conclusion, this statement of the hypotheses will produce the desired outcome if H_0 is rejected. Notice that, although the null hypothesis is stated with an equal sign, it is understood to include any value of μ not specified by the alternative hypothesis. Therefore, failing to reject H_0 does not mean that $\mu = 50$ centimeters per second exactly, but only that we do not have strong evidence in support of H_1.

9-1 Hypothesis Testing

9-1.4 P-Values in Hypothesis Tests

Definition

The ***P*-value** is the smallest level of significance that would lead to rejection of the null hypothesis H_0 with the given data.

9-1 Hypothesis Testing

9-1.4 P-Values in Hypothesis Tests

Consider the two-sided hypothesis test for burning rate

$$H_0: \mu = 50 \quad H_1: \mu \neq 50$$

with $n = 16$ and $\sigma = 2.5$. Suppose that the observed sample mean is $\bar{x} = 51.3$ centimeters per second. Figure 9-6 shows a critical region for this test with critical values at 51.3 and the symmetric value 48.7. The P-value of the test is the α associated with this critical region. Any smaller value for α expands the critical region and the test fails to reject the null hypothesis when $\bar{x} = 51.3$. The P-value is easy to compute after the test statistic is observed. In this example

$$\begin{aligned} P\text{-value} &= 1 - P(48.7 < \bar{X} < 51.3) \\ &= 1 - P\left(\frac{48.7 - 50}{2.5/\sqrt{16}} < Z < \frac{51.3 - 50}{2.5/\sqrt{16}}\right) \\ &= 1 - P(-2.08 < Z < 2.08) \\ &= 1 - 0.962 = 0.038 \end{aligned}$$

