Pipelining Techniques for IIR Digital Filters

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ABSTRACT

FIR digital filter sampling rates can be increased dramatically through the use of "pipelining". However, IIR digital filters are generally not amenable to pipelining due to the inherent pipelining delay. Recently several techniques have been introduced to allow pipelining of IIR filters, but these techniques all require pole-zero cancellation.

Pipelined IIR filters can be designed without the need for pole zero cancellation through the use of a constrained optimization program that restricts filter structure to inherent pipelined structures. Similar to linear-phase FIR filter design procedures, these procedures result in optimum pipelined IIR filters which may be operated at the maximum speed of the hardware used, yet they are competitive in complexity and performance to standard IIR digital filters.

Introduction

In 1984 H. H. Loomis and B. Sinha [1] introduced a new approach to IIR filter design based on modifying the filter algorithm in such a way that "pipelining" was possible. This approach results in dramatic improvements in sampling frequencies for IIR filters [2]. M. A. Soderstrand, K. Chopper, and B. Sinha named the previous pipelining technique "time-domain pipelining" and introduced two additional IIR pipelining schemes ("z-domain pipelining" and "frequency-sampling pipelining") at the 1984 Asilomar conference on Systems, Signals, and Computers in Pacific Grove, CA [3]. Parhi and Messerschmitt at University of California have recently examined the time-domain pipelining technique (they call it "clustered look-ahead pipelining") and the z-domain pipelining technique (they call it "scattered look ahead pipelining") and concluded that the z-domain technique offers advantages over the time-domain technique [4].

In this paper all three techniques are reviewed and compared. Particular emphasis will be placed on introducing the concept of IIR pipelining to those who are not aware of its uses and benefits. The main contribution of this paper, however, is the introduction of a new technique for pipelining IIR digital filters that does not suffer from the ills of pole-zero cancellation as the previous techniques do. This new pipelining technique is based on designing the filter using constrained optimization such that the resulting filter is inherently pipelined. These new filters will demonstrate specific filter functions that can be realized in VLSI with a large reduction in hardware over the other pipelining techniques while performing equally well.

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Problems with Pipelining IIR Digital Filters

Pipelining is accomplished by putting delays after each hardware operation so that partial results can be passed from device to device in such a fashion as to keep the devices operating in parallel. In FIR digital filters, the delay associated with pipelining is not a problem. However, in IIR filters, the output must be fed back to the input. Hence, a delay in the appearance of the output cannot be tolerated.

Time Domain Pipelining Technique

The time domain technique for pipelining digital filters [1,2] is based on modifying the difference equation of the desired filter to accommodate the pipelining. For example, consider a simple first-order digital filter:

$$Y(z) = X(z) + z^{-1}rY(z) \tag{1}$$

If we wish to pipeline the multiplier that produces the product $rY(z)$, we will have to increase the delay z^{-1} in order to accommodate the pipelining delay. However, if we multiply both sides of equation (1) by z^{-1} we obtain:

$$z^{-1}Y(z) = z^{-1}X(z) + z^{-2}rY(z) \tag{2}$$

substituting equation (2) for $z^{-1}Y(z)$ in equation (1) yields:

$$Y(z) = X(z) + z^{-1}rX(z) + z^{-2}rY(z) \tag{3}$$

This new equation has a delay of z^{-2} on the multiplier $rY(z)$, thus doubling the time available to pipeline the multiplier. Further substitutions can increase the delay on $rY(z)$ further, at the expense of adding more terms to the numerator $X(z)$. Since the $X(z)$ terms can be realized with a pipelined FIR filter, they do not suffer from pipelining difficulties. However, the original filter transfer function is still being realized, because the extra delay in the feedback path is introducing extra poles that are cancelled by the extra zeros in the $X(z)$ terms [1,2].

Z-Domain Pipelining Technique

The z-domain technique is based on a change of variables on a pre-distorted filter. For example, if we want a pole pair at angle θ and radius r, we start with the following equation:

$$Y(z) = X(z) + z^{-1}2r^{N}\cos(N\theta)Y(z) + z^{-2}r^{2N}Y(z) \tag{4}$$

To increase the delay in the feedback path from z^{-1} to z^{-N}, we substitute z^{-N} for z^{-1} to get:

$$Y(z) = X(z) + z^{-N}2r^{N}\cos(N\theta)Y(z) + z^{-2N}r^{2N}Y(z) \tag{5}$$

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This filter has the desired poles at radius r and angle θ, but it also has superfluous poles introduced by the change of variables. If an FIR filter is used to cancel the superfluous poles, then the correct response is given and the circuit has a delay of N in the feedback loop to accommodate the pipelining delay.

Frequency Sampling Pipelining Technique

The frequency sampling technique is based on designing a pole producing section of the form of one of the two equations below:

$$Y(z) = X(z) + z^{-N}r^{-N}Y(z) \tag{6a}$$

$$Y(z) = X(z) - z^{-N}r^{-N}Y(z) \tag{6b}$$

We select r for the radius of the poles desired and select N greater than the pipelining delay and at a value that places poles at or near the desired poles. Although pole placement is not accurate, this seems to yield reasonable filters that can be pipelined with less hardware than the previous two techniques. However, like the previous techniques, it is usually necessary to cancel some of the extra poles in order to get the desired response. These poles are cancelled, as before, with a pipelined FIR filter.

The New Pipelining Technique

The new design technique is aimed at designing a better FIR filter than the pole cancelling filters of the previous techniques. The new technique can be used with any of the previous techniques, but we shall consider it with the z-domain and frequency sampling techniques.

The starting place for the new design is either the z-domain or frequency sampling pole producing network. With the z-domain network, poles can be placed anywhere in the complex frequency space. With the frequency sampling technique, we are limited to equally spaced pole positions around a circle of radius r for each term we choose in the denominator.

Instead of cancelling the unwanted poles, however, we will design a zero producing network with zeros constrained to the unit circle (this allows us to realize them with only one multiplier) and optimized to give the best filter performance. In this paper, we shall keep r, θ, and N at the same values and only adjust the zeros, however, in future work we plan to allow the optimization program to adjust r and θ in the z-domain technique, and r and N in the frequency sampling technique (we must, of course, constrain N to be large enough to accommodate the pipelining delay). With this optimization, better filters can be designed and the same performance can be achieved with less hardware. Nonetheless, we shall show that very good results can be obtained by adjusting only the zeros.

Results

For comparison purposes we chose a 4th-order Chebyshev IIR digital filter. This filter when realized as a cascade of two 2nd order IIR filters, requires 10 multipliers. We also designed the best linear-phase pipelined FIR filter we could with 10 multipliers using the Parks-McClellan algorithm. This FIR filter can be pipelined with a delay of any required N. Here we assume that we need N=4.

Figure 1 shows an overlay of the Parks-McClellan and the Chebyshev design. As expected, the specifications are met by both in the passband, but the Chebyshev IIR filter has a much superior transition and stop band. However, the Chebyshev IIR filter cannot be pipelined because we do not have the required pipelining delay of N=4 in the feedback path.

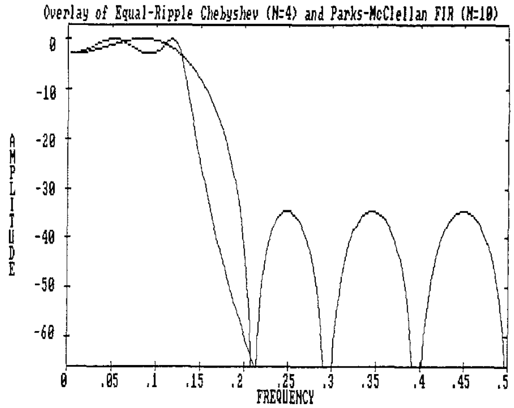


Figure 1 - Comparison of Parks-McClellan FIR filter and Chebyshev IIR Filter

Figure 2 shows the application of the old z-domain IIR filter pipelining technique. Figure 2a shows the block diagram generated using the DSPlay digital signal processing program [5]. In this block diagram, an impulse is applied to two different systems. Across the top of the diagram is the Chebyshev IIR filter that cannot be pipelined because it does not have sufficient delay in the feedback loop (we assume that a delay of N=4 is required). In figure 2b, the top right graph shows the spectrum for the denominator of this filter (Pwr spec Ch D1*D2) and the bottom right graph shows the spectrum for the entire Chebyshev IIR filter (Pwr Spec Chebyshv). In the block diagram of figure 2a, the two denominator pole pairs are shown as D1 and D2,