9-1 Hypothesis Testing

9-1.4 P-Values in Hypothesis Tests

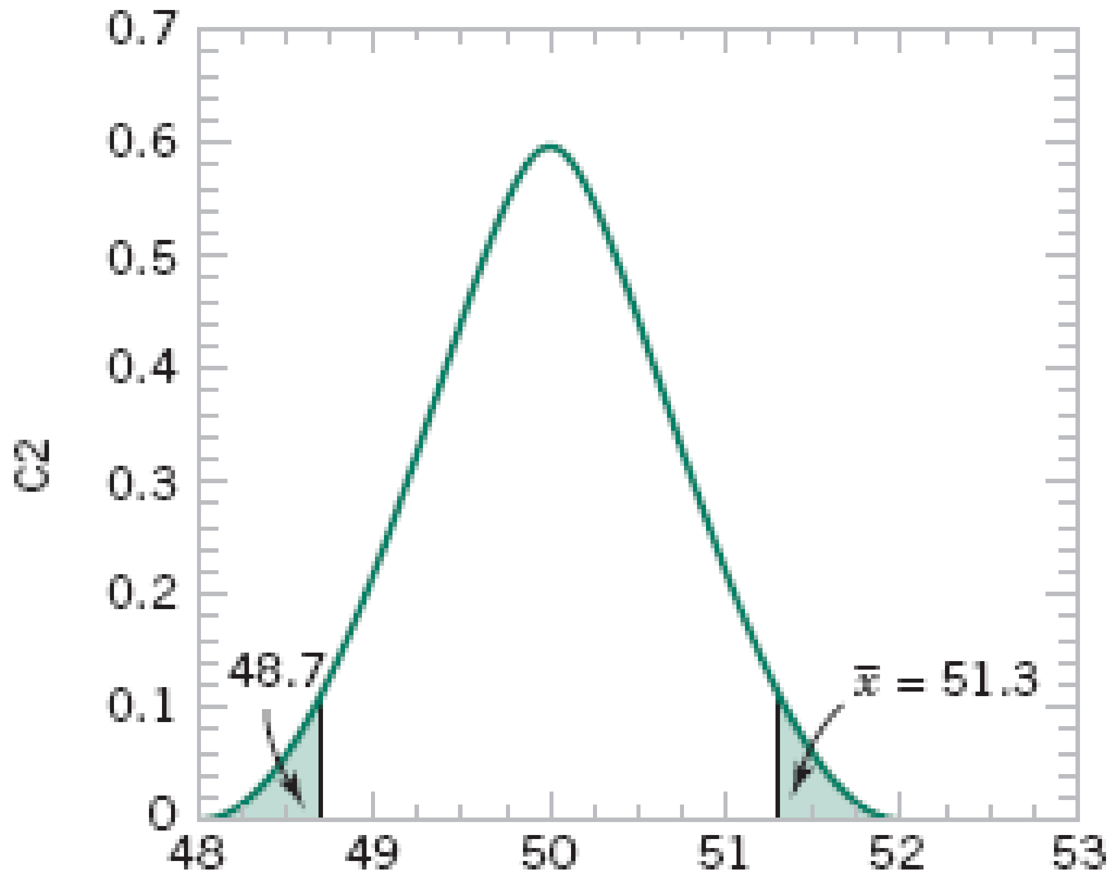


Figure 9-6 *P*-value is area of shaded region when $\bar{x} = 51.3$.

9-1 Hypothesis Testing

9-1.5 Connection between Hypothesis Tests and Confidence Intervals

There is a close relationship between the test of a hypothesis about any parameter, say θ, and the confidence interval for θ. If $[l, u]$ is a $100(1-\alpha)\%$ confidence interval for the parameter θ, the test of size α of the hypothesis

$$H_0: \theta = \theta_0$$
$$H_1: \theta \neq \theta_0$$

will lead to rejection of H_0 if and only if θ_0 is **not** in the $100(1-\alpha)\%$ CI $[l, u]$. As an illustration, consider the escape system propellant problem with $\bar{x} = 51.3$, $\sigma = 2.5$, and $n = 16$. The null hypothesis H_0: $\mu = 50$ was rejected, using $\alpha = 0.05$. The 95% two-sided CI on μ can be calculated using Equation 8-7. This CI is $51.3 \pm 1.96(2.5/\sqrt{16})$ and this is $50.075 \leq \mu \leq 52.525$. Because the value $\mu_0 = 50$ is not included in this interval, the null hypothesis H_0: $\mu = 50$ is rejected.

9-1 Hypothesis Testing

9-1.6 General Procedure for Hypothesis Tests

1. From the problem context, identify the parameter of interest.
2. State the null hypothesis, H_0 .
3. Specify an appropriate alternative hypothesis, H_1.
4. Choose a significance level, α.
5. Determine an appropriate test statistic.
6. State the rejection region for the statistic.
7. Compute any necessary sample quantities, substitute these into the equation for the test statistic, and compute that value.
8. Decide whether or not H_0 should be rejected and report that in the problem context.

9-2 Tests on the Mean of a Normal Distribution, Variance Known

9-2.1 Hypothesis Tests on the Mean

We wish to test:

$$H_0: \mu = \mu_0$$
$$H_1: \mu \neq \mu_0$$

The **test statistic** is:

$$Z_0 = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \qquad (9\text{-}8)$$

9-2 Tests on the Mean of a Normal Distribution, Variance Known

9-2.1 Hypothesis Tests on the Mean

Reject H_0 if the observed value of the test statistic z_0 is either:

$$z_0 > z_{\alpha/2} \text{ or } z_0 < -z_{\alpha/2}$$

Fail to reject H_0 if

$$-z_{\alpha/2} < z_0 < z_{\alpha/2}$$

9-2 Tests on the Mean of a Normal Distribution, Variance Known

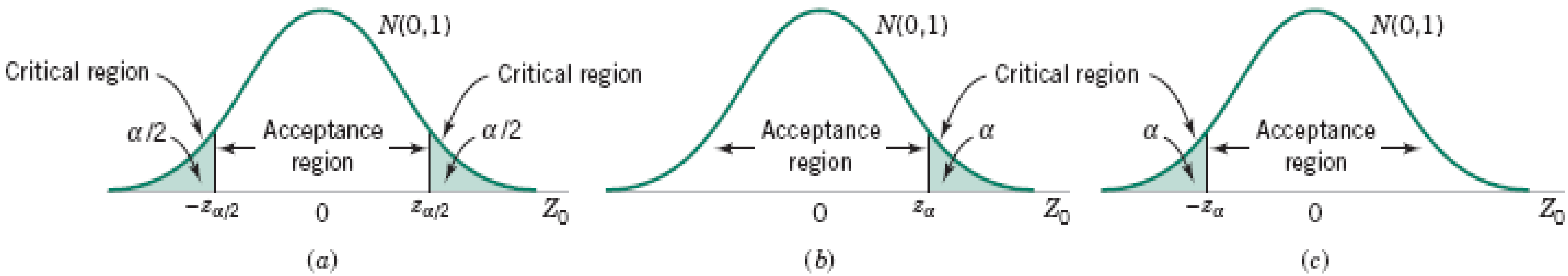


Figure 9-7 The distribution of Z_0 when H_0: $\mu = \mu_0$ is true, with critical region for (a) the two-sided alternative $H_1 : \mu \neq \mu_0$, (b) the one-sided alternative $H_1 : \mu > \mu_0$, and (c) the one-sided alternative $H_1 : \mu < \mu_0$.

9-2 Tests on the Mean of a Normal Distribution, Variance Known

Example 9-2

Aircrew escape systems are powered by a solid propellant. The burning rate of this propellant is an important product characteristic. Specifications require that the mean burning rate must be 50 centimeters per second. We know that the standard deviation of burning rate is $\sigma = 2$ centimeters per second. The experimenter decides to specify a type I error probability or significance level of $\alpha = 0.05$ and selects a random sample of $n = 25$ and obtains a sample average burning rate of $\bar{x} = 51.3$ centimeters per second. What conclusions should be drawn?

9-2 Tests on the Mean of a Normal Distribution, Variance Known

Example 9-2

We may solve this problem by following the eight-step procedure outlined in Section 9-1.4. This results in

1. The parameter of interest is μ, the mean burning rate.
2. H_0: $\mu = 50$ centimeters per second
3. H_1: $\mu \neq 50$ centimeters per second
4. $\alpha = 0.05$
5. The test statistic is

$$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

9-2 Tests on the Mean of a Normal Distribution, Variance Known

Example 9-2

6. Reject H_0 if $z_0 > 1.96$ or if $z_0 < -1.96$. Note that this results from step 4, where we specified $\alpha = 0.05$, and so the boundaries of the critical region are at $z_{0.025} = 1.96$ and $-z_{0.025} = -1.96$.
7. Computations: Since $\overline{x} = 51.3$ and $\sigma = 2$,

$$z_0 = \frac{51.3 - 50}{2/\sqrt{25}} = 3.25$$

8. Conclusion: Since $z_0 = 3.25 > 1.96$, we reject H_0: $\mu = 50$ at the 0.05 level of significance. Stated more completely, we conclude that the mean burning rate differs from 50 centimeters per second, based on a sample of 25 measurements. In fact, there is strong evidence that the mean burning rate exceeds 50 centimeters per second.

9-2 Tests on the Mean of a Normal Distribution, Variance Known

9-2.1 Hypothesis Tests on the Mean

We may also develop procedures for testing hypotheses on the mean μ where the alternative hypothesis is one-sided. Suppose that we specify the hypotheses as

$$H_0\colon \mu = \mu_0 \qquad (9\text{-}11)$$
$$H_1\colon \mu > \mu_0$$

In defining the critical region for this test, we observe that a negative value of the test statistic Z_0 would never lead us to conclude that $H_0\colon \mu = \mu_0$ is false. Therefore, we would place the critical region in the **upper tail** of the standard normal distribution and reject H_0 if the computed value of z_0 is too large. That is, we would reject H_0 if