$$D1(z) = \frac{1}{1-1.60532z^{-1}+.717038z^{-2}} \quad \begin{matrix}(r=.84678)\\(\theta=18.58^\circ)\end{matrix} \tag{7a}$$

$$D2(z) = \frac{1}{1-1.37912z^{-1}+.884851z^{-2}} \quad \begin{matrix}(r=.94067)\\(\theta=42.86^\circ)\end{matrix} \tag{7b}$$

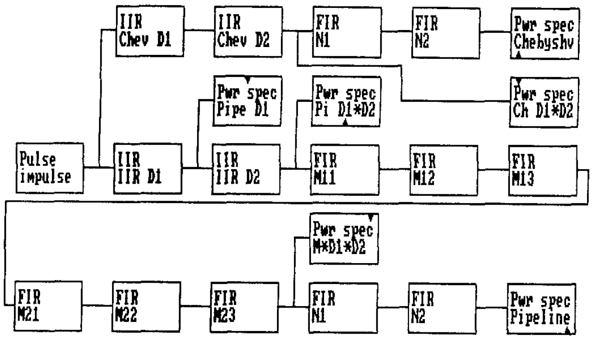
and the two sets of double poles at z=-1 are shown as N1 and N2.

The bottom part of figure 2a shows the old IIR z-domain pipelining technique. The denominator terms labeled IIR D1 and IIR D2 are given by equation (5) with N=4, r=..84678, and θ=18.58° for D1; and r=.94067, θ=42.86° for D2:

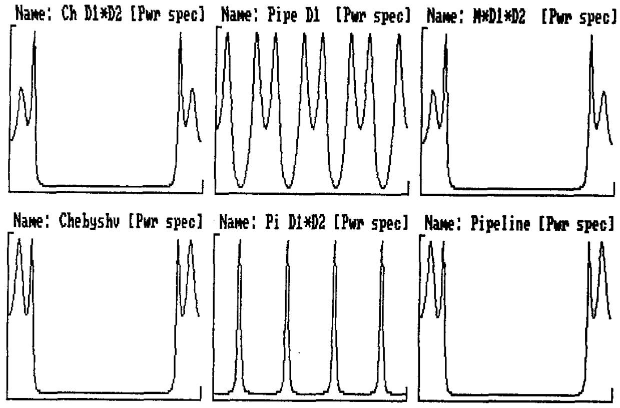
$$\text{IIR } D1(z) = \frac{1}{1-.27811z^{-4}+.264344z^{-8}} \tag{8a}$$

$$\text{IIR } D2(z) = \frac{1}{1+1.54843z^{-4}+.613028z^{-8}} \tag{8b}$$

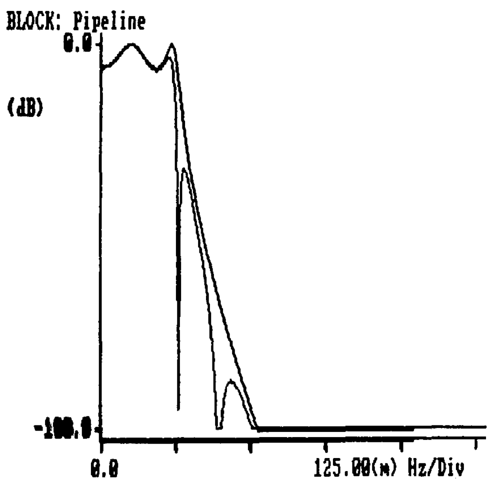
The top middle graph in figure 2b shows the pipelined denominator IIR D1. Since N=4, we obtain images of the pole at ±18.58° at 90±18.58, 180±18.58, and 270±18.58. The bottom middle graph in figure 2b shows the product of the pipelined IIR D1 with IIR D2. Since the radius of D2