$$z_0 > z_\alpha \qquad (9\text{-}12)$$

9-2 Tests on the Mean of a Normal Distribution, Variance Known

9-2.1 Hypothesis Tests on the Mean (Continued)

as shown in Figure 9-7(b). Similarly, to test

$$H_0\colon \mu = \mu_0$$
$$H_1\colon \mu < \mu_0 \tag{9-13}$$

we would calculate the test statistic Z_0 and reject H_0 if the value of z_0 is too small. That is, the critical region is in the **lower tail** of the standard normal distribution as shown in Figure 9-7(c), and we reject H_0 if

$$z_0 < -z_\alpha \tag{9-14}$$

9-2 Tests on the Mean of a Normal Distribution, Variance Known

9-2.1 Hypothesis Tests on the Mean (Continued)

Null hypothesis: $H_0\colon \mu = \mu_0$

Test statistic: $$Z_0 = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}}$$

Alternative hypothesis	Rejection criteria
$H_1\colon \mu \neq \mu_0$	$z_0 > z_{\alpha/2,n-1}$ or $z_0 < -z_{\alpha/2,n-1}$
$H_1\colon \mu > \mu_0$	$z_0 > z_{\alpha,n-1}$
$H_1\colon \mu < \mu_0$	$z_0 < -z_{\alpha,n-1}$

9-2 Tests on the Mean of a Normal Distribution, Variance Known

P-Values in Hypothesis Tests

The ***P*-value** is the smallest level of significance that would lead to rejection of the null hypothesis H_0 with the given data.

$$P = \begin{cases} 2[1 - \Phi(|z_0|)] & \text{for a two-tailed test: } H_0\text{: } \mu = \mu_0 \quad H_1\text{: } \mu \neq \mu_0 \\ 1 - \Phi(z_0) & \text{for a upper-tailed test: } H_0\text{: } \mu = \mu_0 \quad H_1\text{: } \mu > \mu_0 \\ \Phi(z_0) & \text{for a lower-tailed test: } H_0\text{: } \mu = \mu_0 \quad H_1\text{: } \mu < \mu_0 \end{cases} \qquad (9\text{-}15)$$

9-3 Tests on the Mean of a Normal Distribution, Variance Unknown

9-3.1 Hypothesis Tests on the Mean

One-Sample *t*-Test

Null hypothesis: $H_0\colon \mu = \mu_0$

Test statistic: $$T_0 = \frac{\overline{X} - \mu_0}{S/\sqrt{n}}$$

Alternative hypothesis	Rejection criteria
$H_1\colon \mu \neq \mu_0$	$t_0 > t_{\alpha/2,n-1}$ or $t_0 < -t_{\alpha/2,n-1}$
$H_1\colon \mu > \mu_0$	$t_0 > t_{\alpha,n-1}$
$H_1\colon \mu < \mu_0$	$t_0 < -t_{\alpha,n-1}$

9-3 Tests on the Mean of a Normal Distribution, Variance Unknown

9-3.1 Hypothesis Tests on the Mean

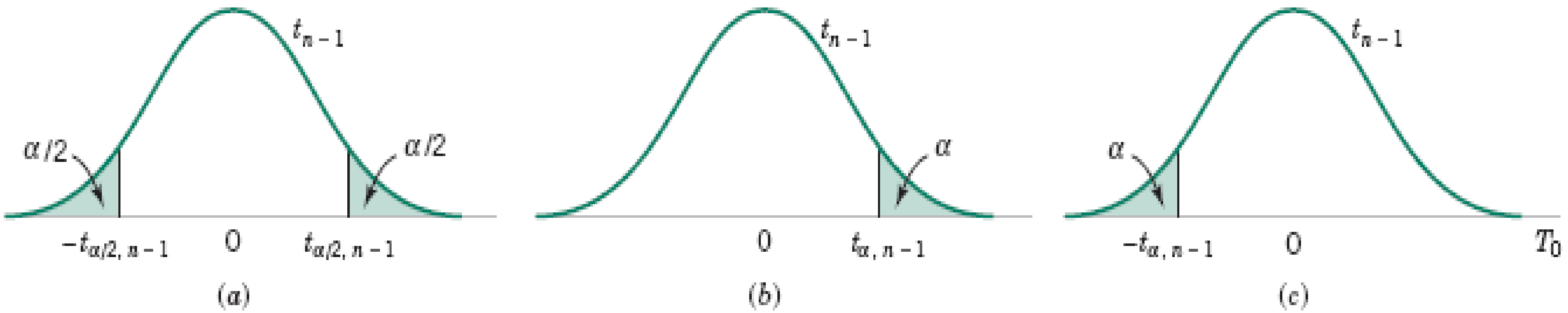


Figure 9-9 The reference distribution for H_0: $\mu = \mu_0$ with critical region for (a) H_1: $\mu \neq \mu_0$, (b) H_1: $\mu > \mu_0$, and (c) H_1: $\mu < \mu_0$.

9-3 Tests on the Mean of a Normal Distribution, Variance Unknown

Example 9-6

The increased availability of light materials with high strength has revolutionized the design and manufacture of golf clubs, particularly drivers. Clubs with hollow heads and very thin faces can result in much longer tee shots, especially for players of modest skills. This is due partly to the "spring-like effect" that the thin face imparts to the ball. Firing a golf ball at the head of the club and measuring the ratio of the outgoing velocity of the ball to the incoming velocity can quantify this spring-like effect. The ratio of velocities is called the coefficient of restitution of the club. An experiment was performed in which 15 drivers produced by a particular club maker were selected at random and their coefficients of restitution measured. In the experiment the golf balls were fired from an air cannon so that the incoming velocity and spin rate of the ball could be precisely controlled. It is of interest to determine if there is evidence (with $\alpha = 0.05$) to support a claim that the mean coefficient of restitution exceeds 0.82. The observations follow:

0.8411	0.8191	0.8182	0.8125	0.8750
0.8580	0.8532	0.8483	0.8276	0.7983
0.8042	0.8730	0.8282	0.8359	0.8660

9-3 Tests on the Mean of a Normal Distribution, Variance Unknown

Example 9-6

The sample mean and sample standard deviation are $\bar{x} = 0.83725$ and $s = 0.02456$. The normal probability plot of the data in Fig. 9-9 supports the assumption that the coefficient of restitution is normally distributed. Since the objective of the experimenter is to demonstrate that the mean coefficient of restitution exceeds 0.82, a one-sided alternative hypothesis is appropriate.

The solution using the eight-step procedure for hypothesis testing is as follows:

1. The parameter of interest is the mean coefficient of restitution, μ.
2. H_0: $\mu = 0.82$
3. H_1: $\mu > 0.82$. We want to reject H_0 if the mean coefficient of restitution exceeds 0.82.
4. $\alpha = 0.05$
5. The test statistic is