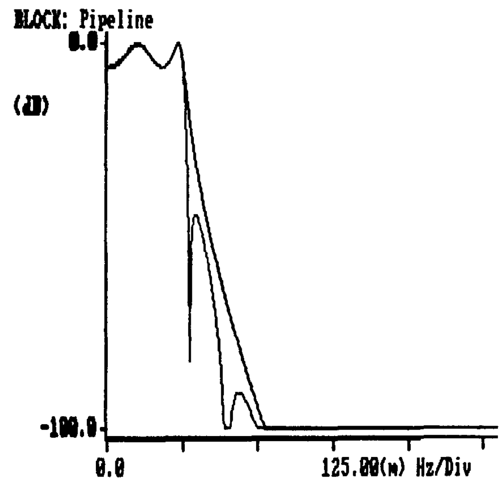
2a. Block Diagram for z-Domain Technique



2b. Simulation Results for z-Domain Technique



2c. Moving the Zeros to the Unit Circle



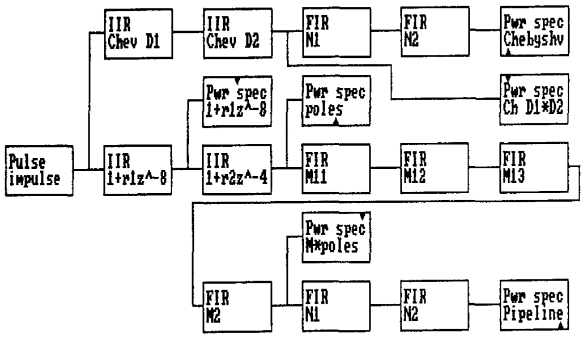
2d. Optimizing the Zero Placement

Figure 2 - Applying the New Procedure to the z-Domain Pipelining Technique

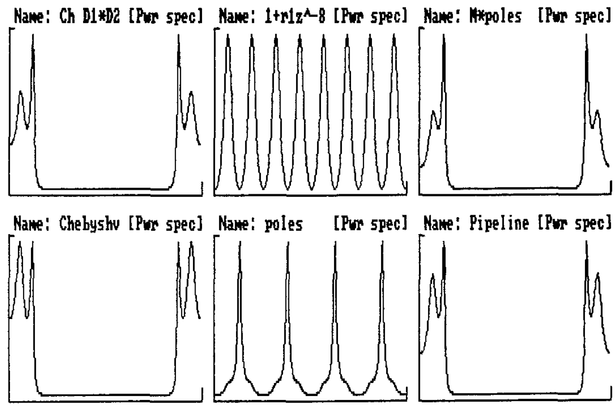
is much closer to the unit circle, it dominates. However, since 42.86° is so close to the image at 90-42.86° (47.14°), they appear to merge together as one pole. These superfluous image poles are cancelled by placing zeros on top of them with M11, M12, and M13 canceling pole pairs at ±71.42°, ±108.58°, and ±161.42° respectively. Similarly, M21, M22, and M23 cancel pole pairs at ±47.14°, ±132.86°, and ±137.14° respectively. The result of cancelling these superfluous poles is to obtain the desired Chebyshev response exactly as shown in the far right column of graphs in figure 2b. Although this does result in an IIR filter that can be pipelined, it requires 20 multipliers (twice that of the IIR Chebyshev) and high precision to guarantee adequate pole-zero cancellation.