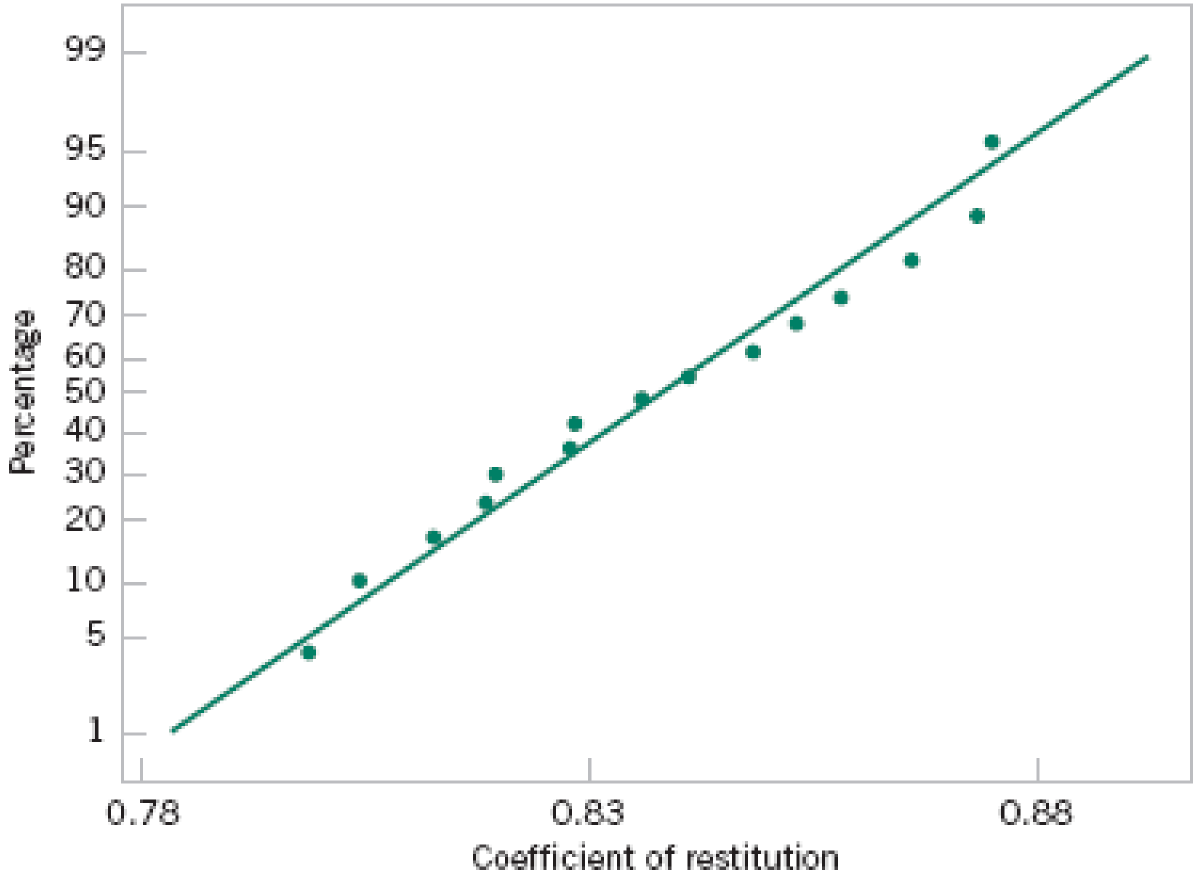
$$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

6. Reject H_0 if $t_0 > t_{0.05,14} = 1.761$

9-3 Tests on the Mean of a Normal Distribution, Variance Unknown

Example 9-6

Figure 9-10 Normal probability plot of the coefficient of restitution data from Example 9-6.



9-4 Hypothesis Tests on the Variance and Standard Deviation of a Normal Distribution

9-4.1 Hypothesis Test on the Variance

Suppose that we wish to test the hypothesis that the variance of a normal population σ^2 equals a specified value, say σ_0^2, or equivalently, that the standard deviation σ is equal to σ_0. Let $X_1, X_2, \ldots, X_n$ be a random sample of n observations from this population. To test

$$H_0: \sigma^2 = \sigma_0^2$$
$$H_1: \sigma^2 \neq \sigma_0^2 \tag{9-26}$$

we will use the test statistic:

$$X_0^2 = \frac{(n-1)S^2}{\sigma_0^2} \tag{9-27}$$

9-4 Hypothesis Tests on the Variance and Standard Deviation of a Normal Distribution

9-4.1 Hypothesis Test on the Variance

If the null hypothesis $H_0: \sigma^2 = \sigma_0^2$ is true, the test statistic X_0^2 defined in Equation 9-27 follows the chi-square distribution with $n - 1$ degrees of freedom. This is the reference distribution for this test procedure. Therefore, we calculate χ_0^2, the value of the test statistic X_0^2, and the null hypothesis $H_0: \sigma^2 = \sigma_0^2$ would be rejected if

$$\chi_0^2 > \chi^2_{\alpha/2, n-1} \qquad \text{or if} \quad \chi_0^2 < \chi^2_{1-\alpha/2, n-1}$$

where $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-\alpha/2,n-1}$ are the upper and lower $100\alpha/2$ percentage points of the chi-square distribution with $n - 1$ degrees of freedom, respectively. Figure 9-10(a) shows the critical region.

9-4 Hypothesis Tests on the Variance and Standard Deviation of a Normal Distribution

9-4.1 Hypothesis Test on the Variance

The same test statistic is used for one-sided alternative hypotheses. For the one-sided hypothesis

$$H_0\colon \sigma^2 = \sigma_0^2$$
$$H_1\colon \sigma^2 > \sigma_0^2 \tag{9-28}$$

we would reject H_0 if $\chi_0^2 > \chi_{\alpha,n-1}^2$, whereas for the other one-sided hypothesis

$$H_0\colon \sigma^2 = \sigma_0^2$$
$$H_1\colon \sigma^2 < \sigma_0^2 \tag{9-29}$$

we would reject H_0 if $\chi_0^2 < \chi_{1-\alpha,n-1}^2$. The one-sided critical regions are shown in Figure 9-10(b) and (c).

9-4 Hypothesis Tests on the Variance and Standard Deviation of a Normal Distribution

9-4.1 Hypothesis Test on the Variance

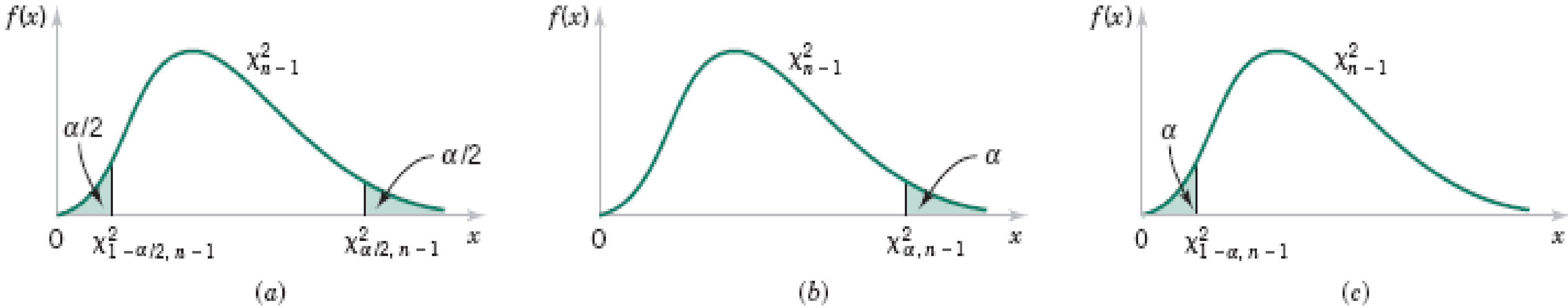


Figure 9-11 Reference distribution for the test of H_0: $\sigma^2 = \sigma_0^2$ with critical region values for (a) H_1: $\sigma^2 \neq \sigma_0^2$, (b) H_1: $\sigma^2 > \sigma_0^2$, and (c) H_1: $\sigma^2 < \sigma_0^2$.

9-4 Hypothesis Tests on the Variance and Standard Deviation of a Normal Distribution

Example 9-8

An automatic filling machine is used to fill bottles with liquid detergent. A random sample of 20 bottles results in a sample variance of fill volume of $s^2 = 0.0153$ (fluid ounces)^2. If the variance of fill volume exceeds 0.01 (fluid ounces)^2, an unacceptable proportion of bottles will be underfilled or overfilled. Is there evidence in the sample data to suggest that the manufacturer has a problem with underfilled or overfilled bottles? Use $\alpha = 0.05$, and assume that fill volume has a normal distribution.