If we now apply the new procedure, all of the pole-cancelling zeros are moved to the unit circle at the same angle as before. This simple move results in the performance shown in figure 2c. This filter has been overlaid with the Chebyshev filter and we see that it matches the Chebyshev filter quite well except for right at the passband edge, where it cuts off a bit too soon. In figure 2d we have optimized the location of the zeros resulting in the zero nearest the passband edge moving from 47.14° to approximately 49° $(1-1.3125z^{-1}+z^{-2})$. The resulting filter shown in figure 2d matches exactly in the passband and is better than the Chebyshev filter in the transition and stop bands. Furthermore, since each of the zeros is on the unit circle, they can be realized with but one multiplier, resulting in a total of 12 multipliers in the realization, only two more than the Chebyshev IIR filter.

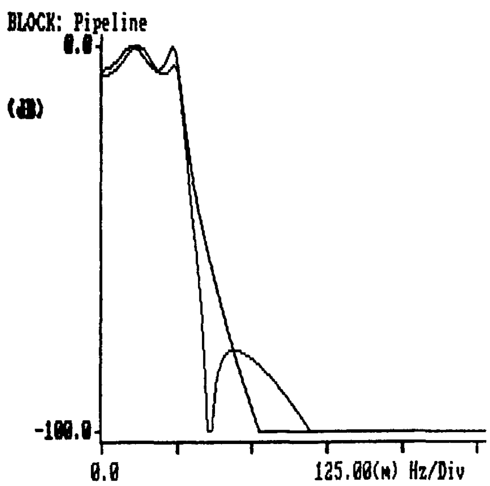
Figure 3 shows the new frequency sampling pipelining technique. As before, the top line in the block diagram of figure 3a is the ideal Chebyshev IIR filter. The remaining block diagram is the new technique. First poles are placed at 22.5° (as close to 18.58° as we can) with the denominator $(1+r_1^8z^{-8})$ and at 45° (as close to 42.86° as we can get) with the denominator $(1+r_2^4z^{-4})$. Zeros are then place on the unit circle at angles ±67.5°, ±112.5°, and ±157.5° by the blocks M11, M12, and M13 respectively to zero the effect of the superfluous poles. Similarly, M2 cancels the effect of the superfluous pole



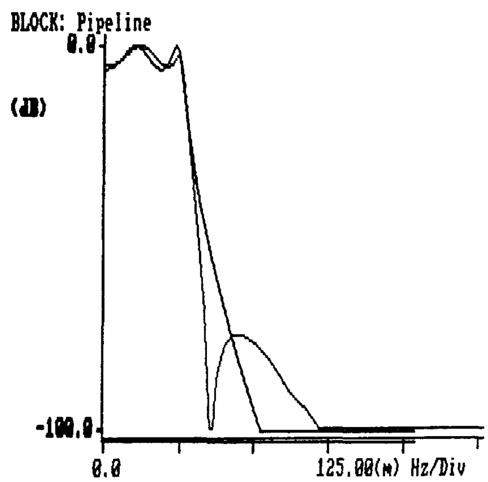
3a. Block Diagram for Frequency-Sampling Technique



3b. Simulation Results for Frequency-Sampling Technique



3c. Moving the Zeros to the Unit Circle



3d. Optimizing the Pole Radii

Figure 3 - Applying the New Procedure to the Frequency-Sampling Pipelining Technique

at ±135°. Figure 3b shows the results of the simulation. As before, the left most column of graphs represents the ideal Chebyshev IIR filter. The top middle graph shows the 8 equally spaced poles at radius r_1=.84678 created by $(1+r_1 8z^{-8})$. The bottom middle graph in figure 3b shows the complete set of poles which is dominated by the 4 equally spaced poles with radius r_2=.94067 created by $(1+r_2 4z^{-4})$. The right-most column of graphs represent the spectrum of the poles (top) and the entire filter (bottom). However, since the poles are not located at the correct locations, we do not get a very good response.

If we take one of the zeros at z=-1 and move it to angle 64° (N1), we get the frequency response shown in figure 3c. Figure 3d shows the result of adjusting the r1 IIR block from $(1+.264343z^{-8})$ to $(1-.2z^{-8})$ which lowers the radius r_1 from .84678 to .81777. Although this does not give quite as good results as the z-domain technique, it requires only 8 multipliers, two less than the IIR Chebyshev filter, and does meet the passband and transition band specifications!

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