Using the eight-step procedure results in the following:

1. The parameter of interest is the population variance σ^2.
2. H_0: $\sigma^2 = 0.01$
3. H_1: $\sigma^2 > 0.01$
4. $\alpha = 0.05$
5. The test statistic is

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

9-4 Hypothesis Tests on the Variance and Standard Deviation of a Normal Distribution

Example 9-8

6. Reject H_0 if $\chi_0^2 > \chi^2_{0.05,19} = 30.14$.

7. Computations:

$$\chi_0^2 = \frac{19(0.0153)}{0.01} = 29.07$$

8. Conclusions: Since $\chi_0^2 = 29.07 < \chi^2_{0.05,19} = 30.14$, we conclude that there is no strong evidence that the variance of fill volume exceeds 0.01 (fluid ounces)^2.

9-5 Tests on a Population Proportion

9-5.1 Large-Sample Tests on a Proportion

Many engineering decision problems include hypothesis testing about *p*.

$$H_0\colon p = p_0$$
$$H_1\colon p \neq p_0$$

An appropriate **test statistic** is

$$Z_0 = \frac{X - np_0}{\sqrt{np_0(1-p_0)}} \qquad (9\text{-}32)$$

and reject $H_0\colon p = p_0$ if

$$z_0 > z_{\alpha/2} \quad \text{or} \quad z_0 < -z_{\alpha/2}$$

Example 9-10

A semiconductor manufacturer produces controllers used in automobile engine applications. The customer requires that the process fallout or fraction defective at a critical manufacturing step not exceed 0.05 and that the manufacturer demonstrate process capability at this level of quality using $\alpha = 0.05$. The semiconductor manufacturer takes a random sample of 200 devices and finds that four of them are defective. Can the manufacturer demonstrate process capability for the customer?

We may solve this problem using the eight-step hypothesis-testing procedure as follows:

1. The parameter of interest is the process fraction defective p.
2. H_0: $p = 0.05$
3. H_1: $p < 0.05$
   This formulation of the problem will allow the manufacturer to make a strong claim about process capability if the null hypothesis H_0: $p = 0.05$ is rejected.
4. $\alpha = 0.05$

9-5 Tests on a Population Proportion

Example 9-10

5. The test statistic is (from Equation 9-32)

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1 - p_0)}}$$

where $x = 4$, $n = 200$, and $p_0 = 0.05$.

6. Reject H_0: $p = 0.05$ if $z_0 < -z_{0.05} = -1.645$

7. Computations: The test statistic is

$$z_0 = \frac{4 - 200(0.05)}{\sqrt{200(0.05)(0.95)}} = -1.95$$

8. Conclusions: Since $z_0 = -1.95 < -z_{0.05} = -1.645$, we reject H_0 and conclude that the process fraction defective p is less than 0.05. The P-value for this value of the test statistic z_0 is $P = 0.0256$, which is less than $\alpha = 0.05$. We conclude that the process is capable.

9-5 Tests on a Population Proportion

Another form of the test statistic Z_0 is

$$Z_0 = \frac{X/n - p_0}{\sqrt{p_0(1 - p_0)/n}} \quad \text{or} \quad Z_0 = \frac{\hat{P} - p_0}{\sqrt{p_0(1 - p_0)/n}}$$

IMPORTANT TERMS AND CONCEPTS

α and β
Connection between hypothesis tests and confidence intervals
Critical region for a test statistic
Hypothesis test
Inference
Null distribution
Null hypothesis
One- and two-sided alternative hypotheses
Operating characteristic (OC) curves
Power of a test
P-value
Reference distribution for a test statistic
Sample size determination for hypothesis tests
Significance level of a test
Statistical hypotheses
Statistical versus practical significance
Test for goodness of fit
Test for homogeneity
Test for independence
Test statistic
Type I and type II errors